離散數學 107-2

Homework 01

姓名: 葉子瑄。學號: 107590041

截止收件: 2019.03.20 (Wednesday) 23:59 pm

Week-05

Homework 01 題目

(Prob. 1) page 16, chapter 1.1 Exercise 34(e) 小題
(Prob. 2) page 24, chapter 1.2 Exercise 8
(Prob. 3) page 38, chapter 1.3 Exercise 10(c) 小題
(Prob. 4) page 58, chapter 1.4 Exercise 28
(Prob. 5) page 69, chapter 1.5 Exercise 10
(Prob. 6) page 82, chapter 1.6 Exercise 6
(Prob. 7) page 96, chapter 1.7 Exercise 30
(Prob. 8) page 113, chapter 1.8 Exercise 6

注意事項

(a) 要熟悉 LaTeX 請翻閱 lshort。

(b) 記得在最後一頁，回報完成作業小時數 (估算，取整數)。

(c) 將檔案夾命名為 hw01_107820xxx，將檔案夾壓縮成 hw01_107820xxx.zip，上傳到網路學園。

(d) LaTeX 數學符號請查此表: List of LaTeX mathematical symbols。

(e) 作業抄襲，以零分計。作業提供給他人抄襲，以零分計。

(f) 作業遲交一週內成績打五折，作業遲交超過一週以零分計。

Problem 1 (1.1 Exercise 34(e) 小題)

Table: Truth Table for the Compound Propositions

p	q	$\neg p$	$(q \to \neg p)$	$(p \leftrightarrow q)$	$(q \to \neg p) \leftrightarrow (p \leftrightarrow q)$
T	T	F	F	T	F
T	F	F	T	F	F
F	T	T	T	F	F
F	F	T	T	T	T

Problem 2 (1.2 Exercise 8)

(a) $r \wedge \neg p$

(b) $q \rightarrow (p \wedge r)$

(c) $\neg q \rightarrow \neg r$

(d) $(\neg p \wedge r) \rightarrow q$

Problem 3 (1.3 Exercise 10(c) 小題)

Proof.

$$
\begin{aligned}
(p \to \neg q) \to (\neg p \to q) &\equiv \neg(p \to \neg q) \lor (\neg p \to q) \\
&\equiv \neg(\neg p \lor \neg q) \lor (\neg p \to q) \\
&\equiv (p \land q) \lor (p \lor q) \\
&\equiv (p \lor q) \\
&\equiv \neg p \to q
\end{aligned}
$$

◀

Problem 4 (1.4 Exercise 28)

let $R(x)$ be "x is in the correct place,"
let $E(x)$ be "x is in excellent condition,"
let $T(x)$ be "x is a [or your] tool,"
and let the domain of discourse be all things.

(a) $\exists x \, \neg R(x)$

(b) $\forall x \, (R(x) \wedge E(x))$

(c) $\forall x \; T(x)(R(x) \wedge E(x))$

(d) $\exists x \, (\neg(R(x) \wedge E(x))$

(e) $\neg\exists x \; T(x)(R(x) \wedge \neg E(x))$

Problem 5 (1.5 Exercise 10)

(a) $\forall x\ F(x, \text{Fred})$

(b) $\forall y\ F(\text{Evelyn}, y)$

(c) $\forall x\ \exists y\ F(x, y)$

(d) $\neg\exists x\ \forall y\ F(x, y)$

(e) $\exists x\ \forall y\ F(x, y)$

(f) $\neg\exists x\ (F(x, \text{Fred}) \wedge F(x, \text{Jerry}))$

(g) $\exists y\ \exists z\ (F(\text{Nancy}, y) \wedge (F(\text{Nancy}, z) \wedge x \neq y \wedge \forall w\ (F(\text{Nancy}, w) \rightarrow (w = y \vee w = z)))$

(h) $\exists y\ (\forall x\ F(x, y) \wedge (\forall z\ ((\forall w\ F(w, z)) \rightarrow z = y))$

(i) $\forall x\ \neg F(x, x)$

(j) $\exists x\ \exists y\ (F(x, y) \wedge (forallz\ (F(x, z) \rightarrow z = y \vee z = x))$

Problem 6 (1.6 Exercise 6)

let r be the proposition "It rains,"
let f be the proposition "It is foggy,"
let s be the proposition "The sailing race will be held,"
let l be the proposition "The life saving demonstration will go on," and
let t be the proposition "The trophy will be awarded."

Step	推導	Reason
1	$\neg t$	Hypothesis
2	$s \rightarrow t$	Hypothesis
3		Modus tollens using (1) and (2)
4	$(\neg r \vee \neg f) \rightarrow (s \wedge l)$	Hypothesis
5	$\neg(s \wedge l) \rightarrow \neg(\neg r \vee \neg f)$	Contrapositive of (4)
6	$negs \vee \neg l) \rightarrow (r \wedge f)$	De Morgan's law and double negative
7	$\neg s \vee \neg l)$	Addition, using (3)
8	$r \wedge f)$	Modus ponens using (6) and (7)
9	◦	Simplification using (8)

Problem 7 (1.7 Exercise 30)

For the "if" part, there are two cases.
If $m = n$
$m - n = 0$
If $m = -n$
$m + n = 0$

For the "only if" part, we suppose that $m^2 = n^2$.
$(m - n)(m + n) = 0$

Problem 8 (1.8 Exercise 6)

題目: Use a proof by cases to show that $\min(a, \min(b, c)) = \min(\min(a, b), c)$ whenever a, b, and c are real numbers.

Proof.

Case 1: If a is smallest.

$a \leq \min(b, c), a \leq b, a \leq c$

$\min(a, b = a$

$a = \min(\min(a, b), c) = \min(a, c) = a$

Case 2: If a is smallest.

$b \leq \min(b, c), b \leq b, b \leq c \min(a, b = b$

$b = \min(\min(a, b), c) = \min(b, c) = b$

Case 3: If c is smallest.

$c \leq \min(b, c), c \leq b, c \leq c \; c \leq \min(a, b)$

$c = \min(\min(a, b), c) = c$

◀

完成作業小時數

完成作業小時數:共 5 小時(估算，取整數)