Lab 9 - OJ
Divide and Conquer (p2)

CS208 Algorithm Design and Analysis

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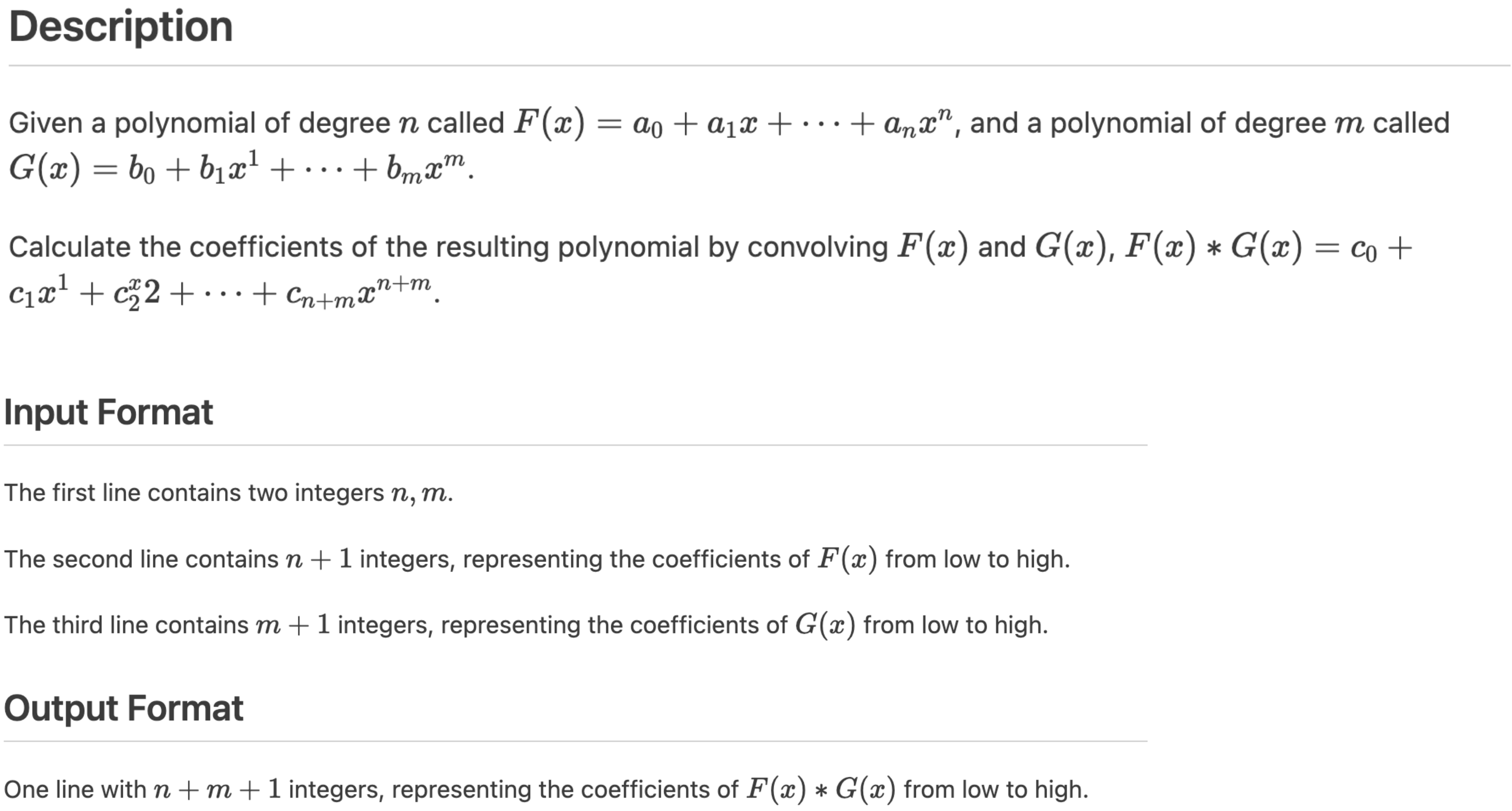
Q1: FFT



Description

Given a polynomial of degree n called $F(x) = a_0 + a_1x + \cdots + a_nx^n$, and a polynomial of degree m called $G(x) = b_0 + b_1x^1 + \cdots + b_mx^m$.

Calculate the coefficients of the resulting polynomial by convolving $F(x)$ and $G(x)$, $F(x) * G(x) = c_0 + c_1x^1 + c_2^x2 + \cdots + c_{n+m}x^{n+m}$.

Input Format

The first line contains two integers n, m.

