5 Introduction to Factorial Designs

5.1 Basic Definitions and Principles

Many experiments involve the study of the effects of two or more factors. In general, factorial designs are most efficient for this type of experiment. By a factorial design, we mean that in each complete trial or replication of the experiment all possible combinations of the levels of the factors are investigated.

The effect of a factor is defined to be the change in response produced by a change in the level of the factor – main effect. Consider the example.

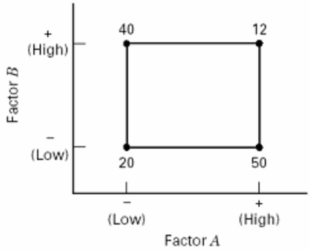


Figure 5-2 A two-factor factorial experiment with interaction.

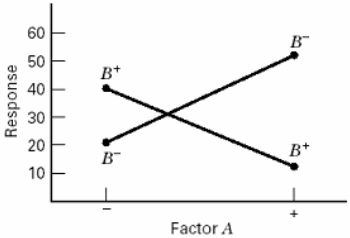


Figure 5-4 A factorial experiment with interaction.

There is another way to illustrate the concept of interaction. Suppose that both of design factors are quantitative. Then a regression model representation of the two-factor experiment could be written as

$$y = \beta_0 + \beta_1 x_1 + \beta_2 x_2 + \beta_{12} x_1 x_2 + \epsilon$$

where y is the response, the β's are parameters whose values are to be determined, x_1 is a variable that represents factor A, x_2 is a variable that represents factor B.

The parameter estimates in this model turn to be related to the effects estimates. For example, from previous example, we have

$$A^- B^- = 20, A^- B^+ = 30, A^+ B^- = 40, A^+ B^+ = 52, A = 21, B = 11, AB = 1$$

and thus,

$$\beta_1 = A/2 = 10.5, \beta_2 = B/2 = 5.5, \beta_{12} = AB/2 = 0.5, \beta_0 + (20 + 30 + 40 + 52)/4 = 35.5$$

Now we have

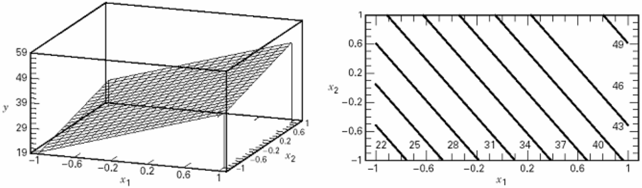
$$\hat{y} = 35.5 + 19.5 x_1 + 5.5 x_2 + 0.5 x_1 x_2$$

Since β_{12} is small, by dropping it give the model:

$$\hat{y} = 35.5 + 19.5 x_1 + 5.5 x_2$$

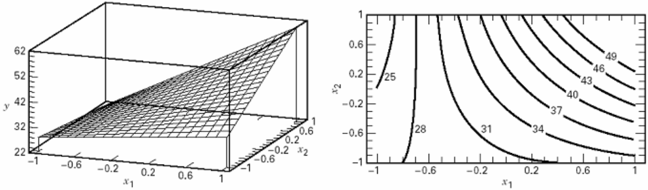
Now suppose that the interaction was not negligible, Figure 5.6 presents the response surface and contour plot for model

$$\hat{y} = 35.5 + 10.5x_1 + 5.5x_2 + 8x_1x_2$$



(a) The response surface (b) The contour plot

Figure 5-5 Response surface and contour plot for the model $\hat{y} = 35.5 + 10.5x_1 + 5.5x_2$.



(a) The response surface (b) The contour plot

Figure 5-6 Response surface and contour plot for the model $\hat{y} = 35.5 + 10.5x_1 + 5.5x_2 + 8x_1x_2$.

5.2 The advantage of Factorials

Suppose that we have tow factors A and B, each at two levels. We denote the levels of the factors by A^-, A^+, B^-, B^+. The effect of changing factor A is given by $A^+B^- - A^-B^-$, and the effect of changing factor B is given by $A^-B^+ - A^-B^-$. It is desirable to take two observations. Thus, the total of six observation are required. If the factorial design has been performed, an additional treatment combination, A^+B^+ would have been taken.

