Lecture: Face Recognition and Feature Reduction

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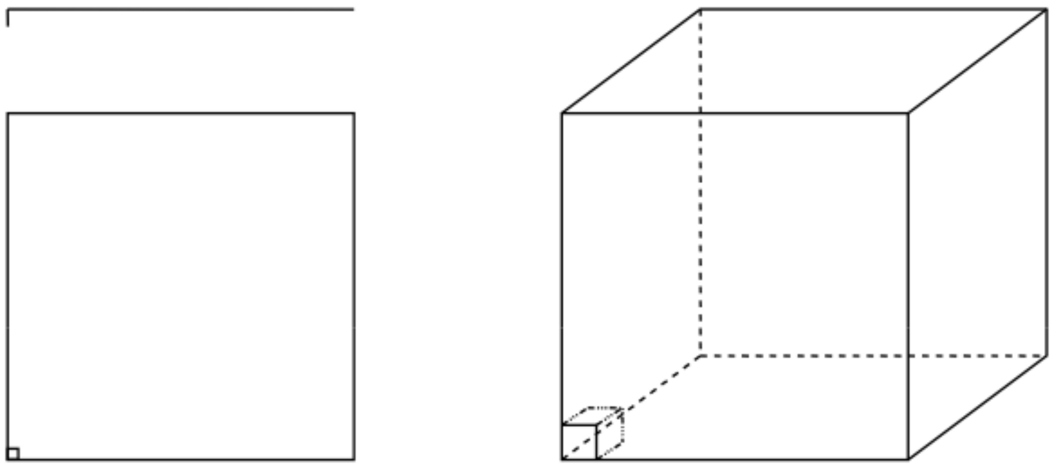
CS 131 Roadmap

Pixels	Segments	Images	Videos	Web
Convolutions Edges Descriptors	Resizing Segmentation Clustering	Recognition Detection Machine learning	Motion Tracking	Neural networks Convolutional neural networks

Recap - Curse of dimensionality

- Assume 5000 points uniformly distributed in the unit hypercube and we want to apply 5-NN. Suppose our query point is at the origin.
  - In 1-dimension, we must go a distance of 5/5000=0.001 on the average to capture 5 nearest neighbors.
  - In 2 dimensions, we must go $\sqrt{0.001}$ to get a square that contains 0.001 of the volume.
  - In d dimensions, we must go $(0.001)^{1/d}$



What we will learn today

- Singular value decomposition
- Principal Component Analysis (PCA)
- Image compression

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Singular Value Decomposition (SVD)

- There are several computer algorithms that can “factorize” a matrix, representing it as the product of some other matrices
- The most useful of these is the Singular Value Decomposition.
- Represents any matrix **A** as a product of three matrices: $\mathbf{U\Sigma V^T}$
- Python command:
  - [U,S,V]= numpy.linalg.svd(A)

Singular Value Decomposition (SVD)

$$\mathbf{U\Sigma V^T = A}$$

- Where **U** and **V** are rotation matrices, and **Σ** is a scaling matrix. For example:

$$\underset{U}{\begin{bmatrix} -.40 & .916 \\ .916 & .40 \end{bmatrix}} \times \underset{\Sigma}{\begin{bmatrix} 5.39 & 0 \\ 0 & 3.154 \end{bmatrix}} \times \underset{V^T}{\begin{bmatrix} -.05 & .999 \\ .999 & .05 \end{bmatrix}} = \underset{A}{\begin{bmatrix} 3 & -2 \\ 1 & 5 \end{bmatrix}}$$

Singular Value Decomposition (SVD)

- Beyond 2x2 matrices:
  - In general, if **A** is *m* x *n*, then **U** will be *m* x *m*, **Σ** will be *m* x *n*, and $\mathbf{V}^\mathbf{T}$ will be *n* x *n*.
  - (Note the dimensions work out to produce *m* x *n* after multiplication)

$$\overset{U}{\begin{bmatrix} -.39 & -.92 \\ -.92 & .39 \end{bmatrix}} \times \overset{\Sigma}{\begin{bmatrix} 9.51 & 0 & 0 \\ 0 & .77 & 0 \end{bmatrix}} \times \overset{V^T}{\begin{bmatrix} -.42 & -.57 & -.70 \\ .81 & .11 & -.58 \\ .41 & -.82 & .41 \end{bmatrix}} = \overset{A}{\begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix}}$$

Singular Value Decomposition (SVD)

- **U** and **V** are always rotation matrices.
  - Geometric rotation may not be an applicable concept, depending on the matrix. So we call them "unitary" matrices – each column is a unit vector.
- **Σ** is a diagonal matrix
  - The number of nonzero entries = rank of **A**
  - The algorithm always sorts the entries high to low

$$\overset{U}{\begin{bmatrix} -.39 & -.92 \\ -.92 & .39 \end{bmatrix}} \times \overset{\Sigma}{\begin{bmatrix} 9.51 & 0 & 0 \\ 0 & .77 & 0 \end{bmatrix}} \times \overset{V^T}{\begin{bmatrix} -.42 & -.57 & -.70 \\ .81 & .11 & -.58 \\ .41 & -.82 & .41 \end{bmatrix}} = \overset{A}{\begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix}}$$

SVD Applications

- We’ve discussed SVD in terms of geometric transformation matrices
- But SVD of an image matrix can also be very useful
- To understand this, we’ll look at a less geometric interpretation of what SVD is doing

SVD Applications

$$\begin{array}{cccccccc} U & & \Sigma & & V^T & & A \\ \begin{bmatrix} -.39 & -.92 \\ -.92 & .39 \end{bmatrix} & \times & \begin{bmatrix} 9.51 & 0 & 0 \\ 0 & .77 & 0 \end{bmatrix} & \times & \begin{bmatrix} -.42 & -.57 & -.70 \\ .81 & .11 & -.58 \\ .41 & -.82 & .41 \end{bmatrix} & = & \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix} \end{array}$$

- Look at how the multiplication works out, left to right:
- Column 1 of **U** gets scaled by the first value from **Σ**.

$$\begin{array}{cccc} U\Sigma & & V^T & A_{partial} \\ \begin{bmatrix} -3.67 & -.71 & 0 \\ -8.8 & .30 & 0 \end{bmatrix} & \times & \begin{bmatrix} -.42 & -.57 & -.70 \\ .81 & .11 & -.58 \\ .41 & -.82 & .41 \end{bmatrix} & \begin{bmatrix} 1.6 & 2.1 & 2.6 \\ 3.8 & 5.0 & 6.2 \end{bmatrix} \end{array}$$

- The resulting vector gets scaled by row 1 of $\mathbf{V}^\mathbf{T}$ to produce a contribution to the columns of **A**

