


A cluster of various spheres in white, gold, and blue with gold and blue stripes, arranged in a group on the left side of the slide.

Computer Organization

Lab7 Floating-Point Processing

A blue pill-shaped box with a small orange circle to its left, containing the text 'MIPS(4) Floating-Point'.

MIPS(4)
Floating-Point



Topic

- **Floating-Point Number**
 - **IEEE 745 On Floating-Point Number**
- **Registers of Coprocessor 1**
- **Floating-Point Instructions**
 - **Load & Store, Move**
 - **Computational**
 - **Relational and Branch ...**



IEEE 745 On Floating-Point Number

$$\pm 1.xxxxxxx_2 \times 2^{yyyy}$$

single: 8 bits

double: 11 bits

single: 23 bits

double: 52 bits

| | | |
|---|----------------------|-----------------|
| S | Exponent (yyyy+Bias) | Fraction (xxxx) |
|---|----------------------|-----------------|

$$x = (-1)^S \times (1 + \text{Fraction}) \times 2^{(\text{Exponent} - \text{Bias})}$$

For single-precision float data:

Exponents(8bit): 0000_0000 and 1111_1111 reserved

Bias in Exponent: 0111_1111

For double-precision float data:

Exponents(11bit): 000_0000_0000 and 111_1111_1111 reserved

Bias in Exponent: 011_1111_1111



IEEE 745 On Floating-Point Number continued

```
.data
fneg1:    .float   -1
wneg1:    .word    -1
fpos1:    .float    1
wpos1:    .word     1
```

| Label | Address ▲ |
|--------------|------------|
| float_rw.asm | |
| fneg1 | 0x10010000 |
| wneg1 | 0x10010004 |
| fpos1 | 0x10010008 |
| wpos1 | 0x1001000c |

$$\pm 1.xxxxxxx_2 \times 2^{yyyy}$$

single: 8 bits
double: 11 bits

single: 23 bits
double: 52 bits

| | | |
|---|----------------------|-----------------|
| S | Exponent (yyyy+Bias) | Fraction (xxxx) |
|---|----------------------|-----------------|

$$x = (-1)^S \times (1 + \text{Fraction}) \times 2^{(\text{Exponent} - \text{Bias})}$$

$$\text{> } -1 = (-1)^1 \times (1 + 0) \times 2^0$$

s: 1; exponent: 0 + 0111_1111; fraction: 0

$$\text{> } 1 = (-1)^0 \times (1 + 0) \times 2^0$$

s: 0; exponent: 0 + 0111_1111; fraction: 0

| Data Segment | | | | |
|--------------|------------|------------|------------|------------|
| Address | Value (+0) | Value (+4) | Value (+8) | Value (+c) |
| 0x10010000 | 0xbf800000 | 0xffffffff | 0x3f800000 | 0x00000001 |



Infinite vs NaN (Floating-Point)

| | 31 | 30 | 23 | 22 | 0 | | | | | |
|---------------|------|----------|------|----------|------|------|------|------|--|--|
| | Sign | Exponent | | Mantissa | | | | | | |
| 93000000 | 0 | 0001 | 1010 | 101 | 1000 | 1011 | 0001 | 0001 | | |
| 0 | 0 | 0000 | 0000 | 000 | 0000 | 0000 | 0000 | 0000 | | |
| +Infinity | 0 | 1111 | 1111 | 000 | 0000 | 0000 | 0000 | 0000 | | |
| -Infinity | 1 | 1111 | 1111 | 000 | 0000 | 0000 | 0000 | 0000 | | |
| Quiet NaN | x | 1111 | 1111 | 0xx | xxxx | xxxx | xxxx | xxxx | | |
| Signaling NaN | x | 1111 | 1111 | 1xx | xxxx | xxxx | xxxx | xxxx | | |

Q1. Which one will get an infinite value, A or B?

Q2. Which one will get the NaN, A or B?

