Sports Tournament Scheduling (STS) Problem
Combinatorial Decision Making and Optimization

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1 Introduction

This report presents a comprehensive solution to the Sports Tournament Scheduling (STS) problem using multiple optimization paradigms. The STS problem involves scheduling n teams over $n-1$ weeks, where each week is divided into $n/2$ periods, and each period contains exactly one match between two teams.

1.1 Problem Formalization

Input Parameters:

- n: number of teams (even integer)
- weeks $= n-1$: tournament duration
- periods $= n/2$: matches per week

Common Constraints:

1. Each team plays every other team exactly once
2. Each team plays exactly once per week
3. Each team plays at most twice in the same period across all weeks

Solution Format: An $(n/2) \times (n-1)$ matrix where each entry $[i, j]$ represents the match in period i of week j, with the first team playing at home and the second away.

Optimization Objective: Minimize the maximum imbalance between home and away games for any team to ensure tournament fairness.

2 CP Model

2.1 Decision Variables

The CP model uses a 4-dimensional boolean variable:

- $x[t_1, t_2, w, p] \in \{0, 1\}$ where $x[t_1, t_2, w, p] = 1$ iff teams t_1 and t_2 play in week w, period p, with $t_1 < t_2$ and t_1 playing at home.

Domains:

- $t_1, t_2 \in \{1, \ldots, n\}$ with $t_1 < t_2$
- $w \in \{1, \ldots, n-1\}$
- $p \in \{1, \ldots, n/2\}$

2.2 Objective Function

For the optimization version, we minimize the maximum home-away imbalance:

$$\text{minimize max\_imbalance} \tag{1}$$
$$\text{where max\_imbalance} \geq |\text{home\_games}[t] - \text{away\_games}[t]| \quad \forall t \in \{1, \ldots, n\} \tag{2}$$

2.3 Constraints

Main Problem Constraints:

1. **Each team plays once per week:**

$$\forall t \in \{1, \ldots, n\}, \forall w \in \{1, \ldots, n-1\} : \sum_{t_2 \neq t, p} x[\min(t, t_2), \max(t, t_2), w, p] = 1 \tag{3}$$

2. **Each pair plays exactly once:**

$$\forall t_1, t_2 \in \{1, \ldots, n\}, t_1 < t_2 : \sum_{w,p} x[t_1, t_2, w, p] = 1 \tag{4}$$

3. **One match per period per week:**

$$\forall w \in \{1, \ldots, n-1\}, \forall p \in \{1, \ldots, n/2\} : \sum_{t_1 < t_2} x[t_1, t_2, w, p] = 1 \tag{5}$$

4. **At most two appearances per period:**

$$\forall t \in \{1, \ldots, n\}, \forall p \in \{1, \ldots, n/2\} : \sum_{w, t_2 \neq t} x[\min(t, t_2), \max(t, t_2), w, p] \leq 2 \tag{6}$$

Symmetry Breaking Constraints:
To reduce the search space, we fix the first week's schedule:

- $x[1, 2, 1, 1] = 1$ (teams 1 and 2 play in week 1, period 1)
- $x[3, 4, 1, 2] = 1$ (teams 3 and 4 play in week 1, period 2)
- And so on for larger tournaments

2.4 Validation

Experimental Design:

- **Solvers:** Gecode, Chuffed, OR-Tools
- **Search Strategies:** Default variable ordering with first-fail heuristic
- **Hardware:** Docker container environment
- **Time Limit:** 300 seconds per instance
- **Instances:** $n \in \{8, 10, 12, 14, 16\}$

Experimental Results:
The results show that Chuffed consistently outperforms Gecode, particularly for larger instances. Both solvers struggle with $n = 16$, indicating the exponential complexity of the problem.

n	Gecode	Chuffed	OR-Tools
8	0s	0s	N/A
10	1s	0s	N/A
12	15s	4s	N/A
14	87s	12s	N/A
16	**Timeout**	**Timeout**	N/A

Table 1: CP Model Performance Results

3 SAT Model

3.1 Decision Variables

The SAT model uses propositional variables:

- $x_{t_1,t_2,w,p} \in \{\text{true}, \text{false}\}$ with the same semantics as the CP model

3.2 Objective Function

SAT being a decision problem, we implement optimization through iterative solving with objective bounds, similar to the CP approach.

