Case Study 16: Training Data vs. Test Data

In our most recent Case Studies, we have begun to explore how to fit models to our data. We have incorporated different forms of predictor variables for these models, including adding multiple predictor variables to our model and adding interaction terms.

Now that we've created many possible models that we can use, how do we pick the "best" model, or the one single model? We'll start to answer this question with the next few Case Studies for the semester, introducing some of the possible methods that we could use in order to select the "best" model.

In this Case Study, we will continue modeling the approval of the President's Foreign Policy with Age, Sex, and Political Affiliation.

Specifically, we'd like to address: What is a model that is good at predicting approval for the President's Foreign Policy based on Age, Sex, and Political Affiliation with new data?

Suppose we work at a political advertising agency. Rather than seek to **understand the relationship** between approval for the president's foreign policy with sex, age, and political affiliation, we would like build a model that will give us the **best predictions** for adults living in the U.S. in which we *don't know what they think about the president's foreign policy*.

We can assume that this agency has the age, sex, political affiliation, and address of all registered voters in the state. So one goal that this political advertising agency might have is to use this data to make predictions about whether a given person that lives at a particular house approves of the president's foreign policy. They could then use that information to decide whether to mail political advertising pamphplets to this address.

Python Libraries and Packages

Python libraries:

```
statsmodels.api, statsmodels.formula.api, scikit-learn
```

Imports

```
import pandas as pd
import numpy as np
import seaborn as sns
import matplotlib.pyplot as plt

import statsmodels.api as sm
import statsmodels.formula.api as smf
```

Different Goals for Building a Regression Model

We have two primary goals for building a regression model:

• **predicting** a value for a new observation

• **understanding** a phenomenon or the relationship between two variables

28.2

29.9

29.9

Predicting a value is all about *using* the model in order to estimate a new value. Understanding a phenomenon focuses more on the *coefficient values*, and often has a special focus on inference.

We will look at an example in lecture with the body dimensions dataset with two different goals. Below, you can find the models that we fit in order to approach these two different goals.

Read the body dimensions dataset.

40.1

44.3

42.5

```
In [2]:
          df=pd.read csv('bdims.csv')
          df.head()
Out[2]:
             biacromial_diameter pelvic_breadth bitrochanteric_diameter chest_depth chest_diameter elbow_diamete
          0
                                            26.0
                            42.9
                                                                    31.5
                                                                                  17.7
                                                                                                  28.0
                                                                                                                   13
          1
                            43.7
                                            28.5
                                                                    33.5
                                                                                  16.9
                                                                                                 30.8
                                                                                                                  14.
```

33.3

34.0

34.0

20.9

18.4

21.5

31.7

28.2

29.4

13.

13.

15.

5 rows × 26 columns

2

3

4

Out[4]:

	bicep_girth	age	sex	weight	height
0	32.5	21	Male	65.6	174.0
1	34.4	23	Male	71.8	175.3
2	33.4	28	Male	80.7	193.5
3	31.0	23	Male	72.6	186.5
4	32.0	22	Male	78.8	187.2
•••					
482	30.3	29	Female	71.8	176.5
483	30.1	21	Female	55.5	164.4
484	27.4	33	Female	48.6	160.7
485	30.6	33	Female	66.4	174.0
486	33.2	38	Female	67.3	163.8

487 rows × 5 columns

```
In [5]:
           results=smf.ols('bicep girth~age+sex+weight+height', data=df).fit()
           results.summary()
                               OLS Regression Results
Out[5]:
              Dep. Variable:
                                                                    0.831
                                  bicep_girth
                                                    R-squared:
                    Model:
                                        OLS
                                                Adj. R-squared:
                                                                    0.829
                   Method:
                                                                    590.9
                               Least Squares
                                                    F-statistic:
                     Date: Wed, 21 Apr 2021 Prob (F-statistic): 2.94e-184
                     Time:
                                    22:52:06
                                                Log-Likelihood:
                                                                  -963.88
          No. Observations:
                                        487
                                                          AIC:
                                                                    1938.
              Df Residuals:
                                        482
                                                          BIC:
                                                                    1959.
                  Df Model:
                                          4
          Covariance Type:
                                   nonrobust
                         coef std err
                                                P>|t| [0.025 0.975]
            Intercept 31.4253
                                 2.032 15.465 0.000
                                                      27.432 35.418
          sex[T.Male]
                       3.4235
                                 0.235 14.590 0.000
                                                        2.962
                                                                3.885
                      -0.0132
                                 0.009
                                       -1.547 0.123
                                                       -0.030
                                                               0.004
                 age
              weight
                       0.2475
                                 0.009 26.789 0.000
                                                        0.229
                                                                0.266
               height -0.1088
                                 0.013
                                        -8.129 0.000
                                                       -0.135
                                                              -0.083
                Omnibus: 13.978
                                    Durbin-Watson:
                                                        1.993
          Prob(Omnibus):
                           0.001 Jarque-Bera (JB):
                                                       15.394
                   Skew:
                           0.347
                                          Prob(JB): 0.000454
                Kurtosis:
                           3.526
                                          Cond. No. 4.78e+03
```

