09-15-Questions

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Question 2.1. 给定 *3* 个点，绘制圆弧 α, β, γ

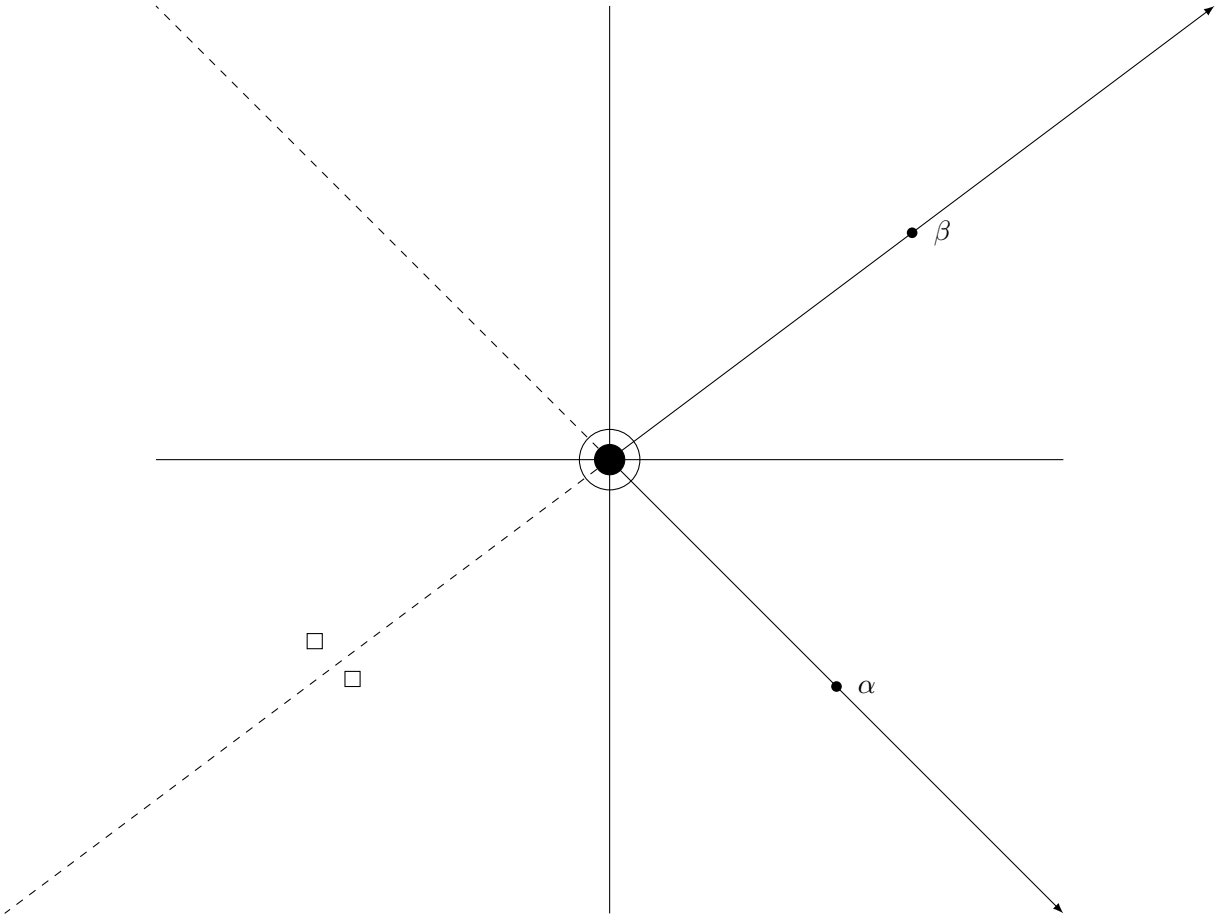

Figure 2.1: 【number of solution】

S1 *solution 1* 作为一维问题，*use algorithm1.*

S2 *solution 2*

升高一个维度，不利用极坐标表示，避免 $0 = 2\pi$ 的问题。*We test the of the point* β *of the line* $\vec{O\alpha}$*, noted as* $S_{\beta\alpha} = \vec{OZ} \times \vec{O\alpha} \cdot \vec{O\beta}$*, and for* $t > 0$*, the left side, means anticlockwise; for*

Algorithm 1: JudgeAlphaBeta

Input: α, β
Output: + for anticlockwise, - for clockwise, 0 for invalid

```
if β > α then
    if β − α > π then
        return JudgeAlphaBeta(α, β − 2π);
    return +1 or β − α;
else if α > β then
    if α − β > π then
        return JudgeAlphaBeta(α − 2π, β);
    return −1 or β − α > π;
end
return 0;
```

