# ASN7

Wednesday, March 13, 2024 2:09 PM

1) 215V, 2hp, 8 pole, 145 Hz, Y-connected

$Slip_{FL} = 14\%$ (s)

a)

Synchronous speed: $n_{rps} = \frac{f}{p} = \frac{145}{8/2} = 36.25 \text{ rps}$

$n_{rpm} = 60\, n_{rps} = 2175 \text{ rpm}$

$n_{rad/s} = 2\pi\, n_{rps} = 227.765 \text{ rad/s}$

b) Rotor Speed

$= (1-s)\, n_{rpm} = 1870.5 \text{ rpm}$

c) Frequency

$= s \cdot f = 20.3 \text{ Hz}$

d) Torque $(P = \tau \omega_o)$

$= \frac{P}{\omega_o} = \frac{2hp \times 746}{\text{rotor speed} \times \frac{2\pi}{60}} = 7.617 \text{ N}_m$

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2) 450 V, 45 Hz, 85 $hp_{hp}$, 132.07 A @ 0.8 pf lagging

$P_{cu\_stator} = 9470.32W$ ; $P_{cu\_rotor} = 5682.19W$

$P_{fric} = 2272.88W$; $P_{core} = 1515.25$ ; $P_{stray} \approx 0$

$P_{in} = \sqrt{3}\, V \cdot I \cdot pf = 82.350 \text{ kW}$

$P_{air\_gap} = P_{in} - (P_{cu\_stator} + P_{core}) = 71.365 \text{ kW}$

$P_{conv} = P_{in} - (P_{cu\_stator} + P_{cu\_rotor} + P_{core}) = 65.683 \text{ kW}$ $P_{in} - P_{air\_gap} - P_{cu\_rotor}$

$P_{output} = P_{hp} \times 746 = 63.410 \text{ kW}$

$\eta = \frac{P_{output}}{P_{in}} = 77\%$

3) 350V, 132 hp, 140 Hz, 4 poles, Y connected (ϕ = php, 2 = pairs)

$P_{rot} = 1200\,W$ ; $S = 4\%$

$R_1 = 0.641\,\Omega$, $R_2 = 0.332\,\Omega$, $X_m = 26.3\,\Omega$

$X_1 = 1.106\,\Omega$, $X_2 = 0.464\,\Omega$

## Motor Speed

$$n = 60 \cdot \frac{f}{pairs} = 4200$$

$$n_{rpm} = n(1-s) = 4032 \text{ rpm}$$

$$n_{rps} = \frac{n_{rpm}}{60} = 67.2 \text{ rps}$$

$$n_{rad/s} = n_{rps} \times 2\pi = 422.23 \text{ rad/s}$$

## Stator Current

$$V_s = \frac{V}{\sqrt{3}} = 202.07\,V$$

$$I_s = \frac{V_s}{(R_1 + jX_1 + RR(jX_m, jX_2 + \frac{R_2}{s}))} = 20.627 - j\,9.93\ A$$

$$I_{stator} = |I_s| = 22.89\ A$$

## Power Factor

$$pf = \cos(-ARG(I_s)) = 0.901$$

## Converted Power

$$V_r = V_s - I_s(R_1 + jX_1) = 177.865 - j\,16.447\ V$$

$$I_r = \frac{V_r}{(jX_2 + \frac{R_2}{s})} = 21.252 - j\,3.170\ A$$

$$P_{conv} = 3\left(\frac{1-s}{s}\right) R_2 |I_r|^2 = 11036.697\ W$$

## Output Power

$$P_{out} = P_{conv} - P_{rot} = 9836.7\ W$$

## Efficiency

$$P_{in} = \sqrt{3}\, V (I_{stator}) \times pf = 12504\ W$$

$$\eta = \frac{P_{out}}{P_{in}} = 78.67\%$$

4) 2 pole, 230 Hz, 46kW supply @ 13759 rpm

Slip

$$n_s = 60 \frac{f}{pair} = 13800$$

$$s = \frac{n_s - n_m}{n_s} = 3.116E-3 \text{ p.u.}$$

Induced Torque

$$\tau = pw \times \frac{30}{n_m \times \pi} = 31.93 \text{ Nm}$$

$$\tau_{lbft} = \frac{\tau}{1.356} = 23.548 \text{ lbft}$$

TORQUE DOUBLED

New Speed

$$n = (1 - 2s) \times n_s = 13714 \text{ rpm}$$

Power

$$P = 2\tau \cdot n \cdot \frac{\pi}{30} = 91912.437 \text{ W}$$

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5) 180V, 246 hp, 175 Hz, 18 pole, Y

