THE JOURNAL OF FINANCE • VOL. LIII, NO. 6 • DECEMBER 1998

Volume, Volatility, Price, and Profit When All Traders Are Above Average

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ABSTRACT

People are overconfident. Overconfidence affects financial markets. How depends on who in the market is overconfident and on how information is distributed. This paper examines markets in which price-taking traders, a strategic-trading insider, and risk-averse marketmakers are overconfident. Overconfidence increases expected trading volume, increases market depth, and decreases the expected utility of overconfident traders. Its effect on volatility and price quality depend on who is overconfident. Overconfident traders can cause markets to underreact to the information of rational traders. Markets also underreact to abstract, statistical, and highly relevant information, and they overreact to salient, anecdotal, and less relevant information.

MODELS OF FINANCIAL MARKETS are often extended by incorporating the imperfections that we observe in real markets. For example, models may not consider transactions costs, an important feature of real markets; so Constantinides (1979), Leland (1985), and others incorporate transactions costs into their models.

Just as the observed features of actual markets are incorporated into models, so too are the observed traits of economic agents. In 1738 Daniel Bernoulli noted that people behave as if they are risk averse. Prior to Bernoulli most scholars considered it normative behavior to value a gamble at its expected value. Today, economic models usually assume agents are risk averse, though, for tractability, they are also modeled as risk neutral. In reality, people are not always risk averse or even risk neutral; millions of people engage in regular risk-seeking activity, such as buying lottery tickets. Kahne-

* University of California, Davis. This paper is based on my dissertation at the University of California, Berkeley. I thank Brad Barber, Minder Cheng, Simon Gervais, David Hirshleifer, Bill Keirstead, Hayne Leland, Mark Rubinstein, Paul Ruud, Hersh Shefrin, René Stulz, Richard Thaler, two anonymous referees, and seminar participants at UC Berkeley and at the Russell Sage Foundation Summer Institute in Behavioral Economics for their comments. I especially thank Brett Trueman for his numerous suggestions and comments. Financial support from the Nasdaq Foundation and from the American Association of Individual Investors is gratefully acknowledged.

man and Tversky (1979) identify circumstances in which people behave in a risk-seeking fashion. Most of the time, though, most people act risk averse, and most economists model them so.[1]

This paper analyzes market models in which investors are rational in all respects except how they value information.[2] A substantial literature in cognitive psychology establishes that people are usually overconfident and, specifically, that they are overconfident about the precision of their knowledge. As is the case with risk-aversion, there are well-known exceptions to the rule, but most of the time people are overconfident. Psychologists also find that people systematically underweight some types of information and overweight others.

The paper looks at what happens in financial markets when people are overconfident. Overconfidence is a characteristic of people, not of markets. It would be convenient if each person's overconfidence had the same effect on markets. But this is not so. Some measures of the market, such as trading volume, are affected similarly by the overconfidence of different market participants; other measures, such as market efficiency, are not. The effects of overconfidence depend on how information is distributed in a market and on who is overconfident. Because analyzing the overconfidence of only one type of trader presents an incomplete and perhaps misleading picture, I look at the overconfidence of different traders: price takers in markets where information is broadly disseminated, strategic-trading insiders in markets with concentrated information, and marketmakers. I also examine markets where information is costly. Three different models are employed to facilitate this multifaceted analysis of overconfidence. These are modifications of Diamond and Verrecchia (1981) and Hellwig (1980), Kyle (1985), and Grossman and Stiglitz (1980).

The main results presented are:

- Trading volume increases when price takers, insiders, or marketmakers are overconfident. This is the most robust effect of overconfidence. Anecdotal evidence suggests that in many markets trading volume is excessive (Dow and Gorton (1997)). Recent empirical studies (Odean (1998a), Statman and Thorley (1998)) indicate that overconfidence generates trading. From a modeling perspective, overconfidence can facilitate orderly trade even in the absence of noise traders.

[1] Another place where observed behavior has found wide acceptance in economic models is in the discounted utility of future consumption. Nineteenth-century economists such as Senior, Jevons, and Böhm-Bawerk believed that, ideally, the present and the future should be treated equally; yet they observed that generally people value present consumption more highly than future (Loewenstein (1992)). Today, when it may affect the predictions of models, economists usually assume that people discount the utility of future consumption. And people usually do discount the future—but not always. They will, for example, "bite the bullet" and get an unpleasant experience over with, which they could otherwise delay. Loewenstein and Prelec (1991) identify circumstances in which people demonstrate negative, rather than the usual positive, time preference.

[2] In the first model presented here, investors also behave with less than full rationality by trading myopically.

- Overconfident traders can cause markets to underreact to the information of rational traders, leading to positive serially correlated returns. Returns are also positively serially correlated when traders underweight new information and negatively serially correlated when they overweight it. The degree of this under- or overreaction depends on the fraction of all traders who under- or overweight the information. A review of the psychology literature on inference finds that people systematically underweight abstract, statistical, and highly relevant information, and overweight salient, anecdotal, and extreme information. This may shed some light on why markets overreact in some circumstances, such as initial public offerings (IPOs) (Ritter (1991)), and underreact in others, such as earnings announcements (Bernard and Thomas (1989, 1990)), dividend initiations and omissions (Michaely, Thaler, and Womack (1995)), open-market share repurchases (Ikenberry, Lakonishok, and Vermaelen (1995)), and brokerage recommendations (Womack (1996)).
- Overconfidence reduces traders' expected utility. Overconfident traders hold underdiversified portfolios. When information is costly and traders are overconfident, informed traders fare worse than uninformed traders. And, as Barber and Odean (1998) find to be true for individual investors, those who trade more actively fare worse than those who trade less. Overconfidence may also cause investors to prefer active management (Lakonishok, Shleifer, and Vishny (1992)) despite evidence that it subtracts value.
- Overconfidence increases market depth.
- Overconfident insiders improve price quality, but overconfident price takers worsen it.
- Overconfident traders increase volatility, though overconfident marketmakers may dampen this effect. Excess volatility in equity markets has been found by some researchers (Shiller (1981, 1989), LeRoy and Porter (1981)), though others have questioned these findings (Kleidon (1986), Marsh and Merton (1986)).

The rest of the paper is organized as follows: Section I reviews related work. Section II describes some of the literature on overconfidence and on inference and discusses why we should expect to find overconfidence in financial markets. Section III presents the models. Section IV discusses the results. And the final section concludes. Formal statements of the propositions, proofs, and the derivations of equilibria are presented in the Appendixes. Table I provides a summary of notation used in the models.

I. Related Work

A number of researchers have modeled economies in which traders hold mistaken distributional beliefs about the payoff of a risky asset. In Varian (1989) traders' priors have different means. Varian notes that the dispersion of posterior beliefs caused by differing distributional assumptions motivates trade. Harris and Raviv (1993) investigate a multiperiod economy in which

Table I

Notation

	Price Takers Model	Insider Model	Costly Information Model
Overconfidence parameter	$\kappa \geq 1$	$\kappa \geq 1$	$\kappa \geq 1$
Parameter underweighting priors	$\eta \leq 1$	$\eta \leq 1$	$\eta \leq 1$
Parameter underweighting signals of others	$\gamma \leq 1$		
Number of traders	$i = 1, \ldots, N$	1 insider, 1 marketmaker	$i = 1, \ldots, N$
Time	$t = 0, \ldots, 4$	$t = 0, 1$	$t = 0,1$
Number of distinct signals	$m = 1, \ldots, M$	1	1
Terminal value of risky asset	$\tilde{v} \sim N(0, h_v^{-1})$	$\tilde{v} \sim N(0, h_v^{-1})$	$\tilde{v} \sim N(0, h_v^{-1})$
Signals	$\tilde{y}_{ti} = \tilde{v} + \tilde{\epsilon}_{tm}$	$\tilde{y} = \tilde{v} + \tilde{\epsilon}$	$\tilde{y} = \tilde{v} + \tilde{\epsilon}$
Error term in signals	$\tilde{\epsilon}_{tm} \sim N(0, h_\epsilon^{-1})$	$\tilde{\epsilon} \sim N(0, h_\epsilon^{-1})$	$\tilde{\epsilon} \sim N(0, h_\epsilon^{-1})$
Noise trader demand		$\tilde{z} \sim N(0, h_z^{-1})$	$\tilde{z} \sim N(0, h_z^{-1})$
Coefficient of absolute risk aversion	a		a
Per capita supply of risky asset	$\bar{x}$		$\bar{x}$
Price of risky asset	P_t	P	P
i's endowment of risky asset	x_{0i}		x_{0i}
i's demand for risky asset	x_{ti}	x	x_{ti}
i's endowment of riskless asset	f_{0i}		f_{0i}
i's demand for riskless asset	f_{ti}		f_{ti}
i's wealth	W_{ti}		W_{ti}
Trader i's information set	Φ_{ti}		
Fraction of traders who buy information			λ

risk-neutral traders disagree about how to interpret a public signal. The model of price-taking traders presented here differs from Harris and Raviv in that my traders are risk averse and disagree about the interpretation of private signals. Furthermore, the nature of this disagreement is grounded in psychological research. In Kandel and Pearson (1995), risk-averse traders disagree about both the mean and the variance of a public signal. In this case, the public signal may motivate increased trading even when it does not change price. De Long et al. (1990) show in an overlapping generations model that oveconfident traders who misperceive the expected price of a risky asset may have higher expected returns, though lower expected utilities, than rational traders in the same economy. Roll (1986) suggests that overconfidence (hubris) may motivate many corporate takeovers. Hirshleifer, Subrahmanyam, and Titman (1994) argue that overconfidence can promote herding in securities markets. Figlewski (1978) finds that the influence of traders with different posterior beliefs on prices depends on wealth, risk aversion, and overall willingness to trade; Feiger (1978) also points out that a trader's influence on price depends on her wealth. Jaffe and Winkler (1976) find that the probability of trading is a function of the precision of an investor's information.

Shefrin and Statman (1994) develop a model in which traders infer, from past observations, the transition matrix governing changes in the dividend growth rate. In their model, some traders are true Bayesians; others make one of two common errors: They weight recent observations too heavily, thus underweighting prior information, or they commit a gambler's fallacy, expecting recent events to reverse so that short runs of realized events more closely resemble long-term probabilities. When all traders are rational, the market behaves as if it had a "single driver" and prices are efficient. Biased traders can introduce a "second driver," thereby distorting prices and, over time, increasing volatility while decreasing market efficiency.

Benos (1998), Kyle and Wang (1997), and Wang (1995) look at overconfidence in models based on Kyle (1985), but with two informed traders. In Benos, traders are overconfident in their knowledge of the signals of others; they also can display extreme overconfidence in their own noisy signal, believing it to be perfect. Kyle and Wang (1997) and Wang (1995) model overconfidence similarly to how it is modeled in this paper—that is, as an overestimation of the precision of one's own information.[3] Gervais and Odean (1997) develop a multiperiod model in which a trader's endogenously determined level of overconfidence changes dynamically as a result of his tendency to disproportionately attribute his success to his own ability. In Daniel, Hirshleifer, and Subrahmanyam (1998) rational risk-averse traders trade with risk-neutral traders who overreact to private signals, properly weight public signals, and grow more overconfident with success. This results in return-event patterns that are consistent with many market anomalies. My paper differs from these others in that it examines how the effects of over-

[3] I learned of Kyle and Wang's work after developing the models in this paper.

confidence depend on who, in a market, is overconfident and on how information in that market is disseminated; it also relates market under- and overreactions to the psychological literature on inference.

II. Overconfidence

A. The Case for Overconfidence

Studies of the calibration of subjective probabilities find that people tend to overestimate the precision of their knowledge (Alpert and Raiffa (1982), Fischhoff, Slovic and Lichtenstein (1977); see Lichtenstein, Fischhoff, and Phillips (1982) for a review of the calibration literature). Such overconfidence has been observed in many professional fields. Clinical psychologists (Oskamp (1965)), physicians and nurses, (Christensen-Szalanski and Bushyhead (1981), Baumann, Deber, and Thompson (1991)), investment bankers (Staël von Holstein (1972)), engineers (Kidd (1970)), entrepreneurs (Cooper, Woo, and Dunkelberg (1988)), lawyers (Wagenaar and Keren (1986)), negotiators (Neale and Bazerman (1990)), and managers (Russo and Schoemaker (1992)) have all been observed to exhibit overconfidence in their judgments. (For further discussion, see Lichtenstein et al. (1982) and Yates (1990).)

The best established finding in the calibration literature is that people tend to be overconfident in answering questions of moderate to extreme difficulty (Fischhoff et al. (1977), Lichtenstein et al. (1982), Yates (1990), Griffin and Tversky (1992)). Exceptions to overconfidence in calibration are that people tend to be underconfident when answering easy questions, and they tend to be well calibrated when predictability is high and when performing repetitive tasks with fast, clear feedback. For example, expert bridge players (Keren (1987)), race-track bettors (Dowie (1976), Hausch, Ziemba, and Rubinstein (1981)) and meteorologists (Murphy and Winkler (1984)) tend to be well calibrated.

Miscalibration is only one manifestation of overconfidence. Researchers also find that people overestimate their ability to do well on tasks and these overestimates increase with the personal importance of the task (Frank (1935)). People are also unrealistically optimistic about future events. They expect good things to happen to them more often than to their peers (Weinstein (1980); Kunda (1987)). They are even unrealistically optimistic about pure chance events (Marks (1951), Irwin (1953), Langer and Roth (1975)).[4]

People have unrealistically positive self-evaluations (Greenwald (1980)). Most individuals see themselves as better than the average person and most individuals see themselves better than others see them (Taylor and Brown

[4] Ito (1990) reports evidence that participants in foreign exchange markets are more optimistic about how exchange rate moves will affect them than how they will affect others. Over two years the Japan Center for International Finance conducted a bimonthly survey of foreign exchange experts in forty-four companies. Each was asked for point estimates of future yen/dollar exchange rates. Those experts in import-oriented companies expected the yen to appreciate (which would favor their company);, those in export-oriented companies expected the yen to fall (which would favor their company).

(1988)). They rate their abilities and their prospects higher than those of their peers. For example, when a sample of U.S. students—average age 22—assessed their own driving safety, 82 percent judged themselves to be in the top 30 percent of the group (Svenson (1981)).[5] And 81 percent of 2994 new business owners thought their business had a 70 percent or better chance of succeeding but only 39 percent thought that any business like theirs would be this likely to succeed (Cooper et al. (1988)). People overestimate their own contributions to past positive outcomes, recalling information related to their successes more easily than that related to their failures. Fischhoff (1982) writes that "they even misremember their own predictions so as to exaggerate in hindsight what they knew in foresight." And when people expect a certain outcome and the outcome then occurs, they often overestimate the degree to which they were instrumental in bringing it about (Miller and Ross (1975)). Taylor and Brown (1988) argue that exaggerated beliefs in one's abilities and unrealistic optimism may lead to "higher motivation, greater persistence, more effective performance, and ultimately, greater success." These beliefs can also lead to biased judgments.

