Digital Image Processing

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Homework3

1.

a.

F(0,0) is the average value of all pixels which is: 85.

b.

DC coefficient (which is equal to F(0,0)) means the average value of all values to be transformed. It is indeed the 0 Hz term. The terminology does indeed come from AC/DC electricity - all the non-zero bins correspond to non-zero frequencies, i.e. "AC components" in an electrical context, whereas the zero bin corresponds to a fixed value, the mean of the signal, or "DC component" in electrical terms.

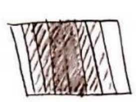
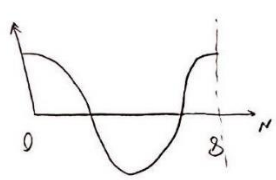
c.

$$h(m,n,u,v) = h(m,n,0,2) = \frac{\sqrt{2}}{8} \cos 0 \, \cos \frac{(2n+1)\pi}{8}$$

$$\rightarrow h(m,n)_{u,v=0,2} = \frac{\sqrt{2}}{8} \cos \frac{(2n+1)\pi}{8}$$

$$\Longrightarrow \frac{2\pi}{\frac{2\pi}{8}} = \boxed{8 = N} \rightarrow \text{Period of the axis } V$$

Since the $h(m,n)_{u,v=0,2}$ function has the phase of $\frac{\pi}{8}$ and it is not a function of U, the basis will be:

2D:  or in 1D: 

d.

F(u)*e^{ju5}

e.

$\frac{1}{2\pi}$F(u)*F(u) ('*' means convolution)

f.

F(-u)

g.

$\frac{1}{|5|}F(u/5).e^{j(u/5)8}$

h.

2π.x(-u)

i.

The DFT is as the following:

```
array([[16.5 +0.j  ,  5.   -5.j  ,  5.   -0.j  ,  4.99+4.99j],
       [ 0.49-0.51j, -0.   -0.j  , -0.   -0.j  ,  0.   -0.01j],
       [ 0.5 -0.01j, -0.   -0.j  ,  0.   -0.j  ,  0.   -0.j  ],
       [ 0.52+0.48j,  0.   -0.01j,  0.01-0.01j,  0.01+0.j  ]])
```

j.

The DFT of the transformed matrix is as the following:

```
array([[ 0.   +0.j  ,  0.   -0.j  ,  0.5 +0.j  , -0.   -0.j  ],
       [ 0.   -0.j  ,  0.   +0.j  ,  0.5 +0.49j,  0.   -0.01j],
       [ 5.   +0.01j,  4.98+5.01j, 16.5 +0.06j,  5.02-5.j  ],
       [-0.01-0.01j,  0.   -0.01j,  0.48-0.52j, -0.01-0.j  ]])
```

k.

Values of 2^{nd} matrix are just shifted (in other words, first matrix is centered). For example, value of 16.5 is shifted from (0,0) to (2,2) coordinates.

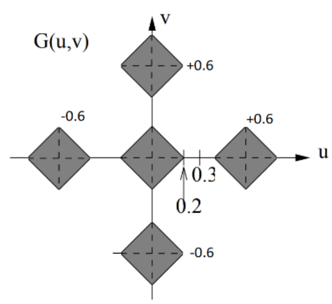
l.

```
array([[ 0.44+0.j  , -0.06-0.29j, -0.06+0.29j],
       [ 0.28+0.48j,  0.28-0.1j , -0.22+0.19j],
       [ 0.28-0.48j, -0.22-0.19j,  0.28+0.1j ]])
```

m.

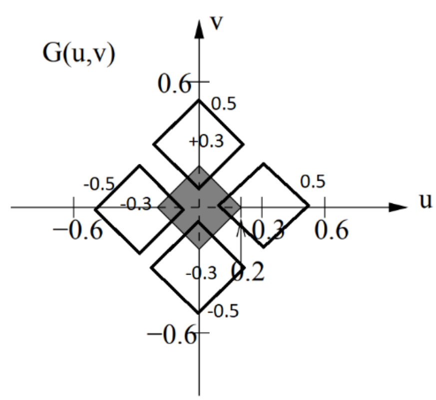
DFT will be replicated by distance of (1/ Δ) in both directions. The result is as follows:

1/ Δ = 0.6



n.

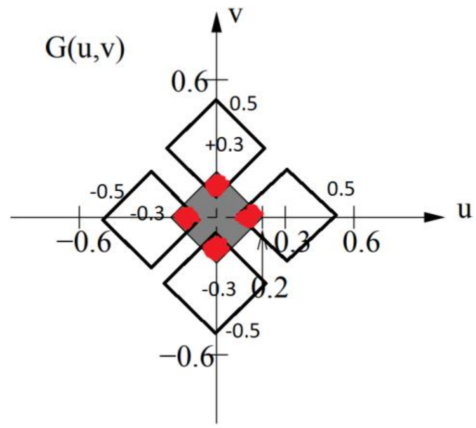
This time: **1/ Δ = 0.3**



o.

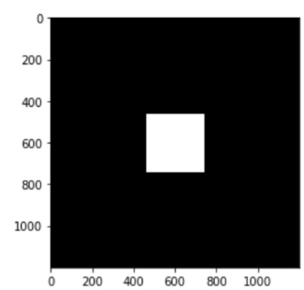
Function h(x,y) shows aliasing. Any values of “Δ” which makes (1/ Δ) to be lower than 0.4 (consequently any value of “Δ” greater than 1/0.4) will result in aliasing.

Aliasing regions are marked by red color:

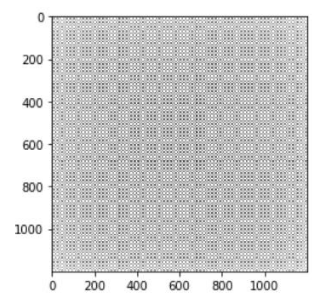


p.

The rotated shape is a square centered at origin:



And the 2D Fourier transform will be as follows:



q.

Just the same as what mentioned in part 'o', any values of "Δ" which makes (1/ Δ) to be lower than, approximately, 0.28 (consequently any value of "Δ" greater than 1/0.28) will result in aliasing.

2.

a.

Equivalent of $H_2(u,v)$ in frequency domain: Guassian low pass filter

Equivalent of $H_2(u,v)$ in frequency domain: Gaussian high pass filter

b.

H_1 and H_2 are low pass and high pass respectively.

c.

Firstly, we have to compute $H_1(x,y)$ and $h_2(x,y)$ in a 3x3 domain matrix:

$$H_1 = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$$H_2 = \begin{bmatrix} 0 & 0.43j & 0.04j \\ 0.43j & 0 & 0.86j \\ -0.43j & -0.86j & 0 \end{bmatrix}$$

Secondly, we have to take inverse of DFT to obtain 3x3 operator in spatial domain:

$$h_1 = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$$h_2 = \begin{bmatrix} 0 & 0.25 & -0.25 \\ -0.25 & 0 & 0 \\ 0.25 & 0 & 0 \end{bmatrix}$$

d.

The first filter turns all values of any array into zero.

The second filter is probably high pass filter since sum of its values are equal to zero.

e.

V_1 is a high pass filter. The crucial point is the weight each frequency gets. This filter has greater absolute weight for corresponding higher frequencies. Also, it is about zero around the origin.

f.

V_2 is a high pass filter. Since it has larger values at higher frequencies (u and v) and it has smaller values near the origin.

g.

V_3 is a low pass filter. Since, it has value of about 1 at the origin (low frequencies) and less value (about zero) at higher frequencies.

h.

V_4 is a low pass filter. Since the function tends to zero at average and high frequencies and has greater value at closer frequencies closer to the origin by comparison.

i.

V_5 is a low pass filter as well. Since the function tends to one at frequencies near the origin and it gets smaller and smaller as frequencies increase.

Parts j, k, l and m are remained unsolved.

3.

a.

i.

To find zero real part of Fourier Transform we have to seek for antisymmetric images in the origin as:

$$\cos(x) = \cos(-x) \rightarrow f(x,y) = -f(-x,-y)$$

Therefore, image (e) have zero real part for all u and v.

ii.

To find zero imaginary part of Fourier Transform we have to seek for symmetric images in the origin as:

$$\sin(x) = -\sin(-x) \rightarrow f(x,y) = f(-x,-y)$$

Therefore, images (a), (b), (c), (f) and (h) have zero imaginary part for all u and v.

iii.

???

iv.

F(0,0) is DC component which means integration of f(x,y). Hence, we have to seek for image with zero integration.

Therefore, images (e) and (h) have zero DC component.

v.

F(u,v) has circular symmetry if f(x,y) has circular symmetry which is only exists in image (f).

b.

(Related script is p3b)

image 1

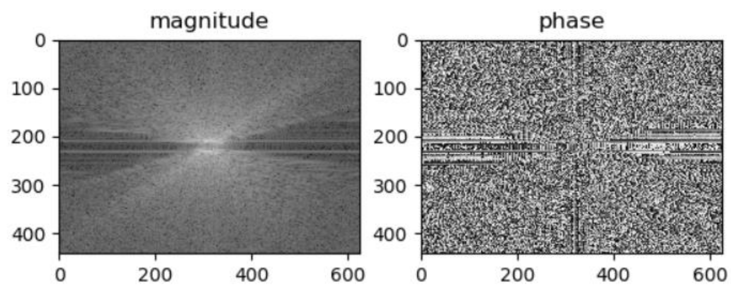


image 2

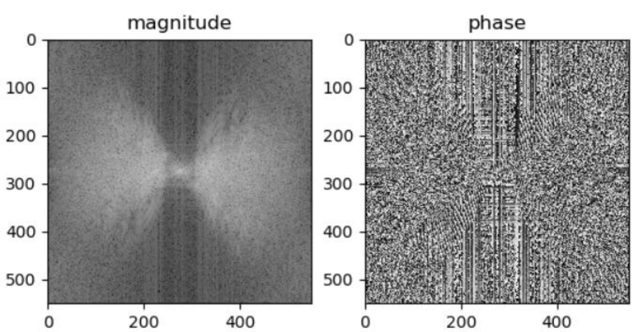


image 3

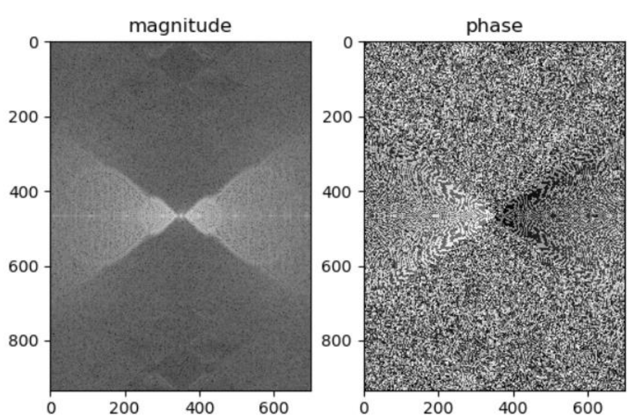


image 4

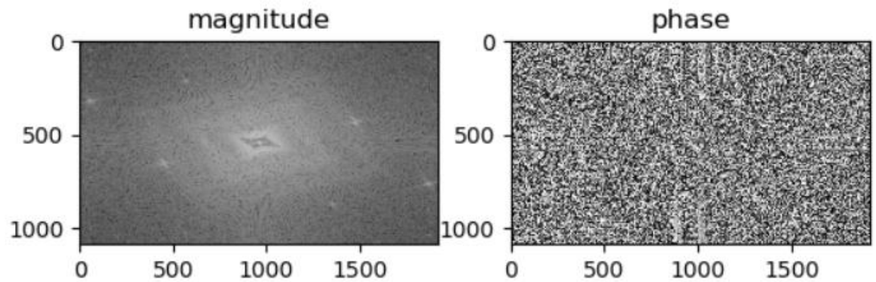


image 5

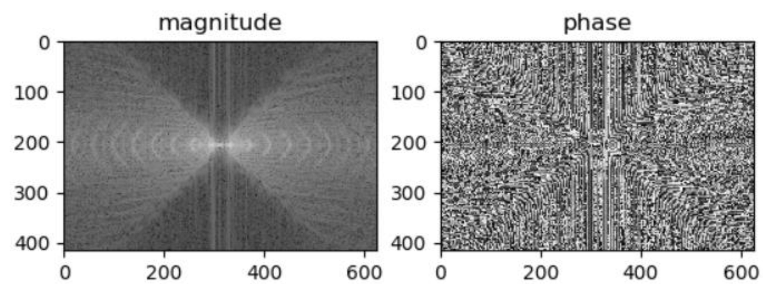
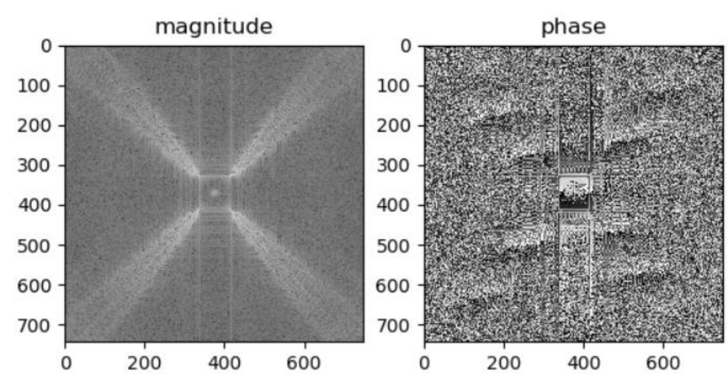