5.3 The two-Factor Factorial Design

5.3.1 An Example

An engineering is designing a battery for use in a device that will be subjected to some extreme variation in temperature. Another factor is the plate material for battery. The engineering decides to test all three materials at three temperature levels, $15, 70, 125^\circ$.

Table 5-1 Life (in hours) Data for the Battery Design Example

Material Type	Temperature (°F)					
	15		70		125	
1	130	155	34	40	20	70
	74	180	80	75	82	58
2	150	188	136	122	25	70
	159	126	106	115	58	45
3	138	110	174	120	96	104
	168	160	150	139	82	60

This is a 3^2 **Factorial design**.

1. What effects do material type & temperature have on life?

2. Is there a choice of material that would give uniformly long life regardless of temperature (a robust product)?

This design is a specific example of the general case of a two-factor factorial. There are several ways to write the model for a factorial design. The effects model is

$$y_{ijk} = \mu + \tau_i + \beta_j + (\tau\beta)_{ij} + \epsilon_{ijk}, \begin{cases} j = 1, 2, ..., a \\ j = 1, 2, ..., b \\ k = 1, 2, ..., n \end{cases}$$

Both factor are fixed and the constraints are

$$\sum_{i=1}^{a} \tau_i = 0, \quad \sum_{j=1}^{b} \beta_j = 0, \quad \sum_{i=1}^{a} (\tau\beta)_{ij} = \sum_{j=1}^{b} (\tau\beta)_{ij} = 0$$

Another possible model is the means model:

$$y_{ijk} = \mu_{ij} + \epsilon_{ijk}, \begin{cases} j = 1, 2, ..., a \\ j = 1, 2, ..., b \\ k = 1, 2, ..., n \end{cases}$$

where the mean of the ijth cell is

$$\mu_{ij} = \mu + \tau_i + \beta_j + (\tau\beta)_{ij}$$

Table 5-2 General Arrangement for a Two-Factor Factorial Design

		Factor B			
		1	2	...	b
Factor A	1	$y_{111}, y_{112}, \ldots, y_{11n}$	$y_{121}, y_{122}, \ldots, y_{12n}$		$y_{1b1}, y_{1b2}, \ldots, y_{1bn}$
	2	$y_{211}, y_{212}, \ldots, y_{21n}$	$y_{221}, y_{222}, \ldots, y_{22n}$		$y_{2b1}, y_{2b2}, \ldots, y_{2bn}$
	$\vdots$				
	a	$y_{a11}, y_{a12}, \ldots, y_{a1n}$	$y_{a21}, y_{a22}, \ldots, y_{a2n}$		$y_{ab1}, y_{ab2}, \ldots, y_{abn}$

In two-factor factorial, both row and column factors, A and B, are of equal interest. So the hypotheses tested are

$$H_0 : \tau_1 = \tau_2 = \cdots = \tau_a = 0, \quad \text{vs.} \quad H_1 : \text{ at least one } \tau_i \neq 0$$

$$H_0 : \beta_1 = \beta_2 = \cdots = \beta_b = 0, \quad \text{vs.} \quad H_1 : \text{ at least one } \beta_j \neq 0$$

and

$$H_0 : (\tau\beta)_{ij} = 0 \quad \text{for all} i, j, \quad \text{vs} \quad H_1 : \text{ at least one } (\tau\beta)_{ij} \neq 0$$

5.3.2 Statistical Analysis of the Fixed Effects Model

Notation:

$$y_{i..} = \sum_{j=1}^{b}\sum_{k=1}^{n} y_{ijk}, \qquad \bar{y}_{i..} = \frac{y_{i..}}{bn}, \qquad i = 1, 2, ..., a$$

$$y_{.j.} = \sum_{i=1}^{a}\sum_{k=1}^{n} y_{ijk}, \qquad \bar{y}_{.j.} = \frac{y_{.j.}}{an}, \qquad j = 1, 2, ..., b$$

$$y_{ij.} = \sum_{k=1}^{n} y_{ijk}, \qquad \bar{y}_{ij.} = \frac{y_{ij.}}{n}, \qquad \begin{cases} i = 1, 2, ..., a \\ j = 1, 2, ..., b \end{cases}$$

$$y_{...} = \sum_{i=1}^{a}\sum_{j=1}^{b}\sum_{k=1}^{n} y_{ijk}, \qquad \bar{y}_{...} = \frac{y_{..}}{abn}$$