SVD Applications

$$\overset{U\Sigma}{\begin{bmatrix} \boxed{-3.67} & -.71 & 0 \\ \boxed{-8.8} & .30 & 0 \end{bmatrix}} \times \overset{V^T}{\begin{bmatrix} \boxed{-.42 \quad -.57 \quad -.70} \\ .81 \quad .11 \quad -.58 \\ .41 \quad -.82 \quad .41 \end{bmatrix}} \qquad \overset{A_{partial}}{\begin{bmatrix} 1.6 & 2.1 & 2.6 \\ 3.8 & 5.0 & 6.2 \end{bmatrix}}$$

$$+ \overset{U\Sigma}{\begin{bmatrix} -3.67 & \boxed{-.71} & 0 \\ -8.8 & \boxed{.30} & 0 \end{bmatrix}} \times \overset{V^T}{\begin{bmatrix} -.42 \quad -.57 \quad -.70 \\ \boxed{.81 \quad .11 \quad -.58} \\ .41 \quad -.82 \quad .41 \end{bmatrix}} \qquad \overset{A_{partial}}{\begin{bmatrix} -.6 & -.1 & .4 \\ .2 & 0 & -.2 \end{bmatrix}}$$

$$= \qquad \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix}$$

- Each product of (*column i of **U***)·(*value i from **Σ***)·(*row i of **V**^{T}*) produces a component of the final **A**.

SVD Applications

$$\overset{U\Sigma}{\begin{bmatrix} -3.67 & -.71 & 0 \\ -8.8 & .30 & 0 \end{bmatrix}} \times \overset{V^T}{\begin{bmatrix} -.42 & -.57 & -.70 \\ .81 & .11 & -.58 \\ .41 & -.82 & .41 \end{bmatrix}} \quad \overset{A_{partial}}{\begin{bmatrix} 1.6 & 2.1 & 2.6 \\ 3.8 & 5.0 & 6.2 \end{bmatrix}}$$

$$\overset{A}{\begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix}}$$

$$\overset{U\Sigma}{\begin{bmatrix} -3.67 & -.71 & 0 \\ -8.8 & .30 & 0 \end{bmatrix}} \times \overset{V^T}{\begin{bmatrix} -.42 & -.57 & -.70 \\ .81 & .11 & -.58 \\ .41 & -.82 & .41 \end{bmatrix}} \quad \overset{A_{partial}}{\begin{bmatrix} -.6 & -.1 & .4 \\ .2 & 0 & -.2 \end{bmatrix}}$$

- We're building **A** as a linear combination of the columns of ***U***
- Using all columns of ***U***, we'll rebuild the original matrix perfectly
- But, in real-world data, often we can just use the first few columns of ***U*** and we'll get something close (e.g. the first $\boldsymbol{A}_{\boldsymbol{partial}}$, above)

SVD Applications

$$\overset{U\Sigma}{\begin{bmatrix} -3.67 & -.71 & 0 \\ -8.8 & .30 & 0 \end{bmatrix}} \times \overset{V^T}{\begin{bmatrix} -.42 & -.57 & -.70 \\ .81 & .11 & -.58 \\ .41 & -.82 & .41 \end{bmatrix}} \quad \overset{A_{partial}}{\begin{bmatrix} 1.6 & 2.1 & 2.6 \\ 3.8 & 5.0 & 6.2 \end{bmatrix}}$$

$$\overset{A}{\begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix}}$$

$$\overset{U\Sigma}{\begin{bmatrix} -3.67 & -.71 & 0 \\ -8.8 & .30 & 0 \end{bmatrix}} \times \overset{V^T}{\begin{bmatrix} -.42 & -.57 & -.70 \\ .81 & .11 & -.58 \\ .41 & -.82 & .41 \end{bmatrix}} \quad \overset{A_{partial}}{\begin{bmatrix} -.6 & -.1 & .4 \\ .2 & 0 & -.2 \end{bmatrix}}$$

- We can call those first few columns of ***U*** the *Principal Components* of the data
- They show the major patterns that can be added to produce the columns of the original matrix
- The rows of $\mathbf{V}^\mathbf{T}$ show how the *principal components* are mixed to produce the columns of the matrix

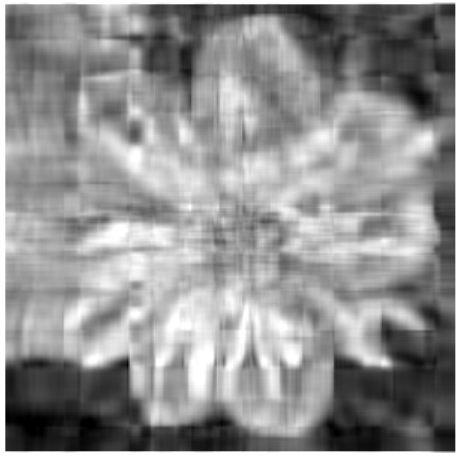
SVD Applications

$$\overset{U}{\begin{bmatrix} -.39 & -.92 \\ -.92 & .39 \end{bmatrix}} \times \overset{\Sigma}{\begin{bmatrix} 9.51 & 0 & 0 \\ 0 & .77 & 0 \end{bmatrix}} \times \overset{V^T}{\begin{bmatrix} -.42 & -.57 & -.70 \\ .81 & .11 & -.58 \\ .41 & -.82 & .41 \end{bmatrix}} = \overset{A}{\begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix}}$$

We can look at **Σ** to see that the first column has a large effect

while the second column has a much smaller effect in this example

SVD Applications



- For this image, using **only the first 10** of 300 principal components produces a recognizable reconstruction
- So, SVD can be used for image compression

SVD for symmetric matrices

- If A is a symmetric matrix, it can be decomposed as the following:

$$A = \Phi \Sigma \Phi^T$$

- Compared to a traditional S\ ıd is an orthogonal matrix.

Principal Component Analysis

$$\overset{U\Sigma}{\begin{bmatrix} -3.67 & -.71 & 0 \\ -8.8 & .30 & 0 \end{bmatrix}} \times \overset{V^T}{\begin{bmatrix} -.42 & -.57 & -.70 \\ .81 & .11 & -.58 \\ .41 & -.82 & .41 \end{bmatrix}} \quad \overset{A_{partial}}{\begin{bmatrix} 1.6 & 2.1 & 2.6 \\ 3.8 & 5.0 & 6.2 \end{bmatrix}}$$

- Remember, columns of ***U*** are the *Principal Components* of the data: the major patterns that can be added to produce the columns of the original matrix
- One use of this is to construct a matrix where each column is a separate data sample
- Run SVD on that matrix, and look at the first few columns of ***U*** to see patterns that are common among the columns
- This is called *Principal Component Analysis* (or PCA) of the data samples

Principal Component Analysis

$$\overset{U\Sigma}{\begin{bmatrix} -3.67 & -.71 & 0 \\ -8.8 & .30 & 0 \end{bmatrix}} \times \overset{V^T}{\begin{bmatrix} -.42 & -.57 & -.70 \\ .81 & .11 & -.58 \\ .41 & -.82 & .41 \end{bmatrix}} \quad \overset{A_{partial}}{\begin{bmatrix} 1.6 & 2.1 & 2.6 \\ 3.8 & 5.0 & 6.2 \end{bmatrix}}$$

- Often, raw data samples have a lot of redundancy and patterns
- PCA can allow you to represent data samples as weights on the principal components, rather than using the original raw form of the data
- By representing each sample as just those weights, you can represent just the “meat” of what’s different between samples.
- This minimal representation makes machine learning and other algorithms much more efficient

How is SVD computed?