Notes:

- [1] Standard Errors assume that the covariance matrix of the errors is correctly specified.
- [2] The condition number is large, 4.78e+03. This might indicate that there are strong multicollinearity or other numerical problems.

Method:Least SquaresF-statistic:784.7Date:Wed, 21 Apr 2021Prob (F-statistic):3.19e-185Time:22:52:06Log-Likelihood:-965.09

No. Observations: 487 AIC: 1938.

Df Model: 3 **Covariance Type:** nonrobust coef std err P>|t| [0.025 0.975] 1.984 15.486 0.000 26.829 34.627 **Intercept** 30.7279 **sex[T.Male]** 3.3844 0.234 14.487 0.000 2.925 3.843 **weight** 0.2449 0.009 26.922 0.000 0.227 0.263 height -0.1060 0.013 -7.980 0.000 -0.132 -0.080 **Omnibus:** 14.566 **Durbin-Watson:** 1.991 Prob(Omnibus): 0.001 Jarque-Bera (JB): 16.497 **Prob(JB):** 0.000262 Skew: 0.345 Kurtosis: 3.581 Cond. No. 4.60e+03

483

Notes:

Df Residuals:

- [1] Standard Errors assume that the covariance matrix of the errors is correctly specified.
- [2] The condition number is large, 4.6e+03. This might indicate that there are strong multicollinearity or other numerical problems.

Testing and Training Data

Data Setup

We will be using a portion of our 2017 random sample Pew dataset to train a logistic regression model that predicts the probability that an adult living in the U.S. supported the president's foreign policy given sex, age, and political affiliation.

BIC:

1955.

We will start by preparing the data, including loading it, cleaning it, and creating any additional variables.

party	q5cf1	sex	age		Out[7]:
Independent	NaN	Female	80.0	0	
Democrat	Disapprove	Female	70.0	1	
Independent	Disapprove	Female	69.0	2	
Republican	NaN	Male	50.0	3	
Democrat	Disapprove	Female	70.0	4	

Let's first drop the rows in this dataset with missing values.

```
df.head()
 Out[8]:
              age
                                q5cf1
                                            party
                      sex
           1 70.0 Female
                           Disapprove
                                         Democrat
           2 69.0 Female
                          Disapprove
                                      Independent
           4 70.0 Female
                           Disapprove
                                         Democrat
             89.0 Female
                           Disapprove
                                      Independent
             92.0 Female
                              Approve
                                        Republican
          Report the size of the data.
 In [9]:
           n=df.shape[0]
          679
 Out[9]:
          We also will create a 0/1 response variable value for the logistic regression model where:
           approve =1 and

 disapprove =0.

In [10]:
           df['y'] = df['q5cf1'].map({'Disapprove':0,'Approve':1})
           df.head()
Out[10]:
              age
                      sex
                                q5cf1
                                            party y
           1 70.0
                   Female
                           Disapprove
                                         Democrat 0
           2 69.0 Female
                          Disapprove
                                     Independent 0
             70.0 Female
                          Disapprove
                                         Democrat 0
             89.0
                   Female
                          Disapprove
                                      Independent 0
```

Creating the Training and Test Dataset

Approve

Next, we split the data into the:

7 92.0 Female

df = df.dropna()

- training dataset: where we randomly select 80% of observations from Pew dataset and the
- **test data set**: comprised of the remaining 20% of observations from Pew dataset.

Republican 1

It's usually best to have your training dataset have much more observations than your test dataset!