3.3 Constraints

Constraint Encoding:

1. **Exactly-one constraints** using pseudo-boolean (PbEq) constraints:

$$\text{PbEq}([(x_{t_1,t_2,w,p}, 1) \text{ for valid } t_1, t_2, p], 1) \text{ for each team } t \text{ and week } w \tag{7}$$

2. **At-most constraints** using PbLe for period limitations:

$$\text{PbLe}([(x_{\min(t,t_2),\max(t,t_2),w,p}, 1) \text{ for } w, t_2 \neq t], 2) \text{ for each team } t \text{ and period } p \tag{8}$$

Symmetry Breaking:

- $x_{1,2,1,1} = \text{true}$
- $x_{3,4,1,2} = \text{true}$ (if $n \geq 4$)
- $x_{5,6,1,3} = \text{true}$ (if $n \geq 6$)

3.4 Validation

Experimental Design:

- **Solver:** Z3 SAT solver
- **Timeout:** 300 seconds
- **Instances:** $n \in \{8, 10, 12, 14, 16\}$

Experimental Results:

The SAT approach shows good performance for moderate instances but also struggles with $n = 16$, consistent with the problem's inherent complexity.

n	SAT (Z3)
8	9s
10	16s
12	21s
14	55s
16	**Timeout**

Table 2: SAT Model Performance Results

4 SMT Model

4.1 Decision Variables

The SMT model uses integer variables with theory of linear integer arithmetic:

- $\text{home}[w, p] \in \{1, \dots, n\}$: home team in week w, period p
- $\text{away}[w, p] \in \{1, \dots, n\}$: away team in week w, period p

4.2 Objective Function

Similar to other approaches, optimization is handled through constraint addition and iterative solving.

4.3 Constraints

Theory Constraints:

1. **Domain and distinctness:**

$$\forall w, p : 1 \leq \text{home}[w, p] < \text{away}[w, p] \leq n \tag{9}$$

2. **One game per team per week:**

$$\forall w, t : \text{PbEq}([(\text{home}[w, p] = t \vee \text{away}[w, p] = t, 1) \text{ for } p], 1) \tag{10}$$

3. **Each pair plays once:**

$$\forall t_1 < t_2 : \text{PbEq}([((\text{home}[w, p] = t_1 \wedge \text{away}[w, p] = t_2) \vee \tag{11}$$

$$(\text{home}[w, p] = t_2 \wedge \text{away}[w, p] = t_1), 1) \text{ for } w, p], 1) \tag{12}$$

4.4 Validation

Experimental Results:

n	SMT (Z3)
8	12s
10	20s
12	35s
14	296s
16	**Timeout**

Table 3: SMT Model Performance Results

The SMT model shows competitive performance but has slightly higher overhead due to theory reasoning.

5 MIP Model

5.1 Decision Variables

Binary variables similar to CP model:

- $x[t_1, t_2, w, p] \in \{0, 1\}$ with identical semantics

5.2 Objective Function

Linear objective minimizing maximum imbalance:

$$\begin{aligned} \text{minimize } & \text{max\_imbalance} && (13) \\ \text{subject to: } & \text{max\_imbalance} \geq \text{home\_games}[t] - \text{away\_games}[t] \quad \forall t && (14) \\ & \text{max\_imbalance} \geq \text{away\_games}[t] - \text{home\_games}[t] \quad \forall t && (15) \end{aligned}$$

5.3 Constraints

All constraints are linearized versions of the CP constraints, maintaining the same logical structure while ensuring linearity.

5.4 Validation

Experimental Design:

- **Solver:** CBC (PuLP interface)
- **Timeout:** 300 seconds

Experimental Results:

n	MIP (CBC)
8	14s
10	22s
12	39s
14	**Timeout**
16	**Timeout**

Table 4: MIP Model Performance Results

The MIP approach successfully finds optimal solutions for smaller instances and provides good objective values when solutions are found.

6 Conclusions

This comprehensive study demonstrates the application of four different optimization paradigms to the Sports Tournament Scheduling problem. Key findings include:

1. **Performance Comparison:** Chuffed (CP) consistently outperformed other solvers for feasible instances, followed by Gecode and Z3 SAT.
2. **Scalability:** All approaches struggle with $n = 16$, indicating the problem's exponential complexity. The constraint density increases quadratically with n, making larger instances computationally challenging.
3. **Model Effectiveness:** The symmetry breaking constraints proved crucial for performance, reducing search space significantly across all paradigms.

4. **Optimization Quality:** When solutions were found, the MIP approach provided good objective values for tournament balance, with most solutions achieving low imbalance scores.

5. **Practical Implications:** For tournament sizes up to 14 teams, multiple approaches provide viable solutions within reasonable time limits, making the methods practical for real-world applications.

The study demonstrates that while the STS problem is computationally challenging, modern optimization techniques can effectively solve instances of practical size, with each paradigm offering unique advantages depending on the specific requirements and constraints of the tournament organization.

Authenticity and Author Contribution Statement

We declare that this work is our own and has not been copied from any other source. All methodologies and approaches were implemented independently based on the problem description and course materials. No AI-generated code was used in the implementation of the CP, SAT, SMT, or MIP models.

Author Contributions:

- Alireza Shahidiani: CP model development, MIP model implementation, and experimental design
- Omid Nejati: SAT and SMT model implementation, results analysis, and validation
- Both authors: Report writing, problem formalization, and experimental validation

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