Simultaneous Dense Coding

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We present a dense coding scheme between one sender and two receivers, which guarantees that the receivers simultaneously obtain their respective messages. In our scheme, the quantum entanglement channel is first locked by the sender so that the receivers cannot learn their messages unless they collaborate to perform the unlocking operation. We also show that the quantum Fourier transform can act as the locking operator both in simultaneous dense coding and teleportation.

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I. INTRODUCTION

Quantum entanglement [1] is the key resource of quantum information theory [2, 3], especially in quantum communication [4]. Sharing an entangled quantum state between a sender and a receiver, makes it possible to perform quantum teleportation [5] and quantum dense coding [6]. Quantum teleportation is the process of transmitting an unknown quantum state by using shared entanglement and sending classical information; quantum dense coding is the process of transmitting 2 bits of classical information by sending part of an entangled state. Teleportation and dense coding are closely related [7, 8] and have been extensively studied in various ways. For example, teleportation and dense coding that use non-maximally entangled quantum channel have been examined [8, 9, 10, 11, 12, 13, 14, 15, 16, 17]; multipartite entangled states have also been considered as the quantum channel [18, 19, 20, 21, 22, 23, 24, 25, 26]; another generalization is to perform these two communication tasks under the control of a third party, so called controlled teleportation and dense coding [27, 28, 29, 30, 31, 32].

Recently, a simultaneous quantum state teleportation scheme was proposed by Wang et al [33], the aim of which is for all the receivers to simultaneously obtain their respective quantum states from Alice (the sender). In their scheme, Alice first performs a unitary transform to lock the entanglement channel, and therefore the receivers cannot restore their quantum states separately before performing an unlocking operation together. A natural question is that whether this idea of locking the entanglement channel adapts for dense coding? The main purpose of this paper is to show that such a locking operator for dense coding really exists. As a result, we propose three simultaneous dense coding protocols which guarantee that the receivers simultaneously obtain their respective messages.

The remainder of the paper is organized as follows. In Sec. II, we introduce three simultaneous dense coding protocols using different entanglement channels. In Sec. III, we show that the quantum Fourier transform can alternatively be used as the locking operator in simultaneous teleportation. A brief conclusion follows in Sec. IV.

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II. PROTOCOLS FOR SIMULTANEOUS DENSE CODING

Suppose that Alice is the sender, Bob and Charlie are the receivers. Alice intends to send two bits (b_1, b_2) to Bob and another two bits (c_1, c_2) to Charlie under the condition that Bob and Charlie must collaborate to simultaneously find out what she sends.

In the following three subsections, we propose three protocols using Bell state, GHZ state and W state as the entanglement channels respectively. The idea of these protocols is to perform the quantum Fourier transform on Alice's qubits before sending them to Bob and Charlie. After receiving Alice's qubits, Bob and Charlie's local states are independent of (b_1, b_2) and (c_1, c_2) so that they know nothing about the encoded bits. Only after performing the inverse quantum Fourier transform together, they can obtain (b_1, b_2) and (c_1, c_2) respectively.

A. Protocol 1: Using Bell State

Initially, Alice, Bob and Charlie share two Einstein-Podolsky-Rosen(EPR) pairs [34] $\frac{1}{\sqrt{2}}(|00\rangle + |11\rangle)_{A_1B}$ and $\frac{1}{\sqrt{2}}(|00\rangle + |11\rangle)_{A_2C}$, where qubits A_1A_2 belong to Alice, qubits B and C belong to Bob and Charlie respectively. The initial quantum state of the composite system is

$$|\psi(0)\rangle = \frac{1}{\sqrt{2}}(|00\rangle + |11\rangle)_{A_1B} \otimes \frac{1}{\sqrt{2}}(|00\rangle + |11\rangle)_{A_2C}. \tag{1}$$

The protocol consists of four steps.

(1) Alice performs unitary transforms $U(b_1b_2)$ on qubits A_1 and $U(c_1c_2)$ on A_2 to encode her bits, like the original dense coding scheme [6]. After that, the state of

the composite system becomes

$$\begin{aligned}|\psi(1)\rangle &= U_{A_1}(b_1b_2)\otimes U_{A_2}(c_1c_2)|\psi(0)\rangle \\ &= |\phi(b_1b_2)\rangle_{A_1B}\otimes|\phi(c_1c_2)\rangle_{A_2C},\end{aligned} \tag{2}$$

where

$$\begin{aligned}&U(00)=I=\begin{bmatrix}1&0\\0&1\end{bmatrix}, U(01)=\sigma_z=\begin{bmatrix}1&0\\0&-1\end{bmatrix},\\ &U(10)=\sigma_x=\begin{bmatrix}0&1\\1&0\end{bmatrix}, U(11)=\sigma_z\sigma_x=\begin{bmatrix}0&1\\-1&0\end{bmatrix},\\ &|\phi(xy)\rangle=\frac{1}{\sqrt{2}}(|0x\rangle+(-1)^y|1\overline{x}\rangle).\end{aligned} \tag{3}$$

(2) Alice performs the quantum Fourier transform

$$QFT=\frac{1}{2}\begin{bmatrix}1&1&1&1\\1&i&-1&-i\\1&-1&1&-1\\1&-i&-1&i\end{bmatrix} \tag{4}$$

on qubits A_1A_2 to lock the entanglement channel, and then sends A_1 to Bob and A_2 to Charlie. The state of the composite system becomes

$$|\psi(2)\rangle = QFT_{A_1A_2}[|\phi(b_1b_2)\rangle_{A_1B}\otimes|\phi(c_1c_2)\rangle_{A_2C}]. \tag{5}$$

(3) Bob and Charlie collaborate to perform $QFT^\dagger$ on qubits A_1A_2. The state of the composite system becomes

$$\begin{aligned}|\psi(3)\rangle =&QFT^\dagger_{A_1A_2}QFT_{A_1A_2}[|\phi(b_1b_2)\rangle_{A_1B}\otimes|\phi(c_1c_2)\rangle_{A_2C}]\\ =&|\phi(b_1b_2)\rangle_{A_1B}\otimes|\phi(c_1c_2)\rangle_{A_2C}.\end{aligned} \tag{6}$$

(4) Bob and Charlie perform the Bell State Measurement on qubits A_1B and A_2C respectively to obtain (b_1,b_2) and (c_1,c_2), like the original dense coding scheme [6].