We use the **train_test_split()** function from the **sklearn.model_selection** package to do this. The parameters for this function are:

- the dataframe we wish to randomly split into a training dataset and a test dataset
- the **test_size**= the percent of the dataset we would like to be allocated to the test dataset
- we an also supply a random_state number.

```
In [11]: from sklearn.model_selection import train_test_split
```

```
df train, df test = train test split(df,
                                    test size=0.20,
                                    random state=123)
```

Let's inspect the newly created training dataset.

```
In [12]:
           df train
Out[12]:
                 age
                         sex
                                  q5cf1
                                               party
                                                      У
            725 39.0 Female Disapprove
                                            Democrat 0
           836 67.0 Female Disapprove
                                            Democrat
            961 51.0
                        Male Disapprove
                                           Democrat 0
           348 72.0
                        Male
                                          Republican
                                Approve
           1025
                 61.0 Female
                              Disapprove
                                           Democrat
           205 90.0 Female
                                Approve
                                          Republican
           693 20.0
                        Male
                                Approve
                                         Independent
           838 68.0
                                Approve
                                          Republican
                        Male
            791 56.0
                        Male Disapprove Independent
           1115 45.0
                        Male
                                Approve Independent
```

543 rows × 5 columns

We can double check that this training dataset contains about 80% of the observations from df.

```
In [13]:
          df train.shape[0]/df.shape[0]
         0.7997054491899853
Out[13]:
```

Let's inspect this new test dataset

	Let S II	et's inspect this new test dataset.						
In [14]:	df_test							
Out[14]:	age sex q5cf1 party y							
	337	79.0	Female	Approve	Republican	1		
	424	30.0	Female	Disapprove	Independent	0		
	751	46.0	Male	Disapprove	Independent	0		
	1423	77.0	Male	Disapprove	Democrat	0		
	1367	58.0	Male	Approve	Independent	1		
	•••		•••					
	872	42.0	Female	Approve	Republican	1		
	915	52.0	Male	Disapprove	Democrat	0		
	535	22.0	Male	Disapprove	Independent	0		
	1075	69.0	Female	Disapprove	Democrat	0		

	age	sex	q5cf1	party	У
933	74.0	Male	Disapprove	Independent	0

136 rows x 5 columns

We can double check that this test dataset contains about 20% of the observations from df.

```
In [15]:
          df test.shape[0]/df.shape[0]
          0.20029455081001474
Out[15]:
```

Fit (i.e. train) the model to training data.

Next we will train our logistic regression model with the training dataset only.

```
In [16]:
           pewmod = smf.logit('y ~ party + age + sex',
                                  data=df train).fit()
           pewmod.summary()
          Optimization terminated successfully.
                     Current function value: 0.402672
                     Iterations 7
                               Logit Regression Results
Out[16]:
              Dep. Variable:
                                          y No. Observations:
                                                                     543
                    Model:
                                       Logit
                                                  Df Residuals:
                                                                     536
                   Method:
                                       MLE
                                                     Df Model:
                                                                       6
                     Date: Wed, 21 Apr 2021
                                               Pseudo R-squ.:
                                                                  0.3899
                     Time:
                                   22:52:06
                                               Log-Likelihood:
                                                                  -218.65
                converged:
                                        True
                                                      LL-Null:
                                                                 -358.39
           Covariance Type:
                                   nonrobust
                                                  LLR p-value: 2.035e-57
                                           coef std err
                                                              z P>|z| [0.025 0.975]
                              Intercept -4.6644
                                                  0.535 -8.719 0.000
                                                                        -5.713
                                                                               -3.616
                   party[T.Independent]
                                         2.1964
                                                  0.352
                                                         6.232 0.000
                                                                        1.506
                                                                                2.887
           party[T.No preference (VOL.)]
                                         2.7477
                                                  0.722
                                                         3.805 0.000
                                                                         1.332
                                                                                4.163
             party[T.Other party (VOL.)]
                                         4.0648
                                                  1.230
                                                          3.306
                                                                0.001
                                                                         1.655
                                                                                6.475
                    party[T.Republican]
                                         4.4606
                                                  0.388 11.498 0.000
                                                                         3.700
                                                                                 5.221
                            sex[T.Male]
                                         0.9140
                                                  0.252 3.633 0.000
                                                                         0.421
                                                                                1.407
                                   age
                                          0.0271
                                                  0.007
                                                         3.840 0.000
                                                                         0.013
                                                                                0.041
```

Test the model's predictive accuracy with the test dataset.

