CSE 101:

Introduction to Computational Thinking

Unit 7:

Random Numbers;
Object-oriented Programming

Games of Chance

- Many games involve chance of some kind:
 - Card games with drawing cards from a shuffled deck
 - Rolling dice to determine how many places we move a piece on a game board
 - Spinning a wheel to randomly determine an outcome
- We expect these outcomes to be **random** or unbiased in other words, *unpredictable*
- Computers can be programmed to generate *apparently* "random" sequences of numbers and other quantities for such games and other applications

Games of Chance

- In this unit we will explore algorithms for generating values that are apparently random and unpredictable
- We say "apparently" because we need to use mathematical formulas to generate sequences of numbers that at the very least appear to be random
- Since we will use an algorithm to generate "random" values, then we really can't say the sequence of values is truly random
- We say instead that a computer generates pseudorandom numbers

Pseudorandom Numbers

- Randomness is a difficult property to quantify
 - Is the list [3, 7, 1, 4] more or less random than [4, 1, 7, 3]?
- The algorithm that generates pseudorandom numbers is called a **pseudorandom number generator**, or PRNG
- The goal is for the algorithm to generate numbers without any kind of apparent predictability
- Python has built-in the capability to generate random values through its random module
- To generate a random integer in the range 1 10:
 import random
 num = random.randint(1, 10) # 10, not 11!

- The mod operator, denoted % in Python, will be a key part of generating pseudorandom numbers
- Suppose we wanted to generate a seemingly random sequence of numbers, all in the range 0 through 11
- Let's start with the number 0 and store it in a new list named **t**:

$$t = [0]$$

• One basic formula for generating numbers involves (1) adding a value to the previously-generated number and then (2) performing a modulo operation

- For our particular example, we could use 7 as our added value and then mod by 12
- Conveniently, the Python language lets us write **t[-1]** to mean "retrieve the last element of list **t**"
- We can write t[-2] to get the second-to-last element,
 t[-3] to get the third-to-last element, and so on
- So in general we can write t.append((t[-1]+7)%12) to generate and store the "next" pseudorandom number
- If we put this code inside of a loop we can generate a series of random values and store in them in the list

```
t = [0]
for i in range(15):
    t.append((t[-1] + 7) % 12)
```

The above code will generate the list:

```
[0,7,2,9,4,11,6,1,8,3,10,5,0,7,2,9]
```

- How "random" are these numbers?
 - They look pretty random, but we notice that eventually they start to repeat
- Can we improve things?
 - Part of the issue is the divisor of 12, but the formula itself is a little too simplistic

- A more general formula for generating pseudorandom numbers is $x_{i+1} = (a \cdot x_i + c) \mod m$
 - x_{i+1} is the "next" random number
 - x_i is the most recently generated random number
 - *i* is the position of the number in the list
 - *a*, *c* and *m* are constants called the *multiplier*, *increment* and *modulus*, respectively
- If the values a, c and m are chosen carefully, then every value from 0 through m-1 will appear in the list exactly once before the sequence repeats
- The number of items in the repetitive part of the list is called the **period** of the list

- We want the period to be as long as possible to make the numbers as unpredictable as possible
- We will implement the above formula, but first we need to explore some new programming concepts

Numbers on Demand

- One possibility for working with random numbers is to generate as many as we need and store them in a list
 - Often, however, in real applications we don't know exactly how many random numbers we will ultimately need
 - Also, in practice we might not want to generate a very long list of random numbers and store them
- Typically, we need only one or just a few random numbers at a time, so generating thousands or even millions of them at once is a waste of time and memory
- Rather than building such a list, we can instead generate the numbers one at a time, *on demand*

Numbers on Demand

- We will define a function **rand()** and a **global variable x** to store the most recently generated random number
 - A global variable is a variable defined outside functions and is available for use by any function in a .py file
- The value of a global variable is preserved between function calls, unlike local variables, which disappear when a function returns
- If we want a function to change the value of a global variable, we need to indicate this by using the **global** keyword in the function
- If we are only reading the global variable, we do not need to use the **global** keyword

The rand () Function (v1)

• Let's consider a function for generating random numbers that uses the formula we saw earlier:

```
x = 0  # global variable
def rand(a, c, m):
    global x
    x = (a * x + c) % m
    return x
```

