ELEC 301

Mini-Project 1 Report

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1. Introduction

This report investigates the accuracy of Miller's Theorem and the method of open-circuit and short-circuit time constants to give an approximation for the poles and zeros of a circuit. For this investigation, three circuits were modeled through Analog Device's LTSpice software.

2. Mini Project

Part 1

1a.

The value for the gain can be calculated through equation 3, which was derived from equations 1 and 2.

$$V_1 = \frac{R_2}{R_1 + R_2} V_s \tag{1}$$

$$V_o = -G(R_3 // R_4) V_s \tag{2}$$

$$\frac{V_o}{V_s} = A_m = K = -47.619 \tag{3}$$

Using Miller's Theorem, we get C_a, the capacitance equivalent on the left side of the circuit, in equation 5a, and C_b, the capacitance equivalent on the right side of the circuit, in equation 5b.

$$Z_a = Z\left(\frac{1}{1-K}\right);\ Z_b = Z\left(\frac{K}{1-K}\right) \qquad \textit{(4a), (4b)}$$

$$C_a = -C_3(1-K) = 972.38\, pF \tag{5a}$$

$$C_b = C_3\left(\frac{K-1}{K}\right) = 20.42\, pF \tag{5b}$$

Calculating for the poles of the circuit at low frequencies:

$$\tau_{C1}^{SC} = (R_1 + R_2)C_1 = 4.2\ ms \tag{6}$$

$$\omega_{LP1} = 238.095\ rad/s \tag{7}$$

$$\tau_{C4}^{SC} = (R_3 + R_4)C_4 = 4\ ms \tag{8}$$

$$\omega_{LP2} = 250\ rad/s \tag{9}$$

Calculating for the poles of the circuit at high frequencies:

$$\tau_{C2}^{SC} = (R_1\ ||\ R_2)(C_2 + C_a) = 48.685\ ns \tag{10}$$

$$\omega_{HP1} = 20.540\ Mrad/s \tag{11}$$

$$\tau_{Cb}^{SC} = (R_3\ ||\ R_4)C_b = 20.42\ ns \tag{12}$$

$$\omega_{HP2} = 48.972\ Mrad/s \tag{13}$$

The transfer function will then be:

$$\begin{aligned} T(s) &= A_m \times F_L(s) \times F_H(s) \\ &= (-47.619)\left[\left(\frac{s}{s+238.1}\right)\left(\frac{s}{s+250}\right)\right]\left[\left(\frac{20.5E6}{s+20.5E6}\right)\left(\frac{49.0E6}{s+49E6}\right)\right] \end{aligned} \tag{14}$$

And the 3dB points are:

$$\omega_{L3db} = \sqrt{\omega_{LP1}^2 + \omega_{LP2}^2} = 345.238\ rad/s \tag{15}$$

$$f_{L3db} = \frac{\omega_{L3db}}{2 \times \pi} = 54.946\ Hz \tag{16}$$

$$\tau_{H3db} = \sqrt{\tau_{HP1}^2 + \tau_{HP2}^2} = 52.794\ ns \tag{17}$$

$$\omega_{H3db} = \frac{1}{\tau_{H3db}} = 18.941M\ rad/s \tag{18}$$

$$f_{H3db} = \frac{\omega_{H3db}}{2 \times \pi} = 3.015\ MHz \tag{19}$$

The circuit for part I of the project is simulated in LTSPICE below as shown in Figure 1.

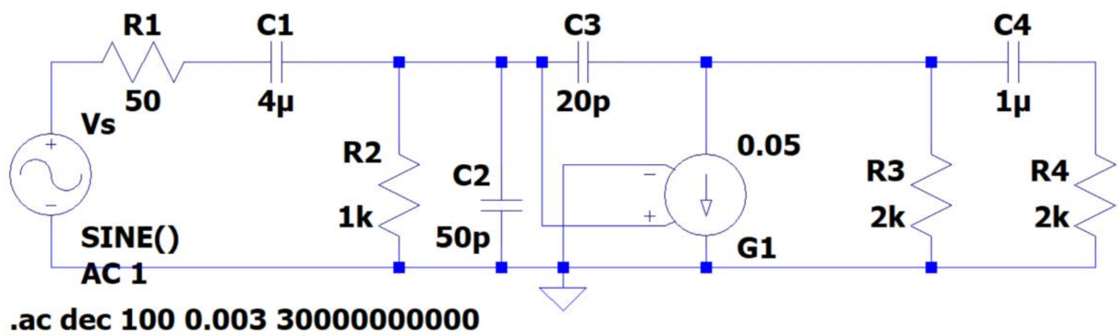

Figure 1. Circuit schematic for part 1 of the project.