Finally, in order to get an idea as to how well our trained logistic regression model with perform with new data (that was not factored in to the optimal selection of $\hat{\beta}_0,\hat{\beta}_1,\ldots,\hat{\beta}_p$) we will calculate various metric that assess the predictive performance of our model with the test dataset including the:

AUC

1423

1367 58.0

77.0

Male

Male

Disapprove

Democrat

Approve Independent

0

0.159691

0.505054

sensitivity and specificity for a few selected predictive probability thresholds.

First, get the predictive probabilities of the test dataset with this trained model.

The predict function uses the fitted model to extract any exogenous variables it needs from the test data. We do not have to specify which variables. We just provide the whole test data frame. Compare the following two code cells and results.

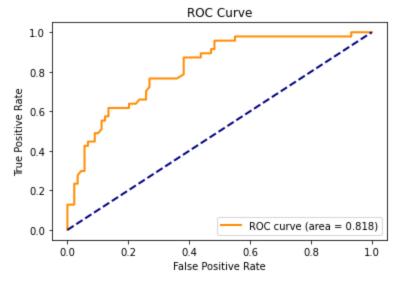
```
In [17]:
           # predictive probabilities - explicit method
          phat test = pewmod.predict(exog=df test[['age', 'sex', 'party']])
          phat test.head(10)
                  0.874386
Out[17]:
          424
                  0.160607
                  0.424221
                 0.159691
         1423
         1367
                 0.505054
         440
                  0.079614
         801
                  0.850883
         1279
                  0.890355
                  0.082286
         187
         342
                  0.057777
         dtype: float64
In [18]:
           # predictive probabilities - implicit method
          phat test = pewmod.predict(exog=df test)
          phat test.head(10)
          337
                  0.874386
Out[18]:
          424
                  0.160607
         751
                 0.424221
                 0.159691
         1423
         1367
                  0.505054
         440
                  0.079614
         801
                 0.850883
         1279
                  0.890355
         187
                  0.082286
                  0.057777
         342
         dtype: float64
In [19]:
          df test['phat test'] = phat test
          df test
         <ipython-input-19-c185c916a8e2>:1: SettingWithCopyWarning:
         A value is trying to be set on a copy of a slice from a DataFrame.
         Try using .loc[row indexer, col indexer] = value instead
         See the caveats in the documentation: https://pandas.pydata.org/pandas-docs/stable/user gu
          ide/indexing.html#returning-a-view-versus-a-copy
            df_test['phat_test']=phat_test
Out[19]:
                age
                                q5cf1
                       sex
                                           party
                                                  y phat_test
               79.0 Female
                              Approve
                                                     0.874386
                                       Republican
                                                  1
               30.0 Female Disapprove
                                      Independent
                                                      0.160607
           751 46.0
                            Disapprove
                                      Independent
                                                  0
                                                      0.424221
                       Male
```

	age	sex	q5cf1	party	у	phat_test
•••						•••
872	42.0	Female	Approve	Republican	1	0.718312
915	52.0	Male	Disapprove	Democrat	0	0.087940
535	22.0	Male	Disapprove	Independent	0	0.277510
1075	69.0	Female	Disapprove	Democrat	0	0.057777
933	74.0	Male	Disapprove	Independent	0	0.611701

136 rows × 6 columns

Next, we generate the ROC curve and calculate the AUC for the test dataset.





Interpretation:

Evaluation: The AUC for the **test dataset** is 0.818.

What can we use it form: This gives us a sense of how good our logistic regression model (which has been trained with the **training dataset**) would be at predicting the probability that an adult living in the U.S.

approves of the president's foreign policy with *new data* (in which we don't know the actual answer of whether they disapprove or approve.

<u>Interpreting AUC</u>: Because the AUC is somewhat high (ie. closer to 1 than it is to 0.5), this tells us that there does exist some predictive probability threshold that gets somewhat close to giving us the ideal scenario of a model with a false positive rate of 0 and a true positive rate of 1 with new data.

Finding a "good" (FPR, TPR) combination.

