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Invited Review

Linear Programming: A Mathematical Tool for Analyzing and Optimizing Children's Diets During the Complementary Feeding Period

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ABSTRACT
During the complementary feeding period, children require a nutrient-dense diet to meet their high nutritional requirements. International interest exists in the promotion of affordable, nutritionally adequate complementary feeding diets based on locally available foods. In this context, two questions are often asked: 1) is it possible to design a diet suitable for the complementary feeding period using locally available food? and 2) if this is possible, what is the lowest-cost, nutritionally adequate diet available? These questions are usually answered using a "trial and error" approach. However, a more efficient and rigorous technique, based on linear programming, is also available. It has become more readily accessible with the advent of powerful personal computers. The purpose of this review, therefore, is to inform pediatricians and public health professionals about this tool. In this review, the basic principles of linear programming are briefly examined and some practical applications for formulating sound food-based nutritional recommendations in different contexts are explained. This review should facilitate the adoption of this technique by international health professionals. ***JPGN 36:12–22, 2003.***

INTRODUCTION

After 6 months of age, breastfed children receive a substantial part of their energy from breast milk; however, complementary nutrient-dense foods are required to cover additional energy and their requirements for several micronutrients, notably iron, zinc, calcium, and vitamin A (1). These foods are complementary in that they provide additional energy and other nutrients during the period when breastfeeding alone is inadequate to meet nutritional needs. This period is recommended to start at 6 months of age (2) and extend for many months thereafter, up to 2 years according to recent recommendations (1). Often, it is assumed that suitable nutrient-dense foods are locally available to provide a nutritionally adequate complementary diet, even in developing countries. At least, this is implied in several WHO publications, including the Code for marketing infant foods (3) and in the most recent guidelines on complementary feeding (4). Yet, children in this age group have high nutritional requirements compared with adults and are vulnerable to multiple nutritional deficiencies. Even in rich developed countries, iron-deficiency anemia could only be eliminated, as a public health problem in infants and young children, with iron-fortified infant foods (5,6). This suggests that even in an environment of food abundance, designing a diet fulfilling all nutritional needs of young children may be more difficult than is often assumed. In developing countries, these difficulties are compounded by the need to design a nutritionally sound diet at low cost.

Curiously, the possibility of providing an affordable balanced diet, based on a combination of family foods, during the complementary feeding period has never been rigorously evaluated. This possibility, however, can be examined using a mathematical approach called linear programming (7,8). More specifically, this technique can address two questions: 1) Is it possible to design a diet suitable for the complementary feeding period using locally available foods? 2) If this is possible, what is the minimum budget needed for designing a diet covering the nutritional requirements of at least 97% of children?

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The problem of cost minimization of a balanced diet was originally posed in 1945 by the Nobel Prize-winner George Stigler (9). Soon afterward, George Dantzig formally solved the problem, using the simplex method he had just developed (10). This led to the theory of linear programming defined by Dantzig as "the maximisation of a linear function subject to linear inequality constraints" (11). Since this time, linear programming has been used in many fields of applied research, but rarely to answer questions related to human nutrition. This is partially attributable to the complexity of mathematically modelling the underlying structure of food selection practices. Yet, constraints related to palatability can be introduced in linear programming models, as was noted by Smith as early as 1959 (12). Also, the time-consuming calculations needed for linear programming were impossible to carry out in practice without readily accessible computer technology. However, this situation has now changed radically with the advent of powerful personal computers with a linear programming function available in commonly used programmes such as Microsoft Excel®. A user-friendly diet analysis program, Nutrisurvey, with a linear programming module, is also now available for free downloading from the Web (13). It seems, therefore, timely to describe this method so that it can be used more widely when formulating diets during the complementary feeding period.

In this review, we first describe the rationale for using linear programming. Then we will discuss the general principles underlying this approach, as well as its specific use for evaluating and improving diets recommended for young children during the complementary feeding period. Additional research applications of linear programming in the field of human nutrition will not be described here. Our objective is only to describe, in sufficient detail, this useful tool so that it can be used by pediatricians and nutrition intervention program planners to make informed decisions in relation to complementary feeding.

DIET DESIGN WITH AND WITHOUT LINEAR PROGRAMMING ASSISTANCE

To better understand the rationale of using linear programming for formulating diets, we need first to examine the method currently used to design diets during the complementary feeding period, namely a "trial and error" approach (Fig. 1). Before beginning the "trial and error" approach, some background information is needed: a list of available foods, their energy and nutrient composition, their price, and an estimated maximum daily portion size that can be reasonably recommended for children. The traditional "trial and error" approach is then iterative, whereby different food combinations are repeatedly tried, based on informed guesses. This approach requires multiple backwards steps, and repeated diet designs to arrive at a solution that may or may not be optimal. Moreover, in the case of repeated failures to develop an adequate low-cost diet, a clear directive for diet modification (*i.e.*, introduction of new foods, especially fortified foods) is not provided. In contrast, once the background information is collected, and if formulating a diet is possible, linear programming quickly and efficiently provides an optimal solution (Fig. 1). If formulating a suitable diet is not feasible, this is clearly shown by the analysis, indicating with certainty that new foods must be introduced into the diet. Linear programming also indicates the types of foods that should be introduced to balance the diet.

