Wind Statistics

Tuesday, March 19, 2024 4:00 PM

Recap: Power in the wind: $P_w = \frac{1}{2}\rho A v^2$

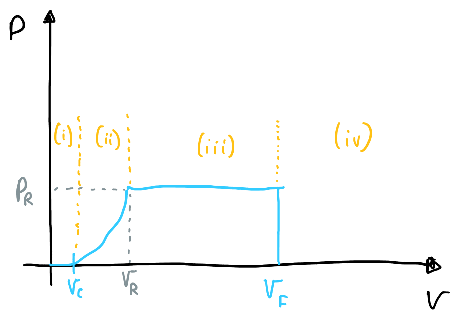
Power produced by turbine: $P = \frac{1}{2}\rho A v^3 \cdot C_p$ "efficiency"

metric for "roughness around turbine"

Also, $\left(\frac{V}{V_0}\right) = \left(\frac{H}{H_0}\right)^{\alpha}$, $\frac{P_w}{P_{w_0}} = \left(\frac{H}{H_0}\right)^{3\alpha} = \left(\frac{V}{V_0}\right)^3$

Relationship between wind speed and electrical power

→ ideal power curve



(i) $0 \leq v \leq v_c$: $P = 0$

- not enough power to overcome friction/inertia of turbine
- v_c is the minimum speed needed to produce net positive generation

(ii) $v_c < v \leq v_R$

- $P \propto v^3$

(iii) $v_R < v \leq v_F$: $P = P_R$

- reaches generator rated power
- must shed power via mechanical alterations of turbine (7.2 in textbook)

(iv) $v > v_F$: $P = 0$

- winds are too strong

winds are too strong

- must shut down to avoid damage to equipment

Q: How much energy to expect from turbine?

(i) $v \le v_c$? easy $\rightarrow$ 0!

(iv) $v > v_F$? also easy $\rightarrow$ 0!

(ii) $v_R < v < v_F$? $P = P_R \rightarrow E = P_R \times t$

(iii) $v_c < v \le v_R$? P is not constant.

$$P = \underbrace{\frac{1}{2}\rho A v^3}_{P_w} \cdot C_p$$

- Try average power:

$$P_{ave} = \left(\frac{1}{2}\rho A v^3\right)_{density_{ave}} = \frac{1}{2}\rho A v_{ave}^3$$

$$= \frac{1}{2}\rho A \left(v^3\right)_{ave}$$

- Need to look at wind power statistics

Ex 10-h period: 3hr no wind, 3hr 5m/s, 4hr 10m/s

$$v_{AVE} = \frac{3\times 0 + 3\times 5 + 4\times 10}{10} = 5.5\,m/s$$

$$\left(v^3\right)_{AVE} = \frac{3\times 0^3 + 3\times 5^3 + 4\times 10^3}{10} = 437.5\,(m/s)^3 \neq 5.5^3$$

OR

$$v_{AVE} = \underset{v_1}{0} \times \underset{P\{v=0\}}{\frac{3}{10}} + \underset{v_2}{5} \times \underset{P\{v=5\}}{\frac{3}{10}} + \underset{v_3}{10} \times \underset{P\{v=10\}}{\frac{4}{10}}$$

probability

$$\boxed{v = \sum_i v_i P\{v = v_i\}}$$

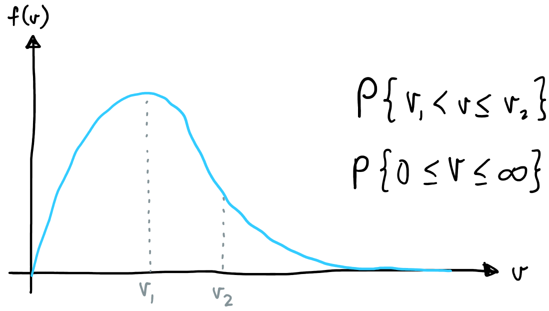
probability

$$\boxed{v_{AVE} = \sum_i v_i P\{v = v_i\}}$$

$$\boxed{(v^3)_{AVE} = \sum_i v_i^3 P\{v = v_i\}}$$

probability mass function — pmf
(discrete)