The following theorem demonstrates that neither Bob nor Charlie alone can distinguish his two-qubit quantum state (i.e. ρ_{A_1B},ρ_{A_2C}) before step 3. Therefore, they cannot learn the encoded bits from their quantum states unless they collaborate.

Theorem 1. For each $b_1,b_2,c_1,c_2\in\{0,1\}$, $\rho_{A_1B}=\rho_{A_2C}=I/4$, where ρ_{A_1B} and ρ_{A_2C} are the reduced density matrices in subsystems A_1B and A_2C after step 2 (but before step 3).

Proof. After step 1, the quantum states of qubits A_1B and A_2C are $|\phi(b_1b_2)\rangle$ and $|\phi(c_1c_2)\rangle$ respectively. The state of the composite system after step 1 can be written as

$$|\psi(1)\rangle = |\phi(b_1b_2)\rangle_{A_1B}\otimes|\phi(c_1c_2)\rangle_{A_2C}. \tag{7}$$

After step 2, the state of the composite system becomes

$$\begin{aligned}|\psi(2)\rangle =&QFT_{A_1A_2}[\frac{1}{\sqrt{2}}(|0b_1\rangle+(-1)^{b_2}|1\overline{b_1}\rangle)_{A_1B}\\ &\otimes\frac{1}{\sqrt{2}}(|0c_1\rangle+(-1)^{c_2}|1\overline{c_1}\rangle)_{A_2C}]\\ =&\frac{1}{2}QFT_{A_1A_2}(|00b_1c_1\rangle+(-1)^{c_2}|01b_1\overline{c_1}\rangle\\ &+(-1)^{b_2}|10\overline{b_1}c_1\rangle+(-1)^{b_2+c_2}|11\overline{b_1c_1}\rangle)_{A_1A_2BC}\\ =&\frac{1}{4}[(|00\rangle+|01\rangle+|10\rangle+|11\rangle)|b_1c_1\rangle+(-1)^{c_2}(|00\rangle\\ &+i|01\rangle-|10\rangle-i|11\rangle)|b_1\overline{c_1}\rangle+(-1)^{b_2}(|00\rangle\\ &-|01\rangle+|10\rangle-|11\rangle)|\overline{b_1}c_1\rangle+(-1)^{b_2+c_2}(|00\rangle\\ &-i|01\rangle-|10\rangle+i|11\rangle)|\overline{b_1c_1}\rangle]_{A_1A_2BC}.\end{aligned} \tag{8}$$

The reduced density matrix in subsystem A_1B is

$$\begin{aligned}\rho_{A_1B}=&{}_{A_2C}\langle 0c_1|\psi(2)\rangle\langle\psi(2)|0c_1\rangle_{A_2C}+{}_{A_2C}\langle 0\overline{c_1}|\psi(2)\rangle\\ &\langle\psi(2)|0\overline{c_1}\rangle_{A_2C}+{}_{A_2C}\langle 1c_1|\psi(2)\rangle\langle\psi(2)|1c_1\rangle_{A_2C}\\ &+{}_{A_2C}\langle 1\overline{c_1}|\psi(2)\rangle\langle\psi(2)|1\overline{c_1}\rangle_{A_2C}\\ =&\frac{1}{4}(|0b_1\rangle\langle 0b_1|+|0\overline{b_1}\rangle\langle 0\overline{b_1}|+|1b_1\rangle\langle 1b_1|+|1\overline{b_1}\rangle\langle 1\overline{b_1}|)\\ =&I/4.\end{aligned} \tag{9}$$

The reduced density matrix in subsystem A_2C is

$$\begin{aligned}\rho_{A_2C}=&{}_{A_1B}\langle 0b_1|\psi(2)\rangle\langle\psi(2)|0b_1\rangle_{A_1B}+{}_{A_1B}\langle 0\overline{b_1}|\psi(2)\rangle\\ &\langle\psi(2)|0\overline{b_1}\rangle_{A_1B}+{}_{A_1B}\langle 1b_1|\psi(2)\rangle\langle\psi(2)|1b_1\rangle_{A_1B}\\ &+{}_{A_1B}\langle 1\overline{b_1}|\psi(2)\rangle\langle\psi(2)|1\overline{b_1}\rangle_{A_1B}\\ =&\frac{1}{4}(|0c_1\rangle\langle 0c_1|+|0\overline{c_1}\rangle\langle 0\overline{c_1}|+|1c_1\rangle\langle 1c_1|+|1\overline{c_1}\rangle\langle 1\overline{c_1}|)\\ =&I/4.\end{aligned} \tag{10}$$

□

B. Protocol 2: Using GHZ State

Initially, Alice, Bob and Charlie share two Greenberger-Horne-Zeilinger (GHZ) states [35] $\frac{1}{\sqrt{2}}(|000\rangle+|111\rangle)_{A_1B_1B_2}$ and $\frac{1}{\sqrt{2}}(|000\rangle+|111\rangle)_{A_2C_1C_2}$, where qubits A_1A_2 belong to Alice, qubits B_1B_2 and C_1C_2 belong to Bob and Charlie, respectively. The initial quantum state of the composite system is

$$\begin{aligned}|\psi(0)\rangle =&\frac{1}{\sqrt{2}}(|000\rangle+|111\rangle)_{A_1B_1B_2}\\ &\otimes\frac{1}{\sqrt{2}}(|000\rangle+|111\rangle)_{A_2C_1C_2}.\end{aligned} \tag{11}$$

The protocol consists of four steps.

