Interest Rate Models

Homework Assignment #7

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The purpose of this assignment is to implement a simple 1-factor LIBOR market model as described in Lecture Notes #7 and #8 (LN7 and LN8). We consider the following 1-factor normal LMM:

$$\begin{aligned} dL_j(t) &= \Delta_j(t, L(t))dt + \sigma_j(t)dZ(t), \\ L_j(0) &= L_{j,0}. \end{aligned} \tag{1}$$

where $\sigma_j(t)$ is a deterministic instantaneous volatility function which should be calibrated to the market. To simplify the project, we make the following assumptions:

(i) We assume that $\sigma_j(t) = 0.0085$, for all t and j (thus circumventing the issue of calibrating the model...). Means no need to calibrate

(ii) The LIBOR / OIS basis is zero, and so the model is a single curve model.

(iii) The dynamics is written under the terminal forward measure, and thus the drift terms Δ_j are given by the appropriate formulas stated in LN7.

For the initial value of each of the SDEs (1) you should use the corresponding LIBOR forward calculated by means of the curve that you have built in Homework Assignment #1.

I also suggest that you take into account the following points:

(i) Ideally the implementation should be done in Python (or C++). Make sure that the Gaussian random numbers are generated by means of a quality algorithm (such as described in LN8).

(ii) Use the spectral decomposition algorithm to simulate a Brownian motion.

Problems

1. Implement the model using Euler's scheme (note that for the normal LMM, Euler's and Milstein's schemes are identical). For drift term calculations, implement the ability to do both:

   (i) the exact calculation, and

   (ii) the frozen curve approximation.

2. Apply your model to a 1Y into 10Y European receiver swaption struck at 3.872%. Use 2,000 simulated paths to carry out the calculation. If you wish to implement a variance reducing method, you may consider using antithetic variables.

   (i) How accurate is your calculation? Compare against a run with 5,000 simulated paths.

   (ii) Compare the performance and accuracy of both drift term calculation methods.

This assignment is due on May 8.