The total corrected sum of squares may be written as

$$\sum_{i=1}^{a}\sum_{j=1}^{b}\sum_{k=1}^{n}(y_{ijk} - \bar{y}_{...})^2 = bn\sum_{i=1}^{a}(\bar{y}_{i..} - \bar{y}_{...})^2$$

$$+ an\sum_{j=1}^{b}(\bar{y}_{.j.} - \bar{y}_{...})^2 + n\sum_{i=1}^{a}\sum_{j=1}^{b}(\bar{y}_{ij.} - \bar{y}_{i..} - \bar{y}_{.j.} + \bar{y}_{...})^2 + \sum_{i=1}^{a}\sum_{j=1}^{b}\sum_{k=1}^{n}(y_{ijk} - \bar{y}_{ij.})^2$$

Further we have

$$\underbrace{SS_T}_{abn-1} = \underbrace{SS_A}_{a-1} + \underbrace{SS_B}_{b-1} + \underbrace{SS_{AB}}_{(a-1)(b-1)} + \underbrace{SS_E}_{ab(n-1)}$$

$$E(MS_A) = \sigma^2 + \frac{bn\sum_{i=1}^{a}\tau_i^2}{a-1}, \quad E(MS_B) = \sigma^2 + \frac{an\sum_{j=1}^{b}\beta_j^2}{b-1}$$

$$E(MS_{AB}) = \sigma^2 + \frac{n\sum_{i=1}^{a}\sum_{j=1}^{b}(\tau\beta)_{ij}^2}{(a-1)(b-1)}, \quad E(MS_E) = \sigma^2$$

Table 5-3 The Analysis of Variance Table for the Two-Factor Factorial, Fixed Effects Model

Source of Variation	Sum of Squares	Degrees of Freedom	Mean Square	F_0
A treatments	SS_A	$a-1$	$MS_A = \frac{SS_A}{a-1}$	$F_0 = \frac{MS_A}{MS_E}$
B treatments	SS_B	$b-1$	$MS_B = \frac{SS_B}{b-1}$	$F_0 = \frac{MS_B}{MS_E}$
Interaction	SS_{AB}	$(a-1)(b-1)$	$MS_{AB} = \frac{SS_{AB}}{(a-1)(b-1)}$	$F_0 = \frac{MS_{AB}}{MS_E}$
Error	SS_E	$ab(n-1)$	$MS_E = \frac{SS_E}{ab(n-1)}$	
Total	SS_T	$abn-1$		

Example (5.1 The Battery Design Experiment)

From Table 5.1, we have

$$y_{11.} = 539, y_{12.} = 229, y_{13.} = 230, y_{21.} = 623, y_{22.} = 479, y_{23.} = 198$$

$$y_{31.} = 576, y_{32.} = 583, y_{33.} = 342,$$

$$y_{1..} = 998, y_{2..} = 1300, y_{3..} = 1300, y_{.1.} = 1738, y_{.2.} = 1291, y_{.3.} = 770, y_{...} = 3799$$

Response: Life in hours
ANOVA for Selected Factorial Model
Analysis of variance table [Partial sum of squares]

Source	**Sum of Squares**	**DF**	**Mean Square**	***F* Value**	**Prob > *F***	
Model	59416.22	8	7427.03	11.00	<0.0001	significant
A	*10683.72*	*2*	*5341.86*	*7.91*	*0.0020*	
B	*39118.72*	*2*	*19559.36*	*28.97*	*<0.0001*	
AB	*9613.78*	*4*	*2403.44*	*3.56*	*0.0186*	
Residual	18230.75	27	675.21			
Lack of Fit	*0.000*	*0*				
Pure Error	*18230.75*	*27*	*675.21*			
Cor Total	77646.97	35				

Std. Dev.	25.98	R-Squared	0.7652
Mean	105.53	Adj R-Squared	0.6956
C.V.	24.62	Pred R-Squared	0.5826
PRESS	32410.22	Adeq Precision	8.178

