Final Project Write up

Shenyuan Gao

gaos74

Topic: Projectile Motion

Introduction:

Projectile motion has long been a subject of significant interest in physics and engineering due to its relevance in sports, aerospace, and various fields of science. This project presents a detailed simulation of projectile motion, incorporating the effects of gravity, aerodynamic drag, Magnus force, and seam orientation.

This study utilizes the **Runge-Kutta method**, a numerical technique for solving differential equations, to accurately compute the projectile's trajectory over time. Users can adjust various parameters, such as the initial velocity, drag coefficient, Magnus coefficient, and spin rate, to investigate their influence on the projectile's flight.

In this project, We will first validate my projectile motion model by introducing an simple case of projectile motion which is mathematically calculatable. Then I will extend My project into a 3 dimensional case where the Magnus effect will be considered and we will be testing the seam angle (ranging from 0-90) and figure out which degree can cause the maximum swing for projectile motion. Finally we will extend our project to test how the seam angle affects the lateral movement of the projectile.

Method:

This is the ODEs that we are using during the whole experiment

$$\begin{cases} \frac{dx}{dt} = v_x \\ \frac{dy}{dt} = v_y \\ \frac{dz}{dt} = v_z \\ \frac{dv_x}{dt} = -c_d \cdot v_x \cdot \text{speed} \\ \frac{dv_y}{dt} = -g - c_d \cdot v_y \cdot \text{speed} \\ \frac{dv_z}{dt} = -c_d \cdot v_z \cdot \text{speed} + c_m \cdot \text{spin\_rate} \cdot v_y \cdot \text{seam\_effect} \end{cases}$$

x: Horizontal distance along the ground (usually the forward direction of travel).

y: Vertical height from the ground (measured upwards).

z: Lateral deviation (sideways direction relative to the initial trajectory).

v_x**: Velocity along the horizontal (forward) direction.**

v_y**: Velocity along the vertical direction.**

v_z**: Velocity along the lateral direction (sideways movement).**

C_d**: drag coefficient**. It quantifies the resistance of the air as the ball moves through it. A higher value of cd means greater drag force.

C_m**: Magnus coefficient**. It quantifies the effect of the Magnus force, which is the lift force experienced by a spinning ball due to differential pressure on different sides of the ball.

Spin_Rate: The **spin rate** of the ball, measured in radians per second. The Magnus force is proportional to the spin rate, meaning a higher spin rate will lead to more significant movement due to the Magnus effect.

seam angle: factor that represents the impact of the seam angle of the ball on the lateral force (Magnus effect).

The **total speed** = $\sqrt{{v_x}^2 + {v_y}^2 + {v_z}^2}$

Validation Test:

In the validation test section , we will perform 1 Validation test by setting the Projectile Motion without considering Magnus effect:

Initial speed V_0 = 30m/s

Initial launch angle θ = 45 degrees

C_m = 0

And we will also not consider the air resistance so we set:

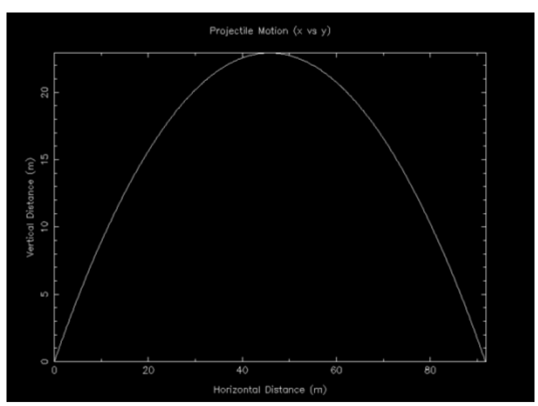
C_d = 0

Since the Magnus effect is ignored :

Spin_rate = 0

The Validation Test is shown below:

```
Please input the Initial speed (Ex: 30):
30
Please input the Magnus coefficient (Ex: 0.05):
0
Please input the Drag coefficient (Ex: 0.25):
0
Please input the Spin Rate (in radians per second, Ex: 10):
0
```



We can use the following mathematic calculation to verify our test result:

Initial speed V_0= 30m/s, Initial launch angle(θ) = 45

The horizontal velocity is :

$$v_{0x} = v_0 \cdot \cos(\theta) = 30 \cdot \cos(45^\circ) = 30 \cdot \frac{\sqrt{2}}{2} = 21.21\,\mathrm{m/s}$$

The vertical velocity is:

$$v_{0y} = v_0 \cdot \sin(\theta) = 30 \cdot \sin(45^\circ) = 30 \cdot \frac{\sqrt{2}}{2} = 21.21\,\mathrm{m/s}$$

Calculate the time:

$$t_{\mathrm{up}} = \frac{v_{0y}}{g} = \frac{21.21}{9.81} = 2.16\,\mathrm{s}$$
$$T = 2 \cdot t_{\mathrm{up}} = 2 \cdot 2.16 = 4.32\,\mathrm{s}$$

The horizontal range :

$$R = v_{0x} \cdot T = 21.21 \cdot 4.32 = 91.65\,\mathrm{m}$$

The maximum height :

$$H = \frac{v_{0y}^2}{2g} = \frac{21.21^2}{2 \times 9.81} = \frac{449.41}{19.62} = 22.91\,\mathrm{m}$$

The calculation above great corresponded to the graph.So our validation test is succeed.

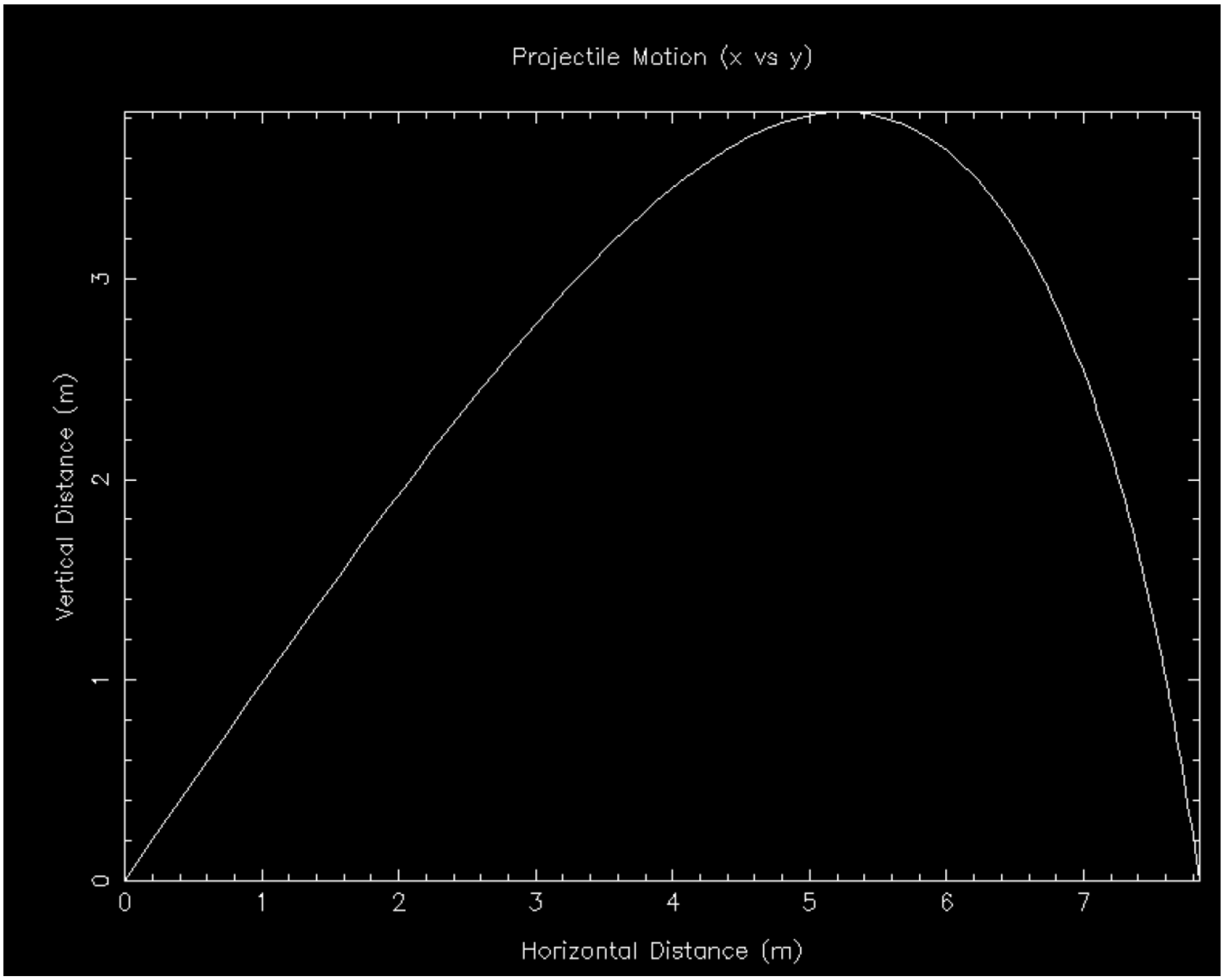
Next we will perform an more realistic experiment where the magnus effect is considered.