$R_1 = 0.641\ \Omega$ $R_2 = 0.332\ \Omega$ $X_m = 26.3\ \Omega$

$X_1 = 1.106\ \Omega$ $X_2 = 0.464\ \Omega$

$$Z_{Th} = RR(R_1 + jX_1, jX_m) + jX_2 = 0.58998 + j\,1.5392$$

$$n_s = 60 \frac{f}{p} = 1166.67 \text{ rpm}$$

$$V_{Th} = \frac{\frac{V}{\sqrt{3}} X_m}{\sqrt{(X_m + X_1)^2 + R_1^2}} = 99.7018 \text{ V}$$

$$R_{Th} = RE(Z_{Th})$$

$X_{Th} = Im(Z_{Th})$

$\tau_{max} = \frac{3V_{Th}^2}{\omega_s} \frac{\sqrt{R_{Th}^2 + X_{Th}^2}}{(R_{Th} + \sqrt{R_{Th}^2 + X_{Th}^2})^2 + X_{Th}^2} = 54.5248\ N_m$

$S_{max} = \frac{R_2}{|Z_{Th}|} = 0.2014$

$n_{max} = n_s(1 - S_{max}) = 931.6865\ rpm$

$\omega_s = n_s \times 2\pi \left(\frac{1}{60}\right) = 122.173\ rpm$

$\tau_{start} = \frac{3V_{Th}^2}{\omega_s} \frac{R_2}{(R_{Th} + R_2)^2 + X_{Th}^2} = 25.1743\ N_m$

$S_{max2} = \frac{2R_2}{|Z_{Th}|} = 0.4028$

$n_{max2} = n_s(1 - S_{max2}) = 696.7064\ rpm$

$\tau_{start\,2} = \frac{3V_{Th}^2}{\omega_s} \frac{2R_2}{(R_{Th} + 2R_2)^2 + X_{Th}^2} = 41.1205\ rpm$

6. $R_{Th} = 35$, $X_{Th} = 32$

$R = \sqrt{R_{Th}^2 + X_{Th}^2} = 47.4236\ \Omega$

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7. 400V, 142hp, 215 Hz, 10pole (5pairs)

$S = 0.05$

$R_1 = 0.2\ \Omega$ $R_2 = 0.12\ \Omega$

$X_1 = 0.41\ \Omega$ $X_2 = 0.41\ \Omega$ $X_m = 15\ \Omega$

$P_{core} = 180W$ $P_{FW} = 250W$ $P_{stray} = 0W$

$V_1 = \frac{V}{\sqrt{3}} = 230.9401V$

$Z_r = RR(jX_m, jX_2 + \frac{R_2}{S}) = 2.2201 + j0.7449$

$V_r = \frac{Z_r}{Z_r + (R_1 + jX_1)} V_1 = 200.1882 - j24.4495$

$I_{Stator} = \frac{V_1 - V_r}{R_1 + jX_1} = 79.7255 - j37.0897$

$I_{line} = |I_{stator}| = 86.1215\ A$

$P_{Scu} = 3 I_{line}^2 R_1 = 4450.1453W$

$P_{in} = 3 V_1 Re(I_{Stator}) = 53849.8333\ W$

$P_{gap} = P_{in} - P_{scu} = 49399.6881\ W$ $3(I_{Line})^2 RE(Z_r)$

$I_{rotr} = \frac{V_r}{R_2/S + jX_2} = 79.3555 - j23.7439$

$P_{conv} = 3\left(\frac{1-S}{S}\right) R_2 |I_{rotr}|^2 = 46929.7037\ W$

$\omega_o = 2\pi \frac{f}{p} = 270.1770\ rad/s$

$\tau_{ind} = \frac{P_{gap}}{\omega_o} = 182.8420\ Nm$

$P_{out} = P_{conv} - (P_{core} + P_{FW}) = 46499.7037\ W$

$\frac{f}{\quad}$ $\frac{2\pi}{\quad}$

$P_{out} = P_{conv} - (P_{core} + P_{FW}) = 46499.7037\ W$

$n_s = 60\frac{f}{p} = 2580 \rightarrow \omega_o = n_s \times \frac{2\pi}{60} = 270.1920\ rad/s$

$n_m = n_s(1-s) = 2451$

$\omega_m = \frac{2\pi}{60} n_m = 256.668$

$\tau_{LOAD} = \frac{P_{out}}{\omega_m} = 181.1667\ N_m$

$\eta = \frac{P_{out}}{P_{in}} \times 100\% = 86.3507\%$

Speed: $n_m = 2451$ rpm, $n_{s_{new}} = 40.85$ rps, $\omega_{s_{new}} = n_m \times \frac{2\pi}{60} = 256.6681$ rad/s

