06-03-Mechanics

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2.1 Classical Mechanics

2.1.1 Center of Mass

Algebra defination $\boldsymbol{r}_c := \frac{\sum m_i \boldsymbol{r}_i}{\sum m_i}$, it relays to the choice of the coordinate system. If the map of the two coordinate systems is $T : S \to S'$, the matrix is orthogonal matrix, and m_i has no inflence to the space, $\boldsymbol{r}'_c = T\boldsymbol{r}_c$.

The operator $\boldsymbol{r}_c$ simplifies the calculation, $(\sum m_i)\boldsymbol{r}_c = \sum m_i \boldsymbol{r}_i$. We define the energy of the system like this, the whole energy $E_{all} := \sum \frac{1}{2} m_i \boldsymbol{v}_i^2$, the explicit energy $E_{ext} = \frac{1}{2}(\sum m_i)\boldsymbol{v}_c^2$, the intrenal energy $E_{int} := \sum \frac{1}{2} m_i (\boldsymbol{v}_i - \boldsymbol{v}_c)^2$, because $\sum m_i \boldsymbol{v}_c^2 - 2\sum m_i \boldsymbol{v}_i \boldsymbol{v}_c = -\boldsymbol{v}_c^2 \sum m_i$, we have $E_{int} = E_{all} - E_{ext}$.

2.1.2 Momentum

$\sum m_i \ddot{\boldsymbol{r}}_i = \sum(\boldsymbol{F}_i + \sum \delta_{ij} \boldsymbol{f}_{i \leftarrow j}) = \sum \boldsymbol{F}_i$. We define momentum $\sum \boldsymbol{P}_i = \sum m_i \dot{\boldsymbol{r}}_i$, therefore, $\frac{d}{dt} \sum \boldsymbol{P}_i = \sum \boldsymbol{F}_i$. [So the speed of light is always a const number, means that no other force can be applied to light?]

$\int \frac{d\boldsymbol{p}}{dt} = \boldsymbol{F}$, so $\int \boldsymbol{dp} = \int \boldsymbol{F} dt$.

2.1.3 Rigid Body

A rigid body changes from one position and orientation to another, the general movement can be represented as $\boldsymbol{v} = \boldsymbol{\omega} \times \boldsymbol{r}$.

Proposition 2.1 (One fixed rotation axis)**.** *Proof that the transformation of a rigid body has one fixed rotation axis.*

As shown in the figure, supposed that point A and B are on a sphere, and are transformed to A' and B', and two perpendicular bisector lines meets at point N, the point N can be transformed to N', because that $\triangle ABN \cong \triangle A'B'N'$, and moving on the orientable surface the triangular *cannot transfered from $\triangle ABC$ into $\triangle ACB$, we say $N = N'$, the line ON is the axis that doesn't move during the transformation.*

The basis of the coordinate system $\boldsymbol{Z}^T := [e_1, e_2, e_3]$, represented as one line and three columns. The transformation of the basis $\boldsymbol{Z}' = \boldsymbol{Z}\boldsymbol{\Gamma}$, $\boldsymbol{Z}\boldsymbol{\Gamma}\boldsymbol{r}' = \boldsymbol{Z}\boldsymbol{r}$, so $\boldsymbol{r}' = \boldsymbol{\Gamma}^{-1}\boldsymbol{r}$, $\boldsymbol{\Gamma}\boldsymbol{\Gamma}^{-1} = \boldsymbol{I}$ is easy to say when we write left in detail and considering that the $\boldsymbol{\Gamma}$ is the unit orthogonal base. Then the $\boldsymbol{\Gamma} \in SO_3$.

