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alg-bipartite-matching.txt

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// Algorithm to find maximum matchings in bipartite graphs
Input:  Graph G=(V,E)
Output: Matching M of maximum cardinality
// Raises exception if G is not bipartite
// Algorithm works even if G is not connected

1. Use a maximal spanning forest of G to classify nodes as outer and inner:
   Mark a node as outer, its neighbors as inner, and their neighbors
   as outer, etc.  If there is any edge of G that connects two outer
   nodes or two inner nodes, then raise exception "Not bipartite"

2. Initialize: msize <-- 0;  for each u in V do u.mate <-- null

3. Find a maximal matching of G using a greedy algorithm:
   for each edge e=(u,v) in E do
       if u.mate = null and v.mate = null then
           u.mate <-- v;  v.mate <-- u;  msize++

4. // Repeat: find augmenting path, increase size of matching
   while true do
       Create an empty queue Q of vertices  // for outer nodes only
       for each u in V do
           u.seen <-- false;  u.parent <-- null
           if u.mate = null and u is an outer node then
               u.seen <-- true;  Q.Enqueue(u)

       while Q is not empty do
           u <-- Q.Dequeue()
           for each edge (u,v) in adj[u] do
               if not v.seen then
                   v.parent <-- u;  v.seen <-- true
                   if v.mate = null then // augmenting path has been found
                       processAugPath(v)
                       go to the beginning of Step 4
                   else
                       x <-- v.mate;  x.seen <-- true;  x.parent <-- v
                       Q.Enqueue(x)
           // Q got empty, but no augmenting path was found
           Break out of the outer while loop of Step 4 and go to Step 5

5. Output current matching as a maximum matching

//Helper function to increase size of matching, using an augmenting path
processAugPath(u)
// u is a free inner node with an augmenting path to the root of the tree
p <-- u.parent;  x <-- p.parent
u.mate <-- p;  p.mate <-- u
while x != null do // go up two steps from outer node x
    nm x <-- x.parent;  y <-- nm x.parent
    x.mate <-- nm x;  nm x.mate <-- x
    x <-- y
msize ++ // Matching size increases by 1
return
```