Assignment 3

Please find uploads on GitHub via: https://github.com/SherryCactus/ACST886-Assignment-3.git

Question 1

Death Rate

```
qx_d <- c()
qx_d[55:57] <- c(0.001046, 0.001199, 0.001375)
px_d <- 1-qx_d
```

Different interest rates

```
i <- 0.06
c <- 0.0192308
IER <- 0.08
RDR <- 0.12
```

Expected Annuity Payments on Valuation Basis at EOY

```
R.EOY <- rep(0,3)
for(t in 1:3){
  R.EOY[t] <- 15000*(1+c)^(t-1) * prod(px_d[55:(55+t-1)])
}
R.EOY

## [1] 14984.31 15254.16 15526.13
```

Expected Expenses at EOY

```
E.EOY <- rep(0,3)
E0 <- 100 + 40250*0.015
f <- 0.05
for(t in 1:3){
  E.EOY[t] <- 10*(1+f)^(t-1) * prod(px_d[55:(55+t-1)])
}
E.EOY

## [1]  9.98954 10.47644 10.98514
```

Expected Policy Value, Interest and Transfer at IER at EOY

The policy value in this case at the end of year t is

$${}_tV = Ra_{55+t:\overline{n-t}|}$$

```
V.BOY <- rep(0,3)
V.EOY <- rep(0,3)
I.EOY <- rep(0,3)
```

```r
T.EOY <- rep(0,3)

V.EOY[1] <- (15000*(1+c)/(1+i) *px_d[56] + 15000*(1+c)^2/(1+i)^2 *px_d
[56]*px_d[57])*px_d[55]
V.EOY[2] <- (15000*(1+c)^2/(1+i) *px_d[57])*px_d[55]*px_d[56]

V.BOY[1] <- (15000/(1+i) *prod(px_d[55:55]) + 15000*(1+c)/(1+i)^2 *prod
(px_d[55:56]) + 15000*(1+c)^2/(1+i)^3 *prod(px_d[55:57])) - 40250
I.EOY[1] <- (40250-E0)*IER
T.EOY[1] <- 40250 - E0 - R.EOY[1] - E.EOY[1] + I.EOY[1] - V.EOY[1]
for(t in 2:3){
  V.BOY[t] <- V.EOY[t-1]
  I.EOY[t] <- V.BOY[t]*IER
  T.EOY[t] <- V.BOY[t] + I.EOY[t] - E.EOY[t] - R.EOY[t] - V.EOY[t]
}
I.EOY

## [1] 3163.700 2256.713 1171.783

V.EOY

## [1] 28208.92 14647.29     0.00
```

(a)

(i)

The profit signature is hence the vector of Transfers at EOY

```r
T.EOY

## [1] -493.2660  553.7019  281.9607
```

(ii)

The expected NPV of profits is just the EPV of profit signature

```r
EPV.T_IER <- sum(T.EOY/(1+IER)^(1:3))
EPV.T_IER

## [1] 241.8119
```

Under the risk discount rate, the EPV of profits is

```r
EPV.T_RDR <- sum(T.EOY/(1+RDR)^(1:3))
EPV.T_RDR

## [1] 201.6858
```

Hence the profit margin is

```r
EPV.T_RDR/40250
```

```
## [1] 0.005010827
```

(b)

The IRR can be calculated by finding the root of $NPV(Transfers) = 0$

```r
NPV <- function(yield){
  sum(T.EOY/(1+yield)^(1:3))
}
IRR <- uniroot(NPV, interval = c(0,1))
IRR
```

```
## $root
## [1] 0.5028739
##
## $f.root
## [1] -0.000155385
##
## $iter
## [1] 6
##
## $init.it
## [1] NA
##
## $estim.prec
## [1] 6.548305e-05
```

Hence, the IRR is 50.29%. The reason of a high IRR in this case is that the EPV of the 3-year repayments is greater than its purchase price 40250.

```r
sum(R.EOY/(1+i)^(1:3))
```

```
## [1] 40748.33
```