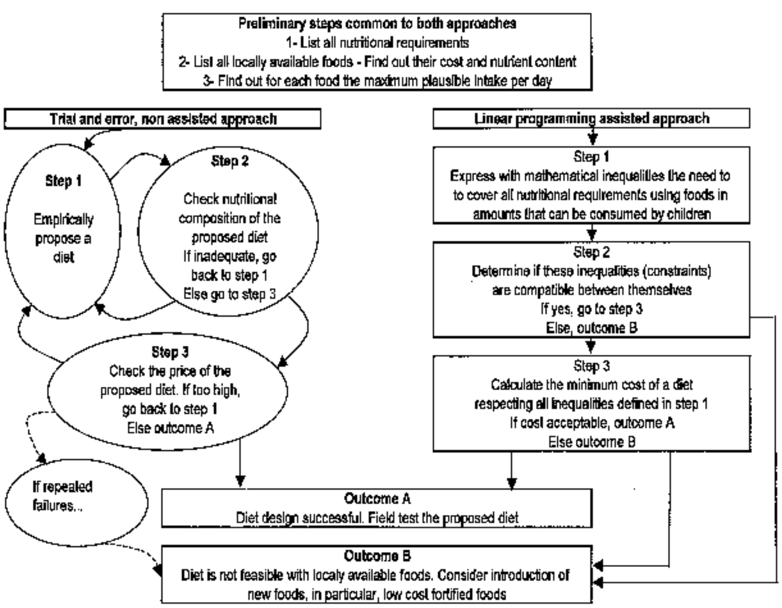

FIG. 1. Flow chart describing diet design with and without the assistance of linear programming.

Even with linear programming, diet design during the complementary feeding period remains a difficult exercise, due to the uncertainties regarding the nutritional requirements during this period and the level of absorption of different micro-nutrients, which varies depending on the interactions of potential enhancers and inhibitors. Also, the quantity of each food a child can reasonably eat in different contexts may be difficult to estimate. Yet, once reasonable estimates are made to define these parameters, linear programming gives a clear answer regarding the feasibility and the cost of designing a nutritionally sound diet. This is in contrast to the currently used "trial and error" method, which is not only time consuming but also prone to error and introduces a major additional source of uncertainty by trying to solve a complex mathematical problem by hand.

WHAT IS LINEAR PROGRAMMING? SOME DEFINITIONS

In mathematical terms, linear programming is a tool to *optimize* (minimize or maximize) a *linear function* of a set of *decision variables* while respecting multiple *linear constraints*.

The function Y to be optimized by linear programming is called the *objective function*. It can be graphically represented by a set of parallel lines or plane surface areas. The variables X_1, X_2,… X_n whose value can be changed to optimise the function Y are called *decision variables*.

A function Y of several variables X_1, X_2, … X_n is linear when it can be expressed in the following way:

$$Y = a_0 + a_1.X_1 + a_2.X_2 \ldots + a_n.X_n$$

where a_0, a_1, a_2 … a_n are constants.

In the same way, a *constraint* on several variables X_1, $X_{2 \ldots}X_n$ is linear when it can be expressed in the following way:

$$b_1.X_1 + b_2.X_2 + \ldots b_n.X_n \geq b_0$$

where b_0, b_1, b_2 … b_n are constants

For interested readers, a simple graphic illustration of the theory underlying linear programming is presented in the appendix to provide a conceptual understanding of the mathematical principles.

DEVELOPING A LINEAR PROGRAMMING MODEL FOR DIET OPTIMIZATION

To develop a linear programming model for diet optimization, the question to be addressed must first be clearly posed. Then, the *objective function* that best answers this question must be formulated and expressed as a linear function of the *decision variables*. Finally, the nutritional and palatability *constraints* required to govern the diet optimization process must be identified. The latter are necessary to ensure a realistic outcome and are described in detail below.

Objective Function and Decision Variables

The objective function mathematically describes the criteria used to select the optimal solution from among all alternative solutions. The objective function chosen will, therefore, depend on the question posed. In relation to the two key questions described above, the objective function will be the energy content of a diet for the first question, whereas it will be the total cost of the diet for the second. In both cases, the objective functions will be minimized, and can be expressed as linear functions of individual food weights X1, X2 … Xn, which are the decision variables. In the first case, designing a balanced diet will be possible only if all the constraints can be fulfilled with a total energy content, chosen as objective function, inferior to the child's energy requirements. In the second case, designing a diet with an acceptable cost will be possible only if the minimum cost, obtained with cost as objective function, is within the limits compatible with a poor family's budget.

The term "food data base" will be used to describe the list of all locally available foods that can, in theory, be incorporated into children's diets, along with their cost and nutritional content. The weight of all locally available foods, which are suitable for feeding young children in a specific environment, are used as decision variables. The term "food basket" will be used to represent the combination of foods selected during the analysis to include in the optimized diet (*i.e.*, all foods with an optimized gram weight greater than zero). This does not imply that each food in the food basket must be consumed in the exact amount indicated by the analysis every day. Instead, like recommended daily intakes, it represents the average amounts of different foods that should be consumed over a short period of time (*e.g.*, a week).