$Z_{Th} = RR(R_1 + jX_1, jX_m) + jX_2 = 0.1895 + j0.8116$

$S_{max} = \frac{R_2}{|Z_{Th}|} = 0.1440$ $\qquad |Z_{Th}| = \sqrt{R_{Th}^2 + X_{Th}^2}$

$V_{Th} = \frac{V_1 X_m}{\sqrt{R_1^2 + (X_1 + X_m)^2}} = 224.9768$

$\tau_{max} = \frac{3V_{Th}^2}{\omega_o} \frac{|Z_{Th}|}{(R_{Th} + Z_{Th})^2 + X_{Th}^2} = 277.2443 N_m$

$R_{22} = |Z_{Th}|$

$R_{add} = R_{22} - R_2 = 0.7134\ \Omega$

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8) 460V ($V_s$), 4 poles ($P=2$), 40.67hp ($P_{FW(convert)}$), 60Hz ($f$), Y

$S_{FL} = 0.038$ @ 60Hz, 460V

$R_1 = 0.2\ \Omega$, $X_1 = 0.67\ \Omega$, $X_m = 33\ \Omega$, $X_2 = 0.58\ \Omega$

$Z_{Th} = RR(R_1 + jX_1, jX_m) + jX_2 = 0.1921 + j1.2378\ \Omega$

$V_{Th} = \frac{V_s}{}$

$$Z_{Th} = RR(R_1 + jX_1, jX_m) + jX_2 = 0.1921 + j1.2378\,\Omega$$

$$V_1 = \frac{V_s}{\sqrt{3}} = 265.5811\ V$$

$$V_{Th} = \frac{V_1 X_m}{\sqrt{R_1^2 + (X_1 + X_m)^2}} = 260.2917\ V$$

$$n_s = 60\frac{f}{p} = 1800$$

$$w_0 = 2\pi\frac{f}{p} = 188.4956\ rpm$$

$$w_m = (1-s)\,w_0 = 181.3327\ rpm$$

$$\frac{P_m}{w_m} = \frac{3V_{Th}^2}{w_0}\frac{R_2/s}{(R_{Th} + \frac{R_2}{s})^2 + X_{Th}^2} \rightarrow R_2 = 0.22\,\Omega$$

$$S_{max} = \frac{R_2}{|Z_{Th}|} = 0.1756$$

$$T_{max} = \frac{3V_{Th}^2}{w_0}\frac{\frac{R_2}{S_{max}}}{(R_{Th} + \frac{R_2}{S_{max}})^2 + X_{Th}^2} = 373.1821\ N_m$$

$$w_{T_{max}} = (1 - S_{max})\,w_0 = 155.3900$$

$$n_{T_{max}} = \frac{60}{2\pi}w_{T_{max}} = 1483.8648\ rpm$$

$$T_{START} = \frac{3V_{Th}^2}{w_0}\frac{R_2'}{(R_{Th} + R_2)^2 + X_{Th}^2} = 139.3805\ N_m$$

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9) 460V, 4 pole, 66.9251 hp, 60 Hz

$R_{line} = 0.5$

$X_{line} = 0.3$

$n = 6$

$S = 0.038$

$R_1 = 0.22\,\Omega$, $X_1 = 0.46\,\Omega$, $X_m = 39\,\Omega$, $X_2' = 0.64\,\Omega$

$$n_s = \frac{60f}{p} = 1800 \qquad V_1 = \frac{V_s}{\sqrt{3}} = 265.5811\ V$$

$$w_s = n_s\frac{2\pi}{60} = 188.4956$$

$$n_m = (1-s)\,n_s = 1731.6$$

$n_m = (1-s)n_s = 1731.6$

$\omega_m = n_m \frac{2\pi}{60} = 181.3327$

$Z_{Th} = RR(R_1 + jX_1, jX_m) + jX_2 = 0.2149 + j\,1.0958$

$V_{Th} = \frac{V_1 X_m}{\sqrt{R_1^2 + (X_1 + X_m)^2}} = 262.4811$

$\text{Solve}\left(\frac{P_w}{\omega_m} = \frac{3V_{Th}^2}{\omega_s} \frac{\frac{R_2}{s}}{(R_{Th} + \frac{R_2}{s})^2 + X_{Th}^2}, R_2\right)$