In this paper overconfidence is modeled as a belief that a trader's information is more precise than it actually is. As a consequence, traders' posterior beliefs are too precise—a result directly supported by the calibration literature cited above. How heavily information is weighted depends not only on overconfidence but also on the nature of the information. Because the overconfident traders in these models believe their information to be more precise than it is, they weight it too heavily when updating their Bayesian posteriors. Relying on these posteriors, they take actions that affect markets. The models can also be used to analyze the effects of overweighting or underweighting information (when updating posteriors) for reasons in addition to overconfidence (e.g., see Proposition 5). To understand such reasons, it is useful to briefly review the psychological literature on inference.

B. Inference

Psychologists find that, when making judgments and decisions, people overweight salient information (i.e., information that stands out and captures attention) (Kahneman and Tversky (1973), Grether (1980)). People also give too much consideration to how extreme information is and not enough to its validity (Griffin and Tversky (1992)); they "often behave as though information is to be trusted regardless of its source, and make equally strong or confident inferences, regardless of the information's predictive value.... Whether the information is accurate and fully reliable or alternatively out-of-date, inaccurate, and based on hearsay may... matter little" (Fiske and Taylor (1991)). They overweight information that is consistent with their existing beliefs, are prone to gather information that supports these beliefs, and readily dismiss information that does not (Lord, Ross, and Lepper (1979),

[5] A modest 51 percent of a group of older Swedish students—average age 33—placed themselves in the top 30 percent of their group.

Nisbett and Ross (1980), Fiske and Taylor (1991)). They are more confident in opinions based on vivid information (Clark and Rutter (1985)) and weigh cases, scenarios, and salient examples more heavily than relevant, abstract, statistical, and base-rate information (Kahneman and Tversky (1973), Bar-Hillel (1980), Hamill, Wilson, and Nisbett (1980), Nisbett and Ross (1980), Bar-Hillel and Fischhoff (1981), Taylor and Thompson (1982), Tversky and Kahneman (1982)). In addition to underweighting base-rate information, people underestimate the importance of sample size (Tversky and Kahneman (1971), Kahneman and Tversky (1972)) and of regression to the mean, that is, the tendency of extreme outcomes to be followed by outcomes closer to the population mean (Kahneman and Tversky (1973)).

In general then, we might expect people to overreact to less relevant, more attention-grabbing information (e.g., an extreme event, a prominent news article with strong human interest, a rumor) while underreacting to important abstract information.[6] In particular, we might expect people to underestimate the importance of single statistics that summarize a large sample of relevant data (e.g., corporate earnings).

C. Information

In the following models, traders update their beliefs about the terminal value of a risky asset, $\tilde{v}$, on the basis of three sources of information: a private signal, their inferences from market price regarding the signals of others, and common prior beliefs. The overconfidence literature indicates that people believe their knowledge is more precise than it really is, rate their own abilities too highly when compared to others, and are excessively optimistic. To be consistent with these patterns, traders in the model must hold posterior beliefs about the distribution of $\tilde{v}$ that are too precise, value their own information more than others' information, and expect higher utility than is warranted. In the models, traders overweight their private signals and, therefore, their posteriors are too precise, their own information is valued more than that of others, and they overestimate their expected utility.

For most of the propositions in this paper to be true, it is sufficient that traders (1) hold posterior beliefs that are too precise and (2) overweight their own information relative to that of others.[7] Both conditions are satisfied if each trader overweights his own signal. These conditions may be further amplified when traders underweight their common prior beliefs or under-

[6] Reacting to how extreme information is rather than how reliable its source is can have dramatic consequences. On April 11, 1997, The *Financial Times of London* reported fraud in connection with an offshore fund called the Czech Value Fund, referring to the fund by the abbreviation CVF. Four days later Castle Convertible fund, a small closed-end fund with a diversified portfolio of convertible stocks and bonds trading on the AMEX under the ticker symbol CVF, plummeted 32 percent in twenty-two minutes. Trading was halted. After the Castle managers assured the exchange that they had no news, trading resumed at close to its preplunge price. Apparently some investors reacted to word of extreme problems rather than to the reliability of that word (*New York Times*, April 20, 1997, p. C1, byline Floyd Norris).

[7] Propositions 2 and 9 require additionally that traders value new information relative to prior information.

weight the signals of others when updating beliefs. Common priors incorporate previous information about security returns' behavior and thus constitute base-rate data that are likely to be underweighted when updating. The signals of others constitute a large sample. Because large sample inferences are usually undervalued, it is likely that if traders err in valuing the signals of others, they will undervalue these. Most of the propositions are proven only for the situation where each trader overweights his own signals. Often, corollaries can be proven for when traders underweight their priors or underweight the signals of others. In the interest of parsimony, these corollaries are not stated formally or proven, though the intuition involved is at times discussed.[8]

The calibration literature discussed above tells us that people overestimate the precision of their information. Overconfidence in one's information is not the only type of overconfidence we might expect to find in the market. Traders could, instead, be overconfident about the way they interpret information rather than about the information itself. For example, traders of a stock might look at signals such as trading momentum, price/earnings ratio, or forecasts of industry trends. These are examples of public information that is available to any trader but is valued differently by different traders. Thus, a Graham-and-Dodd style fundamental investor might be aware of recent changes in a stock's momentum but consider its price/earnings ratio to be a more important signal; a technical trader who follows momentum might believe otherwise. Each is overconfident in his style of analysis and the signal he utilizes. At the same time, each is aware of the beliefs, and perhaps even the signals, of the other.[9] Given people's tendency to reject information that does not fit their beliefs (Fiske and Taylor (1991)), the differing opinions of others are likely to be undervalued.

In the models, traders who believe that their information is more precise than it actually is anticipate greater future utility than it is reasonable to anticipate. In this way these models capture some of the spirit of excessive

[8] When traders in these models underweight new information, the opinions of others, or prior information, the means of their posteriors deviate from the posterior means rational traders would form. As discussed above, these deviations are consistent with how people process different types of information. However, underweighting any of these three sources of information causes traders to underestimate the precision of their posteriors. Such underconfidence is not consistent with generally observed behavior. Even when they discount valid information, people usually maintain strongly held beliefs (e.g., Lord et al. (1979)). Weakly held posteriors do not motivate the results in this paper and, when they arise, should not be considered realistic implications of the models.

[9] Even sophisticated investors may agree to disagree. *The Washington Post* (January 7, 1992, p. C2, byline Allan Sloan) reports that, during the same time period, the nation's most prominent long-term investor, Warren Buffett, and its most prominent short sellers, the Feshbach brothers, held, respectively, long and short positions worth hundreds of millions of dollars in Wells Fargo Bank. (Buffett controls the investments of Berkshire Hathaway Inc.; the Feshbachs run an investment fund.) Ostensibly, Buffett and the Feshbachs disagreed about how much the bank would be hurt by its weak loan portfolio. They also differed in their investment horizons. Despite being right about the loans, the Feshbachs lost $50 million when they had to close their positions. As of January 1992, Buffett was about even.

optimism which psychologists have documented. However, optimism is not limited to an inflated opinion of the precision of unbiased signals. A trader might also have false confidence in a biased (misinterpreted) signal or theory.

D. Overconfidence in Financial Markets

Why might we expect those trading in financial markets to be overconfident? The foremost reason is that people usually *are* overconfident. The exceptions to overconfidence mentioned in Section II.A generally do not apply to financial markets. Most of those who buy and sell financial assets try to choose assets that will have higher returns than similar assets. This is a difficult task and it is precisely in such difficult tasks that people exhibit the greatest overconfidence. Not only novices exhibit overconfidence. Griffin and Tversky (1992) write that when predictability is very low, as in the stock market, experts may even be more prone to overconfidence than novices, because experts have theories and models (e.g., of market behavior) which they tend to overweight.[10]

Securities markets are difficult and slow places in which to calibrate one's confidence. Learning is fastest when feedback is quick and clear, but in securities markets the feedback is often slow and noisy. There may even be a trade-off between speed and clarity of feedback whereby short-term traders get quicker, but noisier, feedback, and long-term traders receive clearer feedback but must wait for it. The problem of noisy feedback can be exacerbated by the endogeneity of the evaluation period. Shefrin and Statman (1985) propose and Odean (1998b) confirms that investors prefer to sell winners and hold losers. If investors judge their original purchase decisions on the basis of the returns realized, rather than those accrued, then, by holding losers, they will judge themselves to have made fewer poor decisions. Furthermore, the feedback from losses will be delayed more than that from gains, further facilitating positive self-evaluations.

Selection bias may cause those participating actively in financial markets to be more overconfident than the general population. People vary in ability and those who believe they have more ability to trade may be more likely to seek jobs as traders or to trade actively on their own account. If people are uncertain judges of their own ability, then we might expect financial markets to be populated by those with the most ability and by those who most overestimate their ability.

Survivorship bias can also lead to overconfidence by market participants. Unsuccessful traders may lose their jobs or choose to drop out of the market; unsuccessful traders who survive will, on average, control less wealth than successful traders. If traders overestimate the degree to which they were responsible for their own successes—as people do in general (Miller and Ross (1975), Langer and Roth (1975); Nisbett and Ross (1980))—successful traders may grow overconfident and more wealth will be controlled by overcon-

[10] This observation may not apply to experts who adhere to computer-based quantitative models (see Dawes, Faust, and Meehl (1989)).

fident traders. In Gervais and Odean (1997) this self-enhancing bias causes wealthy traders, who are in no danger of being driven from the marketplace, to be overconfident. It is not that overconfidence makes them wealthy, but the process of becoming wealthy contributes to their overconfidence. An old Wall Street adage, "Don't confuse brains with a bull market," warns traders of the danger of becoming overconfident during a market rally; no doubt this warning is given for good reason.

This paper finds overconfident traders have lower expected utility.[11] It does not necessarily follow that the overconfident traders lose their wealth and leave the marketplace. An overconfident trader makes biased judgments that may lead to lower returns. However, an overconfident risk-averse trader also chooses a riskier portfolio than he would otherwise hold and may be rewarded for risk-bearing with greater expected returns. It is possible that the profits of greater risk tolerance will more than compensate for the losses of biased judgments. Thus, as a group, overconfident traders could have higher expected returns, though lower expected utility, than properly calibrated traders, as is the case in De Long et al. (1990).

III. The Interaction between Overconfidence and Market Structure

A. Price Takers

Throughout this paper, expectations taken using the distributions that traders believe to be correct are indicated by a subscript "b" (e.g., $\mathrm{var}_{\mathrm{b}}$). Expectations taken using the distributions that are actually correct are indicated by a subscript "a" (e.g., $\mathrm{var}_{\mathrm{a}}$). In equilibrium, overconfident traders believe that they are acting optimally, and so they do not depart from the equilibrium. The traders could, in actuality, improve their expected utilities by acting differently, so the equilibria achieved here are not rational expectations equilibria.

The model of price-taking traders is based on Diamond and Verrecchia (1981) and Hellwig (1980). A riskless asset and one risky asset are exchanged in three rounds of trading at times $t = 1$, $t = 2$, and $t = 3$. Consumption takes place only at $t = 4$, at which time the riskless asset pays 1 unit per share and each share of the risky asset pays $\tilde{v}$, where $\tilde{v} \sim N(\bar{v}, h_v^{-1})$. The riskless interest rate is assumed to be 0. There are N investors ($i = 1, \ldots, N$). As a modeling convenience we analyze the limit economy where $N \to \infty$. Thus each investor correctly assumes that his own demand does not affect prices. At $t = 0$ each trader has an endowment of f_{0i} of the riskless asset and x_{0i} of the risky asset. In trading round t, trader i's demands for the riskless asset and the risky asset are f_{ti} and x_{ti}. $\bar{x}$ is the per capita supply of the risky asset; it is fixed, known to all, and unchanging. This differs from Diamond and Verrecchia (1981) and Hellwig (1980) where a stochastic supply of the risky asset provides an exogenous source of noise. P_t is the price of the risky asset in trading rounds 1, 2, and 3. Trader i's

[11] When objectively measured, expected utility is lower for overconfident traders. However, overconfident traders believe that they are maximizing expected utility.

wealth is $W_{ti} = f_{ti} + P_t x_{ti}$, for $t = 1$, 2, and 3, and $W_{4i} = f_{3i} + \tilde{v} x_{3i}$. There is no signal prior to the first round of trading at $t = 1$. Prior to trading at $t = 2$ and, again, prior to trading at $t = 3$, trader i receives one of M private signals, $\tilde{y}_{ti} = \tilde{v} + \tilde{\epsilon}_{tm}$, where $\tilde{\epsilon}_{tm} \sim N(0, h_\epsilon^{-1})$ and $\tilde{\epsilon}_{21}, \ldots, \tilde{\epsilon}_{2M}, \tilde{\epsilon}_{31}, \ldots, \tilde{\epsilon}_{3M}$ are mutually independent. Each signal is received by the same number of traders. (N is assumed to be a multiple of M.) $\bar{Y}_t = \Sigma_{i=1}^N y_{ti}/N = \Sigma_{m=1}^M y_{tm}/M$ is the average signal at time t.

The assumption that there are $M < N$ signals in any time period is motivated by the observation that when the number of traders is large there are likely to be fewer pieces of information about an asset than there are traders.

Each trader knows that $N/M - 1$ other traders are receiving the same two signals as she is. She believes the precision of these two signals to be κh_ϵ, $\kappa \geq 1$. She believes the precision of the other $2M - 2$ signals to be γh_ϵ, $\gamma \leq 1$. All traders believe that the precision of $\tilde{v}$ is ηh_v, $\eta \leq 1$; that is, traders underestimate, or correctly estimate, the precision of their prior information. Let $\Phi_{1i} = \{\ \}$, $\Phi_{2i} = [y_{2i} \quad P_2]^T$, and $\Phi_{3i} = [y_{2i} \quad y_{3i} \quad P_2 \quad P_3]^T$. Thus, Φ_{ti} represents the information available to trader i (in addition to prior beliefs) at time t. Note that a trader's posterior is more precise than that of a rational trader if, after receiving both of her signals, $\eta h_v + 2(\kappa + (M - 1)\gamma)\ h_\epsilon \geq h_v + 2Mh_\epsilon$.

Trader i's utility function is $-\exp(-aW_{it})$, thus traders have constant absolute risk aversion (CARA) with a risk-aversion coefficient of a. Traders are assumed to be myopic, that is, they look only one period ahead when solving their trading problem. Thus, at times $t = 1$, 2, 3, trader i solves

$$\max_{x_{ti}} \mathrm{E}[-\exp(-a(W_{t+1i})|\Phi_{ti}] \quad \text{subject to } P_t x_{ti} + f_{ti} \leq P_t x_{t-1i} + f_{t-1i}. \tag{1}$$

The traders in this model correctly conjecture that they do not affect prices, thus the only effect of assuming myopia is to eliminate hedging demands (see Brown and Jennings (1989)). As others, including Singleton (1987) and Brown and Jennings (1989), have found, this simplifies the analysis.

