Department of Mechanical Engineering
Indian Institute of Technology Bombay

ME415: Computational Fluid Dynamics & Heat Transfer **Autumn 2016**

Assignment # 1: Computational Heat Conduction for Cartesian Geometry

Weightage: 10% **Instructor**: Prof. Atul Sharma

Date Posted: 22nd Aug. (Monday) **Due Date: 31st Aug. (Wednesday, Early Morning 2 AM)**

ONLINE SUBMISSION THROUGH MOODLE ONLY (No late submission allowed): Create a single zipped file consisting on (a) filled-in answer sheet of this doc file converted into a pdf file and (b) all the computer programs. The name of the zipped file should be **rollnumber_A1**

Note: Both problem and answer sheet are provided below. **SCILAB or MATLAB** should *preferably* be used for programming as well as generating graphical results.

Refer http://spoken-tutorial.org/tutorial-search/?search_foss=Scilab&search_language=English, for getting started to SCILAB for programming as well as generating graphical results. To save figure: Go to "Graphic window number", click on "File", then click on "Export to", select "Windows BMP image" in the "Files of type". Make sure to save the file in the same location where you have this file. More details are given in the next page.

1. **Flux based methodology for CFD development and code-verification for 1D unsteady state heat conduction problem, on a uniform grid.**

   Consider 1D conduction in a long stainless-steel (density ρ: **7750 kg/m3**, specific-heat c_p: **500 J/Kg K**, thermal-conductivity k: **16.2 W/m-K**) sheet of thickness $L=1\ cm$. The sheet is initially at a uniform temperature of 30^oC and is suddenly subjected to a constant temperature of $T_{wb} = 0^oC$ on the west and $T_{eb} = 100^oC$ on east boundary.

   Using the flux based solution methodology of CFD development, a computer program (**A1_1D_Prob1.sci**) for *explicit method on a uniform* 1-D Cartesian grid is given along with this assignment sheet. Present a testing of the code for a volumetric heat generation of 0 and 100, MW/m^3. Consider maximum number of grid points as $imax=12$ and the steady state convergence tolerance as $\varepsilon_{st}=10^{-4}$. Plot the steady state temperature profiles with and without volumetric heat generation and compare with the exact solution.

2. **Flux based methodology for CFD development and code-verification for 2D unsteady state heat conduction problem, on a uniform grid.**

   Consider 2D conduction in a square shaped ($L_1=1m$ and $L_2=1m$) long stainless-steel plate. The plate is initially at a uniform temperature of ***30°C*** and is suddenly subjected to a constant temperature of $T_{wb} = 100^oC$ on the west boundary, $T_{sb} = 200^oC$ on the south boundary, $T_{eb} = 300^oC$ on the east boundary, and $T_{nb} = 400^oC$ on north boundary.

i. Using the flux based solution methodology of CFD development, develop a computer program for *explicit method on a uniform* 2-D Cartesian grid. Use the steady state stopping criterion for non-dimensional temperature, given (slide # 5.58 & 5.59) as

$$\left(\frac{\partial\theta}{\partial\tau}\right)_{i,j} = \frac{l_c^2}{\alpha\Delta T_c}\left(\frac{T_{i,j}^{n+1} - T_{i,j}^{n}}{\Delta t}\right)_{\max \text{ for } i,j} \le \varepsilon_{st}\left\{l_c = L_1 \,\&\, \Delta T_c = T_{nb} - T_{wb}\right\}$$

ii. Present a CFD application of the code for a volumetric heat generation of **0** and **50 kW/m³**. Consider maximum number of grid points as $imax \times jmax = 12\times12$ and the steady state convergence tolerance as $\varepsilon_{st}=10^{-4}$. Plot the steady state temperature contours with and without volumetric heat generation.

