Californian Median Housing Value Predictive Modeling Using Multiple Linear Regression

Introduction

This study examines the multiple factors on the median housing value of Californian properties, with a focus on the geographical and regional information, in order to understand and to make accurate prediction of median housing values.

Materials and Methods

Variable Selection

Applying the power transformation on the predictors and response first, we find their most appropriate transformations. Afterwards, comparison between models determines if the transformation improves the model fitness. Predictors in the fully transformed model, that don't pass the individual significance test, are also compared to models without the particular predictors.

The four selection criteria, adjusted R-squared, Akaike's Information Criterion(AIC), corrected AIC, and Bayesian Information Criterion(BIC), are used to compare model fitness. They measure how well the model fits the dataset, with increasing penalty on model complexity to balance the interpretability of a model with its predictability.

Variance Inflation Factor(VIF) and the correlation coefficients of variables are used to identify and solve multicollinearity issue. We build models with all possible combinations of highly correlated variables, starting with the ones that require the least number of variables to be taken out of the model. From these models that pass the multicollinearity test, we compare the model performance using selection criteria to find the best performing combination of variables.

To decide whether the main effect and interaction terms for the indicator variables, *near_bay*, *near_ocean*, and *oneh_ocean*, should be included in the model, we compare the simple linear models from different categories, specifically the slopes and intercepts.

In order to identify bad leverage points, we apply 4/n as a cut-off of the hat matrix to identify high leverage points, then use a cut off of |standardized residue| >= 4 to identify the outliers among them, which are bad leverage points.

Finally, the individual significance is verified again with the final model. For not individually significant variable, models without the particular variable are compared to a full model using ANOVA test to determine if the variable should be included.

Model Validation

1000 observations are randomly selected from the dataset. It is split by 70/30; the training dataset has 700 observations, while the test dataset has 300 observations. The model is validated with the test dataset by using the same plots for model diagnostics to verify the additional conditions and assumptions.

Model Diagnostics

First, it is to check the two additional conditions on the linearity of the response against the fitted values and linearity between the predictors by appropriate plots. The four assumptions are checked using the residual plots, standardized residual plots, and modified residual plots. Failing the assumptions can greatly affect the credibility and predictability of our model.

Results

Data Description

The sample has 14 variables containing information about 1000 homes in California, randomly selected from a large dataset. From the histograms and Sample Locations plot, most observations are scattered near the San Francisco Bay area or in Los Angeles. The Sample Price plot demonstrates how much the locations of Californian properties determine the median housing prices, with the expensive properties mainly located near ocean, mostly around San Francisco or LA. From the histograms, we can see similar distribution between *total_rooms*, *total_bedrooms*, *populations*, and *households*.

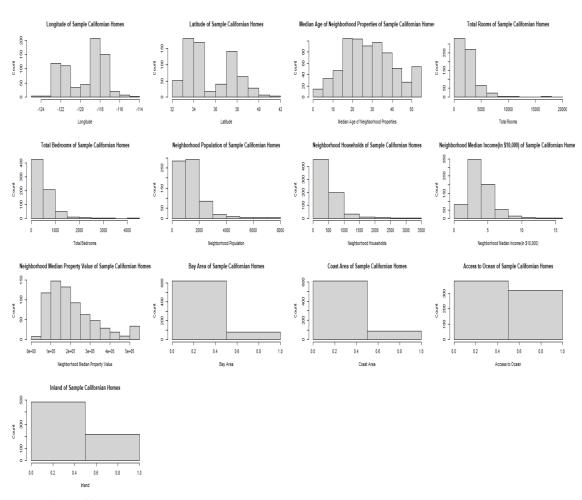


Figure 1 Variable Histograms

Sample Locations

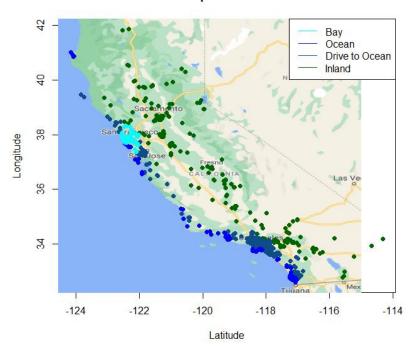


Figure 3 Sample Locations

Sample Price

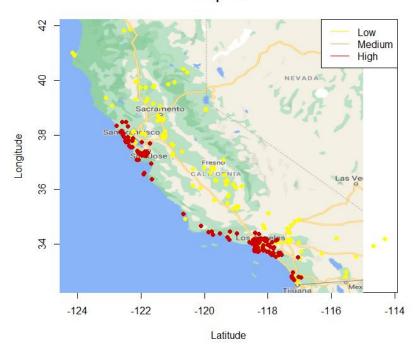


Figure 2 Sample Price

Process of Obtaining the Final Model

Variable	Suggested Power from powerTransform
latitude	-6.2103873
longitude	-5.1456697
median_house_value	0.147994
housing_median_age	0.7533555
total_rooms	0.1445191
total_bedrooms	0.1540388
population	0.1613027
households	0.1747819
median_income	0.161507
near_bay	-1.9938737
near_ocean	-1.7186720
oneh_ocean	-0.1156945

Table 1 Power Transformation Result

The results of power transformation is shown in Table 1. The powers of predictors, *latitude longitude*, and *near_bay* are rounded off to -6, -5, -2 respectively. Because the other predictors and the response have small powers, logarithm transformation is applied instead for interpretability purpose. We add 0.01 to *near_bay*, *near_ocean*, and *oneh_ocean* since they contain zeros.

The results from Table 3 show that predictors *near_bay* and *near_ocean* are best untransformed, while *longitude* and *median_income* should be transformed. Predictor *households* is best to be dropped, because it doesn't pass the significance test.

Model with Highly	Adjusted R^2	AIC	Corrected AIC	BIC
Correlated Predictors				
longitude, population	0.489	-1204.186	-1204.099	-1177.430
longitude, total_bedrooms	0.473	-1182.664	-1182.577	-1155.908
longitude, total_rooms	0.458	-1162.899	-1162.813	-1136.144
latitude, population	0.473	-1182.425	-1182.339	-1155.669
latitude, total_bedrooms	0.458	-1163.364	-1163.278	-1136.608
latitude, total_rooms	0.447	-1149.563	-1149.477	-1122.808

Table 2 Selection Criteria for Models with Multicollinear Variables

Regarding multicollinearity, *inland* is removed due to perfect. One highly correlated group is *latitude* and *longitude*. The second group is *total_rooms*, *total_bedrooms*, *households*, and *population*. Keeping *longitude* and *total_rooms* results in the best non-collinear model from Table 2. Note that the criteria are from models with appropriately transformed variables; the transformation is not included in the table for simplicity purpose.

The analysis on main effect and interaction term from Table 4 indicates that it is best to keep these interaction terms: *longitude* and *near_bay*, *longitude* and *near_ocean*, and *longitude* and

oneh_ocean; it suggests to keep the main effect term *near_bay*. However, the final model doesn't perform better than the previous model, so the changes aren't implemented in the final model.

The filters on bad leverage points show one data point to be a bad leverage point. This point has the maximum <code>housing_median_age</code>, high <code>median_income</code>, and the maximum <code>median_house_value</code>. The property with this specific set of coordinates is situated in the same neighborhood with well known studios. Contextually, it is best to be evaluated separately as an individual case, so it is removed from the train dataset.

From analysis on model conditions and assumptions, *median_income* and *total_rooms* don't satisfy the constant variance assumption. The assumption violation is fixed by applying square root transformation to the response and the two predictors while removing their prior logarithm transformation.

While there are clusters of residuals, they are not separated from the rest of the data, so the errors are independent. From the modified residual plots, the variances of the errors are constant. The points in normal quantile-quantile plot indicates a straight linear relationship, with no noticeable deviation or other pattern; this tells us that the assumption of normally distributed error is satisfied. Therefore, all four assumptions are met for this model.

Goodness of Final Model

Upon inspection, the train and test datasets are fairly similar in distribution. Our final model performs better using the test dataset compared to using the train dataset, with an adjusted r-squared value of 0.667 instead of 0.5905. The model also passes the two additional conditions, upon evaluating the plot from the response against the fitted values and scatterplots of all the predictors. The standardized residual plots and residual plots indicate that the model using the test dataset satisfies the linearity, normality of errors, and independent error assumptions. The modified standard plot show some moderate non-constant variance issues.

Discussion

Final Model Interpretation and Importance

The final model shows that the longitude of the property and its general geographical position in the state of California, and the median age, total number of rooms, median income of the neighborhood housing are significant in relation to the median housing value in a given area. We identify the significant factors and build appropriate model that captures the relationships between the significant factors and median housing value and provides good predictive power.

