Group Project on Financial Data

Initial Setup

```
knitr::opts chunk$set(warning = FALSE, message = FALSE)
library(tidyverse)
## -- Attaching core tidyverse packages ----- tidyverse 2.0.0 --
## v dplyr
               1.1.4
                          v readr
                                       2.1.4
## v forcats
               1.0.0
                                       1.5.1
                          v stringr
## v ggplot2
               3.4.4
                          v tibble
                                       3.2.1
## v lubridate 1.9.3
                          v tidyr
                                       1.3.0
## v purrr
               1.0.2
## -- Conflicts -----
                                                  ----- tidyverse_conflicts() --
## x dplyr::filter() masks stats::filter()
## x dplyr::lag()
                      masks stats::lag()
## i Use the conflicted package (<a href="http://conflicted.r-lib.org/">http://conflicted.r-lib.org/</a>) to force all conflicts to become error
library(readr)
library(quadprog)
set.seed(123)
```

Introduction

In this project, we work with financial time-series data, specifically, the prices of stocks in the S&P 500 index. Our goal in this project is to obtain an "optimal" portfolio of stocks, which we will do in three separate stages.

- 1. Dimension reduction on the stocks to obtain reduced factors/principal portfolios
- 2. Modelling of prices of factors using Gaussian Process Regression
- 3. Portfolio optimisation using Markowitz Mean-Variance Optimization Model

This full, 3 step weight estimation procedure is consolidated into the tested and documented "StockPriceMod" R package which we developed, which is available on github here. This notebook will also explain each of the three stages of the procedure on a sample of the dataset, before performing the full experiment on the year of 2020.

Dataset

We obtain a dataset of stock prices from the Standard and Poor's 500 (S&P 500) index through Yahoo Finance. The total number of dates we obtained is from 2020-01-02 to 2022-12-30, with the Open, High, Low, Close, Adjusted Close Prices (in US dollars), and the Volume and Ticker. We have a total of 499 unique Stock tickers, noting that althought the S&P 500 refers to the 500 largest companies, there may be different classes of shares/stocks issued and due to some data API issues we were not able to obtain the full stock lists.

```
# Load the data from our prepared CSV file
stock_prices <- read_csv("./data/sp500_stock_data.csv",show_col_types = FALSE)
head(stock_prices)</pre>
```

```
## # A tibble: 6 x 8
##
     Date
                                Low Close `Adj Close`
                                                        Volume Ticker
                  Open High
##
     <date>
                 <dbl> <dbl> <dbl> <dbl> <
                                                 <dbl>
                                                          <dbl> <chr>
## 1 2020-01-02
                        180.
                               177.
                                     180
                                                  152. 3601700 MMM
                  178.
## 2 2020-01-03
                  177.
                        179.
                               176.
                                     178.
                                                  151. 2466900 MMM
## 3 2020-01-06
                  177.
                        179.
                               176.
                                     179.
                                                  151. 1998000 MMM
## 4 2020-01-07
                  178.
                        179.
                               177.
                                     178.
                                                  150. 2173000 MMM
## 5 2020-01-08
                 178
                        182.
                               178.
                                     181.
                                                  153. 2758300 MMM
## 6 2020-01-09
                 182.
                        182.
                               180.
                                     181.
                                                  153. 2746300 MMM
tail(stock_prices)
## # A tibble: 6 x 8
##
                                Low Close `Adj Close`
     Date
                  Open High
                                                        Volume Ticker
                                                          <dbl> <chr>
##
                 <dbl> <dbl> <dbl> <dbl>
                                                 <dbl>
     <date>
## 1 2022-12-22
                  144.
                               142.
                                     145.
                                                  144. 1541800 ZTS
                        145.
## 2 2022-12-23
                  145.
                        146.
                               144.
                                     146.
                                                  144. 1017900 ZTS
## 3 2022-12-27
                  146.
                        146.
                               144.
                                     145.
                                                  144.
                                                        957900 ZTS
## 4 2022-12-28
                  145.
                        147.
                               144.
                                     144.
                                                  143. 1443900 ZTS
## 5 2022-12-29
                  145.
                        149.
                               145.
                                     148.
                                                  147. 1298900 ZTS
## 6 2022-12-30
                 147.
                        148.
                               145.
                                     147.
                                                  145. 1249500 ZTS
length(unique(stock_prices$Ticker)) # Number of unique stocks
```

