P1

(i) Mathematically calculating the solution:

Given two points A and B randomly generated in the unit square ($[0, 1]^2$), we want to calculate the probability of A being dominated by B.

Let's denote the coordinates of point A as ((x_A, y_A)) and the coordinates of point B as ((x_B, y_B)).

For A to be dominated by B, the following conditions must be satisfied:

1. ($x_A \leq x_B$)
2. ($y_A \leq y_B$)

The probability of A being dominated by B can be calculated as the ratio of the area where B dominates A to the total area of the unit square.

The area where B dominates A is a triangular region bounded by the line ($x = y$) and the sides of the unit square. Its area can be calculated as

$$\frac{1}{2} \times \frac{1}{2} = \frac{1}{4}$$

Therefore, the probability of A being dominated by B is

$$\frac{1}{4}$$

(ii) Numerically estimating the solution through computer simulations:

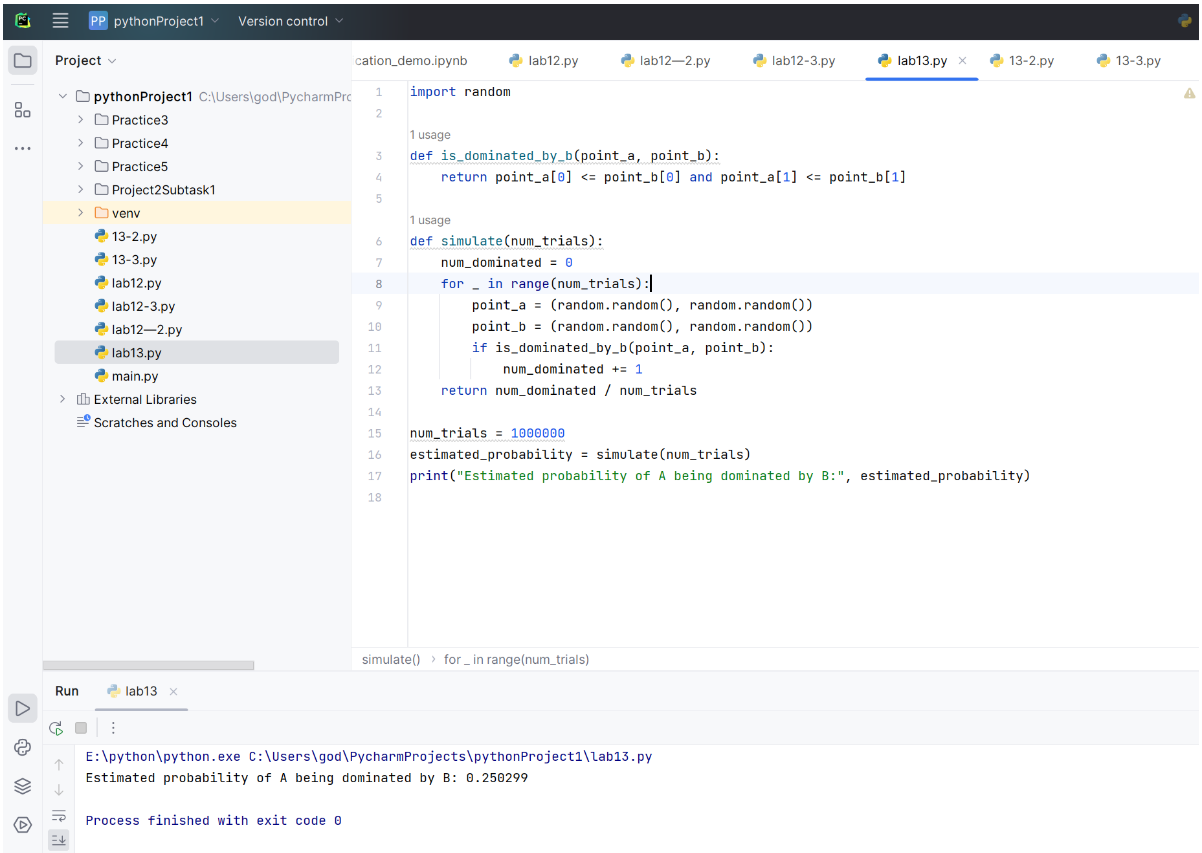
We can simulate this scenario by generating a large number of pairs of random points in the unit square and counting how many times A is dominated by B.