1B.

The simulated poles of the circuit were found through linear approximations, where the Bode plot is converted into several linear approximations and the points where they intersect are considered poles. A side-by-side view of the calculated and simulated values are shown in table 1.

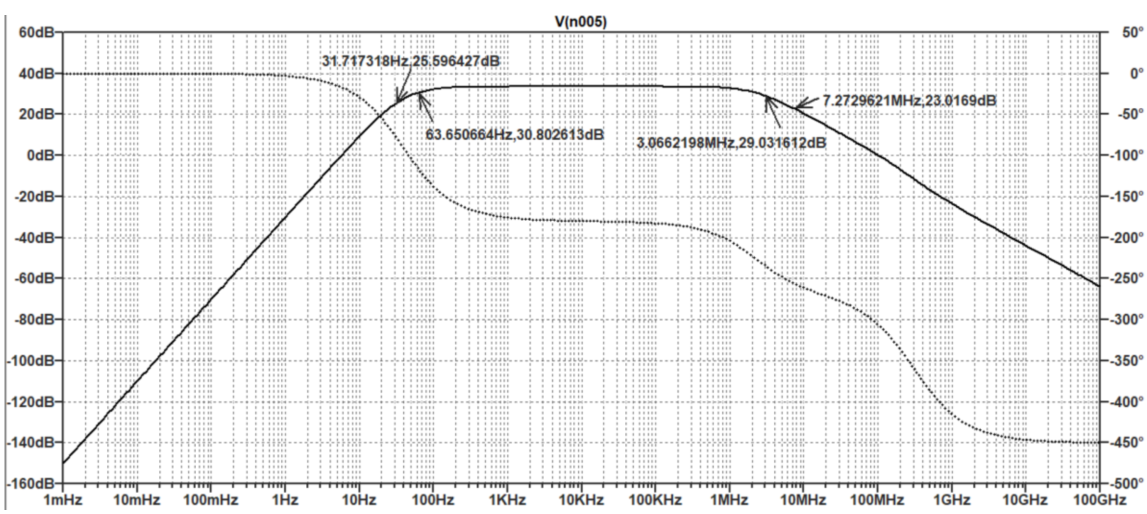

Figure 2. The plot resulting from simulation of the circuit in figure 1. The four poles in the plot have been labeled.

	Calculated	**Simulated**
f_{LP1}	37.894 Hz	31.717 Hz
f_{LP2}	39.789Hz	63.651 Hz
f_{HP1}	3.269 MHz	3.066 MHz
f_{HP2}	7.794 MHz	7.273 MHz

Table 1. A comparison of the calculated and simulated pole locations of the circuit in figure 1.

To calculate for the percent error, equation 20 was used.

$$\% \, Error = \frac{|calculated - simulated|}{simulated} \times 100\% \tag{20}$$

The simulated values of the 3dB point were determined graphically in the plot in figure 2.

	Calculated	**Simulated**	**% Error**
f_{L3dB}	54.946 Hz	50.933 Hz	7.88%
f_{H3dB}	3.015 MHz	2.925 MHz	3.07%

Table 2. Percent error between the calculated and simulated values of the transconductance amplifier in figure 1.

From both tables 1 and 2, there is a greater difference in values between the calculated and the simulated in the lower frequencies, especially for f_{lp2}, so greater care must be used when Miller's Theorem and the method of open and short-circuit time constants are used to approximate a circuit.

Part 2

2A.

For this part, we are looking at the top circuit in figure 3. The poles of the system are located at the intersections of the linear approximations of the Bode plot in figure 4, and are located approximately at 19Hz, 520Hz, 1.5MHz, and 30 MHz. The linear approximations are drawn, from left to right, 40 dB/decade, 20 dB/decade, 0 dB/decade, -20 dB/decade, and finally, -40 dB/decade.

