

**DEPARTMENT OF MECHANICAL ENGINEERING  
INDIAN INSTITUTE OF TECHNOLOGY BOMBAY**

**ME415: Computational Fluid Dynamics & Heat Transfer**

**Assignment # 3:** Computational Fluid Dynamics for Cartesian Geometry on a Uniform

Staggered Grid: **FLUX BASED SOLUTION METHODOLOGY**

**Weightage: 20%**

**Instructor:** Prof. Atul Sharma

**Date Posted:** 4<sup>th</sup> Oct. (Wednesday)

**Due Date:** 19<sup>th</sup> Oct. (Wednesday, Early Morning 2 AM)

**ONLINE SUBMISSION THROUGH MOODLE ONLY (No late submission allowed):** Create a single zipped file consisting on (a) filled-in answer sheet of this doc file converted into a pdf file and (b) all the computer programs. The name of the zipped file should be **rollnumber\_A3**

**Example 9.1 from the book by Sharma (2017):** Lid driven cavity flow probably is the most commonly used problem for testing of an in-house Navier-Stokes solver. This is shown in Fig. 3.1, as a square cavity with the left, right and bottom wall as stationary. The top wall, called here as the lid, acts like a long conveyor-belt and is moving horizontally with a constant velocity  $U_0$ .

The motion results in a lid driven recirculating flow inside the cavity. The cavity is represented by a closed 2D Cartesian square domain of size  $L_1=L_2$ , with all the boundaries as the solid-walls. Figure 3.1 also shows the initial and boundary conditions for the non-dimensional computational set-up of the problem. The simplicity in the shape of the domain and the boundary conditions has led to the wide application of the LDC flow as the

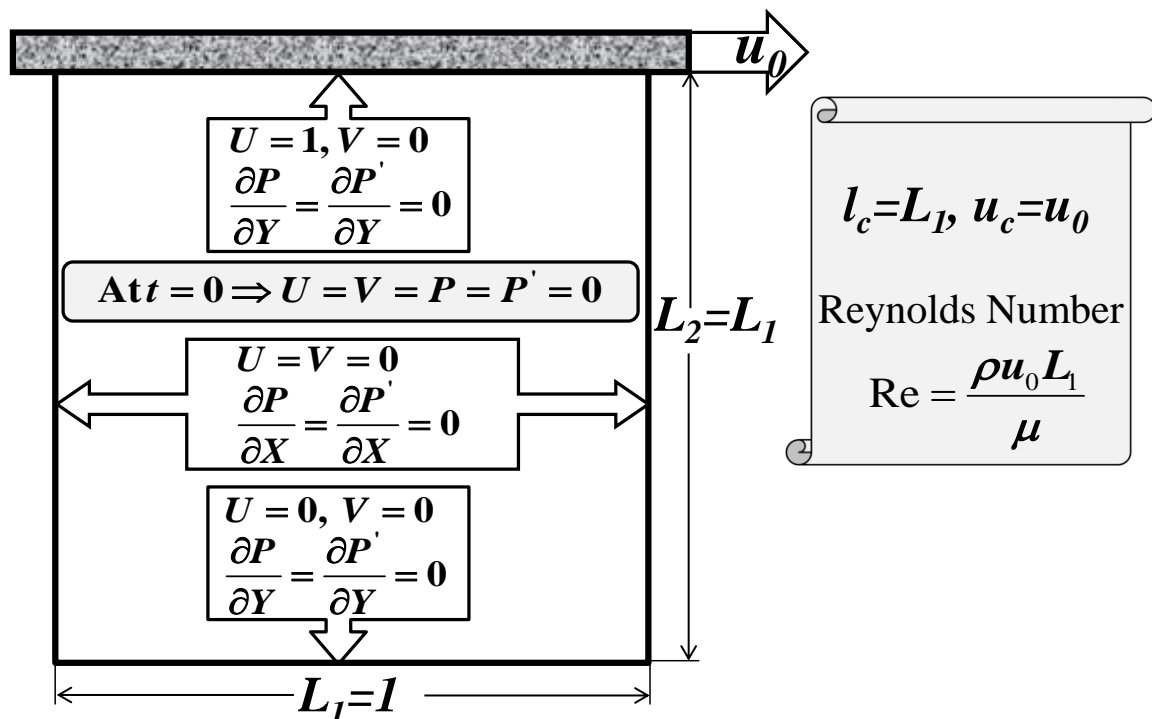


Fig. 3.1: Computational domain and boundary conditions for the lid driven cavity flow.

benchmark problem for a NS solver.