3. **Coefficient of LAEs based methodology for CFD development and code-verification for 1D unsteady state heat conduction problem, on a non-uniform grid.**

Consider 1D conduction in a long stainless-steel sheet of thickness ***L=1 cm***. The sheet is initially at a uniform temperature of **30°C** and is suddenly subjected to a constant temperature of $T_{wb} = 0^oC$ on the west and $h=1000\ W/m^2.K$ and $T_\infty=100^oC$ on east boundary.

i. Generate a non-uniform 1D Cartesian grid, using an algebraic method (can be found in slide no. 5.63 to 5.66). The method involves a transformation of a uniform grid, in a ξ- coordinate based 1D computational domain of unit length, to a *x*- coordinate based physical domain of length L, using an algebraic equation given (*Hoffmann and Chiang, 2000*) as

$$x = L\frac{(1+\beta)\left[(\beta+1)/(\beta-1)\right]^{(2\xi-1)} - (\beta-1)}{2\left\{1+\left[(\beta+1)/(\beta-1)\right]^{(2\xi-1)}\right\}}$$

This equation results in a grid which is finest near the two ends of the domain and gradually become coarser at the middle of the domain. It is called as equal *clustering* of grids at both the ends of the domain. Consider maximum number of grid points as ***imax=12*** and ***β=1.2*** (which controls the non-uniformity in the grid size).

ii. Using the coefficient of LAEs based solution methodology of CFD development, a Gauss-Seidel method based computer program for *implicit method on the non-uniform* 1-D Cartesian grid (**A1_1D_Prob3.sci**) is given along with this assignment sheet. Present a CFD application of the code for a volumetric heat generation of 0 and 100, MW/m³. Consider the convergence tolerance as $\varepsilon_{st}=10^{-4}$ for steady state, and $\varepsilon=10^{-4}$ for iterative solution. Plot the steady state temperature profiles with and without volumetric heat generation, and compare with the exact solution.

4. **Coefficient of LAEs based methodology for CFD development and code-verification for 2D unsteady state heat conduction problem, on a non-uniform grid.**

Consider 2D conduction in a square shaped (***L_1=1m*** and ***L_2=1m***) long stainless-steel plate. The plate is initially at a uniform temperature of ***30^oC*** and is suddenly subjected to a constant temperature of ***T_{wb} = 100^oC*** on the west boundary, insulated on the south boundary, constant incident heat flux of ***q_W = 10 kW/m^2*** on the east boundary, and ***h=100 W/m^2.K*** and ***T_∞ = 30^oC*** on north boundary.

i. Generate a non-uniform 2D Cartesian grid, using an algebraic method, using the equation given above for the non-uniform grid generation in the x-direction. However, this equation is also used in the y-direction to generate the 2D grid. This equation results in a grid which is finest near the two ends (east and west as well as north and south boundary) of the domain and gradually become coarser at the middle of the domain. It is called as equal clustering of grids at both the ends of the domain. Consider maximum number of grid points (for the temperature) as ***imax×jmax=12×12*** and ***β=1.2***.

ii. Using the coefficient of LAEs based solution methodology of CFD development, develop a Gauss-Seidel method based computer program for the *implicit method on a non-uniform* 2-D Cartesian grid. Use the stopping criterion presented in the previous problem, with $\Delta T_c = T_{wb} - T_\infty$.

iii. Using the non-uniform grid and the program, present a CFD application of the code for a volumetric heat generation of **0** and ***50 kW/m^3***. Consider the convergence tolerance as $\varepsilon_{st}=10^{-4}$ for the steady state, and $\varepsilon=10^{-4}$ for iterative solution by the Gauss-Seidel method. Plot the steady state temperature contours with and without volumetric heat generation.

<u>Best Wishes for your success in the insightful field of Computational Fluid Dynamics</u>

Keep Playing with the codes in future also.