2B.

To calculate the 3dB frequencies, a series of general equations were used to get the high frequency poles. When $\tau_{C4}^{OC} < \tau_{C2}^{OC}$, equations 21 and 22 were used. When $\tau_{C4}^{OC} > \tau_{C2}^{OC}$, equations 23 and 24 were used. After getting the poles and following equations 17, 18, and 19, we get the calculated 3dB frequencies. The calculated 3dB frequencies can be found in table 3. Lastly, the method for finding the percent error between the calculated and simulated value can be found in equation 20.

$$\tau_{C2}^{OC} = ((R_1 \,||\, R_2) \ + R_3 + R_4) \times C_2 \tag{21}$$

$$\tau_{C4}^{OC} = (R_1 \,||\, R_2) \times C_4 \tag{22}$$

$$\tau_{C2}^{OC} = ((R_1 \,||\, R_2) \ + R_3) \times C_2 \tag{23}$$

$$\tau_{C4}^{OC} = ((R_1 \,||\, R_2) \,||\, (R_3 \,||\, R_4)) \times C_4 \tag{24}$$

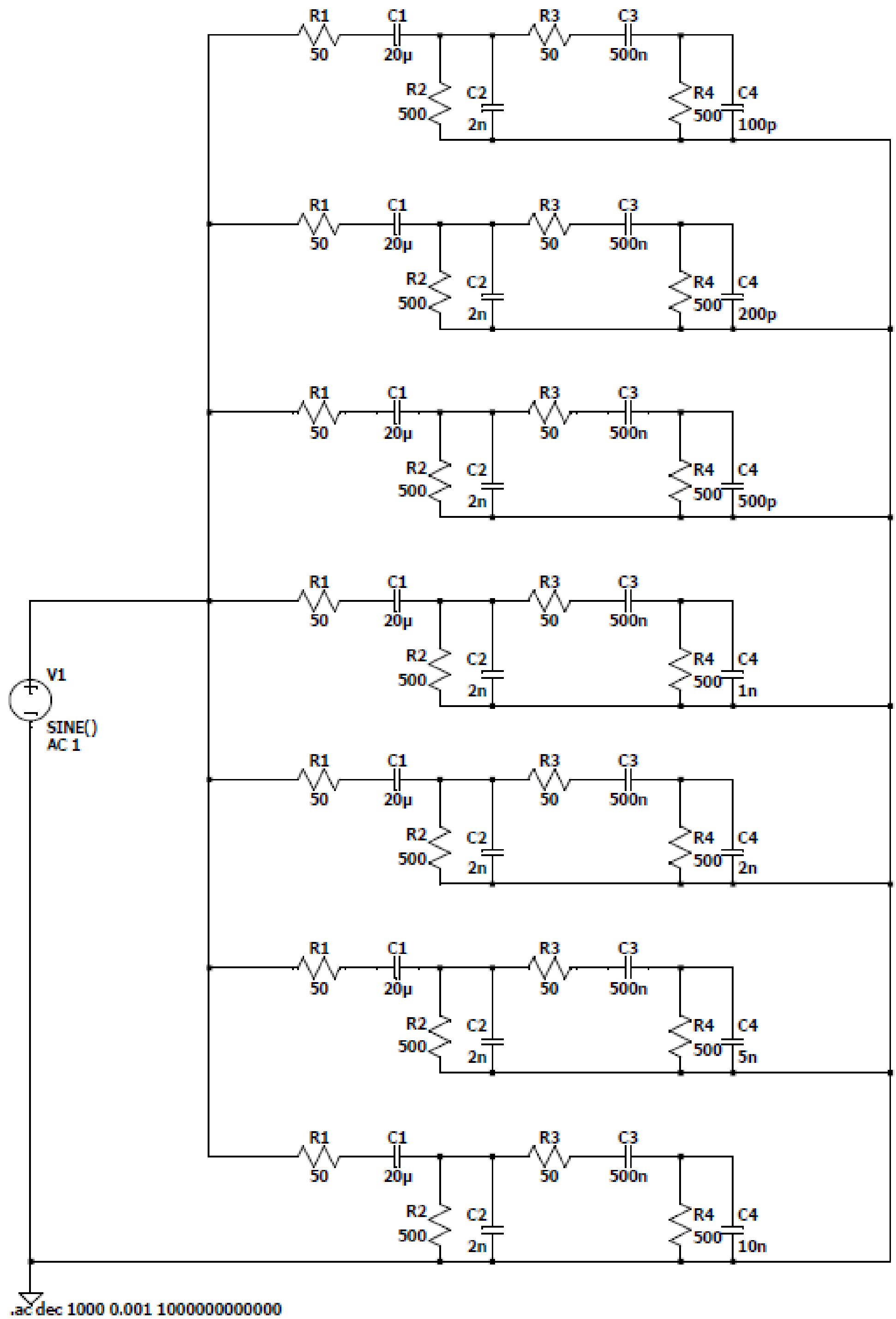

Figure 3. Circuit schematic for part 2 of the project, with varying values for C_4.

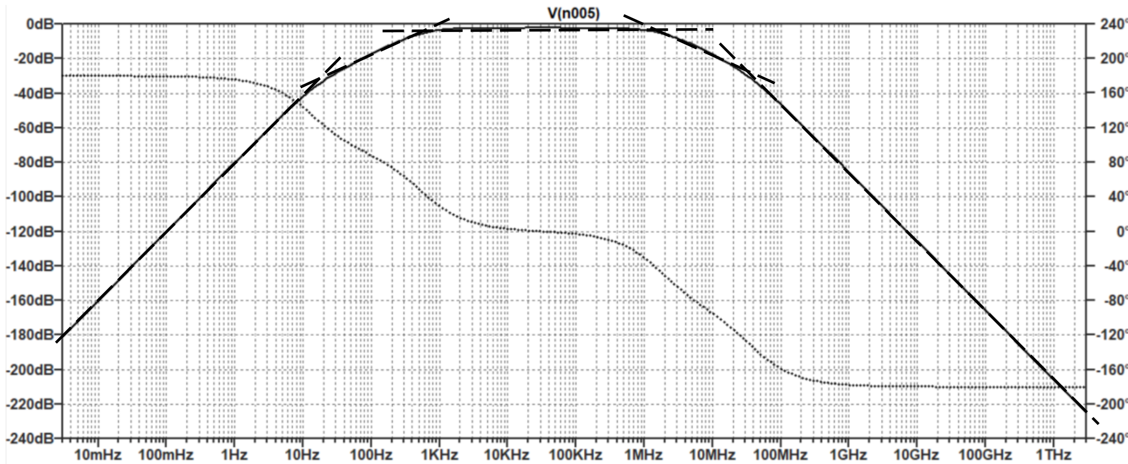

Figure 4. The Bode plot of the circuit in Figure 3. Linear approximations of the plot are shown with dashed lines in the figure.

