ELEC 301

Mini-Project 4 Report

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1. Introduction

In this mini project, we explore active filters, oscillators, and feedback amplifiers. For active filters, we aim to design an active filter to meet specifications in [1] to make it a 2^{nd}-order Butterworth filter. For the oscillator, we aim to understand its operating principles and derive equations that can give good approximations for the oscillation frequency. Lastly, we examine the mixing and sensing scheme of a provided feedback circuit and determine its desensitivity.

2. Mini Project

Part A

A1.

To find the resistances, we use the transfer function in equation 1, equate the denominator in equation 2, solve for its variables in equations 3a and 3b, and solve for the gain in equation 3 through the normalized characteristic of the Butterworth filter in formula 4. This characteristic arises due to the poles being 45° from the real axis. We finally solve equations 5, 6 and 7 to get R_1 = 6306.0194Ω and R_2 = 3693.9806Ω.

$$H(s) = A_M \frac{\left(\frac{1}{RC}\right)^2}{s^2 + s\left(\frac{3-A_M}{RC}\right) + \frac{1}{(RC)^2}} \tag{1}$$

$$s^2 + s\left(\frac{3-A_M}{RC}\right) + \frac{1}{(RC)^2} = s^2 + 2\zeta\omega_n s + \omega_n^2 \tag{2}$$

$$\zeta = \frac{3-A_M}{2};\ \omega_n = \frac{1}{RC} \tag{3a, 3b}$$

$$s^2 + \sqrt{2}s + 1 \tag{4}$$

$$A_M = 3 - \sqrt{2} = 1.5858\frac{V}{V} \tag{5}$$

$$A_M = 1 + \frac{R_2}{R_1} \tag{6}$$

$$R_1 + R_2 = 10k\Omega \tag{7}$$

To get the capacitance, we use the specifications provided in [1] to solve equation 8.

$$C = \frac{1}{2\pi(f_{3dB})(R)} = \frac{1}{2\pi(10kHz)(10k\Omega)} = 1.5915nF \qquad (7)$$

The simulated circuit, using these values, can be seen in figure A1. From figure A2, we see the gain of the circuit to be 4.0046dB and the 3dB frequency f_{3dB} to be 9.3145kHz, close to the specification of 10kHz. Using these obtained values, we get figure A3, showing the location of the complex conjugate poles.

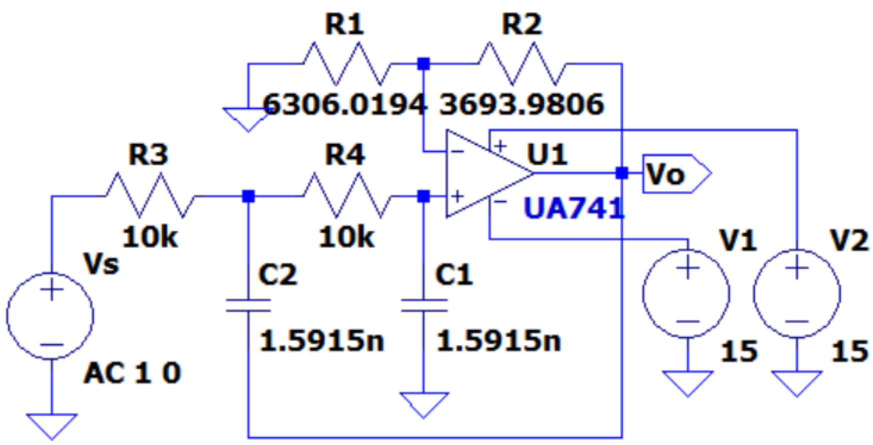

Figure A1. Simulated circuit for the 2nd order Butterworth filter.

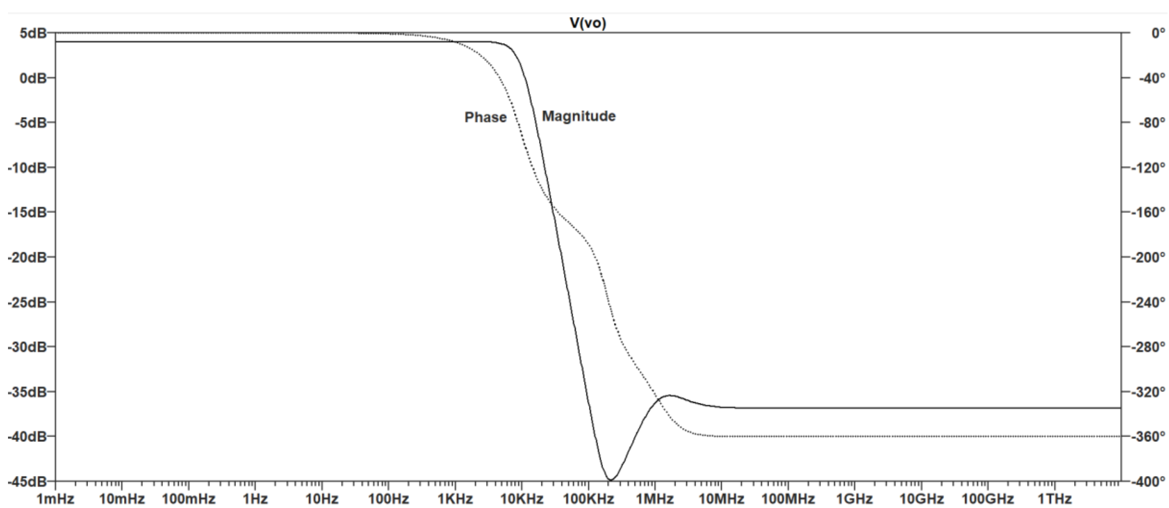

Figure A2. Bode plot for the circuit in figure 1.

