Statistics Homework 1

Mr. Grant

July 11, 2017

1. Do the first two chapters, "Intro to basics" and "Vectors," of the Datacamp R tutorial at https://www.datacamp.com/courses/free-introduction-to-r. Log in with your Facebook or Gmail.

2. Calculate the probability that the NBA championship series between the Golden State Warriors and the Cleveland Cavaliers lasts 5 games or longer, assuming that the games are independent and the probability that Golden State wins an individual game is $\frac{2}{3}$.

3. In class, we saw an example in which a team is more likely to win a game when they've won the previous game, i.e. $\mathrm{P}(G_1) = \frac{2}{3}$ but $\mathrm{P}(G_2|G_1) = \frac{3}{4}$ and $\mathrm{P}(C_1) = \frac{1}{3}$ but $\mathrm{P}(C_2|C_1) = \frac{1}{2}$. Now consider a scenario in which a team is *less* likely to win the second game after winning the first (perhaps losing a game makes the loser motivates to come back and play harder in the next game): $\mathrm{P}(G_2|G_1) = \frac{1}{2}$ and $\mathrm{P}(C_2|C_1) = \frac{1}{4}$. Compute the probability that the first two games end with the teams tied 1-1.

4. **CHALLENGE:** in the previous question, we would say that games are *positively correlated* if winning the first game makes the winner more likely to win the second ($\mathrm{P}(G_2|G_1) > \mathrm{P}(G_1)$ and $\mathrm{P}(C_2|C_1) > \mathrm{P}(C_1)$), and *negatively correlated* if winning the first game makes the winner less likely to win the second ($\mathrm{P}(G_2|G_1) < \mathrm{P}(G_1)$ and $\mathrm{P}(C_2|C_1) < \mathrm{P}(C_1)$). Prove that when the first two games are positively correlated, the chance of a 1-1 tie is lower than when the games are independent, and that when the first two games are negatively correlated, the chance of a 1-1 tie is higher than when the games are independent.

5. **CHALLENGE:** this problem continues the cooties example from class. The probability of the cooties test coming out positive when the person being tested *does not* have cooties is know as the "false positive rate." How low does the false positive rate have to be to make the probability of a positive cooties test being correct greater than 50%?

Solutions

1. N/A

2.

$$\mathrm{P}(5+\text{ games}) = 1 - \mathrm{P}(4\text{ games}) = 1 - (\mathrm{P}(\text{GS in 4}) + \mathrm{P}(\text{Cavs in 4}))$$
$$= 1 - ((\frac{2}{3})^4 + (\frac{1}{3})^4) = 1 - (\frac{16}{81} + \frac{1}{81}) = \frac{64}{81}$$

3.

$$P(\text{1-1 tie}) = 1 - P(\text{GS wins first 2}) - P(\text{Cavs win first 2})$$
$$= 1 - P(G_1 G_2) - P(C_1 C_2) = 1 - P(G_2|G_1)P(G_1) - P(C_2|C_1)P(C_1)$$
$$= 1 - \frac{1}{2}\frac{2}{3} - \frac{1}{4}\frac{1}{3} = 1 - \frac{1}{3} - \frac{1}{12} = \frac{7}{12}$$