

# AIR COMBAT DECISION-MAKING FOR COOPERATIVE MULTIPLE TARGET ATTACK USING HEURISTIC ADAPTIVE GENETIC ALGORITHM

DE-LIN LUO<sup>1</sup>, CHUN-LIN SHEN<sup>1</sup>, BIAO WANG<sup>1</sup>, WEN-HAI WU<sup>2</sup>

<sup>1</sup> College of Automation Engineering, Nanjing University of Aeronautics & Astronautics, Nanjing, 210016, China

<sup>2</sup> Qingdao Branch, Navy Aeronautical Engineering Institute, Qingdao, 266041, China

E-MAIL: luodelin602@163.com, b4893327@public1.ptt.js.cn, wangbiao@nuaa.edu.cn, austin@qingdaonews.com

## Abstract:

The Decision-Making (DM) problem is investigated for Cooperative Multiple Target Attack in air combat. It is to search for a proper attack assignment of  $M$  friendly fighters, with multiple target attack capability, to  $N$  hostile fighters called targets to achieve an optimal missile-target attack effect. Thus, Missile-Target Assignment (MTA) is regarded as the main part of the DM problem and has to be solved firstly. Then, the DM solution is derived from the optimal MTA solution. To the MTA problem, a Heuristic Adaptive Genetic Algorithm (HAGA) is proposed to search for its optimal solution. The HAGA utilizes specific heuristic knowledge to improve the search capability of the Adaptive Genetic Algorithm (AGA). Simulation results show that the HAGA is effective and has much better performance than the AGA.

## Keywords:

Multiple target attack; cooperative air combat; decision-making; heuristic; genetic algorithm

## 1. Introduction

In recent years, with the applications of modern fighter having multiple target attack capability and medium to long range air-to-air missile, Beyond Visual Range (BVR) air combat gradually replaces traditional short range air combat and becomes the main air combat mode. In this mode, a group of friendly fighters can share their information between them through data link system and cooperatively attack multiple air scattered hostile fighters from BVR. To implement Cooperative Multiple Target Attack (CMTA), the friendly fighters must make the decision that who will attack which targets according to the air combat situation. It is known as the air combat Decision-Making (DM) and has become one of the key techniques of airborne fire control system for modern fighters to implement CMTA in BVR air combat<sup>[1]</sup>. In paper [2], Neural Network (NN) method is used to investigate the DM problem. However, it is hard to obtain practical and comprehensive air combat data for NN training. Thus, it is necessary to explore other methods for

the DM problem.

In this paper, the DM problem is considered as an optimization problem for the assignment of  $M$  friendly fighters to  $N$  hostile fighters. It is similar to the extended version<sup>[3]</sup> of general Weapon Target Assignment (WTA) problem<sup>[4][5]</sup>. However, in the DM problem, each target is a hostile fighter that can pose threats to multiple friendly fighters. Since the goal of the DM is to achieve an optimal missile-target attack effect, that is, to minimize the Total Expected Remaining Threats (TERT) of the targets, the Missile-Target Assignment (MTA) problem is regarded as the main part of the DM problem. It can be formulated as a combinatorial optimization problem and is a NP-complete. Usually, it is difficult to solve directly. As general effective optimization methods, Genetic Algorithms (GAs) have been widely used to deal with many complex problems in battlefields<sup>[4][6][7]</sup>. In this paper, a Heuristic Adaptive Genetic Algorithm (HAGA) is proposed to solve the MTA problem of air combat DM for CMTA. In the HAGA, heuristic algorithm is introduced into the Adaptive Genetic Algorithm (AGA) to improve its performance. The heuristic algorithm is developed based on the specific heuristic knowledge, which is obtained by analyzing the air combat tactics for CMTA. Once the optimal MTA solution is obtained, from which, the DM solution can be easily determined.

The rest of the paper is organized as follows. In the next section, the DM problem is analyzed and formulated mathematically including the air combat situation, the MTA problem and the determination of the DM solution. Section 3 presents the HAGA. In section 4, the simulation result of employing the HAGA is given and compared with that of using the AGA. Conclusions are found in the final section.

