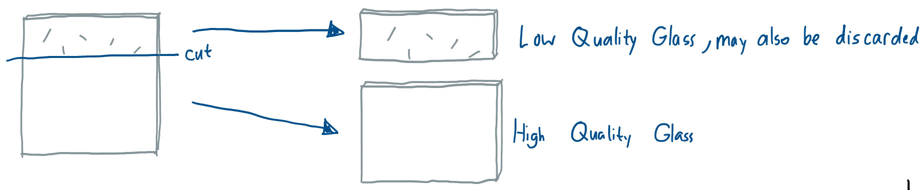
Statistical_Quality_Control_Part1

Tuesday, June 11, 2024 5:51 PM

SQC
- Variables : For such variables like diameter, temperature, anything that gives measurable quantity
- Attributes
  - Defectives : A diode or two in a batch of 100 diodes
  - Defect : # of scratches here and there on sheet of glass, otherwise useable.



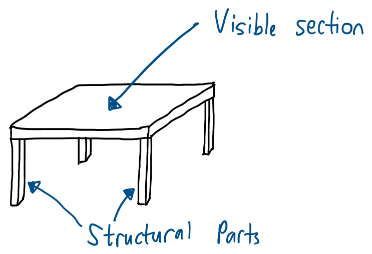
Quality Control for Defects (not Defectives!)

> could be used for "grading" (scoring)

- With wood (timber, or panels)

  High - Quality Grade ⟶ Used for visible sections

  Low - Quality Grade ⟶ Used as structural parts and not visible sections



- Average (mean) is not good enough to describe a distribution
- Some measure of spread is also required
- For example Normal distribution is explained by its mean (μ) and standard deviation

$$f(x) = \frac{1}{\sigma\sqrt{2\pi}} \exp\left[\frac{-(x-\mu)^2}{2\sigma^2}\right] = f(\mu, \sigma)$$

Sampling Techniques

- For example : Sample size $n = 5$ Sample average $\bar{X} = \frac{\sum_{i=1}^{5} X_i}{n}$; $\sigma = \sqrt{\frac{\sum_{i=1}^{5} (X_i - \mu)^2}{n}}$

- Average (mean) is easy to calculate
- Standard deviation is not so easy!
- But instead an alternative measure is used in SQC, called "Range"

$$R = X_{max} - X_{min} \geq 0$$

- Range is shown to have great utility in SQC as it relates to σ and σ can be determined through Range (R)

Sample Number	Measurements	Range
1	$X_1, X_2, X_3 = \bar{X}_1$	$X_{max} - X_{min} = R_1$
2	$X_1, X_2, X_3 = \bar{X}_2$	$X_{max} - X_{min} = R_2$
3	$X_1, X_2, X_3 = \bar{X}_3$	$X_{max} - X_{min} = R_3$
⋮		
N (25)	$X_1, X_2, X_3 = \bar{X}_N$	$X_{max} - X_{min} = R_N$
	$\bar{\bar{X}} = \frac{\sum \bar{X}}{25}$	$\bar{R} = \frac{\sum R_i}{N}$

Sample size for example: $n = 3$

$$\bar{\bar{X}} = \frac{\sum_{i=1}^{N} \bar{X}_i}{N} = \text{taken as} = \mu$$

$$\bar{R} = \frac{\sum_{i=1}^{N} R_i}{N}$$

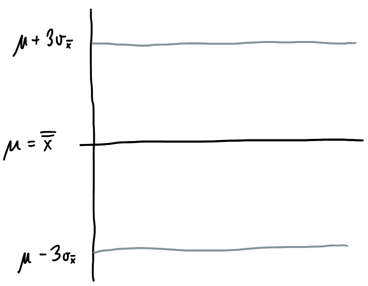
- It has been shown that

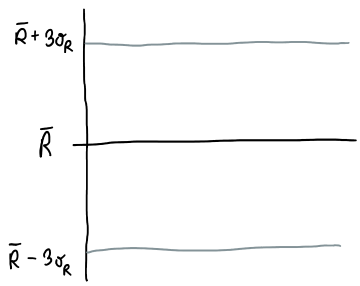
standard deviation of sample range

$$\bar{R} = d_2 \sigma \quad ; \quad \sigma_R = d_3 \sigma$$

> d_2 and d_3 depend on sample size n, and determined through standard statistical table

