Department of Mechanical Engineering
Indian Institute of Technology Bombay

ME415: Computational Fluid Dynamics & Heat Transfer **Autumn 2016**

Assignment # 2: 2D Computational Heat Advection & Convection for Cartesian Geometry

Weightage: 10% **Instructor**: Prof. Atul Sharma

Date Posted: 3rd Sept. (Saturday) **Due Date: 21st Sept. (Wednesday, Early Morning 2 AM)**

ONLINE SUBMISSION THROUGH MOODLE ONLY (No late submission allowed): Create a single zipped file consisting on (a) filled-in answer sheet of this doc file converted into a pdf file and (b) all the computer programs. The name of the zipped file should be **rollnumber_A2**

1. **2-D Computational Heat Advection (CHA) on a uniform grid, with explicit method**: **Flux based methodology**.

Consider a 2D Cartesian (x,y) computational domain of size $L=1m$ and $H=1\ m$, for CHA of a fluid ($\rho=1000\ kg/m^3$ and $c_p=4180\ W/m.K$) moving with a uniform velocity $u=v=1\ m/s$ and an initial temperature of 50^0C. The bottom and left boundary of the domain is subjected to 0^0C and 100^0C, respectively.

Using the flux based solution methodology, develop a computer program "*A2_1_2DAdvection*" for the above problem, run the code for three different advection schemes: (a) FOU, (b) SOU and (c) QUICK. Take the maximum number of grid points in x-and y-direction as $imax=jmax=32$ and convergence criteria as 0.000001. Use the stopping criterion presented in the previous assignment problems, with $\boldsymbol{\Delta T_c=100^oC}$. Report the results as

a) Plot and discuss the steady state temperature contours for the different advection schemes (3 figures).
b) Plot and discuss the temperature profile at the vertical centerline($x=0.5$), $T(y)$, for the different advection schemes (3 figures).

2. **2-D Computational Heat Advection (CHA) on a non-uniform grid, with implicit method**: **Coefficient of LAEs based Methodology.**

Consider the previous problem, as follows:

i. Generate a non-uniform 2D Cartesian grid, using an algebraic method (can be found in slide no. 5.63 to 5.66). The method involves a transformation of a uniform grid in a 1D computational domain (ξ-coordinate) of unit length to a physical domain (x- coordinate) of length L, using an algebraic equation given (*Hoffmann and Chiang, 2000*) as

$$x = L\frac{(1+\beta)\left[(\beta+1)/(\beta-1)\right]^{(2\xi-1)} - (\beta-1)}{2\left\{1+\left[(\beta+1)/(\beta-1)\right]^{(2\xi-1)}\right\}}$$

The above equation is also used in the y-direction to generate the 2D grid. This equation results in a grid which is finest near the two ends (east and west as well as north and south boundary) of the domain and gradually become coarser at the middle of the domain. It is called as equal clustering of grids at both the ends of the domain. Consider maximum number of grid points (for the temperature) as ***imax×jmax=32×32*** and ***β=1.2***.

ii. Using the coefficient of LAEs based solution methodology of CFD development, develop the Gauss-Seidel method based computer program "*A2_2_2DAdvection*" for implicit method on a non-uniform 2-D Cartesian grid.

iii. Using the non-uniform grid and the program, present a CFD application of the code and report the results with plots similar to the previous problem.

3. **2-D Computational Heat Convection on a uniform grid, with explicit method**: **Flux based methodology**.

Consider a 2D Cartesian computational x-y domain of size L=6 unit and H=1 unit, for CHC with a prescribed velocity field. This corresponds to a slug flow (u=1, v=0) of a fluid in a channel; subjected to a non-dimensional temperature of 1 at the inlet and 0 at the walls. At the outlet, fully developed Neumann BC is used. The initial condition for non-dimensional temperature of the fluid is 0.

Using the flux based solution methodology, develop a computer program "*A2_3_2DConvection*" for the above problem, run the code for two different advection schemes: (a) FOU and (b) QUICK; at *Re=10* and *Pr=1 (you can take any value of thermo-physical properties to obtain the given Re and Pr)*. Take the maximum number of grid points in x-and y-direction as *imax=62* and *jmax=22,* respectively; and convergence criteria as 0.000001. Report the results as

i. Plot and discuss the steady state temperature contours for the different advection schemes (2 figures).

ii. Plot and discuss the temperature profile, *T(y)*, at different axial locations (x/L=0.2, 0.4, 0.6, 0.8 and 1), for the different advection schemes (2 figures).