[1] 499

For simplicity, we focus on the dates between 2020-01-02 to 2020-12-31, i.e., all of the days in the year 2020. In particular, due to the effects of the COVID pandemic during this time, we note that the stock prices at this time were highly unstable and exhibited high volatility, and so this time period may serve as an interesting testing ground for our method. We furthermore focus ourselves only on the end of day closing prices for the stocks

```
# Using all of 2020
start_date <- as.Date("2020-01-02")
end_date <- as.Date("2020-12-31")

# Filter for specific date range and select only Date, Close, and Ticker
stock_prices_filtered <- stock_prices %>%
    filter(Date >= start_date & Date <= end_date) %>%
    select(Date, Close, Ticker)

# Reshape data to a wide format
wide_data <- stock_prices_filtered %>%
    pivot_wider(names_from = Ticker, values_from = Close)

wide_data <- wide_data[, !apply(is.na(wide_data), 2, any)]
head(wide_data)</pre>
```

```
## # A tibble: 6 x 495
##
     Date
                          AOS
                                ABT
                                      ABBV
                                             ACN
                                                   ADBE
                                                           AMD
                                                                 AES
                                                                       AFL
                                                                                Α
                                                                                     APD
##
                       <dbl> <dbl> <dbl> <dbl> <
                                                  <dbl> <dbl> <dbl>
                                                                     <dbl> <dbl>
     <date>
                 <dbl>
                                                                                  <dbl>
                         47.8
                               86.9
                                      89.6
                                            210.
                                                   334.
                                                         49.1
                                                                20.0
                                                                      53.3
## 1 2020-01-02
                  180
                                                                             85.9
                                                                                    231.
## 2 2020-01-03
                  178.
                         47.3
                               85.9
                                      88.7
                                            210.
                                                   332.
                                                         48.6
                                                                19.8
                                                                      53.0
                                                                             84.6
                                                                                    226
## 3 2020-01-06
                  179.
                         47.7
                               86.3
                                      89.4
                                            208.
                                                   334.
                                                         48.4
                                                                20.0
                                                                      52.8
                                                                             84.8
                                                                                    226.
## 4 2020-01-07
                  178.
                         47.3
                               85.9
                                      88.9
                                            204.
                                                   333.
                                                         48.2
                                                                20.1
                                                                      52.3
                                                                             85.1
                                                                                    227.
## 5 2020-01-08
                  181.
                         47.3
                               86.2
                                      89.5
                                            204.
                                                   338.
                                                         47.8
                                                                20.1
                                                                      52.5
                                                                             85.9
                                                                                    228.
                                            206.
## 6 2020-01-09
                  181.
                         47.0
                               86.4
                                     90.2
                                                   340.
                                                         49.0
                                                                20.3
                                                                      52.5
                                                                             87.3
```

```
## # i 483 more variables: AKAM <dbl>, ALB <dbl>, ARE <dbl>, ALGN <dbl>,
## # ALLE <dbl>, LNT <dbl>, ALL <dbl>, GOOGL <dbl>, GOOG <dbl>, MO <dbl>,
## # AMZN <dbl>, AMCR <dbl>, AEE <dbl>, AAL <dbl>, AEP <dbl>, AXP <dbl>,
## # AIG <dbl>, AMT <dbl>, AWK <dbl>, AMP <dbl>, AME <dbl>, AMGN <dbl>,
## # APH <dbl>, ADI <dbl>, ANSS <dbl>, AON <dbl>, APA <dbl>, APA <dbl>, AAPL <dbl>,
## # AMAT <dbl>, APTV <dbl>, ACGL <dbl>, ADM <dbl>, ANET <dbl>, AJG <dbl>,
## # AIZ <dbl>, T <dbl>, ATO <dbl>, ADSK <dbl>, ADP <dbl>, AZO <dbl>, AZO <dbl>, ...
nrow(wide_data)
```

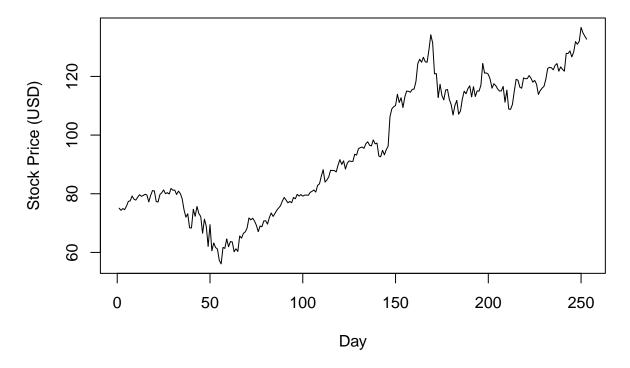
[1] 253

To help in our PCA and further analysis we pivot our data to a "wide" format, i.e., where the rows are the time series days and the columns are the individual stocks. In total, we have an $\mathbb{R}^{253\times495}$ data matrix, where 253 are the number of trading days in the year 2020 and 495 are the number of stocks.

