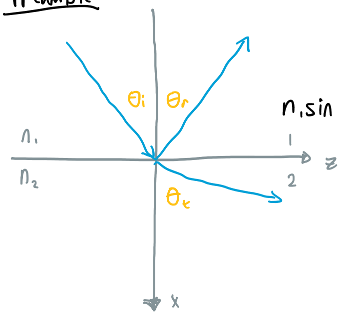
3.1 Optical Dynamics

Friday, September 8, 2023 11:02 AM

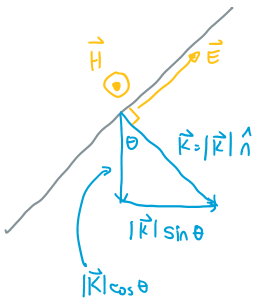
Optical Dynamics

Preamble



$\theta_i = \theta_r$

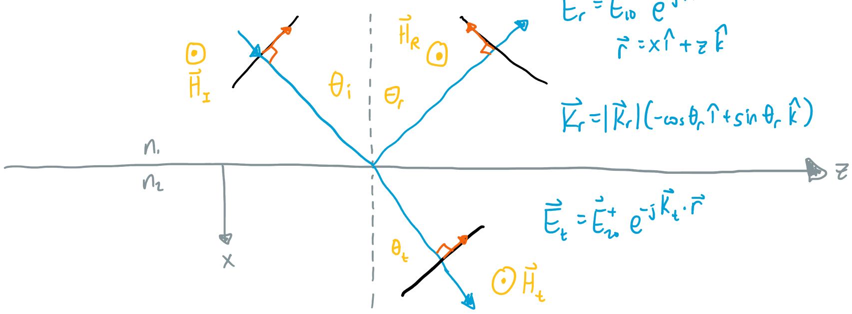
$n_1 \sin\theta_i = n_2 \sin\theta_t$



$\vec{E} = \vec{E}_{10}^{+}\, e^{-j\vec{k}\cdot\vec{r}}$ ← always

Why? $e^{j(\omega t - \vec{k}\cdot\vec{r})} \rightarrow f(z - vt)$ or $f(t - \frac{z}{v})$

Reflection & Transmission



$\vec{E}_r = \vec{E}_{10}^{-}\, e^{-j\vec{k}_r\cdot\vec{r}}$

$\vec{r} = x\hat{i} + z\hat{k}$

$\vec{K}_r = |\vec{K}_r|(-\cos\theta_r\,\hat{i} + \sin\theta_r\,\hat{k})$

$\vec{E}_t = \vec{E}_{t0}^{+}\, e^{-j\vec{k}_t\cdot\vec{r}}$

$\vec{E}$ Field Summary

$\vec{E}_i = \vec{E}_{i0}^{+}\, e^{-j\vec{K}_i\cdot\vec{r}}$ $\qquad$ $\vec{K}_i = |\vec{K}_i|(\cos\theta_i\,\hat{i} + \sin\theta_i\,\hat{k})$

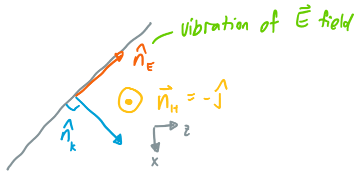
$\vec{E}_r = \vec{E}_{10}^{-}\, e^{-j\vec{K}_r\cdot\vec{r}}$ $\qquad$ $\vec{K}_r = |\vec{K}_r|(\cos\theta_r\,\hat{i} + \sin\theta_r\,\hat{k})$

$\vec{E}_t = \vec{E}_{t0}\, e^{-j\vec{K}_t\cdot\vec{r}}$ $\qquad$ $\vec{K}_t = |\vec{K}_t|(\cos\theta_t\,\hat{i} + \sin\theta_t\,\hat{k})$

$|\vec{K}_i| = |\vec{K}_r| = \omega\sqrt{\mu_1\varepsilon_1}$ $\qquad$ $|\vec{K}_t| = \omega\sqrt{\mu_2\varepsilon_2}$

