09-16-Questions

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Question 2.1. 给定 *3* 个点，绘制圆弧 α, β, γ

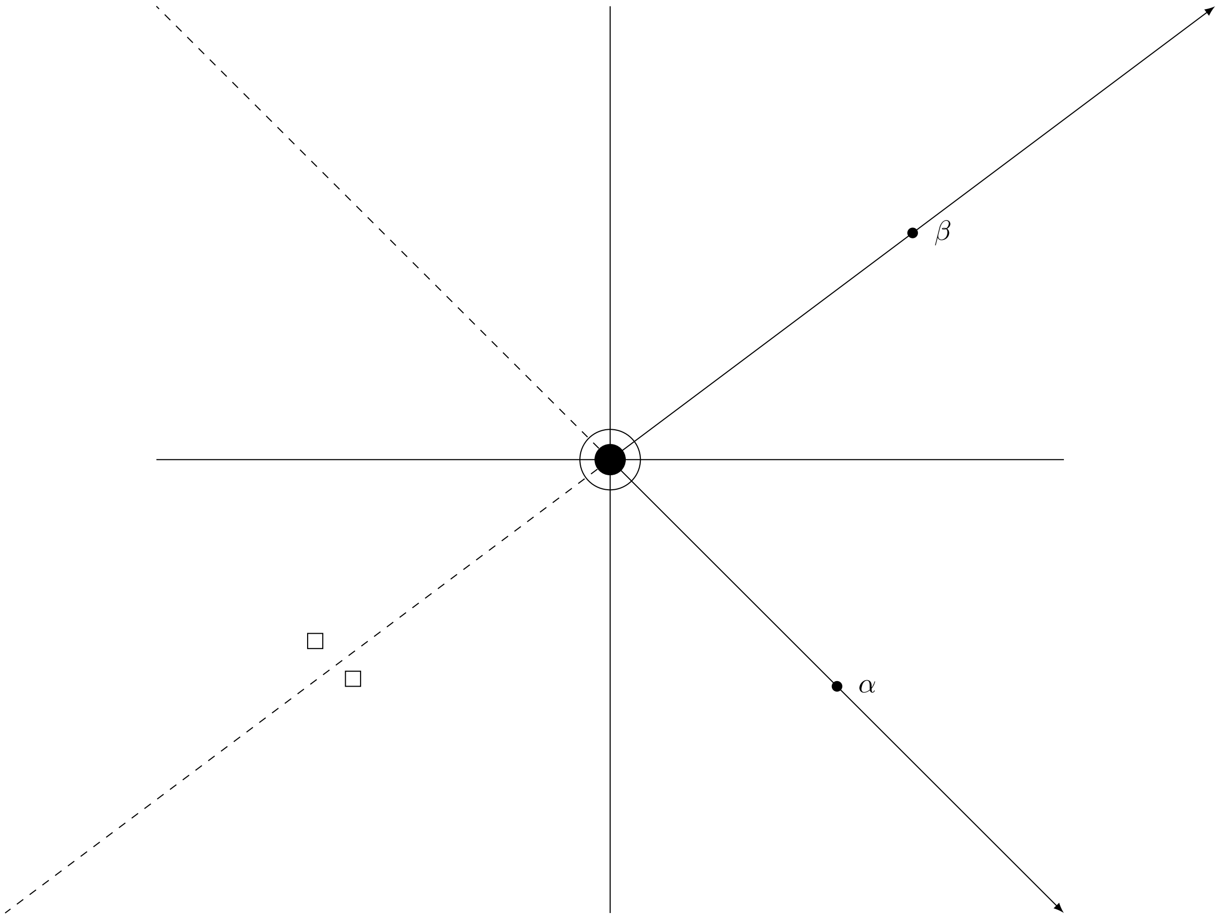

Figure 2.1: 【number of solution】

2.0.0.0.1 S1 *solution 1* 作为一维问题，*use algorithm*??.

2.0.0.0.2 S2 *solution 2*

升高一个维度，不利用极坐标表示，避免 $0 = 2\pi$ 的问题。*We test the of the point* β *of the line* $\vec{O\alpha}$*, noted as* $S_{\beta\alpha} = \vec{OZ} \times \vec{O\alpha} \cdot \vec{O\beta}$*, and for* $t > 0$*, the left side, means anticlockwise; for*

Algorithm 1: JudgeAlphaBeta

Input: α, β
Output: + for anticlockwise, - for clockwise, 0 for invalid

if $\beta > \alpha$ **then**
  if $\beta - \alpha > \pi$ **then**
    return JudgeAlphaBeta($\alpha, \beta - 2\pi$);
  return $+1$ or $\beta - \alpha$;
else if $\alpha > \beta$ **then**
  if $\alpha - \beta > \pi$ **then**
    return JudgeAlphaBeta($\alpha - 2\pi, \beta$);
  return -1 or $\beta - \alpha > \pi$;
end
return 0;

$t < 0$, the right side, means clockwise;

If the sign $S_{\gamma\alpha}, S_{\beta\alpha}$ is the same, and the sign $S_{\gamma\beta}, S_{\alpha\beta}$ is the same, meas that the γ is between α, β. And $S_{\beta\alpha} > 0$, means the arrow of the point α is anticlockwise.

