Problem 1: Count the Surjections[1]

Source filename: `surjections.(cpp|java)`
Input filename: `surjections.in`
Output filename: `surjections.out`

Two Definitions: In mathematics a *function*, *f*, from set *A* to set *B* assigns to each element *a* in *A* exactly one element *b* in *B*. (In general, two different elements in *A* may or may not be assigned to different elements in *B*.)

A *surjection* is a function *f* from a set *A* to a set *B* with the additional property that for each element *b* in *B*, there is some element *a* in *A* with $f(a) = b$. (Surjections are sometimes referred to as "onto" functions.)

Let $S(m,n)$ denote the number of surjections from a set of size *m* into a set of size *n*. **The problem is to compute the formula, $S(m,n)$, given values for *m* and *n*.** A small table of values of $S(m,n)$ is given below:

m	n	S(m,n)
3	1	1
3	2	6
3	3	6
3	4	0
4	3	36
4	4	24
5	4	240

For this problem, you are to write a program, named **`surjections.cpp`**, which reads pairs of integers, *m* and *n*, from a file named **`surjections.in`**, and outputs the value of $S(m,n)$ to the **`surjections.out`**.

Input File (`surjections.in`)
The input file contains several test cases, one per line. Each test case contains two positive integers, *m* and *n*, separated by a single space. The values for *m* and *n* will not exceed 16. The line after the last test case contains two zeroes, separated by a single space, which indicates the end of the input file.

Output File (`surjections.out`)
For each positive pair of integers, *m* and *n*, calculate and print to the output file the number of surjections that can be defined from a set of size *m* to a set of size *n*.

Use the format: "`S(m,n) = s`", where m and n are the two integers read from the file and *s* is the number of surjections. There should be 1 space on either side of the equal sign, but no other spaces should appear in the output.

A formula for calculating $S(m,n)$ may be found on the next page.

Example Input

```
3 1
3 2
3 3
3 4
4 3
4 4
5 4
0 0
```

Corresponding Output

```
S(3,1) = 1
S(3,2) = 6
S(3,3) = 6
S(3,4) = 0
S(4,3) = 36
S(4,4) = 24
S(5,4) = 240
```

[1] A form of this problem first appeared at the ACM South Central Regional Scholastic Programming Contest in 1984.

Given integers, m,n > 0, $S(m,n)$ is given by the following formula:

$$S(m,n) = \begin{cases} 1 & if\ n = 1 \\ 0 & if\ m < n \\ m! & if\ m = n \\ n^m - \sum_{i=1}^{n-1} \left\{ \binom{n}{i} * S(m,i) \right\} & if\ n > 1\ and\ m > n \end{cases}$$

In the above formula, *m*! represents "*m* factorial".

The symbol $\binom{n}{i}$ represents the combination of *n* things taken *i* at a time (where $0 \le i \le n$). One way to calculate the value of $\binom{n}{i}$ is to use the recursive formula:

$$\binom{n}{i} = \begin{cases} 1, & if\ i = 0\ or\ n = 0\ or\ i = n \\ \binom{n-1}{i-1} + \binom{n-1}{i}, & if\ 0 < i < n \end{cases}$$