$\vec{H}_i$ from $\vec{E}_i$

$\vec{H}_i$ from $\vec{E}_i$



$\vec{E}_i = \vec{E}_{10}^{+} e^{-j\vec{k}_i \cdot \vec{r}}$ $\quad \vec{H}_i = \vec{H}_{10}^{+} e^{-j\vec{k}_i \cdot \vec{r}}$ $\quad \vec{k}_i = \hat{n}_k \omega \sqrt{\mu_1 \varepsilon_1}$

$\vec{k}_i \times \vec{E}_{10}^{+} = \omega \mu_1 \vec{H}_{10}^{+}$ $\qquad \vec{E}_{10}^{+} = \hat{n}_E \mathcal{E}_{10}$

$\hat{n}_k \omega \sqrt{\mu_1 \varepsilon_1} \times \hat{n}_E \mathcal{E}_{10} = \omega \mu_1 \hat{n}_H H_{10} \longrightarrow \hat{n}_k \times \hat{n}_E = \hat{n}_H$

$\mathcal{E}_{10} = \sqrt{\frac{\mu_1}{\varepsilon_1}} H_{10} \longrightarrow \eta = \sqrt{\frac{\mu_1}{\varepsilon_1}}$

$H_{10} = \frac{1}{\eta_1} \mathcal{E}_{10}$

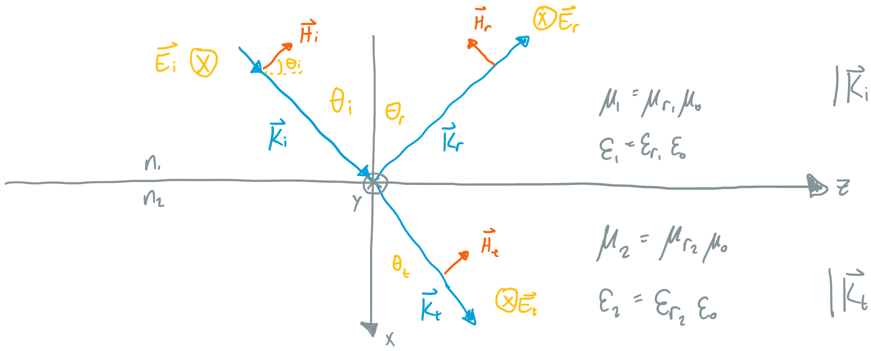
Q. For an EM wave incident obliquely from material 1 and reaching the boundary between materials 1 and 2 where $\eta_1 \neq \eta_2$, which of the following are true?

> $|\vec{k}_i| = |\vec{k}_r|$

$|\vec{k}_r| = |\vec{k}_t|$

$|\vec{k}_i| = |\vec{k}_t|$

Optical Matching



$|\vec{k}_i| = \omega \sqrt{\mu_1 \varepsilon_1} = |\vec{k}_r|$

$|\vec{k}_t| = \omega \sqrt{\mu_2 \varepsilon_2}$

$\vec{H}_i = \hat{n}_{k_i} \times \frac{\vec{E}_i}{\eta}$ $\qquad \eta_i = \sqrt{\frac{\mu_i}{\varepsilon_i}}$

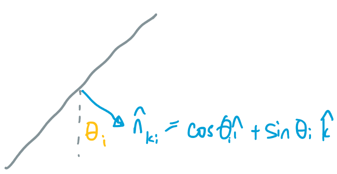
$\vec{E}_i = \hat{j} E_{y_i} e^{-j\phi_i}$ $\qquad \underline{\phi_i = \vec{k}_i \cdot \vec{r}}$ $\qquad \vec{r} = x\hat{i} + z\hat{k}$

plane

Compute $\vec{H}_i$ from $\vec{E}_i$

$\vec{E}_i = \hat{j} E_{y_i} e^{-j\phi_i}$

$\vec{k}_i = k_1 \hat{n}_{k_i}$



$\vec{H}_i = \hat{n}_{k_i} \times \frac{\vec{E}_i}{\eta_i} = (\cos\theta_i \hat{i} + \sin\theta_i \hat{k}) \times \hat{j} \frac{E_{y_i}}{\eta_1} e^{-j\phi_i}$

