

Assignment 6

2015年5月4日 12:37

1. (i) Model (1) is inadequate for prepayment modeling, for the following reasons:

$$\begin{aligned} \text{(ii). } P(\tau > t | \mathcal{F}_t) &= \exp\left(-\int_0^t \lambda(s) ds\right) \\ &= \exp\left(-\int_0^t \lambda r(\lambda s)^{r-1} ds\right) \\ &= \exp(-\lambda t^r), \end{aligned}$$

$$\therefore \text{ survival probability } S(0, t) = e^{-(\lambda t)^r}$$

- (iii) The Price of TBA is given by

$$P(t) = E\left[\sum_j Z(0, T_j) \times (S(0, T_{j-1})C + (S(0, T_{j-1}) - S(T, T_j))B_j)\right]$$

where $Z(0, T_j) = Z(0, \frac{1}{12})$ is the discount factor

$$S(0, T_j) = e^{-(\lambda T_j)^r} = e^{-(\lambda \frac{1}{12})^r}$$

$$B_j = \frac{1-d^{N-j}}{1-d^N}, \quad d = \frac{1}{1+\frac{c}{12}} = \frac{12}{12+c}$$

$$P(t) = \sum_{j=1}^N Z(0, \frac{1}{12}) \left[e^{-(\lambda \frac{1}{12})^r} C + (e^{-(\lambda \frac{1}{12})^r} - e^{-(\lambda \frac{1}{12})^r}) \frac{1 - (\frac{12}{12+c})^{N-j}}{1 - (\frac{12}{12+c})^N} \right]$$

2. (i)

$$\begin{aligned} \text{(ii)} \quad S(0, t) &= \exp\left(-\int_0^t \lambda(s) ds\right) \\ &= \exp\left(-\int_0^t \frac{\lambda r(\lambda s)^{r-1}}{1+(\lambda s)^r} ds\right) \end{aligned}$$

$$\begin{aligned} &\stackrel{z=\lambda s}{=} \exp\left(-\int_0^{\lambda t} \frac{z^{r-1}}{1+z^r} dz\right) \end{aligned}$$

$$\begin{aligned} &\stackrel{x=z^r}{=} \exp\left(-\int_0^{\lambda t^r} \frac{1}{1+x} dx\right) \end{aligned}$$

$$= \exp\left(-\ln(1+(\lambda t)^r)\right)$$

$$= \frac{1}{1+(\lambda t)^r}$$

(iii)

The Price of TBA is given by

$$P(t) = E \left[\sum_j Z(t, T_j) \times (S(t, T_{j-1})C + (S(t, T_{j-1}) - S(t, T_j))B_j) \right]$$

where $Z(t, T_j) = Z(t, T_j)$ is the discount factor

$$S(t, T_j) = \frac{1}{1 + (\lambda + 1)r} = \frac{1}{1 + (\frac{\lambda j}{12})r}$$

$$B_j = \frac{1 - d^{N-j}}{1 - d^N}, \quad d = \frac{1}{1 + \frac{c}{12}} = \frac{12}{12 + c}, \quad T_j = \frac{j}{12}$$

$$P(t) = \sum_{j=1}^N Z(t, \frac{j}{12}) \left(\frac{1}{1 + (\frac{\lambda(j-1)}{12})r} C + \left(\frac{1}{1 + (\frac{\lambda(j-1)}{12})r} - \frac{1}{1 + (\frac{\lambda j}{12})r} \right) \frac{1 - (\frac{12}{12+c})^{N-j}}{1 - (\frac{12}{12+c})^N} \right)$$