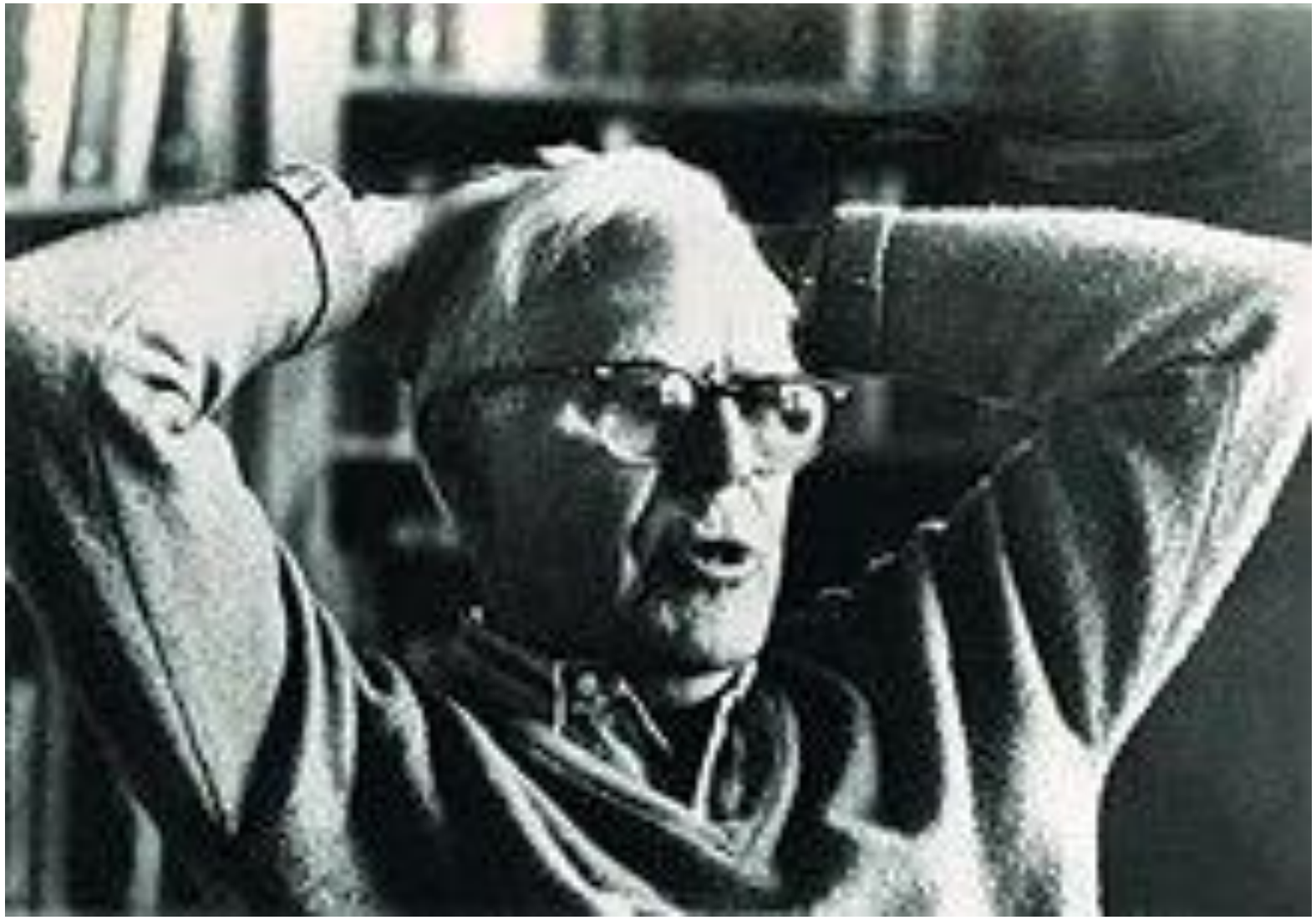
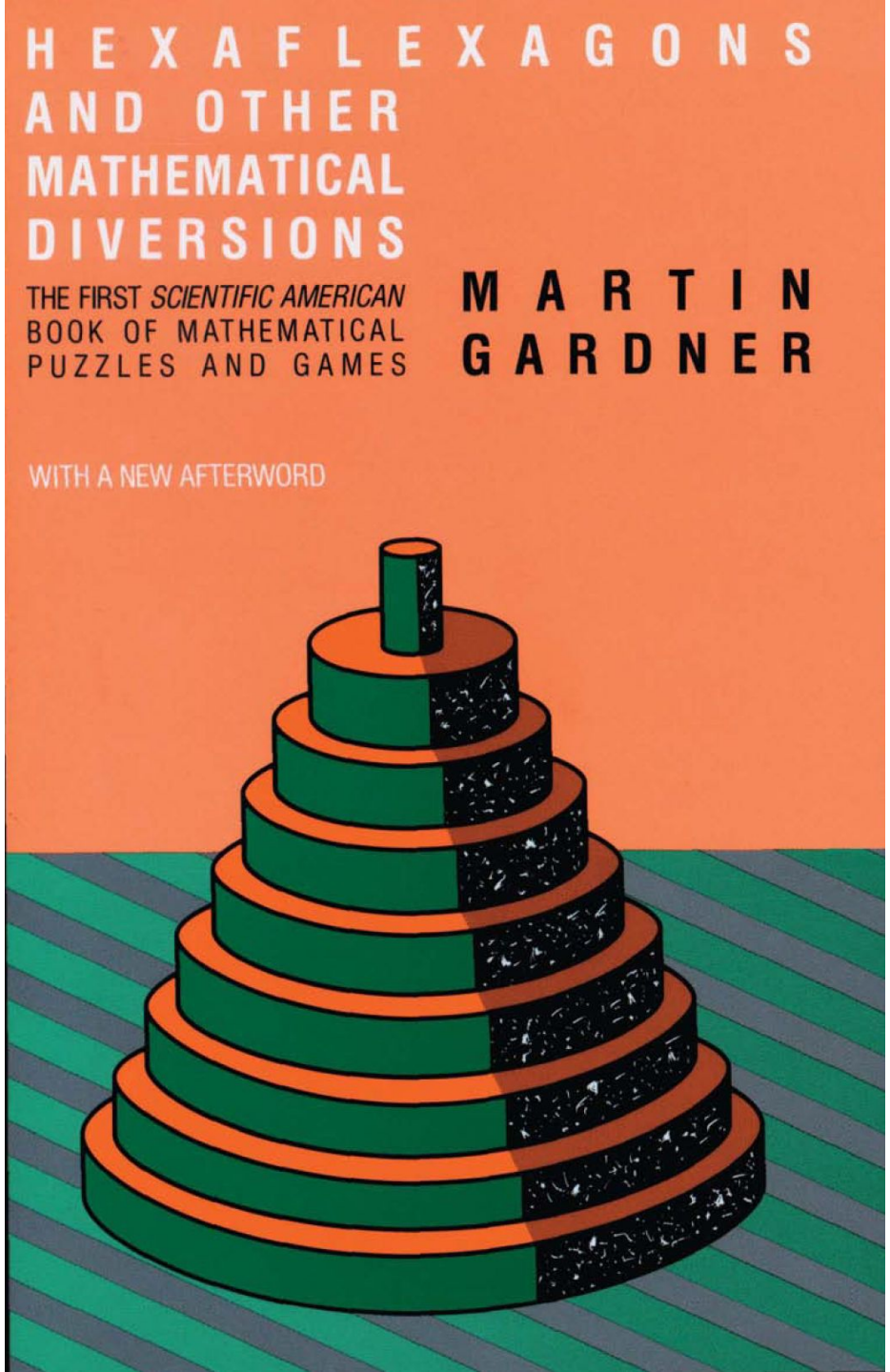