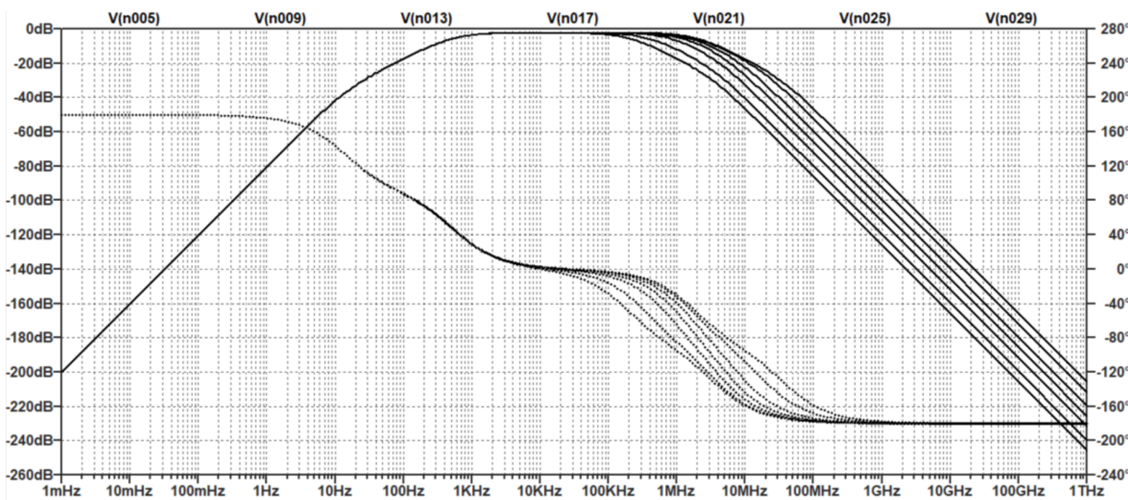

Figure 5. The combined bode plots of all the capacitance values. For both the magnitude and phase plots, the capacitance values, in order from right to left in the graph, are: 100 pF, 200 pF, 500 pF, 1 nF, 2 nF, 5 nF, 10 nF.

ω_{H3dB}	**Capacitor Values** (Frequency values in MHz)						
	100 pF	**200 pF**	**500 pF**	**1 nF**	**2 nF**	**5 nF**	**10 nF**
Simulated	1.8059	1.7012	1.4654	1.1541	0.7375	0.3497	0.1868
Calculated	1.9191	1.8667	1.6855	1.3615	0.8954	0.3937	0.1944
% Error	6.27%	9.73%	15.02%	17.97%	21.41%	12.58%	4.07%

Table 3. A side-by-side comparison between simulated and calculated values of the 3dB frequencies

As seen in table 3, the percentage error is highest when C_4 is at 2 nF.

Part 3

3A.

We have four 1kΩ resistors and two 2kΩ resistors. We want our circuit to have a midband gain of 0.125. Through nodal analysis, we can define three nodes, labeled as V_s, V_1, and V_2 (Figure 6). Using these information, we can show that the gain exhibited by each node-voltage ratio will be $\sqrt[3]{0.125} = 0.5\ or\ \frac{1}{2}$. Thus, from the transfer function in equation 25, we can split our midband gain to equation 26, which is split further into equations 27, 28, and 29.

$$T(s) = \frac{V_o}{V_s} = 0.125 \times \frac{10^5/sec}{s + 10^5/sec} \times \frac{10^6/sec}{s + 10^6/sec} \times \frac{10^7/sec}{s + 10^7/sec} \tag{25}$$

$$A_m = \frac{V_o}{V_2} \times \frac{V_2}{V_1} \times \frac{V_1}{V_s} \tag{26}$$

$$\frac{V_o}{V_2} = \frac{R_6}{R_3 + R_6} = \frac{1}{2} \tag{27}$$

$$\frac{V_2}{V_1} = \frac{R_5 \,||\, (R_3 + R_6)}{R_2 + R_5 \,||\, (R_3 + R_6)} = \frac{1}{2} \tag{28}$$

$$\frac{V_1}{V_s} = \frac{R_4 \,||\, (R_2 + (R_5 \,||\, (R_3 + R_6)))}{R_1 + (R_4 \,||\, (R_2 + (R_5 \,||\, (R_3 + R_6))))} = \frac{1}{2} \tag{29}$$

Solving equations 27, 28, and 29, we get that $R_{1, 2, 3, 6}$ = 1kΩ and $R_{4, 5}$ = 2kΩ. These values were verified through solving equations 30, 31, and 32, giving us the desired midband gain in equation 33.

$$\frac{V_s - V_1}{R_1} = \frac{V_1}{R_4} - \frac{V_1 - V_2}{R_2} \tag{30}$$

$$\frac{V_1 - V_2}{R_2} = \frac{V_2}{R_5} - \frac{V_2 - V_o}{R_3} \tag{31}$$

$$\frac{V_2 - V_o}{R_3} = \frac{V_o}{R_6} \tag{32}$$

$$\frac{V_o}{V_s} = 0.125 \tag{33}$$

To solve for the capacitance values, we use the method of open-circuit and short-circuit time constants. To find the value of C_1, we open both C_2 and C_3 and find the equivalent resistance seen by C_1.