- For this class: tell PYTHON to do it. Use the result.
- But, if you're interested, one computer algorithm to do it makes use of Eigenvectors!

Eigenvector definition

- Suppose we have a square matrix **A**. We can solve for vector x and scalar λ such that Ax= λx
- In other words, find vectors where, if we transform them with **A**, the only effect is to scale them with no change in direction.
- These vectors are called eigenvectors (German for “self vector” of the matrix), and the scaling factors λ are called eigenvalues
- An *m* x *m* matrix will have ≤ *m* eigenvectors where λ is nonzero

Finding eigenvectors

- Computers can find an x such that Ax= λx using this iterative algorithm:
  - X = random unit vector
  - while(x hasn't converged)
    - X = Ax
    - normalize x
- x will quickly converge to an eigenvector
- Some simple modifications will let this algorithm find all eigenvectors

Finding SVD

- Eigenvectors are for square matrices, but SVD is for all matrices
- To do svd(A), computers can do this:
  - Take eigenvectors of AA^T (matrix is always square).
    - These eigenvectors are the columns of **U**.
    - Square root of eigen*values* are the singular values (the entries of $\boldsymbol{\Sigma}$).
  - Take eigenvectors of A^TA (matrix is always square).
    - These eigenvectors are columns of **V** (or rows of $\mathbf{V^T}$)

Finding SVD

- Moral of the story: SVD is fast, even for large matrices
- It's useful for a lot of stuff
- There are also other algorithms to compute SVD or part of the SVD
  - Python's np.linalg.svd() command has options to efficiently compute only what you need, if performance becomes an issue

A detailed geometric explanation of SVD is here:
http://www.ams.org/samplings/feature-column/fcarc-svd

What we will learn today

- Introduction to face recognition
- Principal Component Analysis (PCA)
- Image compression

Covariance

- Variance and Covariance are a measure of the “spread” of a set of points around their center of mass (mean)
- Variance – measure of the deviation from the mean for points in one dimension e.g. heights
- Covariance as a measure of how much each of the dimensions vary from the mean with respect to each other.
- Covariance is measured between 2 dimensions to see if there is a relationship between the 2 dimensions e.g. number of hours studied & marks obtained.
- The covariance between one dimension and itself is the variance

Covariance

$$\text{covariance}(X,Y) = \frac{\sum_{i=1}^{n}(X_i - \bar{X})(Y_i - \bar{Y})}{(n-1)}$$

- So, if you had a 3-dimensional data set (x,y,z), then you could measure the covariance between the x and y dimensions, the y and z dimensions, and the x and z dimensions. Measuring the covariance between x and x , or y and y , or z and z would give you the variance of the x , y and z dimensions respectively

Covariance matrix

- Representing Covariance between dimensions as a matrix e.g. for 3 dimensions

$$C = \begin{bmatrix} cov(x,x) & cov(x,y) & cov(x,z) \\ cov(y,x) & cov(y,y) & cov(y,z) \\ cov(z,x) & cov(z,y) & cov(z,z) \end{bmatrix}$$

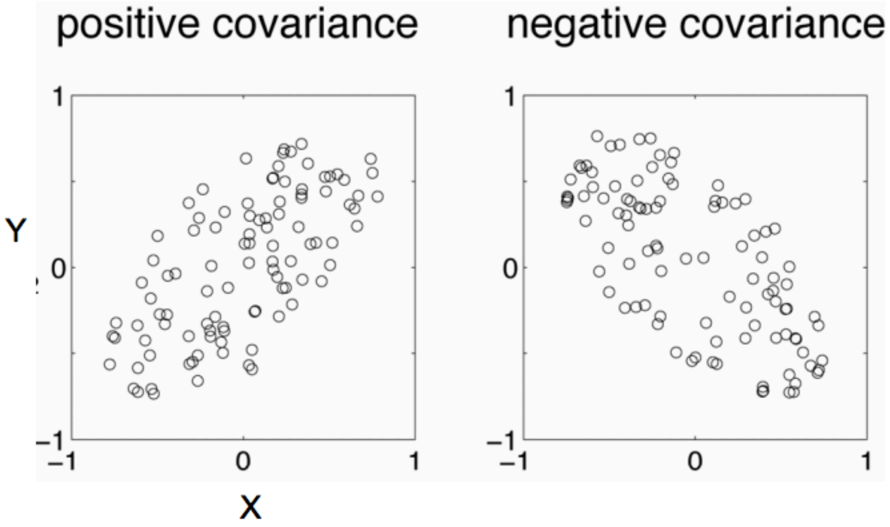
Variances

- Diagonal is the
- cov(x,y) = cov(y,x) hence matrix is symmetrical about the diagonal
- N-dimensional data will result in NxN covariance matrix

Covariance

- What is the interpretation of covariance calculations?
  - e.g.: 2 dimensional data set
  - x: number of hours studied for a subject
  - y: marks obtained in that subject
  - covariance value is say: 104.53
  - what does this value mean?