Martin Gardner, 1914 -- 2010



CHAPTER FOUR

■

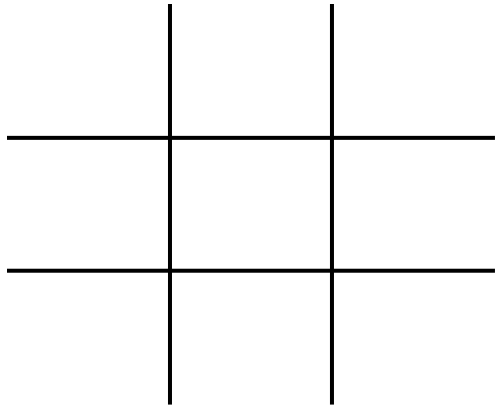
Ticktacktoe

WHO HAS NOT as a child played ticktacktoe, that most ancient and universal struggle of wits of which Wordsworth wrote (*Prelude*, Book I):

> *At evening, when with pencil, and smooth slate*
> *In square divisions parcelled out and all*
> *With crosses and with cyphers scribbled o'er,*
> *We schemed and puzzled, head opposed to head*
> *In strife too humble to be named in verse.*

The game

9 possible starting positions for X?

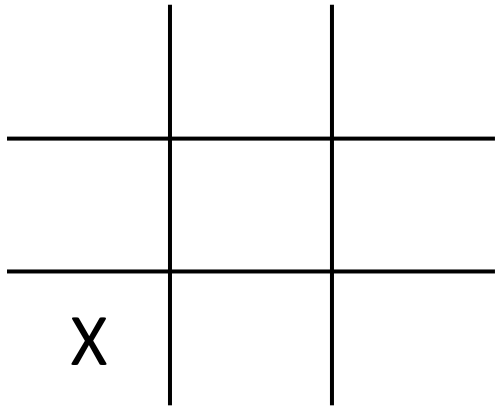


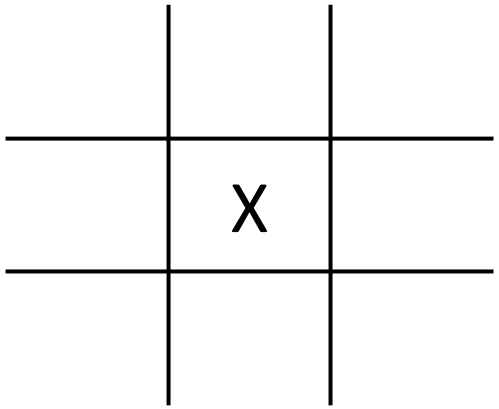
Player 1

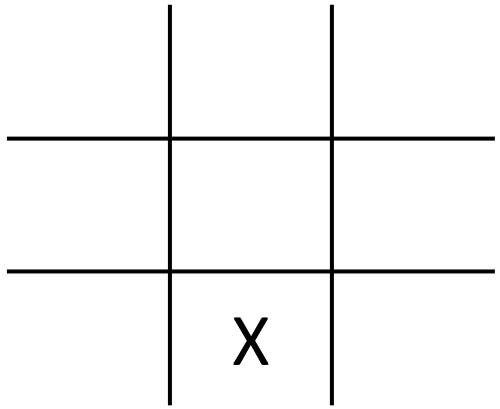
X

Player 2

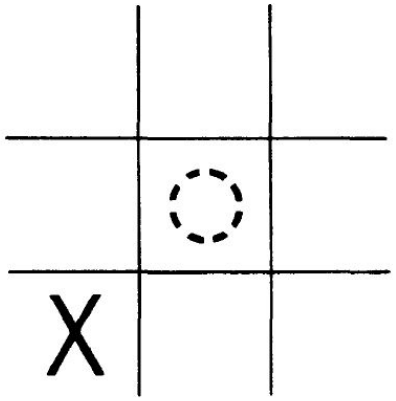
O

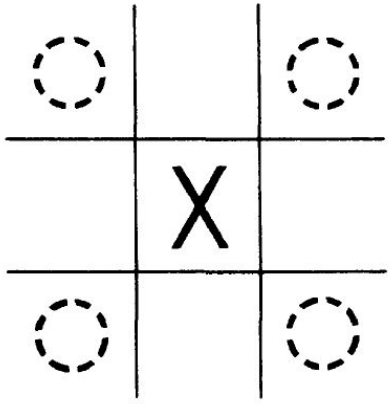


