PUT AN OBJECTIVE SPIN ON SUBJECTIVE CHOICES, MAKE USE OF A MULTIPLE-CRITERIA DECISION ANALYSIS METHOD IN EVERYDAY LIFE

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Let's turn an easy decision into a complicated one (just because we can). Most of the time we make decisions without even putting the criteria on paper because we simply don't have to. But obviously when make an expensive purchase we take time to research our options. Chances are that the more we research the more we will get confused by the overwhelming parameters to consider. Consider the case of purchasing a full frame Digital Single Lens Reflex camera (sensor size: $36mm \times 24mm$). Once you have been fixated on purchasing a full frame DSLR you immediately take a survey of the available options and would probably come up with a list of Canon, Nikon, Sony, Pentax cameras. Well, this just is the beginning of the complexity. Most cameras are very similar in specifications but not exactly the same. What option may be useful to me may not be important for the other buyer. Manufacturers thus consider an ideal buyer and optimize the design. You and I may be on a unique position of the spectrum of buyers. Suppose, for some reason you have to or you like to do a lot of photography in indoor conditions and you value high ISO performance. Another buyer may not find that as a crucial factor because (s)he probably has a nice flash or planning to do flash photography. The point is our priorities can be different and we want to make the best decision based on our needs. However, lets come back to the problem (or happiness) of purchasing a full frame DSLR. After a quick survey on the internet I could compile the following table that includes specifications of four full frame DSLRs. Well I am neither a Nikon nor a Canon fan boy (or am I?). I just wanted to keep the analysis simple and to see how simple we can actually keep the decision making. So, for each camera we will consider 7 parameters/criteria as follows:

- Price. Ahh! the foremost parameter to consider.
- MP. Sensor resolution in terms of Mega Pixels (MP). The higher the resolution the better although, there might be exceptions. On a fixed sensor-size increased mega pixels mean each individual pixel will be smaller in size compared to a low resolution counterpart. This has some ramifications but that is not our concern today.
- FPS. Frames per second (FPS) i.e. how many shots can be taken in a second. For sports photography this can be very important.
- ISO. This measures the sensitivity of the sensor. High ISO usually measures the possibility of being successful in taking pictures in low lights.

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- Video. The ability of the camera in recording videos and the quality of the maximum supported resolution of the video.
- Lens-Ecosystem. This is probably one of the most important criterion to consider. A high end camera body will never give the expected result unless paired with high end lenses. Nikon, Canon both brands offer an impressive array of lenses. Other brands usually offer limited choices in this section.
- Connectivity/ Tethering. The availability of options to connect the camera to smart phones and controlling the camera from smart phones. Nikon offers Wireless Mobile Utility(wmu) app in this regard and the rival Canon offers EOS Remote app.

	Price	MP	FPS	ISO	Video	Lens-Ecosystem	Connectivity/Tethering
Nikon D610	1500.0	24.3	6	$100 - 6400$	$1080p$	*very good*	$wifi + wmu$
Canon 6D ii	1700.0	26.2	6.5	$100 - 40,000$	$1080p$	*best*	$wifi + eos\ remote$
Nikon D850	3296.0	45.7	7	$64 - 100.2k$	$4k$	*very good*	$wifi + wmu$
Canon 5D iv	3200.0	30.4	7	$100 - 32k$	$4k$	*best*	$wifi + eos\ remote$

