2.1 Maxwell's Equations

Friday, September 8, 2023 11:01 AM

2.1) Maxwell's Equations and Bridging Transmission Line Knowledge

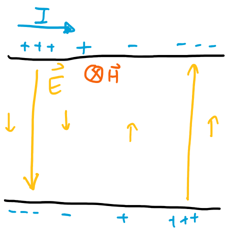
$$\vec{\nabla}\cdot\vec{E} = \frac{\rho_{all}}{\varepsilon_0} \quad \Big| \quad \vec{\nabla}\cdot\vec{B} = 0 \quad \Big| \quad \vec{\nabla}\times\vec{E} = -\mu\frac{\partial \vec{H}}{\partial t} \quad \Big| \quad \vec{\nabla}\times\vec{H} = \vec{J}_{source} + \vec{J}_{conduction} + \vec{J}_{displacement}$$

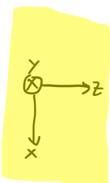
Bridging from TL: TEM Propagation

$$V(z,t) = \overbrace{f_1\left(t-\frac{z}{v}\right)}^{V^+} + \cancel{f_2\left(t+\frac{z}{v}\right)} \to 0 \qquad v = \frac{1}{\sqrt{LC}}$$

backward wave

$$I(z,t) = \underbrace{\frac{1}{Z_0}\left\{f_1\left(t-\frac{z}{v}\right)\right.}_{I^+} - \left.\cancel{f_2\left(t+\frac{z}{v}\right)}\right\} \to 0 \qquad Z_0 = \sqrt{\frac{L}{C}} = Lv$$





$[\vec{E}] = \frac{V}{m}$

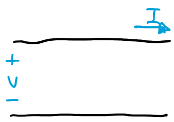
$[\vec{H}] = \frac{A}{m}$

TL Analogs for Plane Wave Properties

Electric Field $\vec{E}$	Voltage (V)
Magnetic Field $\vec{H}$	Current (I)
Permittivity ε	Capacitance (C)
Permeability μ	Inductance (L)

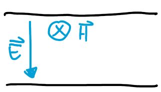
Bridging from TL to TEM Power

TL



$$\langle P\rangle = \frac{1}{2}\,\mathrm{Re}(VI^*)$$

Fields



time averaged RMS

$$\langle P\rangle = \boxed{\frac{1}{2}}\,\mathrm{Re}\underbrace{\left(\vec{E}\times\vec{H}^*\right)}$$

$\langle\vec{S}\rangle$ Poynting vector units: $\frac{W}{m^2}$

Q1 What is the direction of the Poynting Vector $\vec{S}$ if $\vec{E}$ is in $-\hat{x}$ direction and $\vec{H}$ in $-\hat{y}$

$(\vec{S})$ Poynting vector *units: $\frac{W}{m^2}$

Q1 What is the direction of the Poynting Vector $\vec{S}$ if $\vec{E}$ is in $-\hat{x}$ direction and $\vec{H}$ in $-\hat{y}$ direction? $\hat{z}$

Q2 What is the direction of $\vec{S} \times \vec{E}$? Same as $\vec{H}$

Bridging from TL: Waves

T-Line $\frac{\partial V}{\partial z} = -L\frac{\partial I}{\partial t}$ $V \leftarrow E_x$ $\frac{\partial E_x}{\partial z} = -\mu_0 \frac{\partial H_y}{\partial t}$

$\frac{\partial I}{\partial z} = -C\frac{\partial V}{\partial t}$ $I \leftarrow H_y$ $\frac{\partial H_y}{\partial z} = -\varepsilon_0 \frac{\partial E_x}{\partial t}$

Converts to field

$[L] = H/m$ $[V] = \text{Volts}$ $f(z-vt)$ $v = \frac{1}{\sqrt{LC}}$

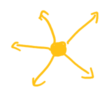
$[C] = F/m$ $[I] = \text{amps}$ $(\alpha(t-\frac{z}{v}))$

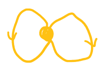
Q If there is nothing in vacuum, why is the vacuum permittivity (ε_0) not just zero?

> Vacuum is full of quantum fluctuations with particles constantly "popping" in and out of existence

> $c = \sqrt{\frac{1}{\mu\varepsilon}}$ can be rewritten as $\varepsilon = \frac{1}{\mu c^2}$. Since neither μ nor c is infinite, ε cannot be 0

Maxwell's Equation — Physical Interpretation


radial


closed

Gauss': $\vec{\nabla} \cdot \vec{E} = \frac{\rho_{all}}{\varepsilon_0}$

source extraction $\vec{\nabla} \cdot \vec{B} = 0$ → magnetic poles always come in pairs. Can't have single magnetic charges but can have single electric charge

Faraday's: $\vec{\nabla} \times \vec{E} = -\mu \frac{\partial \vec{H}}{\partial t}$

Ampère's: $\underbrace{\vec{\nabla} \times}_{\text{rotation extraction}} \vec{H} = \underbrace{\vec{J}_{source}}_{\text{generator}} + \underbrace{\vec{J}_{conduction}}_{\sigma E} + \underbrace{\vec{J}_{displacement}}_{\frac{\partial \vec{D}}{\partial t} = \varepsilon \frac{\partial \vec{E}}{\partial t}}$

generator $= 0$ (in this class)

conductivity σ; σE: Ohm's Law; 0 for insulator → very low σ

$\frac{\partial \vec{D}}{\partial t} = \varepsilon\frac{\partial \vec{E}}{\partial t}$: no physical basis — all intuition. Maxwell's genius

QL Gauss' law given in the form $\vec{\nabla} \cdot \vec{E} = \frac{\rho_{all}}{\varepsilon_0}$ describes ...