Tips:

lwc1 : load word from memory to the register in coprocessor 1

mtc1: move a word from normal register to the register in coprocessor 1

mul.s: floating point multiplication single precision

div.s : floating point division single precision

```
.data          #A
sdata: .word 0xFF7F7FFF
fneg1: .float -1
.text
lw $t0,sdata
mtc1 $t0,$f1
mul.s $f12,$f1,$f1
```

```
li $v0,2
syscall
```

```
lwc1 $f2,fneg1
mul.s $f12,$f12,$f2
```

```
li $v0,2
syscall
```

```
li $v0,10
syscall
```

```
.data          #B
sdata: .word 0xFFFF7FFF
fneg1: .float -1
.text
lw $t0,sdata
mtc1 $t0,$f1
mul.s $f12,$f1,$f1
```

```
li $v0,2
syscall
```

```
lwc1 $f2,fneg1
div.s $f12,$f12,$f2
```

```
li $v0,2
syscall
```

```
li $v0,10
syscall
```




Coprocessor 1 in MIPS

- Q1. What's the difference between 'lwc1' and 'ldc1' ?
Q2. Which demo would trigger the exception?
Q3. Which demo would get the right answer?

| Registers | Coproc 1 | Copro |
|-----------|------------|-------|
| Name | Float | |
| \$f0 | 0x00000000 | |
| \$f1 | 0xbf800000 | |
| \$f2 | 0x00000000 | |
| \$f3 | 0x3f800000 | |

Runtime exception at 0x00400004: first register must be even-numbered

Runtime exception at 0x00400010: all registers must be even-numbered

```
.data          #A
    fneg1:     .float  -1
    fpos1:     .float  1
.text
    lwc1 $f1, fneg1
    lwc1 $f3, fpos1
    add.s $f12, $f1, $f3

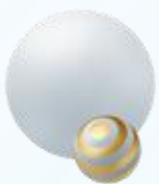
    li $v0, 2
    syscall
    li $v0, 10
    syscall
```

```
.data          #B
    fneg1:     .double -1
    fpos1:     .double  1
.text
    ldc1 $f1, fneg1
    ldc1 $f3, fpos1
    add.d $f12, $f1, $f3

    li $v0, 3
    syscall
    li $v0, 10
    syscall
```

```
.data          #C
    fneg1:     .double -1
    fpos1:     .double  1
.text
    ldc1 $f0, fneg1
    ldc1 $f2, fpos1
    add.d $f11, $f0, $f2

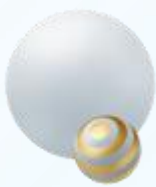
    li $v0, 3
    syscall
    li $v0, 10
    syscall
```



Floating-Point Instructions

| Type | Description | Instructions |
|---------------------|---|--|
| Load and Store | Move data between memory and coprocessor1 register | lwc1, ldc1; swc1, sdc1; ... |
| Move | Move data between nomal register and coprocessor1 register Move data between coprocessor1 registers | mtc1, mfc1; mov.s, mov.d; |
| Computational | Do arithmetic operations on data in coprocessor 1 registers | add.s, add.d; sub.s, sub.d; mul.s, mul.d; div.s, div.d; ... |
| Relational | Compare two floating-point values and set conditional flag | c.eq.s, c.eq.d; c.le.s, c.le.d; c.lt.s, c.lt.d; ... |
| Convert | Convert the data type | floor.w.d, floor.w.s; ceil.w.d, ceil.w.s; round.w.d, round.w.s; cvt.d.s, cvt.d.w, cvt.w.s |
| Conditional jumping | Conditional jump while conditional flag is 0(false)/1(true) | bc1f, bc1t |

| Condition Flags | | | |
|----------------------------|----------------------------|----------------------------|----------------------------|
| <input type="checkbox"/> 0 | <input type="checkbox"/> 1 | <input type="checkbox"/> 2 | <input type="checkbox"/> 3 |
| <input type="checkbox"/> 4 | <input type="checkbox"/> 5 | <input type="checkbox"/> 6 | <input type="checkbox"/> 7 |



