Wave & Tidal

Tuesday, April 9, 2024 4:04 PM

So far: power system/energy intro
↳ load duration/screening curves
efficiency & LCOE as "metrics"

renewable energy systems
↳ main focus: solar & wind

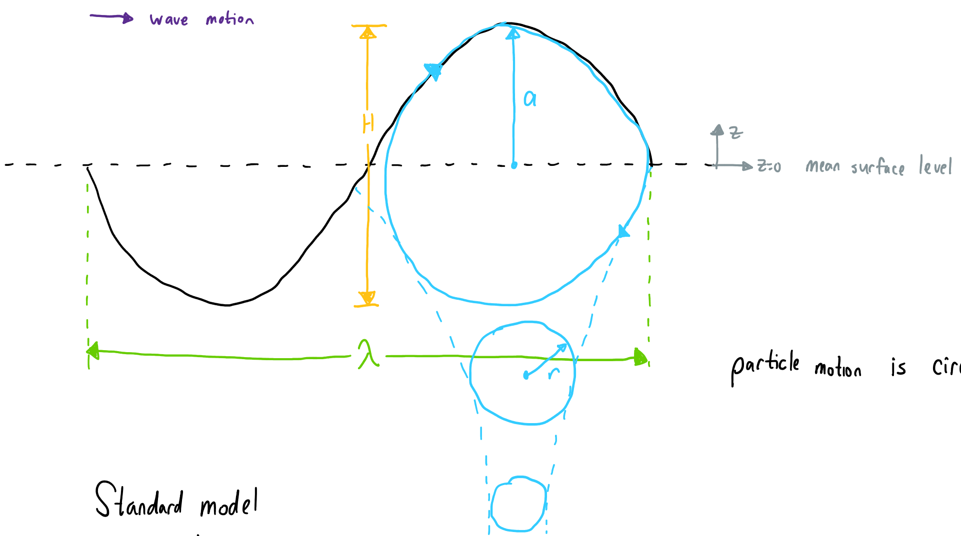
Others

- geothermal
- tidal
- hydro
- biofuel/biomass
- hydrogen fuel cell
- nuclear
- wave energy
- Concentrating solar plants

Wave Energy (8.3 in Masters)

- Solar energy → uneven temperatures and pressures → winds → waves across the ocean surface
- Highly variable, but somewhat predictable
  ↳ forecast days in advance
- Greater lead time for scheduling than solar PV/wind generation

- For modelling, assume ideal sinusoidal wave



particle motion is circular if $D > 0.5\lambda$

depth (D), wavelength (λ)

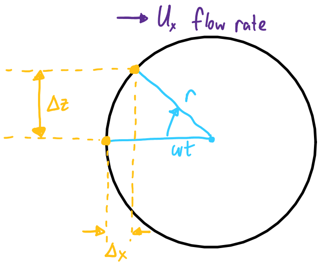
Standard model

$r = ae^{kz}$

k = wave number = $\frac{2\pi}{\lambda}$

z = mean depth below surface (negative)

- Consider "particle" of water below mean surface level.



$$\Delta z = r \sin \omega t = a e^{kz} \sin(\omega t)$$
$$\Delta x = r - r\cos \omega t$$
$$U_x = \frac{d\Delta x}{dt} = r\omega \sin \omega t = \omega a e^{kz} \sin \omega t$$

<u>Aside:</u> power in fluid = pressure × volumetric flow rate

<u>Recall:</u> From wind, $P_L = P_z = -\rho g \Delta z$

For wavefront of length 1m

$$P = \int_{-\infty}^{0} (P_\circ - P_u)\, U_x \, dz = \int_{m}^{0} \rho g \Delta z$$

$$= \rho g \int_{m}^{0} a e^{kz} \sin \omega t \cdot \omega_a e^{kz} \sin \omega t \, dz$$
$$= \rho g^2 a^2 \left(\omega e^{-3kz} \underline{\sin \omega t} \, dz\right)$$
$$= \frac{1}{2} \rho g a^2 \omega \int_{-\infty}^{0} e^{2kz} dz$$

Just use mean value
$\langle \sin^2 \omega t \rangle + \langle \cos^2 \omega t \rangle = 1$
$\langle \sin^2 \omega t \rangle = 1$

$$P = \frac{1}{4} \rho g a^2 + \omega t e^{2kz} \Big|_{\infty}^{0}$$
$$= \frac{1}{4} \rho g a^2 \omega \frac{1}{k} \qquad \omega = \frac{2\pi}{t} \qquad k = \frac{2\pi}{\lambda}$$
$$P = \frac{1}{4} \rho g a^3 \frac{2\pi}{T} \frac{\lambda}{2\pi} = \frac{\rho g a^2 \lambda}{\Delta T}$$

Further, for travelling waves, $\lambda = \frac{2\pi g}{\omega^2} \rightarrow \frac{\lambda}{T} = \frac{2\pi g}{\omega^2} \frac{\omega}{2\pi}$

$\hookrightarrow \frac{2\pi}{\omega}$

$$P = \frac{\rho g a^2}{4} \frac{g}{\omega} = \frac{\rho g^2 a^2 T}{8\pi}$$

$\uparrow \frac{2\pi}{T}$

Finally, $a = \frac{1}{2} H \rightarrow \boxed{P = \frac{\rho g^2 H^2 T}{32\pi}}$ for ideal sinusoidal wave

P = power per meter distance along ridge of wave $[W/m^2]$

ρ = density of water (seawater: 1025 kg/m^3)

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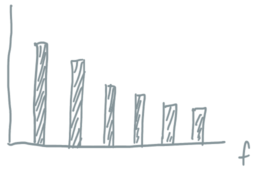
g = acceleration due to gravity (9.8 m/s^2)

H = trough-to-peak wave height [m]

T = period between consecutive waves [s]



- What about real waves? Complex → build off ideal case

$H \rightarrow H_s$ "significant" wave height
$T \rightarrow T_p$ "peak" wave period
} based on wave spectrum modelled as Fourier sum of sinusoidal components

$$\boxed{P \cong 0.86 \frac{\rho g^2}{64\pi} H_s^2 T_p}$$

- See Masters 8.3.2 for WEC technologies
  - only a handful of installations around the world
  - $$$ for now
  - Can be cost-effective to co-locate with offshore wind
    ↳ share cable, converters, substation, etc.

Tidal Power (8.4 in Masters)

- Highly variable, but largely predictable
- Result of gravitational forces exerted by moon
- Technology not yet mature, mainly demonstration only
- Fundamental model is analogous to wind (see Example 8.1 & 8.2 in Masters)

$$P = \frac{1}{2} \rho A v^3$$

surface area

stream speed normal to swept area

density of seawater

- maximum efficiency similarly bounded by Betz Limit