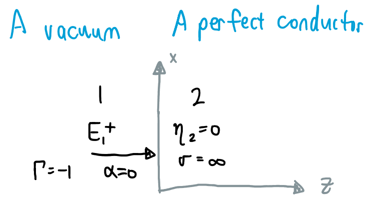
2.4 Reflection at Normal Incidence

Thursday, October 19, 2023 8:23 PM

Relating EM Wave Reflections with TL Equivalents

Transmission Line	EM Waves
$V_s = V_0^+ e^{-jkz}$ (β)	$E^+_{xs1} = E^+_{x10} e^{-jk_1 z}$ ($\alpha + j\beta$)
$I_s = \frac{V_0^+}{Z_0} e^{-jkz}$	$H^+_{ys1} = \frac{E^+_{x10}}{\eta_1} e^{-jk_1 z}$
$Z_0 = \sqrt{\frac{L}{C}}$ $\quad v = \frac{1}{\sqrt{LC}}$	$\eta_1 = \sqrt{\frac{\mu_1}{\varepsilon_{eff}}}$ $\quad v = \frac{\omega}{\beta}$
$\Gamma = \frac{Z_L - Z_0}{Z_L + Z_0}$	$\Gamma = \frac{\eta_2 - \eta_1}{\eta_2 + \eta_1}$
VSWR	SWR



$$E^+_{xs1} = E^+_{x10} e^{-j\beta z}$$

$$E^-_{xs1} = (-1) E^+_{x10} e^{+j\beta z}$$

$$E_{xs1} = E^+_{x10}\left(e^{-j\beta z} - e^{+j\beta z}\right)$$

$$= E^+_{x10}(-2j)\sin(\beta z)$$

$$\mathcal{E}_{x1}(z,t) = \mathrm{Re}\left(e^{j\omega t} E^+_{x10}(-2j)\sin(\beta z)\right)$$

$$= 2E^+_{x10}\sin(\beta z)\sin(\omega t)$$

Standing wave

Q1 $f(z,t) = \sin(\omega t - \beta z)$ Translational

Q2 $f(z,t) = \cos(\omega t)\cos(\beta z)$ Standing

Q3 $f(z,t) = \sin(\omega t) + \sin(\beta z)$ Standing

Q4 In the system described above, the normally incident EM wave, vibrating tangentially to the boundary, reaches the boundary between a vacuum and a perfect conductor and reflects. $\Gamma = -1$ which means

> The reflected E field has the opposite polarity to the incident E field

The reflected E field has the same polarity as the incident E field

There is no reflected E field as the E field is fully transmitted into the conductive medium

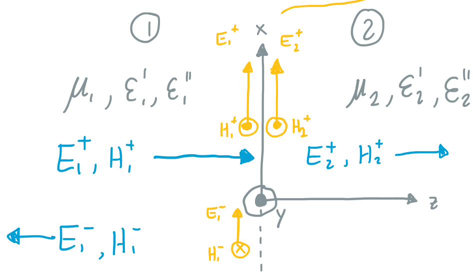
Q5 Continuing from Q4, the reflected E field takes on the behavior of the correct answer because:

▶ The perfect conductor acts like a short circuit, such that the reflected E field must take on the opposite polarity to cancel out the incident E field at the boundary

- The perfect conductor acts like a short circuit, such that the reflected E field must take on the opposite polarity to cancel out the incident E field at the boundary
- The perfect conductor acts like an open circuit, such that the reflected E field must take on the same polarity as the incident E field
- The perfect conductor is so favorable to E fields that all power in the incident wave are transmitted into the perfect conductor
- Regardless of the conductivity of the 2nd material, the reflected E field must take on the opposite polarity

Normal Incidence Reflection

E field in same direction by boundary condition (see in-class 10/15)



$$\varepsilon_{eff} = \varepsilon - j\frac{\sigma}{\omega}$$
$$= \varepsilon' - j\varepsilon''$$
$$\eta_1 = \sqrt{\frac{\mu_1}{\varepsilon_1' - j\varepsilon_1''}}$$ and the same for 2

phasor form

x component, material 1

We know this

E_{xs1}^+	H_{ys1}^+	Incident
E_{xs1}^-	H_{ys1}^-	Reflected
E_{xs2}^+	H_{ys2}^+	Transmitted

amplitudes = ?