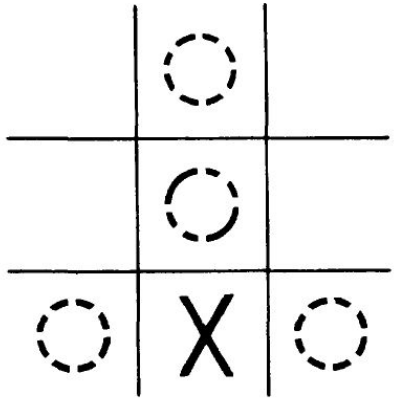


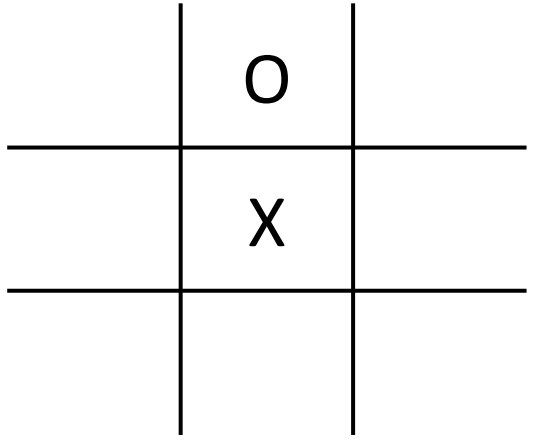


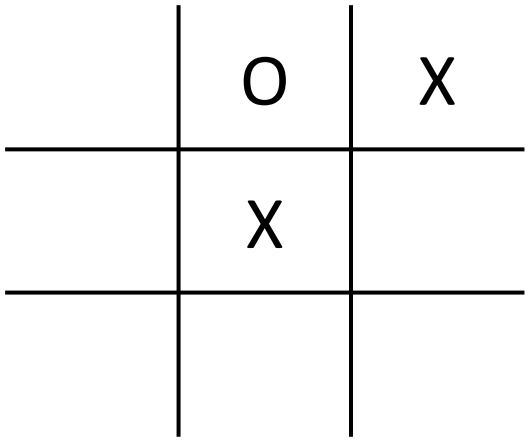
Response to Starting Positions

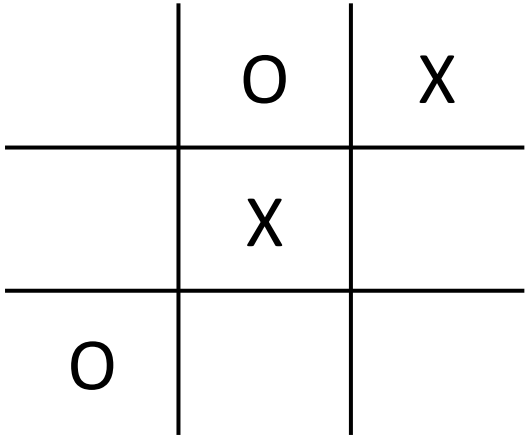


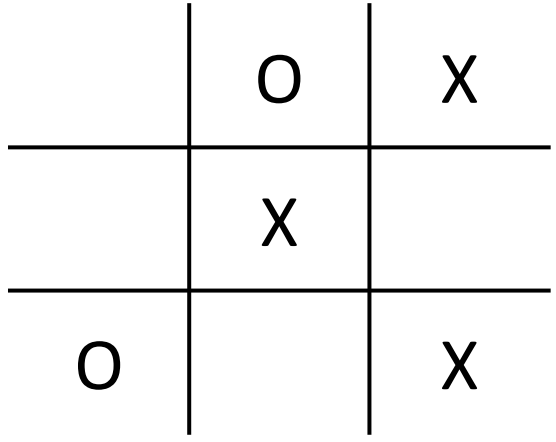












Characteristics

Game board and rules

Two player

Turn-based

Perfect information

Deterministic

Finite

A player can win, lose or draw a game

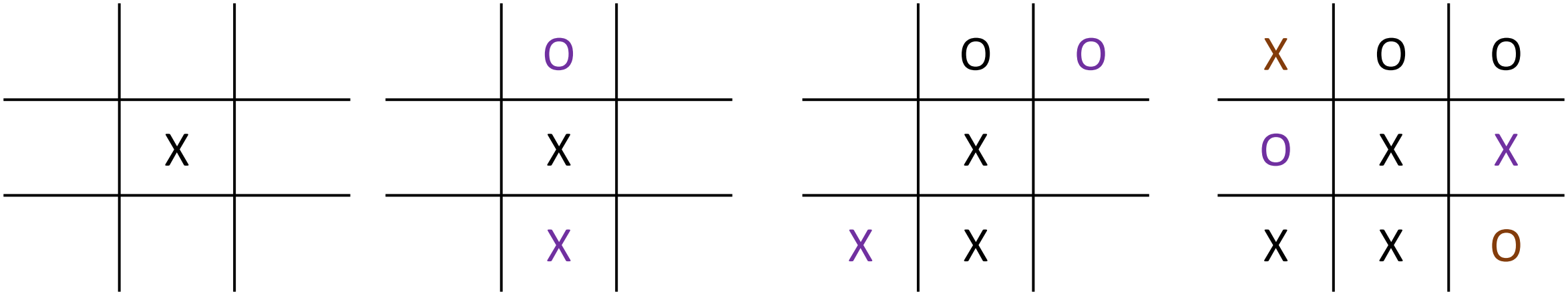
Fundamental theorem: Either the first player or the second can force a win – not both

Zermelo’s theorem: Either the first player can force a win, the second can force a draw or both can at least force a draw

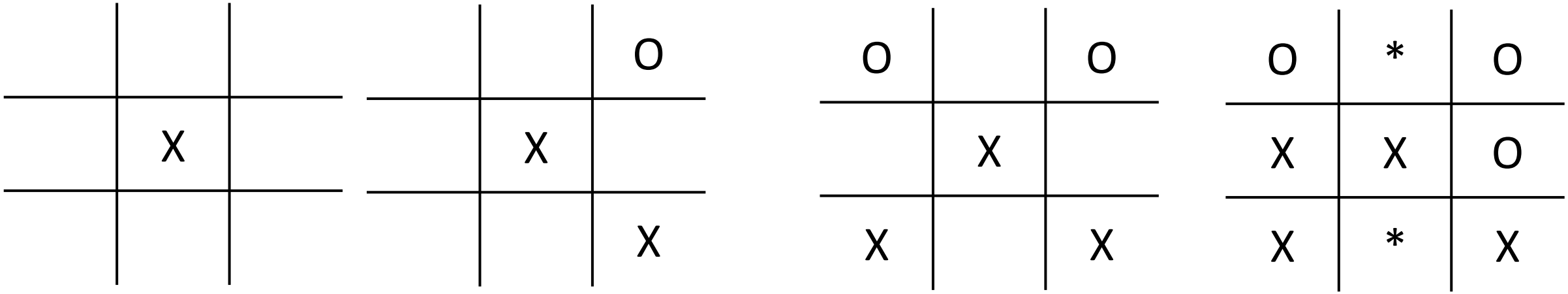
Interesting Variations

Toetacktick

First to 3 in a row loses!



First player can force a draw – can the second?