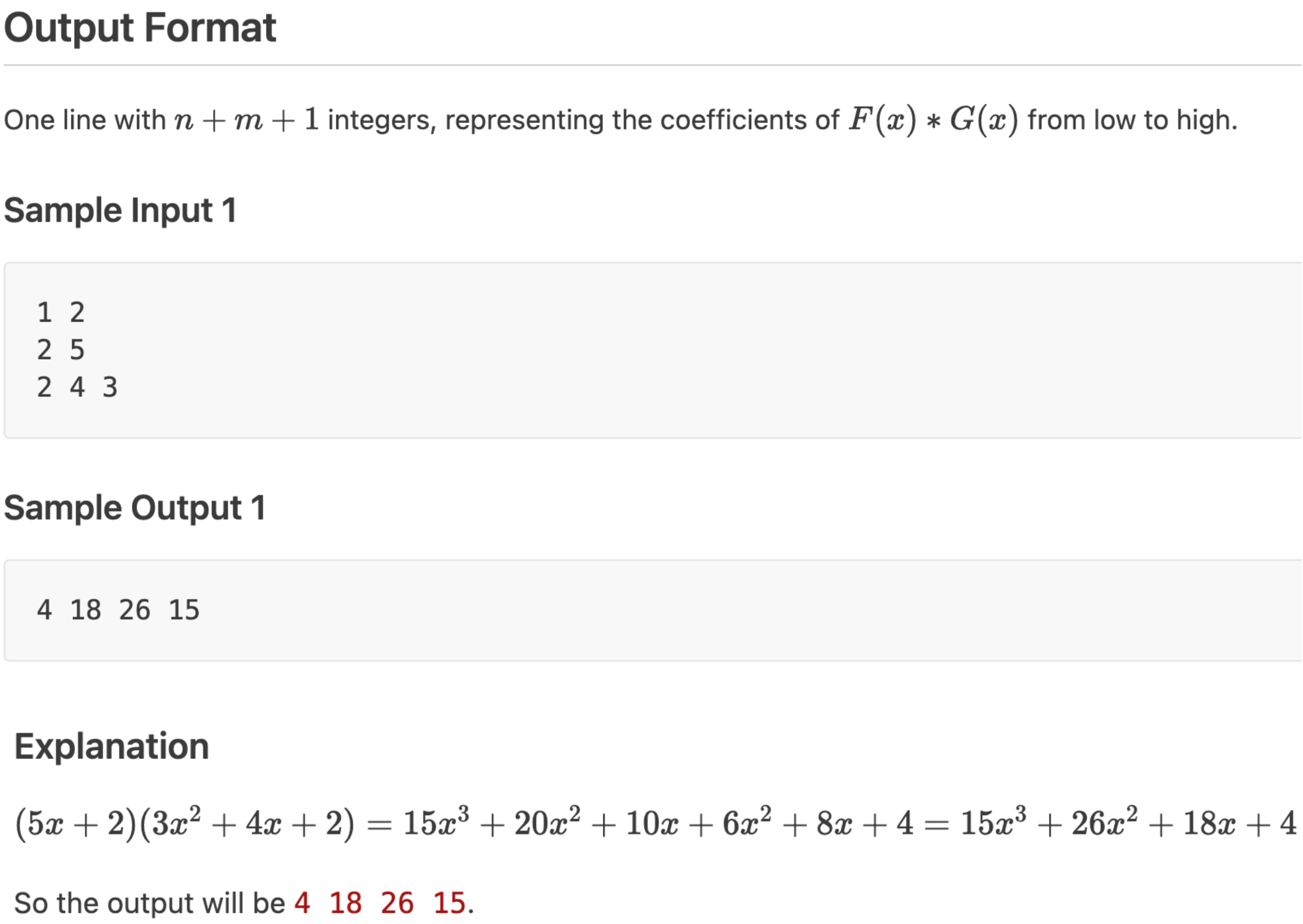
The second line contains $n + 1$ integers, representing the coefficients of $F(x)$ from low to high.

The third line contains $m + 1$ integers, representing the coefficients of $G(x)$ from low to high.

Output Format

One line with $n + m + 1$ integers, representing the coefficients of $F(x) * G(x)$ from low to high.

Q1: FFT



Output Format

One line with $n+m+1$ integers, representing the coefficients of $F(x)*G(x)$ from low to high.

Sample Input 1

```
1 2
2 5
2 4 3
```

Sample Output 1

```
4 18 26 15
```

Explanation

$$(5x+2)(3x^2+4x+2)=15x^3+20x^2+10x+6x^2+8x+4=15x^3+26x^2+18x+4$$

So the output will be 4 18 26 15.