Ideally, we would like to pick a predictive probability threshold that gives us a false positive rate of 0 and true positive rate of 1. However, this ROC curve shows that there does not exist a predictive probability threshold that will give us this ideal combination. So what predictive probability threshold should we choose?

Well, it depends on much a high true positive rate is worth to you vs. a low false positive rate is to you.

Here are a couple options:

<u>Option 1</u>: About (FPR = 0.5, TPR = 0.95)

Notice how that at a FPR of 0.5, the TPR starts to level off in the ROC curve above. By increasing the FPR any more past 0.5, we do not gain much more in the way of a better (higher) TPR. So we could choose the predictive probability threshold that gives us this combination of (FPR = 0.5, TPR = 0.95).

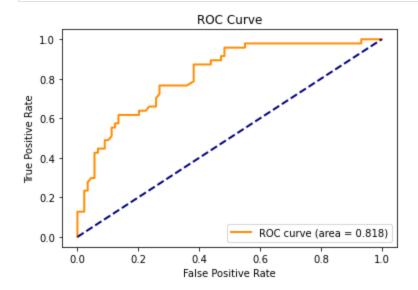
Option 2: About (FPR = 0.1, TPR = 0.6)

Notice how that at a TPR of 0.6, the FPR starts to level off in the ROC curve above. By decreasing the TPR any more past 0.6, we do not gain much more in the way of a better (lower) FPR. So we could choose the predictive probability threshold that gives us this combination of (FPR = 0.1, TPR = 0.6).

What option would a political advertising group choose?

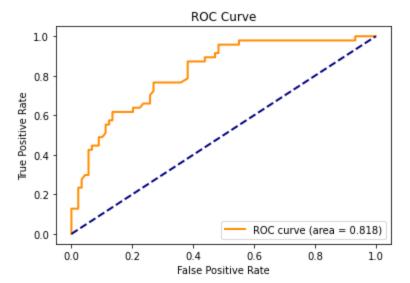
<u>Political Ad Group 1</u>: Suppose this group really values predicting as many people as possible that support the president's foreign policy (ie. are a 1 or positive). Furthermore there is no penalty for mailing ads to houses in which the homeowners don't support the policy (ie. are a 0 or negative).

```
In [23]: plot_roc(fpr_pew, tpr_pew, auc_pew)
```



<u>Political Ad Group 2</u>: Suppose this group would *ideally* like to predict as many people as possible that support the president's foreign policy (ie. are a 1/positive), but have learned that there is a very high backfire effect when they mail ads to houses in which the homeowners don't support the policy (ie. are a 0 or negative).

```
In [24]: plot_roc(fpr_pew, tpr_pew, auc_pew)
```



Finding the predictive probability threshold that corresponds to a (FPR, TPR).

You can use this defined function below to quickly generate the fpr and tpr of a model given:

- y = the actual 0/1 response variable values for a given dataset
- pred_prob = the predictive probabilities for each of the observations of a given dataset
- thresh = a predictive probability threshold value

For instance, the **test dataset** has a tpr = 0.6170 and a fpr = 0.1348 given a predictive probability threshold of $p_0 = 0.5$ with this logistic regression model.

Let's iterate through a series of predictive probability thresholds starting from $p_0=0$ and ending with $p_0=1$ and a step size of 0.01, to see if we can find which predictive probability threshold will give us:

Option 1: About (FPR = 0.5, TPR = 0.95) and

0.5 0.617021 0.134831

0

• Option 2: About (FPR = 0.1, TPR = 0.6).