When solving their maximization problems, traders conjecture that prices are linear functions of the average signals:

$$P_3 = \alpha_{31} + \alpha_{32}\bar{Y}_2 + \alpha_{33}\bar{Y}_3 \tag{2}$$

$$P_2 = \alpha_{21} + \alpha_{22}\bar{Y}_2. \tag{3}$$

The conjectures are identical for all traders and the coefficients determine an equilibrium in which the conjectures are fulfilled. Equilibrium is obtained because traders believe that they are behaving optimally even though, in fact, they are not. This equilibrium and the proofs for this section are presented in Appendix A.

There is no exogenous noise in this model. The purpose of noise is often to keep traders from using price and aggregate demand to make perfect inferences about the information of others. If rational traders with common priors infer the same aggregate signal, they form identical posterior beliefs, and, if their endowments and preferences are also identical, they will not trade. If preferences and endowments differ, trading may occur but it might not occur in response to information, and this runs contrary to what we observe in markets.[12] The absence of exogenous noise in this model demonstrates that, with overconfidence, orderly trading can take place in response to information even when no noise is present. (Varian (1989) has a similar result when traders disagree about the mean of the prior.) Each trader can infer the aggregate signal, but each values his portion of the aggregate differently, arrives at a different posterior belief, and is willing to trade.

In this model, traders can perfectly infer the aggregate signal from price. In practice, traders do not usually make this perfect inference. The certainty in the model would be dispelled if randomly trading noise traders were added to the economy. However, this certainty results not so much from the lack of noise trading as from the conventional assumption that traders are able to know the preferences of all other traders, to know the distributions of all random variables in the economy (though here these are distorted by overconfidence), and to make perfect inferences from their information. In addition to knowing each other's preferences, when traders are not risk neutral and do not have constant absolute risk aversion, they must also know each other's wealth to infer the signals behind trades. As Arrow (1986) points out, the information gathering and computational demands put on traders in models such as this would, in a more realistic setting, "imply an ability at information processing and calculation that is far beyond the feasible and cannot be well justified as the result of learning and adaptation." It may be that the principal source of noise in markets is not that a few (noise) traders do not attempt to optimize their utility, but that most traders are not certain how best to do so.

In its lack of exogenous noise, this model is similar to that of Grossman (1976). But in Grossman's model, a trader can infer the aggregate signal $\bar{y}$ from price and, having done so, can ignore his private signal y_i when determining his demand. As Beja (1976) observes, this creates a paradox in which fully informative prices arise from an aggregate demand function that is without information because if prices are fully informative, traders have no incentive to consider their private signals when formulating their demand. When traders are overconfident, they can still infer the average signal from price, but they do not ignore their own signal when determining their demand. Each trader considers his signal to be superior to those of others, and because the average signal weights all traders' signals equally, it is not a sufficient statistic to determine an individual trader's demand.

[12] See Varian (1989) for a discussion of no-trade theorems.

This model is also related to Figlewski's (1978) model where price-taking traders with different posterior beliefs interact. Figlewski's model does not have an exogenous noise source. To avoid the no-trade dilemma, he assumes traders are unable to infer the information of others from price. Were these traders overconfident, this assumption could be eased and results similar to the ones presented here would follow. In Jaffe and Winkler (1976), risk-neutral informed traders decide to trade after observing a risk-neutral marketmaker's bid and ask. The marketmaker can expect to lose to all rational investors, and so this market is unstable. Jaffe and Winkler suggest that the introduction of liquidity traders or traders who misperceive their ability—such as the overconfident traders modeled here—could stabilize this market.

As discussed above, overconfidence causes traders to have differing posterior beliefs. The more overconfident traders are, the more differing these beliefs. This leads to the first proposition.

PROPOSITION 1: *When traders are price takers, expected volume increases as overconfidence increases (if $M \geq 2$).*

In all of the propositions, expectations are taken over the true probability distributions. Here we see that as overconfidence increases, traders increasingly weight their own signals more heavily than they weight those of others when calculating their posterior beliefs. Their posterior beliefs are therefore more dispersed and more trading takes place.[13] There is one exception to this pattern. If $M = 1$ there is only one (effectively public) signal received by all traders. And because all traders overvalue that signal equally, their beliefs remain homogeneous and no trade takes place though price may change. (If traders varied in their overconfidence in the public signal, they would trade.) Expected volume also increases when traders underweight common priors or the signals of others.

PROPOSITION 2: *When traders are price takers, volatility of prices increases as overconfidence increases.*

When traders are overconfident, each overvalues his own personal signal. This results in the aggregate signal being overvalued relative to the common prior in the pricing functions (equations (2) and (3)) where the coefficients α_{22}, α_{32}, and α_{33} are increasing in κ. Overweighting the error in the aggregate signal increases the volatility of prices. Decreasing η has the same effect and decreasing γ lowers the weight on the aggregate signal and lowers volatility. Another consequence of biased expectations is that they increase the variance of the difference between price and underlying value, $\mathrm{var}(P - \tilde{v})$. Using this variance as a measure of the quality of prices we have the following proposition.

PROPOSITION 3: *When traders are price takers, overconfidence worsens the quality of prices.*

[13] In a dynamic setting, such as that of Shefrin and Statman (1994), volume is determined not simply by differences in beliefs but by the rate of change of those differences (see Karpoff (1986)).

We will see in the next model that when a strategic insider is overconfident, overconfidence can improve the quality of prices. These two models differ in that the next model has noise traders, but, more important, they differ in how information is distributed. Here all traders receive a signal, in the next model information is concentrated in the hands of a single insider. Even if noise is added to the current model, overconfidence will continue to distort prices, not improve them. This is most easily seen when $M = 1$, that is, when there is one public signal. If the signal is public, noisy demand will obviously not affect traders' information, but overconfidence will continue to distort traders' posterior expectations and, thereby, prices. Price quality also worsens when traders underweight common priors or the signals of others.

Distorted expectations reduce expected utilities. When traders are overconfident, their expected utility is lower than when their probabilities are properly calibrated. This is hardly surprising because traders choose their actions in order to optimize expected utility, and when they are overconfident, this attempt to optimize is based on incorrect beliefs. (Similarly, expected utility also declines as η and γ decrease.) And so we have Proposition 4.

PROPOSITION 4: *When price-taking traders are overconfident, their expected utility is lower than if their beliefs are properly calibrated.*

There are no noise traders to exploit in this model, so the aggregate expected returns from trading must be zero. Overconfidence decreases expected utilities because it results in nonoptimal risk sharing. Overconfident traders hold underdiversified portfolios. Those who receive the highest signals hold too much of the risky asset and too little of the risk-free asset; others hold too little of the risky asset and too much of the risk-free asset (given their preferences and the true distributions of signals). Of course each trader believes that she is optimally positioned.[14]

To model overconfidence, I assume that traders overestimate the precision of their private signals. Doing so leads traders to hold differing beliefs and to overestimate the precision of their own posterior beliefs. Diverse posterior beliefs that are held too strongly are sufficient to promote excessive trading, increase volatility, distort prices, and reduce expected utility. For time-series results though, how posteriors are constructed matters. Assuming that people always overweight private signals implies that they always overweight new information. But, as discussed in Section II.B, people do not always overweight new information. They usually overrespond to salient information and underrespond to abstract information. They underweight valuable information and overweight irrelevant information. To examine time-series implications of the model, we therefore look at both over- and underweighted signals. For price-taking traders, overvaluing new information leads to price reversals, undervaluing it leads to price trends.

[14] If traders are overconfident and $M = 1$, expected utilities will not be affected (although prices will change). Beliefs will be homogeneous, albeit mistaken, and traders will hold the same optimal portfolios they would hold if they valued their information correctly.

PROPOSITION 5: *When price-taking traders overvalue (undervalue) new information, price changes exhibit negative (positive) serial correlation.*

Any prediction based on this proposition requires an analysis of the type of information traders are receiving. Note that the serial correlation of returns and of price changes will have the same sign.

Up to this point, all of the traders in this model are overconfident. What would happen if some traders were rational? In general, rational traders would mitigate but not eliminate the effects of the overconfident traders (just as rational traders do not eliminate the effects of trader errors in De Long et al. (1990) or Shefrin and Statman (1994)). In markets such as the one modeled here, traders vote with their dollars. As Figlewski (1978) points out, "a trader with superior information but little wealth may have his information undervalued in the market price." Due to the assumption of constant absolute risk aversion, wealthy traders in the model trade no more than poor ones and so the impact on price of traders with particular viewpoints depends here on their numbers, not wealth. The mere presence of rational traders does not drive price to its rational value. To change price, traders must be willing to trade. Willingness to trade generally depends on strength of beliefs, risk tolerance, and wealth. Though possibly endowed with superior information, rational traders may trust their beliefs no more (and possibly less) than overconfident traders. Their wealth and risk tolerance may not exceed those of others. Introducing rational traders into the model reduces trading volume (per trader), volatility, and the inefficiency of prices. The expected utility of rational traders is greater than that of overconfident traders. Introducing additional overconfident traders who are less overconfident than the existing ones has similar, though less extreme, results.

In the preceding proposition, whether price changes are negatively or positively correlated depends on whether traders overvalue or undervalue new information. In the following proposition, rational traders are added to the economy (as described in Appendix A). When rational traders trade with overconfident traders who undervalue the signals of others ($\gamma \leq 1$), the information of rational traders will be underrepresented in price. Thus prices may trend.

PROPOSITION 6: *When rational traders trade with overconfident traders who (sufficiently) undervalue the signals of others, price changes will be positively serially correlated.*

Positive serial correlated price changes are most likely when the precision of the rational traders' signal is high and when overconfident traders significantly undervalue the signals of others. The specific region where price changes are positively serially correlated is identified in the proof of Proposition 6 (Appendix A).

B. An Insider

This model of insider trading is based on Kyle (1985). Other than notational differences, the only changes made to Kyle's original model are that the insider's private signal of the terminal value is noisy and that the insider is overconfident.

This is a one-period model in which a risk-neutral, privately informed trader (the insider) and irrational noise traders submit market orders to a risk-neutral marketmaker. There are two assets in the economy, a riskless asset and one risky asset. The riskless interest rate is assumed to be 0. The terminal value of the risky asset is $\tilde{v} \sim N(\bar{v}, h_v^{-1})$. $\bar{v}$ is assumed to equal 0; this simplifies notation without affecting the propositions. Prior to trading, a risk-neutral insider receives a private signal $\tilde{y} = \tilde{v} + \tilde{\epsilon}$. $\tilde{\epsilon}$ is normally distributed with mean zero and precision h_ϵ. The insider believes the precision of $\tilde{\epsilon}$ to be κh_ϵ, where $\kappa \geq 1$, and the precision of $\tilde{v}$ to be ηh_v, where $\eta \leq 1$. After observing $\tilde{y}$, the insider demands (submits a market order for) x units of the risky asset. Without regard for price or value, noise traders demand $\tilde{z}$ units of the risky asset, where $\tilde{z} \sim N(0, h_z^{-1})$. The marketmaker observes only the total demand $x + \tilde{z}$ and sets the price (P). The marketmaker correctly assumes that the precision of $\tilde{\epsilon}$ is h_ϵ and that the precision of $\tilde{v}$ is h_v. (The propositions do not change if the marketmaker, like the insider, believes the precision of the prior to be ηh_v.) After trading, the risky asset pays its terminal value $\tilde{v}$. The insider and the marketmaker know the true distribution of $\tilde{z}$ and are aware of each other's beliefs about the precisions of $\tilde{v}$ and $\tilde{y}$.

The insider conjectures that the marketmaker's price-setting function is a linear function of $x + \tilde{z}$,

$$P = H + L(x + \tilde{z}). \tag{4}$$

He chooses x to maximize his expected profit, $x(\tilde{v} - P)$, conditional on his signal, $\tilde{y}$, and given his beliefs about the distributions of $\tilde{v}$, $\tilde{y}$, and $\tilde{z}$ and the conjectured price function. It is assumed, as in Kyle (1985), that the marketmaker earns zero expected profits. The marketmaker conjectures that the insider's demand function is a linear function of $\tilde{y}$,

$$x = A + B\tilde{y}. \tag{5}$$

She sets price to be the expected value of $\tilde{v}$ conditional on total demand ($x + \tilde{z}$), given her beliefs about the distributions of $\tilde{v}$, $\tilde{y}$, and $\tilde{z}$ and the conjectured demand function.

In Kyle's original model, a linear equilibrium always exists in which the conjectured price and demand functions are fulfilled. Given the assumptions of overconfidence made here, a linear equilibrium exists whenever $\kappa h_\epsilon + 2\ \eta h_v > \kappa h_v$. (The equilibrium and the proofs for this section are presented in Appendix B.) The intuition behind the equilibrium condition is the following. The marketmaker sets price to be the expectation of $\tilde{v}$ conditional on the order flow she observes and on her conjecture about the insider's demand function. The insider is trying to maximize his profit. His profit increases if he trades more with the same profit margin or if he trades the same amount with a larger margin. If the insider increases his demand, the marketmaker shifts the price and thus lowers the insider's expected profit margin. Equilibrium exists at the demand-price pair where the insider believes that, if he increases his demand, the negative effect of the lower expected profit margin will just offset the gains of greater trading and, if he lowers demand, the losses from trading less will just offset gains from a higher expected profit margin. If the insider and the marketmaker disagree too much about the relative precisions of the prior and the private signal, there is no equilibrium; for any given insider demand function (A,B), the marketmaker will choose a pricing function (H,L) such that the insider will prefer a yet steeper demand function (i.e., greater B).

As in Kyle's (1985) model, the insider can only influence price through his demand. This assumption is particularly critical when overconfidence is introduced to the model. If the insider could credibly reveal his private signal to the marketmaker, then, due to the different weights each attaches to the prior and to the signal, the insider and the marketmaker would have different posterior beliefs about the expected value of the terminal payoff. And because they are both risk neutral, they would each be willing to trade an infinite amount. Infinite trading is a possible problem whenever risk-neutral traders value common information differently. In Harris and Raviv (1993), risk-neutral traders attribute different density functions to a public signal (Harris and Raviv avoid infinite trading by assuming a fixed number of shares are available and that short sales are not allowed). Jaffe and Winkler (1976) avoid infinite trading by assuming only one asset share can be exchanged. The willingness to trade infinitely is inherent in risk neutrality, not in overconfidence. Risk neutrality is assumed here for tractability.

All of the propositions in this section are true when η is decreasing instead of κ increasing.