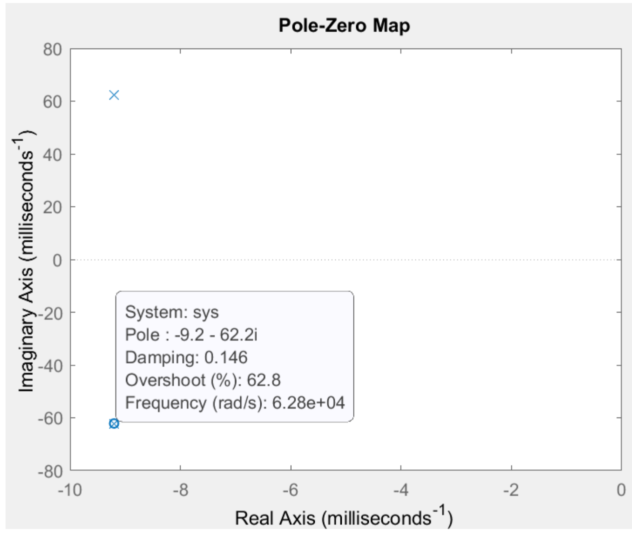

Figure A3. Poles of the circuit in figure A1 in the s-plane.

A2.

From equation 6, we find that increasing R_2 and decreasing R_1 greatly increases A_M, and for A_M = 3, there is no damping constant, and the poles of the Butterworth filter will be on the imaginary axis., as seen in figure A4. To examine the values of A_M that will give the system oscillations, we plot the root locus of the system at $A_M < 3$ and $A_M > 3$, as seen in figures A5 and A6, respectively.

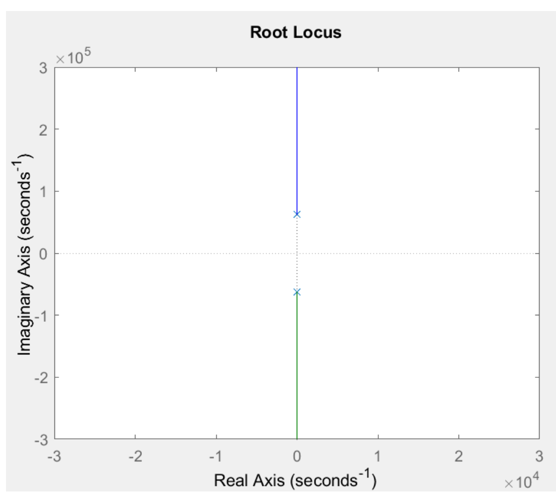

Figure A4. Root locus plot at A_M = 3.0.

The transfer function of the Butterworth is given in equation 8 and is used to get the root locus plot of the system. We get 3 root locus plots: $A_M < 3$, $A_M = 3$, and $A_M > 3$. These plots can be seen in figures A4, A5, and A6, respectively.

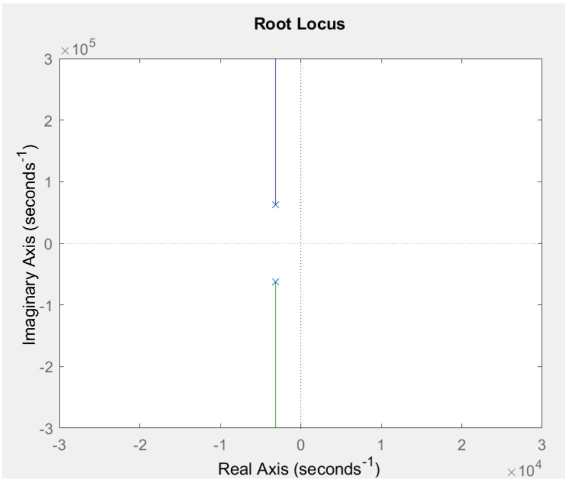

Figure A5. Root locus plot at $A_M < 3$ ($A_M = 2.9$).

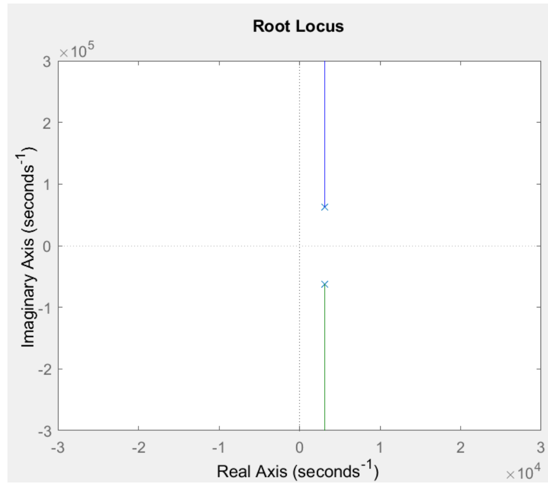

Figure A6. Root locus plot at $A_M > 3$ ($A_M = 3.1$).

$$H(s) = A_M \left(\frac{\frac{1}{(RC)^2}}{s^2 + s\frac{3 - A_M}{RC} + \frac{1}{(RC)^2}} \right) \tag{8}$$

As seen in figures A4, A5, and A6, when A_M is greater than 3, the root locus is at the right-hand side of the s-plane, and when A_M is less than 3, the root locus is at the left-hand side. Thus, for our system to oscillate, we need to ensure that our A_M is greater than or equal to 3, as a value less than that means the system is stable and would not oscillate. As such, we choose R_1 to be 3.333kΩ and R_2 to be 6.667kΩ to maintain the sum of R_1 and R_2 to be 10kΩ. These values were substituted into the circuit in figure A1, and the plot shown in figure A7. Through measuring the wave crests in figure A7, we find the oscillation frequency to be 9.3299kHz.