Using the flux based solution methodology of the CFD development presented in the class for the FOU scheme, develop a computer program for the semi-explicit method based 2D unsteady NS solver on a uniform staggered grid. The code should be written in a **non-dimensional** form, with the length of the cavity  $L_1$  as the characteristic length, and the lid velocity  $U_0$  as the characteristic velocity scale. They are shown in Fig. 3.1, along with the governing-parameter for the isothermal flow as Reynolds number  $Re = \rho U_0 L / \mu$ . The  $Re$  is implemented in the code by a computational set-up as  $\rho = U_0 = L = 1$  and  $\mu = 1/Re$ . Use the developed and tested codes in the previous chapters (for conduction and advection heat transfer) as the generic subroutines in the present code-development for the NS solver. After the CFD development, run the code with a convergence tolerance of  $\varepsilon_{st} = 10^{-3}$  for the steady-state, and  $\varepsilon = 10^{-8}$  for the mass-conservation; and perform the following study:

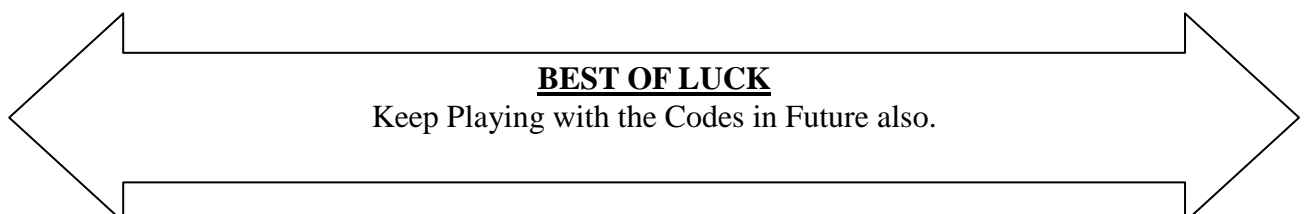
1. **Steady-State flow-patterns:** For a Reynolds number of 100 and a grid size of  $42 \times 42$ , present and discuss a figure for the velocity-vector, U-velocity contour, and pressure-contour in the flow domain (3 figures).
2. **Grid-independence and code-verification study:** For the Reynolds number of 100 and four uniform grid sizes ( $7 \times 7$ ,  $12 \times 12$ ,  $22 \times 22$ , and  $42 \times 42$ ), draw an overlap plot of the results (on all the grid sizes; similar to slide no. 9.66) as follows (1 figure):
  - a) Variation of U-velocity along the vertical centerline.
  - b) Variation of V-velocity along the horizontal centerline.

Discuss on the variation in the results with the change in the grid size. Along with the results for the grid independence study, overlap the benchmark results reported by Ghia et al. (1982) (find attached files: ghia\_dataU.dat & ghia\_dataV.dat, with first column as the coordinate and the second column as the velocity); and discuss the accuracy of the present results (on the grid size of  $42 \times 42$ ) as compared to the benchmark results.

Moreover, during the evaluation/viva, run the problem on a coarser grid size of  $12 \times 12$  and larger steady state convergence criteria of  $10^{-4}$ , due to limited time available during the evaluation; where the simulated results should follow the trend of the benchmark results.

## References

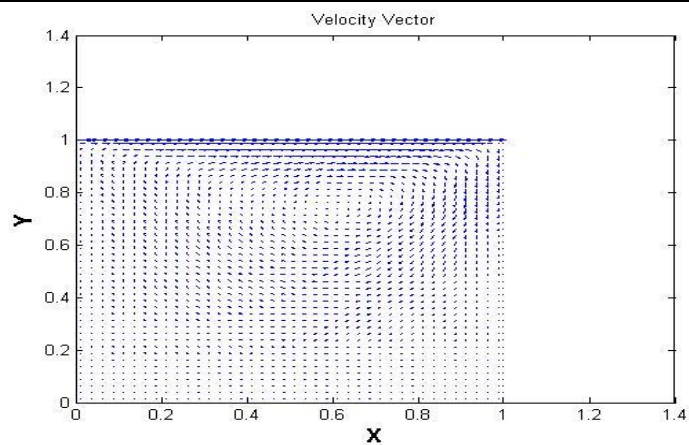
1. **Sharma A., (2017)**, *Introduction to Computational Fluid Dynamics: Development, Application and Analysis*, Athena Academic Limited, London, UK, Chapter 9, pp. 302-306.
2. **Ghia U., Ghia K. N., and Shin C. T. (1982)**. High-Re solutions for incompressible flow using the Navier-Stokes equations and a multigrid method, *J. Comp. Phys.*, vol. 48, pp. 387-411.