- Call the function several times with a=1, c=7, m=12:
 rand(1,7,12) # returns 7 and updates x to 7
 rand(1,7,12) # returns 2 and updates x to 2
 rand(1,7,12) # returns 9 and updates x to 9
- Let's see why x is updated in this way

The rand () Function (v1)

- The key line of code is x = (a * x + c) % m
- Initially, x = 0
- 1. rand(1,7,12): x = (1 * 0 + 7) % 12 = 7
 - So, x becomes 7
- 2. rand(1,7,12): x = (1 * 7 + 7) % 12 = 2
 - So, x becomes 2 -
- 3. rand(1,7,12):x = (1 * 2 + 7) % 12 = 9
 - So, x becomes 9
- The only reason this series of computations works correctly is because the value of x is preserved between function calls

Modules and Encapsulation

- Suppose we wanted to use our new **rand()** function in several files. We have two options:
 - Copy and paste the function to each file (bad idea), or
 - Place the file in a .py by itself (or with other functions) to create a module that can be imported using an import statement (the right way)
- We should place our function in a module, along with the global variable \mathbf{x}
- This global variable will be "hidden" inside the module so that there is no danger of a "name clash", meaning that other modules could have their own global variables named x if they want to

Modules and Encapsulation

- This idea of gathering functions and their related data values (variables) into a single package is called encapsulation
- It's an extension of the concept called **abstraction** we studied earlier in the course
- We know that the math module has some useful functions and constants, like sqrt() and pi
- A module like **math** is an example of a **namespace**, a collections of names that could be names of functions, objects or anything else in Python that has a name
 - A module/namespace is one way of implementing the concept of encapsulation in Python

Modules and Encapsulation

- To create a new module, all we need to do is save the functions and variables of the module in a file ending in .py
 - For example, if we were to save the rand() function in the file prng.py, we could then import the rand() function in a new Python program by typing import prng at the top of the new program
- Below is a revised version of our **rand()** function that encapsulates the function in a module and stores the values of *x*, *a*, *c* and *m* as global variables
- This means the user no longer needs to pass *a*, *c* or *m* as arguments anymore
- We will also add a new function reset() to reset the PRNG to its starting state

The rand () Function (v2)

```
def rand():
x = 0
                          global x
a = 81
                          x = (a * x + c) % m
c = 337
m = 1000
                          return x
def reset(mult, inc, mod):
   global x, a, c, m
   x = 0
   a = mult
   c = inc
   m = mod
```

The rand () Function (v2)

```
a = 81
c = 337
m = 1000
• Examples:
1. rand(): (81 * 0 + 337) % 1000 = 337
2. rand(): (81 * 337 + 337) % 1000 = 634
```

3. rand(): (81 * 634 + 337) % 1000 = 691

x = 0

The rand () Function (v2)

- We can change the values of a, c and m by calling the reset() function. Example: reset(19, 4, 999). reset() also sets x = 0.
- Now we will generate a different sequence of random numbers:
- 1. rand(): (19 * 0 + 4) % 999 = 4
- 2. rand(): (19 * 4 + 4) % 999 = 80
- 3. rand(): (19 * 80 + 4) % 999 = 525

Games with Random Numbers

- Suppose we wanted to simulate the rolling of a six-sided die in a board game
- We would want to generate integers in the range 1 through
 6, inclusive
- Our function rand() generates values outside this range, however
- We can solve this problem using an expression like
 rand() % 6 + 1
- The expression **rand()** % **6** gives us a value in the range 0 through 5, which we can then "shift up" by adding 1

Games with Random Numbers

- If we always initialize *x*, *a*, *c* and *m* to the same values, then every program that uses the **rand()** function will get the same exactly sequence of pseudorandom values
- Instead, we could allow someone using our code to set the starting value of *x*, which we call the **seed** of the pseudorandom number generator
- Another option is we can have the computer pick the seed by using the system clock
- The time module has a function called **time()** which returns the number of seconds since January 1, 1970
- Fractions of a second are also included in the returned value

Games with Random Numbers

 Our revised module shown on the right uses time.time() to pick a random seed

```
import time
a = 81
c = 337
m = 1000
x = int(time.time()) % m
def rand():
   global x
   x = (a * x + c) % m
   return x
```