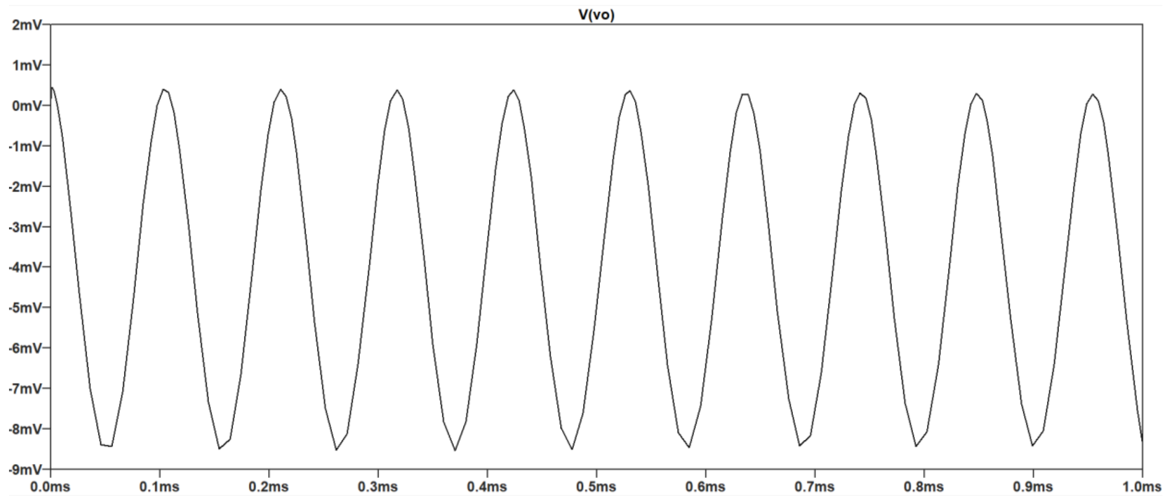

Figure A7. Plot of the oscillating output of the filter in figure A1.

In this part, we turned an active filter to a 2nd-order Butterworth filter according to specifications. On multiplying the 1st-order s term in the denominator of the transfer function in equation 8 to zero, we find that the function's poles become purely imaginary, signifying a non-decaying oscillatory response. Through this, we found the various responses that come from adjusting the gain A_M to be above or below 3, and with examination of their root locus plots, determined the behaviour and stability of the system's transfer function.

Part B

The gain of a phase shift oscillator is given by equation 9. To allow for the circuit to oscillate without an input at a specific frequency, A_f needs to be infinity and A_β needs to be -1, as per the Barkhausen criterion. As such, the complex conjugate poles would be located on the imaginary axis.

$$A_f = \frac{A}{1+A_\beta} \tag{9}$$

To predict the oscillating frequency of the circuit in figure B1, we do a mesh analysis on the circuit in figure B1 in the Laplace domain, with currents I_1, I_2, and I_3 looping clockwise. Solving equations 10 to 13, we get the currents in terms of the input voltage V_i in equations 14 to 16.

$$K = gain = \frac{V_o}{Vi} \tag{10}$$

$$KV_i = V_o = \frac{1}{sc} I_1 + R(I_1 + I_2) \tag{11}$$

$$R(I_2 - I_1) + \frac{1}{sc} I_2 + R(I_2 - I_3) = 0 \tag{12}$$

$$R(I_3 - I_2) + \left(\frac{1}{sc} + R\right) I_3 = 0 \tag{13}$$

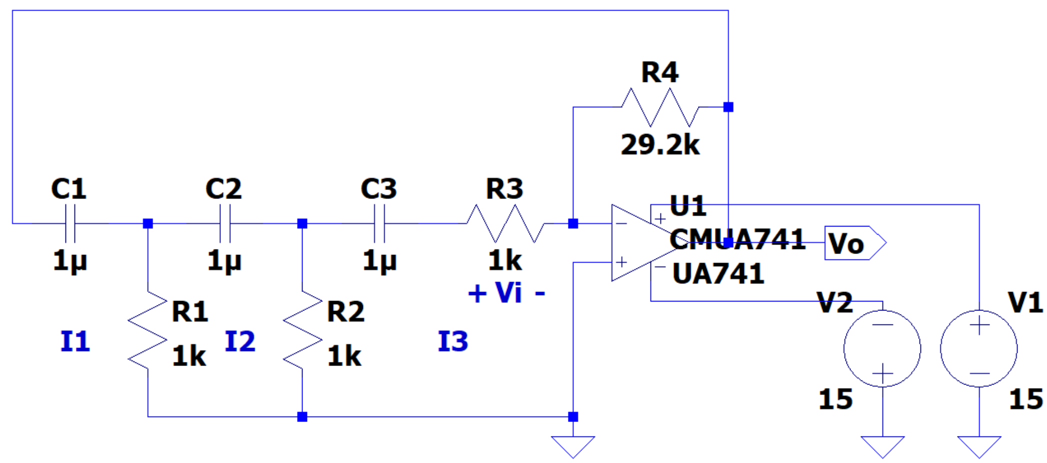

Figure B1. The phase shift oscillator for part B.

$$I_1 = \frac{1}{R}\left(V_i\left(\frac{1+2(sRC)^2}{(sRC)^2} - 1\right)\right) \tag{14}$$

$$I_2 = \frac{V_i}{R}\left(\frac{1 + 2(sRC)}{sRC}\right) \tag{15}$$

$$I_3 = \frac{V_i}{R} \tag{16}$$

Through further substitutions in equation 17 and transferring to the frequency domain in equation 18, we get the formula for the gain of the circuit, and by substituting the gain to unity and phase to 180°, we get the equation for frequency in equation 19 and the value for gain K. The equation for frequency is identical to [2].

$$KV_i = \frac{1 + sRC}{sRC}\left(\frac{1 + 2(sRC)^2}{(sRC)^2} - 1\right)V_i - R\left(\frac{V_i}{R}\left(\frac{1 + 2(sRC)}{sRC}\right)\right) \tag{17}$$

$$K = \frac{[(\omega RC)^3 - 5(\omega RC)] + j(1 - 6(\omega RC)^2)}{(\omega RC)^3} = -29 \tag{18}$$

$$f = \frac{1}{2\pi\sqrt{6}\,RC} \tag{19}$$

Using equation 18, we compute for the calculated oscillation frequency. The simulated values are taken from the distance between wave crests in figures B2, B3, and B4. A side-by-side comparison of these values to the simulated values is given in table 1.