Question 2

Multiple Decrement Table

```r
x <- c(30:40)
qx_d <- c(0.00043, 0.00042, 0.00041, 0.00040, 0.00041, 0.00042, 0.00043,
 0.00045, 0.00049, 0.00053)
qx_w <- c(0.25, 0.2, 0.175, rep(0.15,6), 1)

aqx_d <- qx_d * (1-0.5*qx_w)
aqx_w <- qx_w * (1-0.5*qx_d)

al <- rep(0,11)
ad_d <- rep(0,11)
ad_w <- rep(0,11)
al[1] <- 100000
```

```
ad_d[1] <- al[1]*aqx_d[1]
ad_w[1] <- al[1]*aqx_w[1]
for(t in 2:11){
  al[t] <- al[t-1]-ad_d[t-1]-ad_w[t-1]
  ad_d[t] <- al[t]*aqx_d[t]
  ad_w[t] <- al[t]*aqx_w[t]
}
```

Unit Fund

For an in-force policy

```
P.BOY <- c()
P.BOY[1] <- 5000*(1-0.45)*(1-0.005)
P.BOY[2:10] <- 5000*(1-0.01)*(1-0.005)

VoU.EOY <- c()
VoU.EOY[1] <- P.BOY[1]*(1+0.1)*(1-0.003)
for (t in 2:10){
  VoU.EOY[t] <- (VoU.EOY[t-1]+P.BOY[t])*(1+0.1)*(1-0.003)
}
```

Expected amounts per initial policy

```
EP.BOY <- c()
EVoU.EOY <- c()
for(t in 1:10){
  EP.BOY[t] <- P.BOY[t] * al[t]/al[1]
  EVoU.EOY[t] <- VoU.EOY[t] * al[t+1]/al[1]
}

EI_u.EOY <- c()
EI_u.EOY[1] <- EP.BOY[1]*0.1
for(t in 2:10){
  EI_u.EOY[t] <- (EVoU.EOY[t-1]+EP.BOY[t])*0.1
}
```

Transfers to non-unit fund

```
T_u.EOY <- c()
for(t in 2:10){
  T_u.EOY[1] <- EP.BOY[1]+EI_u.EOY[1]-EVoU.EOY[1]
  T_u.EOY[t] <- EVoU.EOY[t-1]+EP.BOY[t]+EI_u.EOY[t]-EVoU.EOY[t]
}
```

Non-Unit Fund

For an in-force policy

```
PBF.BOY <- 5000*c(0.45, rep(0.01,9))

BSM.BOY <- 0.005*5000*(1-c(0.45, rep(0.01,9)))
```

```
E.BOY <- c(5000*0.45, rep(0,9)) + 58*(1+0.2)^(0:9)

DSB.EOY <- VoU.EOY
```

Expected amounts per initial policy

```
EPBF.BOY <- c()
EBSM.BOY <- c()
EE.BOY <- c()
EDB.EOY <- c()
ESB.EOY <- c()
for(t in 1:10){
  EPBF.BOY[t] <- PBF.BOY[t] * al[t]/al[1]
  EBSM.BOY[t] <- BSM.BOY[t] * al[t]/al[1]
  EE.BOY[t] <- E.BOY[t] * al[t]/al[1]
  EDB.EOY[t] <- DSB.EOY[t] * ad_d[t]/al[1]
  ESB.EOY[t] <- DSB.EOY[t] * ad_w[t]/al[1]
}

EI_nu.EOY <- (EPBF.BOY+EBSM.BOY-EE.BOY)*0.08

T_nu.EOY <- EPBF.BOY+EBSM.BOY-EE.BOY+EI_nu.EOY+T_u.EOY-EDB.EOY-ESB.EOY
```

(a)

The profit signature is

```
T_nu.EOY

##  [1] -38.7603750  23.7783570  21.2621972  18.7990231  16.3686478
##  [6]  13.4784000  10.2381118   6.7380580   3.0520639  -0.7596495
```

The EPV of transfers is

```
sum(T_nu.EOY/(1+0.125)^(1:10))

## [1] 34.67393
```

(b)

We want the transfer from the Unit Fund in year 10 to be higher by 0.7596495.

```
incr <- T_nu.EOY[10]/(1+0.1)
incr

## [1] -0.6905904
```

So in the Unit Fund we want the policy value at time 9 per inforce policy to increase by

```
incr_u <- incr*al[1]/al[10]
incr_u

## [1] -3.713458
```

The new profit signature will be

```
T_nu.EOY_incr <- T_nu.EOY + c(rep(0,8), incr, -T_nu.EOY[10])
T_nu.EOY_incr

##  [1] -38.760375  23.778357  21.262197  18.799023  16.368648  13.4784
00
##  [7]  10.238112   6.738058   2.361473   0.000000
```

(c)

Decreased.

Question 3