$\rightarrow R_2 = 0.12$

$Z_r = RR(R_2 + jX_2, jX_m) = 0.1162 + j\,0.6300$

$V_d = \frac{V_1 Z_r}{Z_r + R_1 + jX_1} = 148.1415 + j\,17.3849$

$I_{line} = \frac{V_1 - V_d}{R_1 + jX_1} = 68.6140 - j\,222.4881$

$|I_{line}| = 232.8279\ A$

$Z_{line} = R_{line} + jX_{line}$

$Z_{eq\,line} = Z_r + R_1 + jX_1 + Z_{line}$

$I_{line2} = \frac{V_1}{|Z_{eq\,line}|} = 163.7236\ A$

$I_{line2} = \frac{V_1}{Z_{eq\,line}} = 84.3942 - j\,140.7963\ A$

$V_t = V_1 - I_{line2} Z_{line} = 181.2951 + j\,44.8299$

$V_{term1} = |V_t|\sqrt{3} = 323.4702\ V$

$V_{1\,new} = nV_1 = 1593.4867$

$Z_{eqn} = \frac{Z_r + R_1 + jX_1}{(\frac{1}{n})^2} = 12.1016 + j\,39.2407$

$Z_{eq\,line\,n} = Z_{eqn} + Z_{line} = 12.6016 + j\,39.5407$

$I_{line\,n} = \frac{V_{1\,new}}{|Z_{eq\,line\,n}|} = 38.3971 A$

$I_{line\,n} = \frac{V_{1\,new}}{Z_{eq\,line\,n}} = 11.6593 - j\,36.5841$

$V_{prim} = V_{1\,new} - Z_{line} I_{line\,n}$

$V_{sec} = V_{prim}(\frac{1}{n})$

$V_{sec2} = \sqrt{3}\, V_{sec}$

$V_{motor} = V_{sec2}(\frac{1}{n})$

$|V_{motor}| = 75.8615\ V$

10) 300V, 60Hz, 8 poles, $n_m$ 850 rpm

$I_{start} = 5.25\, I_{FL}$ ← ratio

$I_R = \frac{s E_2}{\sqrt{R_2^2 + (sX_2)^2}}$ Start: $s=1$

$\rightarrow \frac{E_2}{\sqrt{R_2^2 + X_2^2}} = \frac{5.25 E_2}{\sqrt{R_2^2 + (sX_2)^2}}$

$n_s = 60\frac{f}{p} = 900$

$s = \frac{n_s - n_m}{n_s} = 5.5556\,E{-}2$

Solve $\left(\frac{1}{\sqrt{R_2^2 + X_2^2}} = \frac{5.2s}{\sqrt{R_2^2 + (sX_2)^2}}, R_2\right)$

$\rightarrow \frac{R_2}{X_2} = s_{max} = 0.2993$

$s_{max} = \frac{n_s - n_{m\,min}}{n_s} \rightarrow n_{m\,min} = 630.5920$ rpm

$$\tau = \tau_{max} \frac{2 s_{max} s}{s_{max}^2 + s^2}$$

$\frac{\tau_{max}}{\tau_{FL}} = \frac{s_{max}^2 + s^2}{2 s_{max} s} = 2.7869 = 278.69\%$

$$\frac{\tau_{start}}{\tau_{FL}} = \frac{s_{max}^2 + s^2}{s(s_{max}^2+1)} = 1.5313 = 153.13\%$$

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11) 300V, 75Hz, 16 pole, 166Nm @ 482.5 rpm
> 382.5 rpm, $R_w = 0.1\Omega$

($V_s$, $f$, $p=8$, $n_{m1}$, $n_{m2}$)

$$\frac{S_1 V_1^2}{R_1} = \frac{S_2 V_2^2}{R_2} \rightarrow \frac{S_1}{R_1} = \frac{S_2}{R_2}$$

$$R_2 = \frac{S_2}{S_1} R_1$$

$$n_s = 60\frac{f}{p} = 562.5 \text{ rpm}$$

$$S_1 = \frac{n_s - n_{m1}}{n_s} = 0.1422$$

$$S_2 = \frac{n_s - n_{m2}}{n_s} = 0.32$$

$$R_2 = 0.225\Omega$$

$$R_{add} = R_2 - R_w = 0.125\Omega$$

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12) $\tau_{START} = 1.01959$ p.u.

$\tau_{MAX} = 1.75625$ p.u.

$\tau_{FL} = 1$ p.u.

$$S_{max} = \frac{\tau_{max}}{\tau_{START}}\left(1 - \sqrt{1 - \left(\frac{\tau_{START}}{\tau_{max}}\right)^2}\right) = 0.3200$$

$$\text{Solve}\left(\tau_{FL} = \tau_{MAX}\frac{2 s_{max} S_{FL}}{s_{max}^2 + S_{FL}^2}, S_{FL}\right) \rightarrow S_{FL} = 0.999996$$

$$I_{START} = \sqrt{\frac{\tau_{START}}{S_{FL}}} = 3.1931 \text{ p.u.}$$

$$I_{MAXT} = \sqrt{\tau_{max}\frac{S_{max}}{S_{FL}}} = 2.3707 \text{ p.u.}$$