## 2. The Decision-Making (DM) problem

Suppose in a scenario, there are two opposed sides, blue and red. Assume blue radar spots  $N$  red fighters, called targets, in a certain sky for a coming attack. In

response,  $M$  blue fighters are called to intercept those targets. For simplicity, suppose the blue fighters have uniform performance and the targets also have identical performance. Each blue fighter carries  $L$  air-to-air missiles and has the multiple target attack capability of firing  $L$  missiles to attack  $L$  different targets simultaneously.

### 2.1. The air combat situation

Denote the blue fighter set  $B = \{i, i = 1, 2, \dots, M\}$ , the target set  $R = \{j, j = 1, 2, \dots, N\}$ . Use  $B_i (i = 1, 2, \dots, M)$  to represent the  $i$ -th blue fighter and  $R_j (j = 1, 2, \dots, N)$  to represent the  $j$ -th target. In air combat, the situation between fighter  $B_i$  and target  $R_j$  can be illustrated with figure 1.

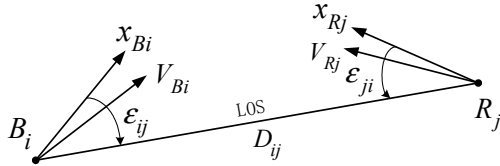


Figure 1. The situation between  $B_i$  and  $R_j$

where, LOS is the Line Of Sight,  $D_{ij}$  is the distance between  $B_i$  and  $R_j$ .  $x_{Bi}$  and  $V_{Bi}$  are the body axis and velocity of  $B_i$ , respectively.  $\epsilon_{ij}$  is the Bore of Sight (BOS) angle of  $R_j$  to  $B_i$ .  $x_{Rj}$ ,  $V_{Rj}$  and  $\epsilon_{ji}$  are defined in similar way.

The threat of  $B_i$  to  $R_j$  can be described as a composite function of its threat factors<sup>[8][9]</sup>

$$th_{ij} = w_1 th_{ij}^{D_{ij}} th_{ij}^{\epsilon_{ij}} + w_2 th_{ij}^{V_{Bi}} \quad (1)$$

where

$$th_{ij}^{D_{ij}} = \begin{cases} 1; & D_{ij} \leq D_M \\ 1 - \frac{D_{ij} - D_M}{D_R - D_M}; & D_M < D_{ij} \leq D_R \\ 0; & D_{ij} > D_R \end{cases} \quad (2)$$

$$th_{ij}^{\epsilon_{ij}} = e^{-\lambda_1 (\pi \epsilon_{ij} / 180)^{\lambda_2}} \quad (3)$$

$$th_{ij}^{V_{Bi}} = \begin{cases} 1; & V_{Rj} < 0.5V_{Bi} \\ 1.5 - V_{Rj} / V_{Bi}; & 0.5V_{Bi} \leq V_{Rj} \leq 1.4V_{Bi} \\ 0.1; & V_{Rj} > 1.4V_{Bi} \end{cases} \quad (4)$$

In Eqs. (1~4),  $w_1$ ,  $w_2$  are non-negative weight coefficients and satisfy

$$w_1 + w_2 = 1 \quad (5)$$

$th_{ij}^{D_{ij}}$  is the distance threat factor<sup>[10]</sup>,  $th_{ij}^{\epsilon_{ij}}$  is the BOS angle threat factor<sup>[9]</sup>, and  $th_{ij}^{V_{Bi}}$  is velocity threat factor<sup>[10]</sup>.  $D_M$  and  $D_R$  are the missile effective range of blue fighter and blue radar maximum track range, respectively.  $\lambda_1$  and  $\lambda_2$  are positive constants.

From Eqs. (2~4), it can be seen  $th_{ij}^{D_{ij}} \in [0, 1]$ ,  $th_{ij}^{\epsilon_{ij}} \in (0, 1]$  and  $th_{ij}^{V_{Bi}} \in [0.1, 1]$ . Thus, there is  $th_{ij} \in [0, 1]$ . So the threat of  $B_i$  to  $R_j$  can be viewed as a probability of  $B_i$  to destroy  $R_j$  by firing one of its missiles.

The threat  $th_{ji}$  of  $R_j$  to  $B_i$  is defined in similar way and has  $th_{ji} \in [0, 1]$ .