PROPOSITION 7: *Expected volume increases as the insider's overconfidence increases.*

Expected volume is measured as the expected value of the sum of the absolute values of insider demand and noise trader demand. When the insider is overconfident, he believes that he has received a stronger private signal, $\tilde{y}$, than is actually the case. In calculating his posterior expectation of the final value of the risky asset, he overweights his signal and derives a posterior expectation farther from the prior than he should. Based on this posterior belief, he trades more aggressively than is optimal, thus increasing expected volume.

PROPOSITION 8: *Market depth increases as the insider's overconfidence increases.*

PROPOSITION 9: *Volatility of prices increases as the insider's overconfidence increases.*

PROPOSITION 10: *The quality of prices improves as the insider's overconfidence increases.*

Overconfidence causes prices to be more sensitive to changes in value ($\tilde{v}$) and in the insider's signal ($\tilde{y}$), and less sensitive to changes in informed demand ($\tilde{x}$) and noise trader demand ($\tilde{z}$). The marketmaker sets price to be the expectation of $\tilde{v}$ conditional on observed orderflow and her conjecture about the insider's demand function. She realizes that the insider is overconfident and that he will trade more in response to any given signal than he would if he were rational. She therefore moves price less in response to changes in total order flow ($\tilde{x} + \tilde{z}$) than she would if the insider were rational. That is, she flattens her supply curve, thereby increasing market depth (which is measured as the inverse of the derivative of price with respect to order flow). Because the overconfident insider trades more in response to any given signal than he would if he were rational, his expected trading increases relative to that of noise traders. Therefore the signal-to-noise ratio in total order flow increases and the marketmaker is able to make better inferences about the insider's signal. This enables her to form a more accurate posterior expectation of $\tilde{v}$ and to set a price that is, on average, closer to $\tilde{v}$. This improves the quality of prices, which is measured, as in the previous model, as the variance of the difference of price (P) and value ($\tilde{v}$). Because the marketmaker can better infer the insider's signal, $\tilde{y}$, the price she sets varies more in response to changes in $\tilde{y}$ than if the insider were rational. This increases the variance of price (volatility). From a different perspective, although the marketmaker has flattened her supply curve, thus dampening volatility for any *given* level of expected order flow, the increased order flow generated by the overconfident insider more than offsets this dampening, and results in increased volatility. Thus both market depth and volatility rise with overconfidence.

PROPOSITION 11: *The expected profits of the insider decrease as his overconfidence increases.*

The insider's expected profits, $\mathrm{E_a}(x(\tilde{v} - P))$, are equivalent to his expected utility because he is risk neutral. The insider submits a demand (to buy or to sell) that is optimal given his beliefs about the distributions of $\tilde{v}$ and $\tilde{y}$, a demand that he believes will maximize his expected profits. He is mistaken about the precision of his knowledge, but conditional on his beliefs he behaves optimally. The demand he submits is not, however, the same demand he would submit were he not overconfident, and it is not optimal given the true distributions of $\tilde{v}$ and $\tilde{y}$. Therefore the insider's expected profits are lower than they would be if he were not overconfident.[15]

[15] Kyle and Wang (1997) show that under particular circumstances when both a rational insider and an overconfident insider trade with a marketmaker, the overconfident insider may earn greater profits than the rational insider. The overconfident insider earns greater profits by "precommitting" to trading more than his share in a Cournot equilibrium. For this result to hold, traders must trade on correlated information, have sufficient resources and risk tolerance to trade up to the Cournot equilibrium, know each other's overconfidence, and trade with a third party (e.g., the marketmaker). Furthermore, if one trader can trade before the other, the result may not hold.

This model includes an overconfident insider, noise traders, and a rational marketmaker who expects to earn zero profits. Whatever profits the overconfident insider gives up are passed on to the noise traders in the form of lower losses. Were the rational marketmaker not constrained by assumption to earn zero profits, she would benefit from the insider's overconfidence. This model (and the next one) require a source of uncertain demand for the risky asset so that the insider's information is not perfectly deducible from total demand. Noise traders who trade randomly and without regard to price (as in Kyle (1985)), though they may lack perfect real world analogues, provide an analytically tractable source of uncertain demand. Overconfident, risk-averse, price-taking traders with private signals, such as the traders described in the previous section, could also provide uncertain demand in a market. In that case, if the insider were not too overconfident, he would profit at the expense of the overconfident price takers. Unfortunately, replacing noise traders with overconfident price takers greatly complicates the model.

C. Marketmakers and Costly Information

The next model examines the behavior of overconfident marketmakers. It also offers an explanation for why active money managers underperform passive money managers: Active managers may be overconfident in their ability to beat the market and spend too much time and money trying to do so.

The model is based on Grossman and Stiglitz (1980). Risk-averse traders decide whether or not to pay for costly information about the terminal value of the risky asset; those who buy information receive a common signal; and a single round of trading takes place. The participants in this trading are the traders who buy information (informed traders), traders who do not buy the information (uninformed traders), and noise traders who buy or sell without regard to price or value.[16] As in the previous models, a riskless asset and one risky asset are traded; the riskless interest rate is assumed to be 0; each share of the risky asset pays $\tilde{v}$, where $\tilde{v} \sim N(\bar{v}, h_v^{-1})$. Traders believe the precision of $\tilde{v}$ to be ηh_v, where $\eta \leq 1$; that is, they undervalue the common prior. There are N investors $(i = 1,\ldots,N)$. As a modeling convenience we analyze the limit economy where $N \to \infty$. Thus each investor correctly assumes that his own demand does not affect prices. Each trader has an endowment of f_{0i} of the riskless asset and x_{0i} of the risky asset. $\bar{x} = (\Sigma_N x_{0i})/N$ is the average endowment. As a notational convenience it is assumed that $\bar{x} = 0$ and $\bar{v} = 0$. Prior to trading, traders choose whether or not to pay cost c in order to receive a signal $\tilde{y} = \tilde{v} + \tilde{\epsilon}$, where $\tilde{\epsilon} \sim N(0, h_\epsilon^{-1})$. Noise trader

[16] In this section "traders" refers to informed traders and to uninformed traders but not to noise traders who are referred to explicitly as "noise traders." As in the insider model, noise traders could be replaced with overconfident price takers, such as those discussed in Section III.A. Overconfidence would motivate trading and the model's results would not change significantly. However replacing noise traders with overconfident price takers greatly complicates the equilibrium without adding much intuition.

demand per (nonnoise) trader is $\tilde{z}$, where $\tilde{z} \sim N(0, h_z^{-1})$. Thus $-\tilde{z}$ is the supply of the risky asset per trader at the time of trading. In equilibrium, λ^* is the fraction of traders who choose to become informed.

All traders, even those who remain uninformed, are overconfident about the signal, which they believe to have precision κh_ϵ, where $\kappa \geq 1$. In the previous models, traders were overconfident about their own signals but not those of others. Here everyone believes the information is better than it is, but some decide the cost is still too high. It is as if all money managers overestimate their ability to manage money actively, but some decide the costs of doing so are too high and so, despite their overconfidence, choose to manage passively.[17] In real markets one would expect traders to hold a spectrum of beliefs about the value of costly information. Those who were more overconfident about the information would be more likely to buy it. One could alternatively specify in this model that those traders who do not buy the signal value it rationally.[18]

Trader i's demand for the risky asset is x_{1i} and for the risk-free asset is f_{1i}. So his final wealth is $W_{1i} = x_{1i}\tilde{v} + f_{1i}$. Trader i's utility function is $U(W_{1i}) = -\exp(-aW_{1i})$, where a is the common coefficient of absolute risk aversion. He maximizes his expected utility by choosing whether or not to become informed, and then, conditional on his information, by choosing his optimal demand subject to the budget constraint. That is, if he is informed, he solves

$$\max_{x_{1I}} \mathrm{E_b}[-\exp(-aW_{1I})|\tilde{y}] \quad \text{subject to } x_{1I}P + f_{1I} \leq x_{0I}P + f_{0I}, \tag{6}$$

and if he is uninformed he solves

$$\max_{x_{1U}} \mathrm{E_b}[-\exp(-aW_{1U})|P] \quad \text{subject to } x_{1U}P + f_{1U} \leq x_{0U}P + f_{0U}, \tag{7}$$

[17] In practice, some practitioners of passive investing tout their own skills as superior active managers. For example, Barclays Global Investment Advisors, the largest manager of index funds, has a Global Advanced Active Group that actively manages more than $70 billion. And George Sauter who oversees $61 billion in stock-index mutual funds at Vanguard Group also actively manages Vanguard Horizon Fund Aggressive Growth Portfolio (*The Wall Street Journal*, February 25, 1997, p. C1, byline Robert McGough).

[18] Assuming that traders who do not purchase the signal value it correctly will result in a range of possible equilibria rather than a single equilibrium point. At one end of the range the same fraction of traders becomes informed as when all traders are rational. Here the rational uninformed traders believe that the fraction of traders who are informed is optimal, and the overconfident informed traders believe that the fraction of traders who are informed is too small. Traders in neither group believe they would benefit from changing groups. At the other end of the range, the same fraction of traders becomes informed as when all traders are overconfident. Here the rational uninformed traders believe that the fraction of traders who are informed is greater than the optimum, and the overconfident informed traders believe that the fraction of traders who are informed is optimal.

where $i = I$ and $i = U$ indicate prototypical informed and uninformed traders and P is the endogenously determined price of the risky asset. In equilibrium all traders believe that the expected utility of the informed traders is equal to that of the uninformed. Because all traders believe the precision of $\tilde{y}$ is κh_ϵ and the precision of $\tilde{v}$ is ηh_v, and because the equilibrium is determined by the traders' beliefs, the equilibrium obtained is the same as would occur in a model without overconfidence where the precision of ϵ was actually κh_ϵ and that of $\tilde{v}$ was ηh_v. Once again equilibrium holds because the traders believe that they are behaving optimally, though, in fact, they are not. The equilibrium and the proofs for this section are presented in Appendix C.

In the previous two models, expected utility drops as overconfidence increases. In this model, where traders are overconfident about a costly signal, it is those who buy the signal who are most hurt by their overconfidence.

PROPOSITION 12: *For many sets of the parameters specifying this economy (and perhaps for all sets), when traders overvalue costly information, the expected utility of informed traders is lower than that of uninformed traders.*

When traders overestimate the value of the costly signal, too many of them are willing to buy it. Its benefits are therefore spread too thin, resulting in lower expected utilities for the informed traders. The proposition states only that this is true for many sets of the parameters that specify the economy. Explicit solutions for the expected utilities of the informed and uninformed traders are given in Appendix C. I evaluate these for a wide variety of parameter values and find that in every case the expected utility of the informed is less than that of the uninformed if $\kappa > 1$ or $\eta < 1$ (and $0 < \lambda < 1$).[19]

When some traders buy information and others do not (i.e., $0 < \lambda < 1$), individual informed traders trade, on average, more than individual uninformed traders. (This is the case even when there is no overconfidence. Uninformed traders as a group, however, may trade more than informed traders as a group when their numbers are sufficiently larger.) When traders are overconfident, the expected utility of informed traders is lower than that of uninformed traders, therefore it follows that the expected utility of those who, on average, trade more is lower than that of those who, on average, trade less. This is consistent with Barber and Odean's (1998) finding that individual investors who turn over their common stocks at higher rates earn, on average, lower net returns.

When some traders buy information and others do not, this model does not offer much intuition about how overconfidence affects total trading volume and volatility. Volume and volatility can increase, decrease, or remain unchanged as overconfidence increases. Even when there is no overconfidence, volume and volatility rise or fall in response to increases in other parameter

[19] If the uninformed traders are rational rather than overconfident, they optimize correctly. In this case it is trivial to show that their expected utility is at least as high as that of informed traders. If it were not, they would become informed.

values such as the coefficient of risk aversion and the precision of $\tilde{y}$. Volume and volatility vary in response to changes in the fraction of investors becoming informed, λ, which is itself extremely sensitive to changes in the various parameter values. These patterns appear to be idiosyncracies of the model rather than generalizations about markets.[20]

When all traders are informed, they act as marketmakers who have some information about the terminal value of the risky asset.[21] The supply schedule they set is: $P = \mathrm{E}_{\mathrm{b}}(\tilde{v}|\tilde{y}) + a\,\mathrm{var}_{\mathrm{b}}(\tilde{v}|\tilde{y})Q$. There are two separate components to this price: $a\,\mathrm{var}_{\mathrm{b}}(\tilde{v}|\tilde{y})Q$ is a response to noise trader demand (since $Q = \tilde{z}$) and hedges traders in their capacity as marketmakers against inventory risk. $\mathrm{E}_{\mathrm{b}}(\tilde{v}|\tilde{y})$ is a response to the signal $\tilde{y}$ and represents traders' speculations about terminal value. If there were no signal, price would be completely determined by inventory risk; if there were a signal, but no noise trader demand, price would be completely determined by the signal. A decrease in η means that these marketmakers perceive themselves as having less reliable prior information and therefore facing greater risk. Thus, lower η steepens the supply curve, decreasing market depth and increasing the inventory-risk component of volatility. On the other hand, when κ increases these marketmakers see themselves as having more reliable information and their perception of the risk of holding inventory is diminished. An increase in κ therefore flattens the supply curve, increasing market depth and decreasing the inventory-risk component of volatility. Overconfidence increases market depth because it lowers a marketmaker's perceived risk. Increasing κ moves $\mathrm{E}_{\mathrm{b}}(\tilde{v}|\tilde{y})$ closer to $\tilde{y}$ and farther from $\bar{v} = 0$, thereby increasing the speculative component of volatility while decreasing its inventory risk component. When expected noise trader demand is low, the speculative component of volatility dominates and increasing κ increases volatility. When expected noise trader demand is high, inventory-risk dominates and increasing κ decreases volatility.

Figure 1 graphs supply curves in two economies. The dashed line represents an economy where $\kappa = 1$ and the solid line an economy where $\kappa = 2$. All other parameter values are the same in both economies and all traders are informed. The supply curves are conditional on traders receiving a signal of $\tilde{y} = 2$ (one standard deviation above the mean signal). The solid line is flat-

[20] To understand why, in this model, expected volume can rise or fall with overconfidence it is helpful to look at boundary cases. When cost is so high that all traders remain uninformed (i.e., $\lambda^* = 0$), the traders do not trade with each other and all trading is done between the uninformed traders and the noise traders. Thus, expected volume equals the expected demand of the noise traders (i.e., $\sqrt{2/\pi h_z}$). When overconfidence increases sufficiently, some traders, but not all, will become informed. Informed and uninformed traders will now trade with each other and they will also continue to fill the demand of the noise traders, so expected volume will rise. As overconfidence continues to rise, all traders may eventually become informed (depending on the other parameter values), in which case expected volume will fall back to the expected demand of noise traders.

[21] Note that when all traders are informed, this model is analogous to a one-period version of the model in Section III.A with noise traders added and $M = 1$.

