Interest Rate Models

Programming Assignment #2

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March 26, 2015

The purpose of this assignment is to implement a simple 1-factor LIBOR market model as described in Lecture Notes 7 and 8 (LN7 and LN8). We consider the following 1-factor normal LMM:

$$\begin{aligned} dL_j(t) &= \Delta_j(t, L(t))dt + \sigma_j(t)dZ(t), \\ L_j(0) &= L_{j,0}. \end{aligned} \tag{1}$$

where $\sigma_j(t)$ is a deterministic instantaneous volatility function which should be calibrated to the market. To simplify the project, we make the following assumptions:

(i) We assume that $\sigma_j(t) = 0.0085$, for all t and j (thus circumventing the issue of calibrating the model...).

(ii) The LIBOR / OIS basis is zero, and so the model is a single curve model.

(iii) The dynamics is written under the terminal forward measure, and thus the drift terms Δ_j are given by the appropriate formulas stated in LN7.

For the initial value of each of the SDEs (1) you should use the corresponding LIBOR forward calculated by means of the curve that you have built in Programming Assignment #1.

I also suggest that you take into account the following points:

(i) Ideally the implementation should be done in C++.

(ii) Use the spectral decomposition algorithm to simulate a Brownian motion. Gaussian random numbers should be generated using quality algorithms as described in LN8.

Problems

1. Implement the model using Euler's scheme (note that for the normal LMM, Euler's and Milstein's schemes are identical). For drift term calculations, implement the ability to do both:

   (i) the exact calculation, and

   (ii) the frozen curve approximation.

2. Apply your model to a spot starting 10 year *knock-out swap*. A knock-out swap is an interest rate swap with a special termination feature. Namely, if, on a fixed leg coupon date (or more precisely, two business days before), the 10 year swap rate sets below a preset barrier B, the swap is terminated. Notice that this is a path dependent derivative and Monte Carlo simulations are an appropriate approach to pricing this product. Use 2,000 simulated paths to carry out the calculation. As a variance reducing method, you may also consider using antithetic variables.

   (i) Assuming $B = 0.95\%$, determine the break-even rate on the fixed leg of the swap.

   (ii) How accurate is your calculation? Compare against a run with 5,000 simulated paths.

   (iii) Analyse the performance of each of the drift terms calculation methods, and the accuracy of the frozen curve approximation.

This assignment is due on May 14.

① How to get swap rate at T=10?

$$S = \frac{\sum L_{j,10} P_{j,10}}{\sum P_{j,10}}$$

② How to price swap at T=30?

$$V = N\left(\text{Fixed} \sum P_{j,30} - \sum L_{j,30} P_{j,30}\right)$$

③ How to calculate $L_{j,t}$.

$$L_{j,t+1} = L_{j,t} + \Delta L_{j,t}$$

④ How to calculate $P_{j,t}$.

$P_{j,t}$