$t < 0$, *the right side, means clockwise;*

If the sign $S_{\gamma\alpha}, S_{\beta\alpha}$ *is the same, and the sign* $S_{\gamma\beta}, S_{\alpha\beta}$ *is the same, meas that the* γ *is between* α, β. *And* $S_{\beta\alpha} > 0$, *means the arrow of the point* α *is anticlockwise.*

For calculate the nearest point of γ, *we can use algorithm 1, test with* $\min |JudgeAlphaBeta(\gamma, \alpha)|, |JudgeAlpha$

Chapter 3 ProbabilityTheory

3.1 状态空间

3.1.1 T0001-连续抛硬币

Q1: with a random sequence of $0, 1$, when comes a subset $1, 0, 0$ A wins, when comes a subset $1, 1, 0$ B wins, otherwise the game keeps. The probability of A wins.

S1 solution 1

We draw the state-transfer graph, and calculate the probability A wins of each state as the initial state.

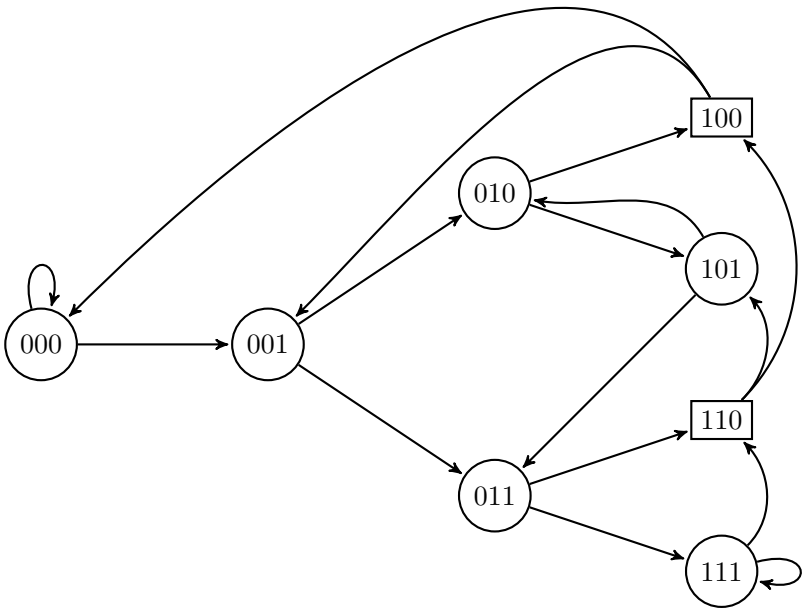

Figure 3.1: 【number of solution】

We calculate the probability of the machines stops at state 100.

$$
\begin{aligned}
P_{100}: &\quad 1 \\
P_{010}: &\quad \frac{1}{2}+\frac{1}{4}\times[\frac{1}{2}+\frac{1}{4}\times[\frac{1}{2}+\frac{1}{4}\times[...]]] \\
&\quad = \lim_{n\to\infty}\sum_{i=0}^{i=n}\frac{1}{2}\times(\frac{1}{4})^i = a_1\frac{1-q^n}{1-q} = \frac{2}{3} \\
P_{001}: &\quad \frac{1}{2}\times P_{010} = \frac{1}{3} \\
P_{000}: &\quad (1-\lim_{i\to\infty}(\frac{1}{2})^i)\times P_{001} = \frac{1}{3} \\
P_{011}: &\quad 0 \\
P_{110}: &\quad 0 \\
P_{111}: &\quad 0 \\
P_{101}: &\quad \frac{1}{2}\times P_{010} = \frac{1}{3} \\
\therefore P_A &= \frac{1}{8}\times(1+\frac{2}{3}+1) = \frac{1}{3}
\end{aligned}
\tag{3.1}
$$

S2 solution 2

we write the squence, which step has two posibilities.

$$
\begin{bmatrix}
\cdots & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \cdots \\
\cdots & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & \cdots
\end{bmatrix}
\longrightarrow
\begin{bmatrix}
 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \cdots \\
1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & \cdots
\end{bmatrix}
\tag{3.2}
$$

It is easily to find that, when we focus to locate the sequence of $1,0,0$, we find that the start index of the sequence is $0,2,4,6,8,\cdots$, because the index 1(which means $1,1,0,0$, and that means B wins.), we can also Analysis the index 3, 5 as the same. While the start index of the sequence is $0,1,2,3,\cdots$, therefore the probability of B wins is two times of A wins.