Limitations of Analysis

The final model doesn't have the best selection criteria compared to some other models we use, for example, logarithm transformation for median_housing_value, total_rooms, and median_income fits the dataset better than square-root transformation. However, models with logarithm transformation don't satisfy the constant variance assumption. Likewise, including more predictors like latitude and total_bedrooms can improve the predictive power of the model but they cause multicollinearity issue.

References

Supplementary Tables

Model	Adjusted R^2	AIC	Corrected AIC	BIC
All Variables Transformed	0.660	-1481.141	-1480.612	-1417.977
Untransformed <i>near_bay</i>	0.660	-1483.025	-1482.572	-1424.412
Untransformed households	0.660	-1481.027	-1480.498	-1417.863
Untransformed near_ocean	0.660	-1483.025	-1482.572	-1424.412
near_bay dropped	0.659	-1480.828	-1480.445	-1426.766
households dropped	0.660	-1483.025	-1482.572	-1424.412
near_ocean dropped	0.657	-1476.410	-1476.028	-1422.348
Untransformed longitude	0.657	-1474.956	-1474.427	-1411.792
Untransformed median_income	0.665	-1492.173	-1491.644	-1429.009

Table 3 Selection Criteria for Models with Varying Transformations

Model	Adjusted R-squared	AIC	Corrected AIC	BIC
Main effect term for	0.592	-1357.08	-1356.826	-1312.127
near_bay and interaction				
terms for near_ocean				
and oneh_ocean				
Interaction terms for	0.591	-1356.630	-1356.370	-1311.670
near_bay, near_ocean,				
oneh_ocean				
Interaction term for	0.593	-1359.790	-1359.530	-1314.830
near_ocean, main effect				
terms for <i>near_bay</i> and				
oneh_ocean				
Interaction term for	0.592	-1357.443	-1357.183	-1312.483
oneh_ocean, main effect				
terms for <i>near_bay</i> and				
near_ocean				
Interaction term for	0.594	-1360.603	-1360.343	-1315.643
near_bay and main				
effect term for				
near_ocean and				
oneh_ocean				
Main effect terms for	0.594	-1361.105	-1360.845	-1316.145
near_bay, near_ocean,				
and oneh_ocean				
Main effect term for	0.602	-1374.095	-1373.776	-1324.584
<i>near_bay</i> and interaction				
terms for <i>near_bay</i> ,				
near_ocean, and				
oneh_ocean				

Table 4 Selection Criteria for Models with Varying Main Effect and Interaction Terms