### 2.2. The Missile-Target Assignment (MTA) problem

In the fore-mentioned scenario, the total missile number of the blue fighters is

$$Z = M \cdot L \quad (6)$$

and suppose

$$N \leq Z \leq 2N \quad (7)$$

Given the  $k$ -th missile of  $Z$  is the  $h$ -th missile of the  $i$ -th blue fighter, the  $k$ -th missile can be defined as

$$k = (i-1) \cdot L + h, \quad h = 1, 2, \dots, L \quad (8)$$

Eq. (8) indicates that the  $k$ -th missile of  $Z$  belongs to the  $i$ -th blue fighter.

In the MTA problem, two assumptions are made. The first one is that each target must be assigned at least one and no more than two missiles. This assumption allows two missiles to be assigned to a target with much threat. The second one is that all missiles must be assigned to targets. In this paper, the TERT of the targets is defined as the threats posed to the blue fighters by the targets after having suffered a cooperative attack from the blue fighters and it is used as the criterion to evaluate the missile-target attack effect. For the blue fighters, the smaller the TERT of the targets, the better the attack effect and the DM solution would be. Use  $th_{kj}$  to represent the probability of the  $k$ -th missile to destroy the  $j$ -th target. If the  $k$ -th missile is assigned to the  $j$ -th target, the  $j$ -th target survives with a probability of  $(1 - th_{kj})$ . After a cooperative attack has been performed, the remaining threat of the  $j$ -th target to the  $i$ -th blue fighter is  $th_{ji} \cdot \prod_{k=1}^Z (1 - th_{kj})^{x_{ij}}$ . Then, the MTA problem is to find a solution  $\pi$  to minimize the following evaluation function

$$E(\pi) = \sum_{j=1}^N \sum_{i=1}^M \left[ th_{ji} \cdot \left( \prod_{k=1}^Z (1 - th_{kj})^{X_{kj}} \right) \right] \quad (9)$$

where, the value of  $X_{kj}$  is 1 or 0.  $X_{kj}=1$  indicates that the  $k$ -th missile is assigned to the  $j$ -th target. Considering the two assumptions for the MTA problem, there are

$$\sum_{j=1}^N X_{kj} = 1, \quad k = 1, 2, \dots, Z \quad (10)$$

$$\sum_{k=1}^Z X_{kj} = 1 \text{ or } 2, \quad j = 1, 2, \dots, N \quad (11)$$

$\pi$  is a feasible assignment vector. It can be represented as  $\pi = (T_1, T_2, \dots, T_k, \dots, T_Z)$ .  $\pi(k) = T_k$  indicates that the  $k$ -th missile is assigned to the target  $T_k$ ,  $T_k \in R$ .

### 2.3. The DM solution

The DM problem is to find out a proper assignment of the  $M$  blue fighters to the  $N$  targets to achieve an optimal attack effect. Suppose the optimal MTA solution  $\pi$  has been obtained. From it, the DM solution can be determined easily according to Eq. (8).

## 3. Solve the MTA problem using the HAGA

### 3.1. The HAGA

It is well known that GAs have fine global but poor local search capability. Besides, GAs are easy to get stuck prematurely in the evolution process. Studies<sup>[11]</sup> show that the probabilities of crossover and mutation play important roles in controlling the GAs performance. To efficiently avoid premature and ensure convergence of the GA in this paper, the adaptive approach recommended in paper [11] is introduced and it is called the Adaptive Genetic algorithm (AGA), in which the probability values of crossover and mutation are varied depending on the fitness values of the solutions. Then a heuristic algorithm is introduced into the AGA and it is called the Heuristic Adaptive Genetic Algorithm (HAGA). The heuristic algorithm is developed based on the specific heuristic knowledge obtained by analyzing the air combat tactics for CMTA and is used as local search approach to enhance the convergent rate to the global optimum of the AGA.

To avoid largely increase computation, only the best solution of a generation is applied with heuristic algorithm to greedily search for a better solution that will be directly inherited to the next generation. The procedure of HAGA is

$\{ t \leftarrow 0 ;$

Initialize the generation number of evolution:  $n$  ;

Initialize the population size:  $s$  ;

Initialize population:  $P(t) = \{\pi_1, \pi_2, \dots, \pi_s\}$  ;

Evaluate  $P(t)$  ;

While (  $t \leq n$  ) do

Choose the best individual in  $P(t)$  :  $\pi_{elutst}$  ;

Apply heuristic search on  $\pi_{elutst}$  to yield  $\pi'$  ;