Linear Constraints in a Complementary Feeding Diet Problem

Nutritional Constraints

Linear constraints establish the boundaries of the optimization process. For the optimized complementary feeding diet problem, the diet selected must meet specific nutrient requirements at a given energy level. Therefore, nutrient constraints are required to ensure that its nutrient content exceeds or is equal to known recommended nutrient intake levels (*i.e.,* covering the requirements of 97% of children). If we want the nutrient content of the diet to be above a minimal value this linear constraint is represented by an inequality (≥). If we want the energy content of the diet to be just equal to average requirements, this linear constraint will be represented by an equality.

Palatability Constraints

Optimization models tend to select combinations of nutrient dense foods, which contain high quantities of several rare nutrients. Liver, legumes, dried fish, and dark green leafy vegetables are examples of foods systematically selected in large quantities, if they are included in the food database. To avoid selecting unrealistic diets, it is important to include a new set of constraints to ensure that the diet selected is palatable and culturally acceptable. Limiting portion sizes, by introducing maximum quantity constraints using a simple inequality constraint (<) for each food in the database, will reject unrealistic diets.

When many foods are available, it is also important to put upper limits on food groups. For instance, in situations where many sorts of legumes or fruit are available, incorporating all these individual foods at their maximum level can also lead to unrealistic diets. This can be avoided by putting an upper limit on the amount of energy contributed by all foods belonging to the same food group.

Ideally, these portion size and food group constraints should be obtained from previous food consumption surveys. If such data are not available, local key informants can instead specify the maximum quantity a child in this age range could consume for each food. These should be confirmed, where possible, by direct observation.

Interpreting the Results

Determining Limiting Nutrients

Linear programming can be used to predict nutrients that are potentially low in a child's diet during the complementary feeding period. Such information can be used to direct nutrition intervention program initiatives, because it suggests that foods rich in (or foods fortified with) these limiting or problem nutrients should be introduced into local diets. These nutrients can be easily identified, because the optimized diet selected by linear programming will contain these nutrients at their minimum required levels. Furthermore, when the constraint on these nutrients is removed, the objective function will decrease. If the objective function is energy, however, these nutrients will only be identified as problem nutrients when the energy content of the optimized diet is at or above the energy requirements of the population of interest.

In most situations, iron will be a limiting nutrient for 6- to 23-month-old children, which may explain why iron deficiency is difficult to eliminate without iron-fortified infant foods. Apart from iron, the limiting nutrients will vary by region and often by season, according to food availability and food price variations in the local market.

What to Do When Constraints Cannot be Met

In some cases, the linear programming analyse shows that either designing a balanced diet is not possible or that the obtained diet has an unacceptably high cost or energy level. In these cases, limiting nutrients must be identified and new foods introduced in the food database. These foods should be selected on the basis of their high content of limiting nutrients in relation to energy or cost, depending on the chosen objective function. If no suitable food can be identified, there is a strong case for using a low-cost fortified food or food supplement, provided this lowers the objective function and makes diet optimization possible at a lower cost.

Adding Nonlinear Constraints: The Case of Energy Density and Absorbed Iron

There may be circumstances when nonlinear nutritional constraints are desirable. For instance, putting a lower limit on the energy density of the whole diet is a nonlinear constraint, when it is defined as a ratio: the mathematical expression of this constraint, which is nonlinear, is:

$$(X_1.E_1 + X_2.E_2 + X_n.E_n) / (X_1 + X_2 + \dots X_n) > ED$$

where $E_{1\dots}$ E_n is the energy content for 100 g of food 1 to n and ED is the desired energy density. Optimization of these nonlinear models often cannot be achieved by solving simple equations or algorithms. Solutions are instead found by an iterative approach, creating a risk that a local optimum is selected instead of an overall general optimal solution. Hence, nonlinear functions should be avoided in the first instance. To achieve this, nonlinear constraints should be reformulated into linear constraints, where possible. For example, the constraint on energy density can be expressed as a linear constraint as follows:

$$X_1(E_1 - ED) + X_2(E_2 - ED) + X_n(E_n - ED) \geq 0$$

These inequalities are linear, and therefore appropriate for a linear programming model. A similar transformation has been described previously for transforming the nonlinear phytate:zinc molar ratio constraint into a linear constraint (8). Keeping the phytate:zinc ratio below a predetermined level may be needed to ensure a reasonable degree of zinc absorption (14).

Some nonlinear constraints cannot be reformulated into a linear form. For example, an algorithm exists to estimate the amount of absorbable iron in a diet using logarithmic and exponential functions of the concentration of vitamin C, phytate, and other inhibitors of iron absorption (15). Arguably, similar complex functions could be used to describe zinc absorption. Nonlinear programming becomes necessary when these functions are optimized.

AN EXAMPLE OF DIET OPTIMIZATION BY LINEAR PROGRAMMING

To illustrate the use of linear programming, we will describe how to formulate diets for 9- to 11-month-old Indian infants after the latest WHO recommendations for complementary feeding (4). In this example, the question we will address is only whether a balanced diet based on family foods is achievable. Therefore, the total energy content of the diet will be the objective function for minimization, and known nutritional requirements and maximum portion sizes will be the constraints. A feasible solution will only exist if the minimum energy content of the food basket (including breast milk) is inferior to the average energy requirements of children of this age (*i.e.*, 688 Kcal) (16). The nutritional requirement constraints used for this age group are presented in Table 1. Protein, vitamin, and mineral requirements were derived from WHO recommendations during the complementary feeding period (1). The calcium requirement corresponds to the latest U.S. recommendations (17).

