Linear Algebra Lecture 4 -Page 1 Chapter 2

Inverse of AB. $A^T \longrightarrow$ transpose.

Product of elimination matrices.

$A = LU$ (no row exchanges) "get elimination in a decent form" matrix form instead of row exchanges

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$AA^{-1} = I = A^{-1}A$.

$(AB)(B^{-1}A^{-1}) = I. = B^{-1}A^{-1}AB$

$AA^{-1} = I$

$(AA^{-1})^T = I^T = I.$

change the order ?!

$= (A^{-1})^T A^T$

this is $(A^T)^{-1}$ (INVERSE OF A^T)

So for inversing and transposing, you can do in either order for a single matrix.

$$\underset{E_{21}}{\begin{bmatrix} 1 & 0 \\ -4 & 1 \end{bmatrix}} \underset{A}{\begin{bmatrix} 2 & 1 \\ 8 & 7 \end{bmatrix}} = \underset{U}{\begin{bmatrix} 2 & 1 \\ 0 & 3 \end{bmatrix}}$$

$$A = L\,U$$

L ... stands for lower triangular

$$\begin{bmatrix} 2 & 1 \\ 8 & 7 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 4 & 1 \end{bmatrix} \begin{bmatrix} 2 & 1 \\ 0 & 3 \end{bmatrix}$$

ones on the diagonal

$$\begin{bmatrix} 1 & 0 \\ 4 & 1 \end{bmatrix} \begin{bmatrix} 2 & 0 \\ 0 & 3 \end{bmatrix} \begin{bmatrix} 1 & \frac{1}{2} \\ 0 & 1 \end{bmatrix}$$

diagonal matrix. in the middle

$A_{3\times3}$: three by three

$E_{32}E_{31}E_{21}A = U$ produced (no row exchanges)

$A = \underline{E_{21}^{-1}E_{31}^{-1}E_{32}^{-1}}\,U$

$= LU$ → ? why this product is better than the former?

(row way)

case:

$$\underset{E_{32}}{\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & -5 & 1 \end{bmatrix}} \underset{E_{21}}{\begin{bmatrix} 1 & 0 & 0 \\ -2 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}} = \begin{bmatrix} 1 & 0 & 0 \\ -2 & 1 & 0 \\ 10 & -5 & 1 \end{bmatrix} = E\,(\text{left of } A)$$

must be this because we only did the elimination from up to downside. (by 4-th way → no way to escape from 0!)

$-2 \times (-5)$ (of new Row 2)

$EA = U$

reverses direction: (reverse order) put E's inverses in the opposite order

$A = LU$

$$\begin{bmatrix} 1 & 0 & 0 \\ 2 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 5 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 2 & 1 & 0 \\ 0 & 5 & 1 \end{bmatrix} = L\,(\text{left of } U)$$

minus

in the right order. the multipliers just sit in the matrix L

keep a record of what those multipliers were, and it's okey

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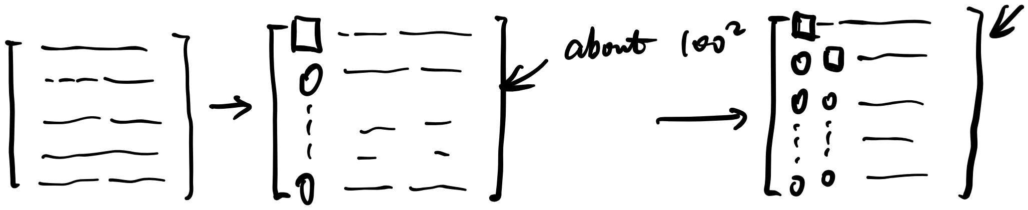
$A = LU$ → as the new insight in elimination that comes from matrix.
doing it in matrix form

If no row exchanges, multipliers go directly into L.
(minus !)

How many operations on $n \times n$ matrix A? ("cost" how expensive?)

$n = 100$. ~~$99 + 98 + \cdots + 1 = 50 \times 99 = 495$.~~

multiply + subtract



about 100^2

about 99^2

Count

$n^2 + (n-1)^2 + \cdots + 1^2$

n cubed.

$\approx \frac{1}{3} n^3$ on A.

cost of b → n^2 on b.

Permutations 3×3 6 P's.

multiplication won't escape. a group on 6 matrices $P^{-1} = P^T$

permutations matrices

$\begin{bmatrix} 1 & & \\ & 1 & \\ & & 1 \end{bmatrix}$, $P_{12} = \begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}$, $P_{13} = \begin{bmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix}$, $P_{23} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix}$, $\begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{bmatrix}$, $\begin{bmatrix} 0 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix}$

their inverse is themselves

Each is inverse of the other one

4×4: how many P's? 24 P's.

So, we could get a more general equation:

$$PA = LU$$