09-15-Questions

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2.1 状态空间

2.1.1 T0001 连续抛硬币

Q1: with a random sequence of $0, 1$, when comes a subset $1, 0, 0$ A wins, when comes a subset $1, 1, 0$ B wins, otherwise the game keeps. The probability of A wins.

S1 solution 1

We draw the state-transfer graph, and calculate the probability A wins of each state as the initial state.

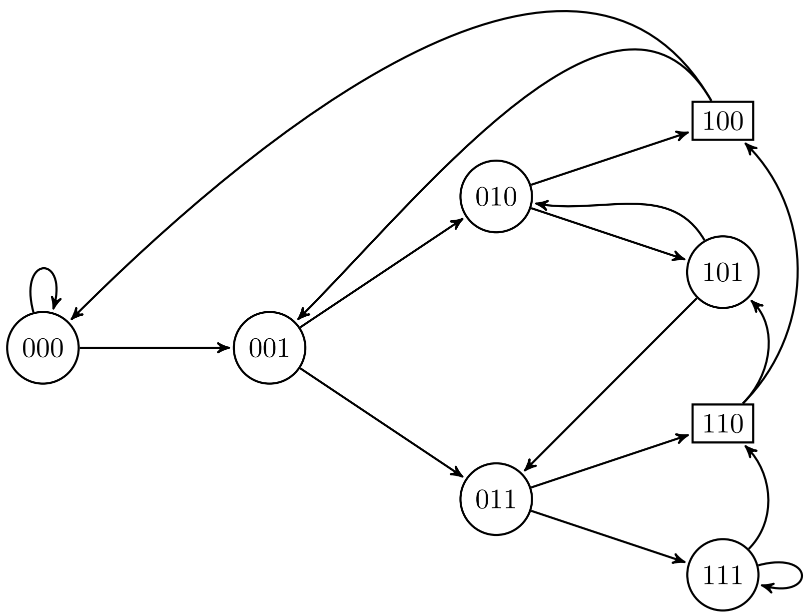

Figure 2.1: 【number of solution】

We calculate the probability of the machines stops at state 100.

$$
\begin{aligned}
P_{100}: &\quad 1 \\
P_{010}: &\quad \frac{1}{2} + \frac{1}{4} \times [\frac{1}{2} + \frac{1}{4} \times [\frac{1}{2} + \frac{1}{4} \times [...]]] \\
&= \lim_{n\to\infty} \sum_{i=0}^{i=n} \frac{1}{2} \times (\frac{1}{4})^i = a_1 \frac{1-q^n}{1-q} = \frac{2}{3} \\
P_{001}: &\quad \frac{1}{2} \times P_{010} = \frac{1}{3} \\
P_{000}: &\quad (1 - \lim_{i\to\infty} (\frac{1}{2})^i) \times P_{001} = \frac{1}{3} \\
P_{011}: &\quad 0 \\
P_{110}: &\quad 0 \\
P_{111}: &\quad 0 \\
P_{101}: &\quad \frac{1}{2} \times P_{010} = \frac{1}{3} \\
\therefore P_A = \frac{1}{8} \times (1 + \frac{2}{3} + 1) = \frac{1}{3}
\end{aligned}
\tag{2.1}
$$

S2 solution 2

we write the squence, which step has two posibilities.

$$
\begin{bmatrix} \cdots & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \cdots \\ \cdots & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & \cdots \end{bmatrix} \longrightarrow \begin{bmatrix} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \cdots \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & \cdots \end{bmatrix} \tag{2.2}
$$

It is easily to find that, when we focus to locate the sequence of $1, 0, 0$, we find that the start index of the sequence is $0, 2, 4, 6, 8, \cdots$, because the index 1(which means $1, 1, 0, 0$, and that means B wins.), we can also Analysis the index 3, 5 as the same. While the start index of the sequence is $0, 1, 2, 3, \cdots$, therefore the probability of B wins is two times of A wins.