Here's a Python code snippet to perform the simulation:

```
import random

def is_dominated_by_b(point_a, point_b):
    return point_a[0] <= point_b[0] and point_a[1] <= point_b[1]

def simulate(num_trials):
    num_dominated = 0
    for _ in range(num_trials):
        point_a = (random.random(), random.random())
        point_b = (random.random(), random.random())
        if is_dominated_by_b(point_a, point_b):
            num_dominated += 1
    return num_dominated / num_trials

num_trials = 1000000
estimated_probability = simulate(num_trials)
print("Estimated probability of A being dominated by B:", estimated_probability)
```

result:

0.250299



P2

(i) Mathematically calculating the solution:

Given two points A and B in the unit hypercube $[0, 1]^4$, we need to calculate the probability of A being dominated by B.

Let's denote the coordinates of A as ((x_1, y_1, z_1, w_1)) and the coordinates of B as ((x_2, y_2, z_2, w_2)).

For A to be dominated by B, the following conditions must be satisfied for each dimension:

1. ($x_1 \leq x_2$)
2. ($y_1 \leq y_2$)
3. ($z_1 \leq z_2$)
4. ($w_1 \leq w_2$)

The probability of A being dominated by B can be calculated as the ratio of the volume of the region where B dominates A to the total volume of the unit hypercube.

The volume where B dominates A is a hyperpyramidal region bounded by the hyperplane (x = y = z = w) and the sides of the unit hypercube. Its volume can be calculated as

$$\frac{1}{2^4} = \frac{1}{16}$$

Therefore, the probability of A being dominated by B is

$$\frac{1}{16}$$

(ii) Numerically estimating the solution through computer simulations:

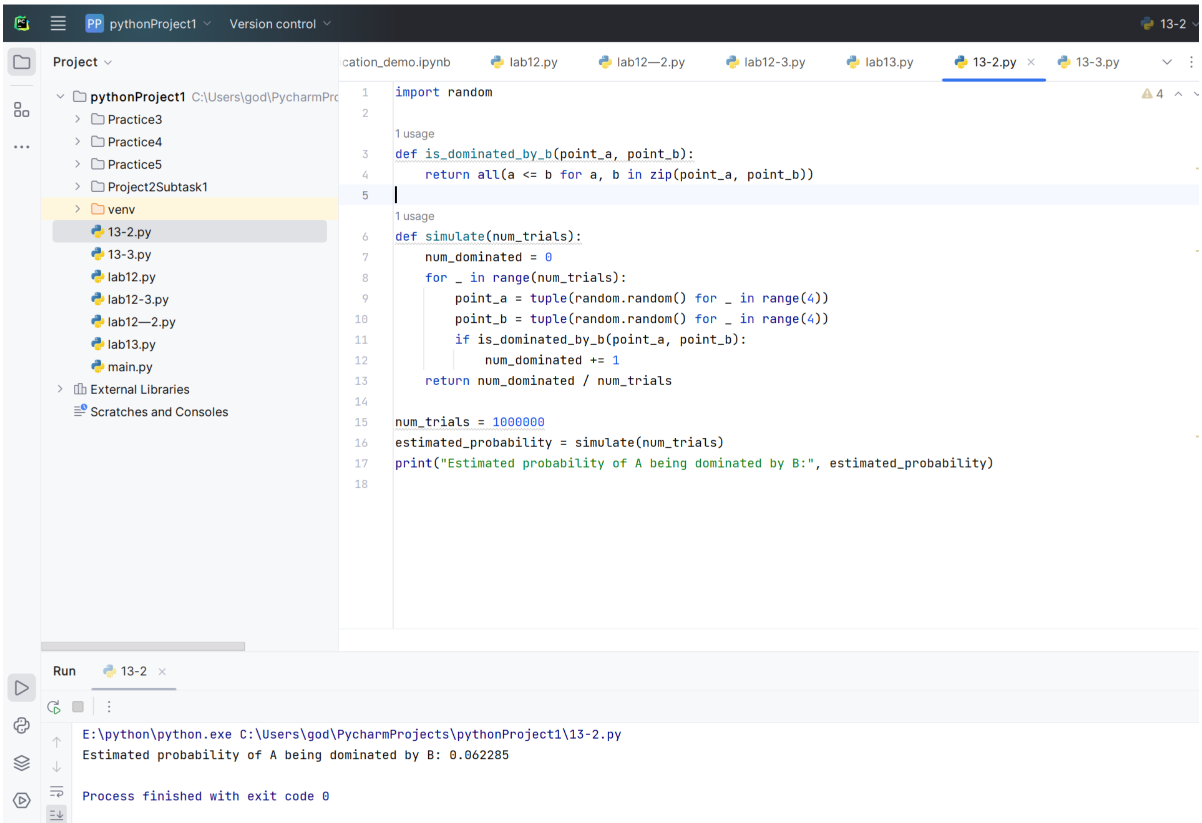
We can simulate this scenario by generating a large number of pairs of random points in the unit hypercube and counting how many times A is dominated by B.

