**PHYS 514: General Relativity Lecture Notes**
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*What is GR?* GR is the modern theory of classical gravity that incorporates the effects of special relativity. The basic idea is the following: gravity is a force. Most forces are described by fields. *What are fields*? We have the following examples of fields:

(1) The Newtonian gravitational Potential, $\Phi$.
(2) For electromagnetism, we have the electromagnetic fields $E$ and $B$.

We call such a theory a **field theory**. A classical field theory has two ingredients that make up a theory of forces described by fields:

(1) **A field equation**: an equation of motion which tells us exactly how the field is determined by some set of forces. This is typically a 2nd order ODE. For example:
    (1) The field equation for Newtonian gravity is $\nabla^2\Phi = 4\pi G\rho$.
    (2) The field equations for E&M are Maxwell's equations.
(2) **Force Law:** an equation which determines how objects move in the presence of a field. For example:
    (1) For Newtonian gravity, this is $F = ma = m\nabla\Phi$.
    (2) For E&M, this is the Lorentz force law $F = q(E + v \times B)$.

The fields are functions of points in space-time. We write them as $\Phi(t, x)$ for the gravitational field or $E(t, x)$ for the electric field. The force law tells us how the objects move, i.e., how they deviate from a straight line ($a = 0$ ). The modern perspective of GR is the following: it is not a field theory; it is something totally different– gravity is not due to a field $\Phi$ that is a function of space-time, but is instead a feature of space-time itself. Hence, we replace the $\Phi$ with a "metric tensor" which describes the curvature / geometry of space-time.

The field equation of GR determines how space-time curves in the presence of matter or energy. This field equation is a differential equation for the metric of space-time that helps us determine how space-time curves in the presence of matter.

The force law is the geodesic equation, which tells us how objects move through curved space-time. In short, this means that objects travel on **geodesics**, which are as straight as possible given the curvature of space-time. For example, on a plane, the shortest way to travel between two points is simply a line. However, for a sphere, the curve that minimises distance is the arc of a great circle.