NOTE: CODE EXECUTION IN SCILAB

<u>To open a *Scilab console*</u>:

Linux: Applications→Programming→Scilab or Applications→Science→Scilab

Windows: Start→All programs→Scilab→Scilab.exe

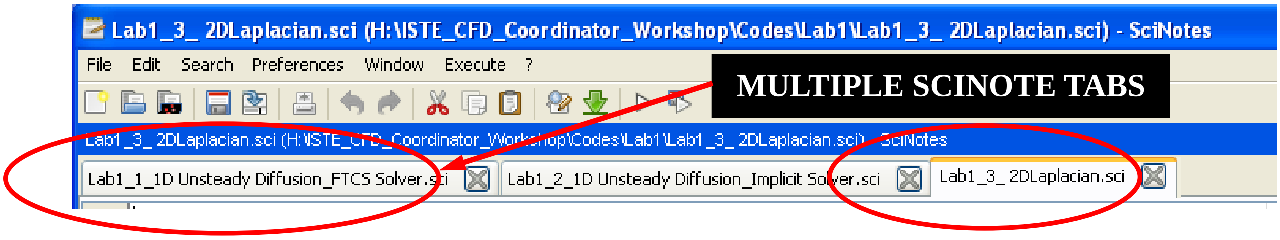
<u>To load a source code</u> (or a *Scinote* file):

In the Scilab console, go to top menu bar

File→Open a file…→(Browse for the *.sci file path)

The source code opens in a new *Scinote* window.

NOTE: Once this window is open, subsequent loading of a new *.sci file will open a new tab in the same *Scinote* window.

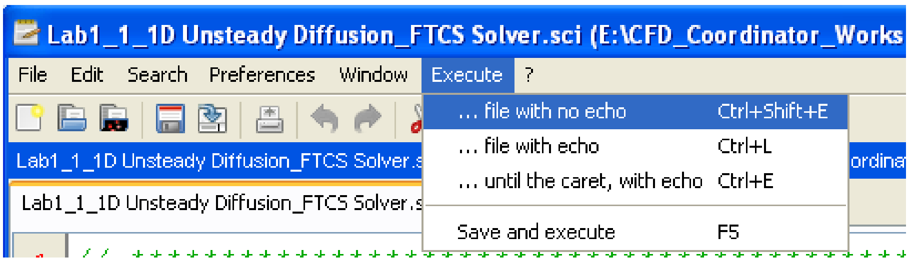


<u>To execute a program written in *Scinote*</u>:

In the Scinote window (with the desired program tab open), go to top menu bar

Execute→…file with no echo (please do not select …file with echo)

The execution begins in the *Scilab console* window.



NOTE: Only one *Scilab* code can be executed at a time. Once an execute command is given and the code is to be stopped at an intermediate stage, use the method given below. If another execute command is given without completing/aborting the previous run, erroneous results may be produced.

