Energy + Economics

Tuesday, March 26, 2024 4:00 PM

Recap: Weibull distribution

$$f(v) = \frac{k}{c}\left(\frac{v}{c}\right)^{k-1} e^{\left(-\left(\frac{v}{c}\right)^k\right)}$$

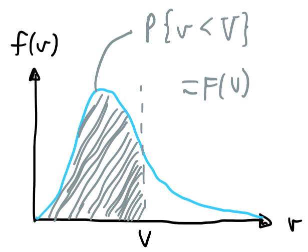
$$P\{v < V\} = F(V) = \int_0^V \frac{k}{c}\left(\frac{v}{c}\right)^{k-1} e^{\left(-\left(\frac{v}{c}\right)^k\right)} dv$$

Let $x = \left(\frac{v}{c}\right)^k$, $dx = k\left(\frac{v}{c}\right)^{k-1}\frac{1}{c}\,dv$

CDF $F(v) = -e^{\left(-\left(\frac{v}{c}\right)^k\right)} + 1 = 1 - e^{\left(-\left(\frac{v}{c}\right)^k\right)}$

Rayleigh Distribution

$k = 2$, $c = \frac{2 v_{AVE}}{\sqrt{\pi}}$



pdf $f(v) = \frac{2v}{\frac{2v_{AVE}}{\sqrt{\pi}}} e^{\left(-\left(\frac{v}{\frac{2v_{AVE}}{\sqrt{\pi}}}\right)^2\right)}$

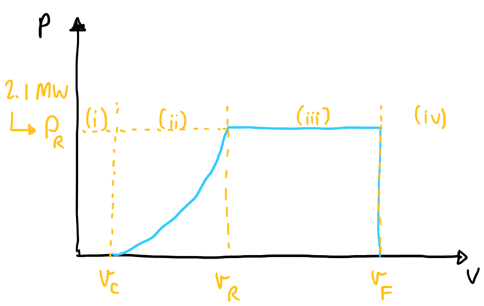
cdf $F(V) = 1 - e^{\left(-\left(\frac{V}{\frac{2v_{AVE}}{\sqrt{\pi}}}\right)^2\right)} = 1 - e^{\left(-\frac{\pi}{4}\left(\frac{V}{v_{AVE}}\right)^2\right)}$

Also, $P\{v < V\} = 1 - P\{v < V\}$

Weibull $P\{v > V\} = e^{\left(-\left(\frac{V}{c}\right)^k\right)}$

Rayleigh $P\{v > V\} = e^{\left(-\frac{\pi}{4}\left(\frac{V}{v_{AVE}}\right)^2\right)}$

Expected Energy from Wind Turbine



Cut-in speed $v_C = 3.5\,m/s$

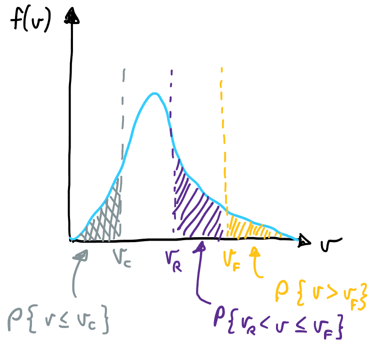
rated speed $v_R = 11\,m/s$

furling speed $v_F = 20\,m/s$

Also we know $v_{AVE} = 7\,m/s$ @ the site

solved in order of difficulty

(i) $0 < v \le v_C$



(i) $0 < v \le v_c$

# of hours with wind speed less than v_c

$$P\{v \le v_c\} = F(v_c)$$
$$= 1 - e^{\left(-\frac{\pi}{4}\left(\frac{v_c}{v_{AVE}}\right)^2\right)}$$
$$= 1 - e^{\left(-\frac{\pi}{4}\left(\frac{3.5}{7}\right)^2\right)} = 0.178$$

hours in a year

$$\text{Hours}\{v \le v_c\} = 8760 \times 0.178 = 1562 \text{ hours/year}$$

→ 0 MW produced due to too-low wind speeds

(iv) $v > v_F$

# of hours with wind speeds above v_F

$$P\{v > v_F\} = 1 - P\{v \le v_F\} = 1 - F(v_F)$$
$$= e^{\left(-\frac{\pi}{4}\left(\frac{v_F}{v_{AVE}}\right)^2\right)}$$
$$\approx e^{\left(-\frac{\pi}{4}\left(\frac{20}{7}\right)^2\right)} = 0.00164$$