$$R_{eq} = R_1 + (R_4 \,||\, (R_2 + R_5 \,||\, (R_3 + R_6))) = 2000\ \Omega \tag{10}$$

$$C_1 = \frac{1}{\omega_{C1} * R_{eq}} = 5\ nF \tag{10}$$

For C_2, we short C_1 and open C_3.

$$R_{eq} = (R_2 \,||\, (R_5 \,||\, (R_3 + R_6))) = 500\ \Omega \tag{10}$$

$$C_2 = \frac{1}{\omega_{C2} * R_{eq}} = 2\ nF \tag{10}$$

Lastly, for C_3, we short both C_1 and C_2.

$$R_{eq} = (R_3 \,||\, R_6) = 500\ \Omega \tag{10}$$

$$C_3 = \frac{1}{\omega_{C3} * R_{eq}} = 0.2\ nF \tag{10}$$

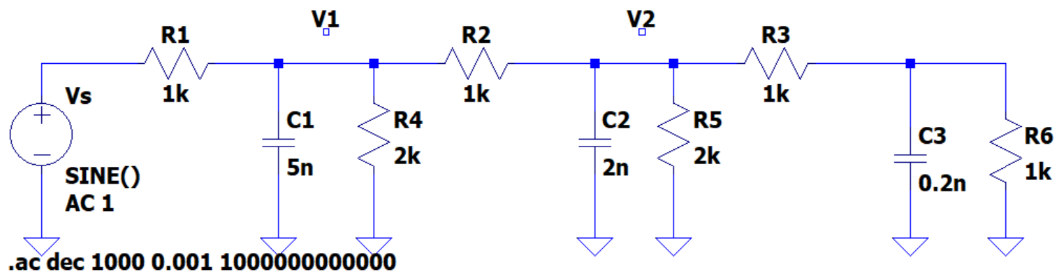

Figure 6. The circuit for Part 3a of the mini-project, including the designed values of the capacitors and resistors. The nodes used to solve the equations for the resistors are V_s, V_1, and V_2 in the figure.

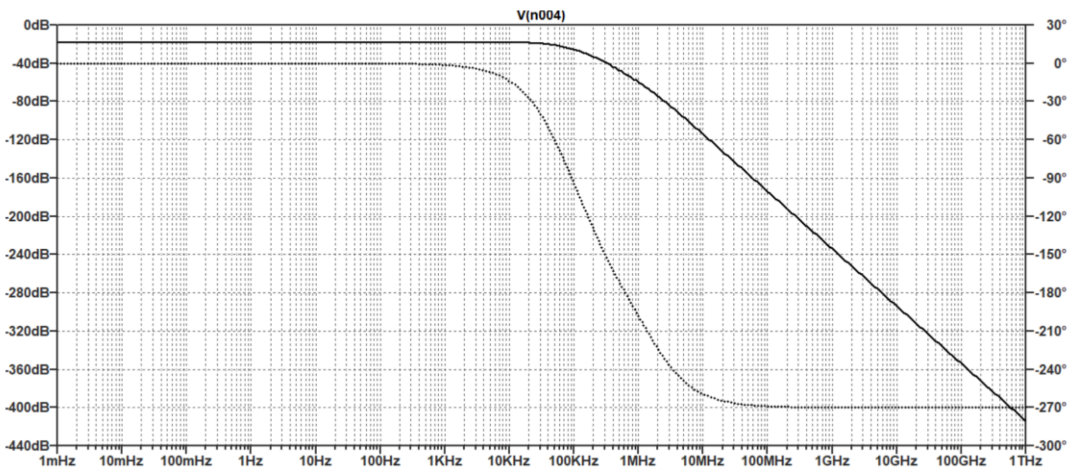

Figure 7. The Bode plots of the circuit designed in Figure 6, including the magnitude and phase plots.

3B.

Using the s-domain function components and the transfer function in equation 25, we simulate the circuit in figure 8 and get the Bode plots in figure 9.

Comparing figures 7 and 9, we can say that the two plots resemble each other, with minor errors potentially caused by significant figures. This shows that transfer functions can be used as a good approximation and a model of a more complex system.