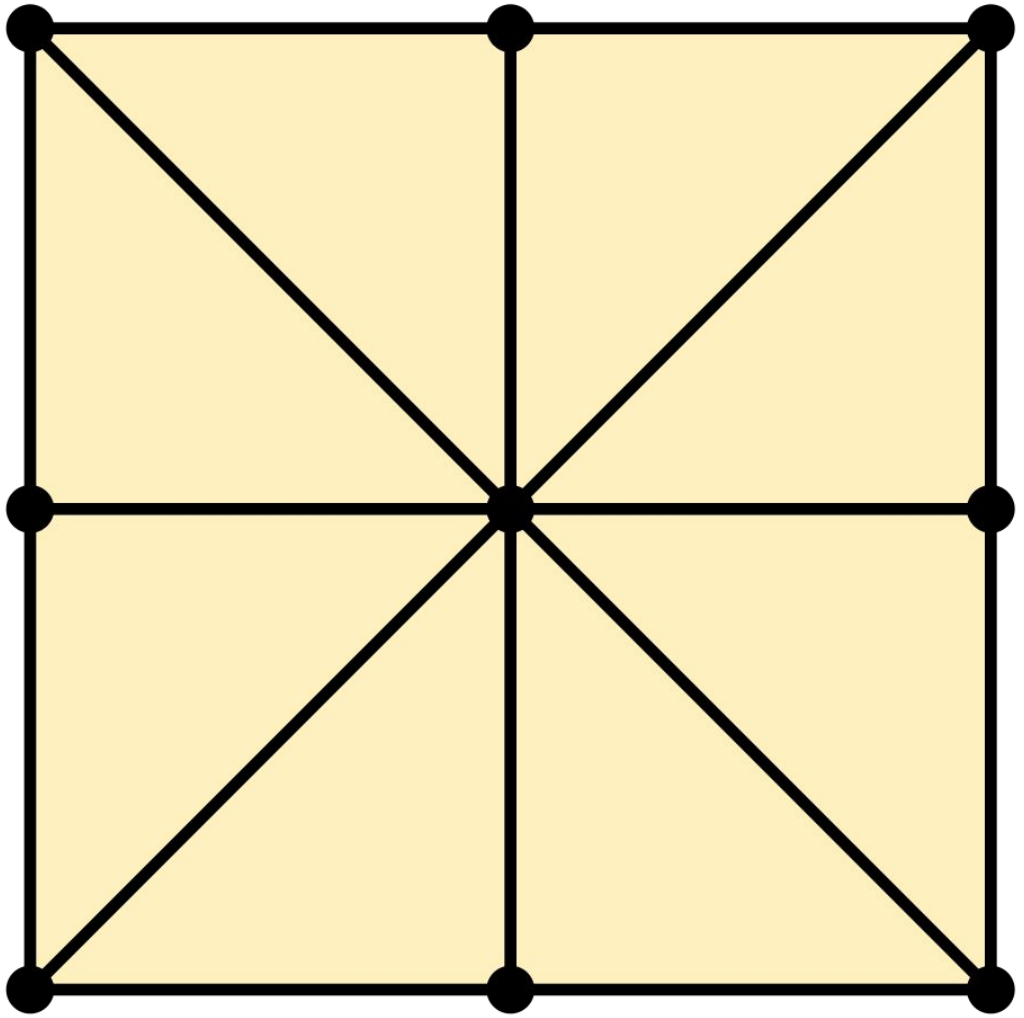


Moving Counters – Three men’s Morris

3 counters each

Once all placed, move counters

Aim to get 3 in a row