0.28 0.765957 0.359551

fpr

tpr

threshold

In [27]: for thresh in np.arange(0,1,.01): print(tpr fpr thresh(df test['y'], df test['phat test'], thresh)) threshold tpr fpr 0 0.0 1.0 1.0 threshold tpr fpr 0.01 1.0 1.0 threshold tpr fpr 0.02 0.978723 0.932584 fpr threshold tpr 0.03 0.978723 0.831461 threshold tpr 0.04 0.978723 0.786517 threshold tpr fpr 0.05 0.978723 0.752809 0 threshold tpr fpr 0.06 0.978723 0.640449 tpr threshold 0.07 0.978723 0.606742 threshold tpr fpr 0.08 0.978723 0.58427 0 threshold tpr fpr 0.09 0.978723 0.561798 0 threshold tpr fpr 0.1 0.978723 0.550562 0 threshold tpr fpr 0 0.11 0.957447 0.550562 threshold tpr fpr 0.12 0.957447 0.539326 0 threshold tpr fpr 0.13 0.957447 0.483146 fpr threshold tpr 0.14 0.93617 0.483146 threshold tpr fpr 0.15 0.914894 0.47191 threshold tpr fpr 0 0.16 0.893617 0.460674 threshold tpr fpr 0.17 0.893617 0.438202 threshold tpr 0.18 0.87234 0.438202 0 threshold tpr fpr 0.19 0.87234 0.438202 0 threshold tpr fpr 0.2 0.87234 0.404494 0 threshold tpr fpr 0.21 0.87234 0.382022 threshold tpr fpr 0 0.22 0.87234 0.382022 threshold tpr fpr 0.23 0.87234 0.382022 threshold tpr fpr 0.24 0.87234 0.382022 threshold tpr fpr 0.25 0.851064 0.382022 threshold tpr 0.26 0.829787 0.382022 threshold tpr fpr 0 0.27 0.787234 0.382022 threshold tpr fpr

threshold tpr fpr	0	0.29	0.765957	0.337079
threshold tpr fpr			=	
threshold tpr fpr	0			
threshold tpr fpr	Ο		-	=
0 0.32 0.765957 0.314607 threshold tpr fpr 0 0.33 0.765957 0.314607 threshold tpr fpr 0 0.34 0.765957 0.292135 threshold tpr fpr 0 0.35 0.723404 0.269663 threshold tpr fpr 0 0.36 0.723404 0.269663 threshold tpr fpr 0 0.36 0.723404 0.269663 threshold tpr fpr 0 0.38 0.659574 0.258427 threshold tpr fpr 0 0.38 0.659574 0.235955 threshold tpr fpr 0 0.39 0.638298 0.213483 threshold tpr fpr 0 0.49 0.617021 0.179775 threshold tpr fpr 0 0.41	O			
0	0		-	=
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threshold tpr fpr	0		-	=
		threshold	tpr	fpr

0	0.62	0.489362	0.089888
	threshold	tpr	fpr
0	0.63	0.468085	0.089888
0	threshold 0.64	tpr 0.446809	fpr 0.089888
O	threshold	tpr	fpr
0	0.65	0.446809	0.078652
	threshold	tpr	fpr
0	0.66	0.446809	0.078652
0	threshold 0.67	tpr 0.446809	fpr 0.078652
Ü	threshold	tpr	fpr
0	0.68	0.446809	0.078652
	threshold	tpr	fpr
0	0.69 threshold	0.446809	0.078652 fpr
0	0.7	tpr 0.446809	0.078652
	threshold	tpr	fpr
0	0.71	0.446809	0.067416
0	threshold	tpr	fpr
0	0.72 threshold	0.425532 tpr	0.067416 fpr
0	0.73	0.425532	0.067416
	threshold	tpr	fpr
0	0.74	0.425532	0.067416
0	threshold 0.75	tpr 0.425532	fpr 0.067416
U	threshold	0.423332 tpr	0.007410 fpr
0	0.76	0.425532	0.067416
	threshold	tpr	fpr
0	0.77	0.425532	0.05618
0	threshold 0.78	tpr 0.404255	fpr 0.05618
O	threshold	tpr	fpr
0	0.79	0.382979	0.05618
	threshold	tpr	fpr
0	0.8 threshold	0.361702	0.05618
0	0.81	tpr 0.319149	fpr 0.05618
	threshold	tpr	fpr
0	0.82	0.297872	0.05618
0	threshold 0.83	tpr	fpr
0	threshold	0.297872 tpr	0.05618 fpr
0	0.84	0.276596	0.033708
	threshold	tpr	fpr
0	0.85	0.255319	0.033708
0	threshold 0.86	tpr 0.234043	fpr 0.022472
O	threshold	tpr	fpr
0	0.87	0.234043	0.022472
	threshold	tpr	fpr
0	0.88 threshold	0.212766	0.022472 fpr
0	0.89	tpr 0.170213	0.022472
	threshold	tpr	fpr
0	0.9	0.12766	0.022472
0	threshold	tpr	fpr
0	0.91 threshold	0.106383 tpr	0.0 fpr
0	0.92	0.085106	0.0
	threshold	tpr	fpr
0	0.93	0.042553	0.0
0	threshold 0.94	tpr 0.042553	fpr
0	threshold	0.042553 tpr fpr	0.0
		-11-	

```
0 0.95 0.0 0.0 threshold tpr fpr 0 0.96 0.0 0.0 threshold tpr fpr 0 0.97 0.0 0.0 threshold tpr fpr 0 0.98 0.0 0.0 threshold tpr fpr 0 0.99 0.0 0.0
```