	Calculated	**Simulated**
R = 1kΩ, C = 1µF	64.9747 Hz	64.1882 Hz
R = 2kΩ, C = 2µF	16.2437 Hz	15.9298 Hz
R = 0.5kΩ, C = 0.5µF	259.8989 Hz	259.8846 Hz

Table 1. Calculated and simulated values of the oscillation frequency.

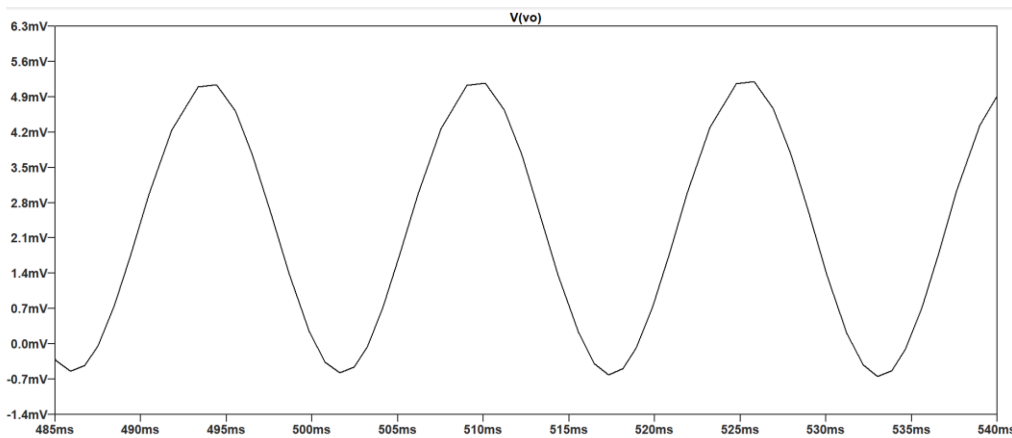

Figure B2. Oscillations of the circuit with R = 1kΩ, C = 1µF.

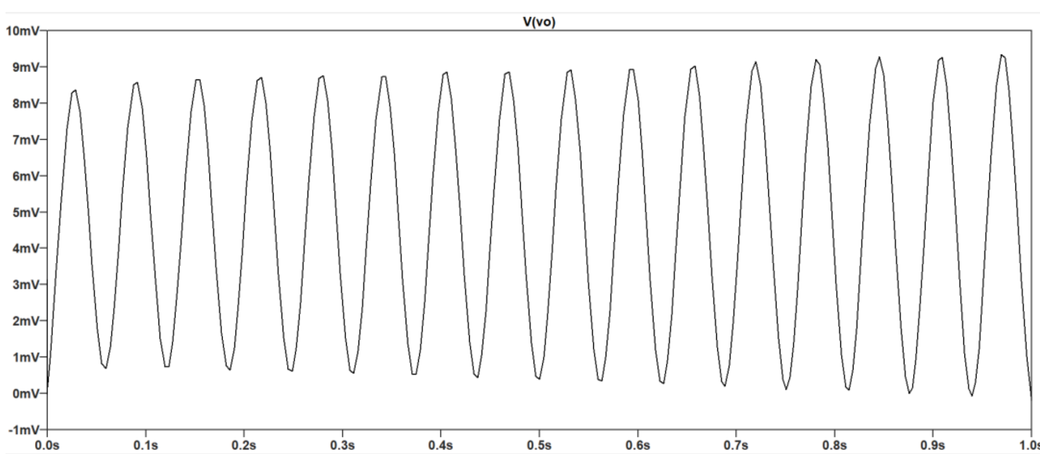

Figure B3. Oscillations of the circuit with R = 2kΩ, C = 2µF.

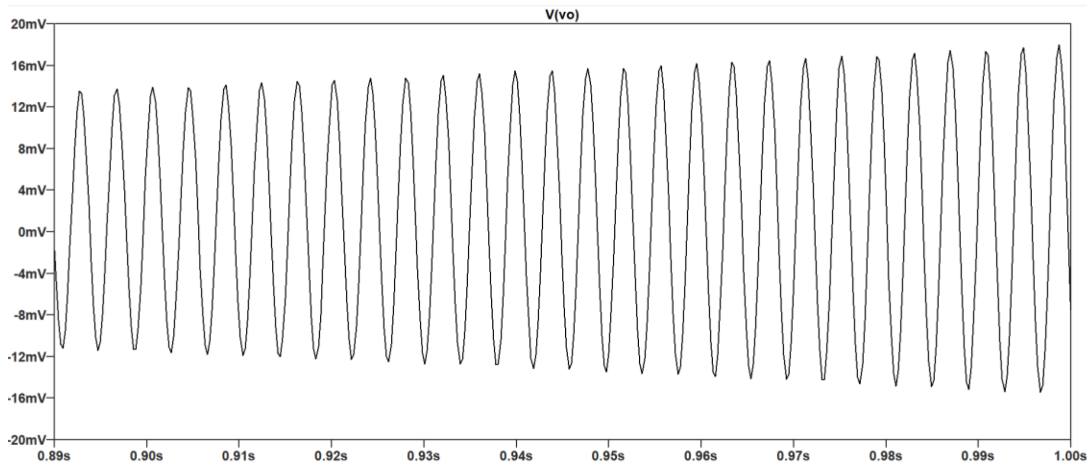

Figure B4. Oscillations of the circuit with R = 0.5kΩ, C = 0.5μF.

As seen in table 1, the calculated and simulated values are close to each other, showing the accuracy of our formula in predicting the oscillation frequency.

A phase shift oscillator works by using the feedback through a system of resistors and capacitors to generate a sinusoidal waveform. The oscillator involves cascading three high-pass filter stages, each of which contributes a 60^o phase shift, totaling a shift of 180^o. The negative feedback from the amplifier adds a -180^o shift to the oscillator, resulting in a 0^o net shift for the system. This satisfies the Barkhausen criterion for sustained oscillations.