A breast milk intake of 663 ml was assumed, based on the latest average estimates provided by the WHO (1). Breast milk was introduced in the food composition table, and an equality constraint (breast milk = 663 g) was used in the model to guarantee its inclusion in the food basket. Hence, in the solution, nutrients contributed by selected complementary foods were added to those present in breast milk. The FAO food composition table of the World Food Dietary System (18) was used for calculations. The linear programming module of Nutrisurvey was used for this analysis (13).

In the recent WHO guidelines, the basic complementary feeding recipe recommended for Indian children aged 12 to 23 months includes 50 g of chapati, 30 g of lentils, 25 g of carrot, 30 g of amaranth leaves, 5 g of ghee, and 50 g of milk (4). These guidelines also state that, for younger children, this recipe can be adapted by reducing quantities. Therefore, our first question was whether nutritional requirements of a 9- to 11-month-old infant could be met using maximum portion size constraints (grams of food per day) twice those specified above, assuming this recipe was consumed twice a day. Linear programming showed that optimization was not possible, *i.e.* a combination of these foods did not exist that fulfilled all nutritional requirements while respecting the maximum portion size constraints (Table 2, Model 1). Indeed, when all food portions were set at their maximum, the total iron, zinc, and niacin contributed by the resulting diet of 4.9, 2.5, and 3.1 mg respectively, were below the requirements shown in Table 1, even though its total energy level of 822 Kcal was in excess of the 688 Kcal acceptable upper level. To complement this recipe, the WHO guidelines recommend the inclusion of various nutrient dense snacks. Inclusion of unlimited amounts of these snacks in the food database for diet optimization led to a feasible solution, but with a total energy level of 731 Kcal, *i.e.*, higher than the upper acceptable limit of 688 Kcal (Table 2, Model 2). Inclusion of iron- and zinc-rich foods, such as chicken meat, liver, and eggs in unlimited amounts made optimization possible, but 43 g of liver was selected, which seems unrealistic (Table 2, Model 3). Fulfilling all nutritional constraints while limiting liver to a more realistic level of 25 g, also provided a feasible solution, although at a higher but still acceptable level of energy (Table 2, Model 4). The energy gap between the diet selected and a child's energy requirement (*i.e.*, 688 kcal − 653 kcal = 35 kcal) can be filled by any low-cost food such as 12 g of chapati.

The models described above showed that a balanced diet made of family foods was indeed achievable for these Indian children, provided that nutrient-dense foods were included in the food basket at levels above those recommended in the WHO guidelines. A cost optimization analysis can also be undertaken to determine whether an affordable diet is feasible (19). This will again change the food basket. Finally, an acceptability trial should be carried out to evaluate whether the obtained quantities can indeed be reasonably recommended on an average daily basis.

This example clearly demonstrates the value of linear programming as a tool for diet development by nutrition intervention program planners. It also highlights the need for nutrient-dense foods during the complementary feed-

TABLE 1. *List of nutritional constraints used in the diet optimization examples (based on the WHO estimated nutritional requirements for a child aged 9 to 11 months (1) except for the modification outlined in the footnotes)*

Protein (g/d)	≥9.6
Vitamins	
Retinol equivalent (μg/d)	≥350
Folate (μg/d)	≥32
Niacin (mg/d)	≥5
Riboflavin (mg/d)	≥0.4
Thiamin (mg/d)	≥0.3
Vitamin B6 (mg/d)	≥0.4
Vitamin B12 (μg/d)	≥0.4
Vitamin C (mg/d)	≥25
Minerals	
Calcium (mg/d)[a]	≥270
Copper (mg/d)	≥0.3
Iron (mg/d)[b]	≥7
Magnesium	≥80
Phosphorus	≥400
Potassium	≥700
Zinc (mg/d)[b]	≥2.8

Constraints were not included for biotin, vitamin D, K, E, pantothenic acid, chloride, fluoride, iodine, manganese, selenium, sodium.
[a] Based on US adequate intake (16)
[b] Assuming high bioavailability

TABLE 2. *Example of diet optimization by linear programming–food basket selection for a breastfed child aged 9–11 months*

	List of allowed foods with maximum amounts (all with breast milk = 663 ml)	Selected food basket	Minimum energy needed to cover nutritional requirements (Upper limit: 688 Kcal[a])
Model 1	Chapati ≤ 100 g Lentils ≤ 60 g Carrot ≤ 50 g Amaranth ≤ 60 g Ghee ≤ 10 g Milk ≤ 100 g	Optimization not possible.	Not applicable
Model 2	Sames foods and limits as in model 1 plus peanuts, banana, avocado, bread, butter, honey, potato, papaya *ad libitum*	Lentils 60 g, carrot 50 g, amaranth 60 g peanut 59 g	873 Kcal Total energy above acceptable level.
Model 3	Same foods and limits as in model 2 with chicken meat, chicken liver, eggs, fish *ad libitum*	Lentils 50 g, amaranth 60 g, milk 6 g, chicken liver 43 g	561 Kcal
Model 4	Same foods as in model 3 but chicken liver limited to 25 g	Lentils 60 g, carrots 50 g, amaranth 60 g, peanuts 41 g, chicken liver 25 g.	653 Kcal

All models included the nutritrional constraints outlined in Table 1. Breast milk was set at 663 ml. The total energy content of the diet was the objective function minimized. Only models 3 and 4 fulfilled all nutritional constraints with a total energy below 688 Kcal corresponding to the total energy needs of children of this age. Model 3's food combination, however include an unrealistically high amount of liver. Eventually, only model 4 seems feasible at this stage.

