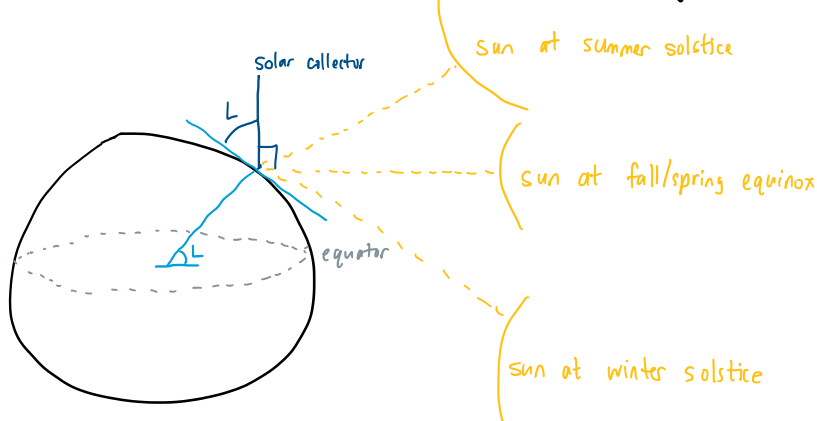


# Solar Position

Thursday, February 1, 2024 3:59 PM

Can we use information from previous lecture to find a good tilt angle for solar collector?

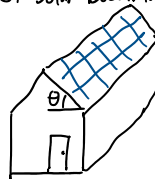


Vancouver:  $L = 49.3^\circ$

- Good rule of thumb: tilt toward the equator by an angle  $L$ 
  - ↳ south in Northern Hemisphere
  - ↳ north in Southern Hemisphere

e.g. Washington D.C.  $L = 38.9^\circ$   
tilt angle  $\cong 38.9^\circ$

2009 Solar Decathlon Challenge



increase tilt angle to optimize for winter

decrease tilt angle to optimize for summer

## Solar Noon

- When the sun is directly over the local meridian, i.e. line of longitude

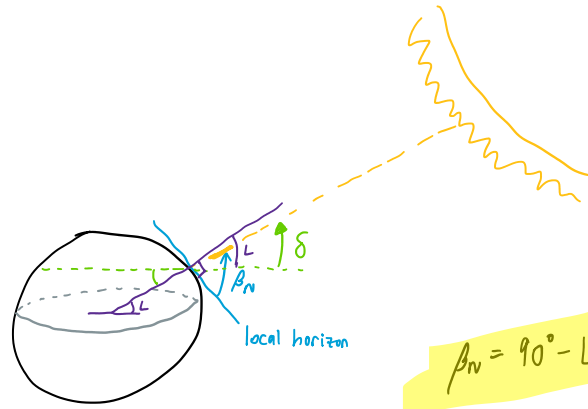
• Recall altitude angle  $\beta$



• Altitude Angle at Solar Noon ( $\beta_n$ )

- Angle between the sun and the local horizon directly beneath the sun

- Angle between the sun and the local horizon directly beneath the sun

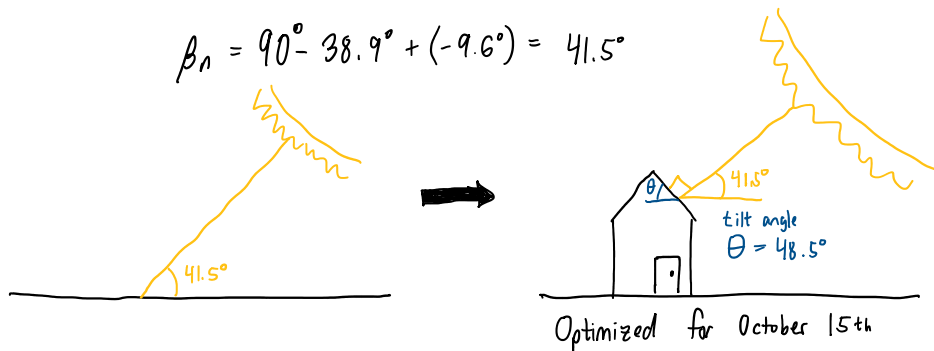


### Zenith

- axis drawn directly overhead at a site
- perpendicular to local horizon
- E.g. Gable Home from 2009 Solar Decathlon Challenge, Washington D.C.
  - Competition takes place on October 15  $\rightarrow$  day number  $n=288$

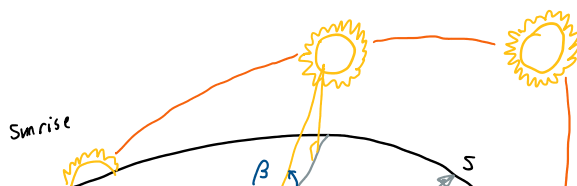
$$\delta = 23.45 \sin\left(\frac{360}{365}(288-81)\right) = -9.6^\circ$$

