

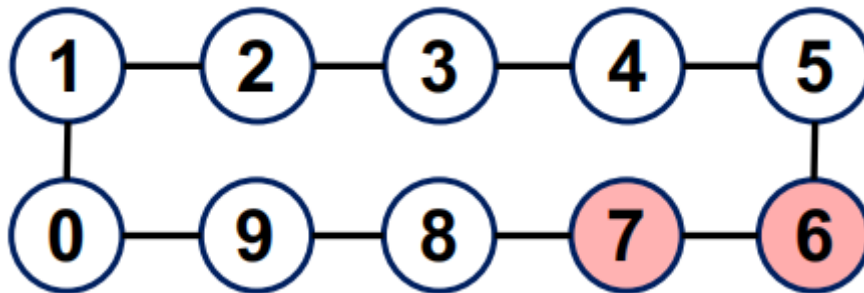
Q 1

When $n \leq 3$, the number of neighbors is 0.

When $n = 4$, the number of neighbors is 2.

When $n > 4$, the number of neighbors is n .

Reason: n cities are following neighborhood structures



For **Adjacent Two-City Change**, firstly we choose one city, we have n kinds. Then choose a adjacent city, we have 2 kinds. But in this process, we repeated the calculation, so we need to divide it by 2. So the result is $n * 2 / 2 = n$.

Q 2

When $n \leq 3$, the number of neighbors is 0.

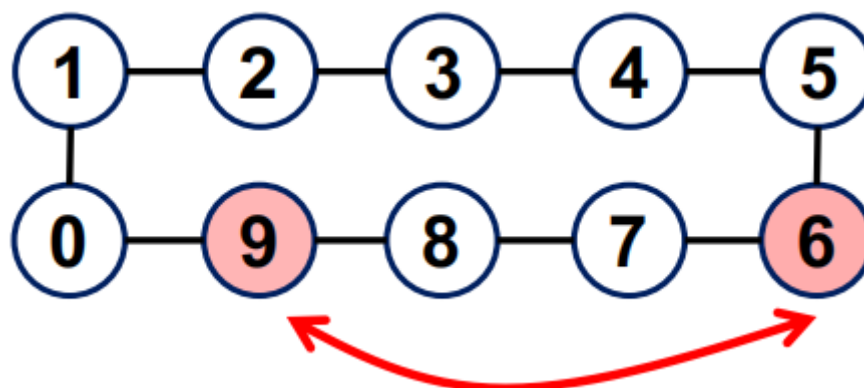
When $n = 4$, the number of neighbors is 2.

When $n > 4$, the number of neighbors is

$$\frac{n * (n - 1)}{2}$$

.

Reason:



For **Arbitrary Two-City Change , including (1) Adjacent Two-City Change**. firstly we choose one city, we have n kinds. Then choose a city (include adjacent city), we have $n-1$ kinds. But in this process, we repeated the calculation, so we need to divide it by 2. So the result is $n * (n-1) / 2$.

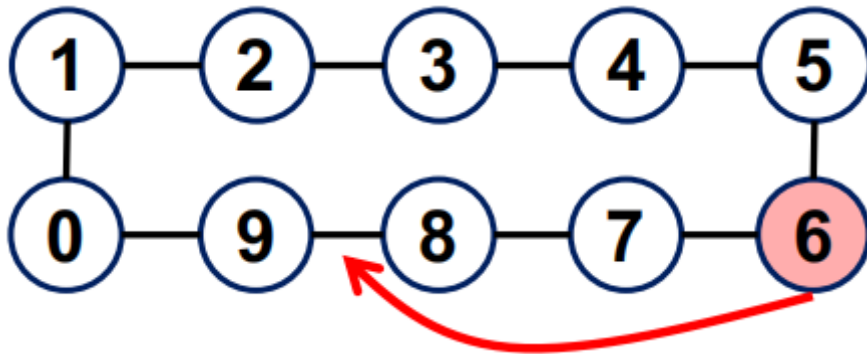
Q 3

Number of neighbors is

$$n * (n - 2) - n$$

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Reason:



For **Insertion (Shift)**, firstly we choose one city which we will insert, we have n kinds. Then there are $n-2$ edges we can insert. But in this process, we repeated the calculation.

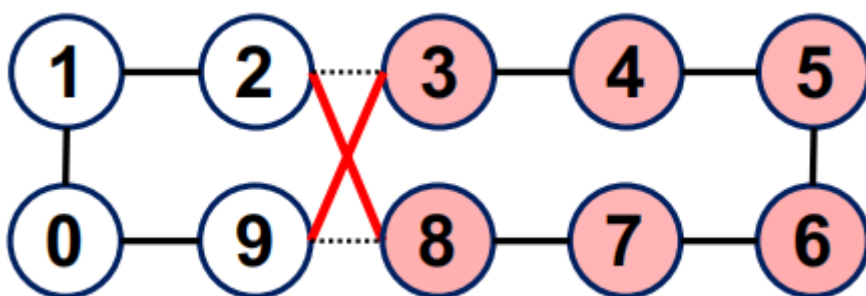
For example, we choose city 1 inserts city 2 and 3 is the same as we choose city 2 inserts city 0 and 1. There are a total of n edges, so we need to subtract n . So the result is $n*(n-2)-n$.

Q 4

Number of neighbors is

$$\frac{n * (n - 3)}{2}$$

Reason:



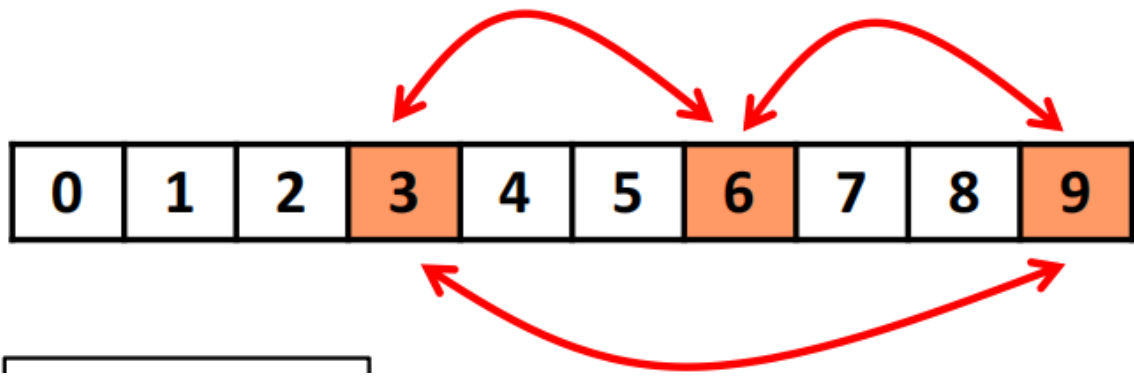
For **Inversion (Arbitrary Two-Edge Change)**, firstly we choose one edge to change, we have n kinds. Then we need to choose another edge to change, and this edge can't be the adjacent edges of the edge that we choose first. So there are $n-3$ edges we can choose. But in this process, we repeated the calculation, so we need to divide it by 2. So the result is $n*(n-3)/2$.

Q 5

Number of neighbors is

$$\frac{2 * n^3 - 3 * n^2 + n}{6}$$

Reason:



For **Arbitrary Three-City Change including two-city change in (1) and (2)** ,we divide this problem into 2 parts.

1: all the 3 numbers changed its index;

2: only 2 numbers changed its index;

So the result is

$$\frac{n * (n - 1) * (n - 2)}{3} + \frac{n * (n - 1)}{2} = \frac{2 * n^3 - 3 * n^2 + n}{6}$$