[a] Based on data from Butt et al, Am J Clin Nutr 2000;72:1558–69 (16).

ing period, and the value of a rigorous approach, such as linear programming, to help design complementary feeding diets for children living in developing communities in different areas of the world.

USING THE COMPUTER FOR SOLVING COMPLEX DIETARY PROBLEMS

Even though linear programming is clearly a valuable tool for international health professionals, its adoption may be hindered by a lack of familiarity with the technique. For this reason, the basic steps required to develop a linear programming model in Microsoft Excel® are presented in this section. The Microsoft Excel® program has been chosen only because it is available on most personal computers and has a solver function that can be used for diet optimization with linear programming. This program allows the inclusion of many constraints and different objective functions, making it a powerful research tool. For field-work purposes, user-friendly programs that organize data automatically, such as Nutrisurvey, are perhaps preferable, although less flexible. To illustrate the use of linear programming and to show how to start using this approach, however, we will discuss a simple example using Excel®, as it is a program widely used by health professionals. In this example, cost was chosen as the objective function with nutritional constraints on energy, calcium, and iron and portion constraints on lentils and liver.

Data Layout

In preparation for linear programming analysis, the first stage is to create a food database, as a table, at the top of the spreadsheet (Fig. 2a). For each food item, the nutrient content, cost per kg, and maximum portion size (grams) allowed per day are included. The maximum daily portion sizes are shown in cells B5:B8. For both lentils and liver, we chose a 60 g maximum portion size. For maize flour and milk, a high arbitrary figure of 999 g was chosen, because they are major components in the children's diets. Afterwards, a second table was created just below the first, which represents the food basket (cells B14:B17) with its price (cells C14:C17) and nutrient content (cells D14:G17). The price and nutrient values for each food in this second table are calculated in the spreadsheet based on the weights of each food item selected. At this stage, arbitrary weights of 50 g for each food were entered in the food basket cells. The cell D14, corresponding to the energy content of 50 g of maize, is highlighted, and its formula ($B14*D5/100) is displayed at the top of the spreadsheet. All other price and nutrient cells are calculated in the same way. At the bottom of this second table, the total cost, as well as energy and nutrient content of the selected food basket are calculated (cells C19:G19). On the last line (cells D21:G21), the values specified by the constraints to define the minimum amounts of energy, protein, calcium, and iron for the whole diet are also displayed. These required minimum values are not calculated but derived from known nutritional requirements.

Activating the Solver Function and Choosing the Objective Function and the Food Variables

In the second stage, the solver function, which is accessed by the "tool" menu, must be activated. In some

