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Section: MATH 302 102

HW 1

1. Let S = {1, {}, c} be a sample space. List all possible events.

{}, {1}, {{}}, {c}, {1, {}}, {1, c}, {{}, c}, {1, {}, c}

2. We roll a fair die until the first 1 comes up. What is the probability that the number of tosses is odd?

In the first throw, there's a $\frac{1}{6}$ chance of getting a one. In the second throw, there's a $\frac{1}{6} \times \frac{5}{6}$ chance of getting a one, where the $\frac{5}{6}$ comes from the first throw. For succeeding throws, we multiply the probability by $\frac{5}{6}$.

As we only consider odd number of throws, we get:

$$\mathbb{P}(odd\ number\ of\ tosses\ to\ get\ a\ one) = \frac{1}{6} + (\frac{5}{6})^2 \times \frac{1}{6} + (\frac{5}{6})^4 \times \frac{1}{6} + ... \tag{1}$$

Using the formula for the infinite geometric series:

$$\mathbb{P}(odd\ number\ of\ tosses\ to\ get\ a\ one) = \frac{a_1}{1-r} = \frac{\frac{1}{6}}{1-(\frac{5}{6})^2} = \frac{6}{11} \tag{2}$$

3. Assuming a fair poker deal, what is the probability of a

(a) royal flush
(b) straight flush
(c) flush
(d) straight
(e) two pair

(a) To get A, K, Q, J, 10, we only have four choices (four suits) out of the $\binom{52}{5} = 2,598,960$ ways to get 5 cards. So, probability is $\frac{4}{2{,}598{,}960}$

(b) In one suit, there are 10 possible combinations. And there are four suits, but a royal flush is not counted. So, the probability is $\frac{4\times(10-1)}{2{,}598{,}960} = \frac{36}{2{,}598{,}960}$

(c) To get a flush, there are $\binom{13}{5} \times 4$ possibilities (for four suits). For each suit, there are 10 straight flushes, which are not counted as flush. Overall, we get $\frac{(\binom{13}{5}-10)\times 4}{\binom{52}{5}} = \frac{5108}{2{,}598{,}960}$

(d) There are ten ways to choose which card will be the highest rank among the five, and four suits to choose from. After picking the first card, you are left with four choices each (representing the four suits) for each of the four remaining unchosen cards for it to be a straight. So, probability is $\frac{(10\times4)\times4\times4\times4\times4}{\binom{52}{5}} = \frac{10{,}240}{2{,}598{,}960}$

(e) There are 13 choices when choosing the first card. Then to complete the first pair, we pick 2 out of the four suits. Same goes for the second pair, only with 12 choices. The fifth card will have 11 choices left, and we choose one suit. So, the probability is $\frac{\binom{13}{1}\binom{4}{2}\times\binom{12}{1}\binom{4}{2}\times\binom{11}{1}\binom{4}{1}}{2\times\binom{52}{5}} = \frac{247{,}104}{2{,}598{,}960}$

4. (a) How many ways are there to deal 52 standard playing cards to four players (so that each player gets 13 cards)?
Clarification: The order of the 13 cards that each player receive does not matter, but the four players are treated as different (e.g., call them player 1, player 2, player 3 and player 4). When in doubt, you should always explain your interpretation of the problem, and unless it makes the problem much easier, you should receive almost a full mark (or a full mark).

(b) Suppose you are world champion in card dealing, and can deal 52 cards in just one second. Compare the number of years it would need you to deal all possible combinations with the current age of the universe (13.77 billion years).

(a) $\binom{52}{13}\binom{39}{13}\binom{26}{13}\binom{13}{13}$. The total number of ways would be 5.4×10^{28}

(b) 5.4×10^{28} seconds is equivalent to 1.7×10^{21} years. Comparing this to the age of the universe, it will take 1.23×10^{11} times longer to finish dealing all possible combinations of the cards.

5. We toss a fair die three times. What is the probability that all tosses produce different outcomes?

On the first throw, we are fine with any outcome, so probability is $\frac{6}{6} = 1$. On succeeding throws, we do not want a number to get repeated, so on the second throw, we get $\frac{5}{6}$, the third $\frac{5}{6}$, and so on.

The probability that all tosses produce different outcomes would be $\frac{6}{6} \times \frac{5}{6} \times \frac{4}{6} = \frac{5}{9}$

6.

7. You own n colours, and want to use them to colour 8 objects. For each object, you randomly choose one of the colours. How large does n have to be so that the probability that there exists a pair of objects which shares the same colour is less than 0.25?

There are $n(n-1)(n-2) \times ... \times (n-7) = \frac{n!}{(n-8)!}$ possibilities to get 8 distinct colours. The probability of choosing 8 distinct colours is $\frac{n!}{n^8(n-8)!}$. Changing this to the possibility that there exists a pair of objects which shares the same colour, this becomes $1 - \frac{n!}{n^8(n-8)!}$, which is supposed to be less than 0.25. This gives us n = 100, with a probability of 0.2497.

So n has to be at least 100.