(continuous)
probability density function — pdf



$$P\{v_1 < v \leq v_2\} = \int_{v_1}^{v_2} f(v)\,dv$$

$$P\{0 \leq v \leq \infty\} = \int_0^{\infty} f(v)\,dv = 1$$

$$\boxed{\begin{aligned} v_{AVE} &= \int_0^{\infty} v\, f(v)\,dv \\ (v^3)_{AVE} &= \int_0^{\infty} v^3 f(v)\,dv \end{aligned}}$$

Cumulative Distribution Function

- Def $\quad F(V) = P\{v \leq V\} = \int_0^{V} f(v)\,dv$

$$F(0) = P\{v \leq 0\} = 0$$

$$F(\infty) = P\{v \leq \infty\} = 1$$

$$P\{v_1 \leq v \leq v_2\} = F(v_2) - F(v_1)$$

Weibull Statistics

$$f(v) = \frac{k}{c}\left(\frac{v}{c}\right)^{k-1} e^{\left(-\left(\frac{v}{c}\right)^k\right)}$$

k: shape parameter

c: scale parameter

k = 2 typically used.

Special case: k = 2 Rayleigh pdf

Special case : $k=2$ Rayleigh pdf

$$f(v) = \frac{2v}{c^2} e^{\left(-\left(\frac{v}{c}\right)^2\right)}$$

c is a parameter that is determined by wind characteristics

↳ Can relate c to v_{AVE}

$$v_{AVE} = \int_0^\infty 2\left(\frac{v}{c}\right)^2 e^{\left(-\left(\frac{v}{c}\right)^2\right)} dv$$

$$= \underset{\dots}{\text{Gaussian Integral}} = c\frac{\sqrt{\pi}}{2} \longrightarrow c = \frac{2v_{AVE}}{\sqrt{\pi}}$$

- So if we know v_{AVE} for a particular site, then we can use Rayleigh pdf to model wind speed stats

$$f(v) = \frac{2v}{\frac{4v_{AVE}^2}{\pi}} e^{\left(-\left(\frac{v}{\frac{2v_{AVE}}{\sqrt{\pi}}}\right)^2\right)}$$

$$\boxed{= \frac{\pi v}{2v_{AVE}^2} e^{\left(-\frac{\pi}{4}\left(\frac{v}{v_{ave}}\right)^2\right)}}$$

- <u>Average Power in Wind</u>
    - Start with v_{AVE} → assume Rayleigh statistics → estimate average power → used in wind farm siting

$$P_{AVE} = \frac{1}{2}\rho A (v^3)_{AVE}$$

- Need $(v^3)_{AVE} = \int_0^\infty v^3 F(v)\, dv$

$$= \int_0^\infty v^3 \cdot \frac{2v}{c^2} e^{\left(-\left(\frac{v}{c}\right)^2\right)} dv$$

$$= \underset{\dots}{\text{"magic"}}$$

$\frac{2v_{AVE}}{\sqrt{\pi}}$

$$= \frac{3}{4}\sqrt{\pi}\, c^3$$

$$= \frac{3}{\cancel{4}}\cancel{\sqrt{\pi}}\, \frac{\cancel{8}^{\,2}\, v_{AVE}^3}{\pi \cdot \cancel{\sqrt{\pi}}} = \frac{6}{\pi} v_{AVE}^3$$

$$\boxed{P_{AVE} = \frac{1}{2}\rho A (v^3)_{AVE} = \frac{6}{\pi}\frac{1}{2}\rho A v_{AVE}^3}$$

☆ With Rayleigh Statistic, P_{AVE} is a function of v_{AVE} only!

So far Found average power in wind ⟶ Energy in wind

$$E = P_{AVE} \times t$$

But Want to find expected energy from turbine

> need to couple wind speed statistics with the power curve