$$\Rightarrow C_n = \begin{bmatrix} 0 & 0 & -a_0 \\ 1 & 0 & -a_1 \\ 0 & 1 & -a_2 \end{bmatrix}$$

$$\Rightarrow p_3(x) = x^3 + a_2 x^2 + a_1 x + a_0$$

$$\Rightarrow \det(x \cdot I - C_3) = \det\left(\begin{bmatrix} x & 0 & a_0 \\ -1 & x & a_1 \\ 0 & -1 & x+a_2 \end{bmatrix}\right)$$

$$= x \cdot \begin{bmatrix} x & a_1 \\ -1 & x+a_2 \end{bmatrix} + a_0 \cdot \begin{bmatrix} -1 & x \\ 0 & -1 \end{bmatrix}$$

$$= x\left[x(x+a_2) - (-1)(a_1)\right] + a_0\left[(-1)(-1)\right]$$

$$= x(x^2 + a_2 x + a_1) + a_0$$

$$= x^3 + a_2 x^2 + a_1 x + a_0$$

$$\boxed{= p_3(x) \quad \blacksquare}$$

1.2 Eigenvalues, Eigenvectors

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$$\Rightarrow \begin{bmatrix} c & c & 0 \\ c & 0 & c \\ 0 & c & c \end{bmatrix}$$

$$\Rightarrow \lambda = c, -c, 2c$$

1 $\det(A - I\lambda) = \det\left(\begin{bmatrix} c-\lambda & c & 0 \\ c & -\lambda & c \\ 0 & c & c-\lambda \end{bmatrix}\right)$

$$= (c-\lambda)\cdot\det\begin{vmatrix} -\lambda & c \\ c & c-\lambda \end{vmatrix} - (c)\cdot\det\begin{vmatrix} c & c \\ 0 & c-\lambda \end{vmatrix}$$

$$= (c-\lambda)\left[(-\lambda)(c-\lambda) - (c)(c)\right] - c(c)(c-\lambda)$$

$$= (\lambda+c)(c-\lambda)(\lambda-2c)$$

$\therefore \lambda = -c, c, 2c$

2. Linear Regression

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$x_i = \{1, 2, 3, 4, 5\}$

$y_i = \{0.7, 2.2, 2.8, 4.4, 4.9\}$

$\Rightarrow \sum x_i = 15$

$\Rightarrow \sum y_i = 15$

$\Rightarrow \sum x_i y_i = 55.6$

$\Rightarrow \sum x_i^2 = 55$

$\Rightarrow \sum y_i = a_0 n + a_1 \sum x_i$

$\Rightarrow \sum x_i y_i = a_0 \sum x_i + a_1 \sum x_i^2$

$\Rightarrow 15 = 5a_0 + 55a_1$

$\Rightarrow 55.6 = 15a_0 + 55a_1$

$\Rightarrow a_0 = -0.18$

$\Rightarrow a_1 = 1.06$

$$\boxed{\begin{aligned} \therefore y &= a_0 + a_1 x \\ &= -0.18 + 1.06x \end{aligned}}$$

$n_1 = 2, \; n_2 = 3, \; I = \int_{-2}^{2} \frac{1}{(x-4)^2} dx$

$\Rightarrow x = \frac{1}{2}\left[(2+2)t + 0\right] \quad \Rightarrow dx = 2dt$

$= 2t$

n_1:

$\Rightarrow I = \int_{-1}^{1} \frac{1}{(2t-4)^2} \cdot 2dt$

$\Rightarrow \quad = f\left(-\frac{1}{\sqrt{3}}\right) + f\left(\frac{1}{\sqrt{3}}\right)$

$\boxed{\approx 0.32231}$

n_2: $I = \int_{-1}^{1} \frac{1}{(2t-4)^2} \cdot 2dt$

$= \frac{8}{9} f(0) + \frac{5}{9} f\left(\frac{\sqrt{15}}{5}\right) + \frac{5}{4} f\left(-\frac{\sqrt{15}}{5}\right)$

$\boxed{\approx 0.33217}$

$\Rightarrow \{(-1,4),(2,5),(3,6)\} \Rightarrow n=2$

1. lagrange

$$\Rightarrow y = y_0 \cdot \frac{(x-x_1)(x-x_2)}{(x_0-x_1)(x_0-x_2)} + y_1 \cdot \frac{(x-x_0)(x-x_2)}{(x_1-x_0)(x_1-x_2)} + y_2 \cdot \frac{(x-x_0)(x-x_1)}{(x_2-x_0)(x_2-x_1)}$$

$$= 4 \cdot \frac{(x-2)(x-3)}{(-1-2)(-1-3)} + 5 \cdot \frac{(x+1)(x-3)}{(2+1)(2-3)} + 6 \cdot \frac{(x+1)(x-2)}{(3+1)(3-2)}$$

$$= \frac{4}{12}(x-2)(x-3) - \frac{5}{3}(x+1)(x-3) + \frac{6}{4}(x+1)(x-2)$$

$$= \frac{1}{3}(x^2-3x-2x+6) - \frac{5}{3}(x^2-3x+x-3) + \frac{3}{2}(x^2-2x+x-2)$$

$$= \frac{1}{6}x^2 + \frac{1}{6}x + 4$$

$$\boxed{= \frac{1}{6}(x^2+x+24)}$$

2. Newton

$$\Rightarrow (x_0, g(x_0)) = (-1,4)$$

$$\Rightarrow (x_1, g(x_1)) = (2,5)$$

$$\Rightarrow (x_2, g(x_2)) = (3,6)$$

$$\Rightarrow g(x_1, x_0) = \frac{g(x_1) - g(x_0)}{x_1 - x_0} = \frac{5-4}{2-(-1)} = \frac{1}{3}$$

