

P1

(i) Mathematically calculating the solution:

Given two points A and B randomly generated in the unit square $([0, 1]^2)$, we want to calculate the probability of A being dominated by B.

Let's denote the coordinates of point A as (x_A, y_A) and the coordinates of point B as (x_B, y_B) .

For A to be dominated by B, the following conditions must be satisfied:

$$1. (x_A \leq x_B)$$

$$2. (y_A \leq y_B)$$

The probability of A being dominated by B can be calculated as the ratio of the area where B dominates A to the total area of the unit square.

The area where B dominates A is a triangular region bounded by the line $(x = y)$ and the sides of the unit square. Its area can be calculated as

$$\frac{1}{2} \times \frac{1}{2} = \frac{1}{4}$$

Therefore, the probability of A being dominated by B is

$$\frac{1}{4}$$

(ii) Numerically estimating the solution through computer simulations:

We can simulate this scenario by generating a large number of pairs of random points in the unit square and counting how many times A is dominated by B.

Here's a Python code snippet to perform the simulation:

```
import random

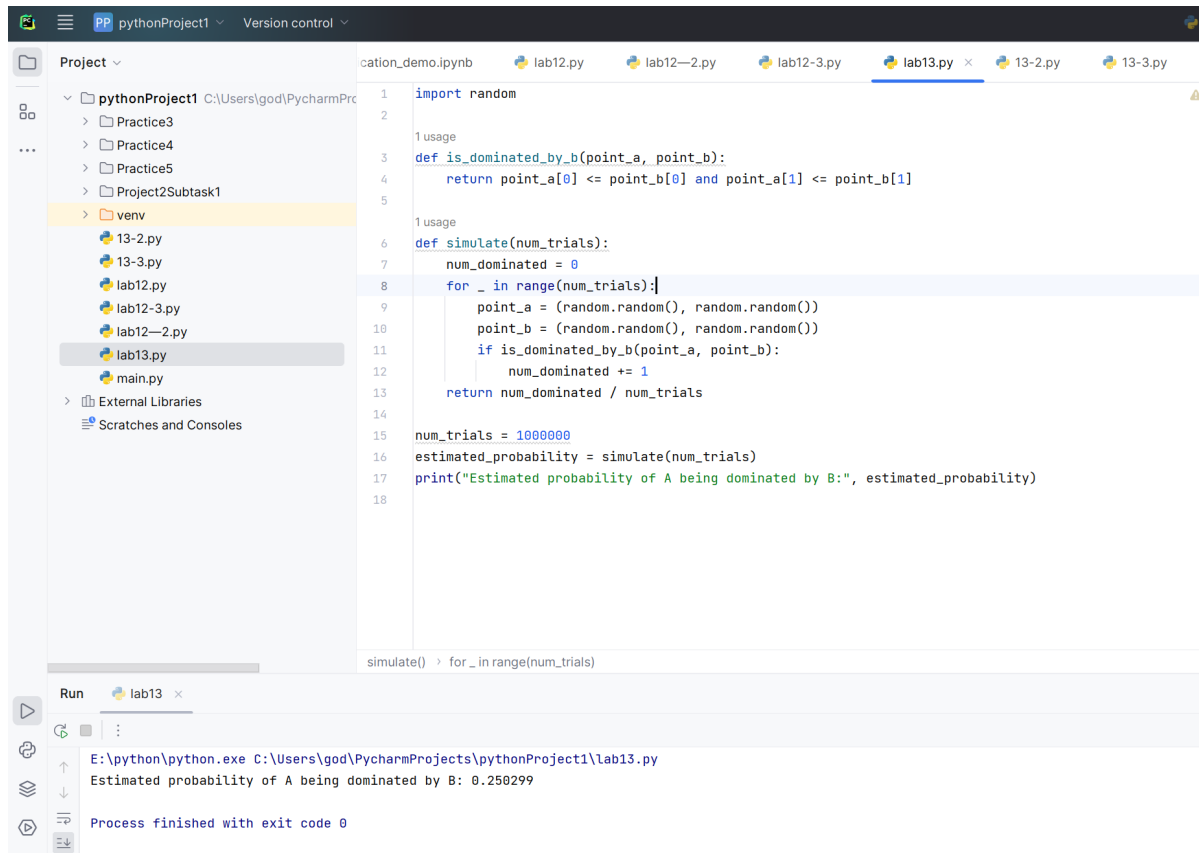
def is_dominated_by_b(point_a, point_b):
    return point_a[0] <= point_b[0] and point_a[1] <= point_b[1]

def simulate(num_trials):
    num_dominated = 0
    for _ in range(num_trials):
        point_a = (random.random(), random.random())
        point_b = (random.random(), random.random())
        if is_dominated_by_b(point_a, point_b):
            num_dominated += 1
    return num_dominated / num_trials

num_trials = 1000000
estimated_probability = simulate(num_trials)
print("Estimated probability of A being dominated by B:", estimated_probability)
```