There can be no electric charges (must be dipoles)

> Electric fields caused by electric charges

> Net outflow of electric field through an enclosed surface

The coupling interaction between $\vec{E}$ and $\vec{H}$ fields

Q2 Faraday's Law given in the form $\vec{\nabla} \times \vec{E} = -\mu \frac{\partial \vec{H}}{\partial t}$ describes...

Time-varying magnetic field induces an electric field

Q3 Ampere's Law given in the form $\vec{\nabla} \times \vec{H} = \varepsilon \frac{\partial \vec{E}}{\partial t}$ (discarding $\vec{J}_{source}$ and $\vec{J}_{conduction}$) describes...

Time-varying electric field induces a magnetic field

Maxwell's Equations — Math Interpretation (uniform materials, constant in space and time)

$\vec{\nabla} \cdot \vec{E} = \frac{\rho_{all}}{\varepsilon_0}$ $[\rho] = \frac{C}{m^3}$, $[\varepsilon_0] = \frac{F}{m}$

$\vec{\nabla} \cdot \vec{B} = 0$ $[\vec{B}] = \frac{Wb}{m^2}$

$\vec{\nabla} \times \vec{E} = -\mu \frac{\partial \vec{H}}{\partial t}$ $[\vec{E}] = \frac{V}{m}$, $[\vec{H}] = \frac{A}{m}$, $[\mu] = \frac{H}{m}$

$\vec{\nabla} \times \vec{H} = \vec{J}_{source} + \vec{J}_{conduction} + \vec{J}_{displacement}$ $[\vec{J}] = \frac{A}{m^2}$

a derivative — a change in position $\left(\frac{\partial}{\partial x}\right)$

$[\vec{\nabla}][\vec{H}] = [J]$

$\frac{1}{m}$ $\frac{A}{m}$ $\frac{A}{m^2}$

Q Consider $\vec{\nabla} \cdot \vec{E} = 0$, $\vec{\nabla} \cdot \vec{B} = 0$, $\vec{\nabla} \times \vec{E} = -\mu \frac{\partial \vec{H}}{\partial t}$, $\vec{\nabla} \times \vec{H} = \varepsilon \frac{\partial \vec{E}}{\partial t}$. Using the identity for curl, $\vec{\nabla} \times (\vec{\nabla} \times \vec{A}) = \vec{\nabla}(\vec{\nabla} \cdot \vec{A}) - \vec{\nabla}^2 \vec{A}$, which of the following is a valid simplification of $\vec{\nabla} \times (\vec{\nabla} \times \vec{E}) = \vec{\nabla}(\vec{\nabla} \cdot \vec{E}) - \vec{\nabla}^2 \vec{E}$?

$0 = -\vec{\nabla}^2 \vec{E}$

> $\vec{\nabla}^2 \vec{E} = \mu \frac{\partial}{\partial t}(\vec{\nabla} \times \vec{H})$

$\vec{\nabla} \times \left(\mu \frac{\partial \vec{H}}{\partial t}\right) = \vec{\nabla} \cdot \vec{E} - \vec{\nabla}^2 \vec{E}$

$0 = \vec{\nabla} - \vec{\nabla}^2 \vec{E}$

$$\vec{\nabla} \times (\vec{\nabla} \times \vec{E}) = \vec{\nabla}(\vec{\nabla} \cdot \vec{E}) - \vec{\nabla}^2 \vec{E}$$

$$\vec{\nabla} \times \left(-\mu \frac{\partial \vec{H}}{\partial t}\right) = -\vec{\nabla}^2 \vec{E}$$

$$-\mu \frac{\partial}{\partial t}(\vec{\nabla} \times \vec{H}) = -\vec{\nabla}^2 \vec{E}$$

$$-\mu \frac{\partial}{\partial t}\left(\varepsilon \frac{\partial \vec{E}}{\partial t}\right) - \vec{\nabla}^2 \vec{E}$$

$$-\mu\varepsilon \frac{\partial^2 \vec{E}}{\partial t^2} - \vec{\nabla}^2 \vec{E}$$

$$\vec{\nabla}^2 \vec{E} - \frac{1}{v^2} \frac{\partial^2 \vec{E}}{\partial t^2} = 0$$

$$\rightarrow v = \frac{1}{\sqrt{\mu\varepsilon}} \rightarrow \mu\varepsilon = \frac{1}{v^2}$$