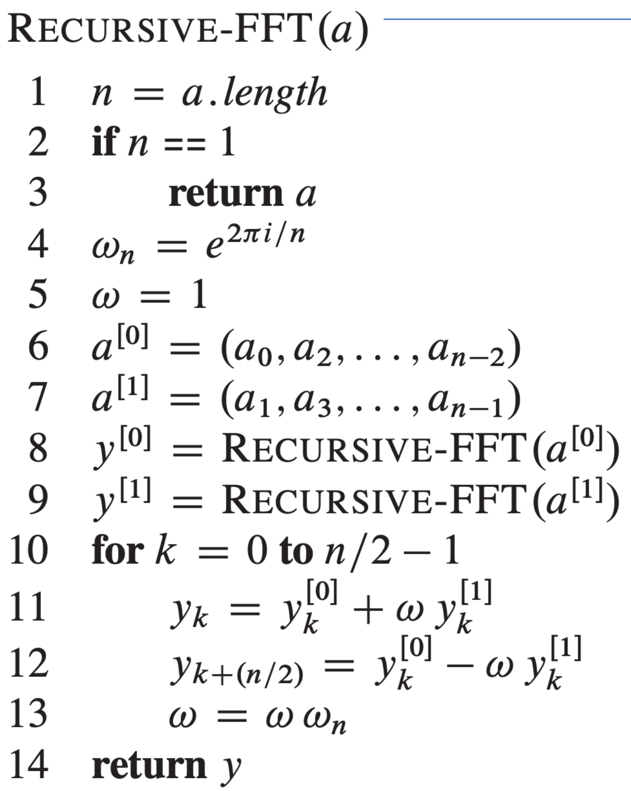
Question: How to deal with the unequal lengths?

$$m \neq n$$

Pad to equal lengths (of 2 to the power of some number)

Q1: FFT

Can add ω_n to the argument



```
RECURSIVE-FFT(a)
n = a.length
if n == 1
    return a
$\omega_n = e^{2\pi i/n}$
$\omega = 1$
$a^{[0]} = (a_0, a_2, \ldots, a_{n-2})$
$a^{[1]} = (a_1, a_3, \ldots, a_{n-1})$
$y^{[0]} =$ RECURSIVE-FFT($a^{[0]}$)
$y^{[1]} =$ RECURSIVE-FFT($a^{[1]}$)
for $k = 0$ to $n/2 - 1$
    $y_k = y_k^{[0]} + \omega\, y_k^{[1]}$
    $y_{k+(n/2)} = y_k^{[0]} - \omega\, y_k^{[1]}$
    $\omega = \omega\, \omega_n$
return y
```

- Let F_A = FFT(A, ω_n) // time O(n log n)
- Let F_B = FFT(B, ω_n) // time O(n log n)
- For i=1 to m, let F_C[i] = F_A[i]*F_B[i] // time O(n)
- Output C = 1/m * FFT(F_C, ω_n^{-1}). // time O(n log n)

Handle complex numbers:
https://introcs.cs.princeton.edu/java/32class/Complex.java.html

Q2: Urban Construction

Sjkmost persuaded Justin to force his citizens to trip with zipline. However, the citizens are not strong enough that they often fell from the zipline. Therefore, they decided to provide the citizens with some cable cars.

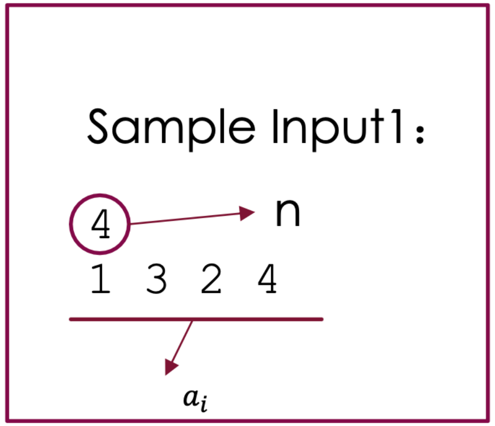
↑ zipline

There are n cable cars on the cable, each has an index a_i. Sijmost is trying to put them in order.

You can spend c $justin$ (a type of currency) to cut down a segment of rope of length c, rotate it and splice back. That is, he can spent $r - l + 1$ $justin$ to rotate the cable cars in an interval.

Sjkmost has a budget of 2×10^7 $justin$. He should sort the cable cars with some operations with a total cost no more than 2×10^7 $justin$. Can you help him?

