Homework 5 solutions

8.9

a. Paris and Marseilles are both in France.

(i) $In(Paris \land Marseilles, France)$.
(2) Syntactically invalid. Cannot use conjunction inside a term.

(ii) $In(Paris, France) \land In(Marseilles, France)$.
(1) Correct.

(iii) $In(Paris, France) \lor In(Marseilles, France)$.
(3) Incorrect. Disjunction does not express "both."

b. There is a country that borders both Iraq and Pakistan.

(i) $\exists\, c \quad Country(c) \land Border(c, Iraq) \land Border(c, Pakistan)$.
(1) Correct.

(ii) $\exists\, c \quad Country(c) \;\Rightarrow\; [Border(c, Iraq) \land Border(c, Pakistan)]$.
(3) Incorrect. Use of implication in existential.

(iii) $[\exists\, c \quad Country(c)] \;\Rightarrow\; [Border(c, Iraq) \land Border(c, Pakistan)]$.
(2) Syntactically invalid. Variable c used outside the scope of its quantifier.

(iv) $\exists\, c \quad Border(Country(c), Iraq \land Pakistan)$.
(2) Syntactically invalid. Cannot use conjunction inside a term.

c. All countries that border Ecuador are in South America.

(i) $\forall\, c \;\; Country(c) \land Border(c, Ecuador) \;\Rightarrow\; In(c, SouthAmerica)$.
(1) Correct.

(ii) $\forall\, c \;\; Country(c) \;\Rightarrow\; [Border(c, Ecuador) \;\Rightarrow\; In(c, SouthAmerica)]$.
(1) Correct. Equivalent to (i).

(iii) $\forall\, c \;\; [Country(c) \;\Rightarrow\; Border(c, Ecuador)] \;\Rightarrow\; In(c, SouthAmerica)$.
(3) Incorrect. The implication in the LHS is effectively an implication in an existential; in particular, it sanctions the RHS for all non-countries.

(iv) $\forall\, c \;\; Country(c) \land Border(c, Ecuador) \land In(c, SouthAmerica)$.
(3) Incorrect. Uses conjunction as main connective of a universal quantifier.

d. No region in South America borders any region in Europe.

(i) $\neg[\exists\, c, d \;\; In(c, SouthAmerica) \land In(d, Europe) \land Borders(c, d)]$.
(1) Correct.

(ii) $\forall\, c, d \;\; [In(c, SouthAmerica) \land In(d, Europe)] \;\Rightarrow\; \neg Borders(c, d)]$.
(1) Correct.

(iii) $\neg\forall\, c \;\; In(c, SouthAmerica) \;\Rightarrow\; \exists\, d \;\; In(d, Europe) \land \neg Borders(c, d)$.
(3) Incorrect. This says there is some country in South America that borders every country in Europe!

(iv) $\forall\, c \;\; In(c, SouthAmerica) \;\Rightarrow\; \forall\, d \;\; In(d, Europe) \;\Rightarrow\; \neg Borders(c, d)$.
(1) Correct.

e. No two adjacent countries have the same map color.

(i) $\forall x, y \;\; \neg Country(x) \lor \neg Country(y) \lor \neg Borders(x, y) \lor$
$\neg(MapColor(x) = MapColor(y))$.

(1) Correct.

(ii) $\forall x, y \;\; (Country(x) \land Country(y) \land Borders(x, y) \land \neg(x = y)) \Rightarrow$
$\neg(MapColor(x) = MapColor(y))$.

(1) Correct. The inequality is unnecessary because no country borders itself.

(iii) $\forall x, y \;\; Country(x) \land Country(y) \land Borders(x, y) \land$
$\neg(MapColor(x) = MapColor(y))$.

(3) Incorrect. Uses conjunction as main connective of a universal quantifier.

(iv) $\forall x, y \;\; (Country(x) \land Country(y) \land Borders(x, y)) \Rightarrow MapColor(x \neq y)$.

(2) Syntactically invalid. Cannot use inequality inside a term.

8.28

a. $W(G, T)$.

b. $\neg W(G, E)$.

c. $W(G, T) \lor W(M, T)$.

d. $\exists\, s \;\; W(J, s)$.

e. $\exists\, x \;\; C(x, R) \land O(J, x)$.

f. $\forall\, s \;\; S(M, s, R) \Rightarrow W(M, s)$.

g. $\neg[\exists\, s \;\; W(G, s) \land \exists\, p \;\; S(p, s, R)]$.