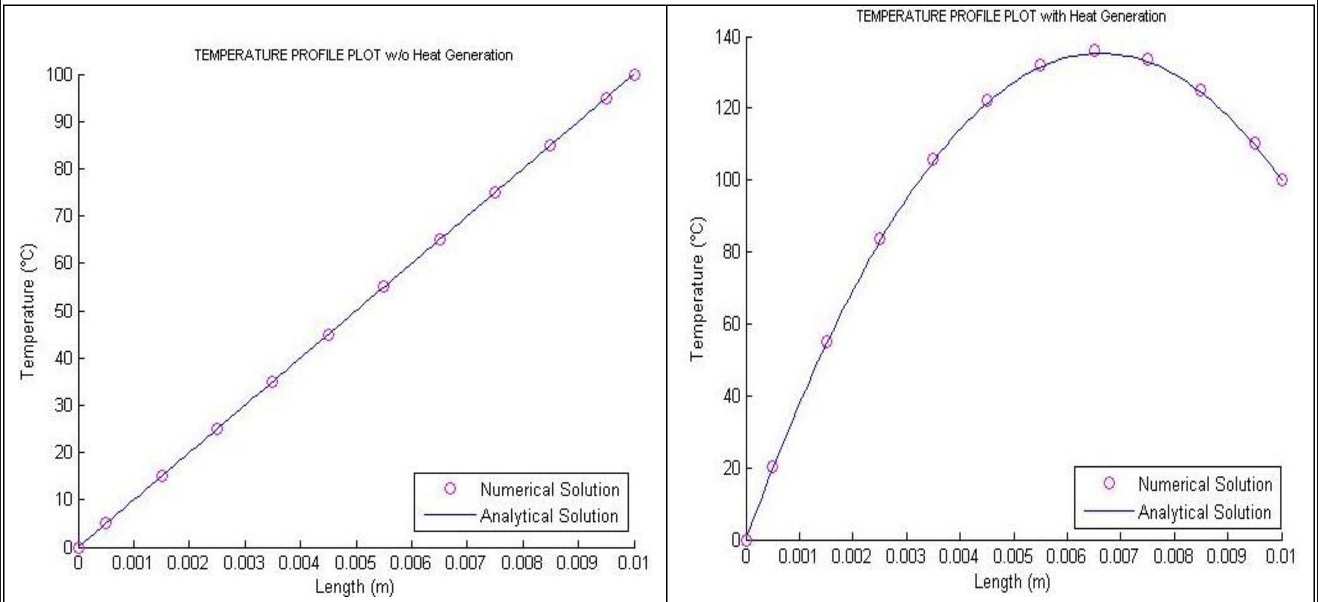
<u>To abort a running program</u>:

Go to the *Scilab console* window and press **CTRL+C**. This interrupts the code execution. A prompt appears asking for user input. Enter "abort" here to stop the code execution.

Answer Sheet

Problem # 1: Flux based methodology, with explicit method and uniform grid: 1D Conduction

Plot the steady state temperature profiles with and without volumetric heat generation and compare with the exact solution. (2 figures).



(a) (b)

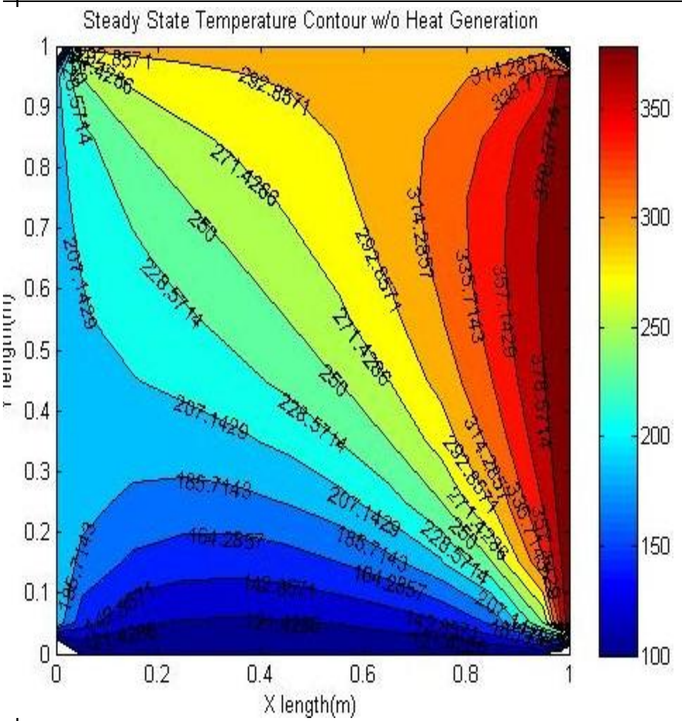
Fig. 1.1: Testing of the explicit method based 1D code on uniform Cartesian grid, for unsteady state heat conduction, (a) without and (b) with heat generation. The figure shows comparison of the numerical with the analytical solution for steady state temperature profile.