See unit07/random_numbers.py

Random Numbers in a Range

- In general, how can we generate random integers from an arbitrary range?
- The formula is surprisingly simple:

```
rand() % (high - low + 1) + low
```

- For example, suppose we wanted to generate a value in the range —5 through 10, inclusive
- The formula indicates we should use this code:

```
rand() % (10 - (-5) + 1) + (-5)
```

- Simplifying gives us: rand() % 16 5
- See unit07/random_numbers.py

List Comprehensions

- Python features a very compact syntax for generating a list called a list comprehension
 - We write a pair of square brackets and inside the brackets put an expression that describes each list item
- For example to make a list of numbers from 1 to 10 write [i for i in range(1,11)]
- To make a list of the first 10 perfect squares we could write [i**2 for i in range(1,11)]
- In general, we write an expression that describes each new item in the new list and a loop that describes a set of existing values to work from
- A list of 10 random numbers: [rand() for i in range(10)]

List Comprehensions

 Suppose we wanted to take a list of words and capitalize them all:

```
names = ['bob', 'DANE', 'mikey', 'ToMmY']
names = [s.capitalize() for s in names]
names would become ['Bob', 'Dane', 'Mikey', 'Dane', 'Dane', 'Mikey', 'Dane', 'Dane', 'Mikey', 'Dane', 'Dane',
```

- names would become ['Bob', 'Dane', 'Mikey', 'Tommy']
- Or perhaps we wanted to extra the first initial of each person and capitalize it:

```
initials = [s[0].upper() for s in names]
```

• initials would be ['B', 'D', 'M', 'T']

Random Shuffles

- Suppose we needed the ability to randomly *permute* (shuffle) a list of items, such as a deck of 52 playing cards
- Let's explore how we might write a function that does exactly this
- First, in Card.py we define a class called Card
- A class defines a new type of object in an object-oriented programming language like Python
- We use a special method called the **constructor** to create (construct) new objects of the class

```
from Card import Card card = Card()
```

The Card Class

- A Card object has a separate rank and suit, which we can query using the rank() and suit() methods, respectively
- The 2 through Ace are ranked 0 through 12
- The suits are mapped to integers as follows:
 - Clubs: 0
 - Diamonds: 1
 - Hearts: 2
 - Spades: 3
- For example, for a **Card** object representing the 9 of Spades, **rank()** would return 7 and **suit()** would return 3

The Card Class

- The ranks and suits are numbered so that we can uniquely identify each card of a standard 52-card deck
 - When calling the constructor to create a **Card** object, we provide a number in the range 0 through 51 to identify which card we want
- Examples:
 - Card(0) and Card(1) are the 2 and 3 of Clubs, respectively
 - Card (50) and Card (51) are the King and Ace of Spaces, respectively
- print(Card(51)) would output A♠ (yes, including that Spades symbol!)

The Card Class

• We can use a list comprehension to generate all 52 cards and store them in a list:

```
deck = [Card(i) for i in range(0,52)]
```

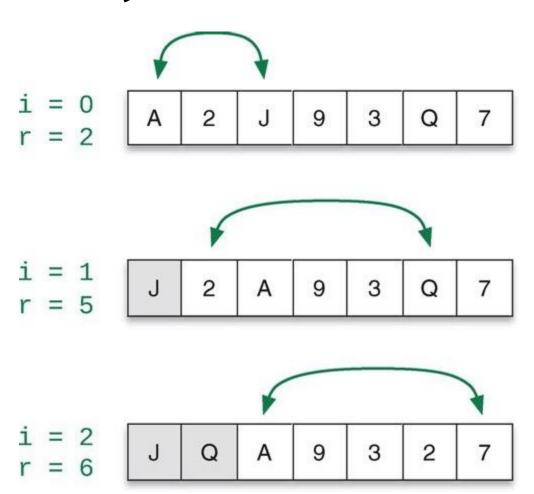
- With slicing we can take a look at just the first 5 by appending [:5] to the name of the variable
 - This notation means "slice out all the elements of the list up to (but not including) the element at index 5"

```
print(deck[:5])
```

• Output: [2♣, 3♣, 4♣, 5♣, 6♣]