R Code

```
setwd(dir = 'C:/Users/i5/Downloads/STA302 Data Analysis I/Final Project')
     data <- read.csv('C:/Users/i5/Downloads/STA302 Data Analysis I/Final Project/housing.csv', header=TRUE)
     set.seed(1004081030)
     rows <- sample(1:nrow(data), 1000, replace=FALSE)
     train <- data[rows[1:700],
    test <- data[rows[701:1000],]
    pred <- c("Longitude", "Latitude", "Median Age of Neighborhood Properties",
"Total Rooms", "Total Bedrooms", "Neighborhood Population",
                   "Neighborhood Households", "Neighborhood Median Income(in $10,000)", "Neighborhood Median Property Value", "Bay Area", "Coast Area",
10
                   "Access to Ocean", "Inland")
11
12 library(car)
13 library(MASS)
14 library(leaps)
15 select_criteria = function(model, n)
16 + {
17
        SSres <- sum(model$residuals^2)
        Rsq_adj <- summary(model)$adj.r.squared
p <- length(model$coefficients) | - 1
18
19
        AIC <- n*log(SSres/n) + 2*p
20
        AIC <- n = \log(33 + 63/11) + 2 p

AICc <- AIC + (2*(p+2)*(p+3)/(n-p-1))

BIC <- n*\log(ssres/n) + (p+2)*\log(n)
21
22
        res <- C(SSres, Rsq_adj, AIC, AICC, BIC)
names(res) <- C("SSres", "Rsq_adj", "AIC", "AIC_c", "BIC")
23
24
25
        return(res)
26 ^ }
27
      # full model
     mod_train <- lm(median_house_value ~ latitude + longitude + housing_median_age
28
                           + total_rooms + total_bedrooms + population + households +
29
30
                             median_income + near_bay + near_ocean + oneh_ocean + inland, data = train)
31
     summary(mod_train) # shows total rooms, households, and near bay to be significant
32
     anova(mod_train) # shows housing_median_age and near_bay to be significant
33
34
     # delete inland given multicollinearity issue
     mdl1 <- lm(median_house_value ~ latitude + longitude + housing_median_age + total_rooms + total_bedrooms + population + households +
35
36
37
                       median_income + near_bay + near_ocean + oneh_ocean, data = train)
38
     summary(mdl1)
     anova(mdl1) # suggest latitude, longitude, total rooms, total bedrooms, population,
     # households, median income, next near_ocean, and oneh_ocean; w/o median_age and near_bay
41
43 # model with transformed values to run individual significance test #
45
     # check with power transformation
     powerTransform(lm(train$median_house_value ~ 1)) # 0.147994
     powerTransform(lm(cbind(train$median_house_value, train$housing_median_age) ~ 1)) # suggest 0.7533555
47
     powerTransform(lm(cbind(train$median_house_value, train$total_rooms) ~ 1)) # suggest power 0.1445191
48
     powerTransform(lm(cbind(trainsmedian_house_value, trainstotal_bedrooms) ~ 1)) # suggest power 0.1540388
50 powerTransform(lm(cbind(trainsmedian_house_value, trainspopulation) ~ 1)) # suggest power 0.1613027
     powerTransform(lm(cbind(trainsmedian_house\_value, trainspopulation) \sim 1)) \ \# \ suggest \ power \ 0.1613027 \ powerTransform(lm(cbind(trainsmedian_house\_value, trainshouseholds) \sim 1)) \ \# \ suggest \ power \ 0.1747819
51
52
     powerTransform(lm(cbind(train$median_house_value, train$median_income) ~ 1)) # suggest power 0.161507
53
    # the other five variables are not strictly positive, so must make adjustment powerTransform(lm(cbind(train%median_house_value, train%latitude) ~ 1)) # suggest power -6.2103873 powerTransform(lm(cbind(train%median_house_value, I(train%near_bay + 0.01)) ~ 1)) # suggest power -1.9938737 powerTransform(lm(cbind(train%median_house_value, I(train%near_ocean + 0.01)) ~ 1)) # suggest power -1.7186720 powerTransform(lm(cbind(train%median_house_value, I(train%oneh_ocean + 0.01)) ~ 1)) # suggest power -0.1156945 powerTransform(lm(cbind(train%median_house_value, abs(train%longitude)) ~ 1)) # suggest power -5.1456697
55
56
57
58
60
61
     # all transformed
    62
63
64
65
66
67
                   data = train)
     anova(md12) # latitude, total_rooms, median_age, total_rooms and oneh_ocean's individual
68
     # significance has improved; households, near_bay near_ocean are worse;
# households, near_bay, near_ocean are not significant
# same significance: longitude, total_bedroom, population, median_income
70
71
72
73
     # near_bay is insignificant with or without transformation; check if drop near_bay
     mdl2a <- lm(I(log(median_house_value)) ~ I(latitude^(-6)) + I(log(total_median_age)) + I(log(total_rooms)) + I(log(total_bedrooms)) + I(log(population)) + I(log(households)) + I(log(median_income)) + I((near_ocean + 0.01)^(-2)) + I(log(oneh_ocean + 0.01)), data = train)
76
77
     anova(mdl2a, mdl2) # shouldn't drop near_bay
    # decide if households should be transformed
# transformed near_bay, transformed near_ocean, original households
```

```
mdl2d \leftarrow lm(I(log(median_house_value)) \sim I(latitude^{-(-6)}) + I(longitude^{-(-5)}) +
                               I(log(housing_median_age)) + I(log(total_rooms)) +
  82
  83
                               I(log(total_bedrooms)) + I(log(population)) + households +
  84
                              I(log(median_income)) + I((near_bay + 0.01) \land (-2)) +
  85
                              I((near\_ocean + 0.01)\land(-2)) + I(log(oneh\_ocean + 0.01)),
  86
                           data = train)
        anova(mdl2d) # household significance 0.367657 compared to 0.066101 as transformed;
  87
        # keep households as transformed
  88
  89
  90
        # compare transformed households vs dropping households
  91
        mdl2e \leftarrow lm(I(log(median_house_value)) \sim I(latitude^(-6)) + I(longitude^(-5)) +
  92
                               I(log(housing_median_age)) + I(log(total_rooms)) +
                              I(log(total_bedrooms)) + I(log(population)) +
  93
  94
                              I(\log(\text{median\_income})) + I((\text{near\_bay} + 0.01) \land (-2)) +
  95
                              I((near\_ocean + 0.01)\land(-2)) + I(log(oneh\_ocean + 0.01)),
  96
                           data = train)
  97
        anova(mdl2e, mdl2) # value 0.7357; should drop
  98
 99
        # decide if near_bay should be transformed
        mdl2f \leftarrow lm(I(log(median_house_value)) \sim I(latitude^(-6)) + I(longitude^(-5)) + I(lon
100
                              I(log(housing_median_age)) + I(log(total_rooms)) +
101
                              I(log(total_bedrooms)) + I(log(population))
102
                              I(log(median_income)) + near_bay +
103
104
                              I((near\_ocean + 0.01)\land(-2)) + I(log(oneh\_ocean + 0.01)),
105
                           data = train)
106
        anova(mdl2f) # 0.296095
107
        mdl2g <- lm(I(log(median_house_value)) ~ I(latitude^(-6)) + I(longitude^(-5)) +
108
                               I(log(housing_median_age)) + I(log(total_rooms)) +
109
                              I(log(total_bedrooms)) + I(log(population)) +
                              I(\log(\text{median\_income})) + I((\text{near\_bay} + 0.01) \land (-2)) +
110
111
                              I((near\_ocean + 0.01)\land(-2)) + I(log(oneh\_ocean + 0.01)),
112
                           data = train)
113
        anova(mdl2g) # 0.296095 as transformed
        # since it doesn't make a difference, will use original for model simplicity
114
115
116 mdl2h <- lm(I(log(median_house_value)) ~ I(latitude^(-6)) + I(longitude^(-5)) +
117
                                I(log(housing_median_age)) + I(log(total_rooms)) +
118
                                I(log(total_bedrooms)) + I(log(population))
119
                                I(log(median_income)) +
120
                                I((near\_ocean + 0.01)\land(-2)) + I(log(oneh\_ocean + 0.01)),
121
                            data = train) # near_bay dropped
        anova(mdl2h, mdl2f) # 0.04218 shouldn't drop near_bay
122
123
124
        # decide if near_ocean should be transformed
125
        mdl2i <- lm(I(log(median_house_value)) ~ I(latitude^(-6)) + I(longitude^(-5)) +
126
                                I(log(housing_median_age)) + I(log(total_rooms)) +
127
                                I(log(total_bedrooms)) + I(log(population)) +
128
                                I(log(median_income)) + near_bay -
129
                                near_ocean + I(log(oneh_ocean + 0.01)),
130
                            data = train)
131
         anova(mdl2i)
         mdl2j <- lm(I(log(median_house_value)) ~ I(latitude^(-6)) + I(longitude^(-5)) +</pre>
132
                                  I(log(housing_median_age)) + I(log(total_rooms)) +
133
134
                                 I(log(total_bedrooms)) + I(log(population))
135
                                 I(log(median_income)) + near_bay +
136
                                  I((near\_ocean + 0.01)\land(-2)) + I(log(oneh\_ocean + 0.01)),
137
                              data = train)
138
         anova(mdl2j) # same p value; thus use a simplified version
139
         mdl2k <- lm(I(log(median_house_value)) ~ I(latitude^(-6)) + I(longitude^(-5)) +</pre>
140
                                 I(log(housing_median_age)) + I(log(total_rooms)) +