Multiple Comparison

Now we illustrate the use of Tukey's test on the battery. Note that for the three material type averages at 70°, we have that

$$\bar{y}_{12.} = 57.25, \bar{y}_{22.} = 119.75, \bar{y}_{32.} = 145.75$$

and

$$T_{0.05} = q_{0.05}(3, 27)\sqrt{\frac{MS_E}{n}} = 3.50\sqrt{\frac{675.21}{4}} = 45.47$$

The pairwise comparisons yield

$$3 \quad vs \quad 1: \quad 145.75 - 57.25 = 88.50 > T_{0.05} = 45.47$$

$$3 \quad vs \quad 2: \quad 145.75 - 119.75 = 26.00 < T_{0.05} = 45.47$$

$$3 \quad vs \quad 1: \quad 119.75 - 57.25 = 62.50 > T_{0.05} = 45.47$$

5.3.3 Model Adequacy Checking

As before, the primary diagnostic tool is residual analysis. The residuals for the two-factor factorial model are

$$e_{ijk} = y_{ijk} - \hat{y}_{ijk} = y_{ijk} - \bar{y}_{ij}$$

.

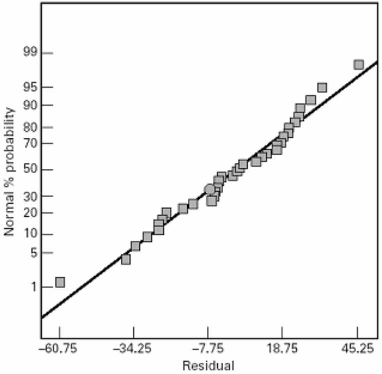


Figure 5-11 Normal probability plot of residuals for Example 5-1.

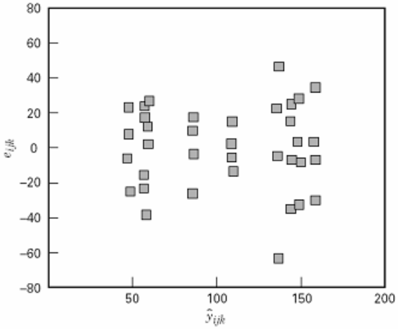


Figure 5-12 Plot of residuals versus $\hat{y}_{ijk}$ for Example 5-1.

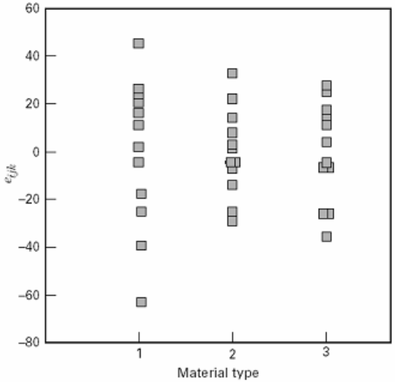


Figure 5-13 Plot of residuals versus material type for Example 5-1.

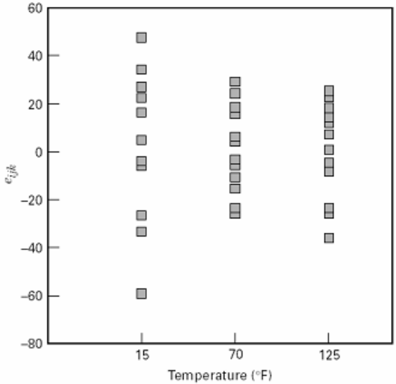


Figure 5-14 Plot of residuals versus temperature for Example 5-1.

5.3.4 Estimating the Model Parameters

The parameters in the effects model for the two-factor factorial

$$y_{ijk} = \mu + \tau_i + \beta_j + (\tau\beta)_{ij} + \epsilon_{ijk}$$

may be estimated by least squares. Under the constraints

$$\sum_{i=1}^{a} \tau_i = 0, \sum_{j=1}^{b} \beta_j = 0, \sum_{i=1}^{a} (\tau\beta)_{ij} = \sum_{j=1}^{b} (\tau\beta)_{ij} = 0$$

the estimates of parameters for the model are

$$\begin{aligned}
\hat{\mu} &= \bar{y}_{...}, \\
\hat{\tau}_i &= \bar{y}_{i..} - \bar{y}_{...}, \qquad i = 1, 2, ..., a, \\
\hat{\beta}_j &= \bar{y}_{.j.} - \bar{y}_{...}, \qquad j = 1, 2, ..., b \\
(\hat{\tau\beta})_{ij} &= \bar{y}_{ij.} - \bar{y}_{i..} - \bar{y}_{.j.} + \bar{y}_{...}, \qquad \begin{cases} i = 1, 2, ..., a \\ j = 1, 2, ..., b \end{cases}
\end{aligned}$$