Discussion on the Fig. 1.1:

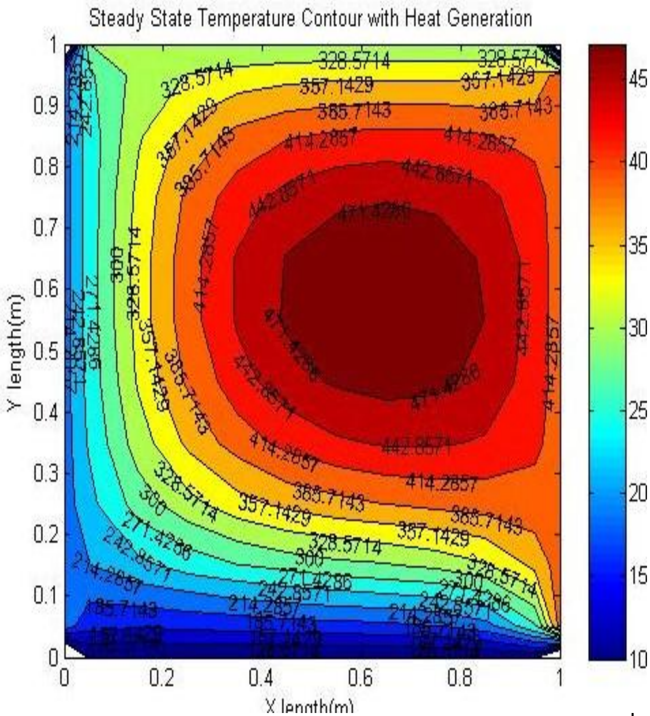
- The continuous line is the analytical solution whereas the points in circle are the numerical solution. The numerical solution is in good conformance with the analytical solution.
- When there is no heat generation, the steady state temperature profile comes out to be a straight line. This is solved using uniform grid points and flux based methodology.
- The boundary conditions of T=0 at x=0 and T=100 at x=0.01 is always satisfied as seen in the graph.
- When there is heat generation, the steady state temperature profile is non linear. The numerical solution is again in good conformance with the analytical solution.
- There is a peak temperature at approx. x=0.007m with T=135 degree celcius before it decreases to 100 degree celcius to meet the boundary condition.
- Hence the heat conduction rate is changing along the length for case(b) whereas it is constant in case(a).
- Both the graphs depict the accuracy of the flux methodology used despite its discrete nature and approximations taken like that for Dx ,Dt,etc.

Problem # 2: Flux based methodology, with explicit method and uniform grid: 2D Conduction

Plot the steady state temperature contours with and without volumetric heat generation. (2 figures).



(a)



(b)

Discussion on the Fig. 1.2: Write your answer, limited inside this text box only

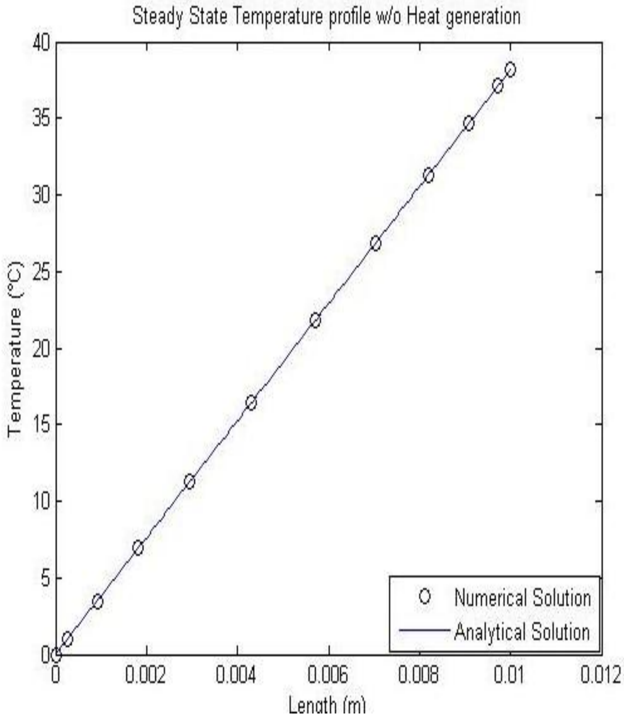
- In case(a),as there is no heat generation, the steady state temperature contour plots are curved.The maximum temperature contour lines occur at the side with 400°C (Note: Because the convention for (i,j) used in code provided by you and (i,j) used by Matlab is different, while plotting the West boundary(T_wb) is not on the left side of this figure. Similarly for North,East and South)Heat transfer is because of the temperature gradient set up. Also the contours appear symmetric about the diagonal.

