

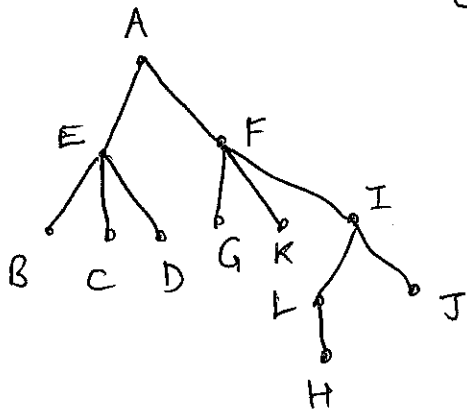
Trees — acyclic, connected graph — composed of nodes & edges.

(i) Unrooted (or Unoriented) tree (ii) Rooted tree.

We will discuss unrooted trees later.

Rooted tree: Recursively defined as root node attached to zero or more subtrees. Nodes with no children are called "leaves". Other nodes are internal nodes.

Example:



Root = A

Leaves = { B, C, D, G, K, H, J }

Children of A = { E, F } — E and F are siblings

Parent of C = E

Descendants of F = { F, G, K, I, H, J, L }

Ancestors of I = { I, F, A }

Proper descendants of node = All descendants of the node except itself

Similarly proper ancestors are all ancestors except itself.

Encoding tree using parent array representation ("Up tree"):

For each node, store its parent.

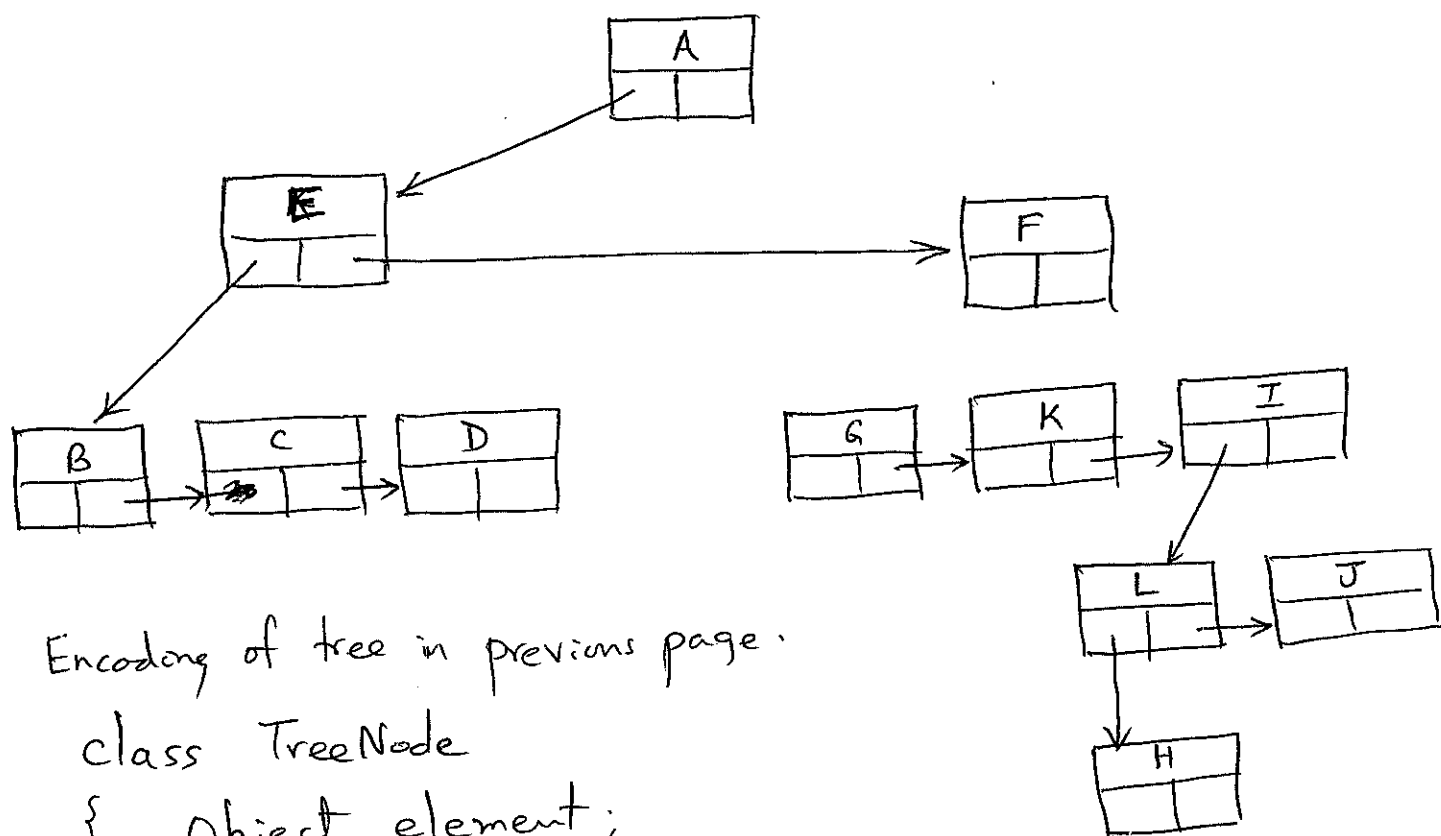
Ex:

A	B	C	D	E	F	G	H	I	J	K	L
-	E	E	E	A	A	F	L	F	I	F	I

← Parent array of above tree

This encoding is rarely used.

More common is to orient the edges of a tree downward, from parents to children. Each node has data and pointers (references) to leftmost child and right sibling.



Encoding of tree in previous page.

```
class TreeNode
```

```
{ Object element;
```

```
  TreeNode firstChild, nextSibling;
```

```
}
```

Path: A sequence of edges to travel from a node to another.

Example: $F \rightarrow I \rightarrow L \rightarrow H$ is a path of length 3 from F to H.

Length of a path is the number of its edges.

Depth of a node = length of the path from the root to that node.

So, Depth (A) = 0, Depth (F) = 1, Depth (L) = 3 in above example.

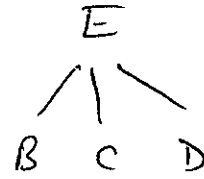
Height of a node = Maximum length of a path from the node to any of its descendants, i.e., length of path from it to a farthest leaf that is a descendant of that node.

Height (A) = 0

Height (I) = 2

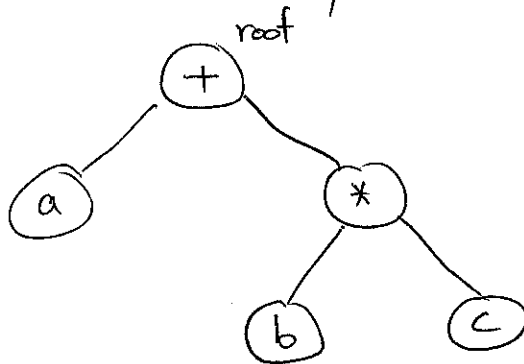
Subtree rooted at a node x is the part of the tree induced by x and its descendants.

Ex: Subtree ~~ind~~ at node E :



Binary Trees: Special subclass of trees in which each node has at most 2 children.

Ex: Arithmetic expression trees:



```

class BinaryTree *
{
    Object element;
    BinaryTree left, right;
}
  
```

Traversals: Visiting all nodes of a tree in some order.

Preorder (r)

```

// Visit a node before
// any of its proper desc
if ( $r \neq \text{null}$ )
{
    Visit  $r$ 
    Preorder( $r.\text{left}$ )
    Preorder( $r.\text{right}$ )
}
  
```

Postorder (r)

```

// visit a node after
// all of its proper descendants
if ( $r \neq \text{null}$ )
{
    Postorder( $r.\text{left}$ )
    Postorder( $r.\text{right}$ )
    Visit  $r$ 
}
  
```

Inorder (r)

```

// visit a node after all
// its proper descendants in
// left subtree but before
// its proper desc in right
// subtree
if ( $r \neq \text{null}$ )
{
    Inorder( $r.\text{left}$ )
    Visit  $r$ 
    Inorder( $r.\text{right}$ )
}
  
```

Output for example above:
+ a * b c

a b c * +

a + b * c