

1 Introduction

Today is 20211211, and I decided to note down all of my knowledge about problems in this notebook. Actually we think for a while whether to separate the knowledge into different documents.

2 Physics

2.1 Energy

2.1.1 T0001

The coordinate of free fall ball ends. With energy equation, we have $GH = \mu G\xi kH$, considering of the backward length, we draw the figure and it can be transferred from the figure $y = |x|$. We have the coordinate at $[1 - |(\mu k)^{-1} \pmod{2} - 1|]kH$.

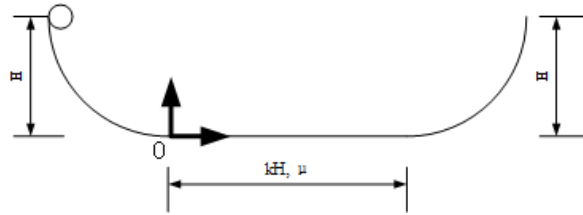


图 1: [T0001: Where the ball ends]

2.1.2 T0002

Suppose the ball free falls tangentially into a circle, at the top of the circle, to maintain the circular movement, $mg + F_{support} = mv^2/R$, since $m(n-2)gR = 1/2mv^2$, we have $F_{support} = mg(2n-5)$.

When $n \leq 1$, it is easy to see that the ball swings. When $n \geq 2.5$, the ball keeps going along the circle. When $n \in (1, 2.5)$, suppose the ball is at the angle θ as shown in the figure, the energy equation shows that $(nR - R + R\cos\theta)mg = 1/2mv^2$, force equation shows that $F_{support} - mg\cos\theta = mv^2/R = 2mg(n-1+\cos\theta)$, so $F_{support} = mg(2n-2+3\cos\theta)$. when $F_{support} = 0$, the ball falls, and it falls at the angle $\cos\theta = \frac{2}{3}(1-n)$.

2.1.3 T0003

When an object separates into several parts, the velocity of each component is different. Using the momentum equation, we have $mv = (-dm)u + (v+dv)(m+dm)$, and we let $u = v + dv - v_{rel}$, so we have $0 = v_{rel}dm + m dv$, so $dv = -v_{rel}\frac{dm}{m}$, the formula is integrated as $v_{final} - v_0 = v_{rel} \ln \frac{m_0}{m_{final}}$. This means that when the rocket accelerates like this, the final mass

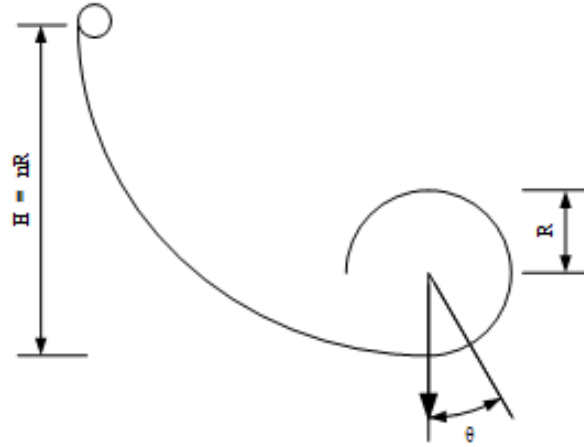


图 2: [T0002: Where the ball falls]

needs to be quite small, because the logarithm function decreases rapidly, also the v_{rel} needs to be big.

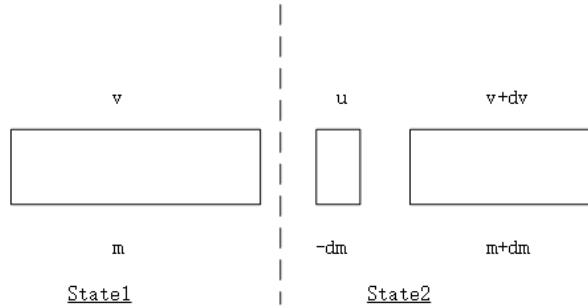


图 3: [T0003: Where the ball falls]

2.1.4 T0004

AF=AE, AB=AC, AB=8.

1) BF = 2, DQ?

2) Prove $\sqrt{2}AC - 2AQ = BG$

3) The right figure, AE=AF=4, the little isosceles triangle rotates, use the middle point M construct another isosceles triangle, when $\min RN, MR$?

Solution:

1) We know how to simplify $\tan(\alpha + \beta)$, it is easy in $\triangle ADQ$, $DQ = \frac{8}{\sqrt{2}} \cdot \frac{1}{1+3} = \frac{4\sqrt{2}}{7}$.

2) The formula is the same as $BG = 2QD$, let $BF := \delta, \angle CBE = \alpha$, we use triangles $\triangle BPF$ and $\triangle BPG$ to get $BG = \frac{\delta \cos(\frac{\pi}{4} - \alpha)}{\cos \alpha}$, since $DQ = BD \tan \alpha$, and with the law of sines, we have

$$\frac{BG}{DQ} = \frac{\delta}{BD} \cdot \frac{\cos(\frac{\pi}{4} - \alpha)}{\sin \alpha} = 2 \cdot \frac{\sin \alpha}{\sin(\frac{7\pi}{4} - \alpha)} \frac{\cos(\frac{\pi}{4} - \alpha)}{\sin \alpha} = 2. \square \quad (1)$$

3) the point M is on the circle with radius $r = 2\sqrt{2}$, we establish the coordinate system with original point at the center of the circle. M is noted as $[x, y]^T$, point B is $[-2r, -2r]^T$, M rotates along B to get N,

$$\begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix} \cdot \begin{bmatrix} x + 2r \\ y + 2r \end{bmatrix} + \begin{bmatrix} -2r \\ -2r \end{bmatrix} = \begin{bmatrix} y \\ -x - 4r \end{bmatrix} \quad (2)$$

so point R is $[y, -2r]^T$, $\min(x + 2r)^2 \Rightarrow [x, y]^T = [-r, 0]^T$, so $MR = \sqrt{(x - y)^2 + (y + 2r)^2} = 2\sqrt{10}$.

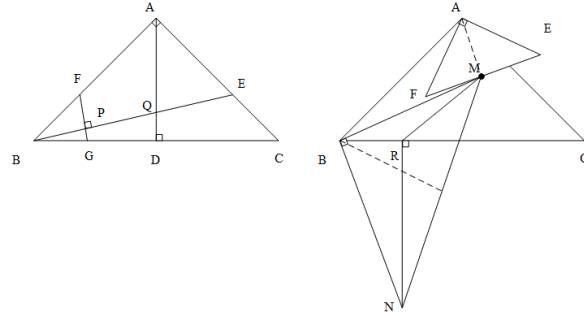


图 4: [T0004:HigniorSchoolGeometry]