Now let's see how we decide in general. Lets compare the price of four listed cameras, {Nikond610: 1500, Canon6D-ii:1700, Nikon D850: 3296, Canon5D-iv: 3199}. If our camera budget is say \$(1500-1700) we will immediately dismiss NikonD850 and Canon5D. What we have done implicitly without much thinking is mostly likely we have done binary classification in our head, something like *if price is not near the budget drop it from the list of options. In fact, we have calculated the distance of the camera price from our target price and then made a yes/no decision.* If you are not happy with this simple yes/no procedure you may want to arrive at the same decision with little bit more math. Like, you can calculate the distance between the item price and your budget e.g. $|1600 - 1700|$, assuming a budget=1600 and item price 1700(Canon 6D ii). We go further and normalize the distance to calculate weight of our choices and obviously the smaller the weight the better the item will fare in affordability. Lets, calculate the normalized distance of all cameras from your budget of 1600. {'nikond610':$|1600 - 1500|/1600$, 'canon6D': $|1600-1700|/1600$, 'nikonD850':$|1600-3296|/1600$, 'canon5d':$|1600-3200|/1600$ } $\equiv$ {'nikond610': 0.0625, 'canon6D': 0.0625, 'nikonD850': 1.06, 'canon5d': 1}. So, we see that there is a tie between NikonD610 and Canon6D-ii however, Nikond610 has a lower price but that did not reflect in our math. So, the conclusion is that this math formulation is not so elegant . Okay lets not torture ourselves to design an elegant mathematical system because there there already are better solutions. Operations-research-Gurus have already devised couple of methods to address such problems. Analytical Hierarchy Process (AHP), Analytic network process (ANP), Best worst method (BWM) to name a few. Analytical Hierarchy Process(AHP) seems to be the most popular but can be a little cumbersome. The latest method that is easier but can perform as good as AHP or better is the Best worst method proposed by Dr. Jafar Rezaei (Rezaei, 2015) of Delft University of Technology. We will see how to apply the Best worst method to make a decision. The target of the Best-worst (BW) method is to find normalize weights for options/criterion based on subjective score we will assign. We will weigh our option twice. First we will compare all options based on our perceived best option. We will assign a score of '1' to the best option and will assign progressively higher scores to less favourable options. So, if we consider 'price' as our criterion we have four options to choose from i.e. Nikon D610, Canon 6D-ii, Nikon D850, and Canon 5D-iv. If we fix our budget at \$1600 obviously Nikon D610 will be the best choice. It has lower price than our target price of 1600. So, we will assign a score of '1' to Nikon D610 on a scale of $1 - 9$ (could also choose another scale with different range). Our next favourable choice will be obviously Canon 6D-ii. Lets put a score 3 on 6D-ii. The least favourable will be Nikon D850 because it has the highest price and it will get a score 9. How about Canon 5D iv? if Nikon D850 gets 9, Canon 5D can get 8 because it has slightly lower price. Remember, **When comparing against the best option: the higher is the practicality/suitability/goodness/usefulness the lower will be the assigned score and a score of '1' will be assigned to the most favourable option**. Here is a snapshot of the all assigned scores when compared to our best option Nikon D610:

Criterion: Price	Nikon D610	Canon 6D ii	Nikon D850	Canon 5D iv
Score against the best(Nikon D610)	1	3	9	8

We move on to the next step. To asses the rationality of assigned scores we will again compare all options against the worst option. The procedure is similar as before. We will assign score while comparing against the worst option. **The worst option will receive a score of '1' and better options will progressively receive higher scores while comparing against the worst option**. So, the Nikon D850 having the highest price will receive a score of

'1'. The best option Nikon D610 will receive a score of 9. Let's assign a score 2 on Canon 5D-iv and 8 on Canon 6D-ii. Summary:

Criterion: Price	Nikon D610	Canon 6D ii	Nikon D850	Canon 5D iv
Score against the worst(Nikon D850)	9	8	1	2

Wait, didn't we assign lowest score to the best options and lower scores to better options when comparing against the best option before? Things are getting confusing here. True, things are little confusing now but if we understand how BWM works confusions will sublimate. In BWM the target is to get normalized weights (w_i) for all the options on a scale of 0-1 such that the summation of weights of all options equal to 1 ($\sum w_i = 1$). So, we will calculate weights, $w_{\text{Nikon D610}}$, $w_{\text{Canon 6D}}$, $w_{\text{Nikon D850}}$, and $w_{\text{Canon 5D}}$. Weights will be calculated in a way such that the distances $|\frac{w_b}{w_i} - \text{score}_i|$ will be minimized. Here, $w_b = w_{\text{Nikon D610}}$. So, the following distances will be minimized while comapring against the best option:

Criterion: Price	Nikon D610	Canon 6D ii	Nikon D850	Canon 5D iv
Score against the best(Nikon D610)	1	3	9	8
Distance to minimize	0	$\lvert\frac{W_{\text{Nikon D610}}}{W_{\text{Canon 6D}}} - 3\rvert$	$\lvert\frac{W_{\text{Nikon D610}}}{W_{\text{Nikon D850}}} - 9\rvert$	$\lvert\frac{W_{\text{Nikon D610}}}{W_{\text{Canon 5D}}} - 8\rvert$