Experiments:

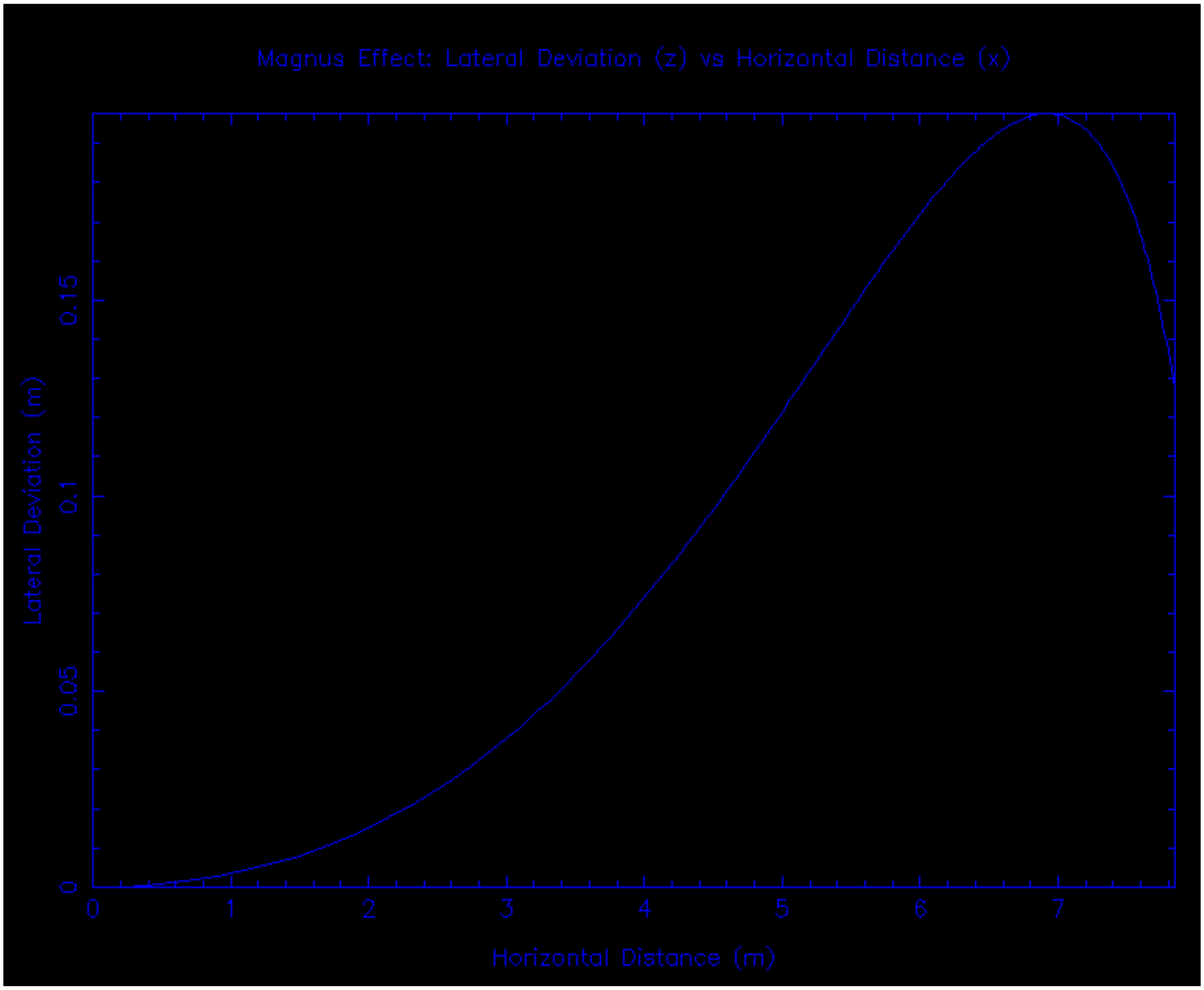
To simulate a realistic model, we will set our experiment augments in the following value:

```
Please input the Initial speed (Ex: 30):
30
Please input the Magnus coefficient (Ex: 0.05):
0.05
Please input the Drag coefficient (Ex: 0.25):
0.25
Please input the Spin Rate (in radians per second, Ex: 10):
10
```