- The order of the cards generated by the list comprehension (i.e., sequential order) is only one particular ordering or **permutation** of the cards
- We want to define a function that will let us permute a list to generate a more random ordering of the items in the list
- A simple algorithm for permuting the items in a list is to iterate over the list and exchange each element with a random element to its right
- This is most easily seen by example, as on the next slide

Iterate over the entire array deck
 (with i as the loop variable and index), swapping a random item to the right of i with deck[i]



- This shuffling algorithm is very easy to implement with the help of a function that will choose a random item to the right of deck[i]
- The function randint(low, high) from the random module generates a random integer in the range low through high (inclusive of both low and high)
- The permute function will shuffle any list of items:
 import random
 def permute(a):
 for i in range(0, len(a)-1):

```
r = random.randint(i, len(a)-1)
a[i], a[r] = a[r], a[i] # swap items
```

```
import random
def permute(a):
    for i in range(0, len(a)-1):
        r = random.randint(i, len(a)-1)
        a[i], a[r] = a[r], a[i] # swap items
```

- r = random.randint(i, len(a)-1) picks the random index, r, that is to the right of i (or might choose i itself, meaning that a[i] doesn't move)
- a[i], a[r] = a[r], a[i] swaps a[i] with the randomly chosen item to its right
- We would call this function with permute (deck) to shuffle our list of Card objects

Defining New Objects

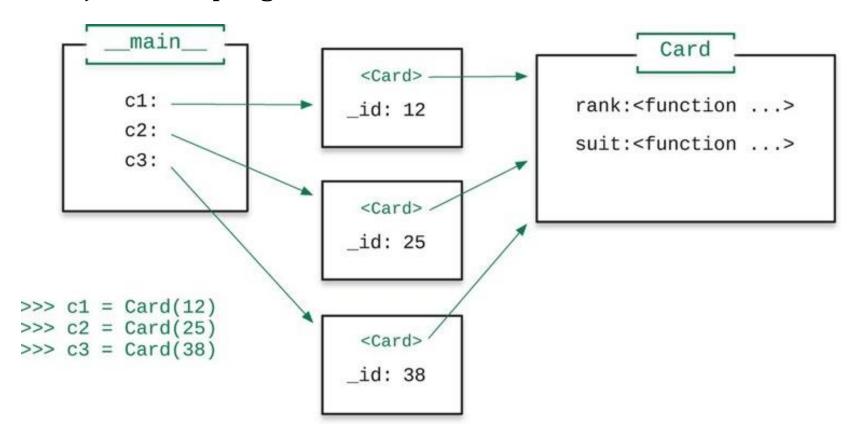
- The **Card** class we have been working with defines a new kind of object we can use in programs
- In object-oriented programming, a class determines the data and operations associated with an object
- For example, for a playing card object we need some way to store the rank and suit of a card; these are its data attributes
- Operations for a playing card might include code that lets us print a playing card on the screen or retrieve the card's rank and suit

Defining New Objects

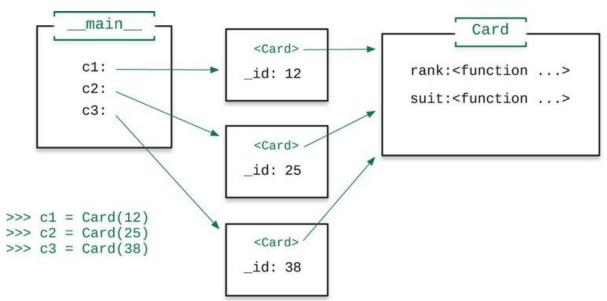
- The data values associated with a particular object are called instance variables
- We say that an object is an **instance** of a class
 - For example, each of the 52 **Card** objects is an independent instance of the **Card** class
 - As such, each **Card** object has its own copies of the instance variable that store the object's rank and suit
- The operations associated with an object are called methods
- So, a class defines the data properties and methods that an object of the class has

Defining New Objects

 Let's see an example where we create three distinct Card objects in a program:

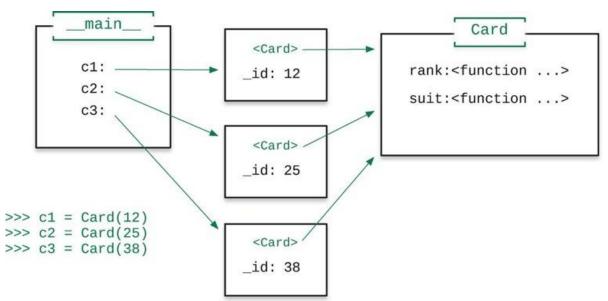


Defining New Objects



- Three **Card** objects were constructed. They are referenced using the variables **c1**, **c2** and **c3**, as shown on the left
- The objects as they might exist in the memory of the computer are shown in the middle of the diagram
- Rather than store the rank and suit separately, they are combined into a single integer called id

Defining New Objects



- Prepending an underscore to a variable indicates that **_id** is an instance variable; this is a naming convention, not a strict rule
- To retrieve the rank or suit, we need to call the methods rank() or suit(), as depicted on the right

Defining New Objects

- To define a new class we usually include the following aspects:
 - One or more instance variables
 - One or more methods that perform some operation or execute some algorithm
 - A <u>__init__</u> method, which initializes (gives starting values to) the instance variables
 - A <u>repr</u> method, which defines a string *representation* of an object that is suitable for printing on the screen
- Let's step through building the Card class from the ground up

- The code we build up in piecemeal fashion will all eventually be saved in a file named Card.py
- We begin by writing a class statement:

class Card:

• Next we write the __init__ method. The **self** keyword refers to the object itself.

```
def __init__(self, n):
    self._id = n
```

• The <u>__init__</u> method is called the class's **constructor** because it is used to construct new objects

- Now we can write the rank () and suit () methods
- They translate the **_id** number into the rank and suit of a card

```
def suit(self):
    return self._id // 13
def rank(self):
    return self. id % 13
```

- This encoding ensures that all 13 cards of a single suit are placed together in consecutive order
- Now let's write a simple __repr__ method

```
def __repr__(self):
    return 'Card #' + str(self. id)
```

• The Card class so far:

```
class Card:
   def init (self, n):
      self. id = n
   def suit(self):
      return self. id // 13
   def rank(self):
      return self. id % 13
   def __repr (self):
      return 'Card #' + str(self. id)
```

- Suppose we created card #43: c1 = Card(43)
- If we go ahead and print out **c1**, we will get output like this: **Card #43**
- That's not very informative, so we'll have to fix it later
- We can write a function **new_deck()** that creates a list of 52 playing-card objects. This function is not part of the **Card** class itself.

```
def new_deck():
    return [Card(i) for i in range(52)]
```

An example call to this function:

```
deck = new deck()
```

- Another improvement we can make is to add special methods that allow us to compare Card objects
- If we want to be able to sort **Card** objects, we must provide the **__lt__()** method, which tells us if one object is "less than" another:

```
def __lt__(self, other):
    return self._id < other._id</pre>
```

• **__eq__()** defines what it means for two **Card** objects to be "equal to" each other:

```
def __eq__(self, other):
   return self._id == other._id
```

• For example, consider the following objects:

```
c1 = Card(1)
c2 = Card(4)
```

- The expression c1 < c2 would be True, but c1 == c2 would be False
- Now that we can compare Card objects, we can sort them using the sorted function
 - sorted makes a copy of a list and then sorts the copy: cards sorted = sorted(cards)

- The **Card** class defines an **application program interface** or API: a set of related methods that other programmers can use to build other software
- Also, we are applying the concept of encapsulation by gathering all the code that defines a Card object in one place
- On that topic, it can be useful to define **class variables**, values that pertain to a particular class but are not instance variables
- For our **Card** class it would be useful if we could print symbols representing the suits and ranks: ♣ ♦ ♥ ♠
- In Python we have access to many thousands of symbols.
- We can access them by giving the correct numeric codes.

- suit_sym = {0: '\u2663', 1: '\u2666', 2: '\u2665', 3: '\u2660'}
- If we were to print **suit_sym**, we would get this output:

```
{0: '♣', 1: '♦', 2: '♥', 3: '♠'}
```

- The codes for various symbols can be found on the Internet by searching for "Unicode characters"
- Likewise, we can define a dictionary for all the ranks:
- rank_sym = {0: '2', 1: '3', 2: '4', 3:
 '5', 4: '6', 5: '7', 6: '8', 7: '9', 8:
 '10', 9: 'J', 10: 'Q', 11: 'K', 12: 'A'}

 We will change our definition of the __repr__ method to this:

• Now, when we print a **Card** object, we will get output like $2 \spadesuit$, $A \spadesuit$, $8 \spadesuit$, $J \spadesuit$, etc.