- We decide on a sample size, n





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> Set up these two charts next to the production line. We take periodic samples & plot the findings on the charts

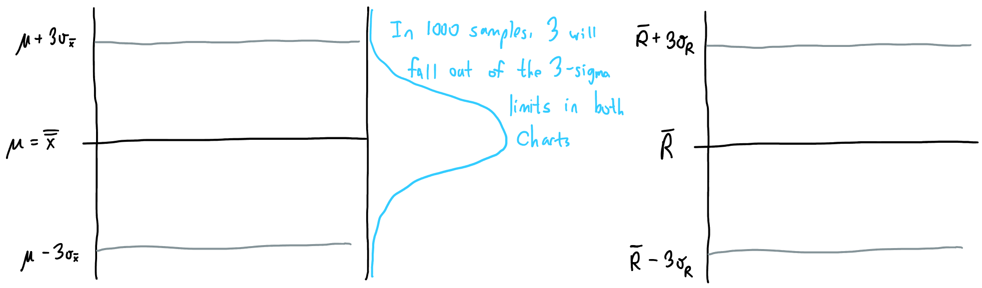
We have to determine $\bar{\bar{X}}$, $\sigma_{\bar{X}}$, $\bar{R}$, and $\sigma_{\bar{R}}$ → Standard deviation of sample range

$\bar{\bar{X}}$ → Average of the main population or average of sample average shown by μ

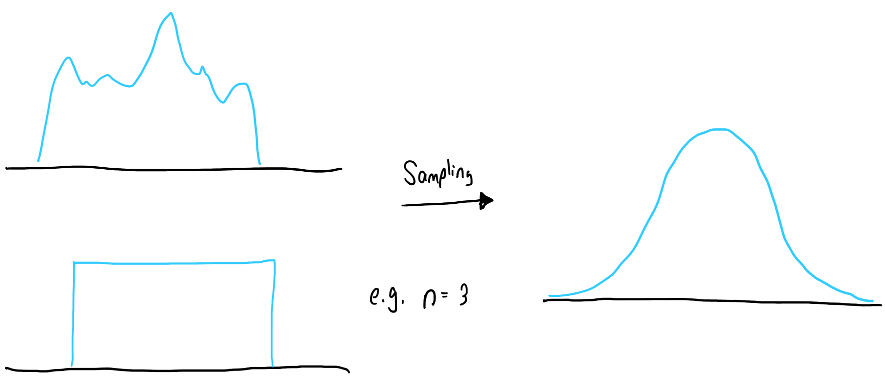
$\sigma_{\bar{X}}$ → Standard deviation of sample average

$\bar{R}$ → Average Range

We have to setup two charts:



$$\frac{X_1 + X_2 + X_3}{n} = X_i$$



* So we have to use Sample average (sample of e.g. 5)

$$\sigma = \sqrt{\frac{\sum_{i=1}^{5} (x_i - \mu)}{n}} \geq 0$$

$$R = X_{max} - X_{min}$$

$$\left\{ \begin{aligned} \bar{R} &= d_2 \sigma \\ \sigma_{\bar{R}} &= d_3 \sigma \end{aligned} \right\}$$

d_2 and d_3 depend on Sample size n (table to be given)

* We also need $\sigma_{\bar{X}}$

$$\sigma = \frac{\bar{R}}{d_2} \quad \dots \quad \sigma_{\bar{X}} = \frac{\sigma}{\sqrt{n}}$$

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$$R = d_2 \sigma \qquad \sigma_{\bar{R}} = d_3 \sigma$$

The Aimed-At Values Method

1. Main population is normally distributed

Production Line

2. Tolerances must be known
3. An acceptable percentage defective also must be specified, e.g. 3%

1. The main population may or may not be normally distributed
   - Two population may have different properties and act similar

$$\begin{pmatrix} \mu_1 , \sigma_1 \\ \mu_2 , \sigma_2 \end{pmatrix}$$

The Estimated-Values Method

- Factors of Production:
  - Good machinery
  - Skilled operation
  - Good Tools
  - Good raw materials
  - ...