141
                                 I(log(total_bedrooms)) + I(log(population)) +
142
                                 I(log(median_income)) + near_bay +
143
                                    + I(log(oneh_ocean + 0.01)),
144
                              data = train)
145 anova(mdl2k, mdl2i) # 0.003605 shouldn't drop near_ocean
```

```
146 mdl2o <- lm(I(log(median_house_value)) ~ I(latitude^(-6)) + longitude +
147
                    I(log(housing_median_age)) + I(log(total_rooms)) +
148
                    I(log(total_bedrooms)) + I(log(population)) + I(log(households)) +
149
                    I(\log(\text{median\_income})) + I((\text{near\_bay} + 0.01) \land (-2)) +
                    I((near\_ocean + 0.01)\land(-2)) + I(log(oneh\_ocean + 0.01)),
150
151
                  data = train)
152
     mdl2p \leftarrow lm(I(log(median_house_value)) \sim I(latitude^(-6)) + I(longitude^(-5)) +
                    I(log(housing_median_age)) + I(log(total_rooms))
153
                    I(log(total_bedrooms)) + I(log(population)) + I(log(households)) +
154
155
                    median_income + I((near\_bay + 0.01) \land (-2)) +
                    I((near\_ocean + 0.01)\land(-2)) + I(log(oneh\_ocean + 0.01)),
156
157
                  data = train)
158
159 # check result with four criterias
160 resultsa <- round(rbind(</pre>
       select_criteria(mdl2, n=nrow(train)),
select_criteria(mdl2f, n=nrow(train)),
select_criteria(mdl2d, n=nrow(train)),
161
162
163
       select_criteria(mdl2i, n=nrow(train)),
164
165
       select_criteria(mdl2h, n=nrow(train)),
       select_criteria(mdl2e, n=nrow(train)),
166
167
       select_criteria(mdl2k, n=nrow(train)),
168
       select_criteria(mdl2o, n=nrow(train)),
169
       select_criteria(mdl2p, n=nrow(train))
170 ),3)
171 rownames(resultsa)<-c("1", "2", "3", "4", "5", "6", "7", "8", "9")
    # 1:all transformed 2: near_bay original 3: households original 4: near_ocean original
172
    # 5: near_bay dropped 6: households dropped 7: near_ocean dropped 8: longitude original
173
174 # 9: median_income original
175 resultsa
176 # drop households; near_ocean and near_bay original; longitude and median_income
177
    # transformed
178
179 # decide if to use original version of longitude, median_income
     mdl2l <- lm(I(log(median_house_value)) ~ I(longitude^(-5)) +
180
                   I(log(housing_median_age)) + I(log(total_rooms)) +
182
                   I(log(median_income)) + near_bay +
183
                   near_ocean + I(log(oneh_ocean + 0.01)),
                 data = train) # reflect result on near_bay, households, and near_ocean
184
185 mdl2o <- lm(I(log(median_house_value)) ~ longitude +
186
                    I(log(housing_median_age)) + I(log(total_rooms)) +
187
                    I(log(median_income)) + near_bay +
188
                    near_ocean + I(log(oneh_ocean + 0.01)),
189
                  data = train)
190 mdl2p <- lm(I(log(median_house_value)) ~ I(longitude^(-5)) +
191
                    I(log(housing_median_age)) + I(log(total_rooms)) +
192
                    median_income + near_bay +
193
                    near_ocean + I(log(oneh_ocean + 0.01)),
194
                  data = train)
     mdl2q \leftarrow lm(I(log(median_house_value)) \sim I(longitude^(-5)) +
195
                    I(log(housing_median_age)) + I(log(total_rooms)) +
196
197
                    I(log(median_income)) + near_bay +
198
                    near_ocean + I(log(oneh_ocean + 0.01)),
                   data = train)
199
200
     resultsb <- round(rbind(
201
        select_criteria(mdl2l, n=nrow(train)),
        select_criteria(mdl2o, n=nrow(train)),
202
203
        select_criteria(mdl2p, n=nrow(train)),
204
        select_criteria(mdl2q, n=nrow(train))
205
     rownames(resultsb)<-c("1", "2", "3", "4")
206
     # 1:all transformed 2: original longitude 3. original median_income
207
208 # 4: 1/median_income instead of log
209 resultsb # 1/median income is bad compared to log or original
210 # result shows that keep longitude and median_income transformed
211
```

```
213 # preconditions check #
215 plot(I(log(train$median_house_value))~fitted(mdl2))
216 abline(a=0,b=1)
    lines(lowess(log(train$median_house_value)~fitted(mdl2)), col="blue") # condition 1 holds
217
    ttrain <- data.frame(train$longitude^(-5),log(train$housing_median_age),
219
                        log(train$total_rooms), log(train$median_income), train$near_bay,
220
                        train$near_ocean, log(train$oneh_ocean + 0.01))
221
    pairs(ttrain) # conditions hold
222
224 # assumptions check based on residual plots#
226 par(mfrow=c(3,3))
227 plot(rstandard(mdl2)~fitted(mdl2), xlab="fitted", ylab="Residuals")
228 - for(i in 1:7){
229
      plot(rstandard(mdl2)~ttrain[,i], xlab=names(ttrain)[i], ylab="Residuals")
230 4 }
231 ggnorm(rstandard(mdl2))
232 qqline(rstandard(mdl2))
233
    plot(I(log(train$median_house_value))~fitted(mdl2))
234
    abline(a=0,b=1)
235 lines(lowess(log(trainsmedian_house_value)~fitted(mdl2)), col="blue")
236 # standardized residue for linearity assumption check
237
    # linearity ok; normality ok
238
239 par(mfrow=c(3,3))
240 plot(mdl2$residuals~fitted(mdl2), xlab="fitted", ylab="Residuals")
241 - for(i in 1:7){
242
      plot(mdl2$residuals ~ ttrain[,i], xlab=names(ttrain)[i], ylab="Residuals")
243 4 }
244 qqnorm(residuals(mdl2))
245 qqline(residuals(mdl2))
246 # regular residue vs predictor to check independent errors and constant variance
247 # constant variance might be violated; independent error is ok, since the clusters
248 # are not separated from the other data; to be sure, we will wait for model validation
249
250 # use modified residue plots to check constant variance again
251 par(mfrow=c(3,3))
252 - for(i in 1:7){
      plot(sqrt(abs(rstandard(mdl2))) ~ ttrain[,i], xlab=names(ttrain)[i],
    ylab="|Standard. Residuals|^0.5", main="|Standard. Residuals|^0.5 vs Predictor",)
253
254
255
      m <- lm(sqrt(abs(rstandard(mdl2))) ~ ttrain[,i])</pre>
      abline(a = m$coefficients[1], b = m$coefficients[2])
256
257 -
258 # constant variance assumption is violated for total_rooms and median_income
259
260 #
261 ttrain1 <- data.frame(train$longitude^(-5),log(train$housing_median_age),
262
                        log(train$total_rooms), log(train$median_income), train$near_bay,
263
                        train$near_ocean, log(train$oneh_ocean + 0.01))
264
    newtrain <- ttrain1
265
    mdl2r <- lm(I(log(median_house_value)) ~ I(longitude^(-5)) +
266
                 I(log(housing_median_age)) + I(log(total_rooms)) +
267
                 I(log(median_income)) + near_bay +
268
                 near_ocean + I(log(oneh_ocean)),
269
                data = train)
270 par(mfrow=c(3,3))
271 - for(i in 1:7){
272
      plot(sqrt(abs(rstandard(mdl2r))) ~ ttrain1[,i], xlab=names(ttrain1)[i],
           ylab="|Standard. Residuals|^0.5", main="|Standard. Residuals|^0.5 vs Predictor",)
273
      m <- lm(sqrt(abs(rstandard(mdl2r))) ~ ttrain1[,i])</pre>
274
275
      abline(a = m$coefficients[1], b = m$coefficients[2])
276 - }
277 # constant variance is much improved
278 round(select_criteria(mdl2r, n=nrow(train))) # 0.568 6192.757 6193.017 6237.716 worst
279
    summary(mdl2r)
280
```

```
281 mdl2s <- lm(I(sqrt(median_house_value)) ~ I(longitude^(-5)) +</pre>
282
                 I(log(housing_median_age)) + I(sqrt(total_rooms)) +
283
                 I(sqrt(median_income)) + near_bay +
284
                 near_ocean + I(log(oneh_ocean)),
285
               data = train)
286 round(select_criteria(mdl2s, n=nrow(train)),3)
287
    # 0.593
              6151.591
                         6151.851
                                  6196.551 better than mdl2r
288
289 par(mfrow=c(3,3))
290 plot(rstandard(mdl2s)~fitted(mdl2s), xlab="fitted", ylab="Residuals")
291 - for(i in 1:7){
292
      plot(rstandard(mdl2s)~ttrain[,i], xlab=names(ttrain)[i], ylab="Residuals")
293 4 }
294 ggnorm(rstandard(mdl2s))
295 qqline(rstandard(mdl2s))
296 plot(I(log(train$median_house_value))~fitted(mdl2s))
297
    abline(a=0,b=1)
298
    lines(lowess(log(train$median_house_value)~fitted(mdl2s)), col="blue")
299
300 par(mfrow=c(3,3))
301 plot(mdl2s$residuals~fitted(mdl2s), xlab="fitted", ylab="Residuals")
302 - for(i in 1:7){
303
     plot(mdl2s$residuals ~ ttrain[,i], xlab=names(ttrain)[i], ylab="Residuals")
304 - }
305 qqnorm(residuals(mdl2s))
306 qqline(residuals(mdl2s))
307 par(mfrow=c(3,3))
308 - for(i in 1:7){
      plot(sqrt(abs(rstandard(mdl2s))) ~ ttrain[,i], xlab=names(ttrain)[i],
           ylab="|Standard. Residuals|^0.5", main="|Standard. Residuals|^0.5 vs Predictor",)
310
311
      m <- lm(sqrt(abs(rstandard(mdl2s))) ~ ttrain[,i])</pre>
      abline(a = m$coefficients[1], b = m$coefficients[2])
312
313 4 }
314 # mdl2r and mdl2s satisfy assumptions
315
316
318 # variable selection #
320 # multicolinearity check #
    vif(mdl2) # given the threshold is 5, latitude, longitude, total_rooms,
321
    # total_bedrooms, and households don't pass the multicolinearity check;
322
323