Further, we have

$$\hat{y}_{ijk} = \hat{\mu} + \hat{\tau}_i + \hat{\beta}_j + (\hat{\tau\beta})_{ij} = \bar{y}_{ij.}$$

5.3.5 Choice of Sample Size

Factor	Φ^2	Numerator Degrees of Freedom	Denominator Degrees of Freedom
A	$\frac{bn\sum_{i=1}^{a}\tau_i^2}{a\sigma^2}\left(\frac{bnD^2}{2a\sigma^2}\right)$	$a-1$	$ab(n-1)$
B	$\frac{an\sum_{j=1}^{b}\beta_i^2}{b\sigma^2}\left(\frac{anD^2}{2b\sigma^2}\right)$	$b-1$	$ab(n-1)$
AB	$\frac{n\sum_{i=1}^{a}\sum_{j=1}^{b}(\tau\beta)_{ij}^2}{\sigma^2[(a-1)(b-1)+1]}\left(\frac{nD^2}{2(\cdots)}\right)$	$(a-1)(b-1)$	$ab(n-1)$

For

Example, $D = 40, \sigma = 25, a = b = 3$ and $\alpha = 0.05$ we have

$$\Phi^2 = \frac{anD^2}{2b\sigma^2} = \frac{n(3)(40)^2}{2(3)(25)^2} = 1.28.$$

n	Φ^2	Φ	$\nu_1 =$ Numerator Degrees of Freedom	$\nu_2 =$Error Degrees of Freedom	β
2	2.56	1.60	2	9	0.45
3	3.84	1,96	2	18	0.18
4	5.12	2.26	2	27	0.06

5.3.6 The Assumption of No Interaction in a Two-Factor Model

The model has the following form:

$$y_{ijk} = \mu + \tau_i + \beta_j + \epsilon_{ijk}, \begin{cases} j = 1, 2, ..., a \\ j = 1, 2, ..., b \\ k = 1, 2, ..., n \end{cases}$$

Source of Variation	Sum of Squares	Degrees of Freedom	Mean Squares	F_0
Material types	10683.72	2	5341.86	5.95
Temperature	39118.72	2	19559.36	21.76
Error	27844.52	31	898.21	
Total	77646.96	35		

5.3.7 One Observation per Cell

This model has the following form

$$y_{ij} = \mu + \tau_i + \beta_j + (\tau\beta)_{ij} + \epsilon_{ij}, \begin{cases} j = 1, 2, ..., a \\ j = 1, 2, ..., b \end{cases}$$

Source of Variation	Sum of Squares	Degrees of Freedom	Mean Squares	Expected Mean Squares
Rows (A)	$\sum_{i=1}^{a} \frac{y_{i.}^2}{b} - \frac{y_{..}^2}{ab}$	$a-1$	MS_A	$\sigma^2 + \frac{b\sum_{i=1}^{a}\tau_i^2}{a-1}$
Column(B)	$\sum_{j=1}^{b} \frac{y_{.j}^2}{a} - \frac{y_{..}^2}{ab}$	$b-1$	MS_B	$\sigma^2 + \frac{a\sum_{j=1}^{b}\beta_j^2}{b-1}$
Residual or AB	Substraction	$(a-1)(b-1)$	$MS_{Residual}$	$\sigma^2 + \frac{\sum_{i=1}^{a}(\tau\beta)_{ij}^2}{(a-1)(b-1)}$
Total	$\sum_{i=1}^{a}\sum_{j=1}^{b} y_{ij}^2 - \frac{y_{..}^2}{ab}$	$ab-1$		

If there is no interaction, we have the model:

$$y_{ij} = \mu + \tau_i + \beta_j + \epsilon_{ij}, \begin{cases} j = 1, 2, ..., a \\ j = 1, 2, ..., b \end{cases}$$

Assume that

$$(\tau\beta)_{ij} = \gamma\tau_i\beta_j$$

the model has the form

$$y_{ij} = \mu + \tau_i + \beta_j + \gamma\tau_i\beta_j + \epsilon_{ij}, \begin{cases} j = 1, 2, ..., a \\ j = 1, 2, ..., b \end{cases}$$