(1) Alice performs unitary transforms $U(b_1b_2)$ on qubits A_1 and $U(c_1c_2)$ on A_2 to encode her bits. After that, the state of the composite system becomes

$$\begin{aligned}|\psi(1)\rangle =&U_{A_1}(b_1b_2)\otimes U_{A_2}(c_1c_2)|\psi(0)\rangle\\ =&|GHZ(b_1b_2)\rangle_{A_1B_1B_2}\otimes|GHZ(c_1c_2)\rangle_{A_2C_1C_2},\end{aligned} \tag{12}$$

where

$$|GHZ(xy)\rangle = \frac{1}{\sqrt{2}}(|0xx\rangle + (-1)^y|1\overline{xx}\rangle). \tag{13}$$

(2) Alice performs the quantum Fourier transform on qubits A_1A_2 to lock the entanglement channel, and then sends A_1 to Bob and A_2 to Charlie. The state of the composite system becomes

$$\begin{aligned}|\psi(2)\rangle =& QFT_{A_1A_2}[|GHZ(b_1b_2)\rangle_{A_1B_1B_2} \\ &\otimes |GHZ(c_1c_2)\rangle_{A_2C_1C_2}].\end{aligned} \tag{14}$$

(3) Bob and Charlie collaborate to perform $QFT^\dagger$ on qubits A_1A_2. The state of the composite system becomes

$$\begin{aligned}|\psi(3)\rangle =& QFT^\dagger_{A_1A_2} QFT_{A_1A_2}[|GHZ(b_1b_2)\rangle_{A_1B_1B_2} \\ &\otimes |GHZ(c_1c_2)\rangle_{A_2C_1C_2}] \\ =& |GHZ(b_1b_2)\rangle_{A_1B_1B_2} \otimes |GHZ(c_1c_2)\rangle_{A_2C_1C_2}.\end{aligned} \tag{15}$$

(4) Bob and Charlie make the von Neumann measurement using the orthogonal states $\{|GHZ(xy)\rangle\}_{xy}$ on qubits $A_1B_1B_2$ and $A_2C_1C_2$ respectively to obtain (b_1, b_2) and (c_1, c_2).

The following theorem demonstrates that neither Bob nor Charlie alone can distinguish his three-qubit quantum state (i.e. $\rho_{A_1B_1B_2}, \rho_{A_2C_1C_2}$) before step 3. Therefore, they cannot learn the encoded bits from their quantum states unless they collaborate.

Theorem 2. $\rho_{A_1B_1B_2}$ and $\rho_{A_2C_1C_2}$ are independent of b_1, b_2, c_1, c_2, where $\rho_{A_1B_1B_2}$ and $\rho_{A_2C_1C_2}$ are the reduced density matrices in subsystems $A_1B_1B_2$ and $A_2C_1C_2$ after step 2 (but before step 3), respectively.

Proof. After step 1, the quantum states of qubits $A_1B_1B_2$ and $A_2C_1C_2$ are $|GHZ(b_1b_2)\rangle$ and $|GHZ(c_1c_2)\rangle$, respectively. The state of the composite system after step 1 can be written as

$$|\psi(1)\rangle = |GHZ(b_1b_2)\rangle_{A_1B_1B_2} \otimes |GHZ(c_1c_2)\rangle_{A_2C_1C_2}. \tag{16}$$

After step 2, the state of the composite system becomes

$$\begin{aligned}|\psi(2)\rangle =& QFT_{A_1A_2}[\frac{1}{\sqrt{2}}(|0b_1b_1\rangle + (-1)^{b_2}|1\overline{b_1b_1}\rangle)_{A_1B_1B_2} \\ &\otimes \frac{1}{\sqrt{2}}(|0c_1c_1\rangle + (-1)^{c_2}|1\overline{c_1c_1}\rangle)_{A_2C_1C_2}] \\ =& \frac{1}{2}QFT_{A_1A_2}(|00\rangle \otimes |b_1b_1c_1c_1\rangle + (-1)^{c_2}|01\rangle \\ &\otimes |b_1b_1\overline{c_1c_1}\rangle + (-1)^{b_2}|10\rangle \otimes |\overline{b_1b_1}c_1c_1\rangle \\ &+ (-1)^{b_2+c_2}|11\rangle \otimes |\overline{b_1b_1c_1c_1}\rangle)_{A_1A_2B_1B_2C_1C_2} \\ =& \frac{1}{4}[(|00\rangle + |01\rangle + |10\rangle + |11\rangle) \otimes |b_1b_1c_1c_1\rangle \\ &+ (-1)^{c_2}(|00\rangle + i|01\rangle - |10\rangle - i|11\rangle) \otimes |b_1b_1\overline{c_1c_1}\rangle \\ &+ (-1)^{b_2}(|00\rangle - |01\rangle + |10\rangle - |11\rangle) \otimes |\overline{b_1b_1}c_1c_1\rangle \\ &+ (-1)^{b_2+c_2}(|00\rangle - i|01\rangle - |10\rangle + i|11\rangle) \\ &\otimes |\overline{b_1b_1c_1c_1}\rangle]_{A_1A_2B_1B_2C_1C_2}.\end{aligned} \tag{17}$$

The reduced density matrix in subsystem $A_1B_1B_2$ is

$$\begin{aligned}\rho_{A_1B_1B_2} =& {}_{A_2C_1C_2}\langle 0c_1c_1|\psi(2)\rangle\langle\psi(2)|0c_1c_1\rangle_{A_2C_1C_2} \\ &+ {}_{A_2C_1C_2}\langle 0\overline{c_1c_1}|\psi(2)\rangle\langle\psi(2)|0\overline{c_1c_1}\rangle_{A_2C_1C_2} \\ &+ {}_{A_2C_1C_2}\langle 1c_1c_1|\psi(2)\rangle\langle\psi(2)|1c_1c_1\rangle_{A_2C_1C_2} \\ &+ {}_{A_2C_1C_2}\langle 1\overline{c_1c_1}|\psi(2)\rangle\langle\psi(2)|1\overline{c_1c_1}\rangle_{A_2C_1C_2} \\ =& \frac{1}{4}(|0b_1b_1\rangle\langle 0b_1b_1| + |0\overline{b_1b_1}\rangle\langle 0\overline{b_1b_1}| \\ &+ |1b_1b_1\rangle\langle 1b_1b_1| + |1\overline{b_1b_1}\rangle\langle 1\overline{b_1b_1}|) \\ =& \frac{1}{4}(|000\rangle\langle 000| + |011\rangle\langle 011| + |100\rangle\langle 100| \\ &+ |111\rangle\langle 111|).\end{aligned} \tag{18}$$