And the generated graph:



From comparation between the graph with magnus effect and the previous graph , we can clearly see that the Magnus effect could alter the symmetry of the parabolic shape. However , the Magnus effect is still not directly visible because the plot is a 2D representation of x vs. y, whereas the Magnus force primarily affects the z (lateral) direction. For better visualizing the Magnus effect, I would plot z (lateral deviation) over horizontal distance x to understand how the projectile moves sideways due to the spin. And here is the desired plotting (z over x):



Result:

The plot(z-x) shows a clear lateral deviation that increases as the projectile moves forward. This lateral deviation is directly caused by the Magnus effect, which is influenced by the spin rate and the Magnus coefficient ($\boldsymbol{C_m}$).

As the projectile moves through the air, the Magnus effect generates a force perpendicular to the projectile's velocity, leading to a lateral deviation. This is why the lateral deviation (z) increases steadily throughout the trajectory.

Initially, the lateral deviation starts at zero and increases continuously, reaching a maximum near the end of the flight. This is consistent with the Magnus effect building up as the projectile travels through the air.

The curve reaches its peak and then slightly drops as the projectile reaches the end of its trajectory. This suggests that as the speed decreases due to drag and gravity, the Magnus effect also weakens slightly, reducing the force acting laterally.

Extended Experiment:

In the extended experiment we will be exploring how the **seam angle affects the lateral movement of the projectile**:

We used all the same input values from previous experiment.