- What if another programmer using our class inadvertently gives a value outside the range 0 through 51 for **n** when constructing a **Card** object?
 - The <u>__init__</u> method will accept the value, but it really shouldn't
- We can solve this problem by adding exception handling to our code
 - An exception is an unexpected event or error that has been detected
 - We say that the program has raised an exception
 - Let's have the <u>__init_</u> method raise an exception if an invalid value is given for **n**

- The new version of __init__ verifies that the argument n is valid
- If not, it raises the exception and includes a diagnostic message of sorts

 Consider a function now that a programmer might use to make new cards that catches any exception that might be thrown by the init method:

```
def make_card(n):
    try:
        return Card(n)
    except Exception as e:
        print('Invalid card: ' + str(e))
    return None
```

• If we call make_card(55), we get this output: Invalid card: Card number must be in the range 0-51.

- This concludes our development of the Card class
- See unit07/Card.py for the completed **Card** class and unit07/playing_cards.py for some tests

Example: Acronym Generator (v1)

- Let's explore a function that will create an acronym from the first letter of each "long" word in a list
- Define a "long" word to be any word with more than two letters
- After studying this first version, we will look at a second version that affords a little extra flexibility in creating acronyms

Example: acronym1.py

```
def acronym(phrase):
    result = ''
    words = phrase.split()
    for w in words:
        if len(w) >= 3: # keep only long words
            result += w.upper()[0]
    return result
```

• Let's trace the execution of this function for one example: acronym('United States of America')

```
def acronym(phrase):
    result = ''
    words = phrase.split()
    for w in words:
        if len(w) > 3:
            result += w.upper()[0]
    return result
```

Variable	Value
phrase	'United States of America'

```
acronym('United States of America')
def acronym(phrase):
    result = ''
    words = phrase.split()
    for w in words:
        if len(w) > 3:
            result += w.upper()[0]
    return result
```

Variable	Value
phrase	'United States of America'
result	T T

```
acronym('United States of America')
def acronym(phrase):
    result = ''
    words = phrase.split()
    for w in words:
        if len(w) > 3:
            result += w.upper()[0]
    return result
```

Variable	Value
phrase	'United States of America'
result	7 7
words	['United', 'States', 'of', 'America']

```
acronym('United States of America')
def acronym(phrase):
    result = ''
    words = phrase.split()
    for w in words:
        if len(w) > 3:
            result += w.upper()[0]
    return result
```

Variable	Value
phrase	'United States of America'
result	7 7
words	['United', 'States', 'of', 'America']
W	'United'

```
acronym('United States of America')
def acronym(phrase):
    result = ''
    words = phrase.split()
    for w in words:
        if len(w) > 3: True
        result += w.upper()[0]
    return result
```

Variable	Value
phrase	'United States of America'
result	7 7
words	['United', 'States', 'of', 'America']
W	'United'

```
acronym('United States of America')
def acronym(phrase):
    result = ''
    words = phrase.split()
    for w in words:
        if len(w) > 3:
            result += w.upper()[0]
    return result
```

Variable	Value
phrase	'United States of America'
result	יטי
words	['United', 'States', 'of', 'America']
W	'United'

```
acronym('United States of America')
def acronym(phrase):
    result = ''
    words = phrase.split()
    for w in words:
        if len(w) > 3:
            result += w.upper()[0]
    return result
```

Variable	Value
phrase	'United States of America'
result	'ט'
words	['United', 'States', 'of', 'America']
W	'States'

```
acronym('United States of America')
def acronym(phrase):
    result = ''
    words = phrase.split()
    for w in words:
        if len(w) > 3: True
        result += w.upper()[0]
    return result
```

Variable	Value
phrase	'United States of America'
result	יטי
words	['United', 'States', 'of', 'America']
W	'States'

```
acronym('United States of America')
def acronym(phrase):
    result = ''
    words = phrase.split()
    for w in words:
        if len(w) > 3:
            result += w.upper()[0]
    return result
```

