離散數學 107-2

Homework 08

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Homework 08 題目

(Prob. 1) page 536, chapter 8.1 Exercise 2
(Prob. 2) page 552, chapter 8.2 Exercise 32
(Prob. 3) page 561, chapter 8.3 Exercise 8
(Prob. 4) page 575, chapter 8.4 Exercise 4(a)
(Prob. 5) page 584, chapter 8.5 Exercise 2
(Prob. 6) page 591, chapter 8.6 Exercise 8

注意事項

(a) 要熟悉 LaTeX 請翻閱 lshort。

(b) 記得在最後一頁，回報完成作業小時數 (估算，取整數)。

(c) 將檔案夾命名為 hw08_107820xxx，將檔案夾壓縮成 hw08_107820xxx.zip，上傳到網路學園。

(d) LaTeX 數學符號請查此表: List of LaTeX mathematical symbols。

(e) 作業抄襲，以零分計。作業提供給他人抄襲，以零分計。

(f) 作業遲交一週內成績打五折，作業遲交超過一週以零分計。

Problem 1 (8.1 Exercise 2)

(a) A permutation of a set with n elements consists of a choice of a first element (which can be done in n ways), followed by a permutation of a set with n $-$ 1 elements.
Therefore $P_0 = 1, P_1 = 1, Pn = nPn{-}1,\ when$ n≥ 2.

Problem 1 (8.1 Exercise 2)

(b)

$$\begin{aligned} P_n &= nP_n - 1 \\ &= n(n-1)P_{n-2} \\ &= n(n-1)(n-2)P_{n-3} \\ &= ..... \\ &= n(n-1)(n-2).....(3)(2)P_1 \\ &= n(n-1)(n-2).....(3)(2)(1) \\ &= n! \end{aligned}$$

Problem 2 (8.2 Exercise 32)

The associated homogeneous recurrence relation is $a_n = 2a_{n-1}$.
We easily solve it to obtain $a_n^{(h)} = 2_n$.
Next we need a particular solution to the given recurrence relation.
By Theorem 6 we want to look for a function of the form $a_n = cn \cdot 2_n$.
We plug this into our recurrence relation and obtain cn
$\cdot 2^n = 2c(n-1)2^{n-1} + 3 \cdot 2^n$.
We divide through by 2_{n-1}.
Obtaining 2cn = 2c(n − 1) + 6, whence with a little simple algebra
c = 3. $a_n = cn \cdot 2^n$. Therefore the particular solution become
$a_n^{(h)} = cn \cdot 2^n = 3n2^n$ So the general solution is the sum the homogeneous solution and this particular solution, namely

$$\begin{aligned} a_n &= a_n^{(h)} + a_n^{(p)} \\ &= \alpha \cdot 2^n + 3n2^n \\ &= (3n + \alpha)2^n \end{aligned}$$

Problem 3 (8.3 Exercise 8)

(a) $f(2) = 2 \cdot 5 + 3 = 13$

(b) $f(4) = 2 \cdot 13 + 3 = 29$
$f(8) = 2 \cdot 29 + 3 = 61$

(c) $f(16) = 2 \cdot 61 + 3 = 125$
$f(32) = 2 \cdot 125 + 3 = 253$
$f(64) = 2 \cdot 253 + 3 = 509$

(d) $f(128) = 2 \cdot 509 + 3 = 1021$
$f(256) = 2 \cdot 1021 + 3 = 2045$
$f(512) = 2 \cdot 2045 + 3 = 4093$
$f(1024) = 2 \cdot 4093 + 3 = 8189$

Problem 4 (8.4 Exercise 4(a))

$$G(x) = a_0 + a_1 x + a_2 x^2 + ... + a_k x^k + ... = \sum_{k=0}^{+\infty} a_k x^k$$

Apparently all the terms are 0 except for the seven -1' s shown.

$$\begin{aligned} G(x) &= -1 - x - x^2 - ... - x^6 + 0x^7 + 0x^8 + ... \\ &= -1 - x - x^2 - ... - x^6 \\ &= \sum_{k=0}^{6} -x^k \\ &= -\sum_{k=0}^{6} x^k \\ &= -\frac{1-x^7}{1-x} \end{aligned}$$

Problem 5 (8.5 Exercise 2)

$$
\begin{aligned}
|C \cup D| &= |C| + |D| - |C \cup D| \\
&= 345 + 212 - 188 \\
&= 369
\end{aligned}
$$

Problem 6 (8.6 Exercise 8)

$$5^7 - C(5,1)\cdot(5-1)^7 + C(5,2)\cdot(5-2)^7 + C(5,3)\cdot(5-3)^7 + C(5,4)\cdot(5-4)^7$$
$$= 16800$$

完成作業小時數

完成作業小時數:共 5 小時(估算，取整數)