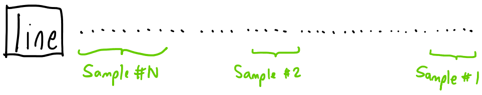
Typically 5

Sample #	Sample size	Sample Average	Sample Range
1	5	$(X_1+X_2+X_3+X_4+X_5)/5$	$X_{max} - X_{min}$
2	5	$(X_1+X_2+X_3+X_4+X_5)/5$	$X_{max} - X_{min}$
⋮	⋮	⋮	⋮
N		$\bar{\bar{X}} = {''} = \frac{\sum_{i=1}^{25} \bar{X}}{}$	$\bar{R} = \frac{\sum R_i}{}$

N

Typically 25

$\bar{\bar{X}} = \mu = \dfrac{\sum_{i=1}^{25} \bar{X}}{25}$ $\qquad$ $\bar{R} = \dfrac{\sum R_i}{25}$



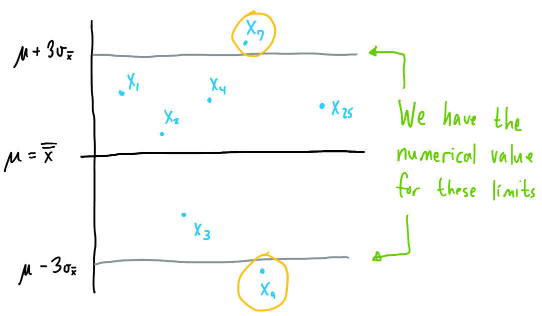
- μ, $\bar{R}$ are calculated

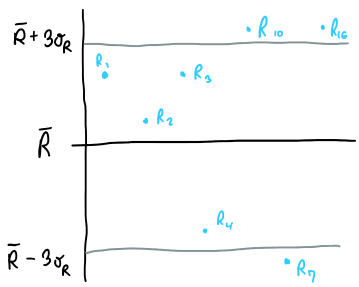
- From the Relation $\bar{R} = d_2 \sigma$ we calculate σ

$$\sigma = \frac{\bar{R}}{d_2} \quad \& \quad \sigma_{\bar{R}} = d_3 \sigma \quad \& \quad \sigma_{\bar{x}} = \frac{\sigma}{\sqrt{n}}$$

$\rightarrow \mu, \sigma_{\bar{x}}, \bar{R} \ \& \ \sigma_{\bar{R}}$

- To verify credibility & stability of data we plot the data on the charts we have established





- In 1000 samples we expect 3 samples to fall outside the 3-sigma limits. For 25 samples we do not expect any data to fall outside limits!

- We recalculate the μ as follows

$$\mu_{new} = \frac{\sum_{i=1}^{25} \bar{X}_i - \bar{X}_7 - \bar{X}_9}{23}$$

- Recalculate the new $\bar{R}$

$$\bar{R}_{new} = \frac{\sum_{i=1}^{25} R_i - R_7 - R_{10} - R_{16}}{22}$$

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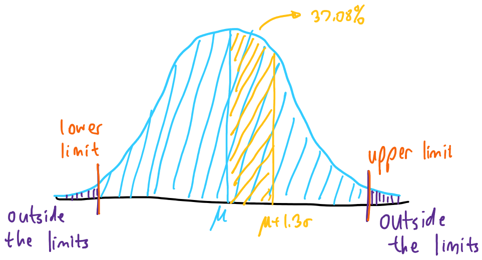
- Recalculate σ

$$\sigma_{new1} = \frac{\bar{R}_{new1}}{d_2}$$ — same as before

$$\sigma_{\bar{x},max1} = \frac{\sigma_{new1}}{\sqrt{n}}$$

$$\sigma_{\bar{R},new1} = d_3 \, \sigma_{new1}$$

- We want to know between UL & LL what percentage of data falls inside the limits



$$UL = \mu + Z_1 \sigma$$

$$LL = \mu - Z_2 \sigma$$

eg: $Z = 1.3 \longrightarrow 0.3708 = 37.08\%$

- For example we know that 99.7% of data is confined between ± 3-sigma limit