We can quickly look at the stock prices for Apple (ticker \$AAPL).

```
plot.ts(wide_data$AAPL, ylab = "Stock Price (USD)", xlab= "Day")
title("Stock Prices for Apple in the year 2020")
```

Stock Prices for Apple in the year 2020



Data Preprocessing

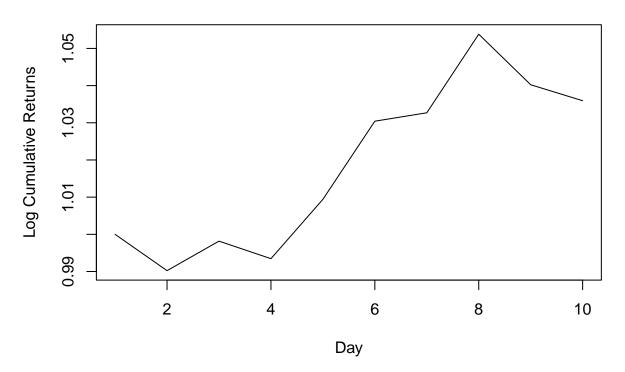
https://gregorygundersen.com/blog/2022/02/06/log-returns/

As opposed to raw prices, it is usually standard in financial analysis to transform to the **return** of an asset (our stock), which we define as being $r_t = \frac{P_t - P_{t-1}}{P_{t-1}} = \frac{P_t}{P_{t-1}} - 1$, where the stock has price P_t at time t and price P_{t-1} at time t-1. In order to obtain unbounded support of our returns, we further transform to the **log-returns**, which is defined as $z_t = \log(1 + r_t) = \log(\frac{p_t}{p_{t-1}})$. Finally, in our case, due to the use of the SSM modelling, we will use the **cumulative log-returns** instead, which we define as:

$$r_{0:t} = 1 + \log \frac{p_t}{p_0} = 1 + \log p_t - \log p_0$$

```
#Looking at first 10 days of the year
start_day <- 1
end day <- 10
# Calculate logarithmic returns
log_cu_returns <- wide_data[start_day:end_day, ] %>%
  mutate(across(-Date, ~log(. / .[1]) + 1)) %>%
  select(-Date)
head(log_cu_returns)
## # A tibble: 6 x 494
      MMM
             AOS
                   ABT
                        ABBV
                               ACN
                                   ADBE
                                           AMD
                                                 AES
                                                       AFL
                                                                Α
                                                                   APD AKAM
     <dbl> <
           1
                 1
                       1
                             1
                                   1
                                         1
                                               1
                                                     1
                                                           1
                                                                  1
                                                                        1
## 2 0.991 0.991 0.988 0.990 0.998 0.992 0.990 0.988 0.993 0.984 0.978 0.995 0.986
## 3 0.992 0.997 0.993 0.998 0.992 0.998 0.985 1
                                                     0.990 0.987 0.977 0.999 0.984
## 4 0.988 0.991 0.987 0.993 0.970 0.997 0.983 1.00  0.981 0.990 0.981 1.03  0.998
## 5 1.00 0.989 0.991 1.00 0.972 1.01 0.974 1.00 0.984 1.00 0.987 1.04 0.980
## 6 1.01 0.985 0.994 1.01 0.981 1.02 0.997 1.01 0.984 1.02 1.01 1.06 1.00
## # i 481 more variables: ARE <dbl>, ALGN <dbl>, ALLE <dbl>, LNT <dbl>,
       ALL <dbl>, GOOGL <dbl>, GOOG <dbl>, MO <dbl>, AMZN <dbl>, AMCR <dbl>,
## #
      AEE <dbl>, AAL <dbl>, AEP <dbl>, AXP <dbl>, AIG <dbl>, AMT <dbl>,
      AWK <dbl>, AMP <dbl>, AME <dbl>, AMGN <dbl>, APH <dbl>, ADI <dbl>,
      ANSS <dbl>, AON <dbl>, APA <dbl>, AAPL <dbl>, AMAT <dbl>, APTV <dbl>,
## #
## #
       ACGL <dbl>, ADM <dbl>, ANET <dbl>, AJG <dbl>, AIZ <dbl>, T <dbl>,
      ATO <dbl>, ADSK <dbl>, ADP <dbl>, AZO <dbl>, AVB <dbl>, AVY <dbl>, ...
plot.ts(log cu returns $AAPL, ylab = "Log Cumulative Returns", xlab= "Day")
title("Log Cumulative Returns for Apple in the year 2020, first 10 days")
```

