Assignment 6

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1\. (i) Model (1) is inadequate for prepayment modeling, for the following reasons:
The Hazard rate in this model is independent with interest rate and coupon rate. If intensity go up, morgage rate should down.

(ii). $P(\tau > t \mid \mathcal{F}_t) = \exp(-\int_0^t \lambda(s)\,ds)$

$$= \exp(-\int_0^t \lambda\gamma(\lambda s)^{\gamma-1} ds)$$

$$= \exp(-\lambda^\gamma t^\gamma)$$

$\therefore$ survial probability $S(0,t) = e^{-(\lambda t)^\gamma}$

(iii) The Price of TBA is given by

$$P(T) = E[\sum_j Z(0,T_j) \times (S(0,T_{j-1})C + (S(0,T_{j-1}) - S(T,T_j))B_j)]$$

where $Z(0,T_j) = Z(0,\frac{j}{12})$ is the discount factor

$$S(0,T_j) = e^{-(\lambda T_j)^\gamma} = e^{-(\lambda\frac{j}{12})^\gamma}$$

$$B_j = \frac{1-d^{N-j}}{1-d^N}, \quad d = \frac{1}{1+\frac{C}{12}} = \frac{12}{12+C}$$

$$P(T) = \sum_{j=1}^{N} Z(0,\tfrac{j}{12})\left[e^{-(\lambda\frac{j-1}{12})^\gamma}C + (e^{-(\lambda\frac{j-1}{12})^\gamma} - e^{-(\lambda\frac{j}{12})^\gamma})\frac{1-(\frac{12}{12+C})^{N-j}}{1-(\frac{12}{12+C})^N}\right.$$

2\. (i) It is alco inadequate
$\lambda(t)$ is independent with montgage rate and coupon rate

(ii) $S(0,t) = \exp(-\int_0^t \lambda(s)\,ds)$

$$= \exp(-\int_0^t \frac{\lambda\gamma(\lambda s)^{\gamma-1}}{1+(\lambda s)^\gamma} ds)$$

$$\overset{\tau=\lambda s}{=} \exp(-\int_0^{\lambda t} \frac{\gamma\tau^{\gamma-1}}{1+\tau^\gamma} d\tau)$$

$$\overset{x=\tau^\gamma}{=} \exp(-\int_0^{(\lambda t)^\gamma} \frac{1}{1+x} dx)$$

$$= \exp(-\ln(1+(\lambda t)^\gamma))$$

$$= \frac{1}{1+(\lambda t)^\gamma}$$

(iii)

The Price of TBA is given by

$$P(T) = E\left[\sum_j Z(0, T_j) \times \left(S(0, T_{j-1}) C + (S(0, T_{j-1}) - S(T, T_j)) B_j\right)\right]$$

where $Z(0, T_j) = Z(0, \frac{j}{12})$ is the discount factor

$$S(0, T_j) = \frac{1}{1+(\lambda t)^r} = \frac{1}{1+\left(\frac{\lambda j}{12}\right)^r}$$

$$B_j = \frac{1-d^{N-j}}{1-d^N}, \quad d = \frac{1}{1+\frac{c}{12}} = \frac{12}{12+c} \qquad T_j = \frac{j}{12}.$$

$$P(T) = \sum_{j=1}^{N} Z\left(0, \frac{j}{12}\right)\left(\frac{1}{1+\left(\frac{\lambda(j-1)}{12}\right)^r} C + \left(\frac{1}{1+\left(\frac{\lambda(j-1)}{12}\right)^r} - \frac{1}{1+\left(\frac{\lambda j}{12}\right)^r}\right) \frac{1-\left(\frac{12}{12+c}\right)^{N-j}}{1-\left(\frac{12}{12+c}\right)^N}\right)$$

(v). model (2) is better, since

model (2)'s shape is more reasonal shape

it has a bell shape, but model (1)'s shape

is monotonal increase, which is not a good estimation

When $t \to \infty$, this is extremely obvious that,

model (1) is too big, but model 2 is reasonable