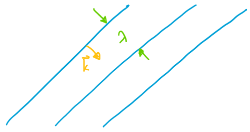
2.7 Wave Dynamics at Oblique Incident Angles

Tuesday, October 31, 2023 10:23 PM

Optics Kinematics

- Wavefront Location, Reflection & Refraction

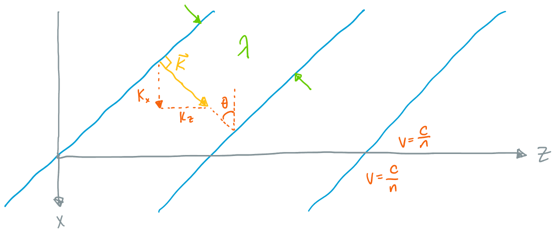


$|\vec{k}| = \frac{2\pi}{\lambda}$ spatial density wave

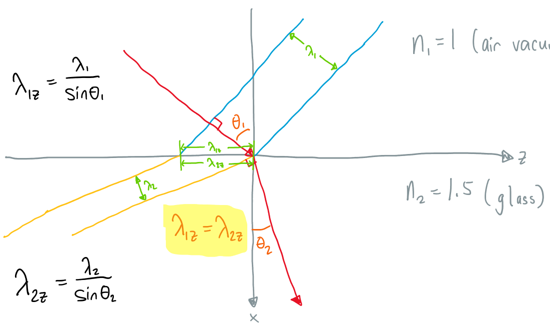
$K_x = |\vec{k}| \cos\theta$

$K_y = |\vec{k}| \sin\theta$

$$\lambda_z = \frac{2\pi}{K_z} = \frac{2\pi}{|\vec{k}|\sin\theta} = \frac{2\pi}{\left(\frac{2\pi}{\lambda}\right)\sin\theta} = \frac{\lambda}{\sin\theta}$$



$\lambda_{1z} = \frac{\lambda_1}{\sin\theta_1}$



$\lambda_{2z} = \frac{\lambda_2}{\sin\theta_2}$

$$\lambda_{1z} = \lambda_{2z} \longrightarrow \frac{\lambda_1}{\sin\theta_1} = \frac{\lambda_2}{\sin\theta_2} \longrightarrow \frac{1}{f}\frac{c}{n_1 \sin\theta_1} = \frac{1}{f}\frac{c}{n_2\sin\theta_2} \longrightarrow \boxed{n_1 \sin\theta_1 = n_2 \sin\theta_2}$$

Snell's Law

$$\left.\begin{aligned} \lambda_1 &= \frac{V_1}{f} = \frac{1}{f}\frac{c}{n_1} \\ \lambda_2 &= \frac{V_2}{f} = \frac{1}{f}\frac{c}{n_2} \end{aligned}\right\}$$

Q1 Using the coordinate system above, which component of λ must stay the same across material boundaries?

> The z component (along the material boundary)

The x component (perpendicular to the material boundary)

Q2 A wave incident from air at an angle of $\theta_i = 45°$ reaches an air-water boundary. If the water has an index of refraction of 1.33, what is the transmitted angle θ_t?

$1 \sin(45°) = 1.33 \sin(\theta)$

$\theta = 32.12°$

Q3 Transmitted Angles in Snell's Law

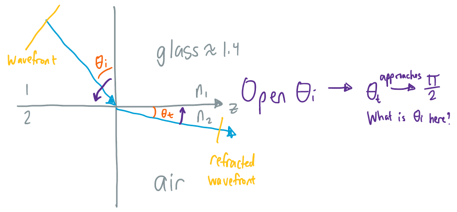
What would you expect the transmission angle to be given the following situations?

- $n_i > n_t$ (e.g. water to air) $\theta_i < \theta_t$

What would you expect the transmission angle to be given the following situations?

- $n_i > n_t$ (e.g. water to air) $\theta_i < \theta_t$
- $n_i < n_t$ (e.g. air to diamond) $\theta_i > \theta_t$
- $n_i = n_t$ $\theta_i = \theta_t$

Critical Angle + TIR (total internal reflection) — Dynamics & Kinematics



Open $\theta_i \rightarrow \theta_t \xrightarrow{\text{approaches}} \frac{\pi}{2}$

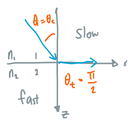
What is θ_i here?