Log Cumulative Returns for Apple in the year 2020, first 10 days



As we can see, the log cumulative returns are significantly more stable and easier to compare across stocks as we use the same starting base for each time window.

Principal Components Analysis

ACN

-0.014926790

ADBE -0.031836784

0.01678336

0.02714022

We will use Principal Component Analysis(PCA) as a dimension reduction technique. Specifically, our goal here is to find some linear dependencies between our stocks, known as **principal components**, which are orthogonal and maximise the variance. We will use the **prcomp** function from the base R stats package. We note that we also need to do centering, and optionally normalisation of the variables could be performed. We find that normalisation helps in the numerical stability of our procedure, so we perform that here, done automatically through the scale and center arguments in prcomp.

As we are in the n < p setting, the number of PCs which we obtain is limited to n = 10. We perform PCA and show some of the components of the 10 PCs.

```
# Perform PCA on logarithmic returns
pca_result <- prcomp(log_cu_returns, scale = TRUE, center= TRUE)</pre>
head(pca_result$rotation)
                                            PC3
##
                 PC1
                              PC2
                                                         PC4
                                                                       PC5
## MMM
         0.040509000
                      0.006555956
                                    0.088461385 -0.006869578
                                                              0.009402407
  AOS
        -0.010308726
                      0.005317343
                                    0.049042489
                                                 0.092321096 -0.107836376
##
## ABT
        -0.004450003
                      0.048321850 -0.030177022 -0.088475842
                                                              0.141678441
  ABBV -0.017639096 -0.013556547
                                   0.040804255 -0.038729233
                                                              0.176263971
##
  ACN
        -0.002574362
                      0.077825551
                                   0.009647837
                                                 0.116963859 -0.017359079
  ADBE
        0.061006932
                      0.001891109
                                   0.033103945
                                                 0.002229658 -0.019466467
##
##
                 PC6
                             PC7
                                           PC8
                                                         PC9
        -0.007785326
## MMM
                      0.06584364
                                  0.082718141 -0.0349762301
                                                              0.063060364
## AOS
        -0.078708715
                      0.05027738 -0.055230154
                                                0.1238552029 -0.006809753
                                                              0.013664600
         0.001763373 -0.01379931
                                  0.017620738
                                                0.0433633798
## ABT
  ABBV -0.033201889 -0.01669778
                                   0.013114268
                                                0.0490620670 -0.083451076
```

As we are interested in both reducing the number of PCs (dimension reduction) and maximising the amount of information, we face a tradeoff in determining the number of PCs to use. We show both the cumulative variance for the 11 PCs and a Scree Plot showing the proportion of variance explained for each PC. For a simple rule of thumb, we choose the number of PCs using the cutoff of 90% of cumulative variance explained, which in this case is the first 5 PCs.