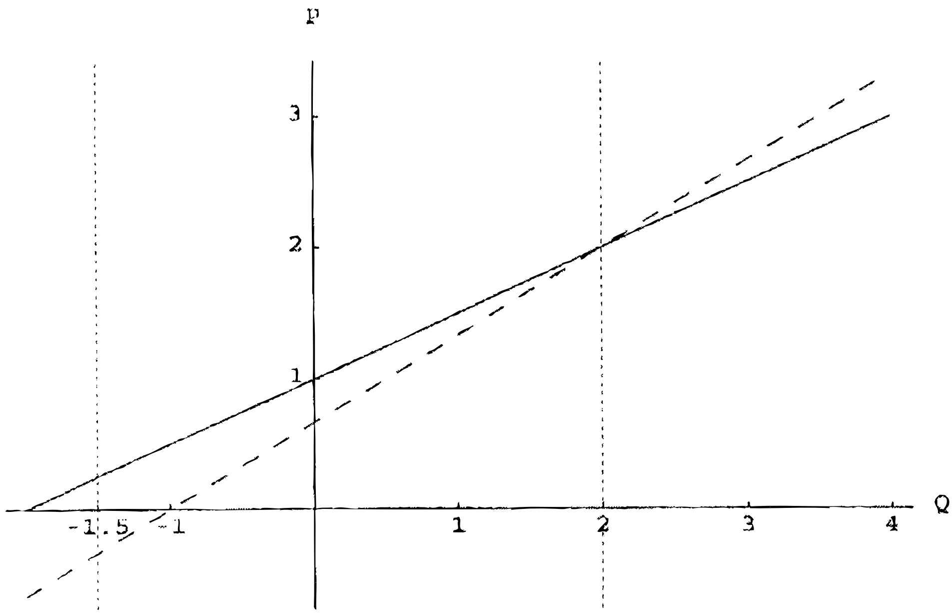

Figure 1. Supply curves when all traders are informed. $P = \mathrm{E_b}(\tilde{v}|\tilde{y}) + a\,\mathrm{var_b}(\tilde{v}|\tilde{y})Q$ where P is price and Q is quantity, for economies in which $\kappa = 2$ (solid line) and $\kappa = 1$ (dashed line) and where, for both economies, signal $\tilde{y} = 2$ has been received, $\eta = 1$, $h_v = 2$, $h_\epsilon = 1$, $h_z = 0.25$, $a = 2$, $c = 0.09$, and $\lambda = 1$.

ter, which means that market depth is greater when $\kappa = 2$. The two supply curves cross at about $Q = 2$. If demand is less than 2 and greater than approximately -1.5, price will be closer to its unconditional expected value, 0, when $\kappa = 1$ than when $\kappa = 2$. But when the magnitude of noise trader demand is high (i.e., $\tilde{z} > 2$ or $\tilde{z} < -1.5$) price will be closer to its expected value when $\kappa = 2$ than when $\kappa = 1$. When expected noise trader demand is low, demand will more often fall into the area where the magnitude (and volatility) of price is smaller for $\kappa = 1$. When expected noise trader demand is high, the economy with $\kappa = 2$ will have lower volatility. The following proposition summarizes the above discussion.

PROPOSITION 13: *Market depth is increasing in the overconfidence of a risk-averse marketmaker. Volatility increases when expected noise trader demand is high and decreases when it is low. (Precise definitions of high and low expected noise trader demand are given in Appendix C.)*

IV. Discussion

This paper examines the effects of overconfidence in a variety of market settings. These settings differ principally in how information is distributed and how prices are determined. For some market measures, such as trading volume, overconfidence has a similar effect in each setting. For others, such as market efficiency, it does not. Which set of predictions is appropriate to a market depends on the informational structure and price setting mechanism of that market. For example, if, for a particular market, crucial information is first obtained by well-capitalized insiders and marketmakers are primarily concerned about trading against informed insiders, then the model of the overconfident insider (Section III.B) is appropriate. However, if relevant information is usually publicly disclosed and then interpreted differently by a large number of traders each of whom has little market impact, the overconfident price-taker model (Section III.A) applies. The observation that overconfident traders will pay too much for information (Section III.C) applies to markets in which traders choose between investing passively and expending resources on information and other costs of active trading. We find the following effects of overconfidence on different market measures.

Overconfidence increases *trading volume*. Overconfident price takers (Section III.A) form differing posterior beliefs and trade speculatively with each other. Were these traders rational they would hold identical posteriors and trade only to initially balance their portfolios. Overconfident insiders (Section III.B) also trade more aggressively than if they were rational. And, as seen in the model of marketmakers (Section III.C), overconfident marketmakers set a flatter supply curve. A flatter supply curve encourages more trading when traders are price sensitive. Thus, in all three settings, overconfidence leads to greater trading volume. Though there is anecdotal evidence of excessive trading—for example, roughly one-quarter of the annual international trade and investment flow is traded each day in foreign exchange markets (Dow and Gorton (1997)); the average annual turnover rate on the New York Stock Exchange is currently greater than 60 percent (*NYSE Fact Book* for 1996)—without an adequate model of what trading volume in rational markets should be, it is hard to prove that aggregate market volume is excessive. Odean (1998a) looks at the buying and selling activities of individual investors at a discount brokerage. Such investors could quite reasonably believe that their trades have little price impact. On average, the stocks these investors buy subsequently underperform those they sell (gross of transactions costs), even when liquidity demands, risk management, and tax consequences are considered. As predicted by the model of price-taker overconfidence, these investors trade too much. However, overconfidence about the precision of private signals alone is not enough to explain why these investors make such poor trading decisions. In addition to overvaluing their information, these investors must also misinterpret it. Statman and Thorley (1998) find that trading volume increases subsequent to market gains. If success in the market leads traders to become overconfident—as Gervais and Odean (1997) find—these increases in volume may be driven by overconfidence.

Whether overconfidence improves or worsens market *efficiency* depends on how information is distributed in the market. On the one hand, when information is distributed in small amounts to many traders or when it is publicly disclosed and then interpreted differently by many traders, overconfidence causes the aggregate signal to be overweighted (Section III.A).

This leads to prices *further from* the asset's true value, $\tilde{v}$, than would otherwise be the case. Though all available information is revealed in such a market, it is not optimally incorporated into price. On the other hand, when information is held exclusively by an insider and then inferred by a marketmaker from order flow (Section III.B), overconfidence prompts the insider to reveal, through aggressive trading, more of his private information than he otherwise would, thereby enabling the marketmaker to set prices *closer to* the asset's true value. If, however, the insider's information is time sensitive and becomes public soon after he trades, this gain in efficiency is short-lived. Given the broad disclosure of information in U.S. equity markets and the brevity of gains in efficiency from overconfident insiders, we would expect overconfidence, in net, to decrease efficiency in these markets.

Traders' overconfidence increases *volatility*; marketmaker's overconfidence may lower it. By overweighting the aggregate signal of the price takers (Section III.A), overconfidence drives price *further from* its true underlying value, $\tilde{v}$, and *further from* its unconditional mean, $\bar{v}$. This results in increased volatility. By prompting the insider to reveal more of his signal (Section III.B), overconfidence enables the marketmaker to move price *closer to* the true underlying value, $\tilde{v}$, and *further from* its unconditional value, $\bar{v}$. This, too, increases volatility. In the first case, overconfidence increases volatility by distorting the prices implied by public, or broadly disseminated, information; in the second, overconfidence increases volatility by moving prices closer to the values implied by highly concentrated, private information. Privately informed, risk-averse marketmakers (Section III.C) flatten their supply curves when they are overconfident, just as they would if they were less risk averse, because overconfidence leads them to perceive less risk in holding inventory. Flattening the supply curve dampens volatility. The influence of a group of traders on price will depend on their numbers, wealth, risk tolerance, overconfidence, and information. In a market with many traders and few marketmakers it is unlikely that dampening of volatility by overconfident marketmakers will offset increases in volatility due to overconfident traders. Some research suggests that market volatility is excessive (Shiller (1981, 1989), LeRoy and Porter (1981)), but this is a difficult proposition to prove (Marsh and Merton (1986), Kleidon (1986)). Pontiff (1997) finds excess volatility for closed-end funds.

Overconfidence increases *market depth*. When an insider is overconfident, he trades more aggressively (for any given signal). The marketmaker adjusts for this additional trading (in response to the same signal) by increasing market depth (Section III.B). Overconfident risk-averse marketmakers perceive that their estimate of the security's true value is more precise than it is and that they face less risk by holding inventory. So they flatten their supply curves, which also increases market depth (Section III.B).

Overconfidence lowers *expected utilities*. Overconfident traders do not properly optimize their expected utilities, which are therefore lower than if the traders were rational. Overconfident traders hold underdiversified portfolios. When information is costly, those who choose to become informed trade more and fare worse than those who remain uninformed (Section III.C). In practice, the cost of active managers' information must be reflected in their fees. Thus, this finding is consistent with many studies of the relative performance of active (informed) and passive (uninformed) money managers.[22] It is also consistent with the lower net returns earned by individual investors whose portfolio turnover is high (Barber and Odean (1998)).

Overconfident traders who discount the opinions of others can cause markets to underreact to the information of rational traders (Section III.A). Markets also underreact when traders underweight their own new information and overreact when they overweight it. The degree of *under-* or *overreaction* depends on what fraction of all traders receives the information and on how willing these traders are to trade. (Bloomfield, Libby, and Nelson (1997) find that traders in experimental markets undervalue the information of others; Bloomfield and Libby (1996) find that the impact of a signal on price, in an experimental market, depends on what fraction of traders receive that signal.) Underreactions occur when all traders undervalue a signal or when only a small fraction of traders overvalue it, but others discount their opinion. Overreactions require that a significant fraction of active traders (those most willing to trade) significantly overvalue a signal.

Some documented market return anomalies indicate overreactions to public events, but most find underreactions.[23] Fama (1997) points out that if markets occasionally overreact and at other times underreact this could be due to simple chance. Like markets, people, too, sometimes overvalue information and at other times undervalue it. Though these valuation errors may appear due to chance, psychologists find that they are systematic. People typically overreact to salient, attention-grabbing information, overvalue cases, anecdotes, and extreme realizations, and overweight irrelevant data. They underreact to abstract statistical information, underestimate the importance of sample size, and underweight relevant data. Markets appear to reflect the same systematic biases as their participants.

Reactions to announcements are considered underreactions when returns in periods following the announcement are of the same sign as returns on the day of the announcement. One of the most robust underreaction anomalies is post-earnings announcement drift (Bernard and Thomas (1989, 1990)).

[22] In an early study, Jensen (1968) finds underperformance by mutual funds. Lakonishok et al. (1992) document that as a group active equity managers consistently underperformed S&P 500 index funds over the period 1983 to 1989. They conclude that, after factoring in management fees, active management subtracts value. Using a variety of benchmarks and benchmarkless tests, Grinblatt and Titman (1993, 1994) find that, at least before fees, some fund managers earn abnormal returns. Malkiel (1995) claims that such results are heavily influenced by survivorship bias. Carhart (1997) also finds little evidence of skilled mutual fund management. Lakonishok et al. ask why pension funds continue to give their money to active managers when index funds outperform active management. They suggest a number of reasons based on agency relationships. They also point out that the pension fund employees may be overconfident in their ability to pick superior money managers.

[23] I wrote the following discussion of market underreactions nearly two and one-half years after the original draft of this paper (November 1994) and subsequent to reading more recent working papers on this topic (Barberis, Shleifer, and Vishny (1997), Daniel et al. (1998), Fama (1997)).

Corporate earnings summarize the operations of a company into a single statistic. This statistic is based on a large sample of information and is highly relevant to the value of the company. It is prototypical of the information that people typically undervalue: Abstract, relevant, and based on a large sample. Markets also underreact to dividends omissions and initiations (Michaely et al. (1995)). The decision to omit dividends is generally made reluctantly and in response to significant corporate difficulties. The omission (or initiation) of dividends is appreciated by investors, but it may not be fully appreciated because the bad (or good) news contained in the omission (or initiation) has been condensed into a single event. We might expect a greater reaction when an omission (or initiation) is accompanied by a well-publicized graphic portrayal of a company's woes (or good fortune). Like dividend initiations, open-market repurchases (Ikenberry et al. (1995)) are positive signals that abstract from a wealth of more salient information. In addition to possibly signaling management's sanguine outlook, the announcement of open-market repurchases states that the supply of shares in a company will be reduced. Investors who do not realize that firms face upward-sloping supply curves when they repurchase shares (Bagwell (1992)), and that price is therefore likely to rise, may underreact to the announcement.

Most of the documented long-run return patterns following information events are underreactions. Fama (1997) classifies the poor long-term performance following initial public offerings (IPOs) (Ritter (1991), Loughran and Ritter (1995)) and seasoned equity offerings (SEOs) (Loughran and Ritter (1995), Spiess and Affleck-Graves (1995)) as "in the over-reaction camp." Using the definition of underreaction given above, however, SEOs would be classified as underreactions because the usually negative market reaction at the announcement of the SEO is followed by underperformance. Using the above definition, IPOs are unclassifiable because no market reaction is observable following the announcement of an IPO. As discussed in Section III.C, price reflects the opinions of those most willing to trade. Generally, if a minority of traders overreacts to information and the majority discounts the minority's opinion or underreacts to the same information, price will underreact. Negative opinions are incorporated into stock prices when investors sell securities they already own and when they sell short. The majority of investors, however, are unwilling to sell short. The first day's price for IPOs is therefore determined by the minority of investors that is most sanguine about a company's prospects: those investors who subscribe to the IPO—some with the intention of flipping it on the first day—and those who buy it on the first day. No one with a bad opinion of the company owns an IPO on the first day and it can be difficult to short the stock so soon. Thus, the high first-day price for an IPO may reflect an overreaction by a minority of market participants to the optimistic stories and scenarios that accompany the IPO's promotion.

Markets often underreact to announcements of abstract, highly statistical, or highly relevant information. Earnings changes, dividend omissions, and brokerage recommendations are all examples of such information. However, behind each of these events lie many concrete, salient stories: new products succeed, others fail, ad campaigns are waged, employees are fired, scandals emerge. Though the sum of these stories is underweighted, the individual parts may, in fact, be overweighted. (As Joseph Stalin put it, "The death of a single Russian soldier is a tragedy. A million deaths is a statistic.") If markets do systematically overreact, they may do so to highly publicized, graphic news and to rumors.[24]

Though brokerage recommendations are delivered in their salient form only to customers, some recommendations are both widely disseminated and attention grabbing. *The Wall Street Journal*'s monthly "Dartboard" column pits the recommendations of four analysts against the random selections of a dart. The analysts, whose portraits are featured, explain the reasons for their picks. Many readers follow this contest, and Barber and Loeffler (1993) show that the market overreacts to these recommendations. Similarly, the market overreacts to recommendations made on the popular TV show *Wall Street Week* (Pari (1987)).