Since the best option will have the highest weight and other better options will get progressively lower weights with the worst option having the lowest weight, it only makes sense to assign lowest score to the best option when thinking about the minimization of weights. Actually in BWM two sets of distances are minimized. $(\frac{w_i}{w_w} - \text{score}_i)$ will be also minimized when comparing against the worst option. Here, the weight of the worst option $w_w = w_{\text{Nikon D850}}$. In a snapshot:

Criterion: Price	Nikon D610	Canon 6D ii	Nikon D850	Canon 5D iv
Score against the worst(Nikon D850)	9	8	1	2
Distance to minimize	$\lvert\frac{W_{\text{Nikon D610}}}{W_{\text{Nikon D850}}} - 9\rvert$	$\lvert\frac{W_{\text{Canon 6D}}}{W_{\text{Nikon D850}}} - 8\rvert$	0	$\lvert\frac{W_{\text{Canon 5D}}}{W_{\text{Nikon D850}}} - 2\rvert$

So, the scoring system makes sense when we think about the rations of weights and distances that needs to be minimized. Ultimately BWM boils down to minimizing the maximum distance between weight ratios and score. So, what are weights that can be calculated? Once, the mathematical minimization problem is solved we will end up with the weights for options based on price criterion, $W_{\text{Nikon D610}} = 0.603$, $W_{\text{Canon 6D}} = 0.251$, $W_{\text{Nikon D850}} = 0.05$, and $W_{\text{Canon 5D}} = 0.094$.

Criterion: Price	Nikon D610	Canon 6D ii	Nikon D850	Canon 5D iv
Final weight of options	0.603	0.251	0.050	0.094

Hmm, the result makes sense. The maximum weight (0.603) is attributed to Nikon D610 because it the most affordable according to our budget (would save us $100) and the worst option has been determined as Nikon D850 because it is the farthest from our target price. All the weight calculations and the solution of this problem were carried out by the python scripts I wrote. These scripts are hosted in my github page: https://github.com/ahsan2github/bw_method. You are welcome to download and use. Okay, now lets think about another criterion e.g. Lens Ecosystem. Both Nikon and Canon has an excellent array of lenses to choose from. From reviews available I have come to learn that the canon 70-200mm is an excellent portrait lens that does not have the focus breathing problem like the Nikon counterpart. Anyway, since there are only to values to consider in this case aka "very good" and "best" we can forgo the weight calculation and just impose values, $W_{\text{Nikon D610}} = 0.2$, $W_{\text{Canon 6D}} = 0.3$, $W_{\text{Nikon D850}} = 0.2$, and $W_{\text{Canon 5D}} = 0.3$. We can also run the BWM algorithm and find the weights. Lets try and see what happens.

Criterion: Lens Ecosystem	Nikon D610	Canon 6D ii	Nikon D850	Canon 5D iv
Score against the best (Canon)	3	1	3	1

Criterion: Lens Ecosystem	Nikon D610	Canon 6D ii	Nikon D850	Canon 5D iv
Score against the worst (Nikon)	1	3	1	3

These scoring results into the following weigh values:

Criterion: Lens Ecosystem	Nikon D610	Canon 6D ii	Nikon D850	Canon 5D iv
Final weight of options	0.125	0.375	0.125	0.375

So, the current scoring in this case favours Canon quite a bit. This is case where we can take in practice a heuristic approach, I think. We can either put arbitrary weights $W_{\text{Nikon D610}} = 0.2$, $W_{\text{Canon 6D}} = 0.3$, $W_{\text{Nikon D850}} = 0.2$, and $W_{\text{Canon 5D}} = 0.3$ (remember, $\sum w_i = 1$) or calculated weights. Now lets think about the sensor resolution aka Mega Pixels (MP). This is a more interesting case and BWM will be of real help to determine weights. Nikon D850 is the king here boasting a handsome 45.7 MP counts. I would score the cameras like the following:

Criterion: MegaPixels (MP)	Nikon D610	Canon 6D ii	Nikon D850	Canon 5D iv
Score against the best(Nikon D850)	8	7	1	3