Covariance interpretation

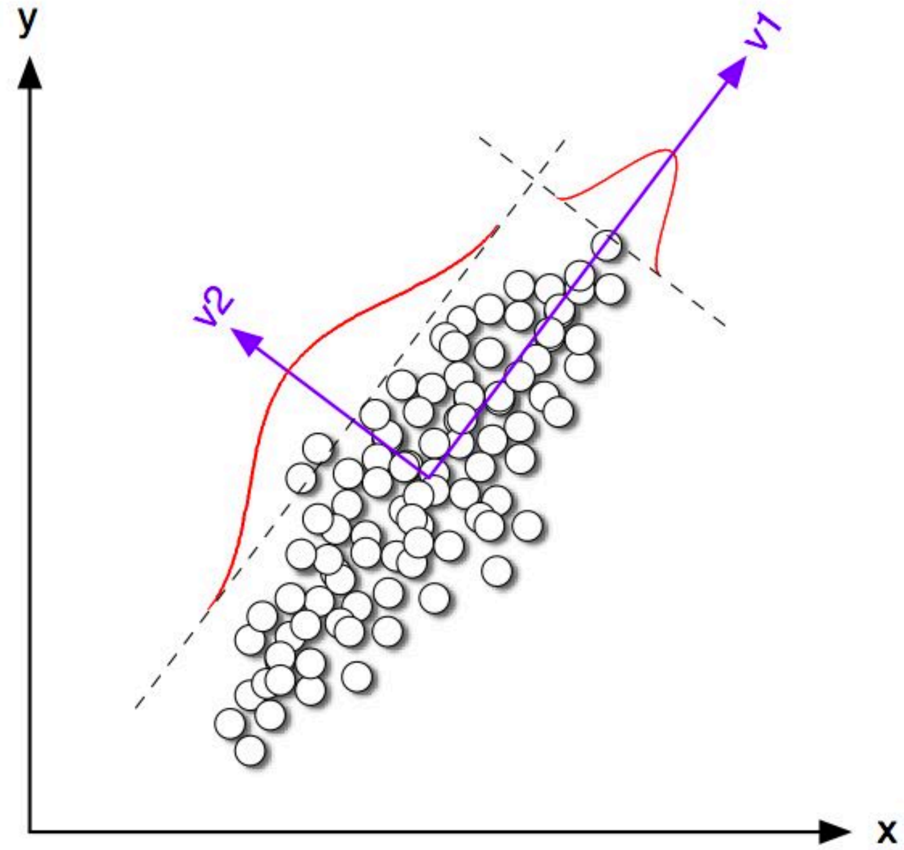


Covariance interpretation

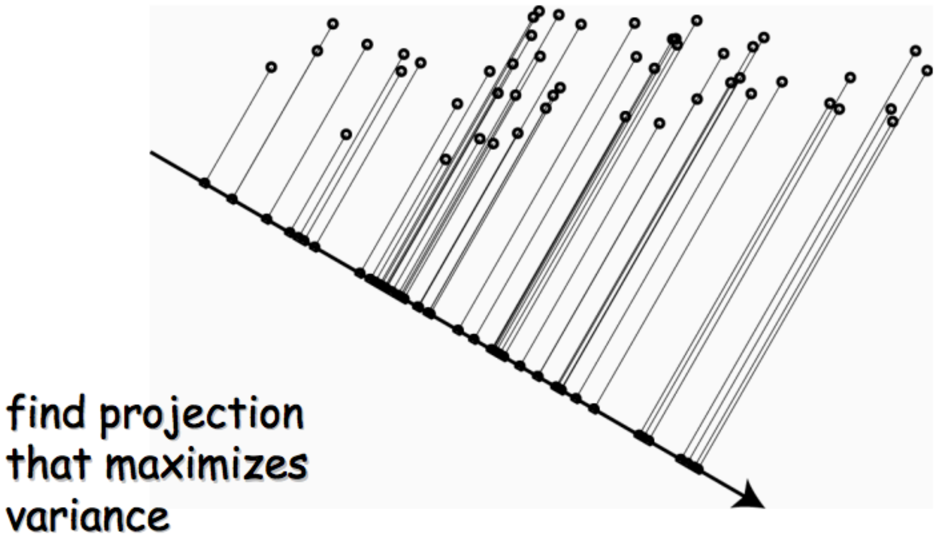
- Exact value is not as important as it's sign.
- A **positive value** of covariance indicates both dimensions increase or decrease together e.g. as the number of hours studied increases, the marks in that subject increase.
- A **negative value** indicates while one increases the other decreases, or vice-versa e.g. active social life at PSU vs performance in CS dept.
- If **covariance is zero**: the two dimensions are independent of each other e.g. heights of students vs the marks obtained in a subject

Example data

Covariance between the two axis is high. Can we reduce the number of dimensions to just 1?



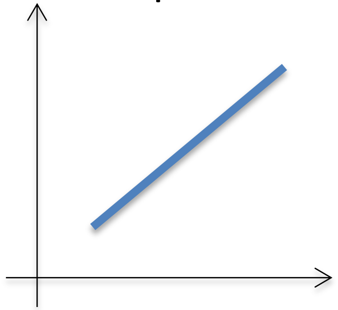
Geometric interpretation of PCA



Geometric interpretation of PCA

- Let's say we have a set of 2D data points x. But we see that all the points lie on a line in 2D.
- So, 2 dimensions are redundant to express the data. We can express all the points with just one dimension.

1D subspace in 2D

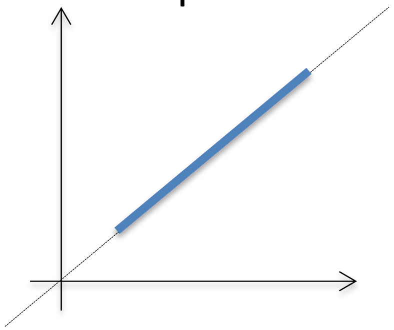


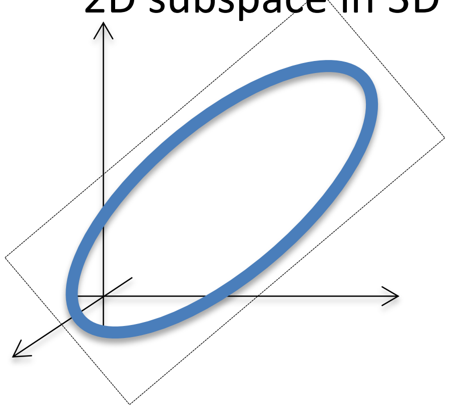
PCA: Principle Component Analysis

- Given a set of points, how do we know if they can be compressed like in the previous example?
  - The answer is to look into the correlation between the points
  - The tool for doing this is called PCA

PCA Formulation

- Basic idea:
  - If the data lives in a subspace, it is going to look very flat when viewed from the full space, e.g.





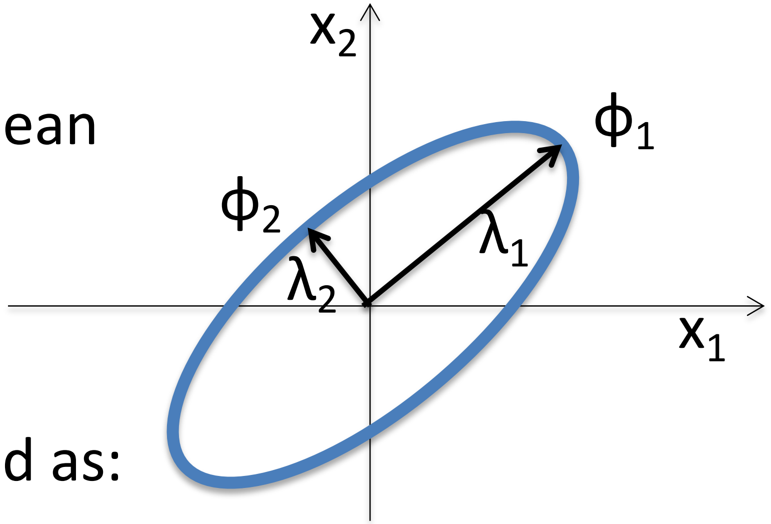
Slide inspired by N. Vasconcelos

PCA Formulation

- Assume x is Gaussian with covariance Σ.