Another signal to which we might expect overreactions is price change. Price change may be the most salient signal received by investors because, unlike other signals such as earnings, it directly, rather than indirectly, contributes to changes in their wealth. It is also the most publicized signal, instantly available on many computer screens, reported daily in newspapers and other media, and mailed monthly to investors in brokerage statements. Furthermore, many investors may overweight the predictive value of price changes; they may see deterministic patterns where none exist,[25] overextrapolate those that do (e.g., momentum), and put too much faith in technical trading rules—though not necessarily the same technical trading rules as each other.

As we saw in the model of price-taking traders, the impact of a private signal depends on how many people receive that signal (and, as Figlewski (1978) points out, on the wealth and risk tolerance of those traders). The impact of traders, even rational traders, depends on their numbers and on their willingness to trade. The mere presence of a few rational traders in a market does not guarantee that prices are efficient; rational traders may be no more willing or able to act on their beliefs than biased traders. It is markets with higher proportions of rational traders that will be more efficient. So, if information processing biases are more pronounced in individ-

[24] There is some evidence of short-term mean-reversion in returns (Lehmann (1990), Jegadeesh (1990)). Such reversions possibly could be due to overreaction to salient news stories, but this has not been shown. Mean reversion has also been found at longer horizons (De Bondt and Thaler (1985, 1987)). Returns tend to be positively serially correlated at intermediate horizons (Jegadeesh and Titman (1993)). Positively serially correlated prices could be the result of underreactions to important information such as earnings changes. They could also be the result of price momentum traders overreacting to price as a signal and purchasing stocks that have risen, thereby driving prices even higher (as in Hong and Stein (1997)).

[25] Gilovich, Vallone, and Tversky (1985) show that people hold strong beliefs that random sequences are nonrandom.

uals than in institutional traders, it should come as no surprise that return anomalies are greatest for small firms (see Fama (1997) for a review) which are traded more heavily by individuals.

V. Conclusion

Overconfidence is costly to society. Overconfident traders do not share risk optimally, they expend too many resources on information acquisition, and they trade too much. These are dead weight losses. Overconfidence increases trading volume and market depth, but decreases the expected utility of overconfident traders. When information is costly, overconfident traders who actively pursue information fare less well than passive traders. Overconfident traders increase volatility, though overconfident marketmakers may dampen it. Price-taking traders, who are overconfident about their ability to interpret publicly disclosed information, reduce market efficiency; overconfident insiders temporarily increase it. When there are many overconfident traders, markets tend to underreact to the information of rational traders. Markets also underreact to abstract, statistical, and highly relevant information and overreact to salient, but less relevant, information. Like those who populate them, markets are predictable in their biases.

Appendix A: Price Takers

LEMMA 1: *An equilibrium exists in which the linear price conjectures, equations (2) and (3), lead to linear demand functions. The coefficients of the price conjectures are*

$$\alpha_{31} = \frac{\eta h_v \bar{v} - a\bar{x}}{\eta h_v + 2(\kappa + \gamma M - \gamma)h_\epsilon}, \tag{A1}$$

$$\alpha_{32} = \alpha_{33} = \frac{(\kappa + \gamma M - \gamma)h_\epsilon}{\eta h_v + 2(\kappa + \gamma M - \gamma)h_\epsilon}, \tag{A2}$$

$$\begin{aligned}\alpha_{21} = {} & \alpha_{31} + (\alpha_{32} + \alpha_{33})\bar{v} - a\bar{x}\,\mathrm{var_b}(P_3|\Phi_2) \\ & + \frac{\bar{v}(\alpha_{33}(\gamma + \gamma M - \kappa)h_\epsilon + \alpha_{32}(\eta h_v + (\kappa + \gamma M - \gamma)h_\epsilon))}{\eta h_v + (\kappa + \gamma M - \gamma)h_\epsilon},\end{aligned} \tag{A3}$$

$$\alpha_{22} = \frac{(\kappa + \gamma M - \gamma)h_\epsilon}{\eta h_v + (\kappa + \gamma M - \gamma)h_\epsilon}, \tag{A4}$$

and

$$P_1 = \alpha_{21} + \alpha_{22}\bar{v} - a\bar{x}\alpha_{22}^2\left(\frac{1}{\eta h_v} + \frac{\gamma + \kappa M - k}{\kappa\gamma h_\epsilon M^2}\right). \tag{A5}$$

Proof: We first solve the equilibrium for the third round of trading. Trader i believes Φ_{3i} has a multivariate normal distribution. We calculate the mean and the covariance matrix of this distribution which are $E_{\mathrm{b}}(\Phi_{3i}) = [\bar{v} \quad \bar{v} \quad \alpha_{21} + \alpha_{22}\bar{v} \quad \alpha_{31} + (\alpha_{32} + \alpha_{33})\bar{v}]^T$ and

$$\Psi = \begin{bmatrix} \frac{1}{\eta h_v} + \frac{1}{\kappa h_\epsilon} & \frac{1}{\eta h_v} & \frac{\alpha_{22}}{\eta h_v} + \frac{\alpha_{22}}{\kappa h_\epsilon M} & \frac{\alpha_{32} + \alpha_{33}}{\eta h_v} + \frac{\alpha_{32}}{\kappa h_\epsilon M} \\ \frac{1}{\eta h_v} & \frac{1}{\eta h_v} + \frac{1}{\kappa h_\epsilon} & \frac{\alpha_{22}}{\eta h_v} & \frac{\alpha_{32} + \alpha_{33}}{\eta h_v} + \frac{\alpha_{33}}{\kappa h_\epsilon M} \\ \frac{\alpha_{22}}{\eta h_v} + \frac{\alpha_{22}}{\kappa h_\epsilon M} & \frac{\alpha_{22}}{\eta h_v} & \frac{\alpha_{22}^2}{\eta h_v} + \frac{\alpha_{22}^2(\gamma + \kappa M - \kappa)}{\kappa\gamma h_\epsilon M^2} & C_1 \\ \frac{\alpha_{32} + \alpha_{33}}{\eta h_v} + \frac{\alpha_{32}}{\kappa h_\epsilon M} & \frac{\alpha_{32} + \alpha_{33}}{\eta h_v} + \frac{\alpha_{33}}{\kappa h_\epsilon M} & C_1 & C_2 \end{bmatrix}, \tag{A6}$$

where

$$C_1 = \frac{\alpha_{22}\alpha_{33}}{\eta h_v} + \alpha_{22}\alpha_{32}\left(\frac{1}{\eta h_v} + \frac{\gamma + \gamma M - \kappa}{\kappa\gamma h_\epsilon M^2}\right),$$

$$C_2 = \frac{(\alpha_{31+}\alpha_{33})^2}{\eta h_v} + \frac{(\alpha_{32}^2 + \alpha_{33}^2)(\gamma + \gamma M - \kappa)}{\kappa\gamma h_\epsilon M^2}. \tag{A7}$$

Let

$$\mathrm{A}^T \equiv \mathrm{cov}_{\mathrm{b}}(\tilde{v}, \Phi_{3i}) = [(\eta h_v)^{-1} \quad (\eta h_v)^{-1} \quad \alpha_{22}(\eta h_v)^{-1} \quad (\alpha_{32} + \alpha_{33})(\eta h_v)^{-1}]^T.$$

Then, by the projection theorem,

$$\begin{aligned} \mathrm{E}_{\mathrm{b}}(\tilde{v}|\Phi_{3i}) &= \bar{v} + A\Psi^{-1}(\Phi_{3i} - \mathrm{E}_{\mathrm{b}}(\Phi_{3i})) \\ &= \frac{(y_{2i} + y_{3i})(\kappa - \gamma)h_\epsilon + (\bar{Y}_2 + \bar{Y}_3)(\gamma h_\epsilon M) + \eta h_v \bar{v}}{\eta h_v + 2(\kappa + \gamma M - \gamma)h_\epsilon} \end{aligned} \tag{A8}$$

and

$$\begin{aligned} \mathrm{var}_{\mathrm{b}}(\tilde{v}|\Phi_3) &= (\eta h_v)^{-1} - A\Psi^{-1}A^T \\ &= \frac{1}{\eta h_v + 2(\kappa + \gamma M - \gamma)h_\epsilon}. \end{aligned} \tag{A9}$$

The conditional variance of $\tilde{v}$ is the same for all traders and so the subscript i is dropped in $\mathrm{var_b}(\tilde{v}|\Phi_3)$. Following Grossman (1976), we can solve equation (1) and get demand function

$$x_{3i} = \frac{\mathrm{E_b}(\tilde{v}|\Phi_{3i}) - P_3}{a\,\mathrm{var_b}(\tilde{v}|\Phi_3)}. \tag{A10}$$

We calculate the average demand per trader and equate this to the per trader supply, $\bar{x}$. Then solving for P_3, we can match coefficients to those of the conjectured second-period price function to obtain α_{31}, α_{32}, and α_{33} as given in equations (A1) and (A2). Thus the linear price conjectures are fulfilled and equilibrium exists at $t = 3$.

To solve the equilibrium at $t = 2$, we again use the projection theorem, calculating

$$\begin{aligned}\mathrm{E_b}(P_3|\Phi_{2i}) &= \frac{2(\kappa+\gamma M-\gamma)h_\epsilon\bar{v} + \eta h_v\bar{v} - a\bar{x}}{\eta h_v + 2(\kappa+\gamma M-\gamma)h_\epsilon} \\ &\quad + \frac{(\kappa+\gamma M-\gamma)(h_\epsilon(\eta h_v + (\kappa+2\gamma M-\gamma)h_\epsilon)(\bar{Y}_2-\bar{v}) + h_\epsilon^2(\kappa-\gamma)(y_{2i}-\bar{v}))}{(\eta h_v + (\kappa+\gamma M-\gamma)h_\epsilon)(\eta h_v + 2(\kappa+\gamma M-\gamma)h_\epsilon)}\end{aligned} \tag{A11}$$

and

$$\begin{aligned}\mathrm{var_b}(P_3|\Phi_2) &= \frac{(\kappa+\gamma M-\gamma)^2 h_\epsilon(\eta(\gamma+\kappa M-\kappa)h_v + ((\kappa-\gamma)^2(M-1) + 2\gamma\kappa M^2)h_\epsilon)}{\gamma\kappa M^2(\eta h_v + (\kappa+\gamma M-\gamma)h_\epsilon)(\eta h_v + 2(\kappa+\gamma M-\gamma)h_\epsilon)^2}.\end{aligned} \tag{A12}$$

Since traders are myopic, trader i's second round demand is

$$x_{2i} = \frac{\mathrm{E_b}(P_3|\Phi_{2i}) - P_2}{a\,\mathrm{var_b}(P_3|\Phi_2)}. \tag{A13}$$

Equating per trader demand and per trader supply, solving for P_2, and matching coefficients gives us the equilibrium values for α_{21} and α_{22} given in equations (A3) and (A4). Using the unconditional expectation and variance of P_2, we can follow the same steps as above to calculate P_1 as given in equation (A5). Q.E.D.

To simplify the exposition, Propositions 1 to 5 are proven for the case of $\eta = 1$ and $\gamma = 1$. Proposition 1 states that expected volume increases as overconfidence increases. This is restated formally and proven in terms of trading round 2 volume. It is also true for round 3 expected volume.

PROPOSITION 1: *If $\kappa > 1$, and $M \geq 2$ then*

$$\mathrm{E_a}\left(\sum_{i=1}^{N} \frac{|x_{2i} - x_{1i}|}{N}\right) \tag{A14}$$

is an increasing function of κ.

Proof: The first step of the proof is to calculate equation (A14), the per capita expected trading volume in trading round 2. Traders have negative exponential utility functions, which means that their demand for the risky asset does not depend on their wealth. They have the same prior beliefs about the distribution of $\tilde{v}$. Because they have the same beliefs as well as the same risk aversion, all traders have the same first period demand: $x_{1i} = \bar{x}$. Coefficients from equations (A3) and (A4) are substituted into equation (3); equations (3), (A11), and (A12) are then substituted into equation (A13) which is substituted into equation (A14). The expectation operator is moved inside the summation and the denominator N is moved outside the expectation. We have then the average expectation of N identical half-normal distributions. Taking expectations and simplifying gives us

$$\begin{aligned}\mathrm{E_a}&\left(\sum_{i=1}^{N} \frac{|x_{2i} - x_{1i}|}{N}\right)\\ &= \sqrt{\frac{2(M-1)h_\epsilon}{M\pi}}\\ &\quad\times \frac{(\kappa-1)\kappa M^2(h_v + 2(\kappa+M-1)h_\epsilon)}{a(\kappa+M-1)((1-\kappa+\kappa M)h_v + ((\kappa^2-2\kappa+1)(M-1)+2\kappa M^2)h_\epsilon)}.\end{aligned} \tag{A15}$$

Bearing in mind that $M \geq 2$, and $\kappa > 1$, one can show that, in the given parameter range, the derivative of equation (A15) with respect to κ is positive and so equation (A15) is increasing in κ. Q.E.D.

Proposition 2 states that volatility increases when overconfidence increases and when traders undervalue their prior beliefs. Three alternative measures of volatility are $\mathrm{var_a}(P_2)$, $\mathrm{var_a}(P_3)$, and $\mathrm{var_a}(P_3 - P_2)$. The proposition is true for all three measures. It is proven for $\mathrm{var_a}(P_3)$.

PROPOSITION 2: *If $\kappa \geq 1$ then* $\mathrm{var_a}(P_3)$ *is an increasing function of κ.*

Proof: Substituting coefficients from equations (A1) and (A2) into equation (A3), we can calculate

$$\mathrm{var_a}(P_3) = \frac{2h_\epsilon(\kappa + M - 1)^2(h_v + 2h_\epsilon M)}{h_v M(h_v + 2h_\epsilon(\kappa + M - 1))^2}. \tag{A16}$$

In the given parameter range, the derivative of equation (A16) with respect to κ is positive. Q.E.D.

The quality of prices can be measured as $\mathrm{var_a}(P_t - \tilde{v})$ for $t = 2$ or 3. As this variance increases, the quality of prices worsens. Proposition 4 and its proof are given here in terms of $t = 3$. The proposition is also true for $t = 2$; the proof is analogous.

PROPOSITION 3: *If* $\kappa \geq 1$, $\mathrm{var_a}(P_3 - \tilde{v})$ *is an increasing function of* κ.

Proof: Substituting coefficients from equations (A1) and (A2) into equation (2), we can calculate

$$\mathrm{var_a}(P_3 - \tilde{v}) = \frac{h_v M + 2h_\epsilon((\kappa - 1)^2 - 2M + 2\kappa M + M^2)}{M(h_v + 2(\kappa - 1 + M)h_\epsilon)^2}. \tag{A17}$$

In the given parameter range, the derivative of equation (A17) with respect to κ is positive. Q.E.D.