image 6



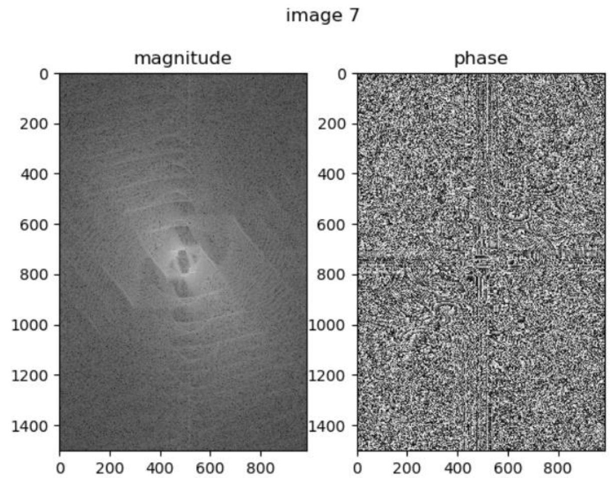
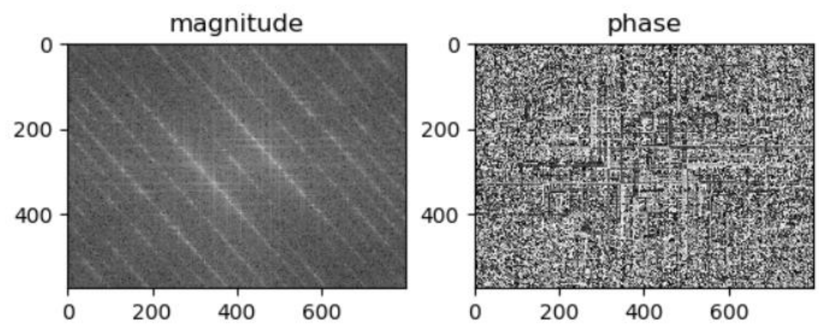


image 8

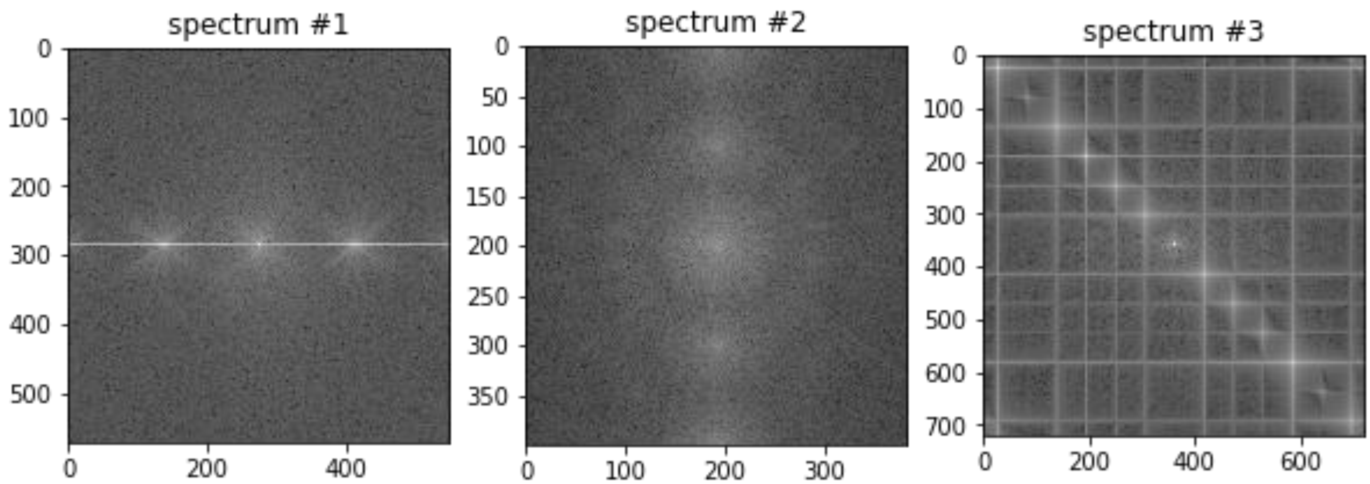


Explanation:

Generally, it is very hard to interpret an image by phase of DFT but some information may be extracted from magnitude illustrations. One of the most significant properties is the existence of dominant bright lines which expresses existence of lines perpendicular to those in DFT spectrum. This property can be sought in all images above.

4.

a.



As it can be observed from the images above, in all three spectrums there are some sharp bright points which have caused sinusoidal noises on raw images. There are 4, 4 and 12 sharp bright points, respectively, to be removed.

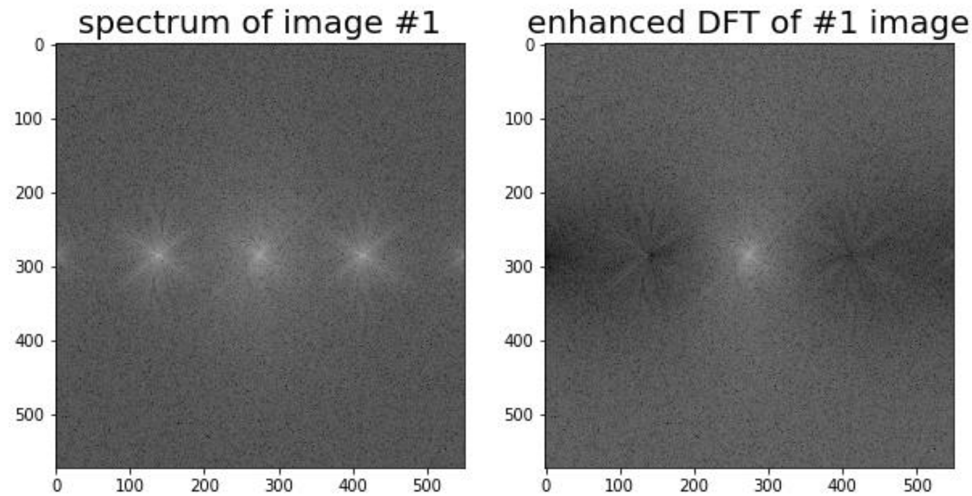
b.

Image #1:

By taking advantage of High-pass gaussian filter at coordinates of the following:

```
In [159]:
bs1 = gaussianHP(100,dfts[0],286,142)
bs2 = gaussianHP(100,dfts[0],286,412)
bs3 = gaussianHP(100,dfts[0],286,1)
bs4 = gaussianHP(100,dfts[0],286,572)
```

Dft will be enhanced as the following:



Then, by taking inverse of DFT and enhancing contrast of final image we obtain the following result:

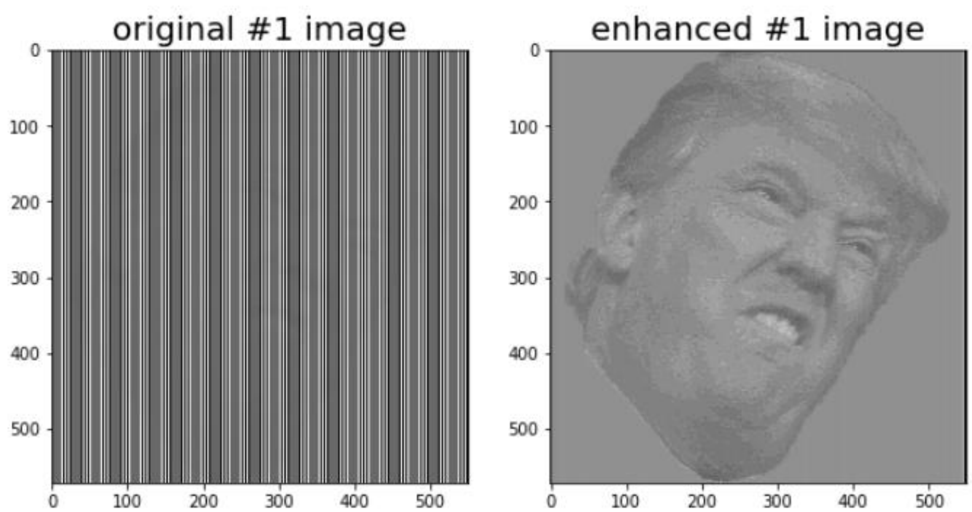
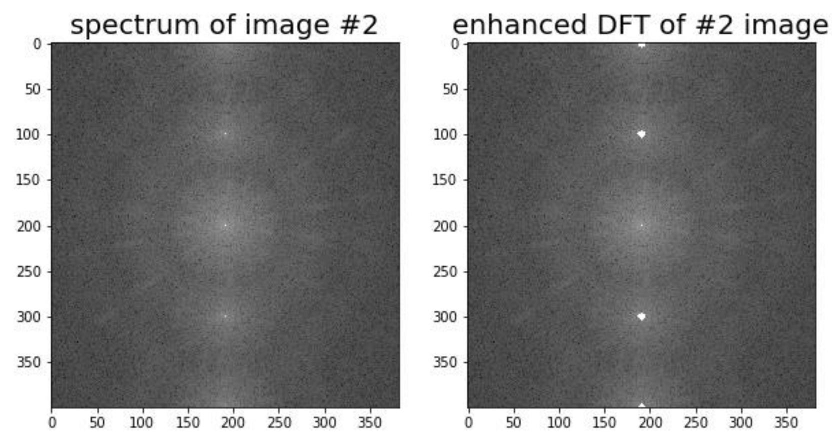


Image #2:

This time by taking advantage of High-pass ideal filter at coordinates of the following:

```
In [325]:
bs1 = idealHP(4,dfts[1],100,191)
bs2 = idealHP(4,dfts[1],300,191)
bs3 = idealHP(4,dfts[1],1,191)
bs4 = idealHP(4,dfts[1],400,191)
```

Dft will be enhanced as the following:



Note: Sharp bright dotes of the right image above have zero values in fact and should be black but due to taking log of zero value for a better visualization they are shown as white dotes.