Demo(1)

```
.include "macro_print_str.asm"
.data
f1: .float 12.625
.text
lwc1 $f0, f1
floor.w.s $f1,$f0 #A
ceil.w.s $f2,$f0 #A
round.w.s $f3,$f0 #A

print_string("original float: ")
print_float($f0)

print_string("\nafter floor:")
print_float($f1)

print_string("\nafter ceil:")
print_float($f2)

print_string("\nafter round:")
print_float($f3)

end
```

```
#add the content to "macro_print_str.asm"

.macro print_float(%fr)

    addi $sp,$sp,-8
    swc1 $f12,4($sp) #B
    sw $v0,0($sp)

    mov.s $f12,%fr #C
    li $v0,2 #C
    syscall

    lw $v0,0($sp)
    lwc1 $f12,4($sp)
    addi $sp,$sp,8 #D

.end_macro
```

Here is a demo which is supposed to get the following output:

```
original float: 12.625
after floor:12
after ceil:13
after round:13
— program is finished running —
```

While running the demo, another result is got as the following snap:

```
original float: 12.625
after floor:1.7E-44
after ceil:1.8E-44
after round:1.8E-44
— program is finished running —
```

Find the reason, and correct the demo. (The tips are marked by A,B,C,D)



Demo(2)

```
##piece 1/2 of code##  
.include "macro_print_str.asm"  
.data  
    str1:    .asciiz  "str1:"  
    fd1:     .float   1.0  
    dd1:     .double  2.0  
.text  
  
##complete code here##  
  
li $v0, 2  
syscall  
  
##complete code here##  
  
bc1t printLe  
j printGt
```

```
##piece 2/2 of code##  
printLe:  
    print_string( " LessOrEqual ")  
j printSecondData  
  
printGt:  
    print_string(" LargerThan ")  
  
printSecondData:  
    li $v0,3  
    syscall  
  
end
```

The output is expected to be like the following screenshot, please complete the code.

```
1.0 LessOrEqual 2.0  
— program is finished running —
```



Practices

1. Calculate the value of e from the infinite series:

$$\sum_{n=0}^{\infty} \frac{1}{n!} = \frac{1}{0!} + \frac{1}{1!} + \frac{1}{2!} + \frac{1}{3!} + \frac{1}{4!} + \frac{1}{5!} + \cdots$$

- Input a double-precision float number which represents a precision threshold.
- Your program should terminate when the difference between two successive iterations is smaller than the precision threshold.
- Print the value of e (as double-precision float).

2. Complete the code on page 9

3. Given a single-precision float number 'x' and a positive integer 'r' . Round up 'x' to a number which keeps 'r' digits after the decimal point. Print the processing results and the final results.

For example, suppose 'x' is 1.5671

- if 'r' is 2, print 1.57;
- if 'r' is 0, print 2;
- if 'r' is 3, print 1.567;



Tips(1)

| | | | | | | | | | | |
|--------|---------------|------|----------|------|----------|------|------|------|------|--|
| Single | | 31 | 30 | 23 | 22 | 0 | | | | |
| | | Sign | Exponent | | Mantissa | | | | | |
| | 93000000 | 0 | 0001 | 1010 | 101 | 1000 | 1011 | 0001 | 0001 | |
| | 0 | 0 | 0000 | 0000 | 000 | 0000 | 0000 | 0000 | 0000 | |
| | +Infinity | 0 | 1111 | 1111 | 000 | 0000 | 0000 | 0000 | 0000 | |
| | −Infinity | 1 | 1111 | 1111 | 000 | 0000 | 0000 | 0000 | 0000 | |
| | Quiet NaN | x | 1111 | 1111 | 0xx | xxxx | xxxx | xxxx | xxxx | |
| | Signaling NaN | x | 1111 | 1111 | 1xx | xxxx | xxxx | xxxx | xxxx | |