We also previously found that the minimum value for gain K is -29, which aligns with the initial design requirement for resistor 29R or known as resistor R4 in figure B1. At the exact 29R value, although the output is oscillating, the signal will die out over time due to a lack of positive feedback. As such, a slight increase in resistance was needed to ensure that the value of $|A_\beta|$ is greater than 1 so that the poles would be on the right-hand side of the s-plane, allowing for oscillations to be sustained.

Part C

C1.

Because we are feeding current into the input and measuring the voltage output, we use the shunt-shunt topology, with y-parameters. We first initialize the circuit in figure C1 to create the largest open-loop gain at 1kHz. This is done through disconnecting the feedback resistor R_f and modifying the value of the variable resistor R_{B2} based on the resulting Bode plots in figure C2. We get R_{B2} to be 20.1kΩ for our design after finer iterations of the resistor's value.

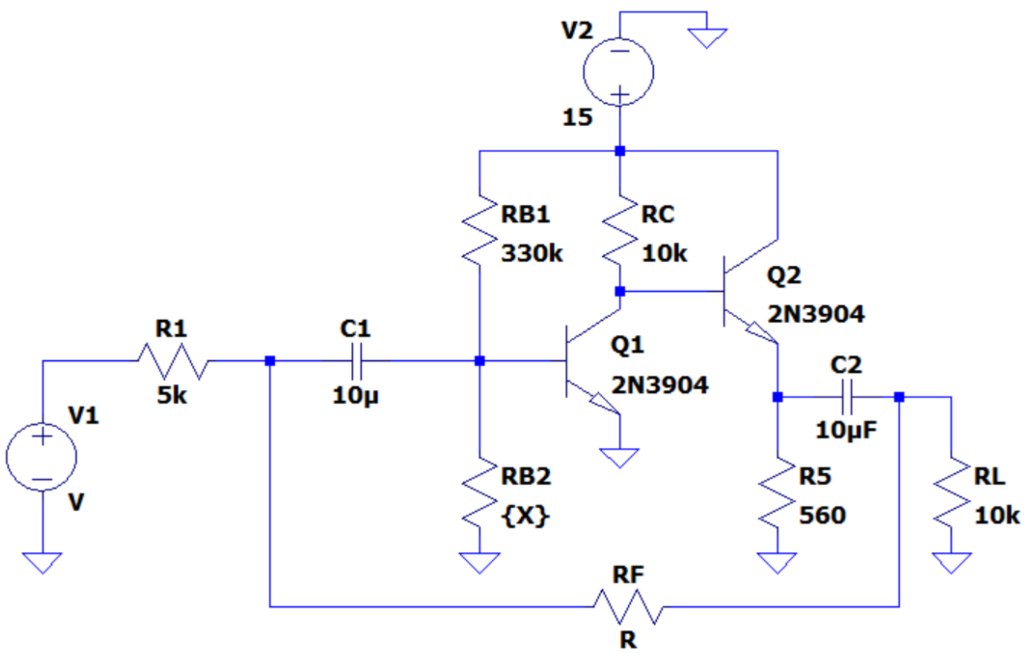

Figure C1. The feedback circuit for part C of the mini project.

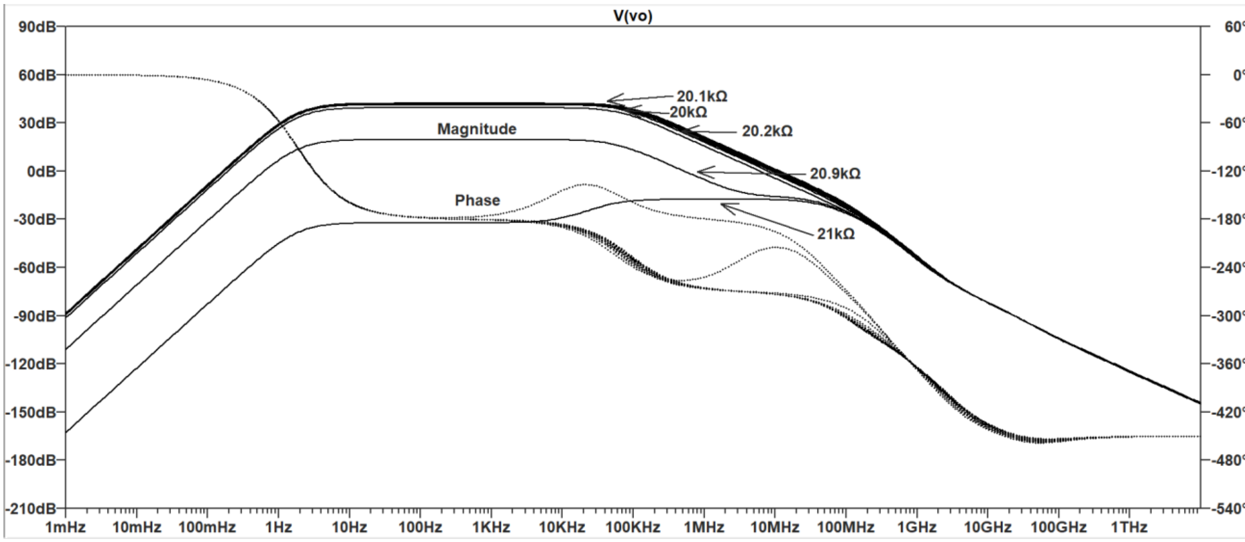

Figure C2. Bode plots for determining values of R_{B2} that gives the highest gain (20.1kΩ).

	I_C	I_B	I_E	V_C	V_B	V_E
Q1	1.3124mA	10.9101μA	1.3233mA	1.7395V	0.6545	0V
Q2	1.9111mA	13.6505μA	1.9248mA	15V	1.7395V	1.0779V

Table 2. D.C. operating point values for transistors Q1 and Q2 in figure C1.