Variable	Value
phrase	'United States of America'
result	'US'
words	['United', 'States', 'of', 'America']
W	'States'

```
acronym('United States of America')
def acronym(phrase):
    result = ''
    words = phrase.split()
    for w in words:
        if len(w) > 3:
            result += w.upper()[0]
    return result
```

Variable	Value
phrase	'United States of America'
result	'US'
words	['United', 'States', 'of', 'America']
W	'of'

```
acronym('United States of America')
def acronym(phrase):
    result = ''
    words = phrase.split()
    for w in words:
        if len(w) > 3: False
            result += w.upper()[0]
    return result
```

Variable	Value
phrase	'United States of America'
result	'US'
words	['United', 'States', 'of', 'America']
W	'of'

```
acronym('United States of America')
def acronym(phrase):
    result = ''
    words = phrase.split()
    for w in words:
        if len(w) > 3:
            result += w.upper()[0]
    return result
```

Variable	Value
phrase	'United States of America'
result	'US'
words	['United', 'States', 'of', 'America']
W	'America'

```
acronym('United States of America')
def acronym(phrase):
    result = ''
    words = phrase.split()
    for w in words:
        if len(w) > 3: True
        result += w.upper()[0]
    return result
```

Variable	Value
phrase	'United States of America'
result	'US'
words	['United', 'States', 'of', 'America']
W	'America'

```
acronym('United States of America')
def acronym(phrase):
    result = ''
    words = phrase.split()
    for w in words:
        if len(w) > 3:
            result += w.upper()[0]
    return result
```

Variable	Value
phrase	'United States of America'
result	'USA'
words	['United', 'States', 'of', 'America']
W	'America'

```
acronym('United States of America')
def acronym(phrase):
    result = ''
    words = phrase.split()
    for w in words:
        if len(w) > 3:
            result += w.upper()[0]
    return result
```



Variable	Value
phrase	'United States of America'
result	'USA'
words	['United', 'States', 'of', 'America']
W	'America'

Example: Acronym Generator (v2)

- Python allows function arguments to have default values
 - If the function is called without the argument, the argument gets its default value
 - Otherwise, the argument's value is given in the normal way
- We have seen a few examples of functions that have optional arguments
- A good example is the **round()** function, which takes two arguments: the value to round and an optional argument that indicates how many digits after the decimal point we want
 - If the second argument is not provided, the number of digits defaults to 0

Example: Acronym Generator (v2)

- The second version of acronym takes an optional argument, include_shorts, that tells the function to include the first letter of all words (including short words), but short words will not be capitalized if they are included
- The first version of acronym simply discarded all short words

Example: acronym2.py

```
def acronym(phrase, include_shorts=False):
    result = ''
    words = phrase.split()
    for w in words:
        if len(w) > 3:
            result += w.upper()[0]
        elif include_shorts:
            result += w.lower()[0]
    return result
```

- By default, the optional argument is **False**, causing short words to be excluded
- When the optional argument is **True** and **w** is a short word, the first letter of the word in lowercase is concatenated to **result**

Example: acronym() (v2)

- Examples:
- acronym('United States of America') still returns 'USA'
- acronym('United States of America', True)
 returns 'USoA'

Optional Arguments

• As another example, suppose we want to make a revised version of the **bmi()** function from earlier in the course:

```
def bmi(weight, height):
  return (weight * 703) / (height ** 2)
```

- This version of **bmi()** assumes weight is given in pounds and height in total inches
- Suppose instead we want to give the programmer the option to use metric or standard (English) units
- We can add a third, optional argument, units, that defaults to metric if the programmer doesn't give a third argument
- Let's see the function on the next slide

Example: bmi_v4.py

```
def bmi(height, weight, units = 'metric'):
   if units == 'metric':
      return weight / height**2
   elif units == 'standard':
      return (weight * 703) / (height ** 2)
   else:
      return None
• Examples:
                                Return Value:
bmi (100, 150, 'standard')
                                10.545
• bmi(100, 150)
                                0.015
• bmi(100, 150, 'metric'):
                                0.015
• bmi(100, 150, 'unknown')
                                None
```