| | High-order word | | | | | Low-order word | | | | | | | |
|---------------|-----------------|----------|------|------|----------|----------------|------|------|------|------|------|------|---|
| | 31 | 30 | | 20 | 19 | 0 | 31 | | | | | | 0 |
| Double | Sign | Exponent | | | Mantissa | | | | | | | | |
| 93000000 | 0 | 000 | 0001 | 1010 | 1011 | 0001 | 0110 | 0010 | 0010 | 1000 | 0000 | | |
| 0 | 0 | 000 | 0000 | 0000 | 0000 | 0000 | 0000 | 0000 | 0000 | 0000 | | | |
| +Infinity | 0 | 111 | 1111 | 1111 | 0000 | 0000 | 0000 | 0000 | 0000 | 0000 | | | |
| -Infinity | 1 | 111 | 1111 | 1111 | 0000 | 0000 | 0000 | 0000 | 0000 | 0000 | | | |
| Quiet NaN | x | 111 | 1111 | 1111 | 0xxx | xxxx | xxxx | xxxx | xxxx | xxxx | | | |
| Signaling NaN | x | 111 | 1111 | 1111 | 1xxx | xxxx | xxxx | xxxx | xxxx | | | | |

reference from "see in MIPS"

| Registers | Coproc 1 | Coproc 0 | |
|---------------------------------------|----------------------------|----------------------------|----------------------------|
| Name | Float | Double | |
| \$f0 | 0x00000000 | 0x0000000000000000 | |
| \$f1 | 0x00000000 | | |
| \$f2 | 0x00000000 | 0x0000000000000000 | |
| \$f3 | 0x00000000 | | |
| \$f4 | 0x00000000 | 0x0000000000000000 | |
| \$f5 | 0x00000000 | | |
| \$f6 | 0x00000000 | 0x0000000000000000 | |
| \$f7 | 0x00000000 | | |
| \$f8 | 0x00000000 | 0x0000000000000000 | |
| \$f9 | 0x00000000 | | |
| \$f10 | 0x00000000 | 0x0000000000000000 | |
| \$f11 | 0x00000000 | | |
| \$f12 | 0x00000000 | 0x4000000000000000 | |
| \$f13 | 0x40000000 | | |
| \$f14 | 0x00000000 | 0x3ff0000000000000 | |
| \$f15 | 0x3ff00000 | | |
| \$f16 | 0x00000000 | 0x0000000000000000 | |
| \$f17 | 0x00000000 | | |
| \$f18 | 0x00000000 | 0x0000000000000000 | |
| \$f19 | 0x00000000 | | |
| \$f20 | 0x00000000 | 0x0000000000000000 | |
| \$f21 | 0x00000000 | | |
| \$f22 | 0x00000000 | 0x0000000000000000 | |
| \$f23 | 0x00000000 | | |
| \$f24 | 0x00000000 | 0x0000000000000000 | |
| \$f25 | 0x00000000 | | |
| \$f26 | 0x00000000 | 0x0000000000000000 | |
| \$f27 | 0x00000000 | | |
| \$f28 | 0x00000000 | 0x0000000000000000 | |
| \$f29 | 0x00000000 | | |
| \$f30 | 0x00000000 | 0x0000000000000000 | |
| \$f31 | 0x00000000 | | |
| Condition Flags | | | |
| <input checked="" type="checkbox"/> 0 | <input type="checkbox"/> 1 | <input type="checkbox"/> 2 | <input type="checkbox"/> 3 |
| <input type="checkbox"/> 4 | <input type="checkbox"/> 5 | <input type="checkbox"/> 6 | <input type="checkbox"/> 7 |

Registers and Flags in Coprocessor 1



Tips(2)

| Service | Code in \$v0 | Arguments | Result |
|--------------|--------------|--------------------------------|----------------------------------|
| print float | 2 | \$f12 = float to print | |
| print double | 3 | \$f12 = double to print | |
| read float | 6 | | \$f0 contains float read |
| read double | 7 | | \$f0 contains double read |

#the content of “macro_print_str.asm”

```
.macro print_string(%str)
    .data
        pstr: .asciiz %str
    .text
        la $a0,pstr
        li $v0,4
        syscall
.end_macro

.macro end
    li $v0,10
    syscall
.end_macro
```