961 51.0

348 72.0

Male Disapprove

Approve

Male

Option 1: It looks like a predictive probability threshold of $p_0=0.13$ will give us a tpr=0.957447 and a fpr=0.483146.

Option 2: It looks like a predictive probability threshold of $p_0=0.50$ will give us a tpr=0.617021and a fpr=0.134831.

Comparing with the Training Data

Just for comparison, let's also create a ROC curve and AUC for this logistic regression model, now using the **training dataset** instead.

Note: this is not something that you would typically do. We are performing this analysis to demonstrate why we split our data into training and testing data.

First, get the predictive probabilities of the *training dataset* with this trained model.

```
In [28]:
          # predictive probabilities - implicit method
          phat train = pewmod.predict(exog=df train)
          phat train.head(10)
         725
                0.026445
Out[28]:
         836
                0.054892
         961
                0.085788
         348
                0.934888
         1025
              0.047031
                0.044657
         251
         73
                 0.477928
         217
                0.572393
                0.922323
                0.237726
         987
         dtype: float64
In [29]:
          df train['phat train']=phat train
          df train
         <ipython-input-29-1a816231d49d>:1: SettingWithCopyWarning:
         A value is trying to be set on a copy of a slice from a DataFrame.
         Try using .loc[row indexer,col indexer] = value instead
         See the caveats in the documentation: https://pandas.pydata.org/pandas-docs/stable/user gu
         ide/indexing.html#returning-a-view-versus-a-copy
           df train['phat train']=phat train
Out[29]:
               age
                      sex
                               q5cf1
                                          party y phat_train
          725 39.0 Female Disapprove
                                       Democrat 0 0.026445
          836 67.0 Female Disapprove
                                       Democrat 0 0.054892
```

Democrat 0

Republican 1

0.085788

0.934888

	age	sex	q5cf1	party	у	phat_train
1025	61.0	Female	Disapprove	Democrat	0	0.047031
•••					•••	
205	90.0	Female	Approve	Republican	1	0.903685
693	20.0	Male	Approve	Independent	1	0.266759
838	68.0	Male	Approve	Republican	1	0.927960
791	56.0	Male	Disapprove	Independent	0	0.491485
1115	45.0	Male	Approve	Independent	1	0.417605

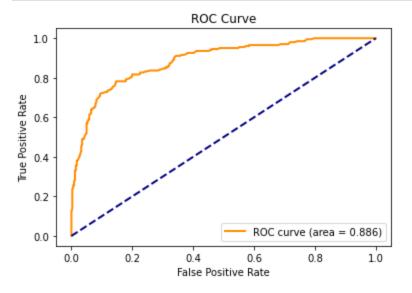
543 rows × 6 columns

Next, we generate the ROC curve and calculate the AUC for the training dataset.

```
In [30]:
    from sklearn.metrics import roc_curve
        from sklearn.metrics import roc_auc_score

        fpr_pew, tpr_pew, score_pew = roc_curve(y_true=df_train['y'], y_score=df_train['phat_train auc_pew = roc_auc_score(y_true=df_train['y'], y_score=df_train['phat_train'])
```





Interpretation:

<u>Evaluation</u>: The AUC for the **training dataset** 0.886, which is higher than it was for the test dataset (ie. AUC = 0.818).!

However, this is to be expected! We would expect to get better predictions from the **training dataset** that we specifically used to pick the values of $\hat{\beta}_0, \hat{\beta}_1, \dots, \hat{\beta}_p$ that would fit the **training dataset** the most.

However, using this AUC of 0.886 to assess how well this model would be at predicting the probability that an adult living in the U.S. supports the president's foreign policy **for new data** would be misleading.

It is much more likely that this model would be slightly worse (with an AUC=0.818) at predicting the probability that an adult living in the U.S. supports the president's foreign policy **for new data**.