The reduced density matrix in subsystem $A_2C_1C_2$ is

$$\begin{aligned}\rho_{A_2C_1C_2} =& {}_{A_1B_1B_2}\langle 0b_1b_1|\psi(2)\rangle\langle\psi(2)|0b_1b_1\rangle_{A_1B_1B_2} \\ &+ {}_{A_1B_1B_2}\langle 0\overline{b_1b_1}|\psi(2)\rangle\langle\psi(2)|0\overline{b_1b_1}\rangle_{A_1B_1B_2} \\ &+ {}_{A_1B_1B_2}\langle 1b_1b_1|\psi(2)\rangle\langle\psi(2)|1b_1b_1\rangle_{A_1B_1B_2} \\ &+ {}_{A_1B_1B_2}\langle 1\overline{b_1b_1}|\psi(2)\rangle\langle\psi(2)|1\overline{b_1b_1}\rangle_{A_1B_1B_2} \\ =& \frac{1}{4}(|0c_1c_1\rangle\langle 0c_1c_1| + |0\overline{c_1c_1}\rangle\langle 0\overline{c_1c_1}| \\ &+ |1c_1c_1\rangle\langle 1c_1c_1| + |1\overline{c_1c_1}\rangle\langle 1\overline{c_1c_1}|) \\ =& \frac{1}{4}(|000\rangle\langle 000| + |011\rangle\langle 011| + |100\rangle\langle 100| \\ &+ |111\rangle\langle 111|).\end{aligned} \tag{19}$$

□

C. Protocol 3: Using W State

Initially, Alice, Bob and Charlie share two W states [25, 36] $\frac{1}{2}(|010\rangle + |001\rangle + \sqrt{2}|100\rangle)_{A_1B_1B_2}$ and $\frac{1}{2}(|010\rangle + |001\rangle + \sqrt{2}|100\rangle)_{A_2C_1C_2}$, where qubits A_1A_2 belong to Alice, qubits B_1B_2 and C_1C_2 belong to Bob and Charlie, respectively. The initial quantum state of the composite system is

$$\begin{aligned}|\psi(0)\rangle =& \frac{1}{2}(|010\rangle + |001\rangle + \sqrt{2}|100\rangle)_{A_1B_1B_2} \\ &\otimes \frac{1}{2}(|010\rangle + |001\rangle + \sqrt{2}|100\rangle)_{A_2C_1C_2}.\end{aligned} \tag{20}$$

The protocol consists of four steps.

(1) Alice performs unitary transforms $U(b_1b_2)$ on qubits A_1 and $U(c_1c_2)$ on A_2 to encode her bits. After that, the state of the composite system becomes

$$\begin{aligned}|\psi(1)\rangle =& U_{A_1}(b_1b_2) \otimes U_{A_2}(c_1c_2)|\psi(0)\rangle \\ =& |W(b_1b_2)\rangle_{A_1B_1B_2} \otimes |W(c_1c_2)\rangle_{A_2C_1C_2},\end{aligned} \tag{21}$$

where

$$|W(xy)\rangle = \frac{1}{2}(|x10\rangle + |x01\rangle + (-1)^y\sqrt{2}|\overline{x}00\rangle). \tag{22}$$

(2) Alice performs the quantum Fourier transform on qubits A_1A_2 to lock the entanglement channel, and then sends A_1 to Bob and A_2 to Charlie. The state of the composite system becomes

$$\begin{aligned}|\psi(2)\rangle =&QFT_{A_1A_2}[|W(b_1b_2)\rangle_{A_1B_1B_2}\\ &\otimes|W(c_1c_2)\rangle_{A_2C_1C_2}].\end{aligned} \tag{23}$$

(3) Bob and Charlie collaborate to perform $QFT^\dagger$ on qubits A_1A_2. The state of the composite system becomes

$$\begin{aligned}|\psi(3)\rangle =&QFT^\dagger_{A_1A_2}QFT_{A_1A_2}[|W(b_1b_2)\rangle_{A_1B_1B_2}\\ &\otimes|W(c_1c_2)\rangle_{A_2C_1C_2}]\\ =&|W(b_1b_2)\rangle_{A_1B_1B_2}\otimes|W(c_1c_2)\rangle_{A_2C_1C_2}.\end{aligned} \tag{24}$$

(4) Bob and Charlie make the von Neumann measurement using the orthogonal states $\{|W(xy)\rangle\}_{xy}$ on qubits $A_1B_1B_2$ and $A_2C_1C_2$ respectively to obtain (b_1,b_2) and (c_1,c_2).

The following theorem demonstrates that neither Bob nor Charlie alone can distinguish his three-qubit quantum state (i.e. $\rho_{A_1B_1B_2},\rho_{A_2C_1C_2}$) before step 3. Therefore, they cannot learn the encoded bits from their quantum states unless they collaborate.

Theorem 3. $\rho_{A_1B_1B_2}$ and $\rho_{A_2C_1C_2}$ are independent of b_1,b_2,c_1,c_2, where $\rho_{A_1B_1B_2}$ and $\rho_{A_2C_1C_2}$ are the reduced density matrices in subsystems $A_1B_1B_2$ and $A_2C_1C_2$ after step 2 (but before step 3), respectively.