The values for g_m, r_π, and h_{FE} are calculated using equations 20, 21 and 22, respectively, with the 1-subscript signifying transistor Q1 and the 2-subscript signifying transistor Q2.

$$g_m = \frac{I_C}{V_T} \rightarrow g_{m1} = 0.05250S, g_{m2} = 0.07644S \tag{20}$$

$$r_\pi = \frac{V_T}{I_{B1}} \rightarrow r_{\pi1} = 2291.4547\Omega, r_{\pi2} = 1831.4347\Omega \tag{21}$$

$$h_{FE} = g_m r_\pi \rightarrow h_{FE1} = 120.2922; h_{FE2} = 140.0022 \tag{22}$$

C2.

Maintaining an open-loop circuit as in part C1, we find the open loop gain of this circuit to be 42.1234dB or A_v = -127.6939V/V, with the negative sign due to our amplifier being inverting. The Bode plot of the circuit, in figure C3, shows the lower 3dB frequency to be 2.8937Hz and the upper 3dB frequency to be 89.1896KHz.

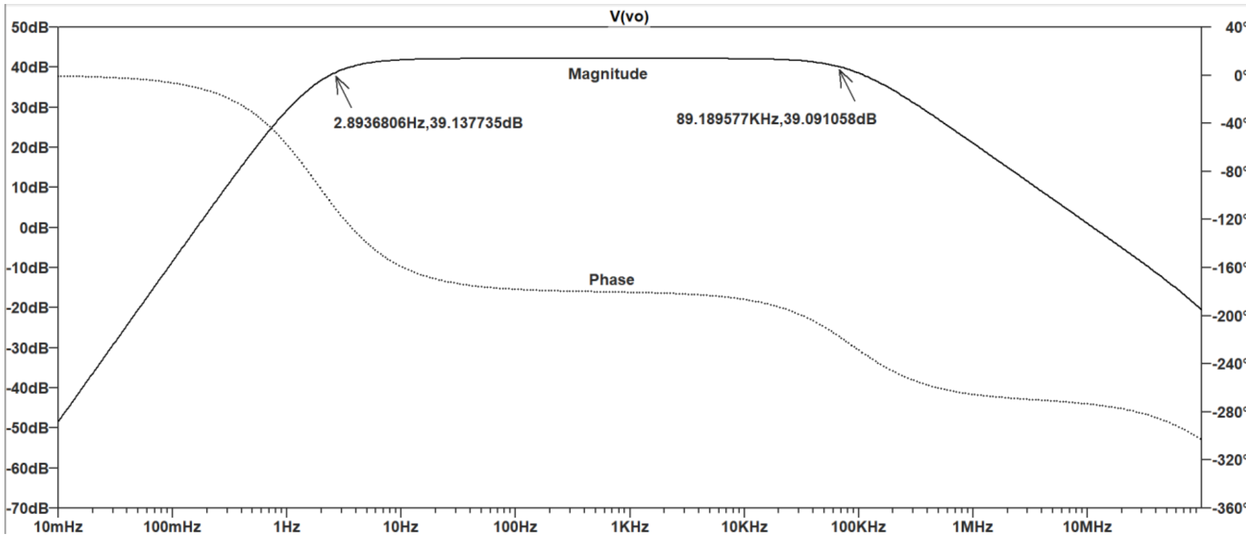

Figure C3. Bode plot of the open-loop version of the circuit in figure C1.

To get the input impedance, we plot V_{test} and I_{test}, as seen in figure C4, which in this case is the current through capacitor C_1 and find R_i at 1kHz. So, we have $R_i = 2.5467k\Omega$. Using a similar method to find R_o, we move the voltage source to the output and get $R_o = 63.3119\Omega$ in figure C5.

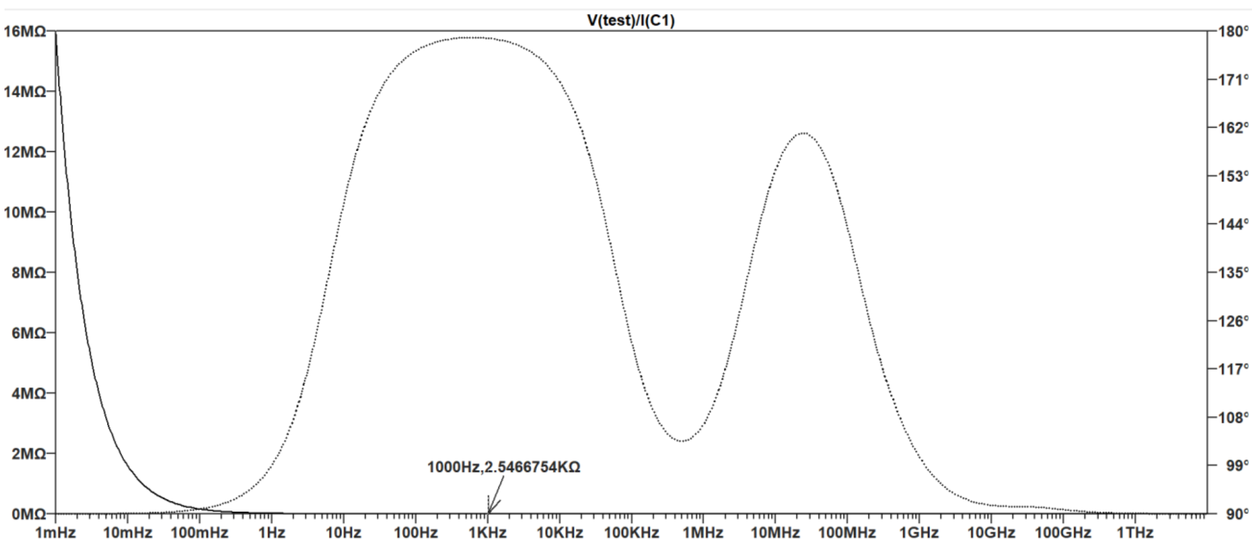

Figure C4. Plot of V_{test} versus I_{test} to find the input resistance.