4. **2-D Computational Heat Convection on a non-uniform grid, with implicit method**: **Coefficient of LAEs based Methodology.**

Consider the previous problem, as follows:

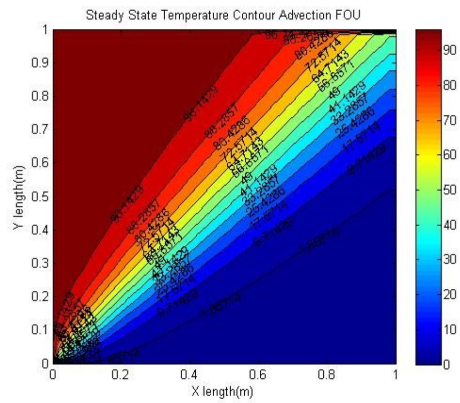
i. Using the coefficient of LAEs based solution methodology of CFD development, develop the Gauss-Seidel method based computer program "*A2_4_2DConvection*" for implicit method on a non-uniform 2-D Cartesian grid, generated for the above Prof. 2, with the maximum number of grid points (for the temperature) as ***imax×jmax=62×22*** and ***β=1.2***

ii. Using the non-uniform grid and the program, present a CFD application of the code and report the results with plots similar to the previous problem.

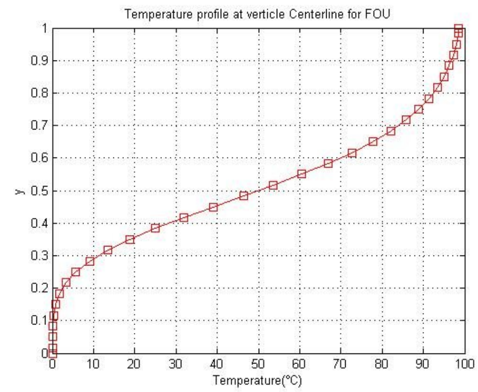
Answer Sheet

Problem # 1: Flux based solution methodology, with explicit method and uniform grid:

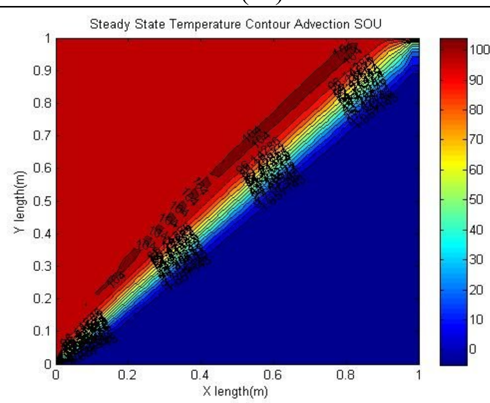
a) Plot and discuss the steady state temperature contours for the different advection schemes (3 figures).
b) Plot and discuss the temperature profile at the vertical centerline(*x=0.5*), *T(y)*, for the different advection schemes (3 figures).



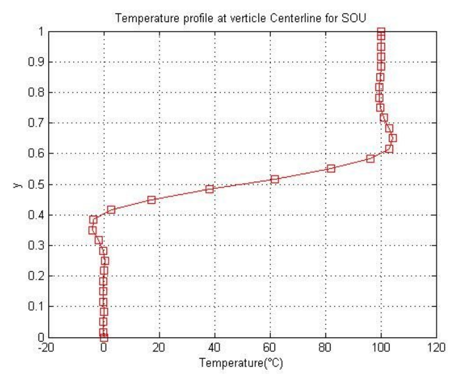
(a1)



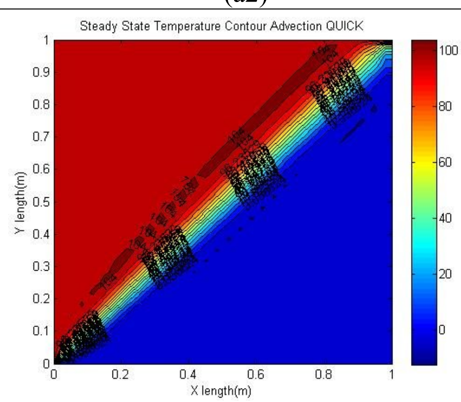
(b1)

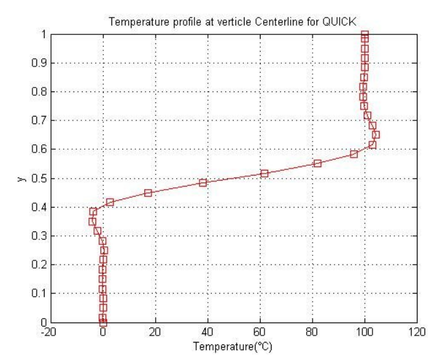


(a2)



