2E. Complex Numbers

All references are to Notes C: Complex Numbers

2E-1. Change to polar form: a) $-1+i$ b) $\sqrt{3}-i$.

2E-2. Express $\dfrac{1-i}{1+i}$ in the form $a+bi$ by two methods: one using the Cartesian form throughout, and one changing numerator and denominator to polar form. Show the two answers agree.

2E-3.* Show the distance between any two complex points z_1 and z_2 is given by $|z_2 - z_1|$.

2E-4. Prove two laws of complex conjugation:

for any complex numbers z and w, a) $\overline{z+w} = \overline{z} + \overline{w}$ b) $\overline{zw} = \overline{z}\,\overline{w}$.

2E-5.* Suppose $f(x)$ is a polynomial with *real* coefficients. Using the results of 2E-4, show that if $a+ib$ is a zero, then the complex conjugate $a-ib$ is also a zero. (Thus, complex roots of a real polynomial occur in conjugate pairs.)

2E-6.* Prove the formula $e^{i\theta}e^{i\theta'} = e^{i(\theta+\theta')}$ by using the definition (Euler's formula (9)), and the trigonometric addition formulas.

2E-7. Calculate each of the following two ways: by changing to polar form, and also by using the binomial theorem.

a) $(1-i)^4$ b) $(1+i\sqrt{3})^3$

2E-8.* By using Euler's formula and the binomial theorem, express $\cos 3\theta$ and $\sin 3\theta$ in terms of $\cos\theta$ and $\sin\theta$.

2E-9. Express in the form $a+bi$ the six sixth roots of 1.

2E-10. Solve the equation $x^4 + 16 = 0$.

2E-11.* Solve the equation $x^4 + 2x^2 + 4 = 0$, expressing the four roots in both the polar form and the Cartesian form $a+bi$.

2E-12.* Calculate A and B explicitly in the form $a+bi$ for the cubic equation on the first page of Notes C, and then show that $A+B$ is indeed real, and a root of the equation.

2E-13.* Prove the law of exponentials (16), as suggested there.

2E-14. Express $\sin^4 x$ in terms of $\cos 4x$ and $\cos 2x$, using (18) and the binomial theorem. Why would you not expect $\sin 4x$ or $\sin 2x$ in the answer?

2E-15. Find $\displaystyle\int e^{2x}\sin x\,dx$ by using complex exponentials.

2E-16. Prove (18): a) $\cos x = \frac{1}{2}(e^{ix} + e^{-ix})$, b) $\sin x = \frac{1}{2i}(e^{ix} - e^{-ix})$.

2E-17.* Derive formula (20): $D(e^{(a+ib)x}) = (a+ib)e^{(a+ib)x}$ from the definition of complex exponential and the derivative formula (19): $D(u+iv) = Du + iDv$.

2E-18.* Find the three cube roots of unity in the $a+bi$ form by locating them on the unit circle and using elementary geometry.