Power in the Wind

Thursday, March 14, 2024 4:07 PM

Power in the Wind

Wind : consequence of moving air

> think of moving air as composed of Small "packets" with finite mass

→ hope that it suffices for modeling purposes

Recall Power is the rate at which energy is transferred

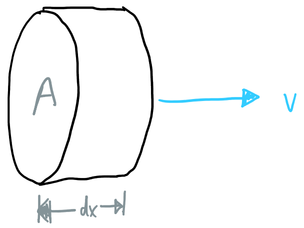
> think of power in wind as the rate at which "packets" of air with certain mass and certain amount of kinetic energy pass through a surface

- Consider a "packet" of air with mass m in kilograms moving at speed v (m/s).

$$KE = \frac{1}{2} m v^2 = E$$

assumption:

$$P = \frac{dE}{dt} = \frac{1}{2}\frac{dm}{dt}v^2 + mv\cancelto{0}{\frac{dv}{dt}}$$



Power passing through surface with cross-sectional area A

density of Air : $\rho = \frac{dm}{dv}$

$\rightarrow dm = \rho dv = \rho A dx = \rho A v\, dt$

$\frac{dm}{dt} = \rho A v$

$\rightarrow \boxed{P = \frac{1}{2}\rho A v^3 \; [W]}$

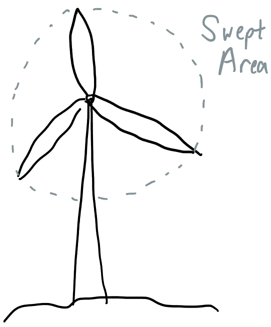
Specific Power (Power Density) $\rightarrow \frac{P_w}{A} = \frac{1}{2}\rho v^3$

1) density ρ
  - varies depending on site

2) area A
  - want large turbines

3) Speed v
  - highest impact (from v^3 term)
  - want higher wind speeds

maximize P_w



Density of Air

- At 15°C, 1 atm : $\rho = 1.225\ kg/m^3$

- Variation with Temperature

Ideal Gas Law : $$\underset{\text{pressure [atm]}}{P}\underset{\text{volume }[m^3]}{V} = \underset{\text{\# of moles (mass-ish)}}{n}\overset{\text{ideal gas constant}}{R}T \leftarrow \text{temperature [K]}$$

- Relate mass m to n by : $m = n \times \overset{\overset{\rightarrow 10^{-3} \times M.W.\ [kg/mol]}{[g/mol]}}{M.W.}$

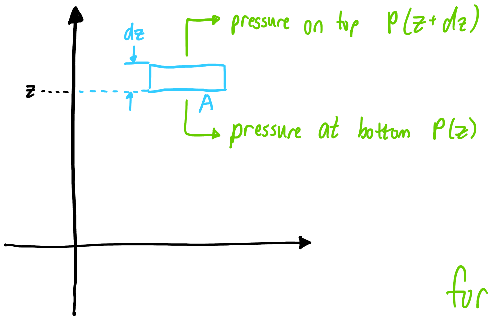
$$\rightarrow n = \frac{m}{10^{-3} \times M.W.}$$

$$PV = \frac{m}{10^{-3} \times M.W.} RT \rightarrow \rho = \frac{m}{V} = \frac{10^{-3} \times M.W.}{RT} P$$

- So $\rho \propto P$ i.e. $P\uparrow \rightarrow \rho\uparrow$

$\rho \propto \frac{1}{T}$ i.e. $T\uparrow \rightarrow \rho\downarrow$

- Variation with altitude
  - intuitively, ↑ altitude → ↓ pressure → ↓ ρ
  - try to develop a model for ρ as function of T and altitude
  - Consider a column of air



gravitational acceleration

Weight of slice: $dW = dm \times g$

$$dW = \rho \underbrace{A\,dz}_{\text{volume}} \times g$$

force exerted per area $dP = \frac{dW}{A} = \rho g\, dz$

$$P(z) = P(z + dz) + dP$$

$$P(z+dz) - P(z) = -dP = -\rho g\, dz$$

$$\frac{P(z+dz) - P(z)}{dz} = -\rho g$$

as $dz \to 0 \to \frac{dP}{dz} = -\rho g = -\frac{10^{-3} \times M.W.}{RT} P$

M.W. of air $= 28.97\ g/mol$ assuming 78.08% N_2 20.95% O_2 etc.

$g = 9.806\ m/s^2$

$R = 8.2056 \times 10^{-5}\ m^3 \cdot atm \cdot K^{-1} \cdot mol^{-1}$

$\to \frac{dP}{dz} = -\frac{0.0342}{T} P$

Assume constant T (<u>NOT</u> true. T↓ with increasing altitude)

1st ODE

$$P(z) = P\, e^{\left(-\frac{0.0342}{T} z\right)}$$

$$P(z) = P_0 e^{\left(-\frac{0.0342}{T} z\right)}$$

boundary condition $z=0$

1 atm (atmospheric pressure at sea level)

$$\boxed{\rho = \frac{10^{-3} \times M.W.}{RT} P_0 \exp\left(-\frac{0.0342}{T} z\right)}$$

- According to model: want low temperature, low elevation

$$P_w = \frac{1}{2}\rho A v^3 \quad [W]$$

> Want to ↑ v for ↑ gains in P_w

- Impact of Tower Height
  - Characterize wind speed with variations in tower height
  - Consider "roughness" of surface near the site

"Smooth" surface : calm sea, flat plains

"rough" surface : buildings, forest

- Common model:

$$\frac{v}{v_0} = \left(\frac{H}{H_0}\right)^{\alpha}$$

roughness

v : wind speed at H height

v_0 : wind speed at H_0 height

α : friction coefficient related to roughness of ground surface

$$\frac{P_w}{} = \frac{\frac{1}{2}\rho A v^3}{} = \left(\frac{v}{}\right)^3 = \left(\frac{H}{}\right)^{3\alpha}$$

$$\frac{P_W}{P_{W_0}} = \frac{\cancel{\frac{1}{2}\rho A} v^3}{\cancel{\frac{1}{2}\rho A} v_0^3} = \left(\frac{v}{v_0}\right)^3 = \left(\frac{H}{H_0}\right)^{3\alpha}$$

- E.g. 50 m vs 80 m tower

  a) Calm water : $\alpha = 0.1$

  $$\frac{P}{P_0} = \left(\frac{v}{v_0}\right)^3 = \left(\frac{H}{H_0}\right)^{3\alpha} = \left(\frac{80}{50}\right)^{0.3} = 1.5$$

  b) large city with tall buildings : $\alpha = 0.4$

  $$\frac{P}{P_0} = \left(\frac{H}{H_0}\right)^{3\alpha} = \left(\frac{80}{50}\right)^{1.2} = 1.76$$ → higher tower is more worth here than on calm water

- Ideal Power Curve

  $P_w = \frac{1}{2}\rho A v^3$ power in the wind

  $P = \frac{1}{2}\rho A v^3 \times C_p$ power delivered

  ↑ efficiency coefficient (more later ...)