1 Introduction

Today is 20211204, and I deciede to note down all of my knowledge about the math in this notebook.

2 Space

2.1 Operation Defination

2.1.1 Element

we define the basic element $\boldsymbol{x} = [x_1, x_2, \dots]^T = \Sigma x_i \boldsymbol{e_i}$, $\boldsymbol{e}_i$ means $x_i = 1, x_j = 0$ for all $j \neq i$. We define Kronecker sign to simply the description of $\boldsymbol{e}_i \cdot \boldsymbol{e}_j$.

$$\delta_{i,j} := \begin{cases} 1, & i = j \\ 0, & i \neq j \end{cases} \tag{1}$$

The set of bases $\{\boldsymbol{e}_i\} \xrightarrow{apply} \boldsymbol{x} \longrightarrow \{x_i\}$.

2.1.2 Dot Product

We define in algebra, $\boldsymbol{x} \cdot \boldsymbol{y} := \sum x_i y_i \boldsymbol{e}_i = \boldsymbol{x}^T \cdot \boldsymbol{y}$.

Then the defination is restricted to the choose of the coordinate system. We take a look a the product with reflect $T : \boldsymbol{x} \rightarrow \boldsymbol{T} \cdot \boldsymbol{x}$,

$$(\boldsymbol{A} \cdot \boldsymbol{B})^T = (a_{ik} b_{kj})^T = c_{ij}^T = c_{ji} = b_{jk} a_{ki} = \boldsymbol{B}^T \cdot \boldsymbol{A}^T \tag{2}$$

we have

$$(\boldsymbol{T} \cdot \boldsymbol{x})^T (\boldsymbol{T} \cdot \boldsymbol{y}) = \boldsymbol{x}^T (\boldsymbol{T}^T \boldsymbol{T}) \boldsymbol{y} = [(\boldsymbol{T}^T \boldsymbol{T}) \boldsymbol{x}]^T \boldsymbol{y} \tag{3}$$

We name T a Contractive mapping when $\boldsymbol{T}^T \boldsymbol{T} \leqslant \theta, 0 \leqslant \theta \leqslant 1$.

2.1.3 geometry Properties

$$\begin{aligned} \parallel \boldsymbol{x} \parallel &:= \sqrt{\boldsymbol{x} \cdot \boldsymbol{x}} \\ \cos \theta_{x,y} &:= \frac{\boldsymbol{x} \cdot \boldsymbol{x}}{\parallel \boldsymbol{x} \parallel \cdot \parallel \boldsymbol{y} \parallel} \end{aligned} \tag{4}$$

2.1.4 Add

$$
\begin{aligned}
\boldsymbol{x} + \boldsymbol{y} &:= \sum (x_i + y_i)\boldsymbol{e}_i \\
k \cdot \boldsymbol{x} &:= \sum k x_i \boldsymbol{e}_i
\end{aligned} \tag{5}
$$

Law $\boldsymbol{x} + \boldsymbol{y} = \boldsymbol{y} + \boldsymbol{x}$, law $(\boldsymbol{x} + \boldsymbol{y}) + \boldsymbol{z} = \boldsymbol{x} + (\boldsymbol{y} + \boldsymbol{z})$ is not obvious in the view of Set Theory.