Task1

P1

$$\textbf{Problem 1:} \quad \text{Minimize} \quad x_1 + x_2$$
$$\text{subject to} \quad x_1 + x_2 \geq 1$$
$$\mathbf{0} \leq \mathbf{x} \leq \mathbf{1}$$

Let's set $x_1 = t$, so $x_2 = 1 - t \; where \; 0 \leq t \leq 1$, so minimize $x_1 + x_2$= 1.

P2

$$\textbf{Problem 2:} \quad \text{Minimize} \quad x_1 + x_2 + x_3 + x_4$$
$$\begin{aligned} \text{subject to} \quad x_1 + x_2 \qquad\qquad &= 1 \\ x_2 + x_3 \qquad &= 1 \\ x_3 + x_4 &= 1 \\ x_1 \qquad\qquad + x_4 &= 1 \end{aligned}$$
$$\mathbf{0} \leq \mathbf{x} \leq \mathbf{1}$$

Let's add the first and third constraint, we can get $x_1 + x_2 + x_3 + x_4 = 1$, so minimum $x_1 + x_2 + x_3 + x_4 = 1$.

Task2

Three LP solvers: **GLPK (GNU Linear Programming Kit), Gurobi, PuLP**

P1 Results:

	x1	x2	Object function
GLPK	1.0	0.0	1.0
Gurobi	1.0	0.0	1.0
Pulp	1.0	0.0	1.0

P2 Results:

	x1	x2	x3	x4	Object function
GLPK	0.0	1.0	0.0	1.0	2.0
Gurobi	1.0	0.0	1.0	0.0	2.0
Pulp	0.0	1.0	0.0	1.0	2.0

Task3

Task 3: Compare the following four methods on your 100-item knapsack problem (used in the lab session on Simulated Annealing) with respect to the solution quality (i.e., the objective function value), and the total computation time.

(i) A heuristic method used to generate an initial solution for SA.

(ii) Simulated algorithm with your parameter setting

(iii) Use of an LP solver (i.e., use of the LP relaxation problem), and create a feasible solution from the LP solutions.

(iv) Use of an integer LP (ILP) solver

Set boundary = 1; Set element = (element<boundary); constraint is satisfied: Set element = (element<boundary) boundary=0.01

	(i)	(ii)	(iii)	(iv)
Running time	0.075ms	256ms	3.15ms	542ms
Object value	1667	2953	2987	3093

Task4

Task 4: You will receive 200-item and 400-item knapsack problems from our TA. Compare the above-mentioned four algorithms on each of those two problems in the same manner as in Task 3.

200-item results:

	(i)	(ii)	(iii)	(iv)
Running time	0.78ms	563ms	7.2ms	1029ms
Object value	4190	6248	6260	6341

400-item results:

	(i)	(ii)	(iii)	(iv)
Running time	2.6ms	1125ms	20.3ms	120456ms
Object value	9841	12918	12929	12936

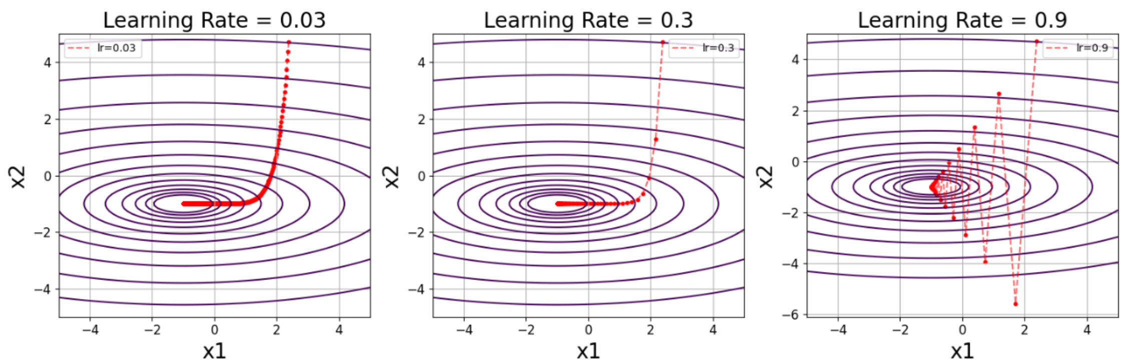
For using an integer LP (ILP) solver, the running time increase very fast when the scale of the problem grows, while using an

LP solver increase nearly linearly.

Task5

Choose an initial solution in [-5, 5]x[-5, 5], specify the step size, and show a sequence of moves from the initial solution using the gradient decent algorithm with the constant step size for the following problem (see below). By iterating these steps using a different initial solution and a different step size, show several sequences of moves. Then, choose the most appropriate step size for this problem. That is, your task is to clearly show the search behavior and to find the best specification of the constant step size.

$$\text{Minimize} \quad f(\mathbf{x}) = (x_1+1)^2/9 + (x_2+1)^2$$



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learning_rates = [0.03, 0.3, 0.9]
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When learning rate is too small, it will take a long time to search at beginning. And if learning rate is too large, it will cause shaking. Therefore, select learning rate with a median value like 0.3 is reasonable .