Crossover:  $P'(t) \leftarrow \text{Crossover}(P(t))$  ;

Mutation  $P''(t) \leftarrow \text{Mutation}(P'(t))$  ;

Evaluate  $P''(t)$  ;

Select  $n-1$  individuals:

$P''(t) = \text{Re production}[P(t) \cup P''(t)]$  ;

Population of the next generation  $P(t) = P''(t) \cup \pi'$  ;

$t \leftarrow t+1$  ;

End}

### 3.2. Realization

#### 1) Individual Encoding

An individual here is defined as a MTA solution. Thus, a solution  $\pi$  is directly used as the gene representation (chromosome) of an individual. The chromosome of an individual can be represented as

$$\pi = \begin{bmatrix} T_1 & T_2 & \dots & T_k & \dots & T_Z \end{bmatrix}$$

It is noted that there are  $Z$  genes in a chromosome, and their corresponding values are integers between 1 and  $N$ , which represent  $N$  kinds of genes.

According to the two assumptions for the MTA problem, there are two corresponding constraints on a chromosome. The first one is that all kinds of genes must exist in a chromosome. The second is that a kind of gene can't appear more than twice in a chromosome.

#### 2) Fitness function

To make select operator more effective, we scale the evaluation function by squaring Eq. (9) and have

$$E'(\pi) = E^2(\pi) \quad (12)$$

Since the smaller the evaluation value of individual  $\pi$  computed by Eq. (12), the larger its fitness is. The fitness function of individual  $\pi$  is defined as

$$fit(\pi) = 1/[E'(\pi) + \varepsilon_0] \quad (13)$$

where,  $\varepsilon_0$  is positive constant.

#### 3) Genetic Operators

##### a). Select operator

Roulette wheel selection is used to reproduce a mating pool. It ensures the selection probability of an individual is proportional to its fitness.

##### b). Crossover operator

One point crossover<sup>[12]</sup> is employed. Considering two

parents, one Crossover Point Position (CPP) is randomly selected. The offspring inherits the part before the CPP of its parent as its corresponding first part. The remaining genes are inherited from the alternate parent by following steps. Step 1: For the alternate parent, the part before CPP is swapped with the remaining part as a whole to yield a Transformed Parent (TP). Step 2: Those kinds of genes not present in the first part of the offspring are inherited from the TP for only once to the remaining part of the offspring in the order in which they firstly appear in the TP. If the length of the offspring is equal to that of its parent, the stated length, the process is completed. Otherwise, step 3 is performed. Step 3: Beginning with the first position of the TP and skipping over those genes which have been inherited in step 2, the feasible gene, whose value appears only once in the current state of the offspring, is inherited to the remaining part with a relative order to the existed genes the same as the order they appear in the TP, until the length of the offspring equals to the stated length. The process is completed.

An example is given to see the procedure.  $A$  and  $C$  are two parents with length 8. There are 5 kinds of genes, integers from 1 to 5. CPP=3, the offspring  $A'$  and  $C'$  are produced as

$$\begin{array}{l} A: 3 \ 1 \ 3 | 5 \ 2 \ 1 \ 4 \ 5 \rightarrow A': 3 \ 1 \ 3 | 1 \ 5 \ 2 \ 5 \ 4 \\ C: 5 \ 2 \ 4 | 3 \ 1 \ 5 \ 3 \ 2 \rightarrow C': 5 \ 2 \ 4 | 5 \ 2 \ 1 \ 4 \ 3 \end{array}$$

Take  $A'$  for instance. First,  $A'$  inherits the part before CPP of  $A$ , 313, as its first part. The remaining genes of  $A'$  are inherited from  $C$ . Step 1: The two parts of  $C$  before and after CCP are swapped into  $C_{TP}=31532524$ . Step 2: Those kinds of genes not present in the first part of  $A'$  are 2, 4 and 5. Thus, the gene  $C_{TP}(3)=5$ ,  $C_{TP}(5)=2$  and  $C_{TP}(8)=4$  are inherited to  $A'$ . Now,  $A'=313524$ , whose length is two positions shorter than 8, so step 3 is preformed. Step 3: First,  $C_{TP}(2)=1$ , then,  $C_{TP}(6)=5$  are inherited to the remaining part of  $A'$  and all the genes in it are kept in an order the same as the relative order they appear in  $C_{TP}$ . Until now, there is  $A'=31315254$ , whose length equals 8. The process to generate  $A'$  is completed. Also in the same way,  $C'$  can be obtained.