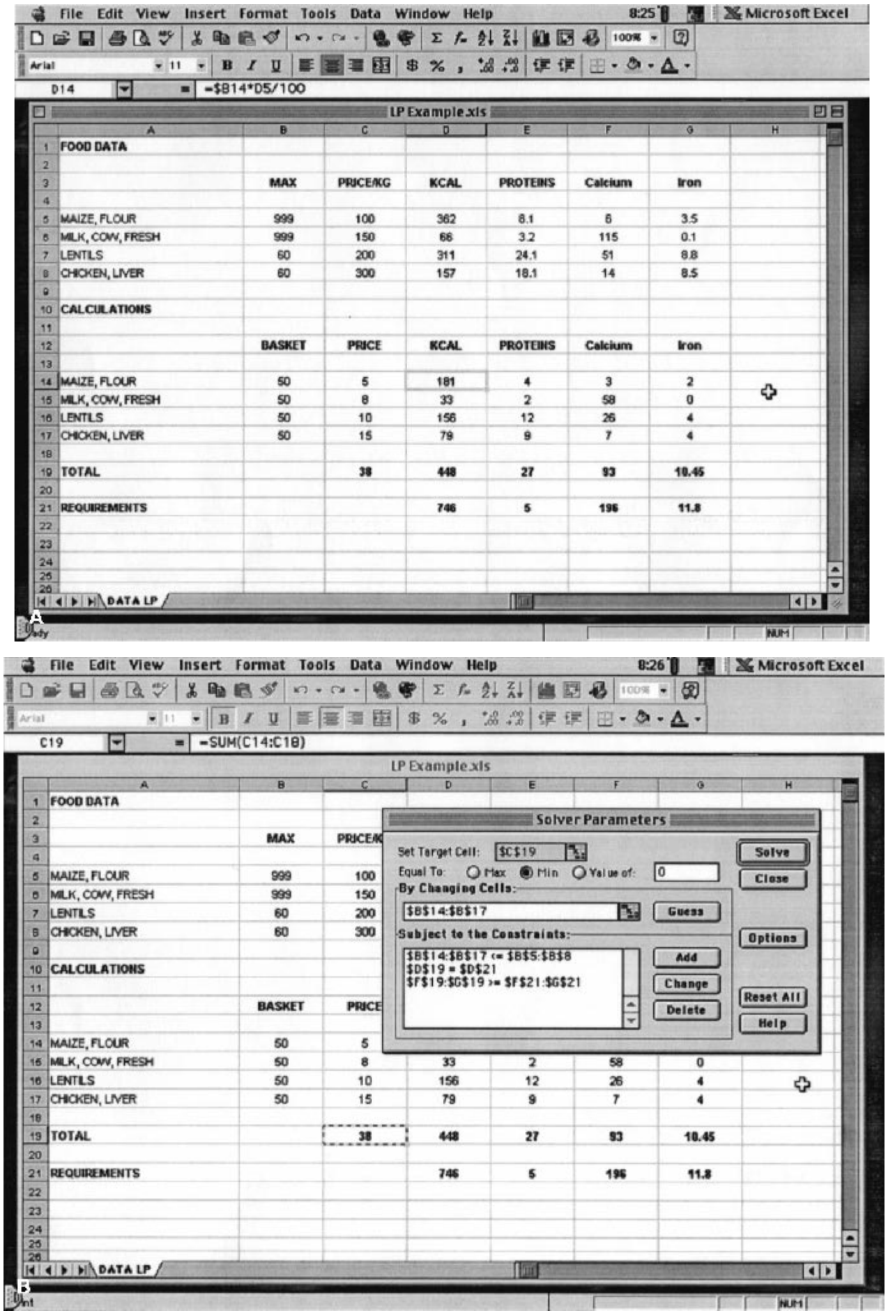

FIG. 2. Optimizing a food basket by linear programming using an Excel spreadsheet.

versions of Excel®, this function first must be installed using the "additional macro" menu. Once in the solver menu, select "options", and once in this menu activate "assume linear model" and "assume non-negative". Then, the parameters of the solver must be entered (Figure 2b). First, a target cell must be chosen. This cell represents the objective function for optimization, which was the minimization of price. Hence, the cell C19 is the target cell for optimization. Then, variable cells must be defined. These cells contain the decision variables that the program will change to select the diet that meets all constraints (nutritional requirements and maximum portion size) at minimal cost. In our example, these variables will be the cells with the weights of maize flour, milk, lentils, and chicken liver ultimately included in the food basket (cells B14:B17).

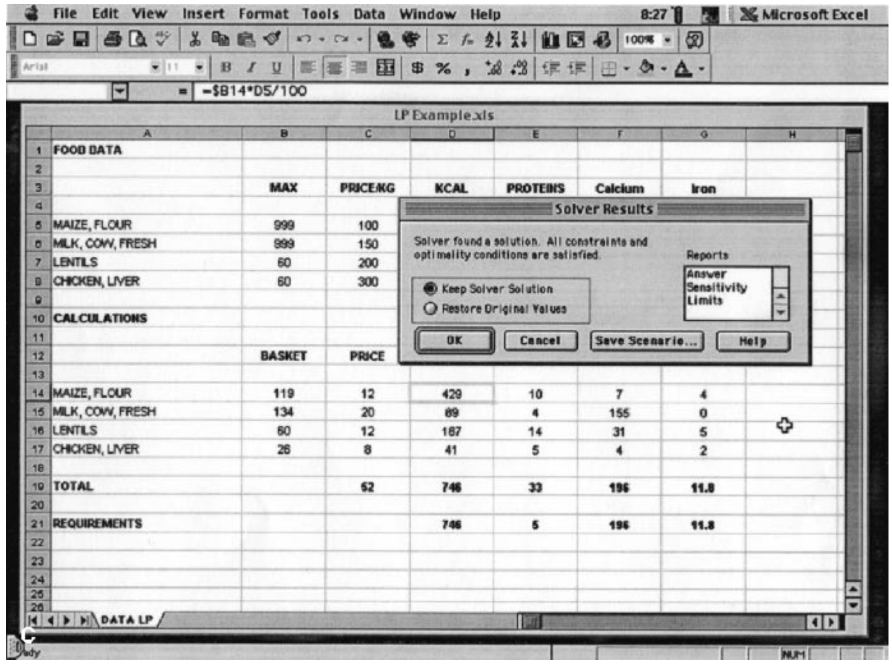

FIG. 2. *Continued*

Setting the Constraints and Solving the Problem

Constraints are included via the “Add” option in the constraint window. Clicking on this option opens an “Add constraint” window where the cells subjected to the constraint are defined. Our first constraint was to limit the quantity of each food in the food basket (cells B14:B17) to less than or equal to the maximum allowed (cells B5:B8). A second constraint was added to ensure the total energy content of the selected foods (cell D19) was equal to the estimated average requirements (cell D21). A third constraint was included to ensure the total protein, calcium, and iron content of the diet was superior to the requirements, *i.e.*, cells E19:G19 superior to cells E21:G21. Activating the “Add” option in the constraint window each time will successively include these constraints, which will then appear in the window “subject to the constraints” (Figure 2b). Once all the parameters of the solver are selected, the problem is solved by clicking on “Solve”. The results of the optimization are shown on Figure 2c. The original arbitrary quantities in the food basket cells (B14:B17) are now replaced by the solution. In our example, all constraints are met at minimal cost with 119 g maize flour, 134 g of milk, 60 g of lentils, and 26 g of liver. The total cost is 52 U.

