**CS208 Algorithm Design and Analysis**
**Theory homework 4**

Total points: 6

**Question 1** (2 points)
Image that you want to encode $n$ characters, whose frequencies are in the pattern of the first $n$ Fibonacci numbers (1, 1, 2, 3, 5, …) using the Huffman codes method. What are the codes for the $n$ th, $n-1$ th, and $n-2$ th characters? (Briefly explain your answer. You can use plots if needed) (2 points)

**Question 2** (2 points)
Suppose you are given two sets $A$ and $B$, each containing $n$ positive integers. You can choose to reorder each set however you like. After reordering, let $a_i$ be the $i$th element in $A$, and $b_i$ be the $i$th element in $B$. You will receive a payoff of $\sum_{i=1}^{n} a_i b_i$.

a) If you reorder $A$ and $B$ into monotonically decreasing order, consider any indices $i$ and $j$ such that $i < j$, which of the two combinations has higher value: $a_i b_i + a_j b_j$ or $a_i b_j + b_i a_j$? Prove your answer. Based on this, describe the optimal way of reordering that maximizes your payoff. (1 point)

b) If you receive payoff of $\prod_{i=1}^{n} a_i^{b_i}$, what is the way of reordering to maximize the payoff? Prove your answer in a similar way to a). (1 point)
<u>*Hint*</u>: You need to show that $a_i^{b_i} a_j^{b_j} \geq a_i^{b_j} a_j^{b_i}$, for any indices $i$ and $j$ such that $i < j$, when $A$ and $B$ into monotonically decreasing order.

**Question 3** (2 points)
For each of the following statements, decide whether it is true or false. If true, give a short explanation; if false, give a counter-example.

a) Let $T$ be a minimum spanning tree of a graph $G$ whose edges are all *positive* and *distinct*. Suppose we replace each edge weight $w_e$ with a its square, $w_e^2$, then $T$ must still be a minimum spanning tree for this new graph.

b) Considering a shortest-path problem on a directed graph $G$, with source node $s$ and destination $t$. Let $P$ be such a shortest *s-t* path. Assume all edge have *positive* and *distinct* weights. Suppose we replace each edge weight $w_e$ with a its square, $w_e^2$, then $P$ must still be a shortest path for this new graph.