PROPOSITION 4: *If* $M \geq 2$ *traders' expected utilities will be lower when* $\kappa > 1$ *than when* $\kappa = 1$.

Proof: In this model, traders can infer the aggregate signal. So if they have perfectly calibrated probability beliefs (i.e., $\kappa = \eta = \gamma = 1$) their posterior beliefs will be identical. Because they have the same beliefs as well as CARA utility functions with the same risk aversion, perfectly calibrated traders will hold the same amount of the risky asset in equilibrium (i.e., $\bar{x}$); this maximizes their utility. This is the position to which traders, whether overconfident or perfectly calibrated, trade in the first round of trading where there is no signal (i.e., $x_{1i} = \bar{x}$). Following steps similar to those used to obtain equation (A15), we can calculate the expected net trading subsequent to the first round of trading. This is

$$\mathrm{E_a}\left(\sum_{i=1}^{N} \frac{|x_{3i} - x_{1i}|}{N}\right) = \frac{2(\kappa - 1)}{a}\sqrt{\frac{(M-1)h_\epsilon}{M\pi}}. \tag{A18}$$

We see that when traders are perfectly calibrated, they do not trade in later rounds and continue to hold their optimal portfolio of the risky asset, $\bar{x}$. However, if traders are overconfident (i.e., $\kappa > 1$) and if $M \geq 2$, they are expected to trade in later rounds, thus departing from their optimal portfolio and reducing their expected utility. Q.E.D.

PROPOSITION 5: $\mathrm{Cov_a}((P_3 - P_2),(P_2 - P_1))$ *is a decreasing function of* κ *and* $\mathrm{cov_a}((P_3 - P_2),(P_2 - P_1)) = 0$ *when* $\kappa = \eta = \gamma = 1$.

Proof: Noting that P_1 is a constant and substituting coefficients from equations (A1), (A2), (A3), and (A4) into equations (2) and (3), we can calculate

$$\begin{aligned}&\mathrm{cov_a}((P_3 - P_2),(P_2 - P_1))\\ &\quad = -\frac{(\kappa - \gamma + \gamma M)^2(\kappa + (M-1)\gamma - (M\eta))h_\epsilon^2}{M(\eta h_v + (\kappa - \gamma + \gamma M)h_\epsilon)^2(\eta h_v + 2(\kappa - \gamma + \gamma M)h_\epsilon)},\end{aligned} \tag{A19}$$

which has the opposite sign of $\kappa - \gamma + M\ (\gamma - \eta)$. So equation (A19) is 0 if $\kappa = \gamma = \eta = 1$, negative if $\kappa - \gamma > M(\eta - \gamma)$, and positive if $\kappa - \gamma < M(\eta - \gamma)$. Note that when $\eta = \gamma = 1$, the sign of expression (A19) is the opposite of $\kappa - 1$. The derivative of equation (A19) with respect to κ is negative. Q.E.D.

Adding rational traders to the economy greatly complicates the expression for covariance. For simplicity, and without altering the basic finding that returns may be positively serially correlated when overconfident traders trade with rational traders, the following proposition is proven for an economy with N overconfident traders and N/M rational traders in which traders receive private signals in period 2 and $\tilde{v}$ is publicly revealed in period 3; therefore $P_3 = \tilde{v}$. As above, in period 2 each overconfident trader receives one of M possible signals, $\tilde{y}_{2i} = \tilde{v} + \tilde{\epsilon}_{2m}$; rational traders receive signal $y_{2r} = \tilde{v} + \tilde{\rho}_2$, where $\tilde{\rho}_t \sim N(0,h_\rho^{-1})$ and $\tilde{\rho}_2$ is independent of $\tilde{\epsilon}_{2m}$ for $m = 1,\ldots,M$. Overconfident traders believe that $\tilde{\rho}_t \sim N(0,(\gamma h_\rho)^{-1})$; rational traders hold correct distributional beliefs about their signals and those of others.

PROPOSITION 6: *Let* $\eta = 0$. *If rational traders trade with overconfident traders, then* $\mathrm{cov_a}(P_3 - P_2, P_2 - P_1)$ *is positive if* $(1-\gamma)(1+\gamma M)\,h_\rho > (\kappa + 1 + \gamma(M-1))(\kappa + \gamma(M-1) - M)h_\epsilon$.

Proof: We can determine the equilibrium as was done in Lemma 1. Then we can calculate

$$\begin{aligned}&\mathrm{cov_a}(P_3 - P_2, P_2 - P_1)\\ &\quad = \frac{M((1-\gamma)(1+\gamma M)h_\rho - (\kappa + \gamma(M-1)+1)(\kappa + \gamma(M-1) - M)h_\epsilon)}{(M(\kappa + \gamma(M-1)+1)h_\epsilon) + (1+\gamma M)h_\rho + (1+M)h_v)^2}\end{aligned} \tag{A20}$$

which is positive when $(1-\gamma)(1+\gamma M)h_\rho > (\kappa+\gamma(M-1)+1)(\kappa+\gamma(M-1)-M)h_\epsilon$. Q.E.D.

Appendix B: An Insider

LEMMA 2: *If $\kappa h_\epsilon + 2\eta h_v > \kappa h_v$, an equilibrium exists in which the insider's linear price conjecture, equation (4), and the market-maker's linear demand conjecture, equation (5), are fulfilled. In equilibrium the coefficients of equations (4) and (5) are*

$$A = 0, \tag{B1}$$

$$B = \sqrt{\frac{\kappa h_v h_\epsilon}{h_z(\kappa h_\epsilon + 2\eta h_v - \kappa h_v)}}, \tag{B2}$$

$$H = 0, \tag{B3}$$

$$L = \frac{1}{2(\eta h_v + \kappa h_\epsilon)}\sqrt{\frac{\kappa h_\epsilon h_z(\kappa h_\epsilon + 2\eta h_v - \kappa h_v)}{h_v}}. \tag{B4}$$

Proof: The insider submits a demand, x, that he believes will maximize his expected profit. To do this he solves

$$\max_x \mathrm{E_b}(x(\tilde{v} - P)|y) = \max_x \mathrm{E_b}(x(\tilde{v} - (H + L(x+z)))|y), \tag{B5}$$

where equation (4) has been substituted for P. Taking first-order conditions and solving for x we have

$$x = \frac{\mathrm{E_b}(\tilde{v}|y) - H}{2L}. \tag{B6}$$

We can calculate

$$\mathrm{E_b}(\tilde{v}|y) = \frac{\kappa h_\epsilon y}{\eta h_v + \kappa h_\epsilon}. \tag{B7}$$

Substituting equation (B7) into equation (B6) we get

$$x = \frac{-H}{2L} + \frac{\kappa h_\epsilon}{2L(\eta h_v + \kappa h_\epsilon)}y. \tag{B8}$$

And so, if the linear conjectures hold,

$$A = \frac{-H}{2L} \quad \text{and} \quad B = \frac{\kappa h_\epsilon}{2L(\eta h_v + \kappa h_\epsilon)}. \tag{B9}$$

The marketmaker sets price equal to the expected value of $\tilde{v}$ given the order flow she observes. We can calculate

$$P = \mathrm{E_a}(\tilde{v}|x + \tilde{z}) = \frac{-ABh_\epsilon h_z}{B^2 h_z(h_\epsilon + h_v) + h_\epsilon h_v} + \frac{Bh_\epsilon h_z}{B^2 h_z(h_\epsilon + h_v) + h_\epsilon h_v}(x + \tilde{z}). \tag{B10}$$

So, if the conjectures hold,

$$H = \frac{-ABh_\epsilon h_z}{B^2 h_z(h_\epsilon + h_v) + h_\epsilon h_v} \quad \text{and} \quad L = \frac{Bh_\epsilon h_z}{B^2 h_z(h_\epsilon + h_v) + h_\epsilon h_v}. \tag{B11}$$

The four equations in (B9) through (B11) have four unknowns. When $\kappa h_\epsilon + 2\eta h_v \geq \kappa h_v$, they have one real nonnegative solution: equations (B1) through (B4). Thus the conjectures are fulfilled and an equilibrium exists. Q.E.D.

Expected trading volume is $\mathrm{E_a}(|x| + |\tilde{z}|)$:

PROPOSITION 7: *If $\kappa \geq 1$, $\eta \leq 1$, and $\kappa h_\epsilon + 2\eta h_v > \kappa h_v$, then $\mathrm{E_a}(|x| + |\tilde{z}|)$ is an increasing function of κ.*

Proof: Substituting equations (B3) and (B4) into equation (B8), and substituting equation (B8) for x, we can calculate

$$\mathrm{E_a}(|x| + |\tilde{z}|) = \sqrt{\frac{2\kappa(h_\epsilon + h_v)}{\pi h_z(\kappa h_\epsilon + 2\eta h_v - \kappa h_v)}} + \sqrt{\frac{2}{\pi h_z}}. \tag{B12}$$

When $\kappa h_\epsilon + 2\eta h_v > \kappa h_v$, the derivative of equation (B12) with respect to κ is positive. Q.E.D.

Market depth is measured as the inverse of the derivative of price with respect to order flow (i.e., $(x + \tilde{z})$):

PROPOSITION 8: *If $\kappa \geq 1$, $\eta \leq 1$, and $\kappa h_\epsilon + 2\eta h_v > \kappa h_v$, then $(dP/d(x + \tilde{z}))^{-1}$ is an increasing function of κ.*

Proof: Substituting equations (B3) and (B4) into equation (4), and differentiating with respect to $(x + \tilde{z})$ gives us

$$\left(\frac{dP}{d(x+\tilde{z})}\right)^{-1} = 1/L. \tag{B13}$$

Substituting equation (B4) for L, the derivative of $1/L$ with respect to κ is positive when $\kappa h_\epsilon + 2\eta h_v > \kappa h_v$. Q.E.D.

Volatility is measured as the variance of price.

PROPOSITION 9: *If $\kappa \geq 1$, $\eta \leq 1$, and $\kappa h_\epsilon + 2\eta h_v > \kappa h_v$, then* $\mathrm{var_a}(P)$ *is an increasing function of κ.*

Proof: Substituting equations (B1) and (B2) into equation (5), and equations (5), (B3), and (B4) into equation (4), we can calculate

$$\mathrm{var_a}(P) = \frac{\kappa h_\epsilon}{2h_v(\kappa h_\epsilon + \eta h_v)}. \tag{B14}$$

The derivative of equation (B14) with respect to κ is positive. Q.E.D.

Quality of prices is measured as the variance of the difference between price and true underlying value.

PROPOSITION 10: *If $\kappa \geq 1$, $\eta \leq 1$, and $\kappa h_\epsilon + 2\eta h_v > \kappa h_v$, then* $\mathrm{var_a}(P - \tilde{v})$ *is a decreasing function of κ.*

Proof: Substituting equations (B1) and (B2) into equation (5), and equations (5), (B3), and (B4) into equation (4), and then into $\mathrm{var_a}(P - \tilde{v})$, we can calculate

$$\mathrm{var_a}(P - \tilde{v}) = \frac{\kappa h_\epsilon + 2\eta h_v}{2h_v(\kappa h_\epsilon + \eta h_v)}. \tag{B15}$$

The derivative of equation (B15) with respect to κ is negative. Q.E.D.

PROPOSITION 11: *If $\kappa \geq 1$, $\eta \leq 1$, and $\kappa h_\epsilon + 2\eta h_v > \kappa h_v$, then* $\mathrm{E_a}(x(\tilde{v} - P))$ *is a decreasing function of κ.*

Proof: Substituting equations (B1) through (B4) into equations (4) and (5), and then into $\mathrm{E_a}(x(\tilde{v} - P))$, we have

$$\mathrm{E_a}(x(\tilde{v} - P)) = \frac{1}{2(\kappa h_\epsilon + \eta h_v)} \sqrt{\frac{\kappa h_\epsilon(\kappa h_\epsilon + 2\eta h_v - \kappa h_v)}{h_v h_z}}. \tag{B16}$$

When $\kappa h_\epsilon + 2\eta h_v > \kappa h_v$, the derivative of (B16) with respect to κ is negative. Q.E.D.

Appendix C: Marketmakers and Costly Information

The following expectation (Toft (1996)) is needed in this section. Let x be a normally distributed random variable with mean μ and variance σ^2, then

$$\mathrm{E}(e^{Ax^2+Bx+C}) = \frac{1}{\sqrt{(1-2A\sigma^2)}} \exp\left\{-\frac{1}{2\sigma^2}\left(\frac{-(\mu+B\sigma^2)^2}{1-2A\sigma^2} + \mu^2\right) + C\right\}. \tag{C1}$$

These expectations can be easily calculated:

$$\mathrm{E_b}(\tilde{v}|\tilde{y}) = \frac{\kappa h_\epsilon}{\eta h_v + \kappa h_\epsilon}\tilde{y} \equiv \mu_\mathrm{b}, \qquad \mathrm{E_a}(\tilde{v}|\tilde{y}) = \frac{h_\epsilon}{h_v + h_\epsilon}\tilde{y} \equiv \mu_\mathrm{a}, \tag{C2}$$

$$\mathrm{var_b}(\tilde{v}|\tilde{y}) = \frac{1}{\eta h_v + \kappa h_\epsilon} \equiv r_{i\mathrm{b}}, \qquad \mathrm{var_a}(\tilde{v}|\tilde{y}) = \frac{1}{h_v + h_\epsilon} \equiv r_{i\mathrm{a}}, \tag{C3}$$

$$\mathrm{var_b}(\mu_\mathrm{b}) = \frac{\kappa h_\epsilon}{(\eta h_v + \kappa h_\epsilon)(\eta h_v)} \equiv S_\mathrm{b}, \qquad \mathrm{var_a}(\mu_\mathrm{b}) = \frac{\kappa^2 h_\epsilon(h_v + h_\epsilon)}{h_v(\eta h_v + \kappa h_\epsilon)^2} \equiv S_\mathrm{a}. \tag{C4}$$

Because $\tilde{y}$ is normally distributed, μ_b is also normally distributed. Define

$$\bar{r} \equiv \frac{r_{\mathrm{ib}}\,\mathrm{var_b}(\tilde{v}|P)}{\lambda\,\mathrm{var_b}(\tilde{v}|P) + (1-\lambda)r_{\mathrm{ib}}}. \tag{C5}$$

LEMMA 3: *There exists an equilibrium in which each informed trader's demand for the risky asset is*

$$x_{1\mathrm{I}} = \frac{\mathrm{E_b}(\tilde{v}|\tilde{y}) - P}{a\,\mathrm{var_b}(\tilde{v}|\tilde{y})}, \tag{C6}$$

each uninformed trader's demand for the risky asset is

$$x_{1\mathrm{U}} = \frac{\mathrm{E}_{\mathrm{b}}(\tilde{v}|P) - P}{a\,\mathrm{var}_{\mathrm{b}}(\tilde{v}|P)}, \tag{C7}$$

price is

$$P = \bar{r}\left(\lambda\frac{\mu_{\mathrm{b}}}{r_{\mathrm{ib}}} + (1-\lambda)\,\frac{\mathrm{E}_{\mathrm{b}}(\tilde{v}|P)}{\mathrm{var}_{\mathrm{b}}(\tilde{v}|P)}\right) + a\bar{r}\tilde{z}, \tag{C8}$$

and the fraction of traders who choose to become informed is

$$\lambda^* = \begin{cases} 0 & \text{if } \left(1 - \dfrac{r_{\mathrm{ib}}(\mathrm{e}^{2ac}-1)}{S_{\mathrm{b}}}\right) \le 0, \\ a\sqrt{\left(1 - \dfrac{r_{\mathrm{ib}}(\mathrm{e}^{2ac}-1)}{S_{\mathrm{b}}}\right)\dfrac{r_{\mathrm{ib}}}{(\mathrm{e}^{2ac}-1)h_z}} & \text{if } \left(1 - \dfrac{r_{\mathrm{ib}}(\mathrm{e}^{2ac}-1)}{S_{\mathrm{b}}}\right) \in \left[0, \dfrac{(\mathrm{e}^{2ac}-1)h_z}{a^2 r_{\mathrm{ib}}}\right], \\ 1 & \text{if } \left(1 - \dfrac{r_{\mathrm{ib}}(\mathrm{e}^{2ac}-1)}{S_{\mathrm{b}}}\right) \ge \dfrac{(\mathrm{e}^{2ac}-1)h_z}{a^2 r_{\mathrm{ib}}}. \end{cases} \tag{C9}$$

Proof: The derivation of this equilibrium roughly follows Grossman and Stiglitz (1980) and Demski and Feltham (1994). Solving equation (6) gives us equation (C6) (see Grossman (1976)). Assume for the moment that, given traders' distributional beliefs and conditional on observing P, $\tilde{v}$ is normally distributed. (We will see below that the assumption is self-fulfilling.) Then solving equation (7) gives us equation (C7).