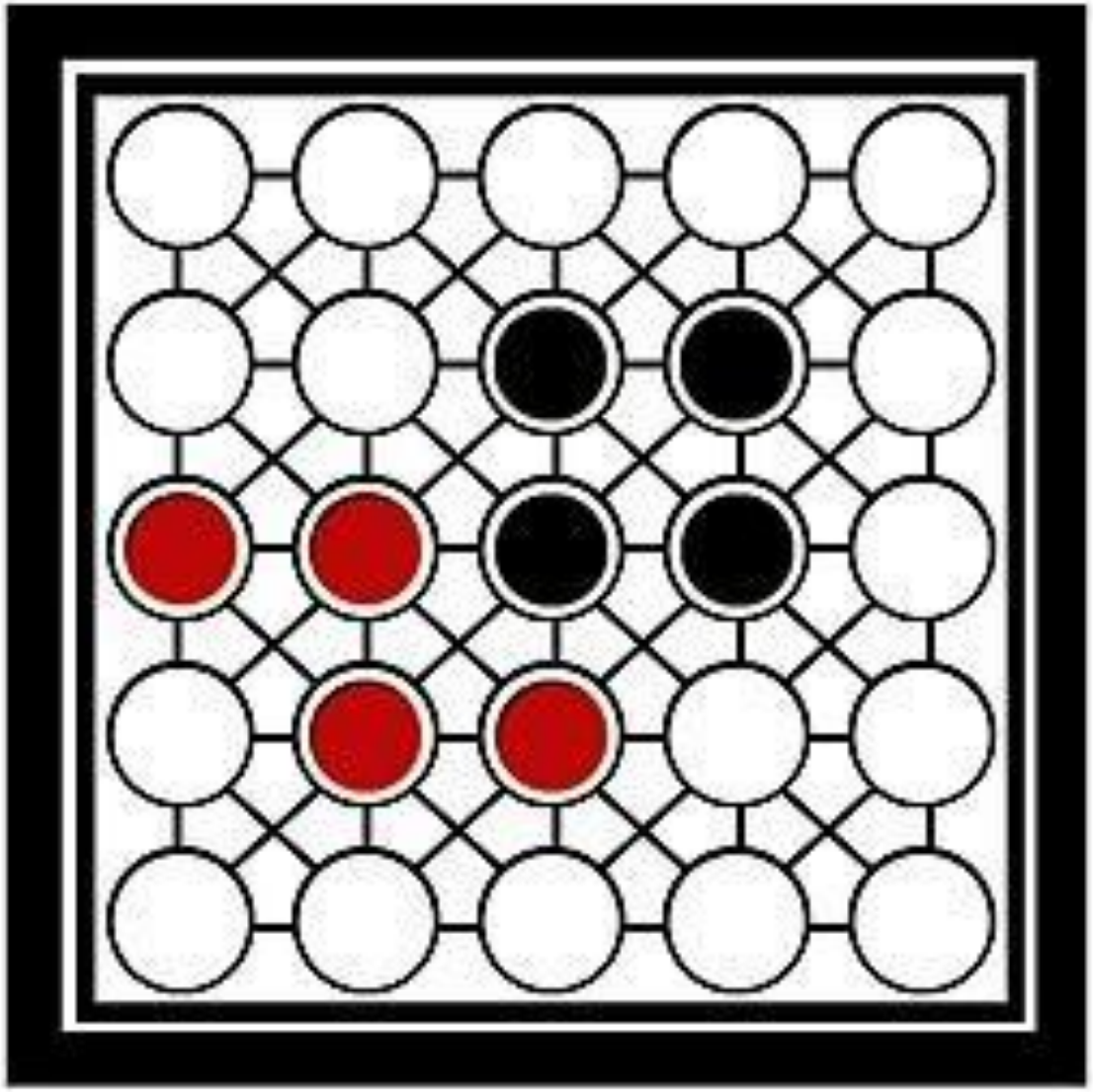
Moving Counters – Teeko

4 markers each

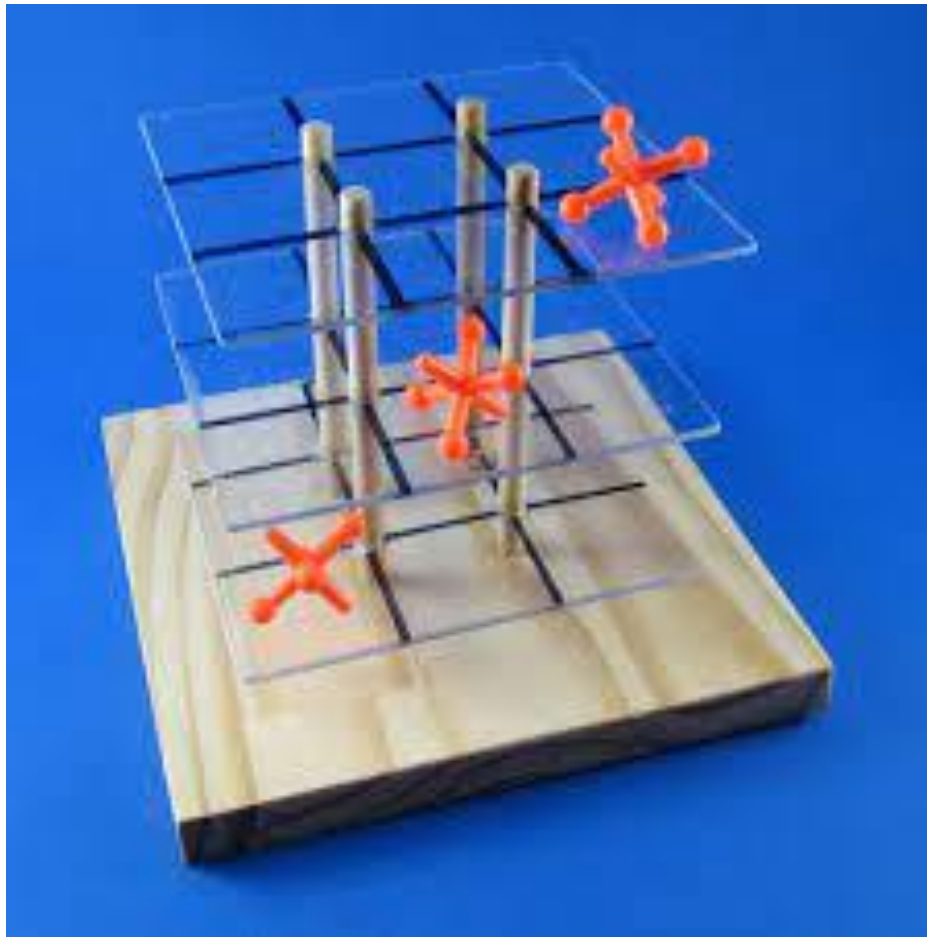
Place all 8

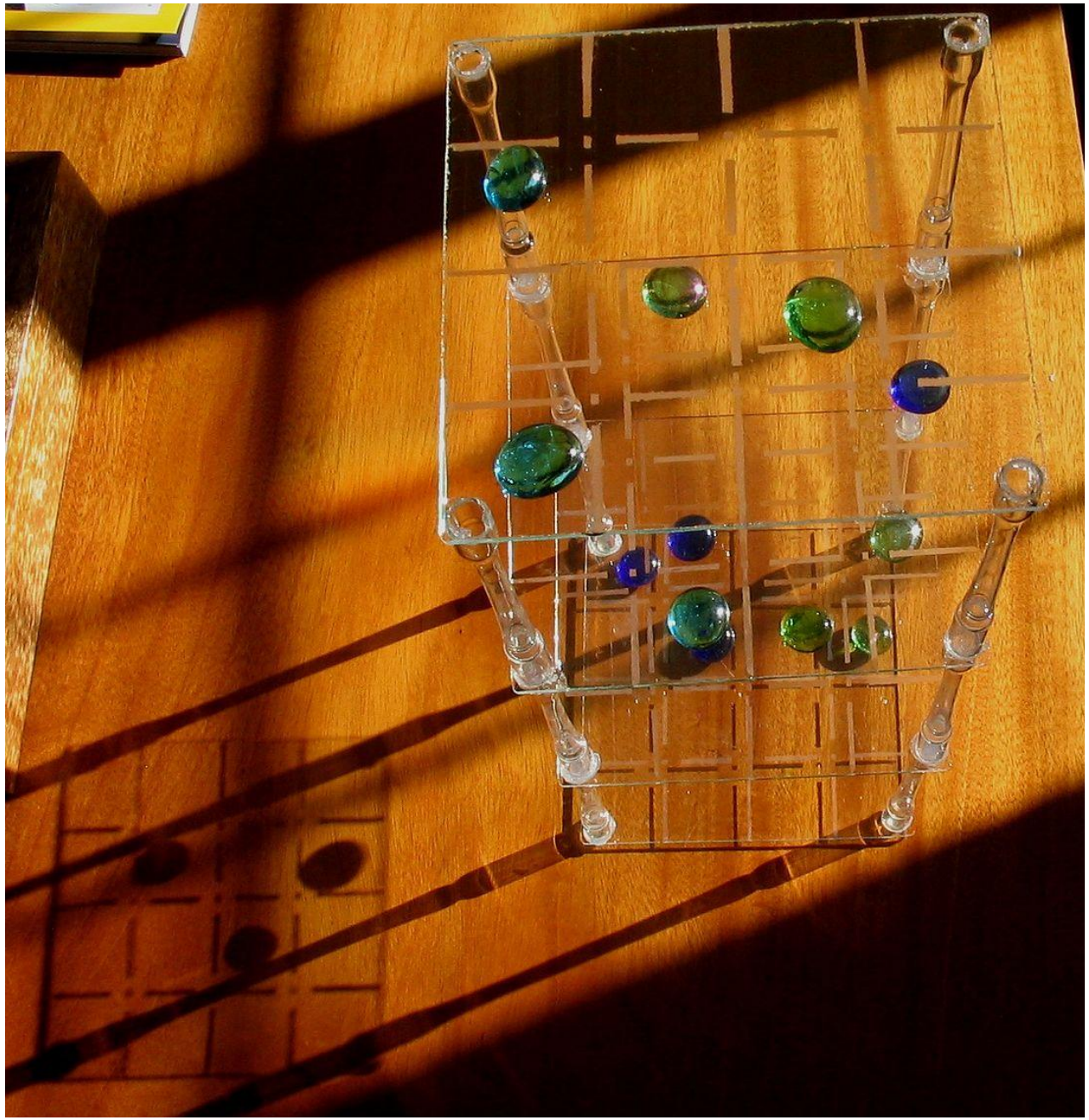