In this case, the test partitions the residual sum of squares into a single-degree-of-freedom component due to nonadditivity and a component for error with $(a-1)(b-1)-1$ degrees of freedom. We have

$$SS_N = \frac{\left[\sum_{i=1}^{a}\sum_{j=1}^{b} y_{ij}y_{i.}y_{.j} - y_{..}\left(SS_A + SS_B + \frac{y_{..}^2}{ab}\right)\right]^2}{abSS_BSS_B}$$

with one degree of freedom, and

$$SS_{Error} = SS_{Residual} - SS_N$$

with (a-1)(b-1)-1 degrees of freedom. To test for the presence of interaction, we compute

$$F_0 = \frac{SS_N}{SS_{Error}/[(a-1)(b-1)-1]}$$

Example (5.2)

Table 5.10 Impurity Data

Temperature (F°)	Pressure 25	30	35	40	45	$y_{i.}$
100	5	4	6	3	5	23
125	3	1	4	2	3	13
150	1	1	3	1	2	8
$y_{.j}$	9	6	13	6	10	$y_{..} = 44$

Table 5.11 Analysis of Variance for Example 5.2

Source of Variation	Sum of Squares	Degrees of Freedom	Mean Squares	F_0	p-value
Temperature	23.33	2	11.67	42.97	0.0001
Pressure	11.60	4	2.90	10.68	0.0042
Nonadditivity	0.0985	1	0.0985	0.36	0.5674
Error	1.9015	7	0.2716		
Total	39.93	14			

5.4 The General Factorial Design

We may consider the factor factorial design model for more than two factors. The three-factor analysis of variance model:

$$y_{ijkl} = \mu + \tau_i + \beta_j + \gamma_k + (\tau\beta)_{ij} + (\tau\gamma)_{ik} + (\beta\gamma)_{jk} + (\tau\beta\gamma)_{ijk} + \epsilon_{ijkl} \quad \begin{cases} i = 1, 2, ...a \\ j = 1, 2, ..., b \\ k = 1, 2, ..., c \\ l = 1, 2, ..., n \end{cases}$$

Assume that A, B, and C are fixed factors with a, b and c levels, respectively, we can decompose the total sum of squares into several sums of squares:

$$\underbrace{SS_T}_{abcn-1} = \underbrace{SS_A}_{a-1} + \underbrace{SS_B}_{b-1} + \underbrace{SS_C}_{c-1} + \underbrace{SS_{AB}}_{(a-1)(b-1)} + \underbrace{SS_{AC}}_{(a-1)(c-1)} + \underbrace{SS_{BC}}_{(b-1)(c-1)} + \underbrace{SS_{ABC}}_{(a-1)(b-1)(c-1)} + \underbrace{SS_E}_{abc(n-1)}$$

Example (5.3 The soft Drink Bottling Problem)

A soft Drink bottler is interested in obtaining more uniform fill heights in the bottles. There are three factors:

A: the pressure carbonation: 10, 12, 14

B: the operating pressure: 25, 30 psi

C: the line speed: 200, 250 Replicates: $n = 2$

Source of Variation	Sum of Squares	Degrees of Freedom	Mean Squares	F_0	p-value
Carbonation(A)	252.750	2	126.375	178.412	< 0.0001
Operating Pressure(B)	45.375	1	45.375	64.059	< 0.0001
Line Speed(C)	22.042	1	22.042	31.118	0.001
AB	5.250	2	2.625	3.706	0.0558
AC	0.583	2	0.292	0.412	0.6713
BC	1.042	1	1.042	1.471	0.2485
ABC	1.083	2	0.542	0.765	0.4867
Error	8.500	12	0.708		
Total	336.625	23			

5.5 Fitting Response Curves and Surfaces It can be useful to fit a response curve to the levels of a quantitative factor, so that the experimenter has an equation that relates the response to the factor. this equation may be used for interpretation.

One quantitative: response

Tow quantitative: response surface

In general, linear regression methods are used to fit these models.