    # correlation of variables with latitude
324
    cors <- NULL
325 - for (i in 1:13){
326
      cors <- c(cors, cor(train$latitude, train[, (i+1)]))</pre>
327 ^ }
328 cdf <- data.frame("Correlation" = cors, "Predictors" = pred)
329 cdf[order(-abs(cdf$Correlation)),] # latitude strongly correlate with longitude
330
    # by -0.91787592, second is -0.42341103 Access to Ocean
331
332
    # correlation of variables with total_rooms
333
    cors <- NULL
334 - for (i in 1:13){
335
      cors <- c(cors, cor(train$total_rooms, train[, (i+1)]))</pre>
336 - }
337
    cdf <- data.frame("Correlation" = cors, "Predictors" = pred)</pre>
338
    cdf[order(-abs(cdf$Correlation)),] # Total Bedrooms: 0.932037674;
    # Neighborhood Households: 0.931694831; Neighborhood Population: 0.883482788
339
340
341
    # model versions: 1. latitude, no longitude 2. longitude, no latitude
    # 3. one of total_rooms, total_bedrooms, population
342
    # 4. two of total_rooms, total_bedrooms, population
343
344
    t1 <- lm(log(train$median_house_value)~., data=newtrain[,-c(1,8)])
345
    vif(t1) # problematic vif with latitude and longitude
346 t2 <- lm(log(train\$median\_house\_value)\sim., data=newtrain[,-c(1,2,8)])
    vif(t2) # latitude < 5 w/o longitude
347
348 t3 <- lm(log(train\$median\_house\_value)\sim., data=newtrain[,-c(1,3,8)])
349 vif(t3) # longitude < 5 w/o latitude
350 # need to remove one from total_rooms,total_bedrooms,and population,
```

```
351 # since vif is still a problem after removing households
       t4 <- lm(log(train$median_house_value)~., data=newtrain[,-c(1,3,5,8)]) # w/o total_rooms
      vif(t4) # bedrooms and population still problematic
t5 <- lm(log(train$median_house_value)~., data=newtrain[,-c(1,3,6,8)]) # w/o total_bedrooms
355 vif(t5) # total_rooms still problematic
356 t6 <- lm(log(train$median_house_value).., data=newtrain[,-c(1,3,7,8)]) # w/o population vif(t6) # total_rooms and total_bedrooms stil problematic
358
        # remove another variable
359 t7 <- lm(log(train$median_house_value)~., data=newtrain[,-c(1,3,5,6,8)])
360 # w/o total_rooms and total_bedrooms
361 vif(t7)
362 t8 <- lm(log(train$median_house_value)~., data=newtrain[,-c(1,3,5,7,8)])
363 # w/o total_rooms and population
364 vif(t8)
365 t9 <- lm(log(train$median_house_value)~., data=newtrain[,-c(1,3,6,7,8)])
366
       # w/o population and total_bedrooms
367 vif(t9)
368 # t7, t8, t9 both pass; same as t2, t3
369 # models: one of
370 # 1. latitude, longitude 2. total_rooms, total_bedrooms, population
372 results <- round(rbind(
          select\_criteria(lm(log(train\$median\_house\_value) \sim., \ data=newtrain[,-c(1,3,5,6,8)]), \ n=nrow(newtrain)),
373
           select\_criteria(lm(log(train\$median\_house\_value) \sim., \ data=newtrain[,-c(1,3,5,7,8)]), \ n=nrow(newtrain)),
374
375
           select\_criteria(lm(log(train\$median\_house\_value) \sim., data=newtrain[,-c(1,3,6,7,8)]), n=nrow(newtrain)), leading(log(train\$median\_house\_value) \sim., data=newtrain[,-c(1,3,6,7,8)]), n=nrow(newtrain)), leading(log(train\$median\_house\_value)), leading(lo
           select_criteria(lm(log(train$median_house_value)~., data=newtrain[,-c(1,2,5,6,8)]), n=nrow(newtrain)),
select_criteria(lm(log(train$median_house_value)~., data=newtrain[,-c(1,2,5,7,8)]), n=nrow(newtrain)),
376
377
           select\_criteria(lm(log(train\$median\_house\_value) \sim., \ data=newtrain[,-c(1,2,6,7,8)]), \ n=nrow(newtrain))
378
379 ),3)
7,37
380 rownames(results)<-c("1", "2", "3", "4", "5", "6")
381 # 1:longitude&population 2: longitude&total_bedrooms 3: longitude$total_rooms
382 # 4: latitude&population 5: latitude&total_bedrooms 6:latitude&total_rooms
383 results # mdl 3 is best with highest adjusted r and lowest other values
        # chosen model: longitude, median_age, total_rooms, median_income;
384
385 # near_bay, near_ocean, oneh_ocean
386 # w/o latitude, total_bedrooms, population, inland
387
       # from individual significance check earlier, drop households, use simplified
388 # near_bay and near_ocean
389 mdl6 <- lm(I(log(median_house_value)) ~ I(longitude^(-5)) +
                          I(log(housing_median_age)) + I(log(total_rooms))
+ I(log(median_income)) + near_bay + near_ocean + oneh_ocean,
390
391
392
                          data = train)
393 summary(mdl6)
394 anova(md16) # longitude, housing_median_age are insignificant
395
      # recheck transformation
396
       mult <- lm(cbind(train$median_house_value, train$housing_median_age, train$total_rooms,
397
398
                                   train$median_income, train$near_bay,
399
                                           train$near_ocean, train$oneh_ocean) ~ 1)
          pow <- powerTransform(mult, family="bcnPower"
400
401
          pow # suggest log for median_house_value, median_age, near_bay, near_ocean, oneh_ocean
           # suggest 1205 for total_rooms, 1.6 for median_income
402
403
404
          # check if dropping longitude
405
          mdl6a <- lm(I(log(median_house_value)) ~ I(log(housing_median_age)) +
406
                                   I(log(total_rooms)) + I(1/median_income)
                                   near_bay + near_ocean + oneh_ocean,
407
                               data = train)
408
          anova(mdl6a,mdl6) # shouldn't drop
409
410
411
          # check if dropping housing_median_age
          mdl6b <- lm(I(log(median_house_value)) ~ I(longitude^(-5)) +
412
413
                                     I(log(total_rooms))
                                + I(1/median_income) +
414
                                   near_bay + near_ocean + oneh_ocean,
415
                               data = train)
416
          anova(mdl6b, mdl6) # 0.007716 shouldn't drop
417
418
          # check if dropping near_bay mdl6c <- lm(I(log(median_house_value)) \sim I(longitude^(-5)) +
419
420
421
                                   I(log(housing_median_age)) + I(log(total_rooms))
422
                                + I(1/median_income) + near_ocean + oneh_ocean,
423
                               data = train)
          anova(mdl6c, mdl6) # shouldn't drop
424
425
426
            # stepwise selection using AIC
427
          stepAIC(lm(log(median_house_value) \sim I(longitude^(-5)) +
428
                                   I(log(housing_median_age)) + I(log(total_rooms))
                                  I(log(median_income)) + near_bay + near_ocean +
log(oneh_ocean + 0.01), data=train, direction = "both", k=2))
429
430
431 # suggest dropping nothing
```

```
434 # Analysis of Covariance #
 435 - ------
 436 # 12 models
 437
                       # near_bay
 438 mod1a <- lm(log(median_house_value) \sim I(longitude^(-5)), data=train[which(train$near_bay==0),]) 439 mod1b <- lm(log(median_house_value) \sim I(longitude^(-5)), data=train[which(train$near_bay==1),])
                       440
 441
                       # different intercept and slope --> near_bay main effect and interaction to longitude
 442
 443
                       444
 445
                       mod2a$coefficients # 12.02524635
mod2b$coefficients # 12.220974293
 446
                                                                                                                                                                                                                                     0.01059813
 447
                                                                                                                                                                                                                                        -0.001511231
                       # similar intercept, slightly different slope
 448
 449
                       450
 451
 452
                       mod3a$coefficients # 10.8942132
                                                                                                                                                                                                             0.1517004
                       mod3b$coefficients # 8.6754424
 453
                                                                                                                                                                                                          0.4732542
 454
                       # slightly different intercept, similar slope
 455
                        \begin{tabular}{ll} mod4a &-& lm(log(median_house_value) &-& I(log(median_income)), data=train[which(trainsnear_bay==0),]) mod4b &-& lm(log(median_house_value) &-& I(log(median_income)), data=train[which(trainsnear_bay==1),]) \\ \end{tabular} 
 456
 457
                       mod4a$coefficients # 10.9670588
 458
                                                                                                                                                                                                     0.8834415
                        mod4b$coefficients # 11.1663574
 459
 460
                       # similar intercept and slope
 461
 462
 463
                       mod5a <- lm(log(median_house_value) ~ I(longitude^(-5)), data=train[which(train$near_ocean==0),])</pre>