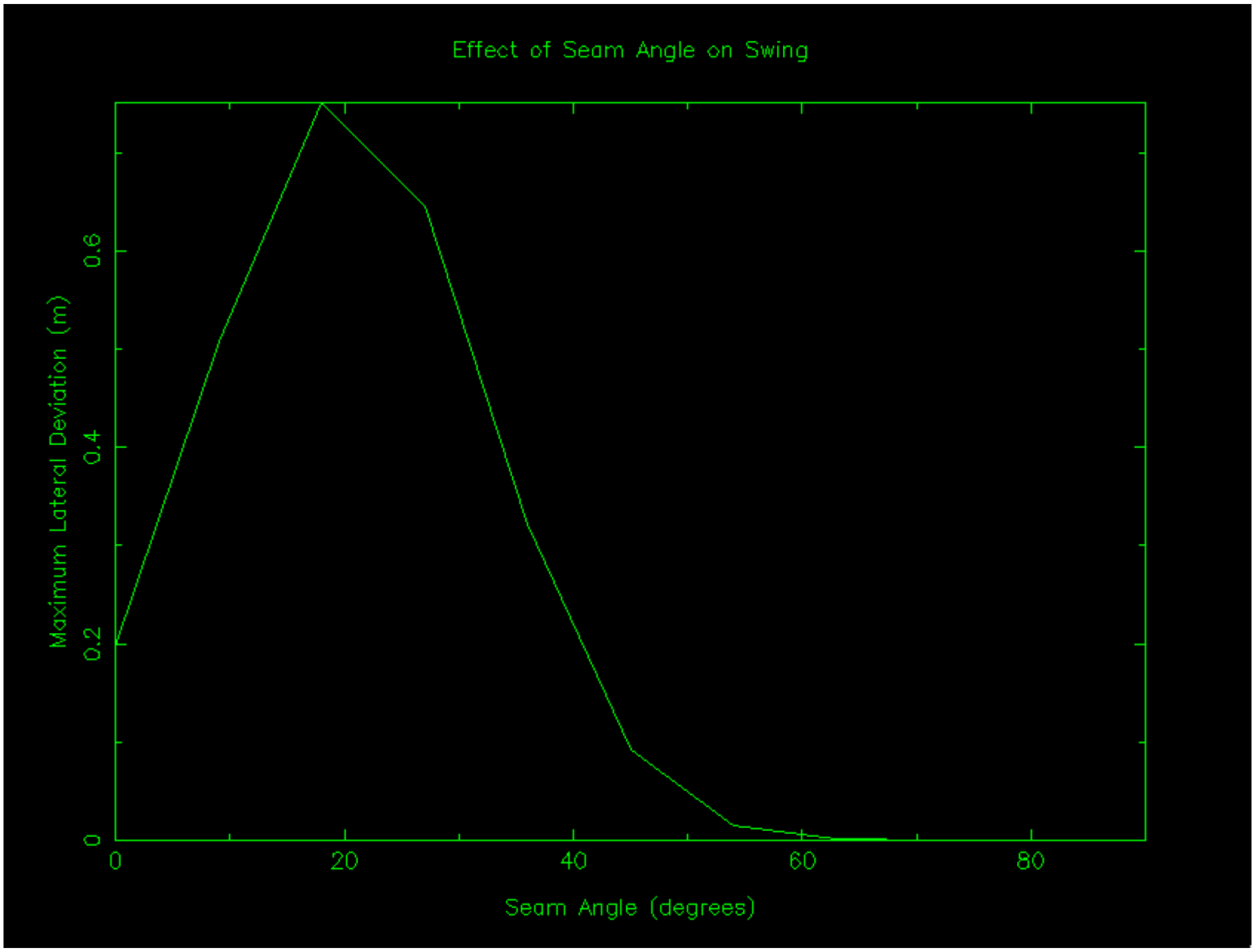
To explore the effect of seam orientation, I varied the seam angle from 0 to 90 degrees in steps of 9 degrees.

For each seam angle, the trajectory was simulated, and the maximum lateral deviation (max_swing) was recorded.

The seam angle influences the effective seam effect and is combined with the Magnus force, determining the amount of swing observed in the projectile's motion.

Here is our result for the extended experiment:

```
Please input the Initial speed (Ex: 30):
30
Please input the Magnus coefficient (Ex: 0.05):
0.05
Please input the Drag coefficient (Ex: 0.25):
0.25
Please input the Spin Rate (in radians per second, Ex: 10):
10
Seam Angle: 0 degrees, Maximum Swing: 0.197628 m
Seam Angle: 9 degrees, Maximum Swing: 0.506906 m
Seam Angle: 18 degrees, Maximum Swing: 0.750683 m
Seam Angle: 27 degrees, Maximum Swing: 0.645661 m
Seam Angle: 36 degrees, Maximum Swing: 0.321524 m
Seam Angle: 45 degrees, Maximum Swing: 0.0923119 m
Seam Angle: 54 degrees, Maximum Swing: 0.0153026 m
Seam Angle: 63 degrees, Maximum Swing: 0.0014659 m
Seam Angle: 72 degrees, Maximum Swing: 8.11548e-05 m
Seam Angle: 81 degrees, Maximum Swing: 2.59655e-06 m
Seam Angle: 90 degrees, Maximum Swing: 4.80119e-08 m
```



Conclusion:

The experiment shows that there is an **optimal seam angle** (around **20 degrees**) where the **swing effect** reaches its maximum. Beyond that point, the lateral deviation starts to decrease and eventually become negligible.