Q2: Urban Construction

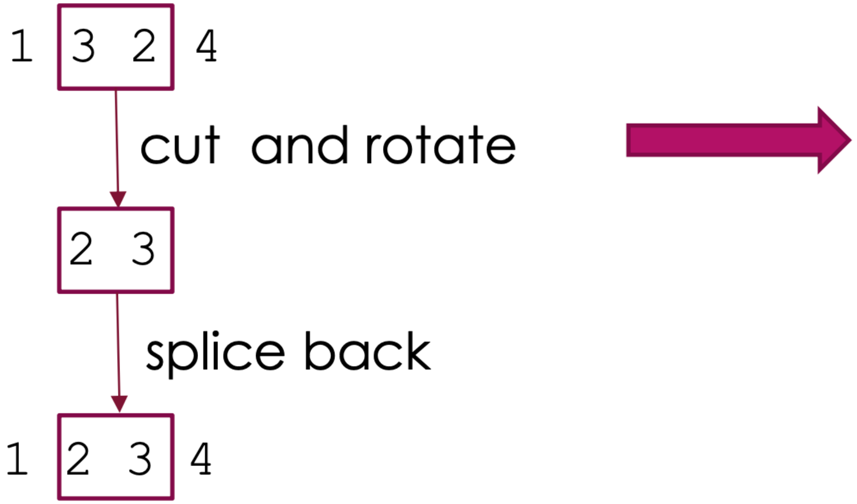


Sample Input1：

4 → n

1 3 2 4

a_i



1 [3 2] 4

cut and rotate

[2 3]

splice back

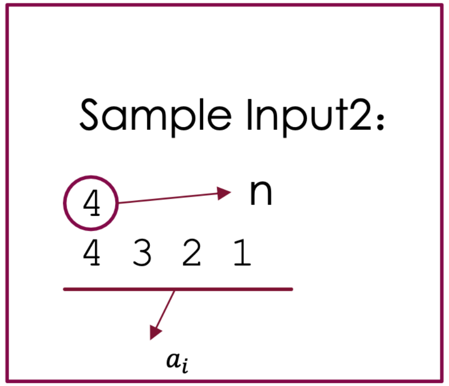
1 [2 3] 4

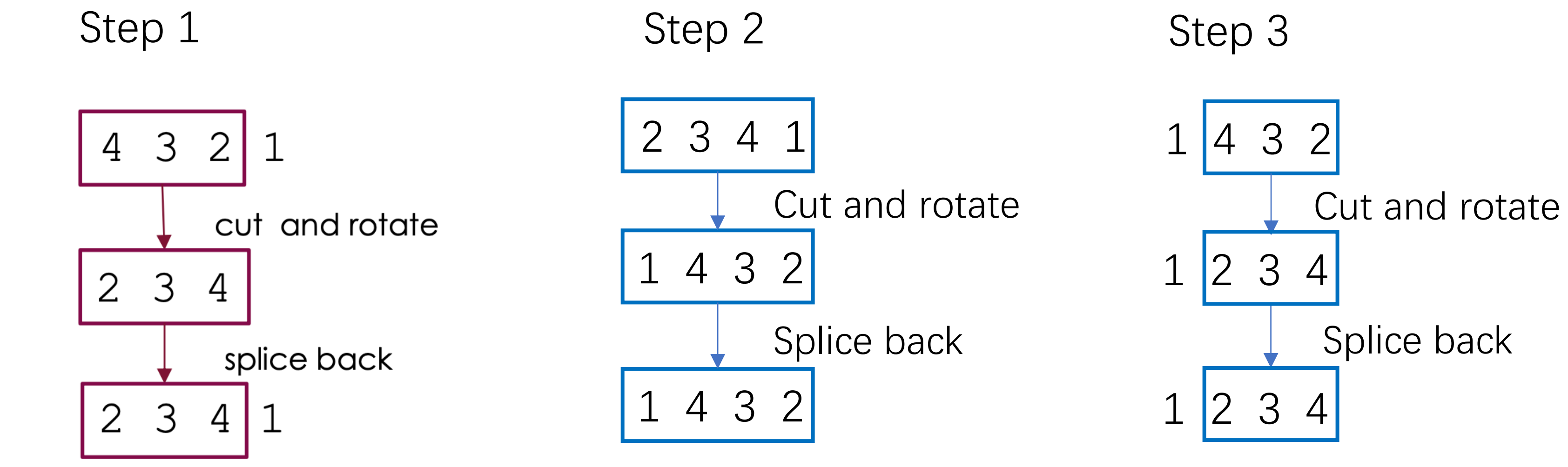
$spend\ 3-2+1 = \ 2\ justin$

$\boldsymbol{Total\ cost: 2 \le \ 2 \times 10^7}$

	Sample output 1:
Left and right ends of the rotation:	2 3
-1 -1 denoting end of output:	-1 -1
An empty line:	

Q2: Urban Construction





$spend\ 3-1+1=3\ justin$

Spend 4-1+1 = 4

Spend 4-2+1 = 3

Sample output 2:

1 3
1 4
2 4
-1 -1

Total cost: 3+4+3=10<2x10^7