                        mod5b <- lm(log(median_house_value) ~ I(longitude^(-5)), data=train[which(trainshear_ocean==1),])</pre>
 464
                       mod5a$coefficients # 1.215589e+01 2.780397e+09
mod5b$coefficients # 1.338566e+01 2.549884e+10
 465
 466
 467
                        # similar intercept, different slope --> near_ocean*longitude
 468
 469
                        mod6a <- lm(log(median_house_value) ~ I(log(housing_median_age)), data=train[which(train$near_ocean==0),])</pre>
 470
                       \verb|mod6b| <- ln(log(median_house_value)| \sim \verb|I(log(housing_median_age))|, | data=train[which(trainsnear_ocean==1),]| > log(housing_median_age)| > log(housi
                     mod6a$coefficients # 11.9801013
mod6b$coefficients # 11.99110614
471
                                                                                                                                                                                                                                       0.019007
472
                                                                                                                                                                                                                                        0.09843133
473
                      # similar intercept and slope
474
                      \begin{tabular}{ll} mod7a <- ln(log(median_house_value) \sim I(log(total_rooms)), data=train[which(train$near_ocean==0),]) \\ mod7b <- ln(log(median_house_value) \sim I(log(total_rooms)), data=train[which(train$near_ocean==1),]) \\ \end{tabular} 
475
476
                      mod7a$coefficients # 10.781933
mod7b$coefficients # 10.586101
                                                                                                                                                                                                          0.164615
477
478
                                                                                                                                                                                                          0.224867
479
                       # similar intercept and slope
480
                     \label{eq:mod8a} $$ \mbox{mod8a} <- \mbox{lm(log(median_house_value)} \sim \mbox{I(log(median_income)), data=train[which(train$near_ocean==0),])} $$ \mbox{mod8b} <- \mbox{lm(log(median_house_value)} \sim \mbox{I(log(median_income)), data=train[which(train$near_ocean==1),])} $$
481
482
                     mod8a$coefficients # 10.9354817
mod8b$coefficients # 11.449247
483
                                                                                                                                                                                                 0.8995935
484
                                                                                                                                                                                                     0.694835
                       # similar intercept and slope
485
486
                 # consider drop near_ocean, test near_ocean and longitude as interact
487
488
                       mod9a <- lm(log(median_house_value) ~ I(longitude^(-5)), data=train[which(train$oneh_ocean==0),])
 489
490
                       mod9b <- lm(log(median_house_value) ~ I(longitude^(-5)), data=train[which(train$oneh_ocean==1),])
                      mod9a$coefficients # 1.282667e+01
mod9b$coefficients # 1.307992e+01
                                                                                                                                                                               2.326147e+10
1.843861e+10
491
492
493
                       # similar intercept, different slope --> oneh_ocean*longitude
494
                     \label{eq:mod10a} $$ \mbox{mod10a} <- \mbox{lm(log(median_house_value)} \sim \mbox{I(log(housing_median_age)), data=train[which(train$oneh_ocean==0),])} \\ \mbox{mod10b} <- \mbox{lm(log(median_house_value)} \sim \mbox{I(log(housing_median_age)), data=train[which(train$oneh_ocean==1),])} \\ \mbox{mod10a$coefficients $\#$ 11.3991593} & 0.1497339 \\ \mbox{mod10b$coefficients $\#$ 12.55622368} & -0.07935795 \\ \mbox{} \mbox{-0.07935795} \\ \mbox{-0.0
495
496
497
498
                       # similar intercept, slightly different slope
 499
 500
                       \begin{tabular}{ll} mod11a <- lm(log(median_house_value) &\sim I(log(total_rooms)), \ data=train[which(trainsoneh_ocean==0),]) mod11b <- lm(log(median_house_value) &\sim I(log(total_rooms)), \ data=train[which(trainsoneh_ocean==1),]) mod11b <- lm(log(median_house_value)) &\sim I(log(total_rooms)), \ data=train[which(trainsoneh_ocean==1),]) mod11b <- lm(log(median_house_value)) &\sim I(log(total_rooms)), \ data=train[which(trainsoneh_ocean==0),]) mod11b <- lm(log(median_house_value)) &\sim I(log(total_rooms)), \ data=train[which(trainsoneh_ocean==1),]) mod11b <- lm(log(median_house_value)) &\sim I(log(total_rooms)), \ data=train[which(trainsoneh_ocean==1),]) mod11b <- lm(log(total_rooms)) &\sim I(log(total_rooms)) &\sim I(log(total_room
 501
 502
                     mod11a$coefficients # 10.2632509
mod11b$coefficients # 11.2553440
 503
                                                                                                                                                                                                             0.2120744
 504
                                                                                                                                                                                                             0.1366175
 505
                       # similar intercept and slope
 506
                     \label{eq:mod12a} $$\operatorname{mod12a} <- \operatorname{lm}(\log(\operatorname{median\_house\_value}) \sim \operatorname{I}(\log(\operatorname{median\_income})), \; \operatorname{data=train}[\operatorname{which}(\operatorname{train}\circ\operatorname{neh\_ocean}==0),]) \\ \operatorname{mod12b} <- \operatorname{lm}(\log(\operatorname{median\_house\_value}) \sim \operatorname{I}(\log(\operatorname{median\_income})), \; \operatorname{data=train}[\operatorname{which}(\operatorname{train}\circ\operatorname{neh\_ocean}==1),]) \\ \\ \\ \operatorname{mod12b} <- \operatorname{lm}(\log(\operatorname{median\_house\_value}) \sim \operatorname{I}(\log(\operatorname{median\_income})), \; \operatorname{data=train}[\operatorname{which}(\operatorname{train}\circ\operatorname{neh\_ocean}==1),]) \\ \\ \operatorname{mod12b} <- \operatorname{lm}(\log(\operatorname{median\_house\_value}) \sim \operatorname{I}(\log(\operatorname{median\_income})), \; \operatorname{data=train}[\operatorname{which}(\operatorname{train}\circ\operatorname{neh\_ocean}==1),]) \\ \\ \operatorname{mod12b} <- \operatorname{lm}(\log(\operatorname{median\_house\_value}) \sim \operatorname{I}(\log(\operatorname{median\_income})), \; \operatorname{data=train}[\operatorname{which}(\operatorname{train}\circ\operatorname{neh\_ocean}==1),]) \\ \\ \operatorname{lm}(\operatorname{log}(\operatorname{median\_house\_value}) \sim \operatorname{I}(\operatorname{log}(\operatorname{median\_income})), \; \operatorname{data=train}[\operatorname{which}(\operatorname{train}\circ\operatorname{neh\_ocean}==1),]) \\ \operatorname{lm}(\operatorname{log}(\operatorname{median\_house\_value}) \sim \operatorname{I}(\operatorname{log}(\operatorname{median\_income})), \; \operatorname{data=train}[\operatorname{which}(\operatorname{train}\circ\operatorname{neh\_ocean}==1),]) \\ \operatorname{lm}(\operatorname{log}(\operatorname{median\_house\_value}) \sim \operatorname{I}(\operatorname{log}(\operatorname{median\_income})), \; \operatorname{log}(\operatorname{log}(\operatorname{median\_income})), \; \operatorname{log}(\operatorname{log}(\operatorname{log}(\operatorname{log}(\operatorname{log}(\operatorname{log}(\operatorname{log}(\operatorname{log}(\operatorname{log}(\operatorname{log}(\operatorname{log}(\operatorname{log}(\operatorname{log}(\operatorname{log}(\operatorname{log}(\operatorname{log}(\operatorname{log}(\operatorname{log}(\operatorname{log}(\operatorname{log}(\operatorname{log}(\operatorname{log}(\operatorname{log}(\operatorname{log}(\operatorname{log}(\operatorname{log}(\operatorname{log}(\operatorname{log}(\operatorname{log}(\operatorname{log}(\operatorname{log}(\operatorname{log}(\operatorname{log}(\operatorname{log}(\operatorname{log}(\operatorname{log}(\operatorname{log}(\operatorname{log}(\operatorname{log}(\operatorname{log}(\operatorname{log}(\operatorname{log}(\operatorname{log}(\operatorname{log}(\operatorname{log}(\operatorname{log}(\operatorname{log}(\operatorname{log}(\operatorname{log}(\operatorname{log}(\operatorname{log}(\operatorname{log}(\operatorname{log}(\operatorname{log}(\operatorname{log}(\operatorname{log}(\operatorname{log}(\operatorname{log}(\operatorname{log}(\operatorname{log}(\operatorname{log}(\operatorname{log}(\operatorname{log}(\operatorname{log}(\operatorname{log}(\operatorname{log}(\operatorname{log