$$\text{Hours}\{v > v_F\} = 8760 \times 0.00164 \approx 14.39 \text{ hrs/yr}$$

→ 0 MW produced due to too-high winds

(iii) $v_R < v \le v_F$

# of hours/yr (kWh/yr) generated when turbine is operating at P_R

$$P\{v_R < v \le v_F\} = P\{v \le v_F\} - P\{v \le v_R\}$$
$$= (\cancel{1} - P\{v > v_F\}) - (\cancel{1} - P\{v > v_R\})$$
$$= P\{v > v_R\} - P\{v > v_F\} \rightarrow 0.00164$$
$$= e^{\left(-\frac{\pi}{4}\left(\frac{v_R}{v_{AVE}}\right)^2\right)} (0.00164)$$
$$\approx e^{\left(-\frac{\pi}{4}\left(\frac{11}{7}\right)^2\right)} (0.00164)$$
$$= 0.1421$$

$$\text{Hours}\{v_R < v \le v_F\} = 8760 \times 0.1421 = 1245 \text{ Hr/yr}$$
$$\text{Energy}\{v_R < v \le v_F\} = 2.1[\text{MW}] \times 1245[\text{Hr/yr}] = 2.615 \text{ GWh/yr}$$

More generally, with constant $P = P_R$

$$\text{Energy}\{v_R < v \le v_F\} = P_R \times \text{Hours}\{v_R < v \le v_F\}$$
$$= P_R \times 8760\, P\{v_R < v \le v_F\}$$
$$= P_R \times 8760 \int_{v_R}^{v_F} f(v)\, dv$$

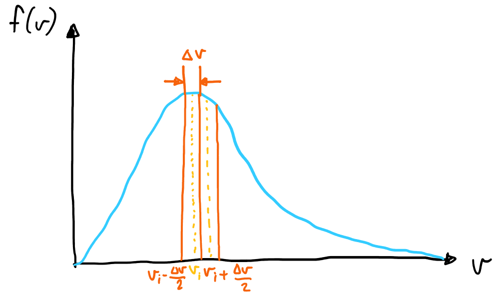
(ii) $v_C < v < v_F$

$P = P(v)$

$$\text{Energy}\{v_C < v \le v_R\} = 8760 \int_{v_C}^{v_R} P(v) f(v)\, dv$$
$$= 8760 \int_{v_C}^{v_R} \left(\frac{1}{2}\rho A C_p v^3\right) \frac{\pi v}{2 v_{AVE}^2}\, e^{\left(-\frac{\pi}{4}\left(\frac{v}{v_{AVE}}\right)^2\right)} dv$$

Difficult / Impossible to solve analytically

Can approximate this as a sum



$$\boxed{\text{Total Energy} = 8760 \times \int_0^\infty P(v) f(v)\, dv}$$

$$P\left\{v_i - \frac{\Delta v}{2} < v < v_i + \frac{\Delta v}{2}\right\} = \int_{v_i - \frac{\Delta v}{2}}^{v_i + \frac{\Delta v}{2}} f(v)\, dv$$
$$\approx f(v_i)\, \Delta v$$

$\text{Energy} \approx 8760 \sum_i P(v_i) f(v_i) \Delta v$ ... more accurate with small Δv

can implement in spreadsheet

can implement in spreadsheet

Wind Turbine Economics

Key factors: fixed O&M, capital cost (turbine, grid connection, construction, ...)

variable O&M

~80% of LCOE is from capital costs

$$\text{LCOE } [\$/\text{kWh}] = \frac{\text{Annual fixed cost} + \text{Annual variable cost } [\$/\text{yr}]}{\text{Annual energy production } [\text{kWh}/\text{yr}]}$$

↳ related to capacitor factor

$$\text{capacity factor} = \frac{\text{Actual energy produced}}{\text{Energy that could have been produced if run @ } P_R \text{ all the time}}$$

Aside: Simple (heuristic) way to estimate CF of wind turbine (7.7.3 in textbook)

$$CF = 0.087\, V_{AVE} - \frac{P_R}{D^2}$$

rotor diameter

Approximately CF is linearly related to V_{AVE} with Rayleigh statistics

Actual energy:

$$E_{out} = 8760\, P_R \times CF$$

$$= 8760\, P_R \left(0.087\, V_{AVE} - \frac{P_R}{D^2}\right)$$

Aside: (to aside)

At optimal: $\frac{dE_{out}}{dP_R} = 0 = 0.087\, V_{AVE} - \frac{2P_R}{D^2}$

↳ where P_{out} is highest

$$\rightarrow P_R^* = \frac{0.087\, V_{AVE}}{2} D^2$$

Can help with quick comparison for different rotor sizes and rated power