$\boxed{\text{Math}}$

$$E_{xs1}^+ = E_{x10}^+ e^{-jk_1 z} \qquad K_1 = \omega\sqrt{\mu_1 \varepsilon_{1\,eff}}$$
$$E_{xs1}^- = E_{x10}^- e^{+jk_1 z} \qquad = \omega\sqrt{\mu_1(\varepsilon_1' - j\varepsilon_1'')}$$
$$E_{xs2}^+ = E_{x20}^+ e^{-jk_2 z}$$

2 unknowns, need 2 conditions → these give us reflection

$$H_{ys1}^+ = \frac{E_{x10}^+}{\eta_1} e^{-jk_1 z} \qquad \eta_1 = \sqrt{\frac{\mu_1}{\varepsilon_1' - j\varepsilon_1''}}$$
$$H_{ys1}^- = -\frac{E_{x10}^-}{\eta_1} e^{+jk_1 z}$$
$$H_{ys2}^+ = \frac{E_{x20}^+}{\eta_2} e^{-jk_2 z}$$

Q The reflected H field in the normal incidence system described above is expressed as $H_{ys1}^- = -\frac{E_{x10}^-}{\eta_1} e^{jk_1 z}$.

Q The reflected H field in the normal incidence system described above is expressed as $H_{ys1}^{-} = -\frac{E_{x10}}{\eta_1} e^{jk_1 z}$.

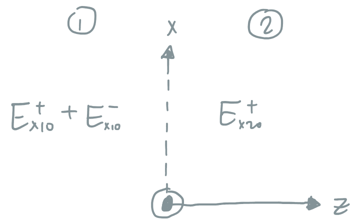
Why is there a negative sign?

> The direction of travel is now opposite, the direction of the reflected H field also has to be flipped to satisfy $\vec{S} = \vec{E} \times \vec{H}$.

Due to specific choice of material properties

A source at the boundary drives the reflected H in the opposite polarity

- How do we calculate E_{x10}^- and E_{x20}^+? (We know E_{x10}^+)



$$E_{x10}^+ + E_{x10}^- = E_{x20}^+ \quad \left(\text{tangential } E\text{'s are continuous at } z=0\right)$$

$$\frac{E_{x10}^+}{\eta_1} - \frac{E_{x10}^-}{\eta_1} = \frac{E_{x20}^+}{\eta_2} \quad \left(\text{tangential } H\text{'s are continuous at } z=0\right)$$

$$E_{x10}^+ + E_{x10}^- = \eta_2\left(\frac{E_{x10}^+}{\eta_1} - \frac{E_{x10}^-}{\eta_1}\right)$$

$$E_{x10}^-\left(1 + \frac{\eta_2}{\eta_1}\right) = E_{x10}^+\left(\frac{\eta_2}{\eta_1} - 1\right)$$

$$\eta_1 E_{x10}^-\left(1 + \frac{\eta_2}{\eta_1}\right) = \eta_1 E_{x10}^+\left(\frac{\eta_2}{\eta_1} - 1\right)$$

$$E_{x10}^-(\eta_1 + \eta_2) = E_{x10}^+(\eta_2 - \eta_1)$$

reflection coefficient!

$$E_{x10}^- = E_{x10}^+\left(\frac{\eta_2 - \eta_1}{\eta_1 + \eta_2}\right) = E_{x10}^+ \Gamma$$

transmission coefficient

$$\tau = 1 + \Gamma = \frac{2\eta_2}{\eta_1 + \eta_2}$$

Consider an air-copper interface with an EM wave incident normally from the air. The copper has a conductivity of $\sigma = 5.96 \times 10^7$ S/m and permeability of μ_0.

Q1 What is the intrinsic impedance of the copper material, η_{copper}, for a 1 GHz wave?