result:

0.250299



The screenshot shows the PyCharm IDE interface. The left sidebar displays the project structure for 'pythonProject1', including folders like 'Practice3', 'Practice4', 'Practice5', 'Project2Subtask1', and a 'venv' environment. The main editor window shows the code for 'lab13.py'. The code defines a function 'is_dominated_by_b' and a 'simulate' function that runs 1,000,000 trials to estimate the probability of point A being dominated by point B. The output window at the bottom shows the command executed: 'E:\python\python.exe C:\Users\god\PycharmProjects\pythonProject1\lab13.py' and the result: 'Estimated probability of A being dominated by B: 0.250299'. The process finished with exit code 0.

```
1 import random
2
3 1 usage
4 def is_dominated_by_b(point_a, point_b):
5     return point_a[0] <= point_b[0] and point_a[1] <= point_b[1]
6
7 1 usage
8 def simulate(num_trials):
9     num_dominated = 0
10    for _ in range(num_trials):
11        point_a = (random.random(), random.random())
12        point_b = (random.random(), random.random())
13        if is_dominated_by_b(point_a, point_b):
14            num_dominated += 1
15    return num_dominated / num_trials
16
17 num_trials = 1000000
18 estimated_probability = simulate(num_trials)
19 print("Estimated probability of A being dominated by B:", estimated_probability)
```

Run lab13 x

E:\python\python.exe C:\Users\god\PycharmProjects\pythonProject1\lab13.py
Estimated probability of A being dominated by B: 0.250299
Process finished with exit code 0

P2

(i) Mathematically calculating the solution:

Given two points A and B in the unit hypercube $[0, 1]^4$, we need to calculate the probability of A being dominated by B.

Let's denote the coordinates of A as (x_1, y_1, z_1, w_1) and the coordinates of B as (x_2, y_2, z_2, w_2) .

For A to be dominated by B, the following conditions must be satisfied for each dimension:

1. $(x_1 \leq x_2)$
2. $(y_1 \leq y_2)$
3. $(z_1 \leq z_2)$
4. $(w_1 \leq w_2)$

The probability of A being dominated by B can be calculated as the ratio of the volume of the region where B dominates A to the total volume of the unit hypercube.

The volume where B dominates A is a hyperpyramidal region bounded by the hyperplane $(x = y = z = w)$ and the sides of the unit hypercube. Its volume can be calculated as

$$\frac{1}{2^4} = \frac{1}{16}$$

Therefore, the probability of A being dominated by B is

$$\frac{1}{16}$$

(ii) Numerically estimating the solution through computer simulations:

We can simulate this scenario by generating a large number of pairs of random points in the unit hypercube and counting how many times A is dominated by B.

