

Math 458: Differential Geometry

Shereen Elaidi

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Contents

1	Introduction	1
1.1	Implicit and Inverse Function Theorems	1
2	Manifolds in \mathbb{R}^3	1
2.1	Definitions	2
2.2	Smooth Maps from $M^m \rightarrow N^n$	2
2.3	Change of Coordinates	2
2.4	Multi-Linear Algebra	2
2.5	Differential Forms in M^n	2
2.6	Change of Variables for Integrals in \mathbb{R}^n	2
2.7	Integrating a n -Form on M^n ($\int_M \omega$)	2
3	Curves	2
3.1	Definitions	2
3.2	Frenet-Serret Frame	2
3.3	Global Properties of Curves	2
3.3.1	The Isoperimetric Inequality	2
3.3.2	Cauchy Crofton Formula	2
4	Surfaces	2
4.1	Definitions	2
4.2	Differentiable Functions on Surfaces	2
4.3	Tangent Plane	2
4.4	First Fundamental Form: Area	2
5	The Gauss Map	2
5.1	Ruled Surfaces and Minimal Surfaces	2
6	The Intrinsic Geometry of Surfaces	2
6.1	Isometries and Conformal Maps	2

1 Introduction

1.1 Implicit and Inverse Function Theorems

2 Manifolds in \mathbb{R}^3

The aim of this part of the course is to build up to integration on manifolds and the invariant Stokes' theorem. The main purpose of this sections is to develop *coordinate-free* calculus, which clarifies the essence of what is happening (sometimes coordinates can be noisy).

2.1 Definitions

2.2 Smooth Maps from $M^m \rightarrow N^n$

2.3 Change of Coordinates

2.4 Multi-Linear Algebra

2.5 Differential Forms in M^n

2.6 Change of Variables for Integrals in \mathbb{R}^n

2.7 Integrating a n -Form on M^n ($\int_M \omega$)

3 Curves

3.1 Definitions

3.2 Frenet-Serret Frame

3.3 Global Properties of Curves

3.3.1 The Isoperimetric Inequality

3.3.2 Cauchy Crofton Formula

4 Surfaces

4.1 Definitions

4.2 Differentiable Functions on Surfaces

4.3 Tangent Plane

4.4 First Fundamental Form: Area

5 The Gauss Map

5.1 Ruled Surfaces and Minimal Surfaces

6 The Intrinsic Geometry of Surfaces

6.1 Isometries and Conformal Maps