- Recall that a gaussian is defined with it's mean and variance:

$$\mathbf{X} \sim \mathcal{N}(\boldsymbol{\mu}, \boldsymbol{\Sigma})$$



- Recall that μ and Σ of a gaussian are defined as:

$$\boldsymbol{\mu} = \mathbf{E}[\mathbf{X}] = [\mathbf{E}[X_1], \mathbf{E}[X_2], \ldots, \mathbf{E}[X_k]]^{\mathrm{T}}$$

$$\boldsymbol{\Sigma} =: \mathbf{E}[(\mathbf{X} - \boldsymbol{\mu})(\mathbf{X} - \boldsymbol{\mu})^{\mathrm{T}}] = [\mathrm{Cov}[X_i, X_j]; 1 \leq i, j \leq k]$$

Dimensionality Reduction

31-Oct-2019

PCA formulation

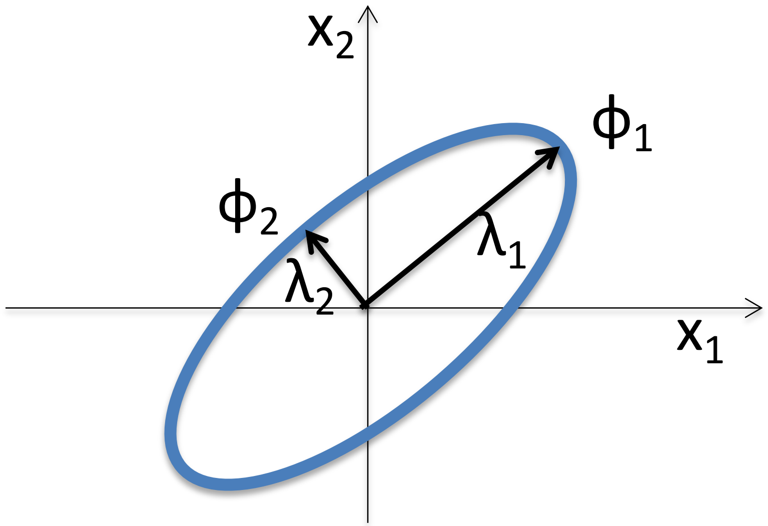
- Since gaussians are symmetric, it's covariance matrix is also a symmetric matrix. So we can express it as:
  - $\mathbf{\Sigma} = \mathbf{U\Lambda U}^T = \mathbf{U\Lambda}^{1/2}(\mathbf{U\Lambda}^{1/2})^T$

$$\mathbf{X} \sim \mathcal{N}(\boldsymbol{\mu}, \mathbf{\Sigma}) \iff \mathbf{X} \sim \boldsymbol{\mu} + \mathbf{U\Lambda}^{1/2}\mathcal{N}(0, \mathbf{I})$$

$$\iff \mathbf{X} \sim \boldsymbol{\mu} + \mathbf{U}\mathcal{N}(0, \mathbf{\Lambda}).$$

PCA Formulation

- If x is Gaussian with covariance Σ,
  - Principal components ϕ_i are the eigenvectors of Σ
  - Principal lengths λ_i are the eigenvalues of Σ



- by computing the eigenvalues we know the data is
  - Not flat if $\lambda_1 \approx \lambda_2$
  - Flat if $\lambda_1 >> \lambda_2$

Slide inspired by N. Vasconcelos

PCA Algorithm (training)

- Given sample $\mathcal{D} = \{\mathbf{x}_1, \ldots, \mathbf{x}_n\}, \;\; x_i \in \mathcal{R}^d$
  - compute sample mean: $\hat{\mu} = \frac{1}{n}\sum_i(\mathbf{x}_i)$
  - compute sample covariance: $\hat{\Sigma} = \frac{1}{n}\sum_i(\mathbf{x}_i - \hat{\mu})(\mathbf{x}_i - \hat{\mu})^T$
  - compute eigenvalues and eigenvectors of $\hat{\Sigma}$

$$\hat{\Sigma} = \Phi\Lambda\Phi^T, \;\; \Lambda = diag(\sigma_1^2, \ldots, \sigma_n^2) \;\; \Phi^T\Phi = I$$

  - order eigenvalues $\sigma_1^2 > \ldots > \sigma_n^2$
  - if, for a certain k, $\sigma_k << \sigma_1$ eliminate the eigenvalues and eigenvectors above k.

Slide inspired by N. Vasconcelos

PCA Algorithm (testing)

- Given principal compoenents $\phi_i, i \in 1, \ldots, k$ and a test sample $\mathcal{T} = \{\mathbf{t}_1, \ldots, \mathbf{t}_n\}, \;\; t_i \in \mathcal{R}^d$
  - subtract mean to each point $\mathbf{t}'_i = \mathbf{t}_i - \widehat{\mu}$
  - project onto eigenvector space $\mathbf{y}_i = \mathbf{A}\mathbf{t}'_i$ where

$$\mathbf{A} = \begin{bmatrix} \phi_1^T \\ \vdots \\ \phi_k^T \end{bmatrix}$$

  - use $\mathcal{T}' = \{\mathbf{y}_1, \ldots \mathbf{y}_n\}$ to estimate class conditional densities and do all further processing on **y**.

Slide inspired by N. Vasconcelos

PCA by SVD

- An alternative manner to compute the principal components, based on singular value decomposition
- Quick reminder: SVD
  - Any real n x m matrix (n>m) can be decomposed as

$$\boxed{A = \mathrm{M}\Pi\mathrm{N}^T}$$

  - Where M is an (n x m) column orthonormal matrix of left singular vectors (columns of M)
  - Π is an (m x m) diagonal matrix of singular values
  - N^T is an (m x m) row orthonormal matrix of right singular vectors (columns of N)

$$\boxed{\mathrm{M}^T\mathrm{M} = I \qquad \mathrm{N}^T\mathrm{N} = I}$$

Slide inspired by N. Vasconcelos

PCA by SVD

- To relate this to PCA, we consider the data matrix

$$X = \begin{bmatrix} | & & | \\ x_1 & \dots & x_n \\ | & & | \end{bmatrix}$$

- The sample mean is

$$\mu = \frac{1}{n}\sum_i x_i = \frac{1}{n}\begin{bmatrix} | & & | \\ x_1 & \dots & x_n \\ | & & | \end{bmatrix}\begin{bmatrix} 1 \\ \vdots \\ 1 \end{bmatrix} = \frac{1}{n}X1$$

Slide inspired by N. Vasconcelos

PCA by SVD

- Center the data by subtracting the mean to each column of X
- The <u>centered data matrix</u> is

$$X_c = \begin{bmatrix} | & & | \\ x_1 & \dots & x_n \\ | & & | \end{bmatrix} - \begin{bmatrix} | & & | \\ \mu & \dots & \mu \\ | & & | \end{bmatrix}$$

$$= X - \mu 1^T = X - \frac{1}{n} X 1 1^T = X\left(I - \frac{1}{n} 1 1^T \right)$$