Here's a Python code snippet to perform the simulation:

```
import random

def is_dominated_by_b(point_a, point_b):
    return all(a <= b for a, b in zip(point_a, point_b))

def simulate(num_trials):
    num_dominated = 0
    for _ in range(num_trials):
        point_a = tuple(random.random() for _ in range(4))
        point_b = tuple(random.random() for _ in range(4))
        if is_dominated_by_b(point_a, point_b):
            num_dominated += 1
    return num_dominated / num_trials

num_trials = 1000000
estimated_probability = simulate(num_trials)
print("Estimated probability of A being dominated by B:", estimated_probability)
```

result:

0.062285

The screenshot shows the PyCharm IDE interface. The top toolbar includes icons for file operations, a search icon, and a 'Run' button. The 'Project' sidebar on the left shows a directory structure with folders 'Practice3', 'Practice4', 'Practice5', and 'Project2Subtask1', and a file '13-2.py'. The main editor window displays the Python code from the previous block. The 'Run' console at the bottom shows the output: 'Estimated probability of A being dominated by B: 0.062285'. The status bar at the bottom indicates 'Process finished with exit code 0'.

P3

(i) Mathematically calculating the solution:

Given two points A and B in the unit hypercube $[0, 1]^{10}$, we need to calculate the probability of A being dominated by B.

Let's denote the coordinates of A as $(x_1, x_2, \dots, x_{10})$ and the coordinates of B as $(y_1, y_2, \dots, y_{10})$.

For A to be dominated by B, the following conditions must be satisfied for each dimension:

1. $(x_1 \leq y_1)$
2. $(x_2 \leq y_2)$
3. ...
4. $(x_{10} \leq y_{10})$

The probability of A being dominated by B can be calculated as the ratio of the volume of the region where B dominates A to the total volume of the unit hypercube.

The volume where B dominates A is a hyperpyramidal region bounded by the hyperplane $(x_1 = x_2 = \dots = x_{10})$ and the sides of the unit hypercube. Its volume can be calculated as

$$\frac{1}{2^{10}} = \frac{1}{1024}$$

Therefore, the probability of A being dominated by B is

$$\frac{1}{1024}$$

(ii) Numerically estimating the solution through computer simulations:

We can simulate this scenario by generating a large number of pairs of random points in the unit hypercube and counting how many times A is dominated by B.