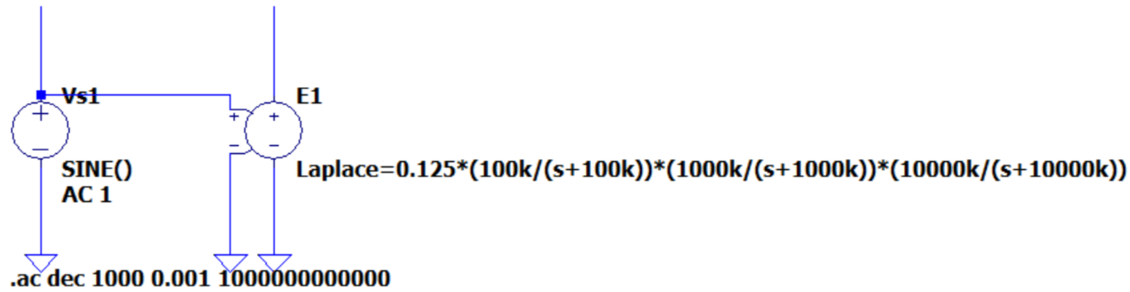

Figure 8. The simulated circuit containing the transfer function in equation 25.

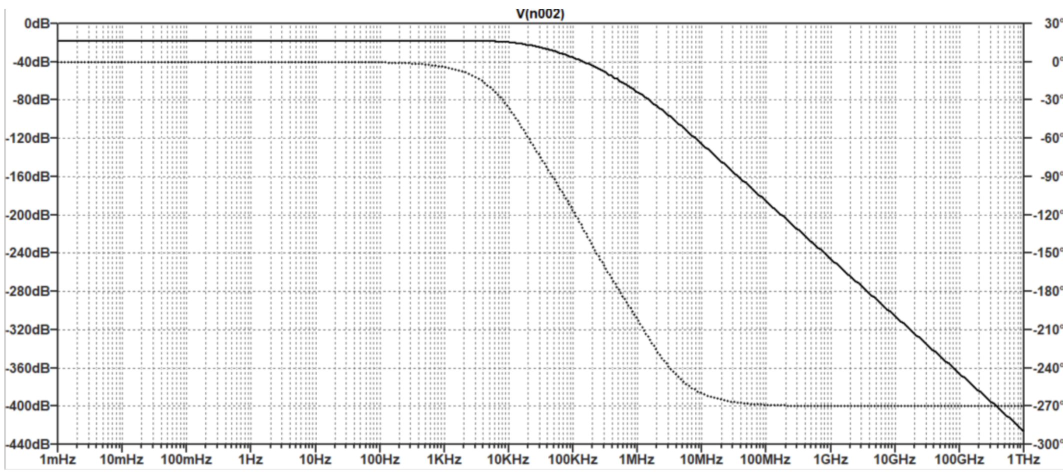

Figure 9. The Bode plots of the circuit designed in Figure 8, including the magnitude and phase plots.

3. Conclusion

In Part 1, we get a higher percent error for lower frequencies compared to higher frequencies. Similar observations can also be seen between the calculated and simulated values of the poles. However, the percent error for the 3dB point is below 10%, which allows us to conclude that the method of open and short-circuit time constants can be a quick way to get an approximation or idea of how a circuit is to behave. This model though may not always be as adequate when we are looking for a more accurate approximations of circuits, as seen in part 2.

In Part 2, a higher percent error was seen when capacitor C_4 was approaching 2 nF in value, with a percent error going beyond 20%, a relatively high number. We also see the behaviour of increasing the value of capacitor C_4, where high frequency poles can be found in increasingly lower frequencies, thus narrowing the bandpass filter.

Lastly, in Part 3, we also used the method of open-circuit and short-circuit time constants to find capacitances of a circuit given a midband gain. We also explored the implications of the use of transfer functions versus circuits in making a simulation. The close resemblance of the Bode plots of both scenarios show the accuracy that transfer functions can give, and explains the popularity of the use of black-box modeling in the field, such as in controls.

4. References

[1] A. Sedra and K. Smith, "Microelectronic Circuits," 5th (or higher) Ed., Oxford University Press, New York.

[2] ELEC 301 Course Notes

[3] LTSpice™ User Manual