- Assumption: $\mu_1 = \mu_2 = \mu_0$

$n_1 \sin\theta_i = n_2 \sin\theta_t$ $\rightarrow \frac{\pi}{2}$

$\theta_c \rightarrow n_1 \sin\theta_c = n_2$

$\sin\theta_c = \frac{n_2}{n_1}$ $\qquad \theta_c \approx 41°$



- What happens when $\theta_i > \theta_c$?

$\frac{n_1}{n_2}\sin\theta_i = 1$ $\qquad \theta_i = \theta_c$

$= \sin\theta_t$ $\qquad \theta_t = \frac{\pi}{2}$

$\theta_i > \theta_c \rightarrow \frac{n_1}{n_2}\sin\theta_i > 1 \rightarrow \sin\theta_t > 1$

θ_t is a complex angle

No energy in downward z direction (T.I.R.)

$\cos\theta_t = \pm\sqrt{1-\sin^2\theta_t}$ negative result!

$= \pm jA$ Which sign to pick? See Dynamics

$\sin\theta_t > 1$ How? Super critical

$\sin\theta_t = \frac{e^{j\theta} - e^{-j\theta}}{2j}$ $\qquad$ Euler

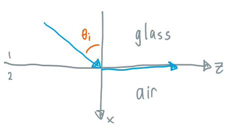
$\theta_t = \alpha + j\beta$

$\rightarrow \sin\theta_t = \frac{e^{j(\alpha+j\beta)} - e^{-j(\alpha+j\beta)}}{2j} = \frac{e^{-\beta}e^{j\alpha} - e^{\beta}e^{-j\alpha}}{2j}$

$e^{\pm j\theta_t} = \cos\theta_t \pm j\sin\theta_t$

$\rightarrow \cos\theta_t = \pm\sqrt{1-\sin^2\theta_t} = \pm jA$

- Applying to Dynamics $\vec{E} \times \vec{H}$



$\vec{E} = \hat{j}\,\underbrace{\tau E_{y_i}}_{E_{y_t}} e^{-j\phi}$ (τ: transmission coefficient)

$\phi = K_2[\cos\theta_t x + \sin\theta_t z]$

$-\oplus A$

$K_2 = \omega\sqrt{\mu_2\varepsilon_2} = \omega\sqrt{\mu_0\varepsilon_2}$

$$\phi = K_2\left[\cos\theta_t x + \sin\theta_t z\right]$$

$\pm j A$

If + is chosen, when we exponentiate, we will get $(-j)(+j)$ which is 1

→ as we increase x, the wave is getting larger (physically non-sensible)

$K_2 = \omega\sqrt{\mu_2\varepsilon_2} = \omega\sqrt{\mu_0\varepsilon_2}$

- $\vec{E}$ and $\vec{H}$

$$\vec{E} = \hat{j}\,\tau E_{y_i} e^{-j\phi}$$

$$\phi = k_2\left[\cos\theta_t x + \sin\theta_t z\right]$$

$$\vec{H} = \frac{\tau E_{y_i}}{\eta_2}\left(-\sin\theta_t \hat{i} + \cos\theta_t \hat{k}\right)e^{-j\phi}$$

> Substitute for $\cos\theta_t = -jA$ to get:

$$\vec{E} = \hat{j}\,\tau E_{y_i} e^{-k_2 A x} e^{-jk_2 \sin\theta_t z}$$

$$\vec{H} = \frac{\tau E_{y_i}}{\eta_2}\left(-\sin\theta_t \hat{i} - jA\hat{k}\right)e^{-k_2 A_x} e^{-jk_2\sin\theta_t z}$$

> We need to compute $\langle\vec{S}\rangle = \frac{1}{2}\mathrm{Re}\left(\vec{E}\times\vec{H}^*\right)$ to get the energy density flow per area

$$\longrightarrow \vec{H} = \frac{\tau^* E_{y_i}^*}{\eta_2}\left(-\sin\theta_t \hat{i} - jA\hat{k}\right)e^{-k_2 A_x} e^{-jk_2\sin\theta_t z}$$

Conjugated

(Assume η in air is real)

> Now for $\langle\vec{S}\rangle$:

$$\langle\vec{S}\rangle = \frac{1}{2}\frac{|\tau|^2 |E_{y_i}|^2}{\eta} e^{-k_2 2A_x} \sin\theta_t \hat{k}$$

direction of $\langle\vec{S}\rangle$ is only along the interface

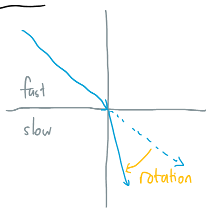
→ No power is delivered downwards

Q1 If a wave incident from water ($n=1.33$) approaches a water-air boundary at an angle greater than the critical angle, what can be observed in the air?