Here's a Python code snippet to perform the simulation:

```
import random

def is_dominated_by_b(point_a, point_b):
    return all(a <= b for a, b in zip(point_a, point_b))

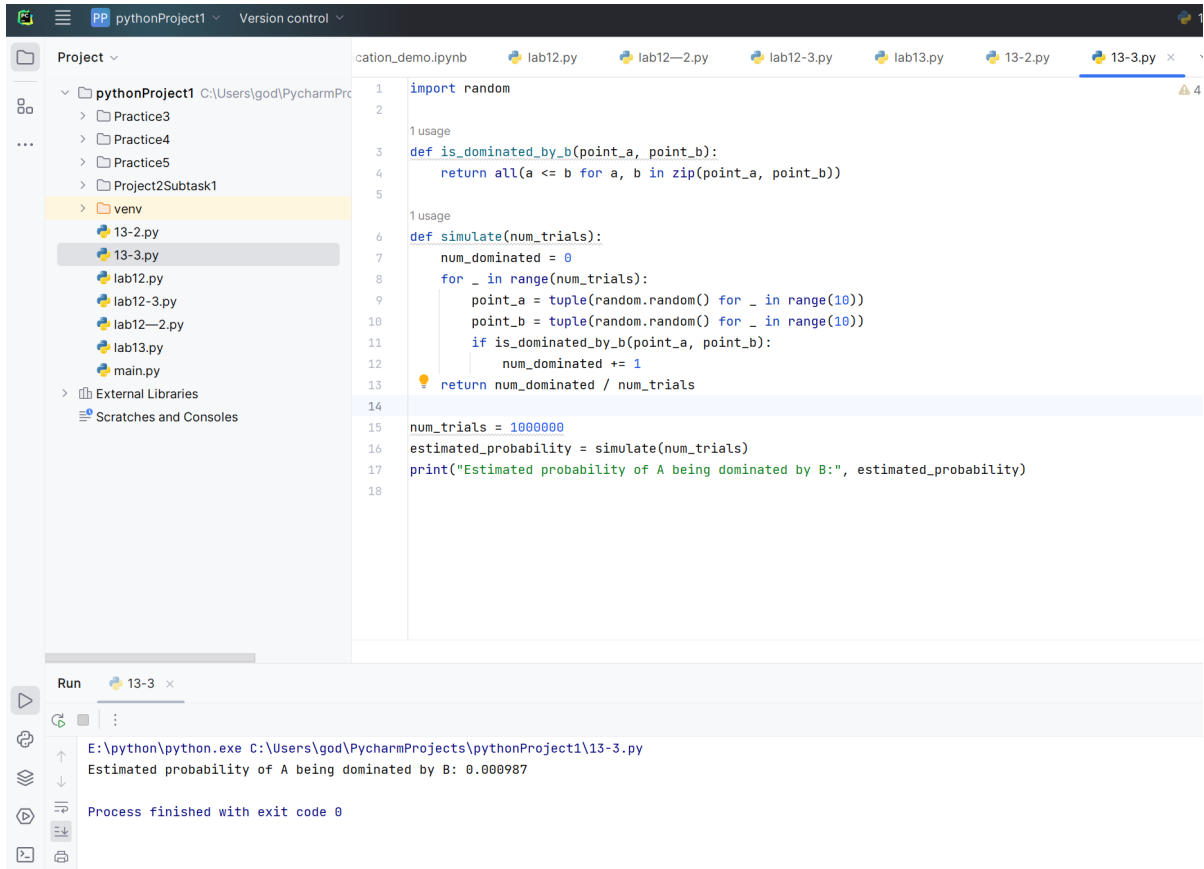
def simulate(num_trials):
    num_dominated = 0
    for _ in range(num_trials):
        point_a = tuple(random.random() for _ in range(10))
        point_b = tuple(random.random() for _ in range(10))
        if is_dominated_by_b(point_a, point_b):
            num_dominated += 1
    return num_dominated / num_trials

num_trials = 1000000
estimated_probability = simulate(num_trials)
```

```
print("Estimated probability of A being dominated by B:", estimated_probability)
```

result:

0.000987



P4

(i) Mathematically calculating the solution:

$$\begin{aligned} & 200 \int_0^1 \int_0^1 [1 - (1-x)(1-y)]^{199} dx dy \\ &= 200 \int_0^1 \int_0^1 [x + y - xy]^{199} dx dy \\ &= \int_0^1 \frac{[1-y]^{200}}{(1-y)} dy \\ &= \int_0^1 1 + y + y^2 + \dots + y^{100} dy \\ &= 1 + 0.5 + \dots + 0.005 \\ &= 5.87 \end{aligned}$$

(ii) Simulation: 5.856 (100 times)

P5

(i) Mathematically calculating the solution:

Similarity as P4:

$$\begin{aligned}
& 2000 \int_0^1 \int_0^1 [1 - (1-x)(1-y)]^{1999} dx dy \\
&= 1 + 0.5 + \dots + 0.0005 \\
&= 8.18
\end{aligned}$$

(ii) Simulation: 8.19 (100 times)

P6

(i) Mathematically calculating the solution:

Similarity :

$$\begin{aligned}
& 200 \int_0^1 \int_0^1 [1 - (1-x)(1-y)]^{199} dx dy \\
& \quad \downarrow \\
& 200 \int_0^1 \cdots \int_0^1 \left[1 - \prod_{i=1}^{10} (1-x_i) \right]^{199} dx_1 dx_2 \cdots dx_{10}
\end{aligned}$$

(ii) Simulation: 180.15 (100 times)

P7

(i) Mathematically calculating the solution:

Similarity as P6:

$$2000 \int_0^1 \cdots \int_0^1 \left[1 - \prod_{i=1}^{10} (1-x_i) \right]^{1999} dx_1 dx_2 \cdots dx_{10}$$

(ii) Simulation: 1380.55 (100 times)