$$\beta_n = 90^\circ - 38.9^\circ + (-9.6^\circ) = 41.5^\circ$$

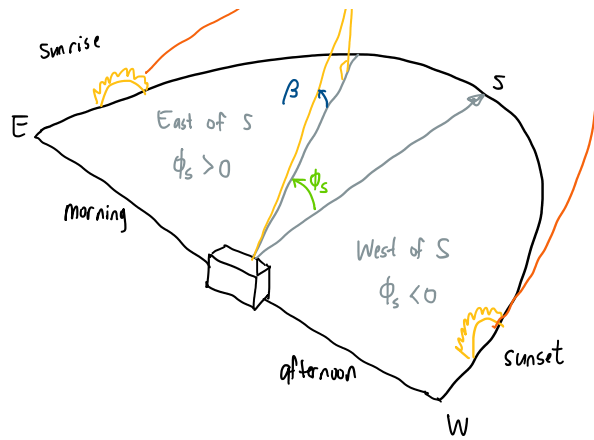


But we have not considered Earth's rotation

- Now, we describe the location of the sun at any time  $\rightarrow$  altitude angle  $\beta$
- $\rightarrow$  azimuth angle  $\phi_s$



$\beta, \phi_s$  depend on



$\beta, \phi_s$  depend on  
 $\hookrightarrow$  day number  
 $\hookrightarrow$  latitude  
 $\hookrightarrow$  hour of day

## Hour angle (H)

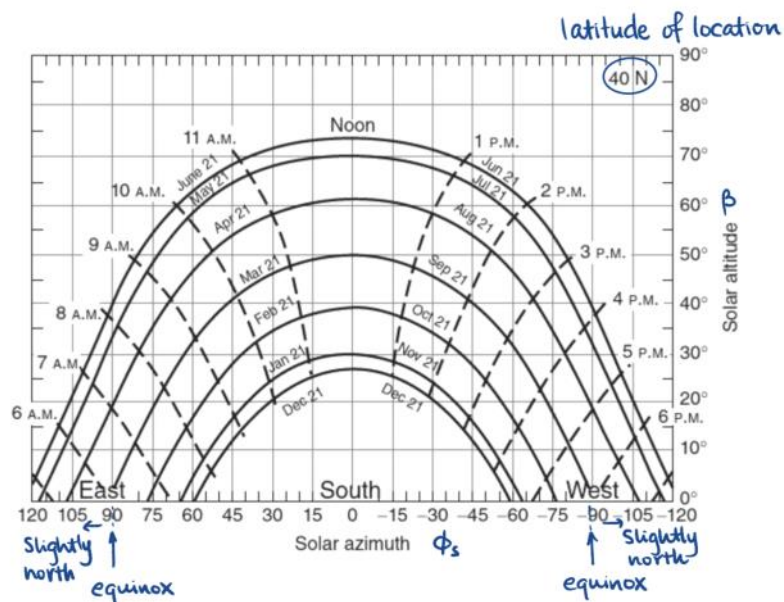
- # of degrees the Earth must rotate to have the Sun directly over the local meridian

$$H = \frac{15^\circ}{\text{hour}} \times \text{hours before solar noon}$$

- Without going through details of derivation,

$$\sin \beta = \cos L \cos \phi_s \cos H + \sin L \sin \delta$$

$$\sin \phi_s = \frac{\cos \delta \sin H}{\cos \beta}$$



- But: in spring and summer, it is possible to have  $\phi_s > 90^\circ$  or  $\phi_s < -90^\circ$  but  $\sin x = \sin(180^\circ - x)$   
 (early morning) (evening)

► We need a test to determine if  $|\phi_s| < 90^\circ$  or  $|\phi_s| > 90^\circ$

If  $\cos H \geq \frac{\tan \delta}{\tan L}$ , then  $|\phi_s| \leq 90^\circ$ . otherwise  $|\phi_s| > 90^\circ$

Example: Find the altitude angle  $\beta$  and azimuth angle  $\phi_s$  for the sun at 3pm solar time in Boulder, CO. ( $L = 40^\circ$ ) on summer solstice.

i)  $\delta = 23.45^\circ$

ii)  $H = 15^\circ(-3) = -45^\circ$  at 3pm solar time

iii)  $\sin \beta = \cos(40^\circ) \cos(23.45^\circ) \cos(-45^\circ) + \sin(40^\circ) \sin(23.45^\circ)$

$\beta = 48.83^\circ$

iv)  $\sin \phi_s = \frac{\cos(23.45^\circ) \sin(-45^\circ)}{\cos(48.83^\circ)} = -0.9854$

$\phi_s = -80.19^\circ$  OR  $260.19^\circ = -99.81^\circ$

↑  
80.19° west of south

↑  
99.81° west of south

v) The test:  $\cos H = \cos(-45^\circ) = 0.707$

$\frac{\tan \delta}{\tan L} = \frac{\tan(23.45^\circ)}{\tan(40^\circ)} = 0.517$

$\cos H > \frac{\tan \delta}{\tan L} \rightarrow |\phi_s| \leq 90^\circ \rightarrow \phi_s = -80.19^\circ$

Summary

$n$  = day number

$\delta$  = solar declination angle

$L$  = latitude

$\beta_n$  = altitude angle at solar noon

$H$  = hour angle

$\beta$  = solar altitude angle

$\phi_s$  = solar azimuth angle

$\delta = 23.45^\circ \sin\left(\frac{360}{365}(n - 81)\right)$

$\beta_n = 90^\circ - L + \delta$

$H = 15^\circ/\text{hr} \times \# \text{ hrs before solar noon}$