For calculate the nearest point of γ, we can use algorithm **??**, *test with* $\min |JudgeAlphaBeta(\gamma, \alpha)|, |JudgeAlph$

Chapter 3 ProbabilityTheory

3.1 状态空间

3.1.1 T0001-连续抛硬币

Q1: with a random sequence of $0, 1$, when comes a subset $1, 0, 0$ A wins, when comes a subset $1, 1, 0$ B wins, otherwise the game keeps. The probability of A wins.

3.1.1.0.1 S1 solution 1

We draw the state-transfer graph, and calculate the probability A wins of each state as the initial state.

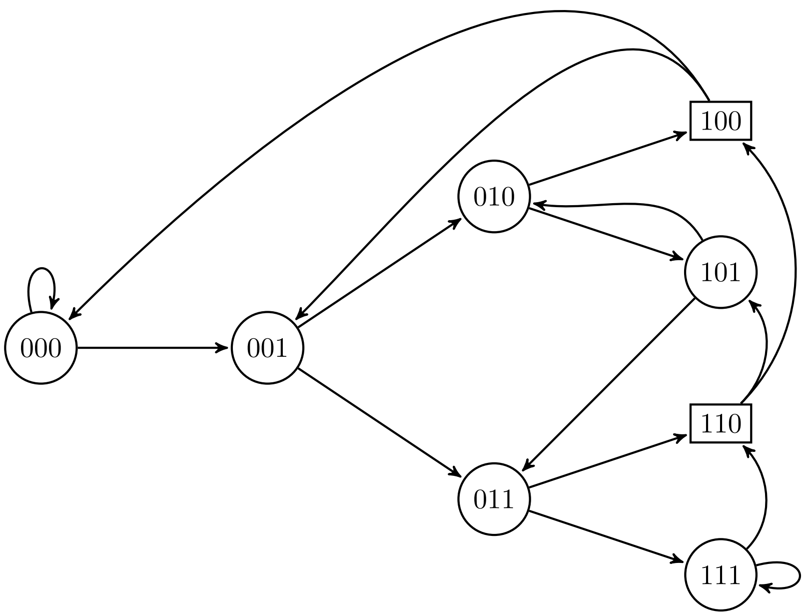

Figure 3.1: 【number of solution】

We calculate the probability of the machines stops at state 100.

$$
\begin{aligned}
P_{100}:&\quad 1\\
P_{010}:&\quad \frac{1}{2}+\frac{1}{4}\times[\frac{1}{2}+\frac{1}{4}\times[\frac{1}{2}+\frac{1}{4}\times[...]]]\\
&= \lim_{n\to\infty}\sum_{i=0}^{i=n}\frac{1}{2}\times(\frac{1}{4})^i = a_1\frac{1-q^n}{1-q}=\frac{2}{3}\\
P_{001}:&\quad \frac{1}{2}\times P_{010}=\frac{1}{3}\\
P_{000}:&\quad (1-\lim_{i\to\infty}(\frac{1}{2})^i)\times P_{001}=\frac{1}{3}\\
P_{011}:&\quad 0\\
P_{110}:&\quad 0\\
P_{111}:&\quad 0\\
P_{101}:&\quad \frac{1}{2}\times P_{010}=\frac{1}{3}\\
\therefore P_A &= \frac{1}{8}\times(1+\frac{2}{3}+1)=\frac{1}{3}
\end{aligned}
\tag{3.1}
$$

3.1.1.0.2 **S2** solution 2

we write the squence, which step has two posibilities.

$$
\begin{bmatrix} \cdots & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \cdots \\ \cdots & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & \cdots \end{bmatrix} \longrightarrow \begin{bmatrix} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \cdots \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & \cdots \end{bmatrix} \tag{3.2}
$$

It is easily to find that, when we focus to locate the sequence of $1, 0, 0$, we find that the start index of the sequence is $0, 2, 4, 6, 8, \cdots$, because the index 1(which means $1, 1, 0, 0$, and that means B wins.), we can also Analysis the index 3, 5 as the same. While the start index of the sequence is $0, 1, 2, 3, \cdots$, therefore the probability of B wins is two times of A wins.