Then, by taking inverse of DFT and enhancing contrast of final image we obtain the following result:

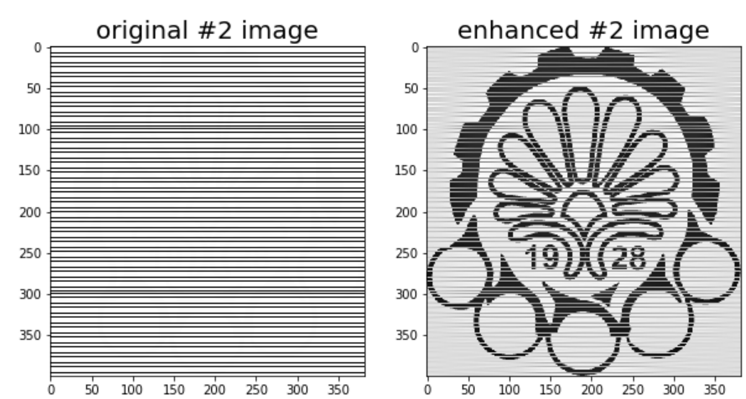
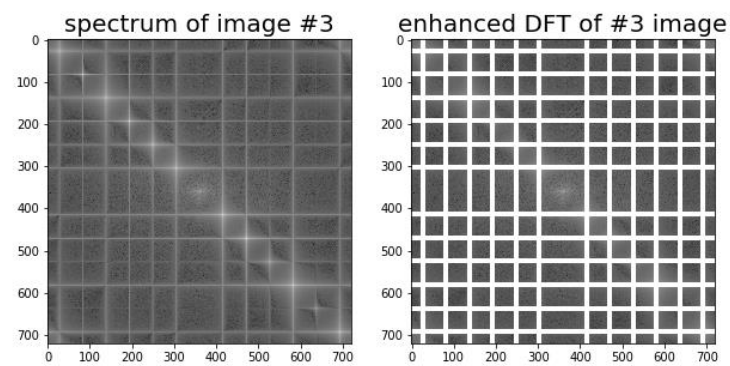


Image #3:

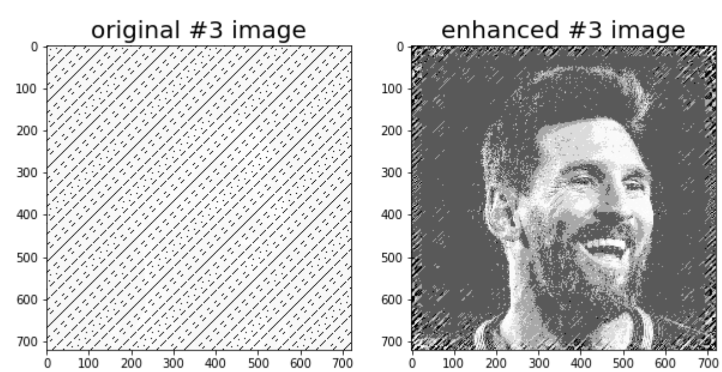
By applying filters in order to set vertical and horizontal bright lines (with coordinates as the following) to zero:

```
In [371]:
bs_ver = np.ones(images[2].shape)
bs_hor = np.ones(images[2].shape)

hor_val = [691,637,581,525,470,413,304,248,193,138,82,27]
ver_val = [28,82,138,192,249,303,415,470,526,580,637,692]
for val in hor_val:
    bs_hor *= hor_line(3,dfts[2],val)
for val in ver_val:
    bs_ver *= ver_line(3,dfts[2],val)

bs = bs_ver*bs_hor
```

Dft will be enhanced as the following:



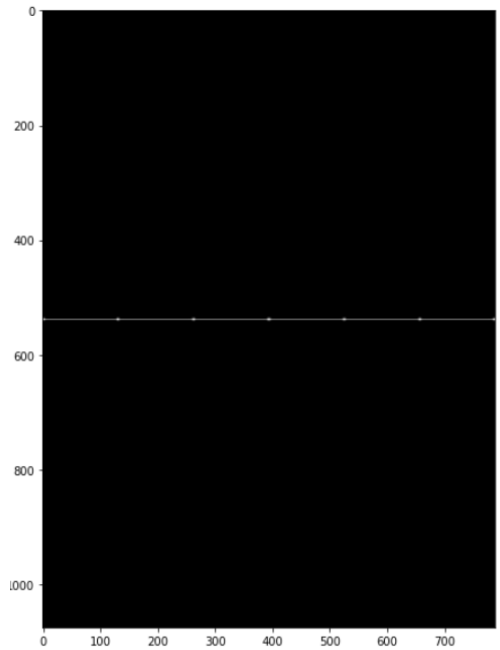
Then, by taking inverse of DFT and enhancing the final image by applying equalization then improving contrast, we obtain the following result:



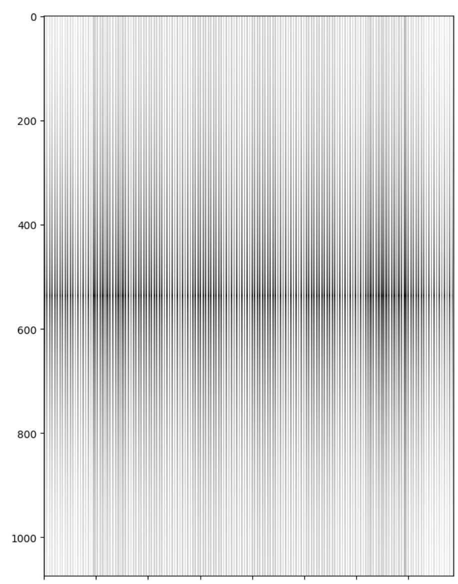
c.

Vertical lines: (for the #1 image)

Creating the following hand-made DFT:

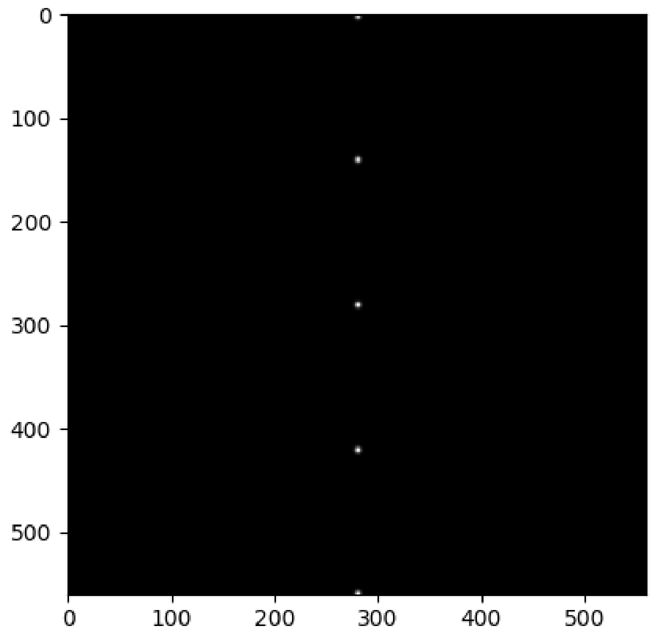


Then taking inverse Fourier transform, following image will be obtained:

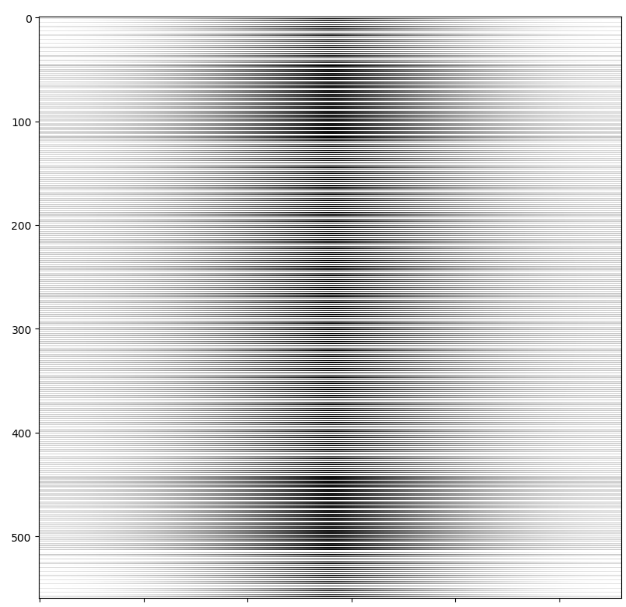


Horizontal lines: (for the #2 image)

Creating the following hand-made DFT:

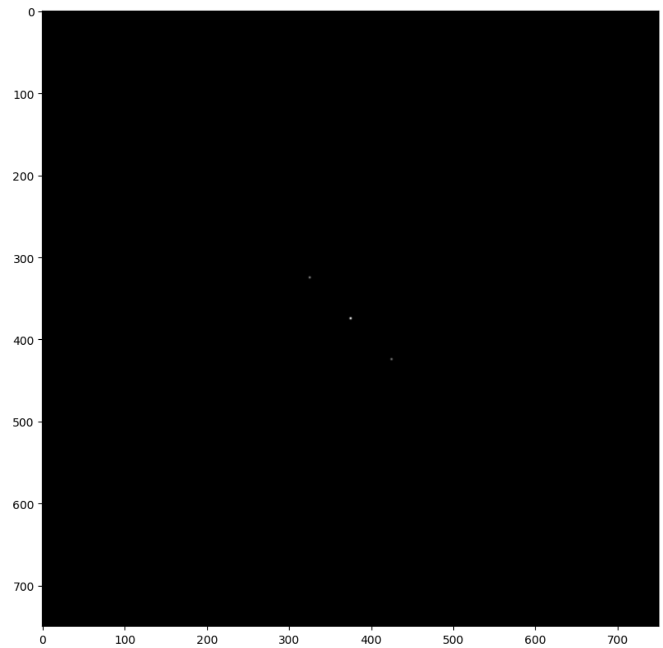


Then taking inverse Fourier transform, following image will be obtained:

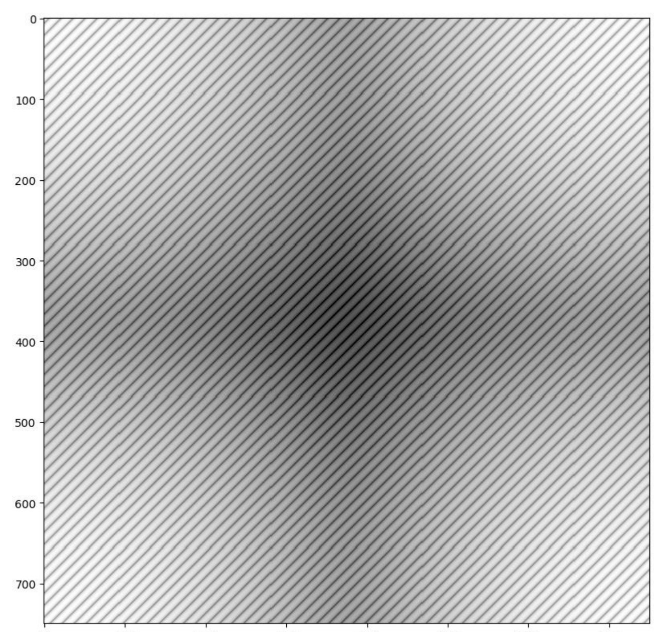


Diagonal lines: (for the #3 image)

Creating the following hand-made DFT:

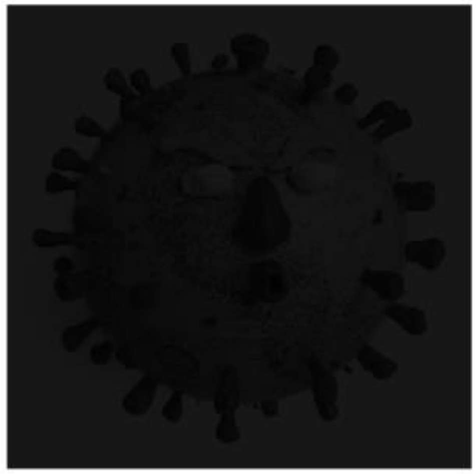


Then taking inverse Fourier transform, following image will be obtained:



d.

Decreasing brightness of images by approximately 90 percent:



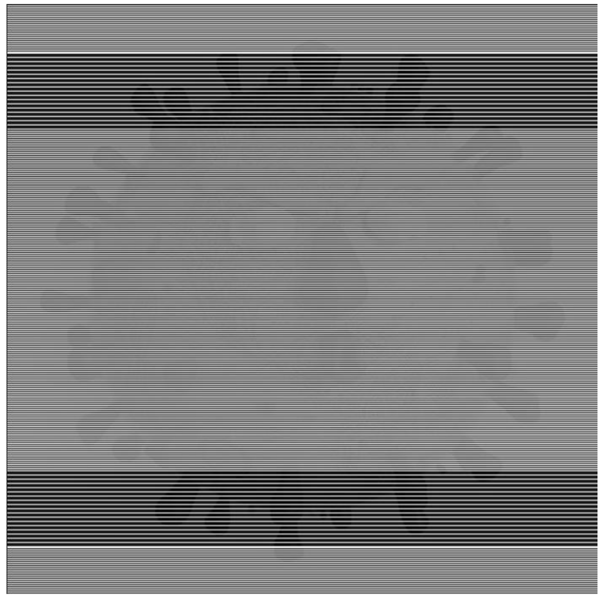
Then, adding dark images to created lines before, the final results are as follows:

(Note that all the images below are saved in a folder named Illusion!)