Criterion: MegaPixels (MP)	Nikon D610	Canon 6D ii	Nikon D850	Canon 5D iv
Score against the worst(Nikon D610)	1	3	9	8

The resulting weights appear to be the following:

Criterion: MegaPixel(MP)	Nikon D610	Canon 6D ii	Nikon D850	Canon 5D iv
Final weight of options	0.052	0.107	0.587	0.251

We can carry on like this and calculate weights of each option entries against each criterion and finally get a table of wights like the following:

	Price	MP	FPS	ISO	Video	Lens-Ecosystem	Connectivity/Tethering
Nikon D610	0.603	0.052	0.042	0.048	0.057	0.125	0.125
Canon 6D ii	0.251	0.107	0.071	0.121	0.089	0.375	0.375
Nikon D850	0.050	0.587	0.442	0.585	0.541	0.125	0.125
Canon 5D iv	0.094	0.251	0.442	0.243	0.312	0.375	0.375

Okay, now that we have all the weights of all options against all criteria which camera should we buy? how should we combine all the information to arrive at the final conclusion? We have one more step left. We need to rank the criteria according to importance to us. In other words, we also need to determine the weights of the criteria. To me price is the most important criterion followed by Mega Pixel/Lens-Ecosystem, ISO, Connectivity, FPS & Video and I would put scores on the criteria somewhat like the following:

	Price	MP	FPS	ISO	video	Lens-Ecosystem	Coonectivity
Score against the best (Price)	1	2	7	4	8	2	5

	Price	MP	FPS	ISO	video	Lens-Ecosystem	Coonectivity
Score against the worst (Video)	9	5	8	3	1	4	5

The determined weights are shown below:

	Price	MP	FPS	ISO	video	Lens-Ecosystem	Coonectivity
Final weights of criteria	0.302	0.211	0.060	0.105	0.022	0.211	0.085

Now we can calculate the final rank for each options. We will multiply option weight against the criteria weight and calculate the summation. An example calculation is shown for Canon 5D-ii:

	Price	MP	FPS	ISO	video	Lens-Ecosystem	Coonectivity
Final weights of criteria	0.302	0.211	0.060	0.105	0.022	0.211	0.085
	×	×	×	×	×	×	×
Weights of option Canon 5D-ii	0.094	0.251	0.442	0.243	0.312	0.375	0.375
Result of multiplication:	0.028	0.052	0.026	0.025	0.006	0.079	0.031
Total (sum):					0.251		

Obviously, we can multiply the option weight matrix with the criteria weight vector and get the final ranks easily.

$$\left(\begin{array}{cccccccc} & \text{Price} & \text{MP} & \text{FPS} & \text{ISO} & \text{Video} & \text{Lens-Ecosystem} & \text{Connectivity/Tethering} \\ \text{Nikon D610} & 0.603 & 0.052 & 0.042 & 0.048 & 0.057 & 0.125 & 0.125 \\ \text{Canon 6D ii} & 0.251 & 0.107 & 0.071 & 0.121 & 0.089 & 0.375 & 0.375 \\ \text{Nikon D850} & 0.050 & 0.587 & 0.442 & 0.585 & 0.541 & 0.125 & 0.125 \\ \text{Canon 5D iv} & 0.094 & 0.251 & 0.442 & 0.243 & 0.312 & 0.375 & 0.375 \end{array}\right)\begin{pmatrix} 0.302 \\ 0.211 \\ 0.060 \\ 0.105 \\ 0.022 \\ 0.211 \\ 0.085 \end{pmatrix}$$

And, this matrix vector multiplication will give the final ranks:

$$\begin{bmatrix} \text{Nikon D610} \\ \text{Canon 6D-ii} \\ \text{Nikon D850} \\ \text{Canon 5D-iv} \end{bmatrix} = \begin{bmatrix} 0.238 \\ 0.228 \\ 0.275 \\ 0.251 \end{bmatrix}$$