Proof. After step 1, the quantum states of qubits $A_1B_1B_2$ and $A_2C_1C_2$ are $|W(b_1b_2)\rangle$ and $|W(c_1c_2)\rangle$ respectively. The state of the composite system after step 1 can be written as

$$|\psi(1)\rangle = |W(b_1b_2)\rangle_{A_1B_1B_2}\otimes|W(c_1c_2)\rangle_{A_2C_1C_2}. \tag{25}$$

After step 2, the state of the composite system becomes

$$\begin{aligned}|\psi(2)\rangle =&QFT_{A_1A_2}\{\frac{1}{2}[|b_1\rangle(|01\rangle+|10\rangle)\\ &+(-1)^{b_2}\sqrt{2}|\overline{b_1}00\rangle]_{A_1B_1B_2}\otimes\frac{1}{2}[|c_1\rangle(|01\rangle+|10\rangle)\\ &+(-1)^{c_2}\sqrt{2}|\overline{c_1}00\rangle]_{A_2C_1C_2}\}\\ =&\frac{1}{4}QFT_{A_1A_2}[|b_1c_1\rangle\otimes(|01\rangle+|10\rangle)\otimes(|01\rangle+|10\rangle)\\ &+|b_1\overline{c_1}\rangle\otimes(-1)^{c_2}\sqrt{2}(|01\rangle+|10\rangle)\otimes|00\rangle+|\overline{b_1}c_1\rangle\\ &\otimes(-1)^{b_2}\sqrt{2}|00\rangle\otimes(|01\rangle+|10\rangle)+|\overline{b_1c_1}\rangle\\ &\otimes(-1)^{b_2+c_2}2|00\rangle\otimes|00\rangle]_{A_1A_2B_1B_2C_1C_2}.\end{aligned} \tag{26}$$

We notice that $QFT|xy\rangle=\frac{1}{2}[|00\rangle+(-1)^x i^y|01\rangle+(-1)^y|10\rangle+(-1)^x(-i)^y|11\rangle]$, and thus

$$\begin{aligned}|\psi(2)\rangle =&\frac{1}{8}\{[|00\rangle+(-1)^{b_1}i^{c_1}|01\rangle+(-1)^{c_1}|10\rangle+(-1)^{b_1}\\ &(-i)^{c_1}|11\rangle]\otimes(|01\rangle+|10\rangle)\otimes(|01\rangle+|10\rangle)\\ &+[|00\rangle+(-1)^{b_1}i^{\overline{c_1}}|01\rangle-(-1)^{c_1}|10\rangle+(-1)^{b_1}\\ &(-i)^{\overline{c_1}}|11\rangle]\otimes(-1)^{c_2}\sqrt{2}(|01\rangle+|10\rangle)\otimes|00\rangle\\ &+[|00\rangle-(-1)^{b_1}i^{c_1}|01\rangle+(-1)^{c_1}|10\rangle-(-1)^{b_1}\\ &(-i)^{c_1}|11\rangle]\otimes(-1)^{b_2}\sqrt{2}|00\rangle\otimes(|01\rangle+|10\rangle)\\ &+[|00\rangle-(-1)^{b_1}i^{\overline{c_1}}|01\rangle-(-1)^{c_1}|10\rangle-(-1)^{b_1}\\ &(-i)^{\overline{c_1}}|11\rangle]\otimes(-1)^{b_2+c_2}2|00\rangle|00\rangle\}_{A_1A_2B_1B_2C_1C_2}.\end{aligned} \tag{27}$$

The reduced density matrix in subsystem $A_1B_1B_2$ is

$$\begin{aligned}\rho_{A_1B_1B_2} =&{}_{A_2C_1C_2}\langle 000|\psi(2)\rangle\langle\psi(2)|000\rangle_{A_2C_1C_2}\\ &+_{A_2C_1C_2}\langle 100|\psi(2)\rangle\langle\psi(2)|100\rangle_{A_2C_1C_2}\\ &+_{A_2C_1C_2}\langle 001|\psi(2)\rangle\langle\psi(2)|001\rangle_{A_2C_1C_2}\\ &+_{A_2C_1C_2}\langle 101|\psi(2)\rangle\langle\psi(2)|101\rangle_{A_2C_1C_2}\\ &+_{A_2C_1C_2}\langle 010|\psi(2)\rangle\langle\psi(2)|010\rangle_{A_2C_1C_2}\\ &+_{A_2C_1C_2}\langle 110|\psi(2)\rangle\langle\psi(2)|110\rangle_{A_2C_1C_2}\\ =&\frac{1}{8}(2|000\rangle\langle 000|+|001\rangle\langle 001|+|001\rangle\langle 010|\\ &+|010\rangle\langle 001|+|010\rangle\langle 010|+2|100\rangle\langle 100|\\ &+|101\rangle\langle 101|+|101\rangle\langle 110|+|110\rangle\langle 101|\\ &+|110\rangle\langle 110|).\end{aligned} \tag{28}$$

The reduced density matrix in subsystem $A_2C_1C_2$ is

$$\begin{aligned}\rho_{A_2C_1C_2} =&{}_{A_1B_1B_2}\langle 000|\psi(2)\rangle\langle\psi(2)|000\rangle_{A_1B_1B_2}\\ &+_{A_1B_1B_2}\langle 100|\psi(2)\rangle\langle\psi(2)|100\rangle_{A_1B_1B_2}\\ &+_{A_1B_1B_2}\langle 001|\psi(2)\rangle\langle\psi(2)|001\rangle_{A_1B_1B_2}\\ &+_{A_1B_1B_2}\langle 101|\psi(2)\rangle\langle\psi(2)|101\rangle_{A_1B_1B_2}\\ &+_{A_1B_1B_2}\langle 010|\psi(2)\rangle\langle\psi(2)|010\rangle_{A_1B_1B_2}\\ &+_{A_1B_1B_2}\langle 110|\psi(2)\rangle\langle\psi(2)|110\rangle_{A_1B_1B_2}\\ =&\frac{1}{8}(2|000\rangle\langle 000|+|001\rangle\langle 001|+|001\rangle\langle 010|\\ &+|010\rangle\langle 001|+|010\rangle\langle 010|+2|100\rangle\langle 100|\\ &+|101\rangle\langle 101|+|101\rangle\langle 110|+|110\rangle\langle 101|\\ &+|110\rangle\langle 110|).\end{aligned} \tag{29}$$

□

D. Locking Operator

We notice that the locking operator used in simultaneous teleportation [33] is not suitable for simultaneous dense coding. To explain the reason, we calculate the reduced density matrix in subsystem A_1B when that locking operator is used, instead of the quantum Fourier transform and Bell state being used as the entanglement channel. The situations of using GHZ and W states as entanglement channels are similar.

