Winter 2020 Semester (Results, Definitions, and Theorems) Lecture: 09

Chapter 9: Metric Spaces (General Properties)

Class: Math 455 (Analysis 4) Date: 9 February 2020 Shereen Elaidi

Abstract

This document contains a summary of all the key definitions, results, and theorems from class. There are probably typos, and so I would be grateful if you brought those to my attention :-).

Syllabus: L^p space, duality, weak convergence, Young, Holder, and Minkowski inequalities, point-set topology, topological space, dense sets, completeness, compactness, connectedness, path-connectedness, separability, Tychnoff theorem, Stone-Weierstrass Theorem, Arzela-Ascoli, Baire category theorem, open mapping theorem, closed graph theorem, uniform boudnedness principle, Hahn Banch theorem.

9.1. Examples of Metric Spaces

Definition 1 (Metric Space)**.** Let X be a non-empty set. A function $\rho : X \times X \to \mathbb{R}$ is called a **metric** if $\forall\ x, y \in X$:

(i) $\rho(x,y) \geq 0$
(ii) $\rho(x,y) = 0 \iff x = y$
(iii) $\rho(x,y) = \rho(y,x)$
(iv) $\rho(x,z) \leq \rho(x,y) + \rho(y,z)$ (**Triangle Inequality**).

A non-empty set together with a metric, denoted (X, ρ) is called a **metric space**.

Definition 2 (Discrete Metric)**.** For any non-empty set X, the **discrete metric** ρ is defined by setting $\rho(x,y) = 0$ if $x = y$ and $\rho(x,y) = 1$ if $x \neq y$.

Definition 3 (Metric Subspace)**.** For any metric space (X, ρ), let $Y \subseteq X$ be non-empty. Then, the restriction of ρ to $Y \times Y$ defines a metric on Y. We define this induced metric space as a **metric subspace**.