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PCA by SVD

- The sample covariance matrix is

$$\boxed{\Sigma = \frac{1}{n}\sum_i (x_i-\mu)(x_i-\mu)^T = \frac{1}{n}\sum_i x_i^c\left(x_i^c\right)^T}$$

where x_i^c is the i^{th} column of X_c

- This can be written as

$$\boxed{\Sigma = \frac{1}{n}\begin{bmatrix} | & & | \\ x_1^c & \dots & x_n^c \\ | & & | \end{bmatrix}\begin{bmatrix} - & x_1^c & - \\ & \vdots & \\ - & x_n^c & - \end{bmatrix} = \frac{1}{n}X_c X_c^T}$$

Slide inspired by N. Vasconcelos

PCA by SVD

- The matrix

$$X_c^T = \begin{bmatrix} - & x_1^c & - \\ & \vdots & \\ - & x_n^c & - \end{bmatrix}$$

is real (n x d). Assuming n>d it has SVD decomposition

$$X_c^T = \mathrm{M}\Pi\mathrm{N}^T \qquad \mathrm{M}^T\mathrm{M} = I \qquad \mathrm{N}^T\mathrm{N} = I$$

and

$$\Sigma = \frac{1}{n} X_c X_c^T = \frac{1}{n} \mathrm{N}\Pi\mathrm{M}^T\mathrm{M}\Pi\mathrm{N}^T = \frac{1}{n} \mathrm{N}\Pi^2\mathrm{N}^T$$

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PCA by SVD

$$\Sigma = \mathrm{N}\left(\frac{1}{n}\Pi^2\right)\mathrm{N}^T$$

- Note that N is (d x d) and orthonormal, and Π^2 is diagonal. This is just the eigenvalue decomposition of Σ
- It follows that
  - The eigenvectors of Σ are the columns of N
  - The eigenvalues of Σ are

$$\lambda_i = \frac{1}{n}\pi_i^2$$

- This gives an alternative algorithm for PCA

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PCA by SVD

- In summary, computation of PCA by SVD
- Given X with one example per column
  - Create the centered data matrix

$$\boxed{X_c^T = \left(I - \frac{1}{n}11^T\right)X^T}$$

  - Compute its SVD

$$\boxed{X_c^T = \mathrm{M}\Pi\mathrm{N}^T}$$

  - Principal components are columns of N, eigenvalues are

$$\boxed{\lambda_i = \frac{1}{n}\pi_i^2}$$

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Dimensionality Reduction

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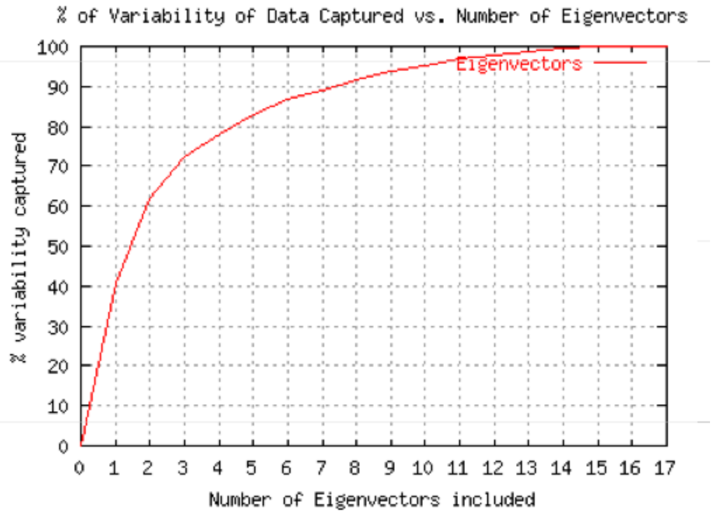
Rule of thumb for finding the number of PCA components

- A natural measure is to pick the eigenvectors that explain p% of the data variability
  - Can be done by plotting the ratio r_k as a function of k



$$r_k = \sum_{i=1}^{k} \lambda_i^2 \Big/ \sum_{i=1}^{n} \lambda_i^2$$

  - E.g. we need 3 eigenvectors to cover 70% of the variability of this dataset

Slide inspired by N. Vasconcelos

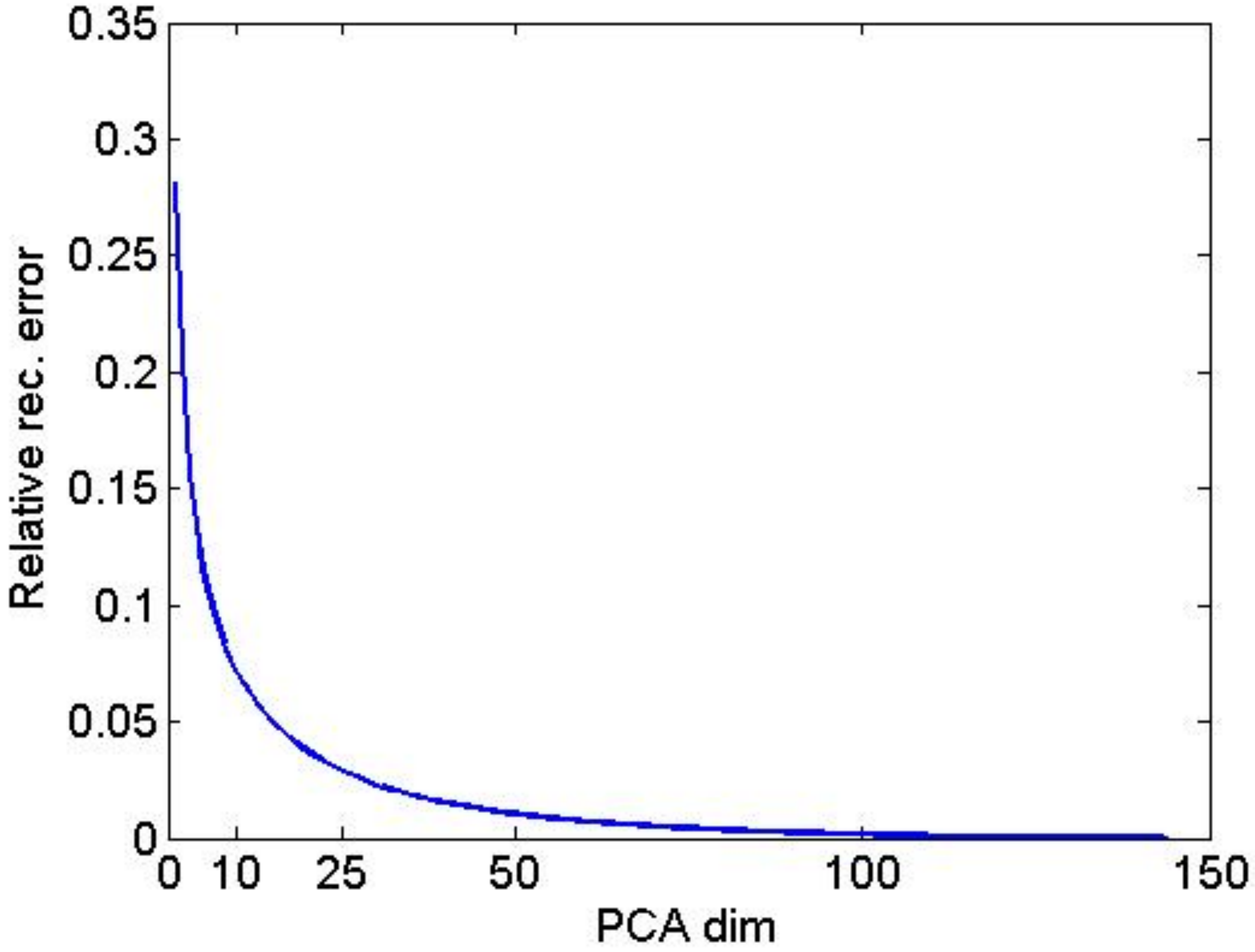
What we will learn today

- Introduction to face recognition
- Principal Component Analysis (PCA)
- Image compression