9.3 Both b and c are sound conclusions; a is unsound because it introduces the previously-used symbol *Everest*. Note that c does not imply that there are two mountains as high as Everest, because nowhere is it stated that *BenNevis* is different from *Kilimanjaro* (or *Everest*, for that matter).

9.6 We use a very simple ontology to make the examples easier:

a. $Horse(x) \Rightarrow Mammal(x)$
$Cow(x) \Rightarrow Mammal(x)$
$Pig(x) \Rightarrow Mammal(x)$.

b. $Offspring(x, y) \land Horse(y) \Rightarrow Horse(x)$.

c. $Horse(Bluebeard)$.

d. $Parent(Bluebeard, Charlie)$.

e. $Offspring(x, y) \Rightarrow Parent(y, x)$
$Parent(x, y) \Rightarrow Offspring(y, x)$.
(Note we couldn't do $Offspring(x, y) \Leftrightarrow Parent(y, x)$ because that is not in the form expected by Generalized Modus Ponens.)

f. $Mammal(x) \Rightarrow Parent(G(x), x)$ (here G is a Skolem function).

9.7

a. Let $P(x, y)$ be the relation "x is less than y" over the integers. Then $\forall\, x \ \ \exists\, y \ \ P(x, y)$ is true but $\exists\, x \ \ P(x, x)$ is false.

b. Converting the premise to clausal form gives $P(x, Sk0(x))$ and converting the negated goal to clausal form gives $\neg P(q, q)$. If the two formulas can be unified, then these resolve to the null clause.

c. If the premise is represented as $P(x, Sk0)$ and the negated goal has been correctly converted to the clause $\neg P(q, q)$ then these can be resolved to the null clause under the substitution $\{q/Sk0, x/Sk0\}$.

d. Suppose you are given the premise $\exists\, x \ \ Cat(x)$ and you wish to prove $Cat(Socrates)$. Converting the premise to clausal form gives the clause $Cat(Sk1)$. If this unifies with $Cat(Socrates)$ then you can resolve this with the negated goal $\neg Cat(Socrates)$ to give the null clause.

9.13 This questions deals with the subject of looping in backward-chaining proofs. A loop is bound to occur whenever a subgoal arises that is a substitution instance of one of the goals on the stack. Not all loops can be caught this way, of course, otherwise we would have a way to solve the halting problem.

a. The proof tree is shown in Figure S9.1. The branch with $\mathit{Offspring}(Bluebeard, y)$ and $Parent(y, Bluebeard)$ repeats indefinitely, so the rest of the proof is never reached.

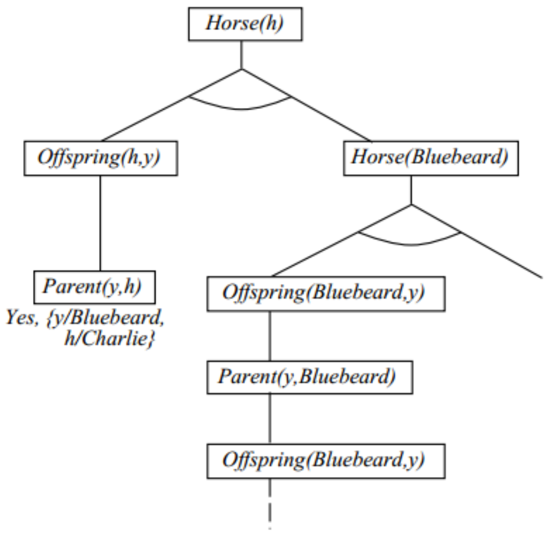

Figure S9.1 Partial proof tree for finding horses.

9.19

a. Results from forward chaining:

(i) $Ancestor(Mother(y), John)$: Yes, $\{y/John\}$ (immediate).

(ii) $Ancestor(Mother(Mother(y)), John)$: Yes, $\{y/John\}$ (second iteration).

(iii) $Ancestor(Mother(Mother(Mother(y))), Mother(y))$: Yes, $\{\}$ (second iteration).

(iv) $Ancestor(Mother(John), Mother(Mother(John)))$: Does not terminate.

b. Although resolution is complete, it cannot prove this because it does not follow. Nothing in the axioms rules out the possibility of everything being the ancestor of everything else.

c. Same answer.