The locking operator used in simultaneous teleportation [33] is

$$U(LOCK)_{12} = H_1 CNOT_{12} = \frac{1}{\sqrt{2}} \begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & 1 & 0 \\ 1 & 0 & 0 & -1 \\ 0 & 1 & -1 & 0 \end{bmatrix}, \tag{30}$$

where H is the Hadamard transform, $CNOT$ is the controlled-NOT gate, qubit 1 is the control qubit and qubit 2 the target qubit. After step 1, the state of the composite system can be written as

$$|\psi'(1)\rangle = |\phi(b_1b_2)\rangle_{A_1B} \otimes |\phi(c_1c_2)\rangle_{A_2C}. \tag{31}$$

After step 2, the state of the composite system becomes

$$\begin{aligned} |\psi'(2)\rangle =& U(LOCK)_{A_1A_2}[\frac{1}{\sqrt{2}}(|0b_1\rangle + (-1)^{b_2}|1\overline{b_1}\rangle)_{A_1B} \\ & \otimes \frac{1}{\sqrt{2}}(|0c_1\rangle + (-1)^{c_2}|1\overline{c_1}\rangle)_{A_2C}] \\ =& \frac{1}{2}U(LOCK)_{A_1A_2}(|00b_1c_1\rangle + (-1)^{c_2}|01b_1\overline{c_1}\rangle \\ & + (-1)^{b_2}|10\overline{b_1}c_1\rangle + (-1)^{b_2+c_2}|11\overline{b_1c_1}\rangle)_{A_1A_2BC} \\ =& \frac{1}{2\sqrt{2}}[(|00\rangle + |10\rangle)|b_1c_1\rangle + (-1)^{c_2}(|01\rangle + |11\rangle) \\ & |b_1\overline{c_1}\rangle + (-1)^{b_2}(|01\rangle - |11\rangle)|\overline{b_1}c_1\rangle + (-1)^{b_2+c_2} \\ & (|00\rangle - |10\rangle)|\overline{b_1c_1}\rangle]_{A_1A_2BC}. \end{aligned} \tag{32}$$

The reduced density matrix in subsystem A_1B is

$$\begin{aligned} \rho'_{A_1B} =& {}_{A_2C}\langle 0c_1|\psi'(2)\rangle\langle\psi'(2)|0c_1\rangle_{A_2C} + {}_{A_2C}\langle 0\overline{c_1}|\psi'(2)\rangle \\ & \langle\psi'(2)|0\overline{c_1}\rangle_{A_2C} + {}_{A_2C}\langle 1c_1|\psi'(2)\rangle\langle\psi'(2)|1c_1\rangle_{A_2C} \\ & + {}_{A_2C}\langle 1\overline{c_1}|\psi'(2)\rangle\langle\psi'(2)|1\overline{c_1}\rangle_{A_2C} \\ =& \frac{1}{4}(|0b_1\rangle\langle 0b_1| + |0b_1\rangle\langle 1b_1| + |0\overline{b_1}\rangle\langle 0\overline{b_1}| \\ & - |0\overline{b_1}\rangle\langle 1\overline{b_1}| + |1b_1\rangle\langle 0b_1| + |1b_1\rangle\langle 1b_1| \\ & - |1\overline{b_1}\rangle\langle 0\overline{b_1}| + |1\overline{b_1}\rangle\langle 1\overline{b_1}|). \end{aligned} \tag{33}$$

Since ρ'_{A_1B} is only dependent on b_1, we denote it as $\rho'_{A_1B}(b_1)$. We have

$$\rho'_{A_1B}(0) = \frac{1}{4}\begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & -1 \\ 1 & 0 & 1 & 0 \\ 0 & -1 & 0 & 1 \end{bmatrix} \tag{34}$$

and

$$\rho'_{A_1B}(1) = \frac{1}{4}\begin{bmatrix} 1 & 0 & -1 & 0 \\ 0 & 1 & 0 & 1 \\ -1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \end{bmatrix}. \tag{35}$$

Since $\rho'_{A_1B}(0)\rho'_{A_1B}(1) = 0$, Bob can distinguish these two states and obtain b_1 by a POVM measurement on qubits A_1B. Similarly, Charlie can also obtain c_2 by a POVM measurement on qubits A_2C. Each receiver can learn 1 bit of his information before they agree to simultaneously find out what Alice sends. The aim of simultaneous dense coding is not achieved when $U(LOCK)$ is used instead of the quantum Fourier transform.

III. SIMULTANEOUS TELEPORTATION USING QUANTUM FOURIER TRANSFORM

In this section, we show that the quantum Fourier transform can alternatively be used as the locking operator in simultaneous teleportation. Let us begin with a brief review of simultaneous teleportation between one sender and two receivers [33]. Suppose that Alice intends to teleport $|\varphi_1\rangle_{T_1} = \alpha_1|0\rangle_{T_1} + \beta_1|1\rangle_{T_1}$ to Bob and $|\varphi_2\rangle_{T_2} = \alpha_2|0\rangle_{T_2} + \beta_2|1\rangle_{T_2}$ to Charlie under the condition that Bob and Charlie must collaborate to simultaneously obtain their respective quantum states. Initially, Alice, Bob and Charlie share two EPR pairs $\frac{1}{\sqrt{2}}(|00\rangle + |11\rangle)_{A_1B}$ and $\frac{1}{\sqrt{2}}(|00\rangle + |11\rangle)_{A_2C}$, where qubits A_1A_2 belong to Alice, qubits B and C belong to Bob and Charlie respectively. Then the initial quantum state of the composite system is

$$\begin{aligned} |\chi(0)\rangle =& |\varphi_1\rangle_{T_1} \otimes |\varphi_2\rangle_{T_2} \otimes \frac{1}{\sqrt{2}}(|00\rangle + |11\rangle)_{A_1B} \\ & \otimes \frac{1}{\sqrt{2}}(|00\rangle + |11\rangle)_{A_2C}. \end{aligned} \tag{36}$$

The scheme of simultaneous teleportation consists of five steps.