Informed trader demand per trader times the fraction of traders who are informed plus uninformed trader demand per trader times the fraction of traders who are uninformed must equal noise trader supply per trader. That is,

$$\lambda\,\frac{\mathrm{E}_{\mathrm{b}}(\tilde{v}|\tilde{y}) - P}{a\,\mathrm{var}_{\mathrm{b}}(\tilde{v}|\tilde{y})} + (1-\lambda)\,\frac{\mathrm{E}_{\mathrm{b}}(\tilde{v}|P) - P}{a\,\mathrm{var}_{\mathrm{b}}(\tilde{v}|P)} = -\tilde{z}. \tag{C10}$$

Solving equation (C10) for P gives us equation (C8). Let

$$\tilde{\zeta} \equiv \tilde{\mu} + \frac{ar_{\mathrm{ib}}}{\lambda}\,\tilde{z}. \tag{C11}$$

Substituting equation (C11) into equation (C8) gives us

$$P = C_1\tilde{\zeta} \quad \text{where} \quad C_1 \equiv \frac{\bar{r}\lambda}{r_{\mathrm{ib}}} + \frac{\bar{r}(1-\lambda)}{\mathrm{var}_{\mathrm{b}}(\tilde{v}|P)}\,G_1,$$

$$G_1 \equiv \frac{S_{\mathrm{b}}}{S_{\mathrm{b}} + D}, \quad \text{and} \quad D \equiv \left(\frac{ar_{\mathrm{ib}}}{\lambda}\right)^2 h_z^{-1}. \tag{C12}$$

If equation (C8) holds, then P is a linear function of $\tilde{\zeta}$ and the two are informationally equivalent. $\tilde{\zeta}$ is a linear combination of two normally distributed random variables and is normally distributed itself. Therefore, given traders' distributional beliefs, $(\tilde{v}|P) = (\tilde{v}|\tilde{\zeta})$ is normally distributed, as was assumed, and has mean and variance.

$$\mathrm{E_b}(\tilde{v}|\tilde{\zeta}) = G_1\tilde{\zeta}, \quad \text{and} \quad \mathrm{var_b}(\tilde{v}|\tilde{\zeta}) = \frac{1}{\eta h_v} - G_1 S_{\mathrm{b}} \equiv r_u. \tag{C13}$$

To solve for λ^* we observe that in equilibrium traders are indifferent between buying and not buying information; thus,

$$\mathrm{E_b}[U(W_{1\mathrm{I}})] = \mathrm{E_b}[U(W_{1\mathrm{U}})]. \tag{C14}$$

Bearing in mind that $\tilde{\zeta}$ has the same information content as P, we can calculate

$$\mathrm{E_b}[U(W_{1i})] = \mathrm{E_b}[\mathrm{E_b}[\mathrm{E_b}[-\exp\{-aW_{1I}\}|\tilde{y}]|\tilde{\zeta},\lambda]] \tag{C15}$$

$$= \mathrm{E_b}\left[\mathrm{E_b}\left[-\exp\left\{-a\mathrm{E_b}[W_{1I}|\tilde{y}] + \frac{a^2}{2}\mathrm{var_b}[W_{1I}|\tilde{y}]\right\}\middle|\tilde{\zeta},\lambda\right]\right] \tag{C16}$$

$$= \mathrm{E_b}\left[\mathrm{E_b}\left[-\exp\left\{-a\left(f_{0I} + x_{0I}P - c + \frac{(\mu_{\mathrm{b}} - P)^2}{2ar_{i\mathrm{b}}}\right)\right\}\middle|\tilde{\zeta},\lambda\right]\right] \tag{C17}$$

$$= \mathrm{E_b}\left[-e^{ac}\sqrt{\frac{r_{i\mathrm{b}}}{r_u}}\exp\left\{-a\left(f_{0i} + x_{0i}P + \frac{(E(\tilde{v}|\tilde{\zeta},\lambda) - P)^2}{2ar_u}\right)\right\}\right] \tag{C18}$$

$$= e^{ac}\sqrt{\frac{r_{i\mathrm{b}}}{r_u}}\,\mathrm{E_b}[U(W_{1U})]. \tag{C19}$$

Equation (C17) is obtained from equation (C16) by substituting $f_{0I} + x_{0I}P + x_{iI}(\tilde{v} - P)$ for W_{1I}, equation (C6) for x_{1i}, and taking expectations. To obtain equation (C18) we multiply out $(\mu_{\mathrm{b}} - P)^2$, apply equation (C1), substitute from (C3) and (C13), and note that $\mathrm{E_b}(\tilde{v}|\tilde{\zeta}) = \mathrm{E_b}(\mu_{\mathrm{b}}|\tilde{\zeta})$. From equations (C14) and (C19) we have

$$1 = e^{ac}\sqrt{\frac{r_{i\mathrm{b}}}{r_u}}. \tag{C20}$$

Substituting for $r_{i\mathrm{b}}$ and r_u in equation (C20), solving for λ, and noting that λ cannot be negative or larger than 1 gives us equations (C9). Q.E.D.

PROPOSITION 12: *There exist values of h_v, h_ϵ, h_z, a, and c, such that if $\eta = 1$ and $\kappa > 1$, or if $\eta < 1$ and $\kappa = 1$, and $0 < \lambda^* < 1$, then* $\mathrm{E_a}[U(W_{1I})] < \mathrm{E_a}[U(W_{1U})]$, *where i = I, and i = U represent prototypical informed and uninformed traders.*

Proof: To calculate the actual expected utility of the informed trader, we take iterative expectations using the actual distributions of $\tilde{v}$ and $\tilde{\epsilon}$. Equation (C1) is used to solve each expectation. The result is:

$$\begin{aligned}\mathrm{E_a}[U(W_{1I})] &= \mathrm{E_a}[\mathrm{E_a}[\mathrm{E_a}[U(W_{1I})|\tilde{y}]|\tilde{\zeta},\lambda]] \\ &= -(1-2C_3\,\mathrm{var_a}(\mu_\mathrm{b}|\tilde{\zeta}))^{-1/2}(1-2C_4\,\mathrm{var_a}(\tilde{\zeta}))^{-1/2} \\ &\quad \times \exp\{a(c-f_{0I}) + (aC_1x_{0I})^2\,\mathrm{var_a}(\tilde{\zeta})/(2(1-2C_4\,\mathrm{var_a}(\tilde{\zeta})))\},\end{aligned} \tag{C21}$$

where

$$C_2 = \left(1 + \frac{(\eta h_v + \kappa h_\epsilon)}{\kappa(h_v + h_\epsilon)}\right)(\eta h_v + \kappa h_\epsilon) - \frac{(\eta h_v + \kappa h_\epsilon)^2}{h_v + h_\epsilon},$$

$$C_3 = \frac{(\kappa-2)(\eta h_v + \kappa h\epsilon)^2}{2\kappa(h_v + h_\epsilon)}, \tag{C22}$$

$$C_4 = \frac{r_{i\mathrm{a}} - 2r_{i\mathrm{b}}}{2r_{i\mathrm{b}}^2}C_1^2 + \frac{(G_2 + C_2C_1\,\mathrm{var_a}(\mu_\mathrm{b}|\tilde{\zeta}))^2}{2\,\mathrm{var_a}(\mu_\mathrm{b}|\tilde{\zeta})(1-\mathrm{C_3}\mathrm{var_a}(\mu_\mathrm{b}|\tilde{\zeta}))} - \frac{G_2^2}{2\,\mathrm{var_a}(\mu_\mathrm{b}|\tilde{\zeta})}, \tag{C23}$$

$$G_2 = \frac{S_\mathrm{a}}{S_\mathrm{a} + D}, \qquad \mathrm{var_a}(\mu_\mathrm{b}|\tilde{\zeta}) = S_\mathrm{a}(1-G_2), \quad \text{and} \quad \mathrm{var_a}(\tilde{\zeta}) = S_\mathrm{a} + D. \tag{C24}$$

Similarly we can calculate the actual expected utility of the uninformed trader:

$$\begin{aligned}\mathrm{E_a}[U(W_{1U})] &= \mathrm{E_a}[\mathrm{E_a}[\mathrm{E_a}[U(W_{1U})|\tilde{y}]|\tilde{\zeta},\lambda]] \\ &= -(1-2C_5\,\mathrm{var_a}(\tilde{\zeta}))^{-1/2} \\ &\quad \times \exp\{-af_{0I} + (aC_1x_{0U})^2\,\mathrm{var_a}(\tilde{\zeta})/(2(1-2C_5\,\mathrm{var_a}(\tilde{\zeta})))\},\end{aligned} \tag{C25}$$

where

$$C_5 = \frac{(G_1 - C_1)^2}{2r_u^2}\,\mathrm{var_a}(\tilde{v}|\tilde{\zeta}) - \frac{(G_1 - C_1)(G_3 - C_1)}{r_u}, \tag{C26}$$

$$G_3 = \frac{\eta S_\mathrm{b}}{S_\mathrm{a} + D}, \quad \text{and} \quad \mathrm{var_a}(\tilde{v}|\tilde{\zeta}) = \frac{1}{h_v} - \frac{\eta^2 S_\mathrm{b}^2}{S_\mathrm{a} + D}. \tag{C27}$$

If equation (C21) is less than equation (C25) for any parameter set for which $\eta = 1$, $\kappa > 1$, and $0 < \lambda^* < 1$, and for any parameter set for which $\eta < 1$, $\kappa = 1$, and $0 < \lambda^* < 1$, then the proposition is true. I evaluate equations (C21) and (C25) for a wide variety of parameter values and find equation (C21) to be less than equation (25) whenever these parameter conditions are true, suggesting that the expected utility of informed traders is always less than than of uninformed traders if traders are overconfident or if they undervalue their prior information. Q.E.D.

In the final proposition λ^* and P are written as functions of the economy's parameter values (i.e., $\lambda^*(\kappa,\eta,h_v,h_\epsilon,h_z,a,c)$ and $P(\kappa,\eta,h_v,h_\epsilon,h_z,a,c)$). As in Proposition 10, market depth is measured as the inverse of the derivative of price with respect to order flow.

PROPOSITION 13: *Let $\kappa_2 > \kappa_1 \geq 1$, then, for any choice of parameter values η, h_v, h_ϵ, h_z, a, and c such that $\lambda^*(\kappa_2,\eta,h_v,h_\epsilon,h_z,a,c) = 1$ and $\lambda^*(\kappa_1,\eta,h_v,h_\epsilon,h_z,a,c) = 1$,*

$$\left(\frac{dP(\kappa_2,\eta,h_v,h_\epsilon,h_z,a,c)}{d\tilde{z}}\right)^{-1} > \left(\frac{dP(\kappa_1,\eta,h_v,h_\epsilon,h_z,a,c)}{d\tilde{z}}\right)^{-1} \tag{C28}$$

and

$$\mathrm{var_a}(P(\kappa_2,\eta,h_v,h_\epsilon,h_z,a,c)) > \mathrm{var_a}(P(\kappa_1,\eta,h_v,h_\epsilon,h_z,a,c)) \quad \text{if } \mathrm{E_a}(|\tilde{z}|) < \sqrt{2\phi/\pi}, \tag{C29}$$

$$\mathrm{var_a}(P(\kappa_2,\eta,h_v,h_\epsilon,h_z,a,c)) = \mathrm{var_a}(P(\kappa_1,\eta,h_v,h_\epsilon,h_z,a,c)) \quad \text{if } \mathrm{E_a}(|\tilde{z}|) = \sqrt{2\phi/\pi}, \tag{C30}$$

and

$$\mathrm{var_a}(\mathrm{P}(\kappa_2,\eta,h_v,h_\epsilon,h_z,a,c)) < \mathrm{var_a}(P(\kappa_1,\eta,h_v,h_\epsilon,h_z,a,c)) \quad \text{if } \mathrm{E_a}(|\tilde{z}|) > \sqrt{2\phi/\pi}, \tag{C31}$$

where

$$\phi = \frac{\eta(h_v + h_\epsilon)((\kappa_1 + \kappa_2)\eta h_v + 2\kappa_1\kappa_2 h_\epsilon)}{a^2(2\eta h_v + (\kappa_1 + \kappa_2)h_\epsilon)}. \tag{C32}$$

Proof: Substituting $\lambda^* = 1$ into equation (C5), and equation (C5) and $\lambda^* = 1$ into equation (C12), we find that $P = \tilde{\zeta}$. Differentiating P with respect to $\tilde{z}$, taking the inverse, and substituting from equation (C3) gives us market depth:

$$\left(\frac{dP}{d\tilde{z}}\right)^{-1} = \frac{\eta h_v + \kappa h_\epsilon}{a}, \tag{C33}$$

which is increasing in κ. Bearing in mind that $E_a(|\tilde{z}|) = \sqrt{2/\pi h_z}$, we find that $P = \tilde{\zeta}$, equations (C4) and (C24), and algebraic manipulations give us equations (C29) through (C31). Q.E.D.

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