



Cairo University

Faculty Of Computers and Artificial Intelligence

Decision and Game Theory

Course code: DS414/DS411

Sequential Game Problem

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Part 1

1.Problem Description:

Three companies, Alpha, Beta, and Gamma, are competing in a market. Each company has to make a decision on whether to maintain (M) or reduce (R) their prices. The payoffs for each company depend not only on the immediate profit but also on the long-term market share and brand positioning.

2.Players:

- **Player 1:** Alpha
- **Player 2:** Beta
- **Player 3:** Gamma

3.Strategies:

- **Maintain (M):** Keep the current price.
- **Reduce (R):** Reduce the price to increase short-term market share.

Player one strategies {M,R}

Player two strategies {MM,MR,RM,RR}

Player three strategies

{MMMM,MMMR,MMRM,MMRR,MRMM,MRMR,MRRM,MRRR,RMMM,RM
MR,RMRM,RMRR,RRMM,RRMR,RRRM,RRRR}

4.Payoffs:

The payoffs are represented as tuples in the form of (Alpha, Beta, Gamma), where each element is the payoff for the respective company.eg (2,2,2) this means that alpha will play m and beta will play m and gamma will play and the payoff of them is (2,2,2)

5.type of game

The game is sequential game, between 3 players players make decisions one after another

6.type of information

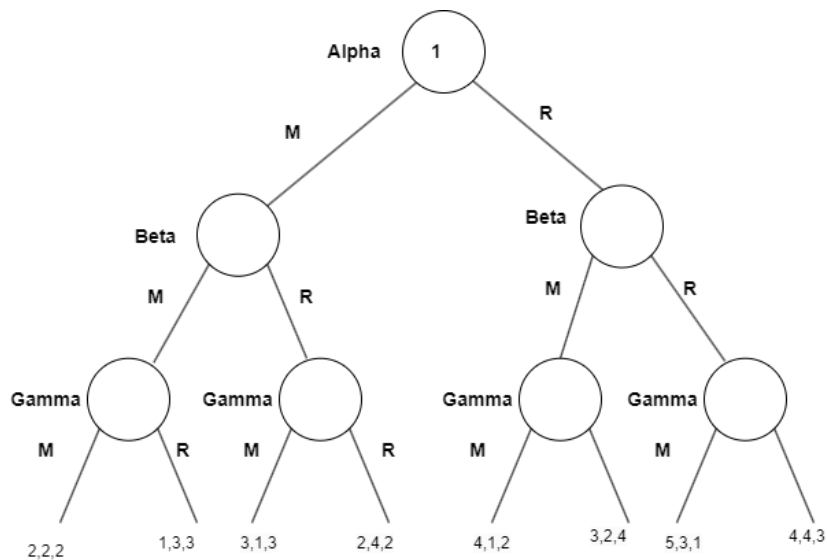
he game is one of perfect information, as each player knows the payoff structure and the available strategies for all players.

Part 2:

1.The payoff matrix

		If alpha choose m		Player 3	Maintain (M)	Reduce (R)
		Maintain (M)			(2, 2, 2)	(1, 3, 3)
		Reduce (R)			(3 1, 3)	(2, 4, 2)
Player 2 (beta)	If alpha play					
	R	Player 3(gamma)	Maintain (M)	Reduce (R)	(player2)	
	Maintain (M)		(4, 1, 2)	(3, 2, 4)		
	Reduce (R)		(5, 3, 1)	(4, 4, 3)		

2.The game tree



Summery of the game

the game has 3 compaines plays an sequential game each p;

An action profile is a combination of strategies chosen by all three players. There are a total of 8 possible action

profiles:

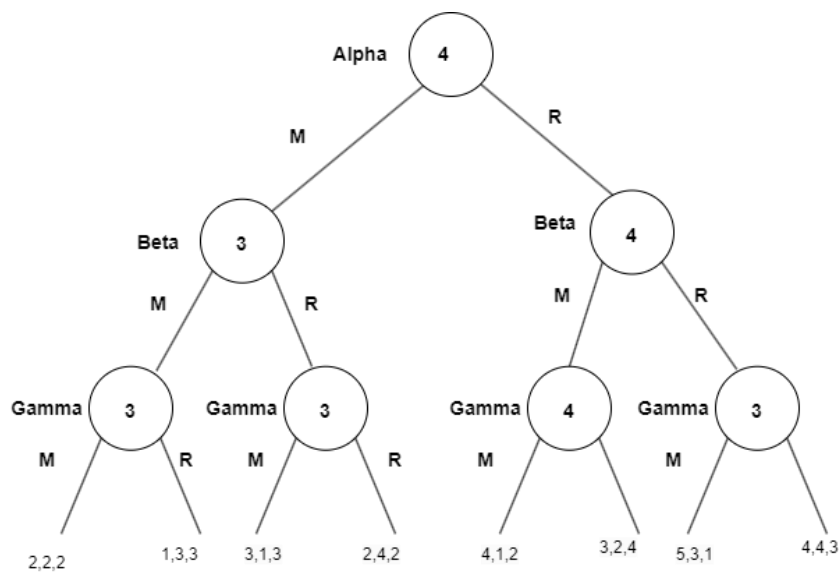
- (M, M, M)
- (M, M, R)
- (M, R, M)
- (M, R, R)
- (R, M, M)
- (R, M, R)