0.0100194337

0.014650997 -0.0005192823 -0.063119661

0.013987827

0.005882421

```
explained_variance <- pca_result$sdev^2 / sum(pca_result$sdev^2)
cumulative_variance <- cumsum(explained_variance)
print(cumulative_variance)

## [1] 0.4880975 0.6749094 0.7958053 0.8577381 0.9045194 0.9381545 0.9629781
## [8] 0.9820628 1.0000000 1.0000000

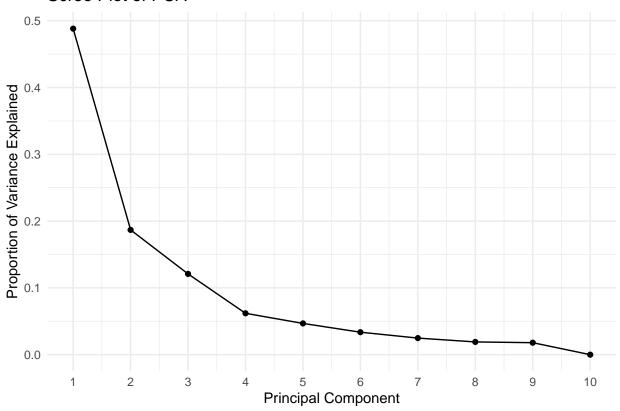
# Find the number of components that explain at least the threshold variance
num_components <- which(cumulative_variance >= 0.9)[1]
print(num_components)

## [1] 5
# Return the principal components
y <- pca_result$x[, 1:num_components]</pre>
```

```
pca_df <- tibble(
    Principal_Component = seq_along(explained_variance),
    Variance_Explained = explained_variance
)

ggplot(pca_df, aes(x = Principal_Component, y = Variance_Explained)) +
    geom_line() +
    geom_point() +
    scale_x_continuous(breaks = seq_along(explained_variance)) +
    theme_minimal() +
    labs(title = "Scree Plot of PCA", x = "Principal Component", y = "Proportion of Variance Explained")</pre>
```

Scree Plot of PCA



As we can see from the Scree Plot, there is a quick dropoff in the proportion of variance explained past the first few PCs, implying that we do not need much PCs to retain the information in the dataset.

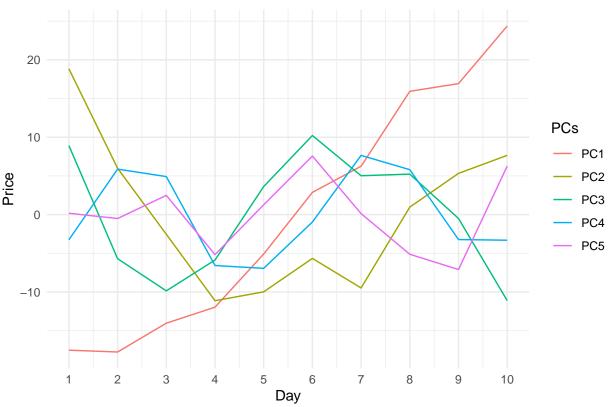
We further plot a time series of the first 5 PCs for the 10 days.

```
y_df <- data.frame(
  Date = 1:10, y
)

long_data <- y_df %>%
  pivot_longer(
  cols = -Date, # -Date means to keep the Date column as is
  names_to = "PCs", # This will be the new column for stock names
  values_to = "Price" # This will be the new column for stock prices
)
```

```
ggplot(long_data, aes(x = Date, y = Price, color = PCs)) +
  geom_line() +
  scale_x_continuous(breaks = 1:max(long_data$Date)) +
  theme_minimal() +
  labs(title = "PC Prices Over Time", x = "Day", y = "Price")
```

PC Prices Over Time



SSM and GPR Modelling In this section we will try to predict the log returns of each PCs using a State Space Model, or equivalently, the Gaussian Process Regression. Each PC's behaviour is viewed as an independent time series (due to the orthogonality of the PCs), and we will fit each time series with the simple linear state-space model:

$$X_t = \phi X_{t-1} + V$$
$$Y_t = X_t + W$$

where $W \sim \mathcal{N}(0, \sigma_w^2)$ and $V \sim \mathcal{N}(0, \sigma_v^2)$.

Kalman Filter

We note that using the Kalman Filter with initial distribution $\mathcal{N}(0,1)$ is equivalent to a Gaussian process regression with the following relation: $\phi = \exp(-\frac{1}{\gamma})$ and $\sigma_v^2 = 1 - \exp(-\frac{2}{\gamma})$.