Example (5.4)

Consider the experiment describe in Example 5.1. Temperature is quantitative, material type is qualitative. In this example, we use the following model:

$$y_{ijk} = \beta_0 + \beta_{11}x_{1i} + \beta_{12}x_{1i}^2 + \beta_{2j} + \beta_{21j}x_{1i} + \beta_{22j}x_{1i}^2 + \epsilon_{ijk}$$

Example

Table 5-15 Design-Expert Output for Example 5-4

Response: Life in hr
ANOVA for Response Surface Reduced Cubic Model
Analysis of variance table [Partial sum of squares]

Source	Sum of Squares	DF	Mean Square	F Value	Prov > F	
Model	59416.22	8	7427.03	11.00	<0.0001	significant
A	*39042.67*	*1*	*39042.67*	*57.82*	<0.0001	
B	*10683.72*	*2*	*5341.86*	*7.91*	*0.0020*	
A^2	*76.06*	*1*	*76.06*	*0.11*	*0.7398*	
AB	*2315.08*	*2*	*1157.54*	*1.71*	*0.1991*	
A^2B	*7298.69*	*2*	*3649.35*	*5.40*	*0.0106*	
Residual	18230.75	27	675.21			
Lack of Fit	*0.000*	*0*				
Pure Error	*18230.75*	*27*	*675.21*			
Cor Total	77646.97	35				

Std. Dev.	25.98	R-Squared	0.7652
Mean	105.53	Adj R-Squared	0.6956
C.V.	24.62	Pred R-Squared	0.5826
PRESS	32410.22	Adeq Precision	8.178

Example

Final Equation in Terms of Actual Factors:

Material type 1

Life =
+169.38017
−2.48860 *Temp
+0.012851 *Temp²

Material type 2

Life =
+159.62397
−0.17901 *Temp
+0.41627 *Temp²

Material Type 3

Life =
+132.76240
+0.89264 *Temp
−0.43218 *Temp²

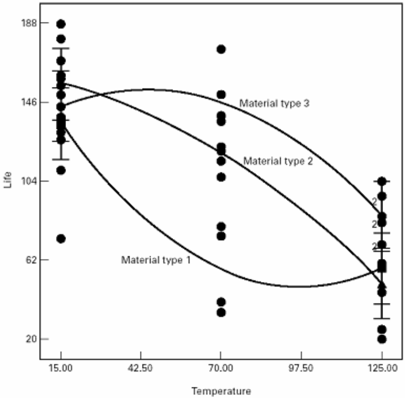


Figure 5-18 Predicted life as a function of temperature for the three material types, Example 5-4.

5.6 Blocking in a Factorial Design

Consider the two-factor factorial model with n replicates:

$$y_{ijk} = \mu + \tau_i + \beta_j + (\tau\beta)_{ij} + \epsilon_{ijk} \quad \begin{cases} i = 1, 2, ...a \\ j = 1, 2, ..., b \\ l = 1, 2, ..., n \end{cases}$$

Now a particular raw material is required in order to run this experiment and also, this raw material is available in batches that are not large enough to all abn treatment combinations to be run from same batch. The batches of raw materials request a

randomization restriction or a block:

$$y_{ijk} = \mu + \tau_i + \beta_j + (\tau\beta)_{ij} + \delta_k + \epsilon_{ijk} \quad \begin{cases} i = 1, 2, ...a \\ j = 1, 2, ..., b \\ l = 1, 2, ..., n \end{cases}$$

δ_k is the effect of the kth block. This model assume that there are no interaction between block and treatments. If these interaction do exist, they cannot be separated from the error component.

Example (5.6)

Table 5.21 Intensity Level at Target Detection

Operators (Blocks)	1		2		3		4	
Filter Type	1	2	1	2	1	2	1	2
Ground Clutter								
Low	90	86	96	84	100	92	92	81
Medium	102	87	106	90	105	97	96	80
High	114	93	112	91	108	95	98	83

Source of Variation	Sum of Squares	Degrees of Freedom	Mean Squares	F_0	p-value
Ground Clutter(G)	335.58	2	167.79	15.13	0.0003
Filter type(F)	1066.67	1	1066.67	96.19	< 0.0001
GF	77.08	2	38.54	3.48	0.0573
Blocks	402.17	3	134.06	12.09	0.0003
Error	166.33	15	11.00		
Total	2047.83	23			