- In case(b), there is heat generation because of which the higher temperature contour is more rounded and takes up most of the place inside the sheet. Here the maximum temperature clearly exceeds 400°C .Here the heat transfer was also because of the extra volumetric heat that is provide.

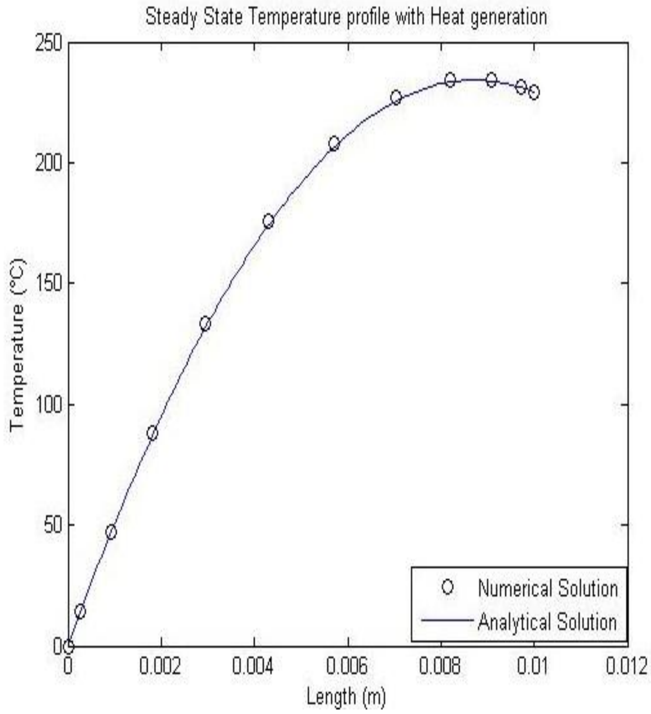
- Uniform grid was used along with Flux methodology. The boundary conditions of fixed temperature on each side is always satisfied.

Problem # 3: Coefficient of LAEs based methodology, with implicit method and non-uniform grid: 1D conduction

Plot the steady state temperature profiles with and without volumetric heat generation and compare with the exact solution. (2 figures).



(a)