The Kalman function takes inputs ϕ , σ_v^2 , σ_w^2 , m_0 , σ_0^2 and the number of prediction days n, it returns a the predicted mean, predicted variance, updated mean and updated variance. It is a general Kalman Filter function and will be then implemented using our relation to GPR.

```
kalman <- function(y,phi,Sigmav,Sigmaw,m0,Sigma0,n=2){
T <- length(y)</pre>
```

```
#initialization
mu.p \leftarrow rep(NA, T+n)
Sigma.p <- rep(NA,T+n)</pre>
mu.f <- rep(NA,T)</pre>
Sigma.f <- rep(NA,T)</pre>
#forward recursion time1
mu.p[1] \leftarrow m0
Sigma.p[1] <- Sigma0</pre>
mu.f[1] \leftarrow m0 + (y[1]-m0)*(Sigma0/(Sigma0+Sigmaw))
Sigma.f[1] <- Sigma0-(Sigma0^2/(Sigma0+Sigmaw))</pre>
#forward recursion time 2:T
for (t in 2:T){
  #prediction
  mu.p[t] \leftarrow phi*mu.f[t-1]
  Sigma.p[t] <- phi^2 * Sigma.f[t-1] + Sigmaw
  #update
  deno <- Sigmaw + Sigma.p[t]</pre>
  mu.f[t] <- Sigmaw*mu.p[t]/deno + Sigma.p[t]*y[t]/deno</pre>
  Sigma.f[t] <- Sigmaw*Sigma.p[t]/deno</pre>
#predict for T+1:T+n
for (t in (T+1):(T+n)){
  if (t == T+1){
    mu.p[t] \leftarrow phi*mu.f[t-1]
    Sigma.p[t] <- phi^2 * Sigma.f[t-1] + Sigmaw
  }
  else{
    mu.p[t] \leftarrow phi*mu.p[t-1]
    Sigma.p[t] <- phi^2 * Sigma.p[t-1] + Sigmaw
  }
}
return (list(mu.f=mu.f,Sigma.f=Sigma.f,mu.p=mu.p,Sigma.p=Sigma.p))
```

We then implement it with GPR:

```
likelihood <- rep(NA,T)

#at time 1
likelihood[1] <- log(dnorm(y[1],mean=m0,sd = sqrt(Sigma0 + Sigmaw)))

#time 2:T
for (t in 2:T){
   likelihood[t] <- log(dnorm(y[t],mean=mu.p[t],sd=sqrt(Sigmaw+Sigma.p[t])))
}
return (sum(likelihood))
}</pre>
```

There are two functions here:

- kf.gp simply applies the Kalman Filter on observed y, with σ_v^2 and ϕ as functions of hyper parameter γ , computing the predictive and updated distributions.
- kf.loglikelihood computes the log-likelihood of the observed y in a iteration manner by noting

$$\log(p(y_{1:T})) = p(y_1) + \sum_{t=2}^{T} p(y_t|y_{1:t-1})$$

and

$$p(y_t|y_{1:t-1}) = \mathcal{N}(y_t; m_{t|t-1}, \sigma_{t|t-1}^2 + \sigma_w^2)$$

Optimization to get hyperparameters' MLE

In real life we never observe the hyper parameters γ and σ_w^2 . We propose using optim to optimize against σ_w^2 and γ with the log-likelihood computed using kf.loglikelihood.

We plot here the SSM/GPR modelling for the log-returns for the first 30 days of Apple stock, along with the 99% upper and lower confidence intervals.

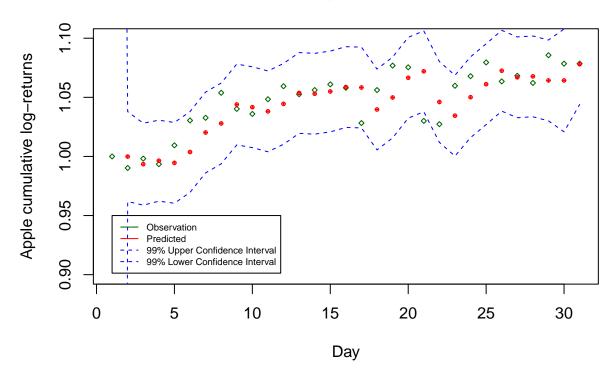
```
log_returns_monthly <- wide_data[1:30, ] %>%
  mutate(across(-Date, ~log(. / .[1]) + 1)) %>%
  select(-Date)

aapl_log_ret <- log_returns_monthly$AAPL
  optim_hyperparam = optim_parm(aapl_log_ret)
  results = kf.gp(aapl_log_ret,gamma=optim_hyperparam$gamma,Sigmaw = optim_hyperparam$Sigmaw,n=0)
  mu.p <- results$mu.p
  Sigma.p <- results$Sigma.p</pre>
```

```
se.p <- sqrt(Sigma.p)

alpha=0.01
cv99 = qnorm(1-alpha/2)
CIupper.p <- mu.p + cv99*se.p
CIlower.p <- mu.p - cv99*se.p
time = 1:(length(aapl_log_ret)+1)
aapl_log_ret <- c(aapl_log_ret,aapl_log_ret[length(aapl_log_ret)])
plot(time,aapl_log_ret,cex=0.5,col='darkgreen',pch=5,ylim=c(.9,1.1),main='SSM model with Apple log-reture
points(time,mu.p,cex=0.5,col='red',pch=10)
points(time,CIupper.p,col='blue',type ='l',lty=2,lwd=1)
points(time,CIlower.p,col='blue',type ='l',lty=2,lwd=1)
legend(1,.95,legend= c('Observation','Predicted','99% Upper Confidence Interval','99% Lower Confidence</pre>
```