Here's a Python code snippet to perform the simulation:

```python
import random

def is_dominated_by_b(point_a, point_b):
    return all(a <= b for a, b in zip(point_a, point_b))

def simulate(num_trials):
    num_dominated = 0
    for _ in range(num_trials):
        point_a = tuple(random.random() for _ in range(4))
        point_b = tuple(random.random() for _ in range(4))
        if is_dominated_by_b(point_a, point_b):
            num_dominated += 1
    return num_dominated / num_trials

num_trials = 1000000
estimated_probability = simulate(num_trials)
print("Estimated probability of A being dominated by B:", estimated_probability)
```

result:

0.062285



P3

(i) Mathematically calculating the solution:

Given two points A and B in the unit hypercube [0, 1]¹⁰, we need to calculate the probability of A being dominated by B.

Let's denote the coordinates of A as ($(x_1,x_2, ..., x_{10})$) and the coordinates of B as ($(y_1, y_2, ..., y_{10})$).

For A to be dominated by B, the following conditions must be satisfied for each dimension:

1. ($x_1 \leq y_2$)
2. ($x_2 \leq y_2$)
3. ...
4. ($x_{10} \leq y_{10}$)

The probability of A being dominated by B can be calculated as the ratio of the volume of the region where B dominates A to the total volume of the unit hypercube.

The volume where B dominates A is a hyperpyramidal region bounded by the hyperplane ($x_1 = x_2 = ... = x_{10}$) and the sides of the unit hypercube. Its volume can be calculated as

$$\frac{1}{2^{10}} = \frac{1}{1024}$$

Therefore, the probability of A being dominated by B is

$$\frac{1}{1024}$$

(ii) Numerically estimating the solution through computer simulations:

We can simulate this scenario by generating a large number of pairs of random points in the unit hypercube and counting how many times A is dominated by B.

