Assignment 6

2015年5月4日 12:37

1. (i) Model (1) is inadequate for prepayment modeling, for the following reasons:

(ii). $P(\tau > t \mid \mathcal{F}_t) = \exp\left(-\int_0^t \lambda(s)\,ds\right)$

$= \exp\left(-\int_0^t \lambda\gamma(\lambda s)^{\gamma-1} ds\right)$

$= \exp(-\lambda^\gamma t^\gamma)$

$\therefore$ survial probability $S(0,t) = e^{-(\lambda t)^\gamma}$

(iii) The Price of TBA is given by

$$P(T) = E\left[\sum_j Z(0,T_j) \times \left(S(0,T_{j-1})C + (S(0,T_{j-1}) - S(T,T_j))B_j\right)\right]$$

where $Z(0,T_j) = Z(0,\frac{j}{12})$ is the discount factor

$$S(0,T_j) = e^{-(\lambda T_j)^\gamma} = e^{-(\lambda \frac{j}{12})^\gamma}.$$

$$B_j = \frac{1-d^{N-j}}{1-d^N}, \quad d = \frac{1}{1+\frac{C}{12}} = \frac{12}{12+C}$$

$$P(T) = \sum_{j=1}^{N} Z(0,\tfrac{j}{12})\left[e^{-\left(\lambda\frac{j-1}{12}\right)^\gamma} C + \left(e^{-(\lambda\frac{j-1}{12})^\gamma} - e^{-(\lambda\frac{j}{12})^\gamma}\right)\frac{1-\left(\frac{12}{12+C}\right)^{N-j}}{1-\left(\frac{12}{12+C}\right)^N}\right]$$

2. (i)

(ii) $S(0,t) = \exp\left(-\int_0^t \lambda(s)\,ds\right)$

$$= \exp\left(-\int_0^t \frac{\lambda\gamma(\lambda s)^{\gamma-1}}{1+(\lambda s)^\gamma} ds\right)$$

$\tau = \lambda s$

$$= \exp\left(-\int_0^{\lambda t} \frac{\gamma\tau^{\gamma-1}}{1+\tau^\gamma} d\tau\right)$$

$x = \tau^\gamma$

$$= \exp\left(-\int_0^{(\lambda t)^\gamma} \frac{1}{1+x} dx\right)$$

$$= \exp\left(-\ln\left(1+(\lambda t)^\gamma\right)\right)$$

$$= \frac{1}{1+(\lambda t)^\gamma}$$

(iii)

The Price of TBA is given by

$$P(T) = E\left[\sum_j Z(0, T_j) \times \left(S(0, T_{j-1})C + (S(0, T_{j-1}) - S(T, T_j))B_j\right)\right]$$

where $Z(0, T_j) = Z(0, \frac{j}{12})$ is the discount factor

$$S(0, T_j) = \frac{1}{1+(\lambda_j)^r} = \frac{1}{1+(\frac{\lambda_j}{12})^r}$$

$$B_j = \frac{1-d^{N-j}}{1-d^N}, \quad d = \frac{1}{1+\frac{C}{12}} = \frac{12}{12+C} \qquad T_j = \frac{j}{12}.$$

$$P(T) = \sum_{j=1}^{N} Z(0, \tfrac{j}{12})\left(\frac{1}{1+(\frac{\lambda(j-1)}{12})^r}C + \left(\frac{1}{1+(\frac{\lambda(j-1)}{12})^r} - \frac{1}{1+(\frac{\lambda j}{12})^r}\right)\frac{1-(\frac{12}{12+C})^{N-j}}{1-(\frac{12}{12+C})^N}\right)$$