}(\operatorname{log}(\operatorname{log}(\operatorname{log}(\operatorname{log}(\operatorname{log}(\operatorname{log}(\operatorname{log}(\operatorname{log}(\operatorname{log}(\operatorname{log}(\operatorname{log}(\operatorname{log}(\operatorname{log}(\operatorname{log}(\operatorname{log}(\operatorname{log}(\operatorname{log}(\operatorname{log}(\operatorname{log}(\operatorname{log}(\operatorname{log}(\operatorname{log}(\operatorname{log}(\operatorname{log}(\operatorname{log}(\operatorname{log}(\operatorname{log}(\operatorname{log}(\operatorname{log}(\operatorname{log}(\operatorname{log}(\operatorname{log}(\operatorname{log}(\operatorname{log}(\operatorname{log}(\operatorname{log}(\operatorname{log}(\operatorname{log}(\operatorname{log}(\operatorname{log}(\operatorname{log}(\operatorname{log}(\operatorname{log}(\operatorname{log}(\operatorname{log}(\operatorname{log}(\operatorname{log}(\operatorname{log}(\operatorname{log}(\operatorname{log}(\operatorname{log}(\operatorname{log}(\operatorname{log}(\operatorname{log}(\operatorname{log}(\operatorname{log}(\operatorname{log}(\operatorname{log}(\operatorname{log}(\operatorname{log}(\operatorname{log}(\operatorname{log}(\operatorname{log}(\operatorname{log}(\operatorname{log}(\operatorname{log}(\operatorname{log}(\operatorname{log}(\operatorname{log}(\operatorname{log}(\operatorname{log}(\operatorname{log}(\operatorname{log}(\operatorname{log}(\operatorname{log}(\operatorname{log}(\operatorname{log}(\operatorname
 507
 508
                      mod12a$coefficients # 10.8223276
  509
                                                                                                                                                                                                         0.9436217
                 mod12b$coefficients # 11.3910752
 510
                                                                                                                                                                                                          0.6718017
                 # similar intercept and slope
                 # consider drop oneh ocean, test interaction term between oneh ocean and longitude
 513
```

```
514 mdl7 <- lm(I(log(median_house_value)) ~ I(longitude^(-5)) + I(longitude^(-5)*near_bay)
515
                             + I(longitude^(-5)*near_ocean) + I(longitude^(-5)*oneh_ocean) +
                                                I(log(housing_median_age)) + I(log(total_rooms))
516
517
                                            + I(log(median_income)) + near_bay, data = train)
518
        summary(mdl7)
519
520 stepAIC(lm(log(median_house_value) ~ I(longitude^(-5)) + I(longitude^(-5)*near_bay)
521
                              + I(longitude^(-5)*near_ocean) + I(longitude^(-5)*oneh_ocean) +
522
                                 I(log(housing_median_age)) + I(log(total_rooms))
                             + I(log(median_income)) + near_bay + near_ocean + oneh_ocean, data=train, direction = 'both', k = 2))
523
524
525
         # suggest dropping longitude
526
         mdl7a <- lm(I(log(median_house_value)) ~ I(longitude^(-5)*near_bay)
527
                             + I(longitude^(-5)*near_ocean) + I(longitude^(-5)*oneh_ocean) +
528
                                 I(log(housing_median_age)) + I(log(total_rooms))
529
                             + I(log(median_income)) + near_bay, data = train)
530
         anova(mdl7a, mdl7) # shouldn't drop
         vif(mdl7a) # longitude and longitude*near_bay too correlated
531
532
         mdl7b <- lm(I(log(median_house_value)) ~ I(longitude^(-5)) + I(longitude^(-5)*near_ocean) +
                                   I(longitude^(-5)*oneh_ocean) + I(log(housing_median_age)) +
533
534
                                   I(log(total_rooms)) + I(log(median_income)) + near_bay, data = train)
535
         vif(md17b)
536
         mdl7c \leftarrow lm(I(log(median_house_value)) \sim I(longitude^(-5)) + I(longitude^(-5)*near_bay)
537
                                + I(longitude^(-5)*near_ocean) + I(longitude^(-5)*oneh_ocean) +
538
                                  I(log(housing_median_age)) + I(log(total_rooms))
539
                                + I(log(median_income)), data = train)
540
        vif(mdl7c)
541
         # either delete longitude*near_bay or near_bay makes VIF test pass
542
         mdl6d <- lm(I(log(median_house_value)) ~ I(longitude^(-5)) +
543
                                   I(\bar{l} \circ l) + 
544
                                   I(log(total_rooms)) + I(log(median_income)) + near_bay + oneh_ocean,
545
                               data = train) # interaction term for near_ocean instead of main effect
546
        mdl6e <- lm(I(log(median_house_value)) ~ I(longitude^(-5)) +
547
                                   I(longitude \(-5) \(\cdot\) oneh_ocean) + I(log(housing_median_age)) +
548
                                   I(log(total_rooms)) + I(log(median_income)) + near_bay + near_ocean,
549
                                data = train) # interaction term for oneh_ocean instead of main effect
550
        mdl6f <- lm(I(log(median_house_value)) ~ I(longitude^(-5)) +
551
                                   I(longitude^(-5)*near_bay) +I(log(housing_median_age)) +
552
                                   I(log(total_rooms)) + I(log(median_income)) + near_ocean + oneh_ocean,
                             data = train) # interaction term for near_bay instead of main effect
553
        # use selection criteria to compare models
554
555
        results1 <- round(rbind(
556
             select_criteria(mdl7b, n=nrow(newtrain)),
557
             select_criteria(mdl7c, n=nrow(newtrain)),
558
             select_criteria(mdl6d, n=nrow(newtrain)),
            select_criteria(mdl6e, n=nrow(newtrain)),
select_criteria(mdl6f, n=nrow(newtrain)),
559
560
561
            select_criteria(mdl6, n=nrow(newtrain))
562
        ),3)
        rownames(results1)<-c("1", "2", "3", "4", "5", "6")
563
        # 1: main effect term for near_bay 2: interaction term for near_bay 3: all main effects
564
565 results1 # mdl6 is best
566
568 # Leverage Points #
570 # leverage points
571 hii1 <- hatvalues(mdl2s)
        # h]p1 <- which(hii1 > 4/nrow(train))
572
573
        # h]p1 # high leverage points
574 # show that there are some high leverage points
575 #standardized residue
576 r1 <- rstandard(mdl2s)
          # use the residues of leverage points to check for outliers among leverage points
578 lr1 <- r1[which(hii1 > 4/nrow(ttrain2))]
579 lrs1 <- which(lr1 >= 4 | lr1 <= -4) # outlier range for large dataset
580 lrs1
```

```
581 # shows point: 478
582 ttrain2[478,]
583 summary(ttrain2)
584 train[478,]
585 summary(train)
586 # 478 very old median_age, high median_income, oneh_ocean
587
   # house value 500001, max of housing value
588
589 # test out models again
590 results1 <- round(rbind(</pre>
591
      select_criteria(mdl7b, n=nrow(newtrain)),
      select_criteria(mdl7c, n=nrow(newtrain)),
592
593
      select_criteria(mdl6d, n=nrow(newtrain)),
594
      select_criteria(mdl6e, n=nrow(newtrain)),
595
      select_criteria(mdl6f, n=nrow(newtrain)),
596
      select_criteria(md]1, n=nrow(newtrain)),
597
      select_criteria(mdl2, n=nrow(newtrain)),
598
      select_criteria(mdl3, n=nrow(newtrain)),
599
      select_criteria(mdl6, n=nrow(newtrain)),
600
      select_criteria(mdl7, n=nrow(newtrain))
601
    ),3)
602 rownames(results1)<-c("1", "2", "3", "4", "5", "6", "7")
603 # 1: main effect term for near_bay 2: interaction term for near_bay
604 # 3: interaction term for near_ocean 4: interaction term for oneh_ocean
605 # 5: interaction term for near_bay
606 # mdl6 all main effect; mdl7 interaction term for near_ocean and oneh_ocean, and
607 # both interaction term and main effect for near_bay
608 results1 # mdl2 is best, after that mdl7, then mdl6; both mdl2 and 7 have correlation
609 # issue, so mdl6 is still best
610
611
613 # check residual plot again #
614
615
    # update mdl8 based on bad leverage
616 mdl8 <- lm(I(sqrt(median_house_value)) ~ I(longitude^(-5)) +
617
                 I(log(housing_median_age)) + I(sqrt(total_rooms)) +
618
                 I(sqrt(median_income)) + near_bay +
619
                 near_ocean + I(1/log(oneh_ocean)), data = train[-478,])
620 round(select_criteria(mdl8, n=nrow(train)),3)
621
    # mdl8 four measures: .520 -1246.194 -1245.933 -1201.234
622 # compared to 0.590
                                                6196.262 from mdl2s; improved!
                          6151.303
                                     6151.563
623 anova(md18)
624 # longitude and housing_median_age are not significant
625
    best <- regsubsets(I(sqrt(median_house_value)) \sim I(longitude\land(-5)) +
                        I(log(housing_median_age)) + I(sqrt(total_rooms)) +
626
627
                        I(sqrt(median_income)) + near_bay +
628
                        near_ocean + I(1/log(oneh_ocean)),
629
                       data = train[-478,], nbest=1)
630 summary(best)
631 # let's plot these for easier digestibility