(b2)

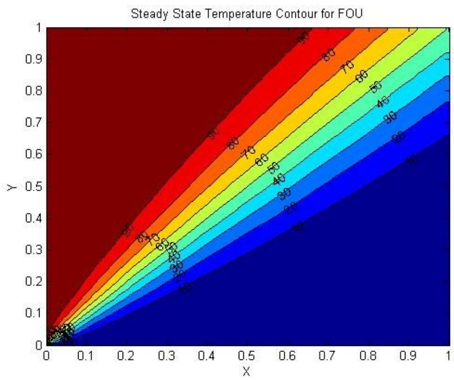




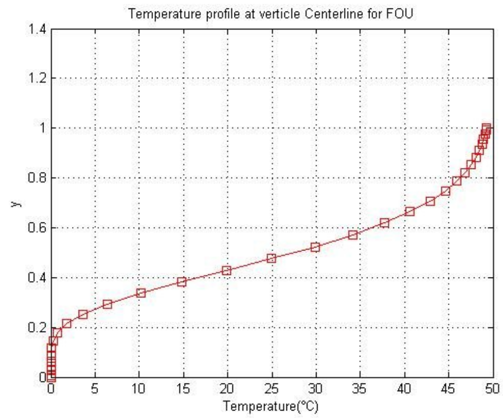
Discussion on the Fig. 2.1: Temperature in FOU scheme is within 0-100 degree celcius whereas in SOU and Quick, temperature vary beyond 0 and 100 degree C.
Also the temperature variation is across a higher width compared for FOU compared to SOU and Quick.
The temperature variation at centerline is smooth in FOU whereas in SOU and QUICK, it remains constant for a while and deviates slightly before increasing further.
Also FOU scheme has been assumed at the north and east boundary since no boundary condition was given there.

Problem # 2: Coefficient of LAEs based solution methodology, with implicit method and non-uniform grid:

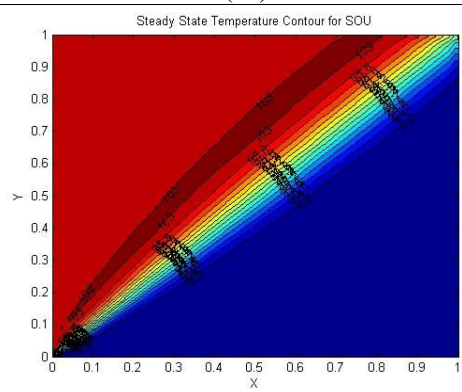
a) Plot and discuss the steady state temperature contours for the different advection schemes (3 figures).
b) Plot and discuss the temperature profile at the vertical centerline($x=0.5$), $T(y)$, for the different advection schemes (3 figures).



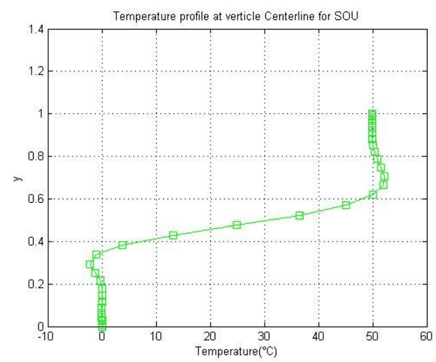
(a1)



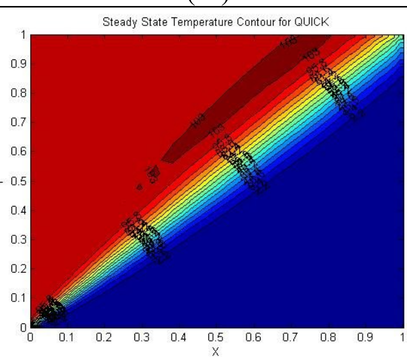
(b1)



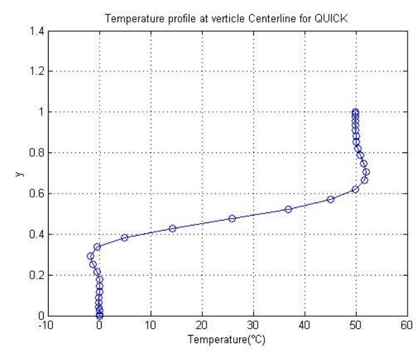
(a2)



(b2)



(a3)