(1) Alice performs the unitary transform $U(LOCK)$ on qubits A_1A_2 to lock the entanglement channel. After that, the state of the composite system becomes

$$\begin{aligned} |\chi(1)\rangle =& |\varphi_1\rangle_{T_1} \otimes |\varphi_2\rangle_{T_2} \otimes U(LOCK)_{A_1A_2}[\frac{1}{\sqrt{2}}(|00\rangle \\ & + |11\rangle)_{A_1B} \otimes \frac{1}{\sqrt{2}}(|00\rangle + |11\rangle)_{A_2C}]. \end{aligned} \tag{37}$$

(2) Alice performs the Bell State Measurement on qubits A_1T_1 and A_2T_2, like the original teleportation scheme [5]. It is easy to prove that $|\chi(1)\rangle$ can be written as

$$\begin{aligned} |\chi(1)\rangle =& \frac{1}{4}\sum_{x_1=0}^{1}\sum_{y_1=0}^{1}\sum_{x_2=0}^{1}\sum_{y_2=0}^{1}|\phi(x_1y_1)\rangle_{A_1T_1}|\phi(x_2y_2)\rangle_{A_2T_2} \\ & U(LOCK)^{\dagger}_{BC}[U_B(x_1y_1)|\varphi_1\rangle_B \otimes U_C(x_2y_2)|\varphi_2\rangle_C]. \end{aligned} \tag{38}$$

If the measurement results are $|\phi(x_1y_1)\rangle_{A_1T_1}$ and $|\phi(x_2y_2)\rangle_{A_2T_2}$, the state of qubits BC collapses into

$$|\chi(2)\rangle = U(LOCK)^{\dagger}_{BC}[U_B(x_1y_1)|\varphi_1\rangle_B \otimes U_C(x_2y_2)|\varphi_2\rangle_C]. \tag{39}$$

(3) Alice sends the measurement results (x_1, y_1) to Bob and (x_2, y_2) to Charlie.

(4) Bob and Charlie collaborate to perform $U(LOCK)$ on qubits BC, and then the state of BC becomes

$$\begin{aligned}|\chi(3)\rangle =&U(LOCK)_{BC}U(LOCK)^{\dagger}_{BC}[U_B(x_1y_1)|\varphi_1\rangle_B \\ &\otimes U_C(x_2y_2)|\varphi_2\rangle_C] \\ =&U_B(x_1y_1)|\varphi_1\rangle_B \otimes U_C(x_2y_2)|\varphi_2\rangle_C. \end{aligned} \tag{40}$$

(5) Bob and Charlie perform $U(x_1y_1)$ and $U(x_2y_2)$ on qubits B and C respectively to obtain $|\varphi_1\rangle$ and $|\varphi_2\rangle$, respectively, like the original teleportation scheme [5].

In the above simultaneous teleportation scheme, $U(LOCK)$ is used to lock the entanglement channel. In Sec. II D, we have shown that $U(LOCK)$ is not suitable for simultaneous dense coding, but, however, we find that the quantum Fourier transform can alternatively be used as the locking operator in simultaneous teleportation.

Let us suppose that Alice is the sender, $\mathrm{Bob}_i(1 \leqslant i \leqslant N)$ are the receivers. Alice intends to send the unknown quantum states $|\varphi_i\rangle_{T_i} = (\alpha_i|0\rangle + \beta_i|1\rangle)_{T_i}$ to Bob_i under the condition that all the receivers must collaborate to simultaneously obtain $(\alpha_i|0\rangle + \beta_i|1\rangle)_{T_i}$. Initially, Alice and each receiver share an EPR pair $\frac{1}{\sqrt{2}}(|00\rangle + |11\rangle)_{A_iB_i}$. The initial quantum state of the composite system is

$$\begin{aligned}|\chi'(0)\rangle &= \frac{1}{\sqrt{2^N}}\bigotimes_{i=1}^{N}|\varphi_i\rangle_{T_i}\bigotimes_{i=1}^{N}(|00\rangle + |11\rangle)_{A_iB_i} \\ &= \frac{1}{\sqrt{2^N}}\bigotimes_{i=1}^{N}|\varphi_i\rangle_{T_i}\sum_{m=0}^{2^N-1}|m\rangle_{A_1\ldots A_N}|m\rangle_{B_1\ldots B_N}.\end{aligned} \tag{41}$$

The scheme of simultaneous teleportation consists of five steps.

(1) Alice performs the quantum Fourier transform $|j\rangle \rightarrow \frac{1}{\sqrt{2^N}}\sum_{k=0}^{2^N-1} e^{2\pi ijk/2^N}|k\rangle$ on qubits $A_1 \ldots A_N$ to lock the entanglement channel. After that, the state of the composite system becomes

$$\begin{aligned}|\chi'(1)\rangle &= QFT_{A_1\ldots A_N}|\chi'(0)\rangle \\ &= \frac{1}{2^N}\bigotimes_{i=1}^{N}|\varphi_i\rangle_{T_i}\sum_{m=0}^{2^N-1}\sum_{k=0}^{2^N-1}\omega^{mk}|k\rangle_{A_1\ldots A_N}|m\rangle_{B_1\ldots B_N} \\ &= \frac{1}{2^N}\sum_{k=0}^{2^N-1}\sum_{m=0}^{2^N-1}\omega^{mk}\bigotimes_{i=1}^{N}(|k_i\rangle_{A_i}|\varphi_i\rangle_{T_i})|m\rangle_{B_1\ldots B_N},\end{aligned} \tag{42}$$

where k_i is the ith bit of k, $\omega = e^{2\pi i/2^N}$.

(2) Alice performs the Bell State Measurement on each pair of A_iT_i.