Original Image

- Divide the original 372x492 image into patches:
  - Each patch is an instance that contains 12x12 pixels on a grid
- View each as a 144-D vector

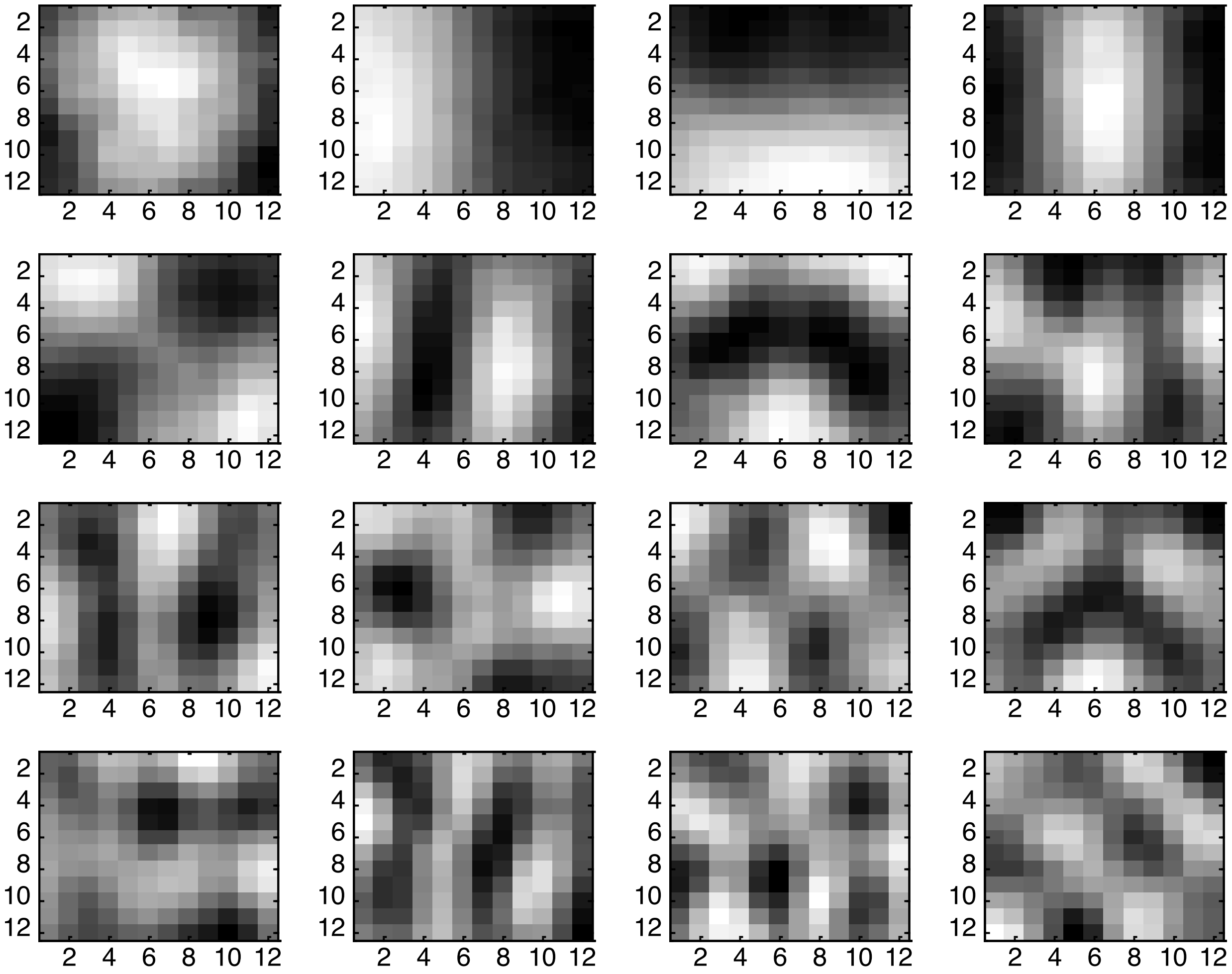
L_2 error and PCA dim



PCA compression: 144D) 60D

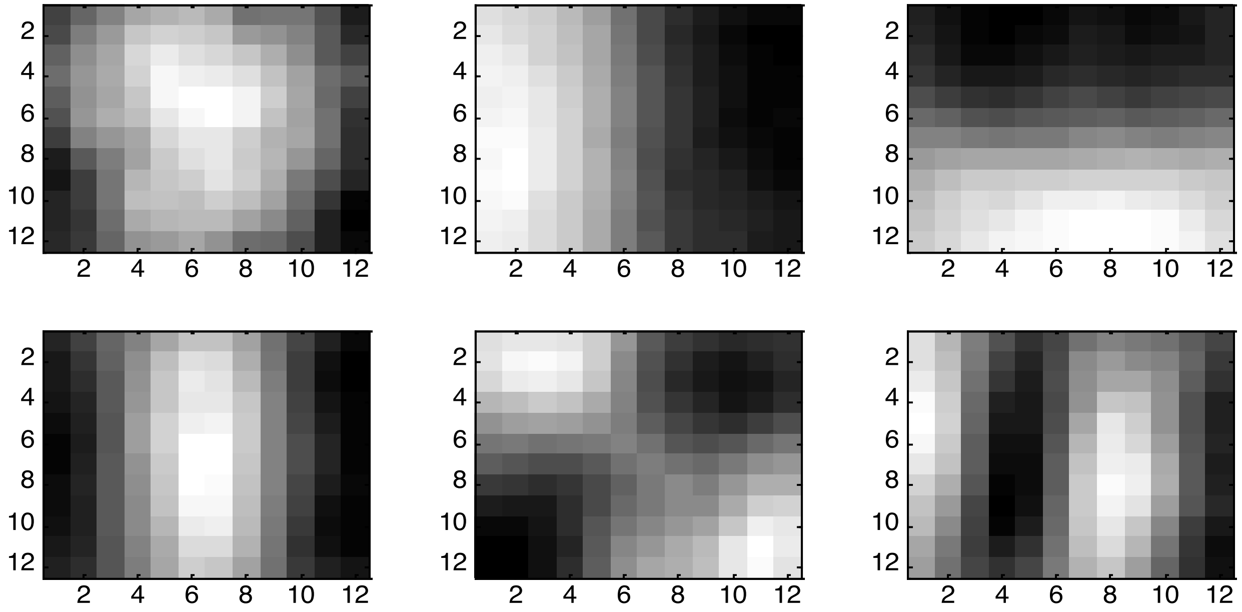
PCA compression: 144D) 16D

16 most important eigenvectors



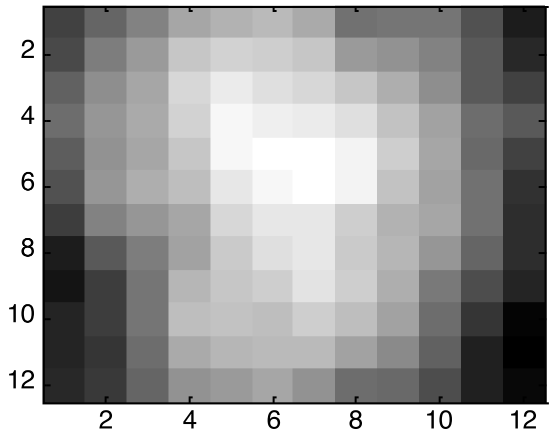