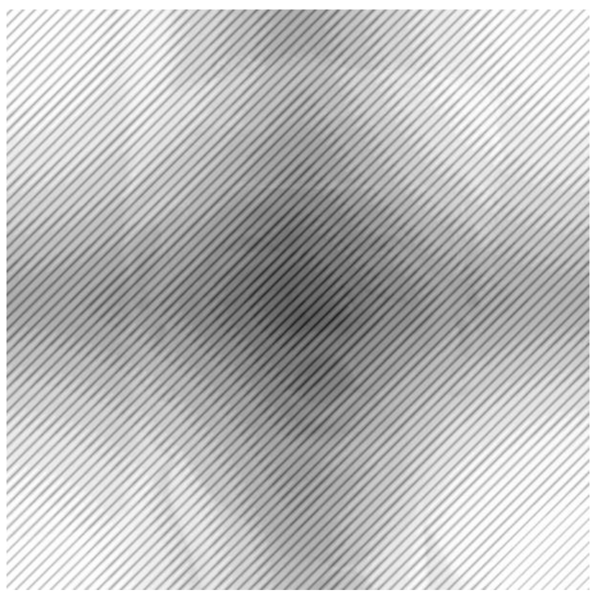
(The images shown below are pasted from their large sized images for better visualization)

Vertical

Horizontal



Diagonal



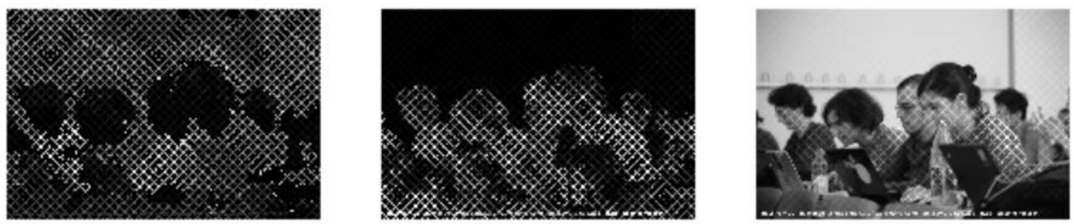
5.

a.

Comparing resultant images of different color spaces, **HSV color space** is chosen for all three images (a), (b) and (c):

Image (a):

channels



dfts of image #1 (HSV)

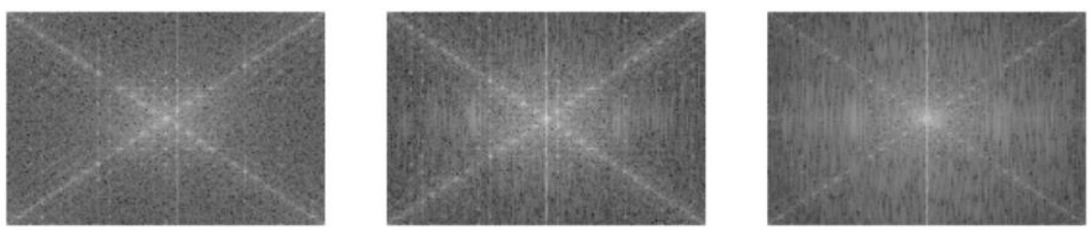
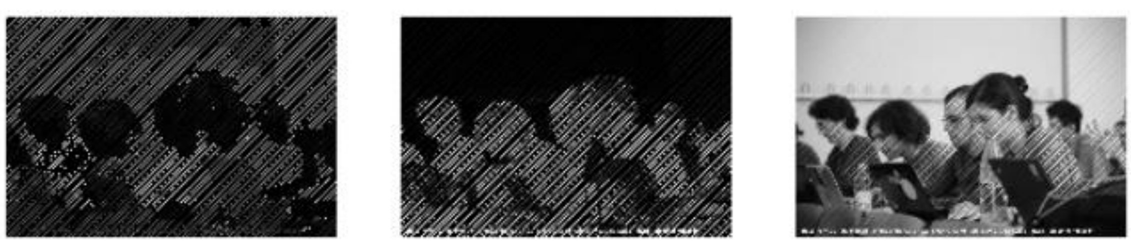


Image (b):

channels



dfts of image #2 (HSV)

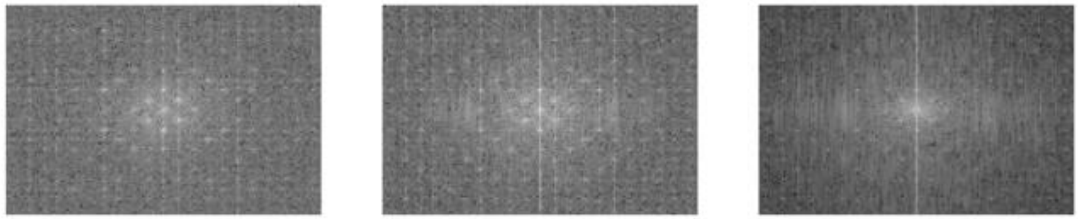
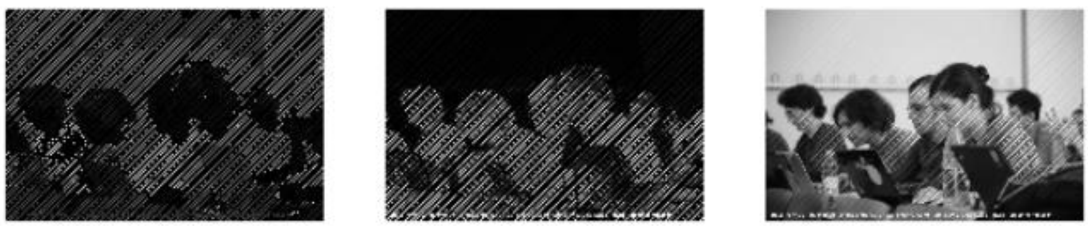
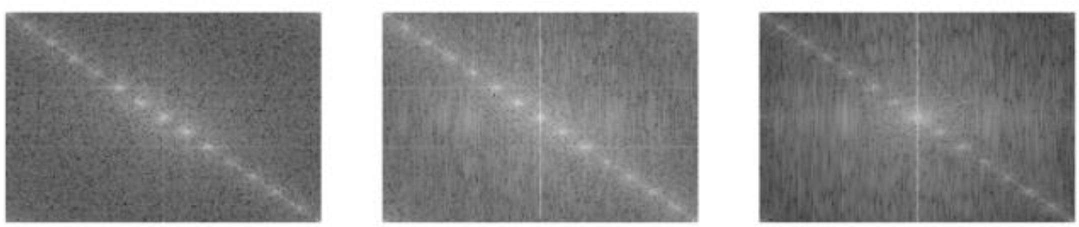


Image (c):

channels



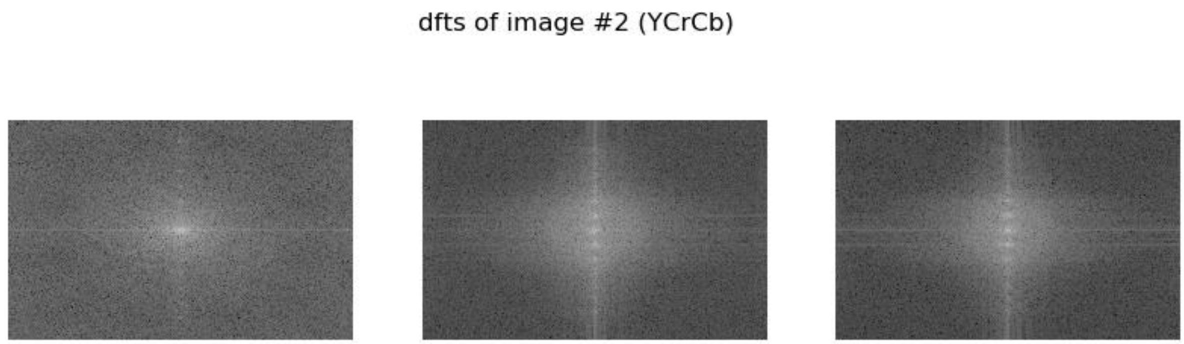
dfts of image #3 (HSV)



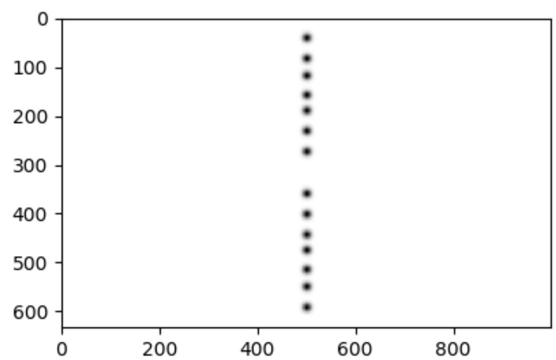
b.

1st image:

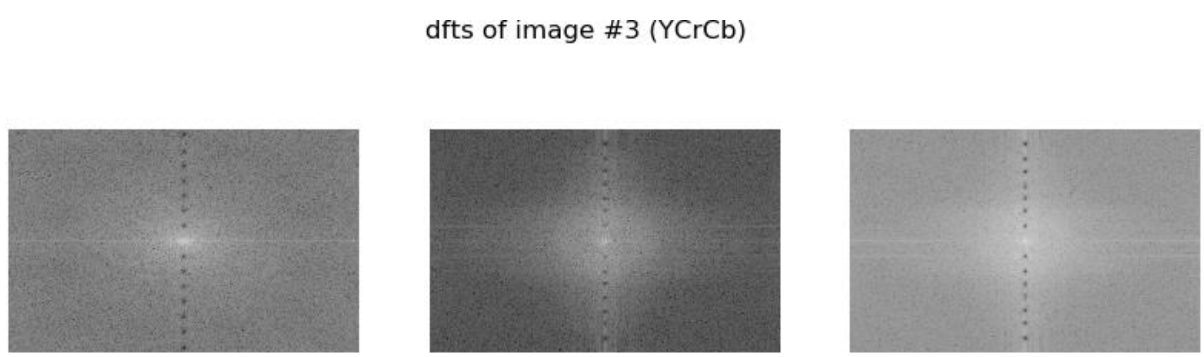
Choosing YCrCb color space, the DFT spectrum of channels are as follows:



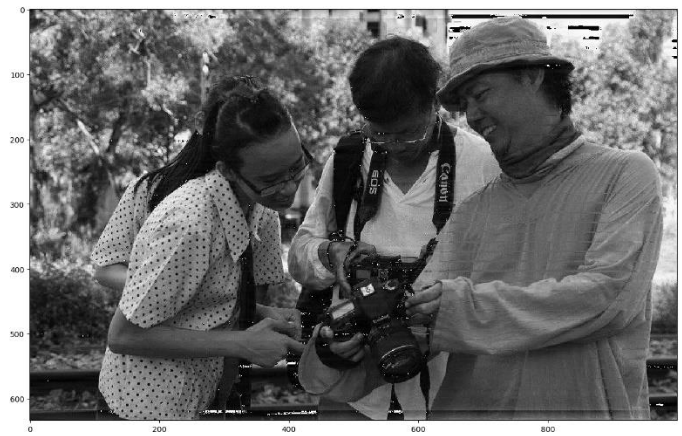
Then, by applying following notch filter:



Resultant DFTs are as follows:



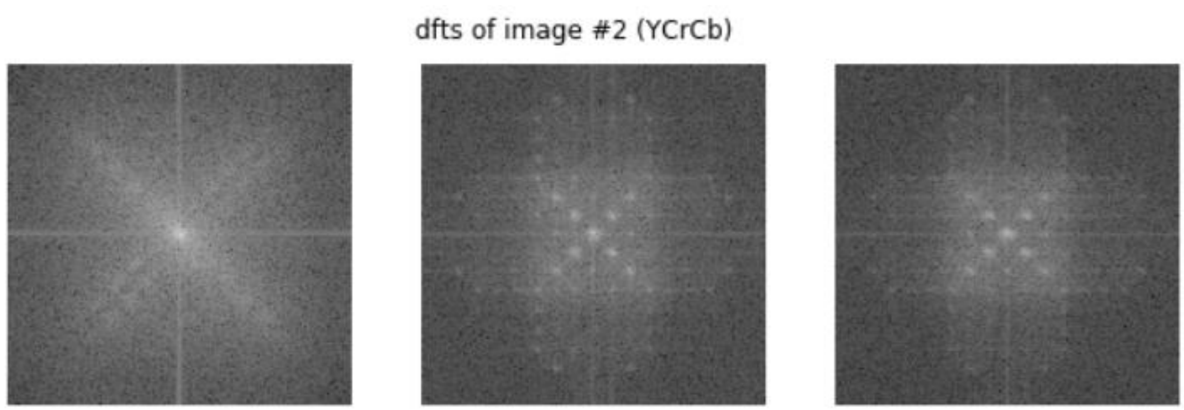
Lastly, the grayscale image obtained (left) vs the grayscale of original image (right) are as follows:



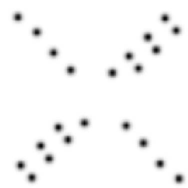
As it can be seen, lines are significantly faded especially on people’s skins.