- (R, R, M)
- (R, R, R)

Part 3

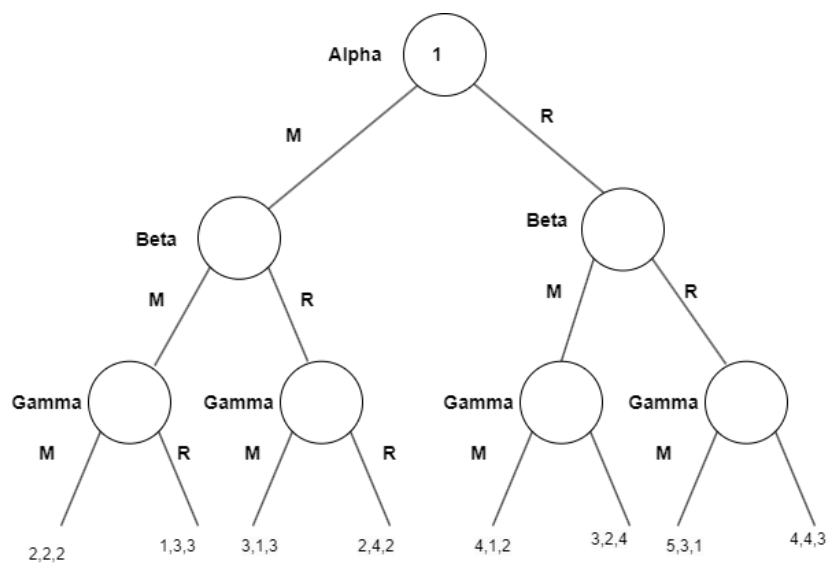
- Solving The Problem Using Backward Induction:

The solution Using Backward is (R , R , R).



- Solving The Problem Using SPNE:

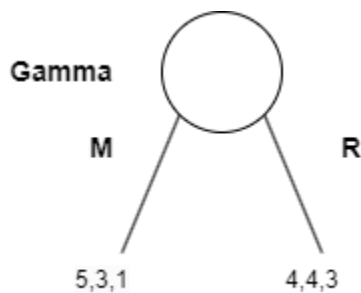
1.First Sub game:



				Alpha = M												
	MMMM	MMMF	MMRM	MMRR	MRMM	MRMR	MRRM	MRRR	RMMM	RMMR	RMRM	RMRR	RRMM	RRMR	RRRM	RRRR
MM	2,2,2	2,2,2	2,2,2	2,2,2	2,2,2	2,2,2	2,2,2	2,2,2	1,3,3	1,3,3	1,3,3	1,3,3	1,3,3	1,3,3	1,3,3	1,3,3
MR	2,2,2	2,2,2	2,2,2	2,2,2	2,2,2	2,2,2	2,2,2	2,2,2	1,3,3	1,3,3	1,3,3	1,3,3	1,3,3	1,3,3	1,3,3	1,3,3
RM	3,1,3	3,1,3	3,1,3	3,1,3	2,4,4	2,4,4	2,4,4	2,4,4	3,1,3	3,1,3	3,1,3	3,1,3	2,4,2	2,4,2	2,4,2	2,4,2
RR	3,1,3	3,1,3	3,1,3	3,1,3	2,4,2	2,4,2	2,4,2	2,4,2	3,1,3	3,1,3	3,1,3	3,1,3	2,4,2	2,4,2	2,4,2	2,4,2
				Alpha = R												
MM	4,1,2	4,1,2	3,2,4	3,2,4	4,1,2	4,1,2	3,2,4	3,2,4	4,1,2	4,1,2	3,2,4	3,2,4	4,1,2	4,1,2	3,2,4	3,2,4
MR	5,3,1	4,4,3	5,3,1	4,4,3	5,3,1	4,3,3	5,3,1	4,4,3	5,3,1	4,4,3	5,3,1	4,4,3	5,3,1	4,4,3	5,3,1	4,4,3
RM	4,1,2	4,1,2	3,3,4	3,2,4	4,1,2	4,1,2	3,2,4	3,2,4	4,1,2	4,1,2	3,2,4	3,2,4	4,4,2	4,1,2	3,2,4	3,2,4
RR	5,3,1	4,4,3	5,3,1	4,4,3	5,3,1	4,4,3	5,3,1	4,4,3	5,3,1	4,4,3	5,3,1	4,4,3	5,3,1	4,4,3	5,3,1	4,4,3

2.Second Subgame:

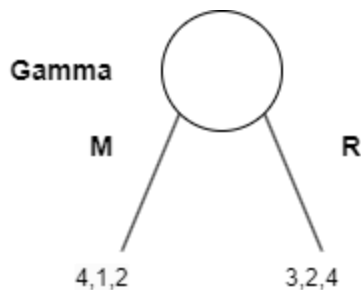
Gamma choose **R**



$$3 > 1$$

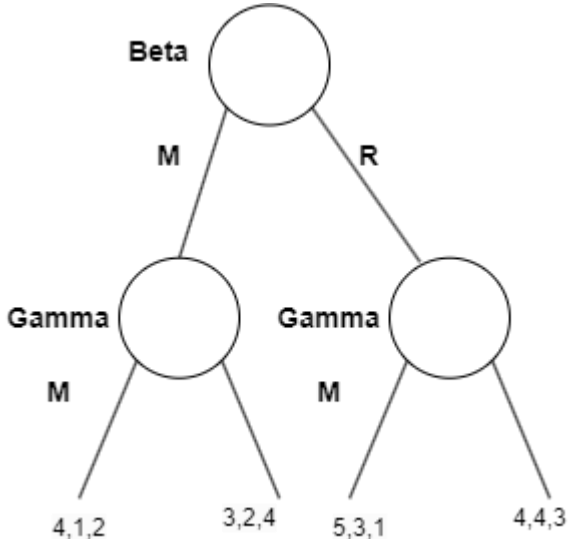
3.Third Subgame:

Gamma choose **R**



$$4 > 2$$

4.Fourth Subgame:

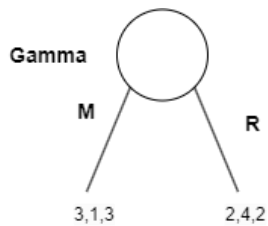


		player 3			
player 2		MM	MR	RM	RR
	M	4,1,2	4,1,2	3,2,4	3,2,4
	R	5,3,1	4,4,3	5,3,1	4,4,3

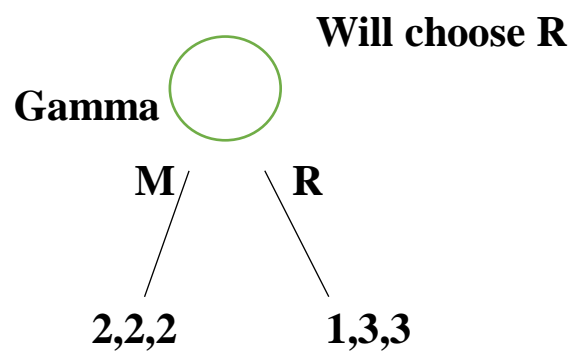
Nash (R , MR) and (R , RR)

5.Fiftth Subgame:

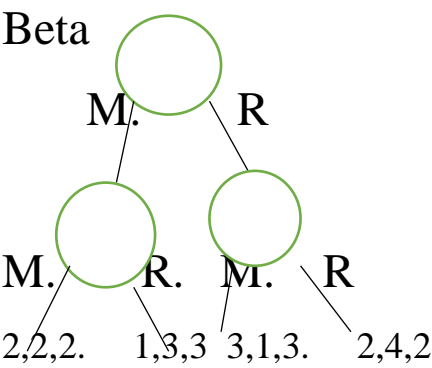
Gamma choose **M**



6.the sixth Subgame:



7.the seventh Subgame:



Player2	MM.	MR	RM
M	2,2,2	2,2,2	1,3,3
R	3,1,3	2,4,2	3,1,3

So Nash (M, RM)

	S1	S2	S3	S4	S5	S6	S7			
MRMMMR	✓	✓	✗	✓	✓	✗	✗			
MRMMRR	✓	✓	✓	✓	✓	✗	✗			
MRMRMR	✓	✓	✗	✗	✗	✗	✗			
MRMRRR	✓	✓	✓	✓	✓	✗	✗			
MRRRMR	✓	✓	✗	✓	✓	✓	✗			
MRRRRR	✓	✓	✓	✓	✓	✓	✓			
MRRRMR	✓	✓	✗	✓	✗	✓	✓			
MRRRRR	✓	✓	✓	✓	✗	✓	✗			
MRMMMR	✓	✓	✗	✓	✓	✗	✓			
MRMMRR	✓	✓	✓	✓	✓	✗	✗			
MRMRMR	✓	✓	✗	✓	✗	✓	✗			

So Nash is (MR , RMRR) and (MR , RMRR)

The nash of this game is (R , MR , RRRR) and (R , RR , RMRR).

Part 4:

- List of references:
 - <https://chatgpt.com/>
 - https://books.google.com.eg/books?hl=en&lr=&id=1WXxDwAAQBAJ&oi=fnd&pg=PP1&dq=example+game+theory&ots=ysmC8aoJ8l&sig=-s5kYGEAqfLmN1h4nMEx1Q2k1zs&redir_esc=y-v=onepage&q=example%20game%20theory&f=false