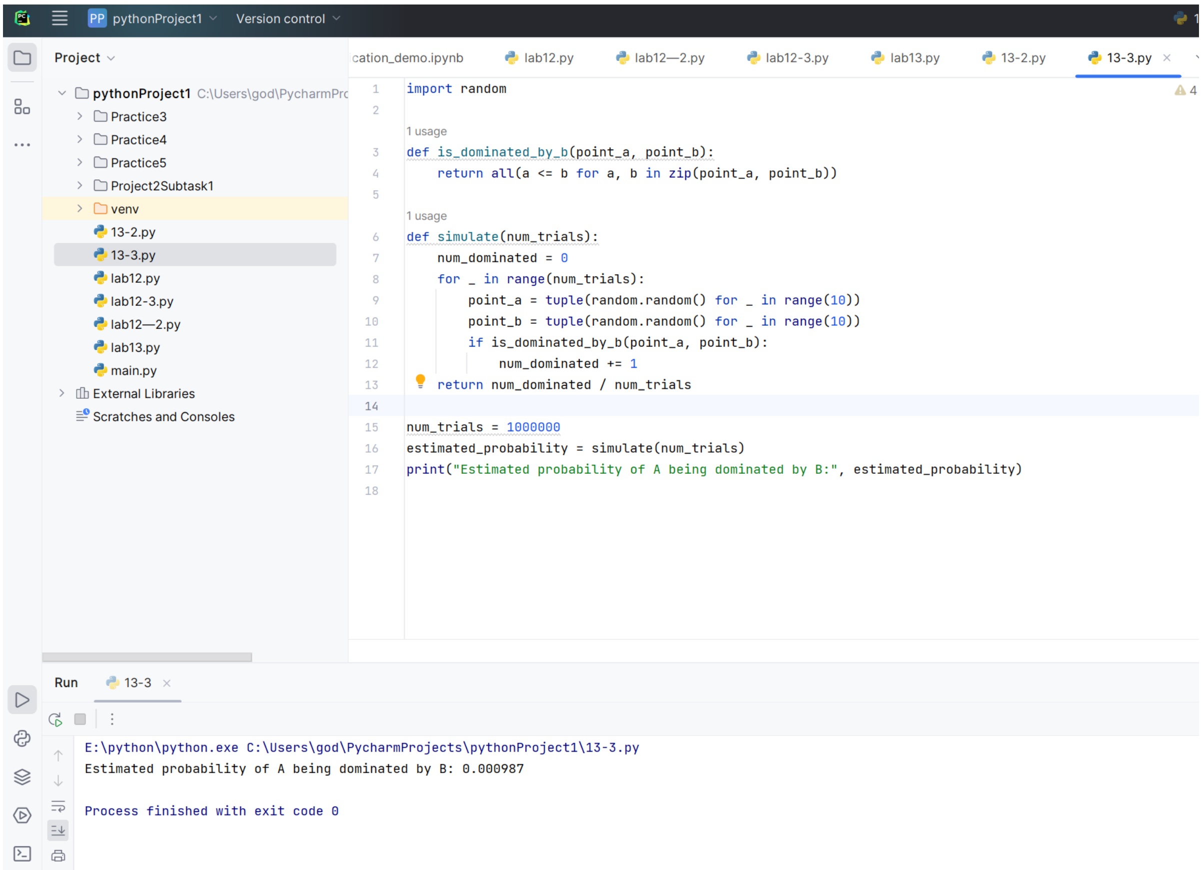
Here's a Python code snippet to perform the simulation:

```
import random

def is_dominated_by_b(point_a, point_b):
    return all(a <= b for a, b in zip(point_a, point_b))

def simulate(num_trials):
    num_dominated = 0
    for _ in range(num_trials):
        point_a = tuple(random.random() for _ in range(10))
        point_b = tuple(random.random() for _ in range(10))
        if is_dominated_by_b(point_a, point_b):
            num_dominated += 1
    return num_dominated / num_trials

num_trials = 1000000
estimated_probability = simulate(num_trials)
```

```
print("Estimated probability of A being dominated by B:", estimated_probability)
```

result:

0.000987



P4

(i) Mathematically calculating the solution:

$$
\begin{aligned}
200\int_0^1\int_0^1 [1-(1-x)(1-y)]^{199}\,dx\,dy \\
=200\int_0^1\int_0^1 [x+y-xy]^{199}\,dx\,dy \\
=\int_0^1 \frac{[1-y]^{200}}{(1-y)}\,dy \\
=\int_0^1 1+y+y^2+\ldots.+y^{100}\,dy \\
=1+0.5+\ldots+0.005 \\
=5.87
\end{aligned}
$$

(ii)Simulation: 5.856 (100 times)

P5

(i) Mathematically calculating the solution:

Similarity as P4:

$$
\begin{aligned}
2000 \int_0^1 \int_0^1 [1-(1-x)(1-y)]^{1999}\, dx\, dy \\
= 1 + 0.5 + \ldots + 0.0005 \\
= 8.18
\end{aligned}
$$

(ii)Simulation: 8.19 (100 times)

P6

(i) Mathematically calculating the solution:

Similarity :

$$
200 \int_0^1 \int_0^1 [1-(1-x)(1-y)]^{199}\, dx\, dy
$$

$$
\downarrow
$$

$$
200 \int_0^1 \cdots \int_0^1 \left[1 - \prod_{i=1}^{10}(1-x_i)\right]^{199} dx_1\, dx_2 \cdots dx_{10}
$$

(ii) Simulation: 180.15 (100 times)

P7

(i) Mathematically calculating the solution:

Similarity as P6:

$$
2000 \int_0^1 \cdots \int_0^1 \left[1 - \prod_{i=1}^{10}(1-x_i)\right]^{1999} dx_1\, dx_2 \cdots dx_{10}
$$

(ii) Simulation: 1380.55 (100 times)