$$\Rightarrow g(x_2, x_1) = \frac{g(x_2) - g(x_1)}{x_2 - x_1} = \frac{6-5}{3-2} = 1$$

$$\Rightarrow g(x_2, x_1, x_0) = \frac{g(x_2, x_1) - g(x_1, x_0)}{(x_2 - x_0)} = \frac{1-\left(\frac{1}{3}\right)}{3-(-1)}$$

$$= \frac{1}{6}$$

$$\Rightarrow f(2) = g(x_0) + g(x_1, x_0)(x - x_0) + g(x_2, x_1, x_0)(x - x_0)(x - x_1)$$

$$= 4 + \left(\frac{1}{3}\right)(x - x_0) + \left(\frac{1}{6}\right)(x+1)(x-2)$$

$$= \frac{1}{6}x^2 + \frac{1}{6}x + 4$$

$$\boxed{= 1/6\,(x^2 + x + 24)}$$

3- both newton's divided difference & lagrange interp' gave the same polynomial:

$$\boxed{\frac{1}{6}(x^2+x+24)}$$

5. ODE IVP

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$\Rightarrow h = 0.25$

$\Rightarrow I = [0, 1]$

$\Rightarrow y'(t) = 5t^4 \cdot y(t)$

$\Rightarrow y_0 = 1$

① $\frac{dy}{dt} = 5t^4 \cdot y$

$\Rightarrow dy \cdot \frac{1}{y} = 5t^4 dt$

$\Rightarrow \int \frac{1}{y} dy = \int 5t^4 dt$

$\Rightarrow \ln|y| = t^5 + C$

$\Rightarrow \ln|y(0)| = t^5 + 0$

$\boxed{\Rightarrow \therefore y(t) = e^{t^5}}$

② $h = \frac{1}{4}, t_0 = 0, y_0 = 1$

$\Rightarrow t_1 = t_0 + h$

$= 0.25$

$\Rightarrow y_1 = y_0 + h \cdot f(t_0, y_0)$

$= 1 + 0.25(0)$

$= 1$

$\Rightarrow t_2 = t_1 + h$

$= 0.50$

$\Rightarrow y_2 = y_1 + hf(t_1, y_1)$

$= 1 + 0.25(0.01953125)$

$= 1.00488$

$\Rightarrow t_3 = t_2 + h$

$= 0.75$

$\Rightarrow y_3 = y_2 + hf(t_2, y_2)$

$= 1.08339$

$\Rightarrow t_4 = t_3 + h$

$= 1$

$\Rightarrow y_4 = y_3 + hf(t_3, y_3)$

$\boxed{= 1.51187}$

3) $h = \frac{1}{4}, t_0 = 0, y_0 = 1$

$\Rightarrow t_1 = 0.25$

$\Rightarrow t_2 = 0.50$

$\Rightarrow t_3 = 0.75$

$\Rightarrow t_4 = 1$

$\Rightarrow y_1 = y_0 + \frac{h}{2}\left[f(t_0, y_0) + f(t_1, y_1)\right]$

$= 1.00244$

$\Rightarrow y_2 = y_1 + \frac{h}{2}\left[f(t_1, y_1) + f(t_2, y_2)\right]$

$= 1.04424$

$\Rightarrow y_3 = y_2 + \frac{h}{2}\left[f(t_2, y_2) + f(t_3, y_3)\right]$

$= 1.30766$

$\Rightarrow y_4 = y_3 + \frac{h}{2}\left[f(t_3, y_3) + f(t_4, y_4)\right]$

$\boxed{= 2.70679}$

4) the exact value gi

$y(1)$ is e (2.7182

method is pretty f

4 the exact value given by $y(1)$ is e (2.71828). Euler's method is pretty far off, having a difference of 1.67404 while the difference in Heun's method is 0.01149, which is pretty close.

6. ODE BVP

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$\rightarrow y''(t) = (2+4t^2)\cdot y(t)$, $y_0 = 1$, $y_1 = e$, $h = 0.25$, $I = [0,1]$

① $i =$
$1 \rightarrow 0$
$2 \rightarrow 0.25$
$3 \rightarrow 0.50$
$4 \rightarrow 0.75$
$5 \rightarrow 1.00$

$\Rightarrow y''(t)\big|_i \cong \dfrac{y_{i+1} - 2y_i + y_{i-1}}{h^2}$

$\Rightarrow \dfrac{y_{i+1} - 2y_i + y_{i-1}}{h^2} - 2 + 4t_i^2$

② $\Rightarrow \dfrac{y_3 - 2y_2 + y_1}{h^2} - (2+4h^2)y_2 = 0$

$\boxed{\Rightarrow 16y_3 - 34.25y_2 + 16y_1}$

③ $\dfrac{y_4 - 2y_3 + y_2}{h^2} - 3y_3 = 0$

$\boxed{\Rightarrow 16y_4 - 35y_3 + 16y_2}$

④ $\Rightarrow \boxed{16y_5 - 36.25y_4 + 16y_3}$

⑤ $\Rightarrow \boxed{y_5 = e = y(1)}$

$$\boxed{\begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 16 & -34.25 & 16 & 0 & 0 \\ 0 & 16 & -35 & 16 & 0 \\ 0 & 0 & 16 & -36.25 & 16 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} y_1 \\ y_2 \\ y_3 \\ y_4 \\ y_5 \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \\ e \end{bmatrix}}$$