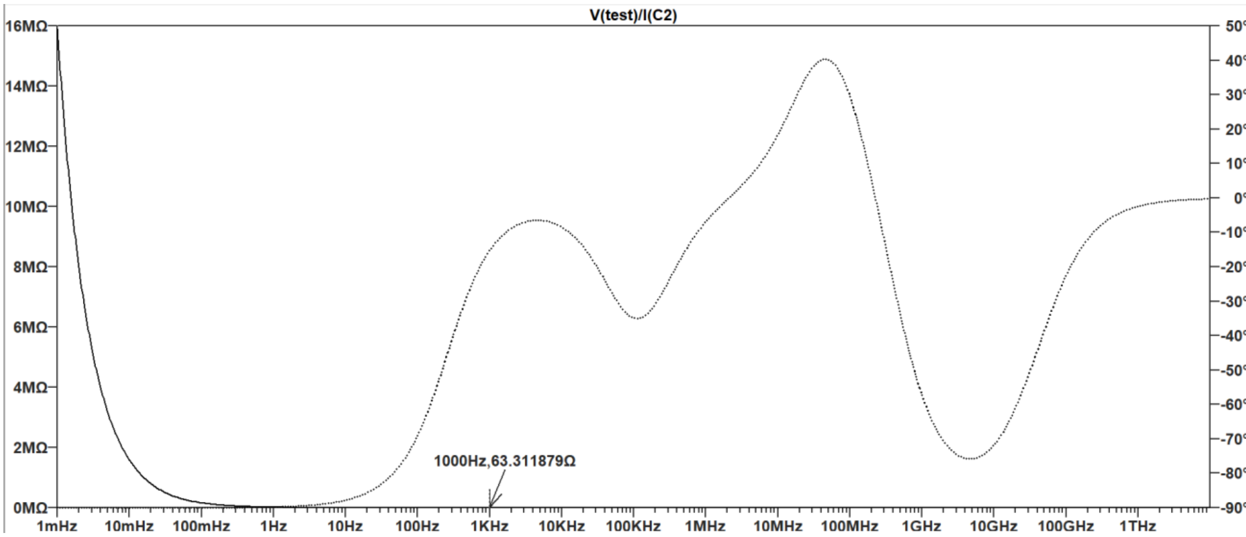

Figure C5. Plot of V_{test} versus I_{test} to find the output resistance.

Using these values, we predict the closed-loop frequency response. Using y-parameters, we get equation 23. The β value for the feedback network comes from the y_{12} parameter and is solved in equation 24.

$$\begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} V_1 \\ V_2 \end{bmatrix} \begin{bmatrix} y_{11} & y_{12} \\ y_{21} & y_{22} \end{bmatrix} \quad (23)$$

$$At\ V_1 = 0, y_{12} = \frac{I_1}{V_2} = -\frac{1}{R_F} = \beta \rightarrow \beta = -10\mu S \quad (24)$$

From this we can get the open-loop gain in equation 25 and calculate the closed-loop gain in equation 26.

$$A = \frac{V_o}{I_i} = R_s \frac{V_o}{V_i} = 5000 \times -127.6939 = -638.4695\ \mathrm{kV/A} \quad (25)$$

$$A_f = \frac{A}{1 + A\beta} = -86.4585\ kV/A \quad (26)$$

The closed-loop input and output impedances are calculated in equations 27 and 28.

$$R_{i,closed} = \frac{R_{i,open}}{1 + A\beta} = 344.8620\Omega \quad (27)$$

$$R_{o,closed} = \frac{R_{o,open}}{1 + A\beta} = 8.5734\Omega \quad (28)$$

Finally, we have the closed-loop 3dB frequency in equations 29 and 30.

$$f_{L3dB,closed} = \frac{f_{L3dB,open}}{1 + A\beta} = 0.3918Hz \quad (29)$$

$$f_{H3dB,closed} = f_{H3dB,open} \times (1 + A\beta) = 658.6376kHz \quad (30)$$

Simulating the closed-loop, we get the simulated values f_{L3dB} = 0.4591Hz and f_{H3dB} = 632.7854kHz. The plot of this closed-loop system can be found in figure C6. Comparing these values to the calculated, we see that they are close to each other.

For the resistances, we measured R_i = 241.152Ω and R_o = 8.538Ω. Comparing these values to the ones obtained in equations 27 and 28, the R_o values are very close to each other, while R_i is a bit further apart. This may be due to the calculations and assumptions made by LTSpice to account for non-ideal behaviour.

C3.

The closed-loop frequency response with varying values for R_f can be seen in figure C6.

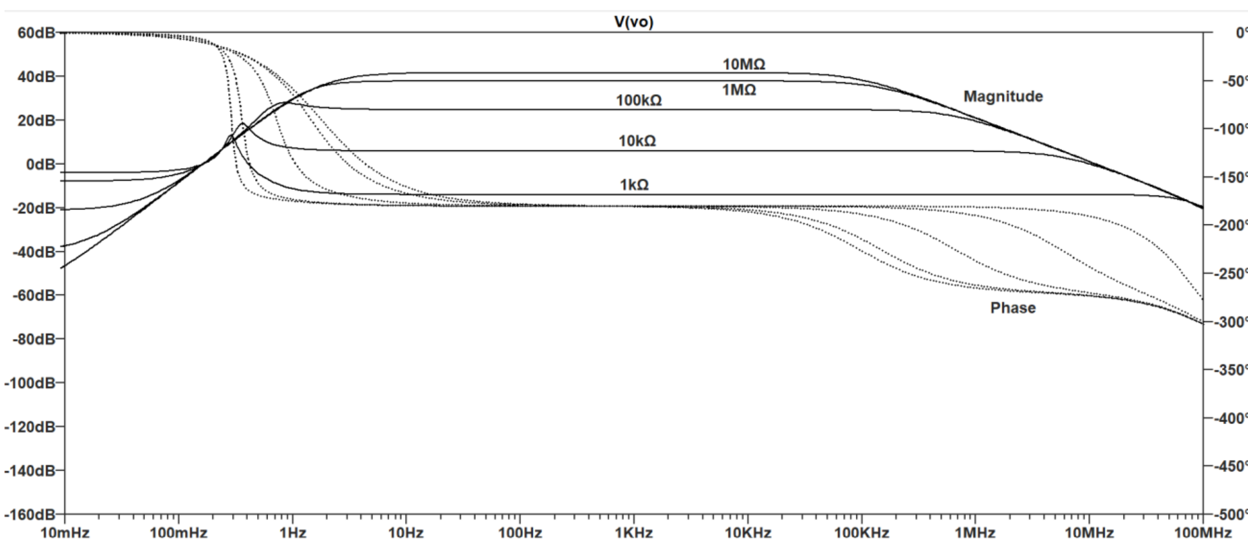

Figure C6. The closed-loop frequency response of part C, with varying R_F values.