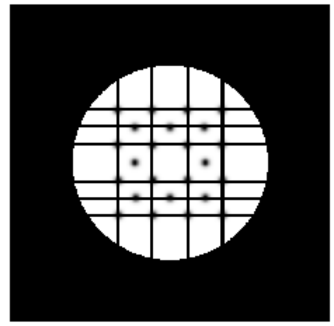
2nd image:

Choosing YCrCb color space, the DFT spectrum of channels are as follows:

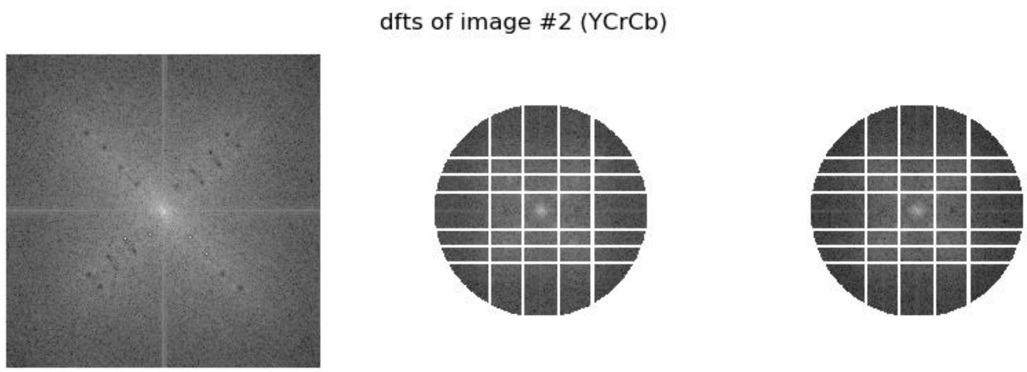


Then, by applying following notch filters:

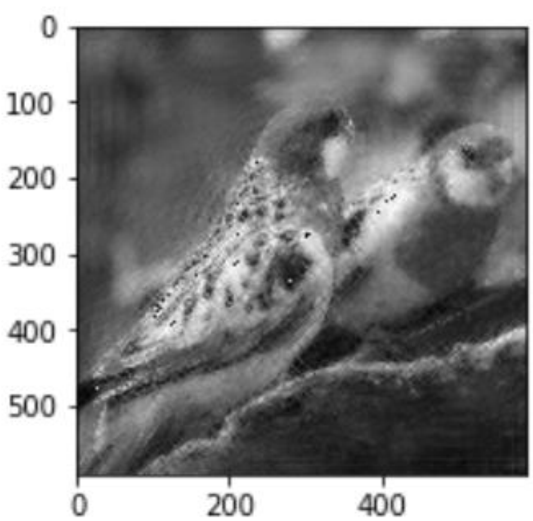


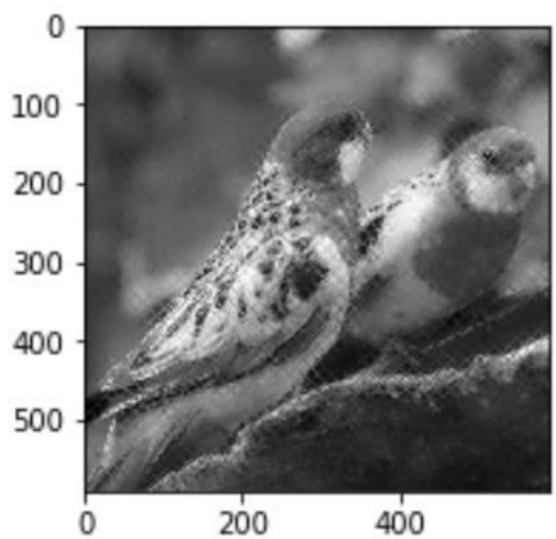


Resultant DFTs are as follows:



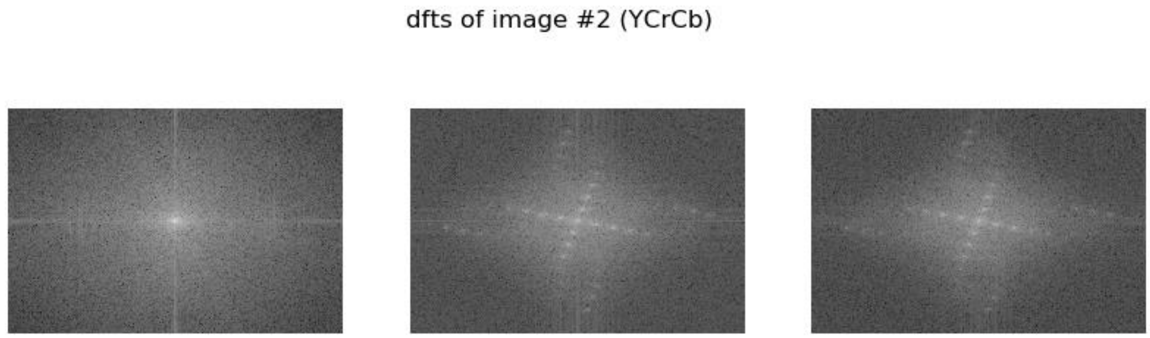
Lastly, the grayscale image obtained (left) vs the grayscale of original image (right) are as follows:



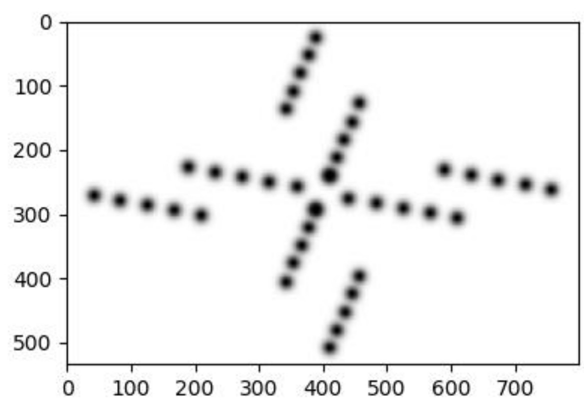


3rd image:

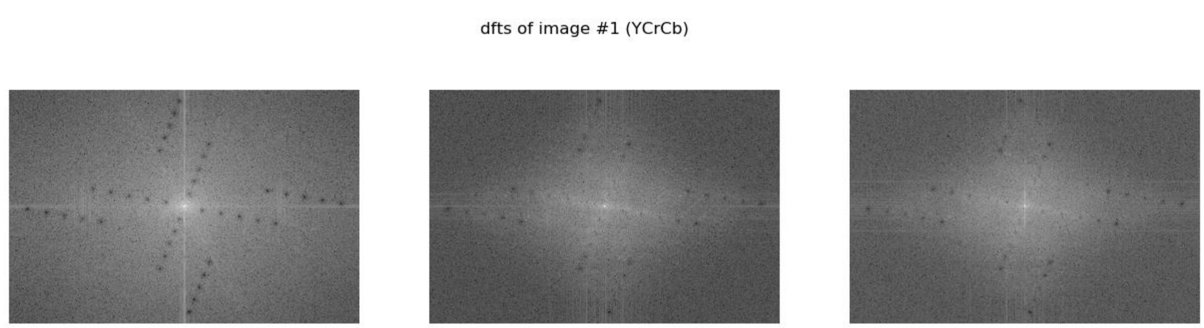
Choosing YCrCb color space, the DFT spectrum of channels are as follows:



Then, by applying following notch filters:



Resultant DFTs are as follows:



Lastly, the grayscale image obtained (up) vs the grayscale of original image (down) are as follows:

As we can see, considering diagonal lines, the filtered image has significantly enhanced compared to the grayscale of original image.

c.

???

d.

???

6. **(Related script is p6)**

a.

Images are aligned based on eyes centers:

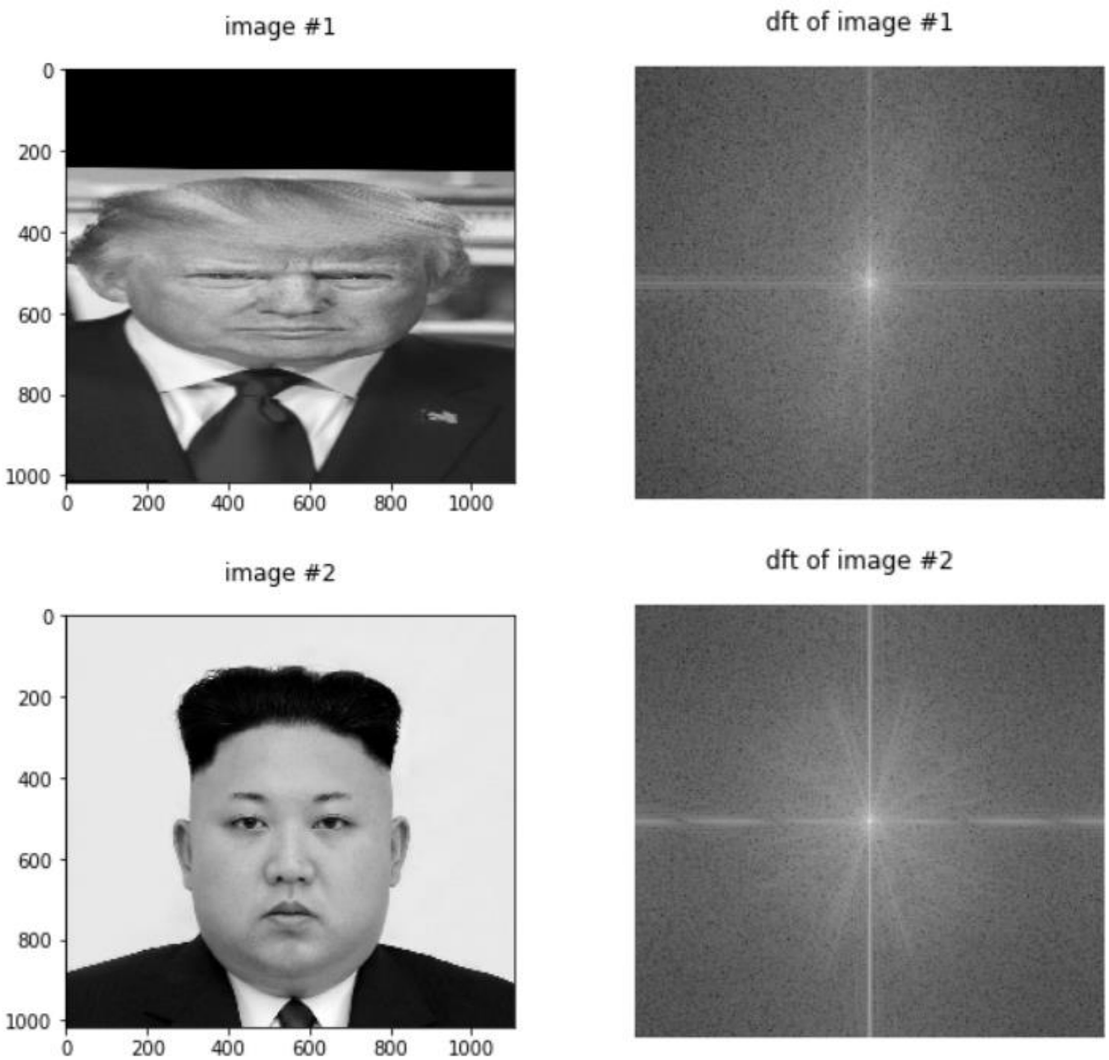
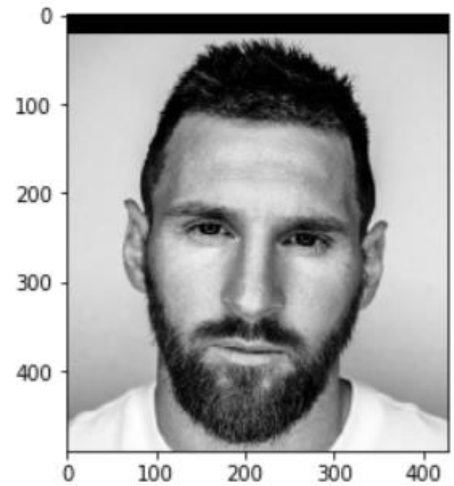