$$\vec{H}_i = \hat{n}_{k_i} \times \frac{\vec{E}_i}{\eta_i} = (\cos\theta_i \hat{\imath} + \sin\theta_i \hat{k}) \times \hat{\jmath} \frac{E_{y_i}}{\eta_1} e^{-j\Phi_i}$$
$$= (\cos\theta_i \hat{k} + \sin\theta_i (-\hat{\imath})) \frac{E_{y_i}}{\eta_1} e^{-j(k_1\cos\theta_i x + k_1 \sin\theta_i z)}$$

$$\boxed{\begin{array}{l|l} \vec{E}_i = \hat{\jmath} E_{y_i} e^{-jk_1(\cos\theta_i x + \sin\theta_i z)} & \vec{H}_i = (-\sin\theta_i \hat{\imath} + \cos\theta_i \hat{k}) \frac{E_{y_i}}{\eta_1} e^{-jk_1(\cos\theta_i x + \sin\theta_i z)} \\ \vec{E}_r = \hat{\jmath} E_{y_r} e^{-jk_1(-\cos\theta_r x + \sin\theta_r z)} & \vec{H}_r = (-\sin\theta_r \hat{\imath} - \cos\theta_r \hat{k}) \frac{E_{y_r}}{\eta_1} e^{-jk_1(-\cos\theta_r x + \sin\theta_r z)} \\ \vec{E}_t = \hat{\jmath} E_{y_t} e^{-jk_2(\cos\theta_t x + \sin\theta_t z)} & \vec{H}_t = (-\sin\theta_t \hat{\imath} + \cos\theta_t \hat{k}) \frac{E_{y_t}}{\eta_2} e^{-jk_2(\cos\theta_t x + \sin\theta_t z)} \end{array}}$$

$$E_{y_i} e^{-jk_1(\cos\theta_i \cdot 0 + \sin\theta_i z)} + E_{y_r} e^{-jk_1(-\cos\theta_i \cdot 0 + \sin\theta_r z)} = E_{y_t} e^{-jk_2(\cos\theta_t \cdot 0 + \sin\theta_t z)}$$
$$E_{y_i} e^{-jk_1 \sin\theta_i z} + E_{y_r} e^{-jk_1(\sin\theta_r z)} = E_{y_t} e^{-jk_2 \sin\theta_t z}$$
$$E_{y_i} + E_{y_r} = E_{y_t} \longrightarrow 1 + \Gamma = \tau$$
$$\Gamma = \frac{E_{yr}}{E_{yi}} \qquad \tau = \frac{E_{yt}}{E_{yi}}$$

$$\frac{E_{yi}}{\eta_1}\cos\theta_i - \frac{E_{yr}}{\eta_1}\cos\theta_i = \frac{E_{yt}}{\eta_2}\cos\theta_t$$
$$\frac{\cos\theta_i}{\eta_1} - \frac{\Gamma\cos\theta_i}{\eta_1} = \frac{\tau}{\eta_2}\cos\theta_t$$
$$\Gamma = \frac{\eta_2\cos\theta_i - \eta_1\cos\theta_t}{\eta_2\cos\theta_i + \eta_1\cos\theta_t}$$
$$\tau = \frac{2\eta_2\cos\theta_i - \eta_1\cos\theta_t}{\eta_2\cos\theta_i + \eta_1\cos\theta_t}$$

<u>Q1</u> The oblique transmission and reflection coefficients presented here take different forms from normal incident cases presented in the past. The reason is

Field components perpendicular to the boundary must be matched

Tangential boundary matching conditions are not maintained in oblique incidence

> <u>None of the above</u>

<u>TE Wave Oblique incidence</u> — A uniform plane wave is incident from air onto glass at an angle from the normal of 30°.

<u>Q2</u> What is the transmission angle into glass given that $\eta_{air} = 1$ and $\eta_{glass} = 1.45$?

$n_1 \sin\theta_1 = n_2 \sin\theta_2 \longrightarrow 1 \sin(30) = 1.45 \sin\theta_2 \longrightarrow \theta_2 = 20.17°$

Q3 What is the reflection coefficient if the incident wave is in the TE polarization?

-0.222