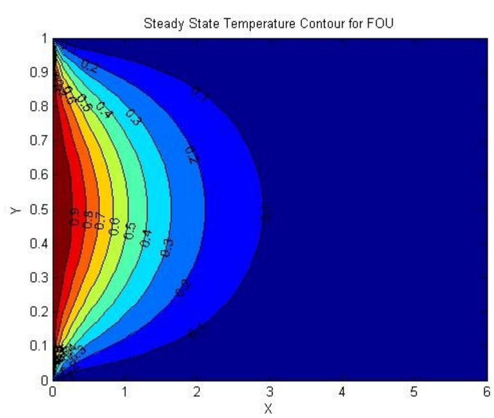
(b3)

Discussion on the Fig. 2.2: The temperature values in FOU advection scheme is within 0 and 100 degree celcius whereas that of SOU and Quick vary beyond 0 and 100 degree celcius.
The width of variation of temperature for FOU is greater compared to SOU and Quick.
The temperature variation at centerline is smooth in case of FOU . In SOU and Quick, temperature remains constant before deviating slightly after which it continues to increase.
The deviation is lesser compared to that in explicit method

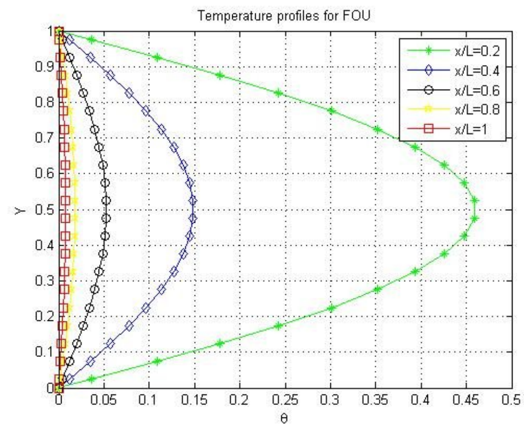
Problem # 3: Flux based solution methodology, with explicit method and uniform grid:

a) Plot and discuss the steady state temperature contours for the different advection schemes (2 figures).

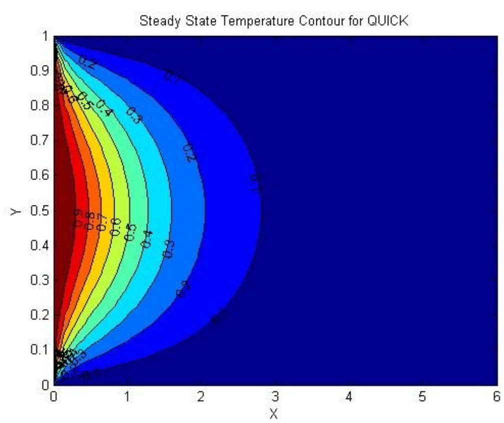
b) Plot and discuss the temperature profile, *T(y)*, at different axial locations (x/L=0.2, 0.4, 0.6, 0.8 and 1), for the different advection schemes (2 figures).



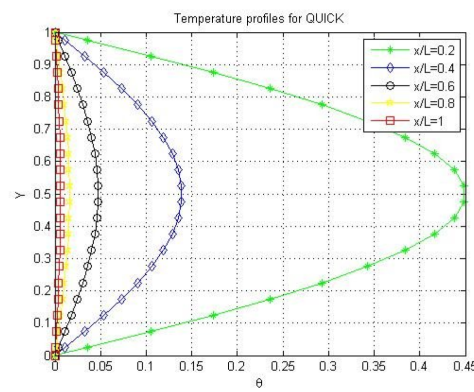
(a1)



(b1)



(a2)



(b2)