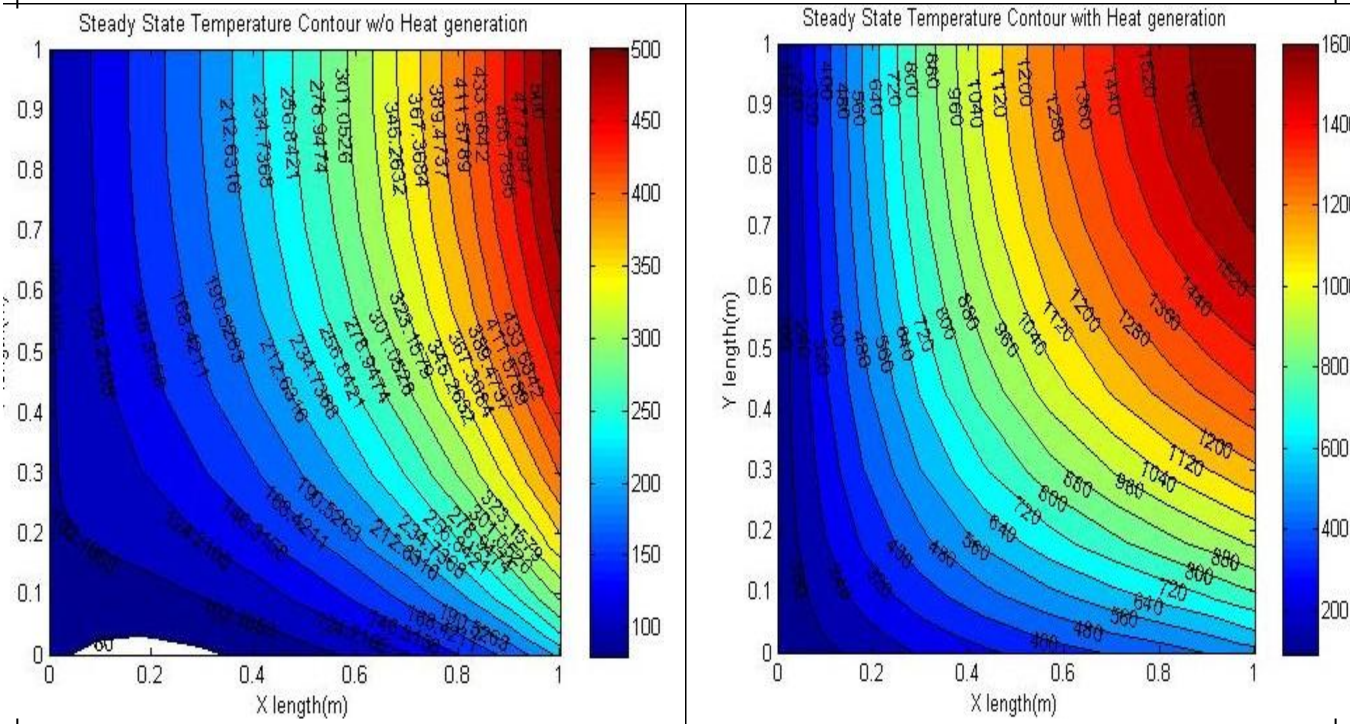
(b)

Discussion on the Fig. 1.3: Write your answer, limited inside this text box only

- The continuous line is the analytical solutions whereas the circles are the numerical solution. In case(a), we can clearly see that both the solutions are in good agreement with each other. The temperature profile is linear as expected when there is no heat generation. Since there is convection at the other end, the temperature did not rise as much as in the constant temperature boundary condition.
- In case(b), since there is volumetric heat generation, the steady state temperature profile is non linear and has a peak at approx. x=0.09 with Tmax=235°C . The rate of heat conduction keeps changing with length. Also at the end x=0.01, heat is lost to the fluid unlike in case(a) where heat was added by the fluid to the steel bar.
- Here the grid is non uniform,calculated using the algebraic equation given. The volumetric heat generation also varies with length since delta x varies with length.Gauss Siedel method of iteration was used to calculate the temperature values.
- Hence the accuracy of this method is again justified inspite of its discrete nature and various approximations incorporated.

Problem # 4: Coefficient of LAEs based methodology, with implicit method and non-uniform grid: 2D conduction

Plot the steady state temperature profiles with and without volumetric heat generation. (2 figures).



(a) (b)

Discussion on the Fig. 1.4: Write your answer, limited inside this text box only

- In case(a), the steady state temperature contour lines are a curve .Without heat generation, the maximum temperature reaches around 500 degree celcius.(Note:Since (i,j) convention for the code provided by sir and the convention used by Matlab is different, while plotting, the West side(T_wb) is not on the left side of the figure. Same goes for North,East and South sides)

- In case(b), with volumetric heat generation the maximum temperature has increased drastically beyond 1400 degree celcius. Also the contour plots look more rounded compared to case(a).

- At every time step, the boundary conditions are always satisfied.

- Here the non uniform grid is created using the algebraic equation given, Also Gauss Siedel Method of iteration was used.