Wow with the highest total points Nikon D850 ranks first! Our calculation is telling us to buy Nikon D850 but our budget is only $1600 and Nikon D850 costs $3296. What just happened! Well, the calculation is not wrong. We had a lot emphasis on MegaPixels. When we put score on criteria we put a lot of emphasis on MegaPixels, see Price has a weight of 0.302 and MP has a weight of 0.211. So, although Nikon D610 ranks over Nikon D850 in terms of price, Nikon D850 just outweighs Nikon D610 in all other aspects and ranks on top. If we can't by any means afford Nikon D850 or we must not spend above $1600 we must not put huge emphasis on MP. Huh!! How cruel reality is, didn't hesitate to point a finger at us. We should probably change scores little bit to reduce importance on MegaPixels (high MegaPixels comes at a much higher cost):

	Price	MP	FPS	ISO	video	Lens-Ecosystem	Coonectivity
Score against the best (Price)	1	5	7	4	8	2	5

	Price	MP	FPS	ISO	video	Lens-Ecosystem	Coonectivity
Score against the worst (Video)	9	5	8	3	1	4	5

With revised scoring we get revised weights:

	Price	MP	FPS	ISO	video	Lens-Ecosystem	Coonectivity
Final weights of criteria	0.346	0.097	0.069	0.121	0.025	0.242	0.097

New weights lead to the following rankings:

$$\begin{bmatrix} \text{Nikon D610} \\ \text{Canon 6D-ii} \\ \text{Nikon D850} \\ \text{Canon 5D-iv} \end{bmatrix} = \begin{bmatrix} 0.232 \\ 0.215 \\ 0.204 \\ 0.220 \end{bmatrix}$$

Disappointed! I like Canon 6D-ii.

Calculation of weights, minimization of the maximum distance

In this section the weight calculation procedure is shown. This part is not necessary to understand in order to use BWM. However, this is not hard to understand either. The distance minimization problem can be turned into a linear optimization problem. This has been discussed extensively by Rezaei (2016). Lets consider the problem of determining the weights of the options against the price criterion. It was shown that the problem in this case boils down to minimizing the following two sets of distances:

$$\left\{ \left|\frac{W_{\text{Nikon D610}}}{W_{\text{Canon 6D}}} - 3\right|,\ \left|\frac{W_{\text{Nikon D610}}}{W_{\text{Nikon D850}}} - 9\right|,\ \left|\frac{W_{\text{Nikon D610}}}{W_{\text{Canon 5D}}} - 8\right| \right\},$$

$$\left\{ \left|\frac{W_{\text{Nikon D610}}}{W_{\text{Nikon D850}}} - 9\right|,\ \left|\frac{W_{\text{Canon 6D}}}{W_{\text{Nikon D850}}} - 8\right|,\ \left|\frac{W_{\text{Canon 5D}}}{W_{\text{Nikon D850}}} - 2\right| \right\}$$

Which in turn can be rewritten in solvable form as:

$$\left\{ \left|\frac{W_{\text{Nikon D610}}}{W_{\text{Canon 6D}}} - 3\right| \leq \zeta,\ \left|\frac{W_{\text{Nikon D610}}}{W_{\text{Nikon D850}}} - 9\right| \leq \zeta,\ \left|\frac{W_{\text{Nikon D610}}}{W_{\text{Canon 5D}}} - 8\right| \leq \zeta \right\},$$

$$\left\{|\frac{W_{\text{Nikon D610}}}{W_{\text{Nikon D850}}}-9|\leq\zeta,\ |\frac{W_{\text{Canon 6D}}}{W_{\text{Nikon D850}}}-8|\leq\zeta,\ |\frac{W_{\text{Canon 5D}}}{W_{\text{Nikon D850}}}-2|\leq\zeta\right\}$$

where, the target is to minimize ζ, such that, $W_{\text{Nikon D610}}+W_{\text{Canon 6D}}+W_{\text{Nikon D850}}+W_{\text{Canon 5D}}=1$
This problem can be linearised and formulated as follows (Rezaei, 2016).