PCA compression: 144D) 6D

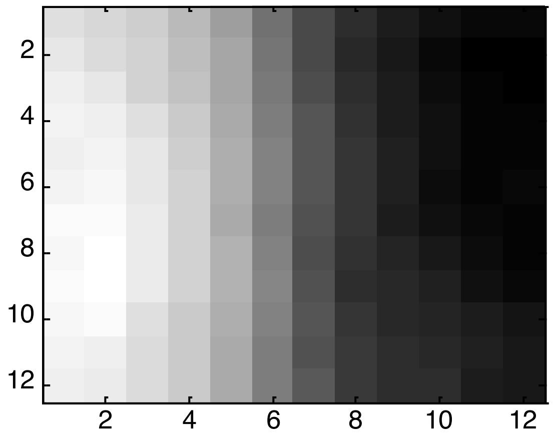
6 most important eigenvectors

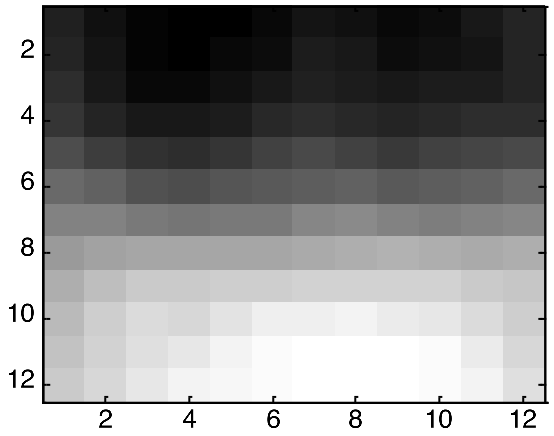


PCA compression: 144D) 3D

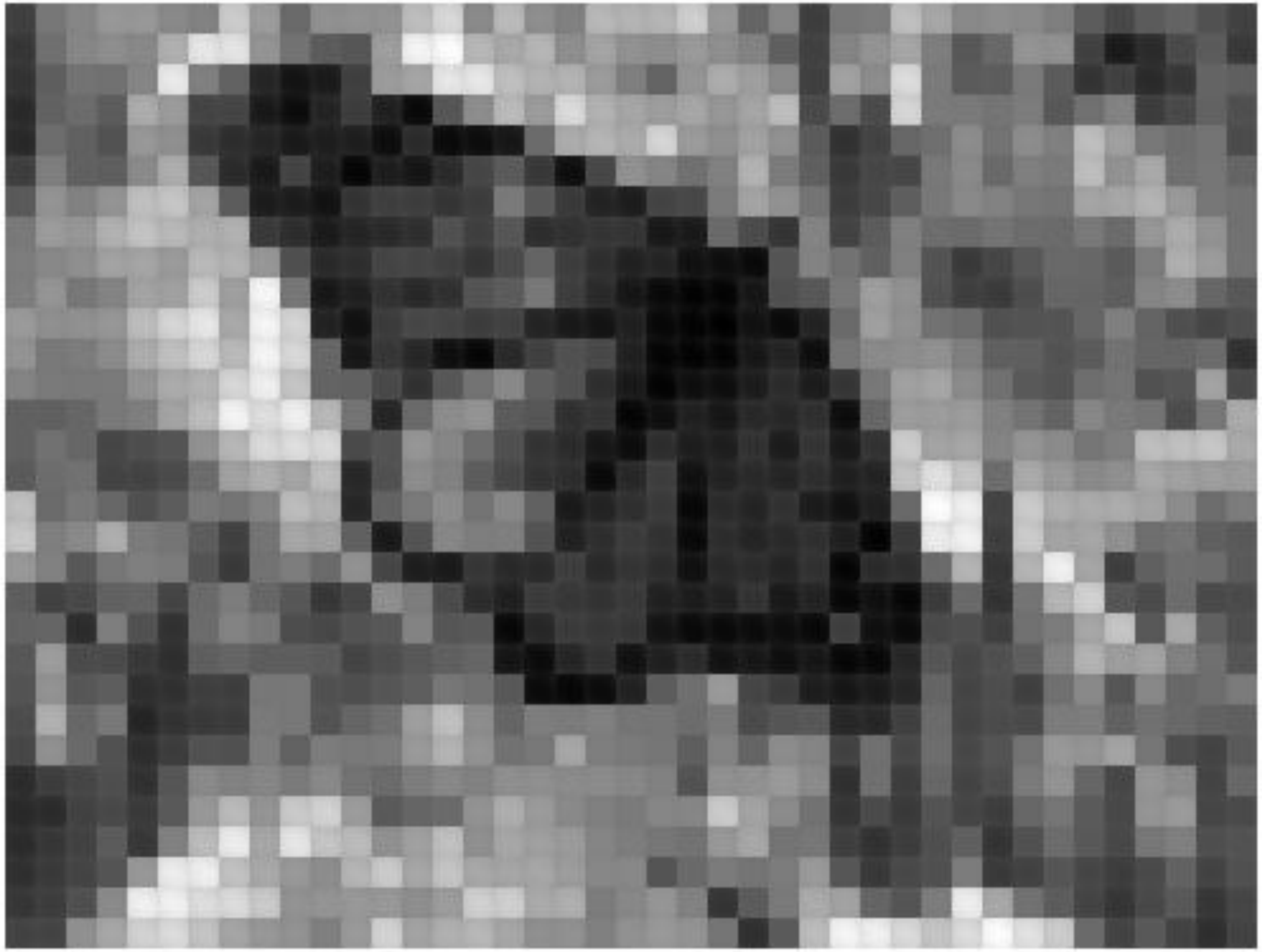
3 most important eigenvectors







PCA compression: 144D) 1D



What we have learned today

- Introduction to face recognition
- Principal Component Analysis (PCA)
- Image compression