SSM model with Apple log-return, first 30 days of 2020



Portfolio Optimisation

 $https://sites.math.washington.edu/{\sim}burke/crs/408/fin-proj/mark1.pdf$

The well-known Markowitz's Mean-Variance Optimisation is the basis of our Portfolio strategy. In this setting, the return on assets are modelled as random variables, and the goal is to choose a portfolio of weighting factors through an optimality criterion. Specifically, we have n stocks which we weight in our portfolio with a set of weighting factors $\{w_i\}_{i=1}^p$. The idea then is to maximize the expected return and to minimize the volatility at the same time. Mathematically speaking, we aim to maximize

$$\mathbb{E}[R] = \sum_{i} w_i \mathbb{E}[R_i]$$

subject to minimize

$$\sigma^2 = \sum_{i,j} w_i w_j \sigma_i \sigma_j \rho_{ij}$$

where $\{R_i\}_i$ is the percentage return on the underlying assets; $\{w_i\}_i$ is the respective proportion that sum to 1; $\{\sigma_i\}$ is the standard deviation of the return on the *i*th underlying asset and ρ_{ij} is the correlation between *i*th return and *j*th return.

We will use the same 5 PCs that we obtained previously in PCA, and over each PC, model with our SSM individually. We predict for the next day and obtain the mean and variance of the prediction.

```
# Initialize an empty dataframe to store the time series data
time_series_df <- data.frame(matrix(ncol = 2, nrow = num_components))</pre>
rownames(time series df) <- paste("PC", 1:num components, sep = "")
colnames(time_series_df)[1] <- "mu"</pre>
colnames(time_series_df)[2] <- "sigma2"</pre>
for (m in 1:num_components) {
  \# Obtain optimized hyperparameters for the m-th component
  optim_hyperparam <- optim_parm(y[, m])</pre>
  # Run the Gaussian Process with Kalman filter
  gp_result \leftarrow kf.gp(y[, m], optim_hyperparam gamma, optim_hyperparam Sigmaw, n = 0)
  # Add the time series to the dataframe
  time_series_df[m, ] <- c(tail(gp_result$mu.p, 1) , tail(gp_result$Sigma.p, 1))
}
time_series_df
##
                         sigma2
                  mu
```

```
## PC1 1.869980e+01 100.92437
## PC2 5.605874e+00 80.23354
## PC3 -6.203526e+00 43.89531
## PC4 -4.155165e-01 14.05056
## PC5 8.510026e-07 10.48528
```

We note here that we readjust the predicted cumulative log-returns $(r_{0:T+1} = 1 + \log \frac{P_{T+1}}{P_0} = 1 + \log(P_{T+1}) - \log(P_0))$ back to the log-returns as follows:

$$z_{T+1} = \log \frac{P_{T+1}}{P_T} = \log(P_{T+1}) - \log(P_T) - \log(P_0) + \log(P_0) = r_{0:T+1} - r_{0:T}$$

As we use the predicted daily log returns z_{T+1} for our portfolio optimisation.

```
pred_returns <- time_series_df$mu - tail(y,1) #z_T+1
covar_mat <- diag(time_series_df[[2]])
Amat <- matrix(0, num_components, num_components)
Amat[,1] <- 1
bvec <- rep(0,num_components)
bvec[1] <- 1
lambda <- 1

QP_result <- solve.QP(2*covar_mat, lambda * pred_returns, matrix(1, nrow=num_components,ncol=1), c(1), sqp_result$solution</pre>
```

[1] 0.02716953 0.05674442 0.18293053 0.50007058 0.23308494

Experiments

We utilise our developed package, "StockPriceMod" which is available on github here to perform our portfolio optimisation over the year 2020. This package can be install with the command devtools::install_github('Shermjj/StockPriceMod').

```
rm(list = ls()) #First clear the environment of the previously defined functions due to conflicts
library(StockPriceMod)
library(tidyverse)
library(readr)
library(quadprog)
set.seed(123)
```