$$\{|W_{\text{Nikon D610}}-3W_{\text{Canon 6D}}|\leq\zeta,\ |W_{\text{Nikon D610}}-9W_{\text{Nikon D850}}|\leq\zeta,\ |W_{\text{Nikon D610}}-8W_{\text{Canon 5D}}|\leq\zeta\}, \tag{1}$$

$$\{|W_{\text{Nikon D610}}-9W_{\text{Nikon D850}}|\leq\zeta,\ |W_{\text{Canon 6D}}-8W_{\text{Nikon D850}}|\leq\zeta,\ |W_{\text{Canon 5D}}-2W_{\text{Nikon D850}}|\leq\zeta\} \tag{2}$$

and the target is to minimize ζ such that $W_{\text{Nikon D610}}+W_{\text{Canon 6D}}+W_{\text{Nikon D850}}+W_{\text{Canon 5D}}=1$. The set 1 of inequalities actually contains six inequalities as follows:

$$\begin{aligned}
W_{\text{Nikon D610}}-3W_{\text{Canon 6D}}&\leq\zeta\\
W_{\text{Nikon D610}}-3W_{\text{Canon 6D}}&\geq-\zeta\\
W_{\text{Nikon D610}}-9W_{\text{Nikon D850}}&\leq\zeta\\
W_{\text{Nikon D610}}-9W_{\text{Nikon D850}}&\geq-\zeta\\
W_{\text{Nikon D610}}-8W_{\text{Canon 5D}}&\leq\zeta\\
W_{\text{Nikon D610}}-8W_{\text{Canon 5D}}&\geq-\zeta
\end{aligned}$$

similarly from set 2 we get the following inequalities:

$$\begin{aligned}
W_{\text{Canon 6D}}-8W_{\text{Nikon D850}}&\leq\zeta\\
W_{\text{Canon 6D}}-8W_{\text{Nikon D850}}&\geq-\zeta\\
W_{\text{Canon 5D}}-2W_{\text{Nikon D850}}&\leq\zeta\\
W_{\text{Canon 5D}}-2W_{\text{Nikon D850}}&\geq-\zeta
\end{aligned}$$

All these inequalities can be arranged as a linear system of inequalities in matrix format and a regular optimization problem can be formulated in terms of the cost function with a target of minimizing the function:

$$\text{cost function}=c^{\text{Transpose}}x\equiv\begin{bmatrix}0\\0\\0\\0\\1\end{bmatrix}^{\text{Transpose}}\begin{bmatrix}W_{\text{Nikon D610}}\\W_{\text{Canon 6D}}\\W_{\text{Nikon D850}}\\W_{\text{Canon 5D}}\\\zeta\end{bmatrix} \tag{3}$$

The inequalities in matrix format would appear as:

$$A_{ub}x\leq b_{ub}\equiv\begin{pmatrix}1&-3&0&0&-1\\-1&3&0&0&-1\\1&0&-9&0&-1\\-1&0&9&0&-1\\1&0&0&-8&-1\\-1&0&0&8&-1\\0&1&-8&0&-1\\0&1&-8&0&-1\\0&-1&8&0&-1\\0&0&-2&1&-1\\0&0&2&-1&-1\end{pmatrix}\begin{pmatrix}W_{\text{Nikon D610}}\\W_{\text{Canon 6D}}\\W_{\text{Nikon D850}}\\W_{\text{Canon 5D}}\\\zeta\end{pmatrix}\leq 0 \tag{4}$$

and, the equality constraint would appear as:

$$A_{eq}x=b_{eq}\equiv\begin{bmatrix}1\\1\\1\\1\\0\end{bmatrix}^{\text{Transpose}}\begin{bmatrix}W_{\text{Nikon D610}}\\W_{\text{Canon 6D}}\\W_{\text{Nikon D850}}\\W_{\text{Canon 5D}}\\\zeta\end{bmatrix}=1 \tag{5}$$

These system of inequalities and equations has been solved to minimize the cost function using scipy's optimization library, particularly suing 'linprog' function. Linprog takes the cost function matrix, the inequality matrix, equality matrix and corresponding right hand side b_{ub}, and b_{eq} vectors as follows:

$$linprog(c, A_{eq} = A_{eq}, b_{eq} = b_{eq}, A_{ub} = A_{ub}, b_{ub} = b_{ub},\ bounds = (0, None),\ options = \{"disp" : False\})$$

More details about the algorithm used in the scipy optimization library or in linprog can be found in the scipy documentation page: https://docs.scipy.org/doc/scipy-0.15.1/reference/generated/scipy.optimize.linprog.html

References

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