#### c). Mutation Operator

Inversion mutation<sup>[4]</sup> is employed here. When mutation occurred on a parent, its offspring is generated by inverse the part between the two randomly selected positions in its parent. For example, mutation is occurred on the part between 2 and 5 positions of  $C'$ . The mutation of  $C'$  to generate its offspring  $C''$  can be described as

$$C': 5 \ 2 \ 4 \ 5 \ 2 | 1 \ 4 \ 3 \rightarrow C'': 5 \ 2 \ 5 \ 4 \ 2 | 1 \ 4 \ 3$$

#### 4) Heuristic Algorithm

For a missile, we expect it to attack a target, which has much threat to the blue fighters, and to which the missile has much threat or kill probability. This missile-target assignment is considered as a good missile-target match. Therefore, for the  $k$ -th missile, it tries to select the  $j$ -th target by making the following assignment function as large as possible

$$ASM(k, j) = th_{kj} \bullet \sum_{i=1}^M th_{ji} \quad (14)$$

When a target is regarded as a great threat to the blue fighters, two missiles can be assigned to it. This two missiles is called a missile pair, they may belong to one blue fighter or two blue fighters. A good pair is in which the two missiles are both good matches to the same target and the difference between the two matches is possible small. For example, a missile pair, including the  $k$ -th and the  $l$ -th missile, is assigned to the  $j$ -th target. The difference between its two missile-target matches is computed as

$$DIF(k, l, j) = |ASM(k, j) - ASM(l, j)| \quad (15)$$

Take solution  $\pi$  for instance. The idea of heuristic algorithm is to find out the worst one of all the missile pairs in  $\pi$  according to Eq. (15) and determine the worse matched missile in which pair. This missile greedily selects a target with which it matches better than with the current one from those targets which are assigned only one missile each in the solution  $\pi$ . For example,  $C''$  is applied with heuristic algorithm to generate  $C'''$  as

$$C'': 5 \ 2 \ 5 \ 4 | 2 | 1 \ 4 \ 3 \rightarrow C''': 5 \ 2 \ 5 \ 4 | 3 | 1 \ 4 \ 3$$

In  $C''$ , It can be seen there are three missile pairs to three different targets. One is  $C''(1)=C''(3)=5$ , which indicates that the pair including missile 1 and 3 is assigned to target 5. The other two pairs are  $C''(2)=C''(5)=2$  and  $C''(4)=C''(7)=4$ . Assume that  $C''(2)=C''(5)=2$  is the worst pair, whose  $DIF(2, 5, 2)$  value is the largest in the three pairs. In this pair, suppose  $C''(5)=2$  is worse match than  $C''(2)=2$ , so  $C''(5)$  will search other target for a better match. Here, the selectable targets are 1 and 3 which appear only once in  $C''$ . Assume  $C''(5)=3$  is the best match in the selectable targets and its original matched target 2 according to Eq. (14). As a result, missile 5 is reassigned to target 3. Specifically,  $C''(5)=2$  is replaced by  $C'''(5)=3$  in the generated  $C'''$ .

#### 5) The optimal MTA solution

When evolution stopped, the best individual in the last

generation is the optimal MTA solution

It is noted that the optimal MTA solution is not unique, for the differences between the missiles on a fighter are not considered here. However, the DM solution is unique.

#### 4. Simulations and results

In the scenario mentioned before, assume  $M = 4$ ,  $N = 14$ ,  $L = 4$ ,  $Z = 16$ ,  $V_{R_j} = 300\text{m/s}$ ,  $(j = 1, 2, \dots, N)$ ,  $V_{B_i} = 350\text{m/s}$ ,  $(i = 1, 2, \dots, M)$ . For  $B_i$ ,  $D_R = 120\text{km}$ ,  $D_M = 70\text{km}$ , both are 1.5 times the values of the corresponding parameters of  $R_j$ . The advantages of  $B_i$  over  $R_j$  in performance can make the blue fighters launch an empty attack on the targets, even in the situation where the targets have not yet spotted their adversaries. Suppose, at a certain time instant, all the targets are in the attackable regions of the blue fighters. The situation of both sides is illustrated with figure 2, in which the blue fighters fly head on with the targets at the same altitude. The blue fighters decide to make a cooperative attack on the targets.