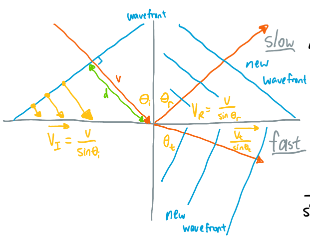
> A wave traveling parallel to the water-air boundary with E field exponentially decaying into the air (evanescence waves)

Q2 For a wave incident from material 1 traveling towards material 2, if an evanescent wave is observed in material 2, the wave intensity decays exponentially into the 2nd material

In-class 11/10/2023



Snell's law is a conservation law.



Δt = travel time to interface

d = wavefront normal = $V\Delta t$

$d_I = \frac{d}{\sin\theta} = V_I \Delta t$

$$\frac{V}{\sin\theta_i} = \frac{V}{\sin\theta_r} = \frac{V_t}{\sin\theta_t}$$

$V_i = V_r = V_1$; $V_t = V_2$

$$\frac{V_1}{\sin\theta_i} = \frac{V_1}{\sin\theta_r} = \frac{V_2}{\sin\theta_t} \longrightarrow \frac{\sin\theta_i}{V_1} = \frac{\sin\theta_r}{V_1} = \frac{\sin\theta_t}{V_2}$$

$n = \frac{c}{v}$

$v \longrightarrow$ speed/velocity

$\frac{1}{v} \longrightarrow$ "slowness"

frequencies of reflected & transmitted & incident need to have same frequencies

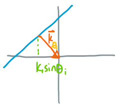
$\lambda = \frac{v}{f}$

$$\frac{w\sin\theta_i}{V_1} = \frac{w\sin\theta_r}{V_1} = \frac{w\sin\theta_t}{V_2}$$

$$\frac{2\pi f}{V_1}\sin\theta_i = \frac{2\pi f}{V_1}\sin\theta_r = \frac{2\pi f}{V_2}\sin\theta_t$$

$$\frac{2\pi}{\lambda_1}\sin\theta_i = \frac{2\pi}{\lambda_1}\sin\theta_r = \frac{2\pi}{\lambda_2}\sin\theta_t$$

$$k_1\sin\theta_i = k_1\sin\theta_r = k_2\sin\theta_t$$



Horizontal component of k is conserved

$$\frac{\sin\theta_i}{V_1} = \frac{\sin\theta_t}{V_2} \longrightarrow \sin\theta_t = \frac{V_2}{V_1}\sin\theta_1$$

$\exists\ \theta_1$ for which $\sin\theta_1 \frac{V_2}{V_1} = 1$

$\rightarrow \theta_t = \frac{\pi}{2}$ for a particular $\theta_i \longrightarrow \theta_c$ critical angle

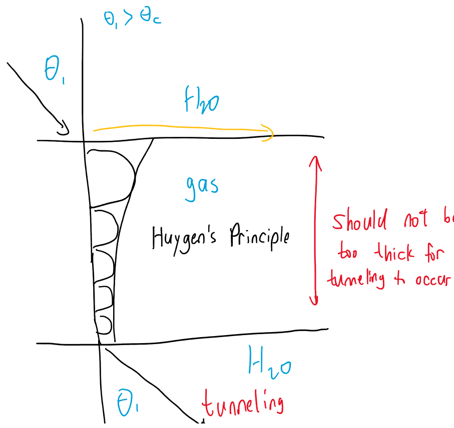
$\sin\theta_t = \frac{V_2}{V_1}\sin\theta_i > 1$

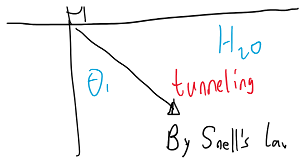
$\hookrightarrow$ a complex number   No energy in downward z direction

$$\sin\theta_t = \frac{e^{j\theta_t} - e^{-j\theta_t}}{2j}$$

In-class 11-17-2023

Picture :





By Snell's Law