CONCLUSIONS

Linear programming represents a powerful tool for designing optimized diets for young children during the complementary feeding period. It is a useful mathematical method that can be used to easily translate internationally accepted nutritional recommendations into sound food-based guidelines based on locally available foods and local market prices. Indeed, linear programming is much more efficient than the empirical “trial and error” approach currently used for formulating diets during the complementary feeding period. As such, it could become an indispensable tool for paediatricians and nutrition program planners.

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APPENDIX—A GRAPHIC INTRODUCTION TO LINEAR PROGRAMMING

The approach used by linear programming to answer the questions raised in the introduction regarding feasibility and cost of diet formulation can be illustrated by a simple example based on a diet made of two foods. This simple, nonrealistic, theoretical example was chosen because it can be represented in a two-dimensional graph. First we will examine graphically how linear programming determines whether or not calcium and iron requirements can be met when a complementary diet is based on cow milk and maize flour. We will show that the calcium requirement can be met, whereas iron requirements cannot be met by giving maize and cow milk. This example, therefore, illustrates the two possible outcomes of a linear programming analysis, namely that designing a diet is or is not feasible. This will be followed by a graphic illustration of the method used by linear programming to identify the combination of two foods meeting two nutritional constraints at the lowest possible cost.

In these examples, we will assume that 12- to 23-month-old breastfed children must obtain, from complementary foods, 746 kcal of energy, 11.8 mg of iron, and 196 mg of calcium. These figures were derived from the 1998 WHO estimates of nutritional requirements during the complementary feeding period for both sexes, assuming low bioavailablity for iron, and that breast milk provides 346 kcal of energy (1). For this and the following examples, the nutritional composition of foods was obtained from the FAO table World Dietary Systems (2) (Table). We also include arbitrary food prices in currency units for the example of cost minimization.

Question 1: Is it Possible to Fulfill Nutritional Requirements?

To answer this question, we must first determine which cow milk and maize flour combination will provide the required amount of energy, *i.e.*, 746 Kcal. These combinations fulfil the following equation:

$$E_{mf}.X_{mf}/100 + E_{cm}.X_{cm}/100 = 746 \qquad \text{(I)}$$

where X_{mf}, X_{cm}, E_{mf}, and E_{cm} represent the weight (in grams) and the energy content (in kcal/100g) of maize flour and cow milk, respectively.

Graphically, these combinations can be represented by a straight line (Fig. 1 Appendix). All combinations located below this line provide less than 746 Kcal and can be incorporated in a diet, the energy gap being filled by any locally available food(s). On the other hand, it is not possible to choose the maize and milk combinations located above this line because they will exceed the average energy needs of the target group.

Likewise, all the cow milk and maize flour combinations that provide 196 mg of Ca are defined by the following equation:

$$Ca_{mf}.X_{mf}/100 + Ca_{cm}.X_{cm}/100 = 196 \qquad \text{(II)}$$

where Ca_{mf} and Ca_{cm} represent the calcium content (per 100g) of maize flour and cow milk respectively. All the combinations providing this amount of calcium are represented by a straight line, and all combinations located on or above this straight line provide 196 mg or more of

TABLE (Appendix). *Nutritional composition and arbitrary prices for foods used in the examples described in the appendix*

	Price (Currency Units/kg)	Energy (Kcal/100g)	Protein (g/100g)	Calcium (mg/100g)	Iron (mg/100g)
Maize flour	100	362	8.1	6	3.5
Cow's milk	150	66	3.2	115	0.1
Lentils (cooked)	120	116	9	19	3.3
Liver	300	157	18.1	14	8.5

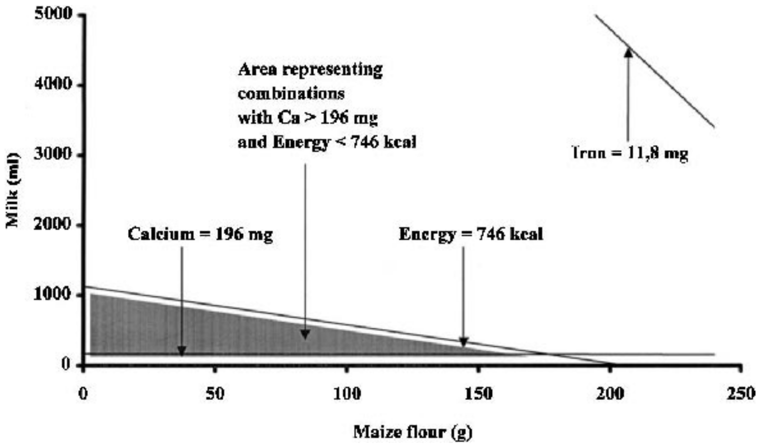

FIG. 1 Annex. Maize flour and milk combinations that provide energy, calcium, or iron in quantities corresponding to the requirements of a 12- to 23-month-old breastfed child. The shaded triangle corresponds to the maize and milk combinations that provide at least 196 mg of calcium and less than 746 Kcal. All combinations providing more than 11.8 mg iron have an energy content that is above the energy requirements of a 12- 23-month-old child.

calcium and can be recommended as a complementary diet.

