Branch_and_Bound

Tuesday, June 11, 2024 5:36 PM

Branch and Bound Algorithm

- Used when the conditions in Johnson's algorithm (i.e., $n/3/P/C_{max}$) don't hold

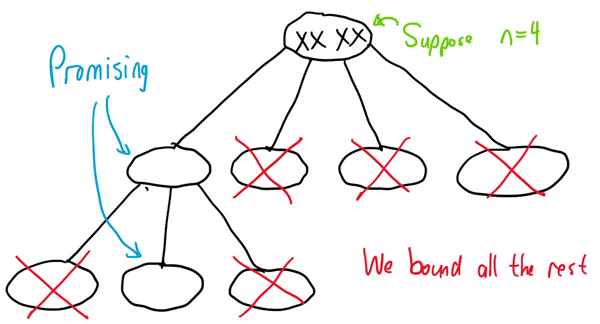
$$\text{Either } \begin{cases} t_{1,n} \geq t_{2,n} \\ t_{3,n} \geq t_{2,n} \end{cases}$$

or for larger problems such as:

$n/4/P/C_{max}$

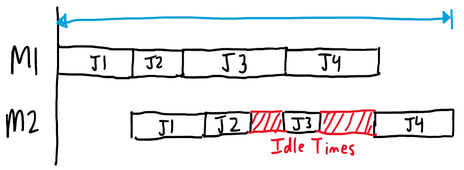
↳ or any # of machines

- Why the name B&B?
  - The algorithm starts like a tree when we branch & bound some branches



- We start with 2-machine case

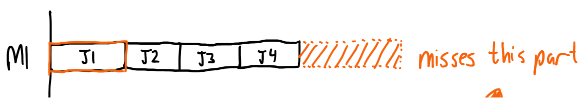
Case 1



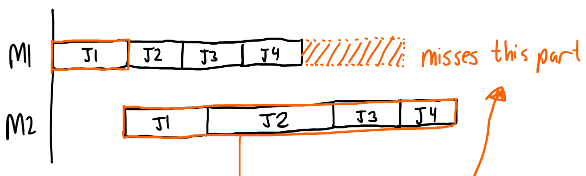
We write to lower bounds

$$\begin{cases} L_1 = \sum_{i=1}^{n} t_{i,M_1} + t_{n,M_2} \rightarrow \text{larger, correct} \\ L_2 = t_{1,M_1} + \sum_{i=1}^{n} t_{i,M_2} \rightarrow \text{misses the idle times} \end{cases}$$

Case 2



Case 2



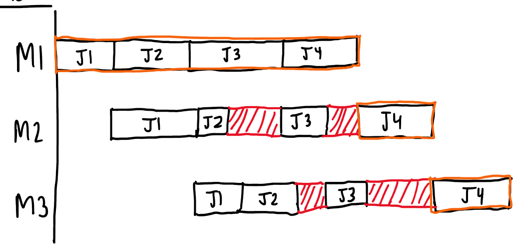
$$\begin{cases} L_1 = \sum_{i=1}^{n} t_{i,M1} + t_{n,M2} \\ L_2 = t_{1,M1} + \sum_{i=1}^{n} t_{i,M2} \end{cases}$$ → larger, correct

- There could be no other cases with 2 machines if we select

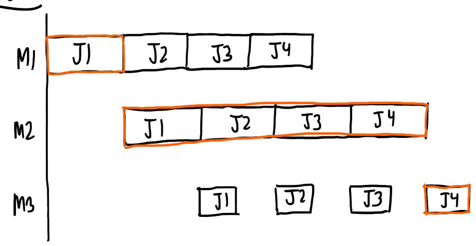
Lower bound on Completion time $L = \text{Max}(L_1, L_2)$

- Continuing with 3-machines

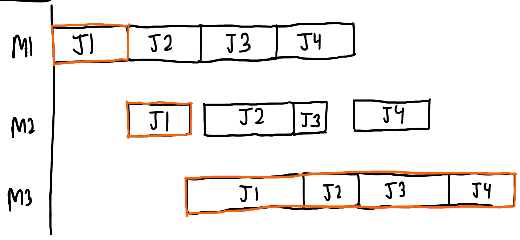
Case 1



Case 2



Case 3



Three equations $(m=3)$

Case 1 $\left\{ L_1 = \sum_{i=1}^{n} t_{i,M1} + t_{n,M2} + t_{n,M3} \right.$

Three equations (m=3)

$$
\begin{cases}
\text{Case 1} & L_1 = \sum_{i=1}^{n} t_{i,M1} + t_{n,M2} + t_{n,M3} \\
\text{Case 2} & L_2 = t_{1,M1} + \sum_{i=1}^{n} t_{i,M2} + t_{n,M3} \\
\text{Case 3} & L_3 = t_{1,M1} + t_{1,M2} + \sum_{i=1}^{n} t_{i,M3}
\end{cases}
$$

Which equation is the largest in any one case?

Case 1 : L_1

Case 2 : L_2

Case 3 : L_3

- There cannot be other cases for 3 machine solutions

Lower bound on completion time (or equally flow time) $L = \text{Max}(L_1, L_2, L_3)$

- We redefine lower bound as follows:

① Define $TIME1(J_r)$, $TIME2(J_r)$ & $TIME3(J_r)$ at which time M1, M2, M3 respectively complete the processing of the last job in the sequence J_r, where J_r contains a particular subset (size r) of n jobs

② A lower bound on completion (makespan) of all schedules that began with the sequence J_r is

from same i

$$
LB(J_r) = \text{Max}
\begin{cases}
L_1 = TIME1(J_r) + \sum_{\bar{J}_r} t_{i,M1} + \underbrace{\min_{\bar{J}_r}\left(t_{i,M2} + t_{i,M3}\right)} \\
L_2 = TIME2(J_r) + \sum_{\bar{J}_r} t_{i,M2} + \min_{\bar{J}_r}\left(t_{i,M3}\right) \\
L_3 = TIME3(J_r) + \sum_{\bar{J}_r} t_{i,M3}
\end{cases}
$$

$$
\min_{\bar{J}_r} \quad (t_{1,M2} + t_{1,M3}) \ \underline{\text{OR}} \ (t_{4,M2} + t_{4,M3})
$$

We have 4 jobs:

$J_r = \{2,3\}$ $\qquad \bar{J}_r = \{1,4\}$

$J_r = \{3,1,2\}$ $\qquad \bar{J} = \{4\}$

$J_r = \{3, 1, 2\}$ $\quad\quad$ $\bar{J} = \{4\}$

$J_r = \{1\} = 144$

$J_r = \{2\} = 128$

$J_r = \{3\} = 136$

$J_r = \{4\} = 126$