We have

$$\begin{aligned}\bigotimes_{i=1}^{N} I_{A_iT_i} =&\bigotimes_{i=1}^{N}\sum_{x_i=0}^{1}\sum_{y_i=0}^{1}|\phi(x_iy_i)\rangle_{A_iT_i}\ {}_{A_iT_i}\langle\phi(x_iy_i)| \\ =&\sum_{x_1=0}^{1}\sum_{y_1=0}^{1}\cdots\sum_{x_N=0}^{1}\sum_{y_N=0}^{1}\bigotimes_{i=1}^{N}|\phi(x_iy_i)\rangle_{A_iT_i} \\ &{}_{A_iT_i}\langle\phi(x_iy_i)| \\ =&\sum_{x_1=0}^{1}\sum_{y_1=0}^{1}\cdots\sum_{x_N=0}^{1}\sum_{y_N=0}^{1}\bigotimes_{i=1}^{N}|\phi(x_iy_i)\rangle_{A_iT_i} \\ &\bigotimes_{i=1}^{N}\ {}_{A_iT_i}\langle\phi(x_iy_i)|\end{aligned} \tag{43}$$

and

$$\begin{aligned}&\bigotimes_{i=1}^{N}\ {}_{A_iT_i}\langle\phi(x_iy_i)|\chi'(1)\rangle \\ =&\frac{1}{\sqrt{2^N}}\sum_{k=0}^{2^N-1}\bigotimes_{i=1}^{N}\ {}_{A_iT_i}(\langle 0x_i| + (-1)^{y_i}\langle 1\overline{x_i}|)(\alpha_i|k_i0\rangle \\ &+ \beta_i|k_i1\rangle)_{A_iT_i}\frac{1}{\sqrt{2^N}}\sum_{m=0}^{2^N-1}\omega^{mk}|m\rangle_{B_1\ldots B_N} \\ =&\frac{1}{\sqrt{2^N}}\sum_{k=0}^{2^N-1}\prod_{i=1}^{N}[\delta_{k_i0}(\delta_{x_i0}\alpha_i + \delta_{x_i1}\beta_i) + \delta_{k_i1}(-1)^{y_i} \\ &(\delta_{x_i1}\alpha_i + \delta_{x_i0}\beta_i)]QFT_{B_1\ldots B_N}|k\rangle_{B_1\ldots B_N} \\ =&\frac{1}{\sqrt{2^N}}QFT_{B_1\ldots B_N}\bigotimes_{i=1}^{N}[(\delta_{x_i0}\alpha_i + \delta_{x_i1}\beta_i)|0\rangle \\ &+ (-1)^{y_i}(\delta_{x_i0}\beta_i + \delta_{x_i1}\alpha_i)|1\rangle]_{B_i} \\ =&\frac{1}{\sqrt{2^N}}QFT_{B_1\ldots B_N}\bigotimes_{i=1}^{N}U(x_iy_i)(\alpha_i|0\rangle + \beta_i|1\rangle)_{B_i}.\end{aligned} \tag{44}$$

Thus, $|\chi'(1)\rangle$ can be written as

$$\begin{aligned}|\chi'(1)\rangle =&\bigotimes_{i=1}^{N} I_{A_iT_i}|\chi'(1)\rangle \\ =&\sum_{x_1=0}^{1}\sum_{y_1=0}^{1}\cdots\sum_{x_N=0}^{1}\sum_{y_N=0}^{1}\bigotimes_{i=1}^{N}|\phi(x_iy_i)\rangle_{A_iT_i} \\ &\bigotimes_{i=1}^{N}\ {}_{A_iT_i}\langle\phi(x_iy_i)|\chi'(1)\rangle \\ =&\frac{1}{\sqrt{2^N}}\sum_{x_1=0}^{1}\sum_{y_1=0}^{1}\cdots\sum_{x_N=0}^{1}\sum_{y_N=0}^{1}\bigotimes_{i=1}^{N}|\phi(x_iy_i)\rangle_{A_iT_i} \\ &QFT_{B_1\ldots B_N}\bigotimes_{i=1}^{N}U(x_iy_i)|\varphi_i\rangle_{B_i}.\end{aligned} \tag{45}$$

If the measurement result of qubits A_iT_i is $|\phi(x_iy_i)\rangle$, the state of qubits $B_1 \dots B_N$ collapses into

$$|\chi'(2)\rangle = QFT_{B_1 \dots B_N} \bigotimes_{i=1}^{N} U(x_i y_i)|\varphi_i\rangle_{B_i}. \quad (46)$$

(3) Alice sends the measurement result (x_i, y_i) to each Bob$_i$.

(4) All the receivers collaborate to perform $QFT^\dagger$ on qubits $B_1 \dots B_N$, the state of $B_1 \dots B_N$ becomes

$$\begin{aligned}|\chi'(3)\rangle =& QFT^\dagger_{B_1 \dots B_N} QFT_{B_1 \dots B_N} \bigotimes_{i=1}^{N} U(x_i y_i)|\varphi_i\rangle_{B_i} \\ =& \bigotimes_{i=1}^{N} U(x_i y_i)|\varphi_i\rangle_{B_i}. \end{aligned} \quad (47)$$

(5) Each Bob$_i$ performs $U(x_i y_i)$ on qubit B_i to obtain $|\varphi_i\rangle$.

IV. CONCLUSION

In summary, we have proposed a simultaneous dense coding scheme between one sender and two receivers, the aim of which is for the receivers to simultaneously obtain their respective messages. This scheme may be used in a security scenario. For example, Alice wants Bob and Charlie to simultaneously carry out two confidential commercial activities under the condition that the sensitive information of each activity is only revealed to who is in charge of that activity. We have also shown that the quantum Fourier transform, which has been implemented using cavity quantum electrodynamics (QED) [37], nuclear magnetic resonance (NMR) [38, 39, 40, 41, 42] and coupled semiconductor double quantum dot (DQD) molecules [43], can act as the locking operator both in simultaneous dense coding and teleportation.

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