Applied Stochastic Process
Assignment 1
SHEN YIQIN 1730014057
Mar /2 /2020

Question 1
The sample space is: $S = [e_1, e_2, e_3, e_4, ..., e_n]$. where $e_n = e_{n-1} = H$, for $i = 1, 2, 3...n-2, e_i \in \{H, T\}$, but if $e_i = H$, then $e_{i+1} = T$.
The probability that it will be tossed exactly four times is $P = 1 \cdot \frac{1}{2} \cdot \frac{1}{2} \cdot \frac{1}{2} = \frac{1}{8}$

Question 2
Since$E \subset F, P(E) \leq P(F), EF^c$is disjoint with E.So,$P(F) = P(E) + P(EF^c) \geq P(E)$

Question 3
It can be proved by using the method of induction
When n=1, the inequality holds obviously.
$P(E_1) \leq P(E_1)$
Assume when n=k, the equality holds:
$P(\bigcup_{i=1}^{k} x_i) \leq \sum_{i=1}^{k} P(x_i)$
When n=k+1,
$P(\bigcup_{i=1}^{k+1} x_i) = P(\bigcup_{i=1}^{k} x_i \cup x_{k+1}) = P(\bigcup_{i=1}^{k} x_{i=1}) + P(x_{k+1}) - P(\bigcup_{i=1}^{k} x_i \cap x_{k+1}))$
Since, $P(\bigcup_{i=1}^{k} x_i \cap x_i)) \geq 0$,
$P(\bigcup_{i=1}^{k+1} x_i) \leq P(\bigcup_{i=1}^{k} x_i) + P(x_{k+1}) = \sum_{i=1}^{k+1} P(x_i)$
The inequality also holds. So, for any positive whole number n, the Boole's inequality holds.

Question 4
The sample space is: $S = \{e_1, e_2, ..., e_n\}, n = 2, 3, 4, ...$, for $i = 1, 2, ..., k, ..., n-1, e_i \in \{E, F, \text{neither E nor F}\}, e_k \in \{E, F\}, e_n = \begin{cases} E & \text{if} \quad e_k = F \\ F & \text{if} \quad e_k = E \end{cases}$
Since E and F are mutually exclusive, $P(E \cup F) = P(E) + P(F)$
We know that eventually E or F will occur, the problem is can be considered as when fisrt time E or F occurs, it will be E.
$P(\text{E before F}) = P(E|E \cup F) = \frac{P(E \cap (E \cup F))}{P(E \cup F)} = \frac{P(E)}{P(E)+P(F)}$.

Question 5
$P(Awins) = p + (1-p)(1-p)p + (1-p)(1-p)(1-p)(1-p)p + ... =$

$\sum_{n=0}^{\infty}(1-p)^{2n}p = \frac{p}{2p-p^2} = \frac{1}{2-p}$
$P(Bwins) = (1-p)p + (1-p)(1-p)(1-p)p + ... = \sum^{\infty} n = 1(1-p)^{2n-1}p = \frac{1-p}{2-p}$
Question 6

$$\begin{aligned} P(E_1E_2...E_n) &= P(E_1)P(E_2E_3...E_n|E_1) \\ &= P(E_1)P(E_2|E1)P(E_3E_4...E_n|E_1E_2) \\ &\quad ... \\ &= P(E_1)P(E_2|E_1)P(E_3|E_1E_2)...P(E_n|E_1E_2...E_n) \end{aligned} \tag{1}$$

Question 7
(a)
$P(E|F) = \frac{P(EF)}{P(F)} = 0$
(b)
$P(E|F) = \frac{P(EF)}{P(F)} = \frac{P(E)}{P(F)} = \frac{0.6}{P(F)}$
(c)
$P(E|F) = \frac{P(EF)}{P(F)} = \frac{P(F)}{P(F)} = 1$

Question 8
(a)
Assume A is the event that Bill hits the target, B is the event that George hits the target, C is the event that one of them hit the target. Obviously A and B are independent where $P(A) = 0.7, P(B) = 0.4$
$P(C) = 0.7 \cdot (1-0.6) + (1-0.7) \cdot 0.4 = 0.42 + 0.12 = 0.64$
$P(B|C) = \frac{P(BC)}{P(C)} = \frac{P(B) \cdot P(C|B)}{P(C)} = \frac{0.4 \cdot (1-0.7)}{0.64} = \frac{2}{9}$
(b)
$P(B|A \cup B) = \frac{P(B)}{P(A \cup B)} = \frac{0.4}{0.7+0.4-0.28} = \frac{20}{41}$

Question 9
Assume E is the event that A is executed, B is the event that knowing B is free, C is the event that knowing C is free $P(A|B) = \frac{P(AB)}{P(B)} = \frac{P(A) \cdot P(B|A)}{P(B)} = \frac{\frac{1}{3} \cdot \frac{1}{2}}{\frac{1}{2}} = \frac{1}{3}$
Similarly, $P(A|C) = P(A|B) = \frac{1}{3}$
So, jailer's reasoning is wrong.