Then can move along lines

Aim to get all 4 in a line



3D Ticktacktoe





3D Ticktacktoe

3x3x3 Board?

27 spaces

1st player gets 14 moves if a draw occurs

No way to place 14 marks without forming a line!

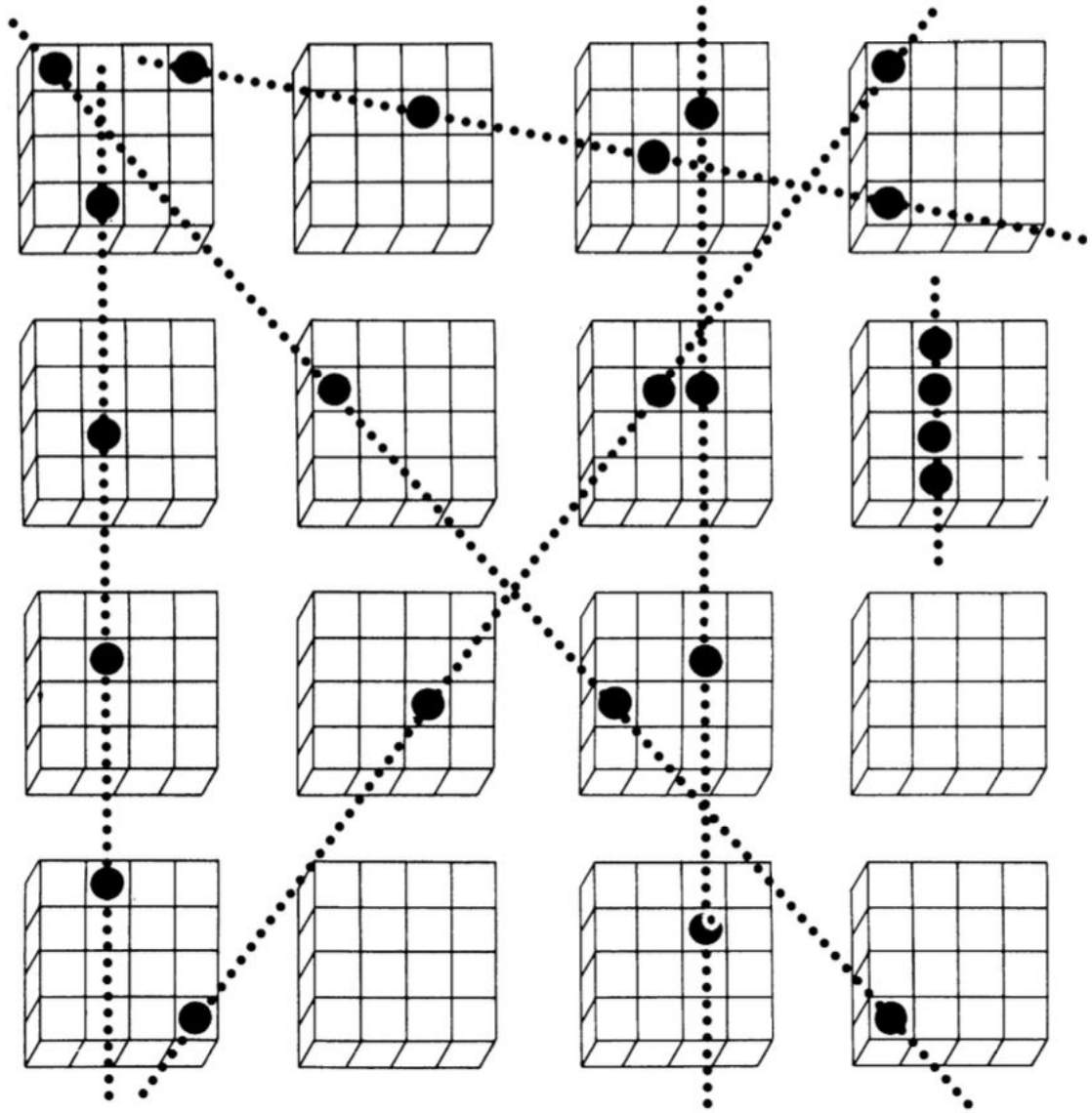


FIG. 19.
Four-dimensional ticktacktoe. Dotted lines show some winning plays.