    subsets(best, statistic="adjr2") # favor keeping all variables
632
633
    # check if dropping longitude
634
     mdl8a <- lm(I(sqrt(median_house_value)) ~
635
                   I(log(housing_median_age)) + I(sqrt(total_rooms)) +
636
                   I(sqrt(median_income)) + near_bay +
637
                   near_ocean + I(1/log(oneh_ocean)), data = train[-478,])
638
    anova(mdl8a,mdl8) # 0.03162
639
     mdl8b <- lm(I(sqrt(median_house_value)) ~ I(longitude^(-5)) +
640
                     I(sqrt(total_rooms)) +
641
                    I(sqrt(median_income)) + near_bay +
642
                    near_ocean + I(1/log(oneh_ocean)), data = train[-478,])
643 anova(mdl8b,mdl8) # 0.0004241
644 # shouldn't drop either variable
```

```
647 # preconditions recheck #
649 plot(I(sqrt(train[-478,]$median_house_value))~fitted(mdl8))
650 abline(a=0,b=1)
651 lines(lowess(sqrt(train[-478,]$median_house_value)~fitted(mdl8)), col="blue")
652
   # condition 1 holds
653 ttrain2 <- data.frame(train$longitude^(-5),log(train$housing_median_age),</p>
                     sqrt(train$total_rooms), sqrt(train$median_income), train$near_bay,
654
655
                     train$near_ocean, 1/log(train$oneh_ocean))
656 newtrain <- ttrain2[-478.]
657
    pairs(newtrain) # conditions hold
658
660 # assumptions recheck based on residual plots#
662 par(mfrow=c(3,4))
663 plot(rstandard(mdl8)~fitted(mdl8), xlab="fitted", ylab="Residuals")
664 - for(i in 1:7){
     plot(rstandard(mdl8)~newtrain[,i], xlab=names(newtrain)[i], ylab="Residuals")
665
666 - }
667 ggnorm(rstandard(mdl8))
668 qqline(rstandard(mdl8))
669 plot(I(sqrt(train[-478,]$median_house_value))~fitted(mdl8))
670 abline(a=0,b=1)
671 lines(lowess(log(train$median_house_value)~fitted(mdl8)), col="blue")
672 # standardized residue for linearity assumption check
673
   # linearity ok; normality ok
674
675 par(mfrow=c(3,3))
676 plot(mdl8$residuals~fitted(mdl8), xlab="fitted", ylab="Residuals")
677 - for(i in 1:7){
678
     plot(mdl8$residuals ~ newtrain[,i], xlab=names(newtrain)[i], ylab="Residuals")
679 - }
680 qqnorm(residuals(mdl8))
681 qqline(residuals(mdl8))
682
   # regular residue vs predictor to check independent errors and constant variance
683 # constant variance might be violated; independent error is ok, since the clusters
684 # are not separated from the other data; to be sure, we will wait for model validation
685
686 # use modified residue plots to check constant variance again
687 par(mfrow=c(3,3))
688 - for(i in 1:7){
   plot(sqrt(abs(rstandard(mdl8))) ~ newtrain[,i], xlab=names(newtrain)[i], ylab="|Standard. Residuals|^0.5", main="|Standard. Residuals|^0.5 vs Predictor",)
689
690
691
     m <- lm(sqrt(abs(rstandard(mdl8))) ~ newtrain[,i])</pre>
692
     abline(a = m$coefficients[1], b = m$coefficients[2])
693 4 }
694 # constant variance assumption is satisfied
695 vif(mdl8)
698 # model validation #
700 summary(train)
701 summary(test)
702 # looks comparable
    # let's fit the same 3 predictor model but using the test data
704
705 mdl8_test <- lm(I(sqrt(median_house_value)) ~ I(longitude^(-5)) +
                 I(log(housing_median_age)) + I(sqrt(total_rooms)) +
706
707
                 I(sqrt(median_income)) + near_bay +
708
                 near_ocean + I(log(oneh_ocean)),
709
               data = test)
710 summary(mdl8_test)
711 summary(mdl8)
712 anova(mdl8_test)
713 # model performed better for test dataset with adjr2 0.667 instead of 0.5905
714 vif(mdl8_test)
```

```
716 # check preconditions
717 plot(I(sqrt(test$median_house_value))~fitted(mdl8_test))
718 abline(a=0,b=1)
719
    lines(lowess(sqrt(test$median_house_value)~fitted(mdl8_test)), col="blue")
720 # condition 1 holds
721 ttest <- data.frame(test$longitude^(-5),log(test$housing_median_age),
722
                        sqrt(test$total_rooms), sqrt(test$median_income), test$near_bay,
723
                        test$near_ocean, log(test$oneh_ocean))
724 newtest <- ttest
725 pairs(newtest) # conditions pass
726
727
    par(mfrow=c(3,4))
    plot(rstandard(mdl8_test)~fitted(mdl8_test), xlab="fitted", ylab="Residuals")
728
729 - for(i in 1:7){
    plot(rstandard(mdl8_test)~newtest[,i], xlab=names(newtest)[i], ylab="Residuals")
730
731 ^ }
732 qqnorm(rstandard(md18_test))
733 gqline(rstandard(mdl8_test))
734 plot(I(sqrt(train$median_house_value))~fitted(mdl8_test))
    abline(a=0,b=1)
735
736 lines(lowess(log(train$median_house_value)~fitted(mdl8_test)), col="blue")
737 # standardized residue for linearity assumption check
738 # linearity ok; normality ok
739
740 par(mfrow=c(3,3))
741 plot(mdl8_test$residuals~fitted(mdl8_test), xlab="fitted", ylab="Residuals")
742 - for(i in 1:7){
743
     plot(mdl8_test$residuals ~ newtest[,i], xlab=names(newtest)[i], ylab="Residuals")
744 - }
745 qqnorm(residuals(mdl8_test))
746 qqline(residuals(mdl8_test))
747
    # regular residue vs predictor to check independent errors and constant variance
748 # constant variance might be violated; independent error is ok, since the clusters
749 # are not separated from the other data; to be sure, we will wait for model validation
750
751
   # use modified residue plots to check constant variance again
752
    par(mfrow=c(3,3))
753 - for(i in 1:7)
754
    plot(sqrt(abs(rstandard(mdl8_test))) ~ newtest[,i], xlab=names(newtest)[i],
755
           ylab="|Standard. Residuals|^0.5", main="|Standard. Residuals|^0.5 vs Predictor",)
      m <- lm(sqrt(abs(rstandard(mdl8_test))) ~ newtest[,i])</pre>
756
757
      abline(a = m$coefficients[1], b = m$coefficients[2])
758 4 }
759 # constant variance not a horizontal line for several variables
760
762 # summary statistics of sample data set #
764 str(train)
765 summary(train)
766 apply(train[,2:14], 2, mean)
767 apply(train[,2:14], 2, sd)
768
769 #histograms
770 par(mfrow=c(4,4))
771 - for (i in 1:9){
772
       hist(as.numeric(train[,(i+1)]), breaks=10, main=sprintf(
          '%s of Sample Californian Homes", pred[i]), xlab=pred[i], ylab= "Count")
773
774 ^ }
775 + for (i in 10:13){
776
       hist(as.numeric(train[,(i+1)]), breaks=2, main=sprintf(
         "%s of Sample Californian Homes", pred[i]), xlab=pred[i], ylab = "Count")
777
778 - }
779
```

```
780 # boxplot 1
781 par(mfrow=c(3,3))
782 - for (i in 1:9){
783
        boxplot(as.numeric(train[,(i+1)]), xlab=pred[i], ylab="Value")
784 - }
785
786 # scatter plot of longitude and latitude
      ocean_pts <- which(train$near_ocean == 1)
787
788 bay_pts <- which(train$near_bay == 1)
     inland_pts <- which(train$inland == 1)
     oneh_pts <- which(train$oneh_ocean == 1)
790
      neither <- which((train$oneh_ocean == 0)&(train$inland == 0))</pre>
791
792
      mix <- sample(train$x, 20, replace = FALSE)</pre>
793
794
      par(mfrow=c(1,1))
795
      library(png)
796
     img <- readPNG('C:/Users/i5/Downloads/STA302 Data Analysis I/Mini project 2/map0.png')
797
      plot(train$longitude, train$latitude, xlab=pred[2], ylab=pred[1],
            main ="Sample Locations", type = "n")
798
      rasterImage(img,xleft=-125, xright=-115, ybottom=32, ytop=42.5)
799
      points(train$longitude[inland_pts],train$latitude[inland_pts], col = "darkgreen",
801
              pch = 16
802
      points(train$longitude[oneh_pts],train$latitude[oneh_pts], col = "dodgerblue4", pch = 16)
      points(train$longitude[ocean_pts], train$latitude[ocean_pts], col = "blue", pch = 16)
803
      points(train$longitude[bay_pts], train$latitude[bay_pts], col = "cyan", pch = 16) legend("topright", legend = c("Bay", "Ocean", "Drive to Ocean", "Inland"), col = c("cyan", "blue", "dodgerblue4", "darkgreen"), lty = 1)
804
805
806
807
808
     #graph in term of price points
      low <- which(train$median_house_value <= 110025)</pre>
810
      average <- which(110025<train$median_house_value <267525)
      high <- which(train$median_house_value >= 267525)
      plot(train$longitude, train$latitude, xlab=pred[2], ylab=pred[1],
812
     main ="Sample Price", type = "n")
rasterImage(img,xleft=-125, xright=-115, ybottom=32, ytop=42.5)
813
814
     points(train$longitude[low],train$latitude[low], col = "yellow", pch = 16)
points(train$longitude[average],train$latitude[average], col = "tan1", pch = 16)
points(train$longitude[high],train$latitude[high], col = "red3", pch = 16)
legend("topright", legend = c("Low", "Medium", "High"), col = c("yellow", "tan1")
815
816
817
81.8
                                                                                  "red3"), lty = 1)
819
820
     # update train based on new dataset
821
822
     # relationship between each variables
      pairs(train[,2:14], lower.panel=NULL)
823
824
825 # relationship with median house values
826 par(mfrow=c(4,4))
827 - for (i in 1:11){
        plot(train[, (i+1)], train$median_house_value, xlab = pred[i], ylab = pred[9])
828
829 ^ }
830
831 # correlation table of all relationships
832 library(xtable)
833 cors <- NULL
834 - for (i in 1:13){
835
         cors <- c(cors, cor(train$median_house_value, train[, (i+1)]))</pre>
836 4 }
     cdf <- data.frame("Correlation" = cors, "Predictors" = pred)</pre>
837
838 cdf[order(-abs(cdf$Correlation)),]
```