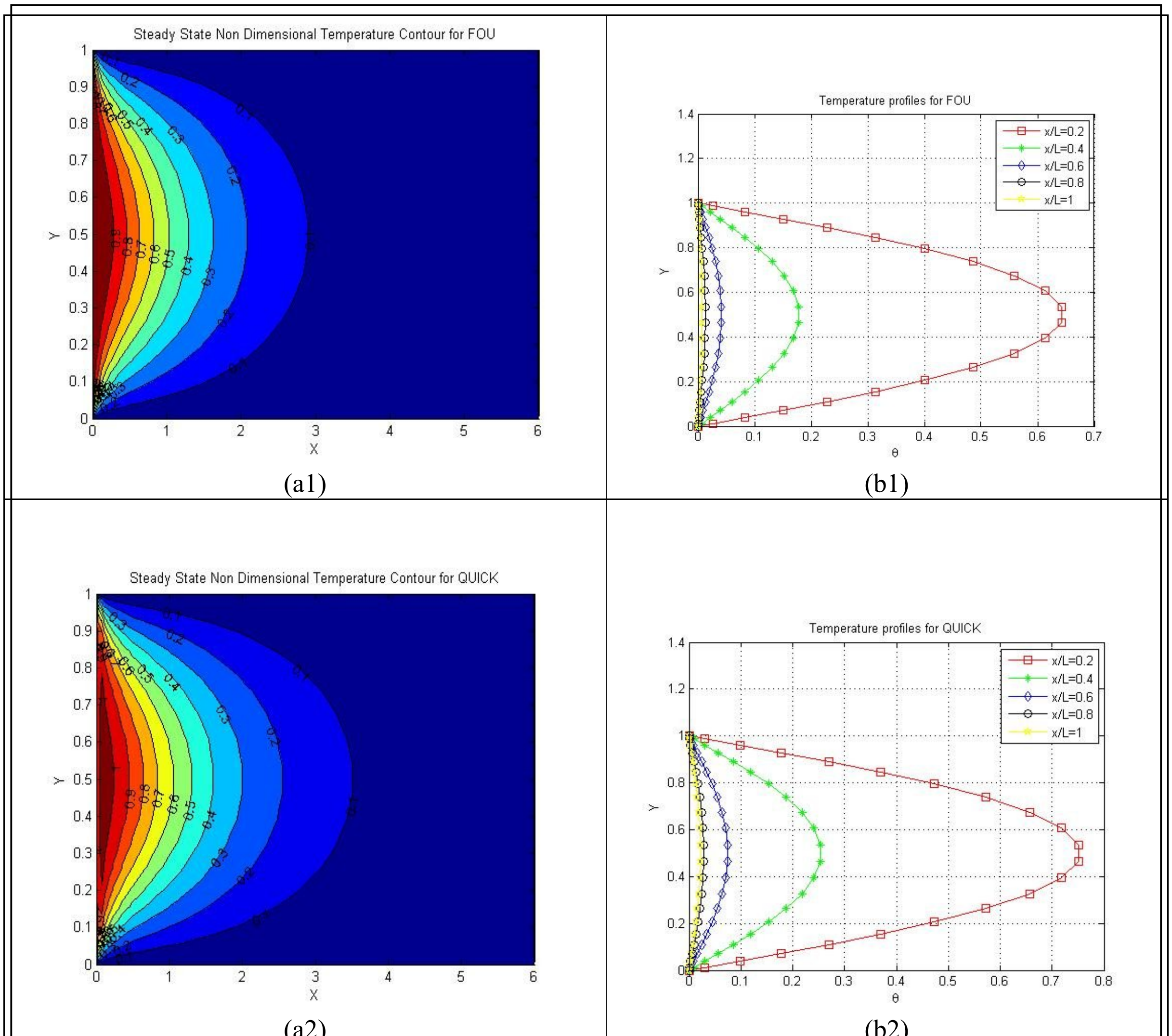
Fig. 2.3: Steady state temperature contours using the (a1) FOU scheme and (a2) QUICK scheme. Temperature profiles using the (b1) FOU scheme and (b2) QUICK scheme

Discussion on the Fig. 2.3: Temperature decreases as the fluid moves from left to right because it moves from higher temperature to lower temperature. Heat transfer is because of advection as well as convection and carries the heat with it.
The temperature contour in FOU is deeper or varies through a larger distance compared to Quick.
As y increases temperature first increases upto a certain maximum at the centre and then decreases. It is symmetric about the centerline.Also as x increases the temperature profile becomes flatter and varies less.This is because at larger distances, it is unaffected by the temperature boundary conditions and temperature does not change drastically.

Problem # 4: Coefficient of LAEs based solution methodology, with implicit method and non-uniform grid:

a) Plot and discuss the steady state temperature contours for the different advection schemes (2 figures).

b) Plot and discuss the temperature profile, *T(y)*, at different axial locations (x/L=0.2, 0.4, 0.6, 0.8 and 1), for the different advection schemes (2 figures).



(a1) (b1)

(a2) (b2)

Fig. 2.4: Steady state temperature contours using the (a1) FOU scheme and (a2) QUICK scheme. Temperature profiles using the (b1) FOU scheme and (b2) QUICK scheme

Discussion on the Fig. 2.4: The temperature values decrease as we move from left to right(along the direction of the fluid flow). This is because fluid is flowing from higher to lower temperature. Hence heat transfer by convection is taking place.

The temperature contour for Quick scheme is deeper than that of FOU i.e temperature varies through a larger distance along the fluid flow.

Along varies vertical lines i.e along y temperature values first increases upto a certain maximum at the centerline and then decreases i.e it is symmetric about the centerline.

The temperature profile varies a lot at the entrance of the flow and then gradually becomes flatter along the direction of the flow.