image #3



dft of image #3

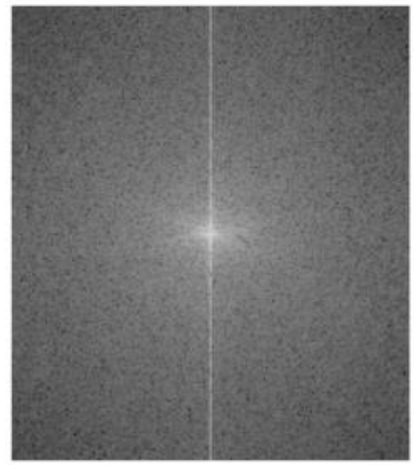
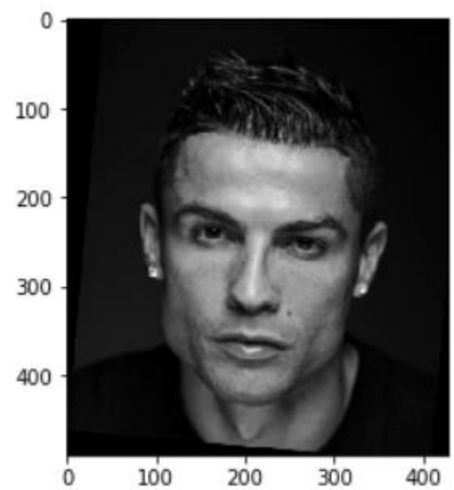
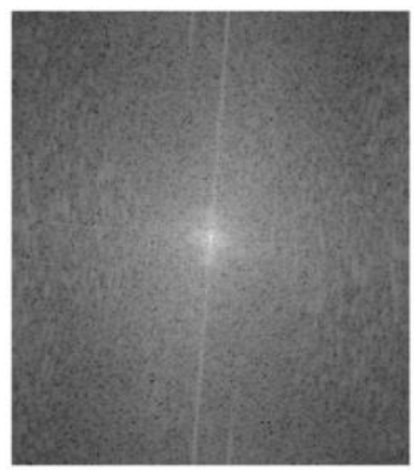


image #4



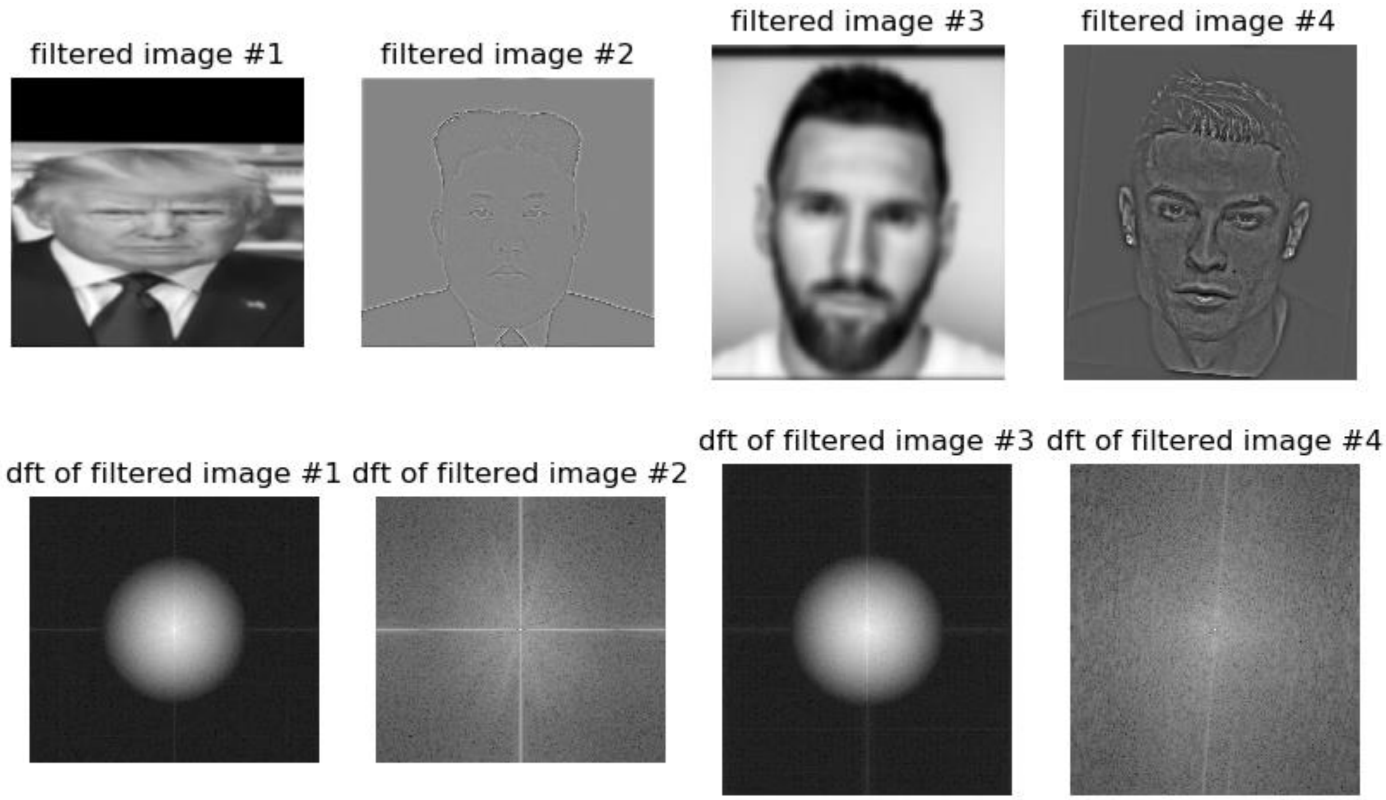
dft of image #4



b.

Chosen cut-off frequencies are as follows respectively:

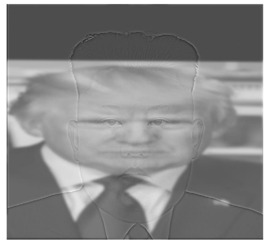
35 for first pair images and 14 for second pair images.

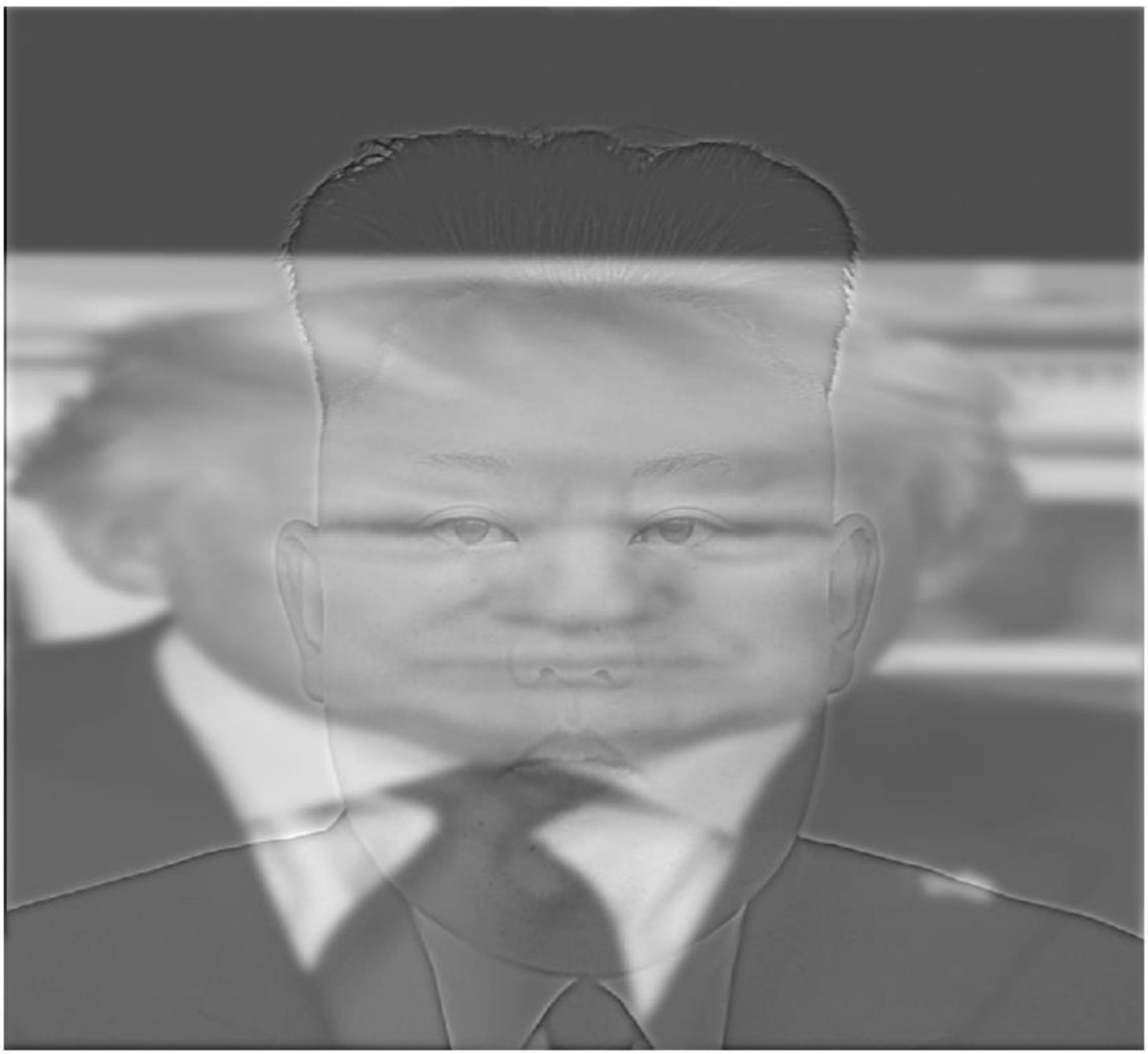


c.

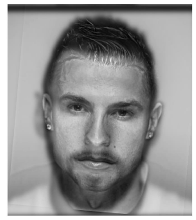
Merge of first pair of images:

(shown in two sizes as a substitution of viewer distances)