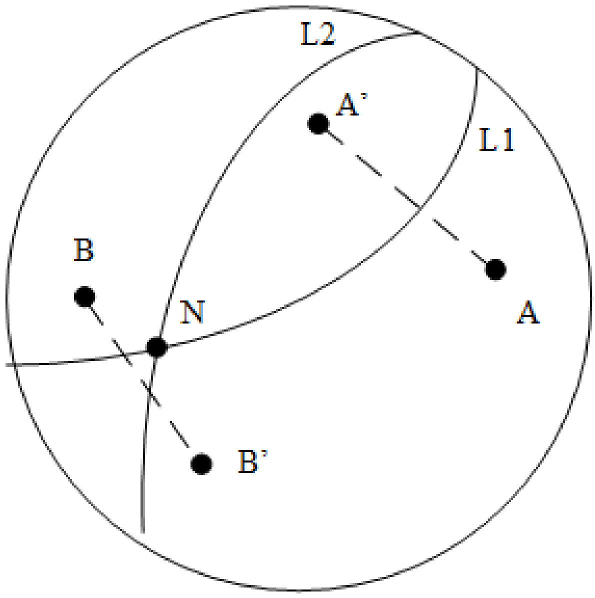

Figure 2.1: [T0004:ProvehasOneAxis]

The eigen value of $\boldsymbol{\Gamma}$, $|\boldsymbol{\Gamma} - \boldsymbol{\lambda}\boldsymbol{I}| = \boldsymbol{0}$, so $-x^3 + \alpha x^2 + \beta x + 1 = 0$, since the image of the function is continuous through $(-\infty, +\infty)$, it must have one root. 1 is one of the root. If the three roots are all real, they are all 1, or 1,-1,-1. If the rest two roots are virtual, they are $e^{i\theta}, e^{-i\theta}$, so $T\phi = e^{i\theta}\phi$.

$$\begin{aligned}
&\Gamma(\phi_1 + i\phi_2) = e^{i\theta}(\phi_1 + i\phi_2),\\
&\Gamma(\phi_1 - i\phi_2) = e^{-i\theta}(\phi_1 - i\phi_2)\\
&therefore:\\
&\Gamma(\phi_1) = \cos(\theta)\phi_1 - \sin(\theta)\phi_2,\\
&\Gamma(\phi_2) = \sin(\theta)\phi_1 + \cos(\theta)\phi_2
\end{aligned} \tag{2.1}$$

θ is the rotation angle.

2.1.4 Anglar Momentum

$\boldsymbol{L}_i := \boldsymbol{r}_i \times \boldsymbol{p}_i$, with the defination of $\boldsymbol{p}$, we have $\sum \boldsymbol{L}_i = \sum m_i \boldsymbol{r}_i \times \boldsymbol{v}_i$, and $\sum \boldsymbol{L}_i = \sum \boldsymbol{r}_i \times \boldsymbol{F}_i + \sum \boldsymbol{r}_i \times \sum_j \boldsymbol{f}_{i \leftarrow j}$. Since $\sum (\boldsymbol{r}_i - \boldsymbol{r}_j) \times \boldsymbol{f}_{i \leftarrow j} = 0$, we have $\sum \boldsymbol{L}_i = \sum \boldsymbol{r}_i \times \boldsymbol{F}_i$. We also call that Torque.

2.2 Reference

2.2.1 Todo

2.2.2 Tmeplate

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Theorem 2.1 (均值不等式)**.** *设 A, B 是两个实数, 则 $2AB \leq 2A^2 + B^2$.*

均值不等式. 设 A, B 是两个实数, 则 $2AB \leq 2A^2 + B^2$.

□

The free fall ball ends at $[1 - |(\mu k)^{-1} \pmod 2 - 1|]kH$

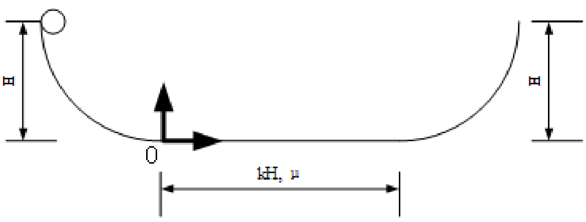

Figure 2.2: [T0001:]

Chapter 3　固体力学

Chapter 4 空气动力学

Chapter 5 流体动力学

Chapter 6 Reference