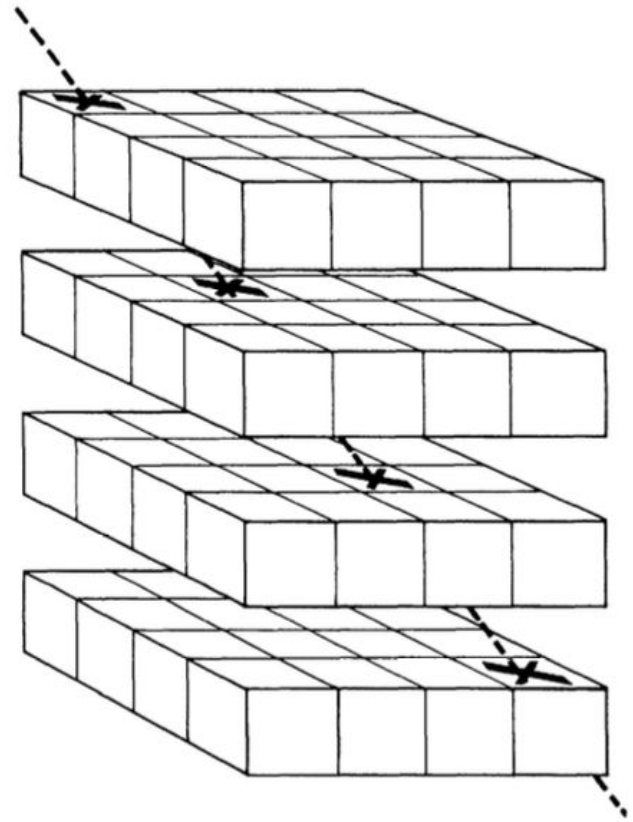
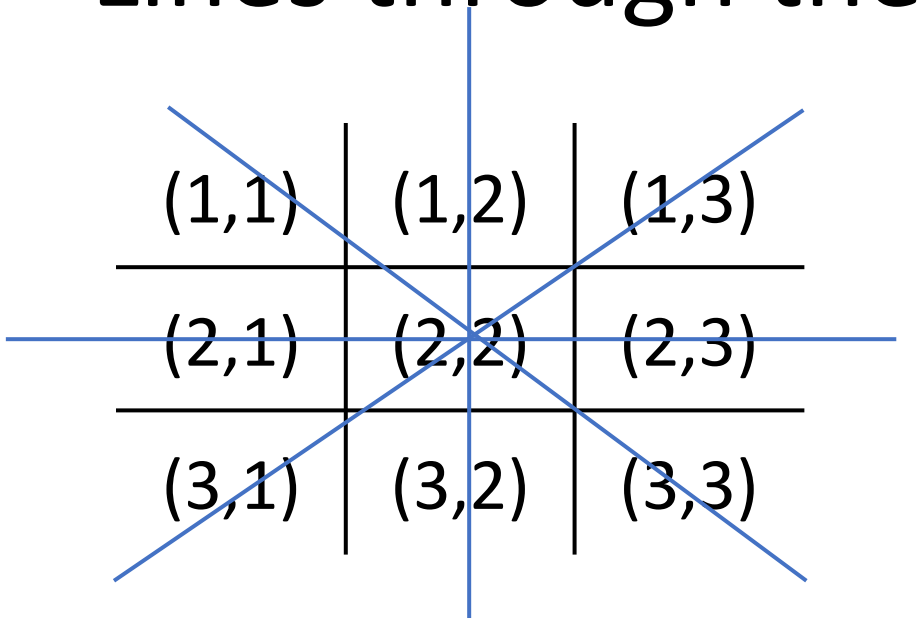
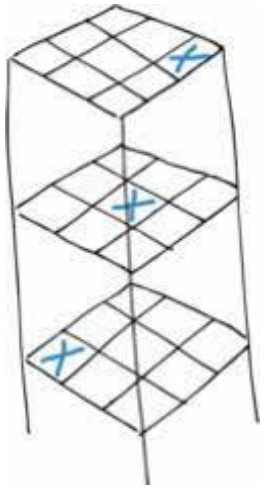


FIG. 20.
The assembled cube.

Lines through the midpoint in 3D ticktacktoe





13 lines

4 in the middle plane

3 +2 each two verticals = 5

4 diagonals

(1,1,1)	(1,2,1)	(1,3,1)
(2,1,1)	(2,2,1)	(2,3,1)
(3,1,1)	(3,2,1)	(3,3,1)

(1,1,2)	(1,2,2)	(1,3,2)
(2,1,2)	(2,2,2)	(2,3,2)
(3,1,2)	(3,2,2)	(3,3,2)

(1,1,3)	(1,2,3)	(1,3,3)
(2,1,3)	(2,2,3)	(2,3,3)
(3,1,3)	(3,2,3)	(3,3,3)