Using algebraic manipulation on equation 9, we get equation 31. Gain A remains as the open-loop gain. From equation 31, we get the measured values of β across different R_f values listed and compared to calculated values in table 3. The calculated values come from equation 24.

$$\beta = \frac{1}{A_v R_s} - \frac{1}{A} \tag{31}$$

R_f	A_V (V/V)	A_f (kV/A)	β	
	Simulated	Simulated	Calculated	Measured
1kΩ	-0.1991	-0.9962	-1mS	-1.0022mS
10kΩ	-1.9638	-9.8707	-100µS	-99.7437µS
100kΩ	-17.2508	-90.3769	-10µS	-9.4985µS
1MΩ	-77.8508	-490.0888	-1µS	4.7420µS
10MΩ	-120.0072	-878.6883	-0.1µS	0.4282µS

Table 3. A_v and the calculated vs simulated β for different values of R_f.

As seen in table 3, the calculated and the measured values of β are close to each other. However, as R_f increases, the difference between them increases, which means less accuracy.

C4.

Using a similar methodology as in part C2, we find the input and output resistances in table 4.

R	R_f		
	10kΩ	**100kΩ**	**1MΩ**
R_i	25.481Ω	241.152Ω	1.289kΩ
R_o	1.305Ω	8.538Ω	39.188Ω

Table 4. Input and output resistances for various R_f values.

The amount of feedback, 1 + Aβ, can be related by joining equations 27 and 28 together to form equation 32. Through this equation, we can compare the estimated and the simulated amount of feedback in table 5.

$$1 + A\beta = \frac{R_{i,open}}{R_{i,closed}} = \frac{R_{o,open}}{R_{o,closed}} \tag{32}$$

	Amount of Feedback		
	10kΩ	**100kΩ**	**1MΩ**
Input (R_i)	99.945	10.561	1.976
Output (R_o)	48.515	7.415	1.616
Predicted	64.847	7.385	1.638

Table 5. Amount of feedback predicted and observed in R_i and R_o.

As seen in table 5, the predicted values of the amount of feedback are closer at a higher value of R_f.

C5.

The desensitivity factor can be found using equation 33. Solving this equation gives us equation 34.

$$\frac{dA_f}{dA} = \frac{1}{(1 + A\beta)^2} \tag{33}$$

$$desensitivity\ factor = (1 + A\beta) \tag{34}$$

As seen in the formula for the desensitivity factor, as R_f goes to infinity, β goes to zero. As such, the desensitivity factor when R_f is infinity is 1. For R_f = 100kΩ, the desensitivity factor can be calculated using thee values of A and β calculated in part C2 and through equation 34. We get the velue to be 7.3847.

To compute for the desensitivity factor from the simulation, we manipulate equation 33 and measure the voltage gain for both $R_f = \infty$ and R_f = 100kΩ. The values for the voltage gain are placed in table 6. Using these values, we solve for the desensitivity factor in equations 35 to 37.

	R_C (kΩ)	A_v (V/V)
$R_f = \infty$	9.9	-127.0691
	10	-127.6897
	10.1	-128.2304
R_f = 100kΩ	9.9	-17.2393
	10	-17.2508
	10.1	-17.2608

Table 6. Voltage gain for varying R_C and R_f.

$$dA = \frac{(-127.6897 - -127.0691) + (-128.2304 - -127.6897)}{2} = -0.5807 \tag{35}$$

$$dA_f = \frac{(-17.2508 - -17.2393) + (-17.2608 - -17.2508)}{2} = -0.0108 \tag{36}$$

$$desensitivity\ factor = \sqrt{\frac{dA}{dA_f}} = 7.3494 \tag{37}$$

Comparing the calculated desensitivity factor, 7.3494, to the theoretical factor, 7.3847, they are very close to one another.

This part of the mini proect shows how much gain is reduced when we go from an open-loop to a closed-loop system and how the midband of the system is widened. Another observation is that at low values of R_f, the gain is larger than the midband. This may be due to the lesser amounts of negative feedback gain obtained, which increases the overall gain of the system. In a circuits perspectives, it is possible that resonance occurred in the capacitor at low frequencies, resulting to capacitors taking less power from the circuit.

Lastly, we also found that with feedback large differences in gain A only caused a slight difference in gain A_f. This indicates the desensitization of the feedback circuit.

3. Conclusion

In this mini project, we saw how an active filter can be made into a 2nd-order Butterworth filter and be made to meet specification and still get a good approximation. This waas done through modifying the A_M gain and changing the values of the resistors.

In addition, we saw how phase shift oscillators worked and how the predicted values from calculations coming from provided equations match closely to the simulated values, demonstrating the accuracy of the predictions.

Lastly, in the feedback amplifier, we determined the topology of our circuit in question and used it to determine and optimize the open-loop gain. We also examined the behaviour of the closed-loop circuit under varying configurations and examined the system's desensitivity.

4. References

[1] ELEC 301 Mini Project 4 Handout

[2] ELEC 301 Course Notes

[3] A. Sedra and K. Smith, "Microelectronic Circuits," 5th (or higher) Ed., Oxford University Press, New York.

[4] LTSpice™ User Manual