Figure 1 Appendix shows it is possible to meet calcium requirements with cow milk and maize flour: the shaded area represents all combinations which provide both at least 196 mg of calcium and less than 746 Kcal of energy: they are above the calcium requirement line and below the energy requirement lines.

Continuing with this example, we will examine all cow milk and maize flour combinations that provides 11.8 mg of iron. In the same way, these combinations fulfil the following equation:

$$Fe_{mf}.X_{mf}/100 + Fe_{cm}.X_{cm}/100 = 11.8 \text{ mg} \quad \text{(III)}$$

where Fe_{mf} and Fe_{cm} represent the iron content (per 100g) of maize flour and cow milk respectively.

All maize and milk combinations that provide 11.8 mg of iron or more are located on or above the straight line defined by this equation and those below do not meet this requirement. Figure 1 Appendix shows it is not possible to cover iron requirements with any cow milk and maize combination that also provides less than 746 Kcal: all those providing the required amount of iron are well above the straight line representing this maximum acceptable level of energy. This indicates that a solution is not feasible when the objective is to design a diet based on maize flour and cow milk to meet the iron requirements of a 12- to 23-month-old child.

Question 2: What is the Nutritionally Adequate Food Combination with the Lowest Cost?

We will now illustrate how linear programming identifies the lowest cost food combination that will meet specified nutritional requirements. We will continue with our simple example using cow milk and maize flour and limit the analysis to selecting the combination of cow milk and maize flour that cover the energy and calcium requirements at the lowest possible cost. Cost will now be used as the objective function. The total cost C_{total} of each maize flour and cow milk combination can be calculated by the following general equation:

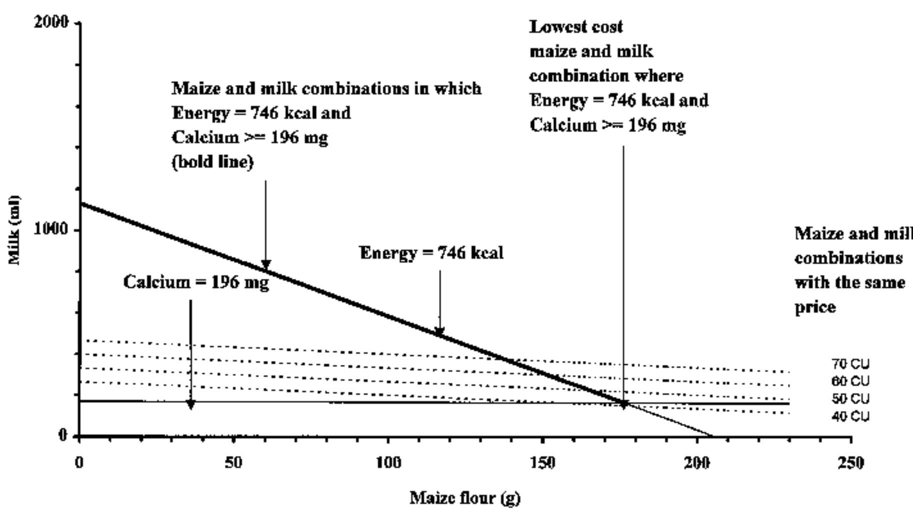

FIG. 2 Annex. Determination of a maize flour and milk combination that provides 746 Kcal and 196 mg of calcium at the lowest cost. Parallel dotted lines represent different maize and milk combinations of identical price.

$$C_{mf}.X_{mf}/1000 + C_{cm}.X_{cm}/1000 = C_{total} \quad (IV)$$

where C_{mf}, C_{cm} represent the cost per kg of maize flour and of cow milk, respectively. Unlike constraints, the objective function is represented by a series of parallel lines defined by equation (IV), each potential total cost being represented by a different straight line, with those in the lowest part of the graph representing the combinations of lowest price. Figure 2 Appendix shows these straight lines for C(tot) values of 70, 60, 50, and 40 arbitrary currency units. This graphic illustration shows that the optimal solution (*i.e.*, the maize and milk combination that provides 746 kcal and at least 196 mg of calcium at the lowest cost) occurs at the intersection of the two lines that represent the requirements for energy and calcium (*i.e.*, 746 kcal and 196 mg). In this example, the combination with the lowest cost was 177 g of maize flour plus 161 g of milk at a cost of 41.2 Units.

There is no limit to the number of foods and the number of constraints that can be included in a linear programming model. Graphic representation of constraints and solutions becomes difficult when the number of food is three, and impossible once the number of foods exceeds three, yet the basic principles remain the same, *viz*. examining the respective position of different surface or volumes in a multidimensional space and then determining whether there is a surface or a volume representing a set of possible solutions. Optimization is always based on identifying the lowest (or highest) point in an area or a volume representing all the acceptable food combinations. The simplicity of the concept contrasts with the complexity of the calculations, which increase rapidly with the number of foods and constraints introduced.

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