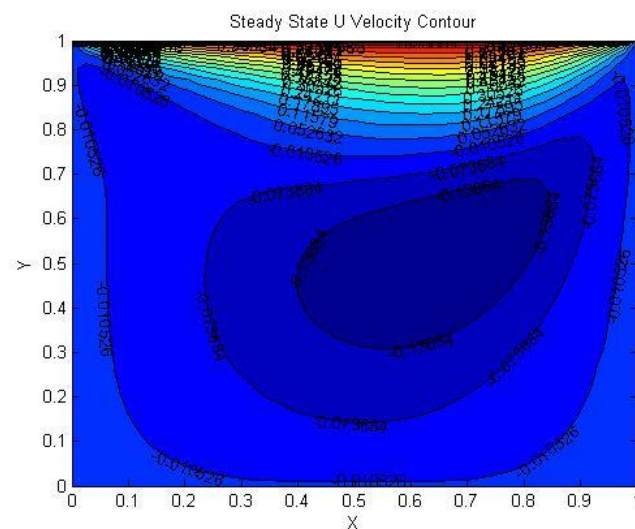
# Answer Sheet

## Problem # 1: Lid Driven Cavity Flow:

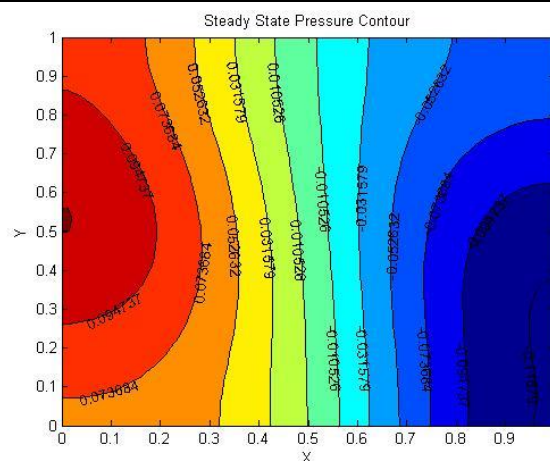
1. **Steady-State flow-patterns:** For a Reynolds number of 100 and a grid size of  $42 \times 42$ , present and discuss a figure for the velocity-vector, U-velocity contour, and pressure-contour in the flow domain (3 figures).



(a)



(b)



- a) Plot and discuss the variation of U-velocity along the vertical and V-velocity along the horizontal centerline of the cavity and its comparison with the benchmark results, on a grid size of  $32 \times 32$ , at  $Re=100$  and  $400$  (2+2 figures). Overlap the results obtained by Ghia et al. (1982), with symbols for published and line for present results.

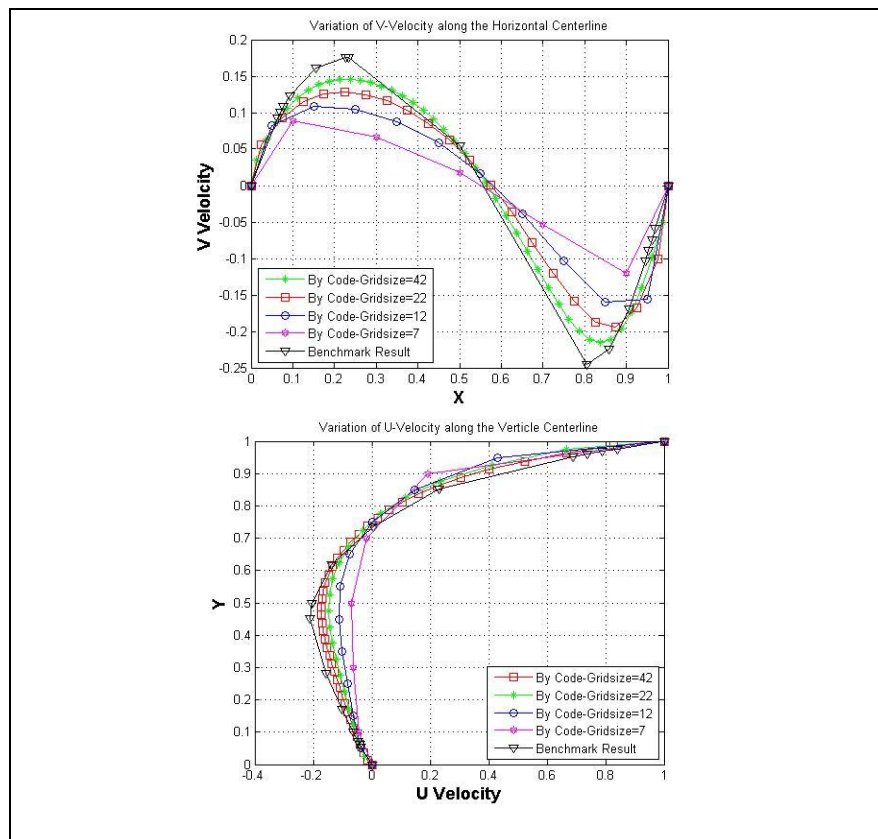


Fig. 3.3: Steady state results obtained for a lid-driven cavity at  $Re=100$ : (a) V-velocity along the horizontal centerline, (b) U-velocity along the vertical centerline; considering the four different uniform grid sizes.. Here, the symbols and lines represent published and present results, respectively.

Discuss Fig. 3.1 and 3.2 here, limited inside this text box only:

- The U velocity contour suggests that it decreases as we move downwards and becomes negative before increasing. A circular flow pattern is observed.
- The velocity vector suggests the boundary conditions that are satisfied. It also depicts closed pattern flow.
- The pressure contour indicates decrease in pressure as we move right.
- The V velocity plot at the horizontal centerline follows a wave (similar to a sinusoidal curve). As the grid size increases, the accuracy of the plot increases, i.e. it starts getting closer to the benchmark results.
- The U velocity plot at the vertical centerline decreases and then increases with increase in  $y$ . Again as the grid size increases, the accuracy of the plot improves, i.e. it starts getting closer to the benchmark results.
- The results for grid size  $42 \times 42$  is very accurate compared to the benchmark results. It is also a lot smoother compared to the benchmark results.

