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HW 4

1. Problem 1

(a) $\mathbb{E}[X] = 1 \times \frac{1}{7} + 2 \times \frac{1}{14} + 3 \times \frac{3}{14} + 4 \times \frac{2}{7} + 5 \times \frac{2}{7} = \frac{49}{14} = \frac{7}{2}$

(b) $\mathbb{E}[|X-2|] = 1 \times \frac{1}{7} + 0 \times \frac{1}{14} + 1 \times \frac{3}{14} + 2 \times \frac{2}{7} + 3 \times \frac{2}{7} = \frac{25}{14}$

k	1	2	3	4	5
k-2	-1	0	1	2	3
|k-2|	1	0	1	2	3

2. Problem 2

(a)

$$P(X=1) = \frac{\binom{5}{0}}{\binom{10}{0}} \times \frac{\binom{5}{1}}{\binom{10}{1}}$$

$$P(X=2) = \frac{\binom{5}{1}}{\binom{10}{1}} \times \frac{\binom{5}{1}}{\binom{9}{1}}$$

From above, we get:

$$P(X=k) = \frac{\binom{5}{x-1}}{\binom{10}{x-1}} \times \frac{\binom{5}{1}}{\binom{10-(x-1)}{1}} = \frac{\binom{5}{x-1}}{\binom{10}{x-1}} \times \frac{\binom{5}{1}}{\binom{11-x}{1}}$$

The probability mass function of X is:

X	P(X = k)	Earnings
1	1/2	\$6
2	5/18	\$4
3	5/36	\$0
4	5/84	-\$8
5	5/252	-\$24
6	1/252	-\$56

(b) $\mathbb{E}[X] = 1 \times \frac{1}{2} + 2 \times \frac{5}{18} + 3 \times \frac{5}{36} + 4 \times \frac{5}{84} + 5 \times \frac{5}{252} + 6 \times \frac{1}{252} = \frac{11}{6}$

(c) $\sum_{k=1}^{6}(Earnings \times P(X=k)) = \$2.94$

3. Problem 3

(a)

$$\begin{aligned}\mathbb{E}[aX+b] &= \sum[aX+b]P(X=k)\\ &= \sum[aXP(X=k)+bP(X=k)]\\ &= \sum[aXP(X=k)]+\sum[bP(X=k)]\\ &= a\sum[X]P(X=k)+b\sum P(X=k)\\ &= a\mathbb{E}[x]+b\\ & QED\end{aligned}$$

(b)

$$\begin{aligned}\mathbb{E}[aX+b-\mathbb{E}[aX+b]]^2 &= \mathbb{E}[aX+b-a\mathbb{E}[X]-b]^2\\ &= \mathbb{E}[a(X-\mathbb{E}[X])]^2\\ &= \mathbb{E}[a^2(X-\mathbb{E}[X])^2]\\ &= a^2\mathbb{E}[(X-\mathbb{E}[X])^2]\\ & QED\end{aligned}$$

(c)

$$\begin{aligned}\mathbb{E}[(X-\mathbb{E}[X])^2] &= \mathbb{E}[(X^2-2X\mathbb{E}[X]+(\mathbb{E}[X]^2)]\\ &= \mathbb{E}[X^2]-\mathbb{E}[2X\mathbb{E}[X]]+\mathbb{E}[\mathbb{E}[X]^2]\\ &= \mathbb{E}[X^2]-2(\mathbb{E}[X\mathbb{E}[X]])+\mathbb{E}[X]^2 \text{ from part a, and the fact that the expected value of the expected value is itself}\\ &= \mathbb{E}[X^2]-2\mathbb{E}[X]\mathbb{E}[X]+\mathbb{E}[X]^2\\ &= \mathbb{E}[X^2]-2\mathbb{E}[X]^2+\mathbb{E}[X]^2\\ &= \mathbb{E}[X^2]-\mathbb{E}[X]^2\\ & QED\end{aligned}$$

4. Problem 4

Jumps occur at X = {1, 2, 5, 100}

P(X = 1) = $\frac{1}{4}$
P(X = 2) = $\frac{1}{3}-\frac{1}{4}=\frac{1}{12}$
P(X = 5) = $\frac{1}{2}-\frac{1}{3}=\frac{1}{6}$
P(X = 100) = $\frac{1}{2}$

5. Problem 5

(a) $\int_{1.9}^{1.9}(t^2-3)\,dt=0$
(b) $(1.9^2-3)=0.61$
(c)

$$F_x'(t)=\begin{cases}0 & t<\sqrt{3} \text{ or } t\geq 2\\ 2t & \sqrt{3}\leq t<2\end{cases}$$

6. Problem 6

Out of n points, we get three vertices ($\binom{n}{3}$), and the probability of the three vertices forming a triangle (complete with the edges) is $\frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} = \frac{1}{8}$.

Let $X_1, X_2, ..., X_{\binom{n}{3}}$ be the triangles formed. Each of these triangles can have complete edges (1) or not (0). So for the first triangle, $\mathbb{E}[X_1] = 1 \times \frac{1}{8} + 0 \times \frac{7}{8} = \frac{1}{8}$.

$$
\begin{aligned}
\mathbb{E}[X_1 + X_2 + ... + X_{\binom{n}{3}}] &= \mathbb{E}[X_1] + \mathbb{E}[X_2] + ... + \mathbb{E}[X_{\binom{n}{3}}] \text{ by linearity} \\
&= \frac{1}{8} + \frac{1}{8} + ... + \frac{1}{8} \\
&= \binom{n}{3} \times \frac{1}{8}
\end{aligned}
$$