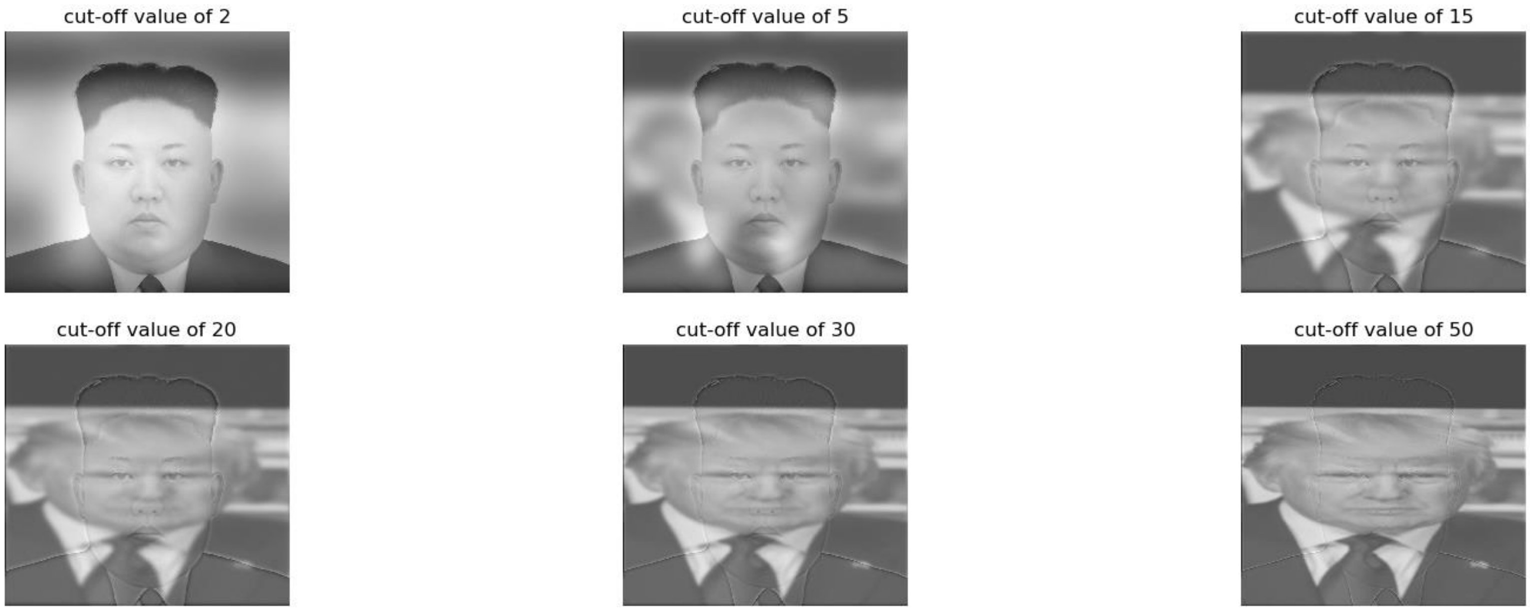


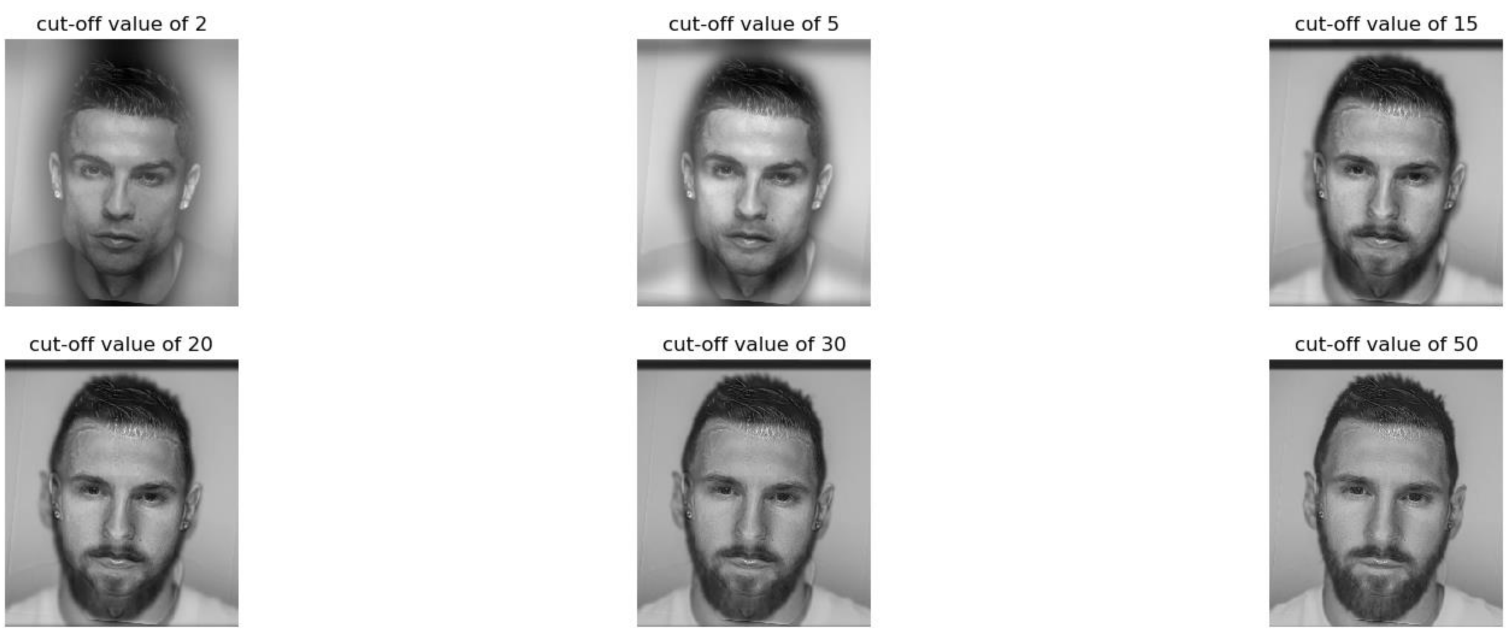


Merge of second pair of images:



d.

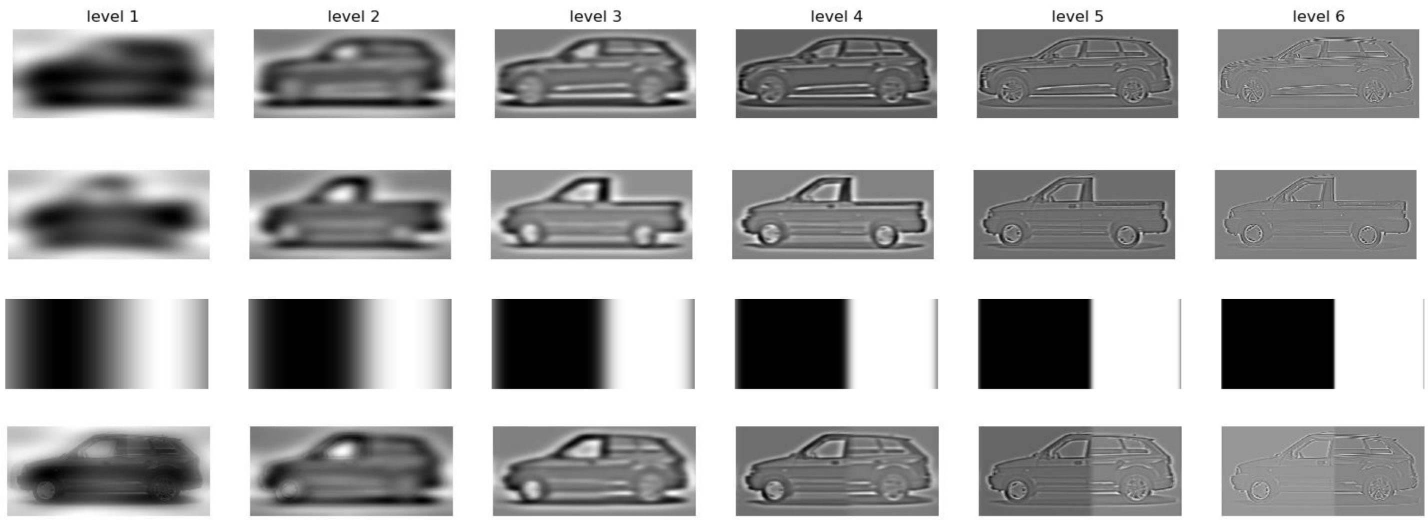

cut-off value of 2
cut-off value of 5
cut-off value of 15
cut-off value of 20
cut-off value of 30
cut-off value of 50


cut-off value of 2
cut-off value of 5
cut-off value of 15
cut-off value of 20
cut-off value of 30
cut-off value of 50

7. **(Related script is p7)**

a.

Laplacian filtered versions of two images along with gaussian filtered versions of mask and combination of two filtered images (for ‘n’ of value 6):



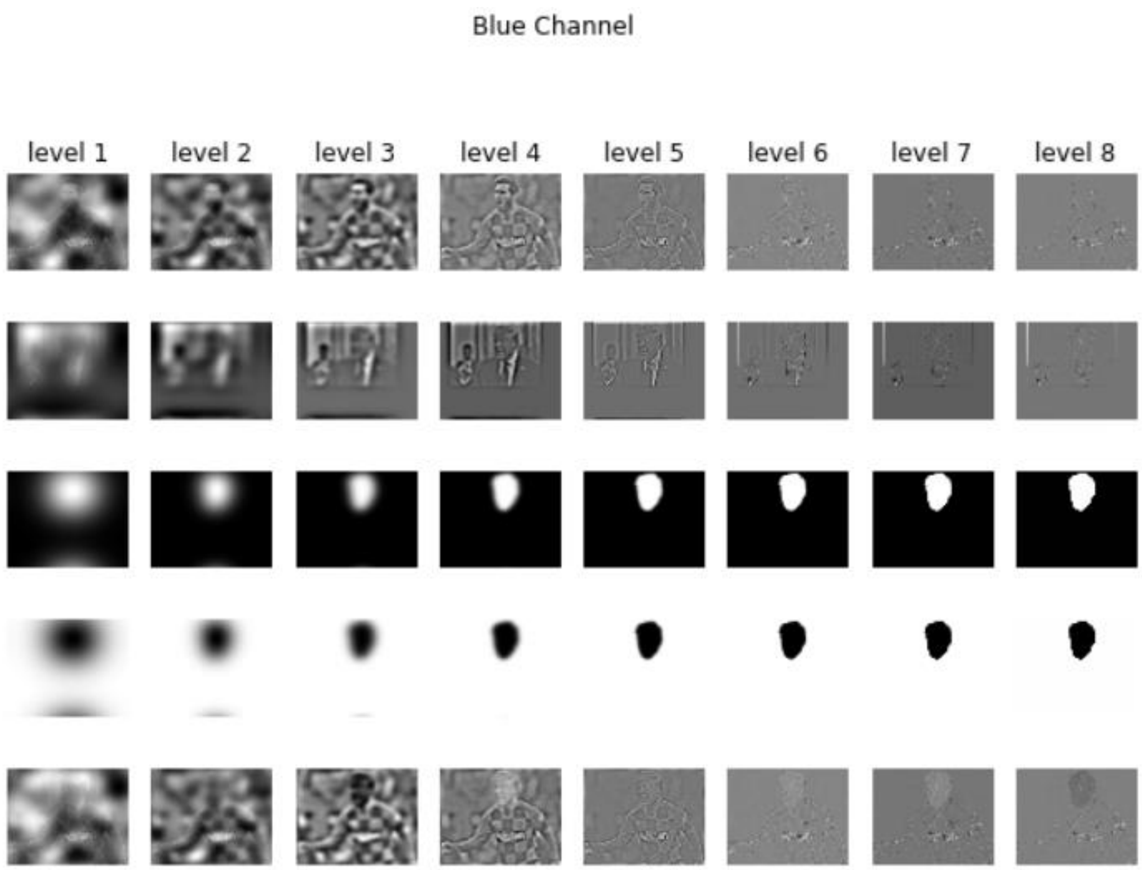
Summing all combinations obtained above, results in the following image:

Some deficiencies of the image above are rooted in imperfection of gaussian filtered versions of mask as can be seen in the images above.

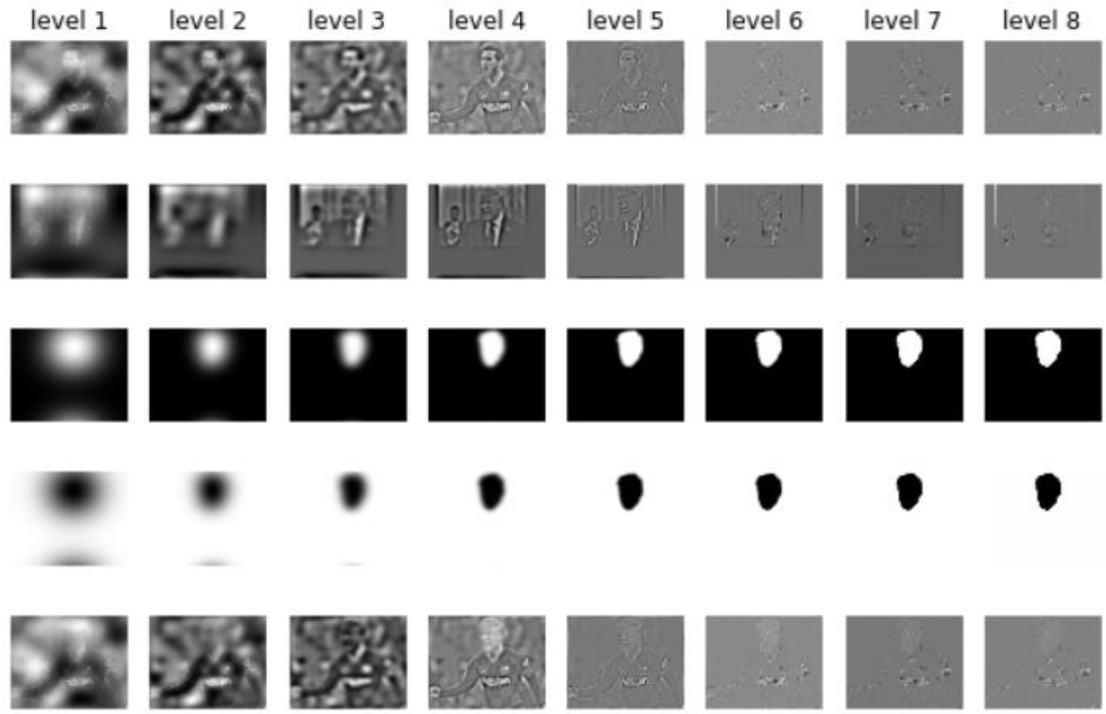
b.