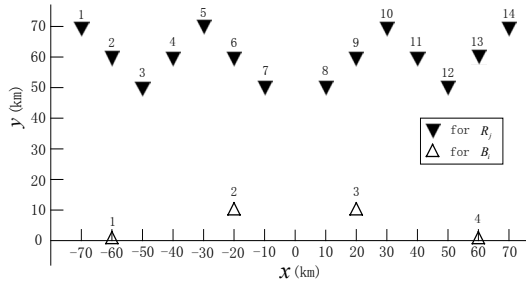


Figure 2. The Air combat situation for CMTA

According to Eq. (8), missile 1 to 4, 5 to 8, 9 to 12 and 13 to 16 belong to blue fighter 1 to 4 respectively. In this simulation, the population size is selected as 50. Employing the HAGA, computation terminated after 50 iterations. Figure 3 shows the evolution process.

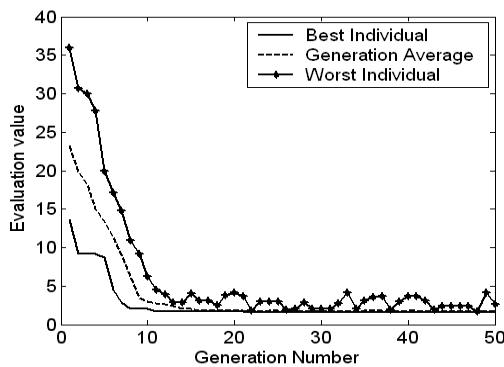


Figure 3. Evolution curves for the HAGA

In figure 3, the evolution converges after 22 generations. One of the best MTA solutions is

$$\pi = (4, 2, 3, 1, 7, 6, 7, 5, 9, 8, 8, 10, 11, 14, 13, 12)$$

which represents that missile 1 is assigned to target 4, missile 2 is assigned to target 2, and so on. The procedure to determine the corresponding DM solution is showed in figure 4.

$\pi$ :	4	2	3	1	7	6	7	5	9	8	8	10	11	14	13	12
Missiles:	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
	$B_1$				$B_2$				$B_3$				$B_4$			

Figure 4. The determination of the DM resolution

It can be seen from figure 4 that the best DM solution is:  $B_1$  to attack  $R_1, R_2, R_3$  and  $R_4$ ,  $B_2$  to attack  $R_5, R_6$  and  $R_7$ , in which  $R_7$  is assigned two missiles,  $B_3$  to attack  $R_8, R_9$  and  $R_{10}$ , in which  $R_8$  is assigned two missiles,  $B_4$  to attack  $R_{11}, R_{12}, R_{13}$  and  $R_{14}$ . We ran 10 trials and obtained the same results.

To test the performance of the HAGA, the AGA is employed for a comparison. The evolution process of the HAGA is compared with that of the AGA in figure 5.

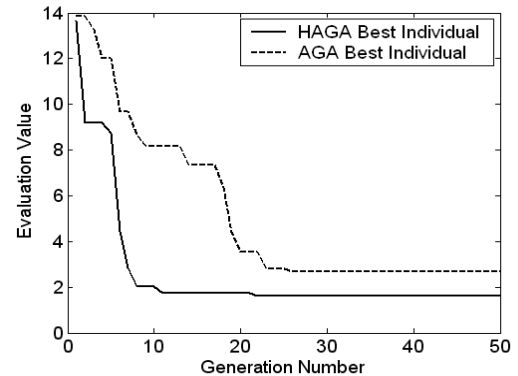


Figure 5. A comparison between the HAGA and the AGA

It can be seen from figure 5 that the convergence rate of the HAGA is much higher than that of the AGA. This is because the HAGA applies effective specific heuristic knowledge to promote local search capability of the AGA and obviously enhances the speed to find out the global optimal solution as a result.

#### 5. Conclusions

In this paper, the DM problem for CMTA is investigated. The MTA is regarded as the main problem of the DM and is solved firstly by using the proposed HAGA. Then, the DM solution can be easily determined from the

optimal MTA solution. The HAGA uses specific heuristic knowledge to improve the local search capability of the AGA. Simulation results show that the HAGA can quickly find the globe optimum and its performance is much better than that of the AGA.

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