Intermediate results of each channels are as follows

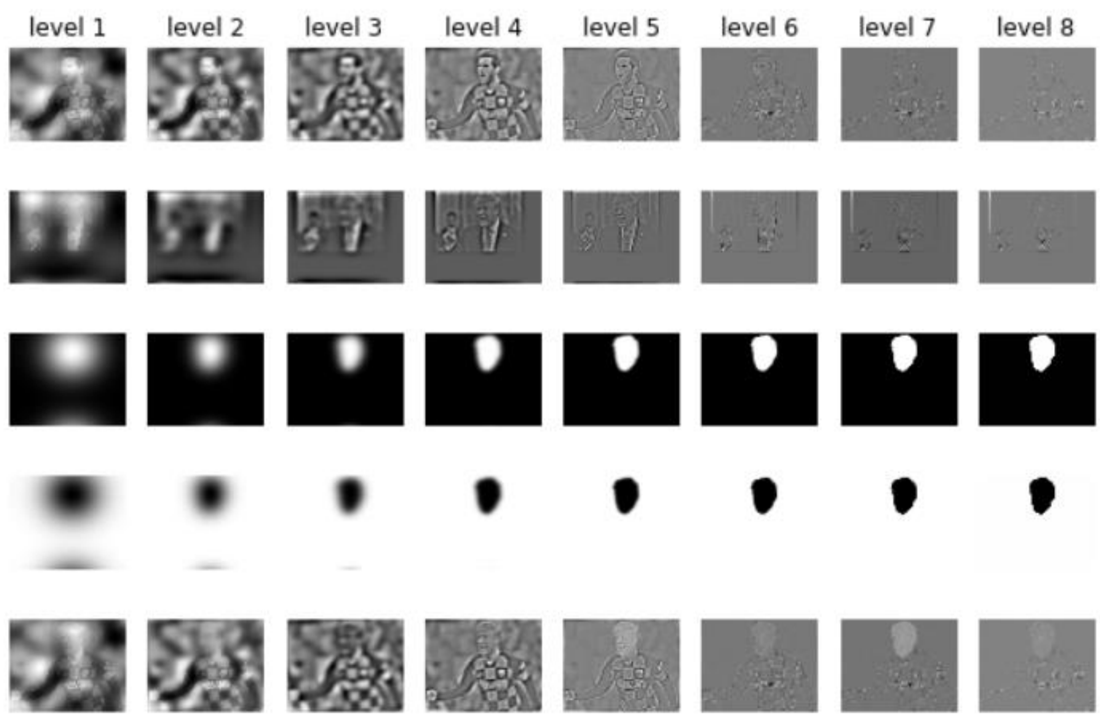
(for ‘n’ of value 8):



Green Channel



Red Channel



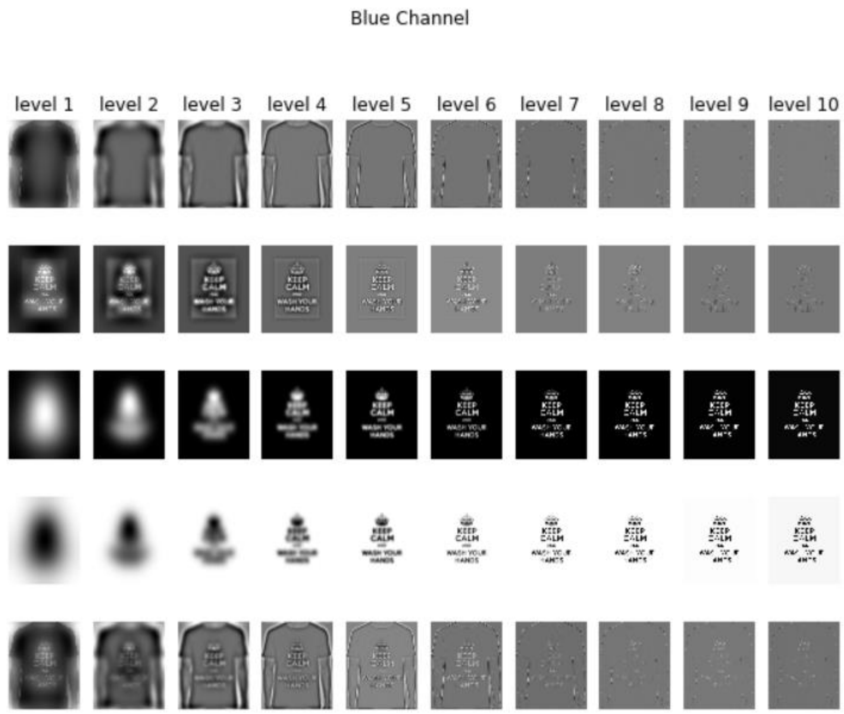
The latest mixed result is as follows:

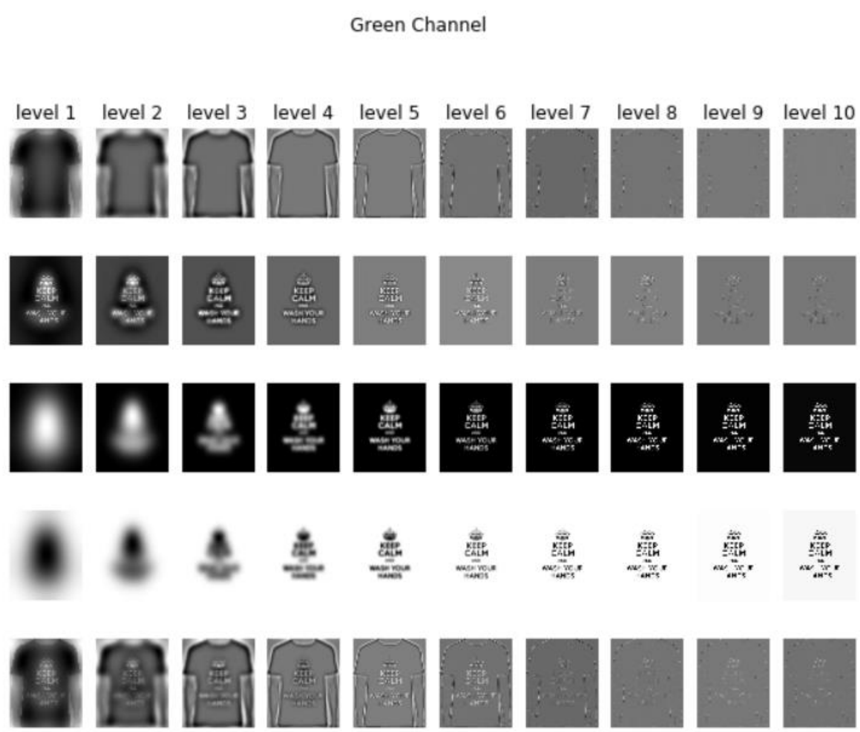
c.

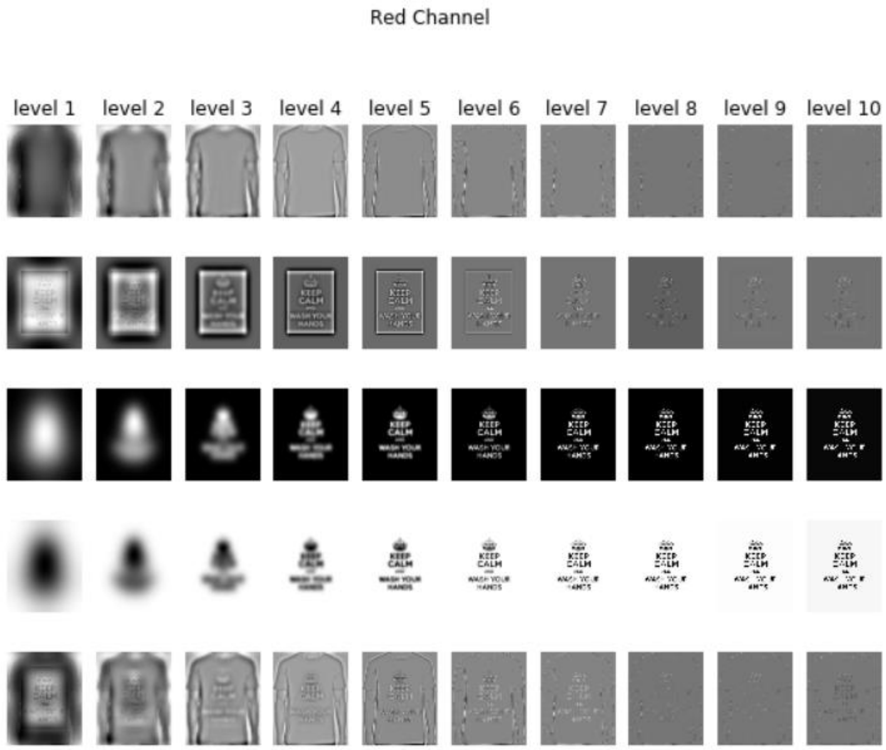
Note image is transformed to the following image (left) and Defined mask of the Note image is as the following (right):

d.

Intermediate results of each channels are as follows (for ‘n’ of value 10):






Red Channel
level 1
level 2
level 3
level 4
level 5
level 6
level 7
level 8
level 9
level 10


KEEP
CALM
AND
WASH YOUR
HANDS

8.

???

9.

a.

Primary key to visualize hidden image behind dense successive black and white lines is to find a way in order to blur the image or generally, blur the vision. This is mainly due to the fact that humans' vision has low dynamic range and needs light sensitivity adjustment to recognize some details. This is called brightness adaption which is done by blurring (in the concept of "Illusion Images") which makes the overall brightness appear to be the average brightness of the black and white. Consequently, this improves light sensitivity and the areas where the pixel tone is different from the average start to stand out. The mentioned blurring operation can be done by multiple ways such as blurring vision, moving image or observer's head rapidly, decreasing image size and so on.

b.

This may be due to the fact that convolution is an expensive and time-consuming operation, whereby, it is not logical to be performed on large number of frames.

c.

Aliasing in images appears in shapes like moire patterns. Moire patterns are patterns made by interference of two very similar patterns which are approximately similar but rather displaced, rotated or having slightly different pitch. Aliasing occurs mostly in the images with repeated curved or diagonal shapes. This phenomenon may change shapes in size or direction or both. Therefore, it may cause diagonal lines to be changed in both size and direction.

d.

DFT is actually sampled function of Fourier Transformation of periodic discrete function which is periodic itself (DTFT).

e.

Generally, aliasing can be avoided only if Nyquist criterion is satisfied and also the function (signal) is unbounded. Since, only unbounded functions (signals) are band limited in frequency domain. Considering what mentioned and also the fact that, generally, practical and actual functions (signals) are bounded and have finite duration, therefore, in most of the conditions, their frequency content is not band limited. Hence, in general, aliasing is not avoidable.

f.

The plus sign in shifted DFT is rooted in one main reason which is discussed as follows:

This reason is named as **edge effect** which can be caused by strong edges inside the image or, more importantly, can be generated by strong edge effects between an image and its neighbors. The latter reason is rooted in the fact that DFT always treats an image as if it were part of a periodically replicated array of identical images extending horizontally and vertically to infinity the same as their DFTs.

g.

Yes, the new image is the rotated image by 180 degrees.

Assuming:

$$DFT(f(x,y)) = F(u,v)$$

By multiplying phase to '-1' we are actually multiplying coefficient of "j" to '-1' in F(u,v). This is actually the process of conjugating F(u,v) since:

$$\cos(-\theta) + j.\sin(-\theta) = \cos(\theta) - j.\sin(\theta) = (\cos(\theta) + j.\sin(\theta))^*$$

Also, we know that:

$$\text{Inverse fourier of } (F(u,v)^*) = f(-x,-y)^*$$

Also considering that f(x,y) should be real to be presentable, we have:

$$f(-x,-y)^* = f(-x,-y)$$

Hence, we have shown that new image is symmetric with respect to the coordinate center (origin) or in other words rotated by 180 degrees.