Q2 What is the reflection coefficient of the boundary?

$$\varepsilon\sigma + j\frac{\sigma}{\omega}$$

$$\eta_{copper} = \sqrt{\frac{\mu_{copper}}{\varepsilon_{eff}}} = \sqrt{\frac{\omega\mu}{2\sigma}} + j\sqrt{\frac{\omega\mu}{2\sigma}} = \sqrt{\frac{(2\pi \cdot 1\times10^9)(4\pi\times10^{-7})}{2(5.96\times10^7)}} + j\sqrt{\frac{(2\pi \cdot 1\times10^9)(4\pi\times10^{-7})}{2(5.96\times10^7)}}$$

$$\Gamma = \frac{\eta_2 - \eta_1}{\eta_2 + \eta_1} = \frac{\eta_{copper} - \eta_{air}}{\eta_{copper} + \eta_{air}}$$

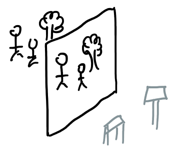
$$= \sqrt{\frac{(2\pi \cdot 1\times10^{9})(4\pi\times10^{-7})}{2(5.96\times10^{7})}} + j\sqrt{\frac{(2\pi \cdot 1\times10^{9})(4\pi\times10^{-7})}{2(5.96\times10^{7})}}$$

$$= 8.139E{-3} + j\,8.139E{-3}$$

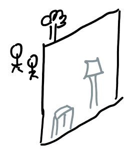
$$= \underline{0.0115 \angle 45^\circ}$$

$$= \underline{-0.9996}$$

In-Class 10/25/23



Day

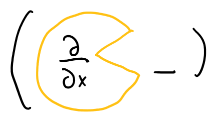


Night

> At night, there's no interference. Physics says the window is not fully transparent. The difference is in material properties, so the waves traveling in one material hits the boundary. The two materials have a difference in impedance — resulting in reflection and refraction

> In optics, glass is an insulator

Prof's PACMAN analogy $\vec{\nabla} = \left(\frac{\partial}{\partial x}, \frac{\partial}{\partial y}, \frac{\partial}{\partial z}\right)$



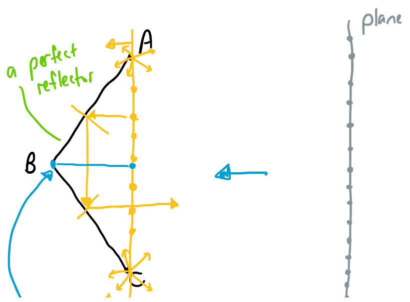
> Need boundary conditions for Maxwell's equations

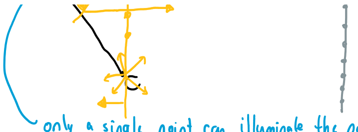
> If you have two materials together, we think of this as "welded" although it's more complex in reality

> The boundary conditions used for this class:

tangential component of $\vec{E}$ has to be continuous across the boundary

" " " $\vec{H}$ " " " " " " "





only a single point can illuminate the point. But point has 0 energy. So we can barely illuminate the corner

> We have enough to do this, but not the edges (A, C)

> Gauss' Law is based on Coulomb's law

> If you sprinkle charges on perfect conductor, they'll disappear

> $\vec{E} = 0$

Quiz

An incident wave reaching the boundary between free space and a perfect conductor would be perfectly reflected

Expressions for reflection & transmission coefficients for normally incident EM waves on an interface b/w 2 different media :

> continuity of tangential components of $\vec{E}$ and $\vec{H}$

3) E, F eliminated — standing wave

$$\vec{E} = E^{+}_{x10}\left(e^{-j\beta z} - e^{j\beta z}\right) \longrightarrow -2j \sin \beta z$$

$$\sin \beta z = \frac{e^{j\beta z} - e^{-j\beta z}}{2j}$$

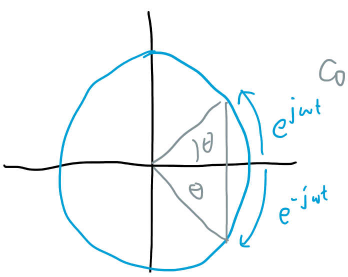
$$= \left(-2j \sin \beta z \; e^{j\omega t}\right)$$

$$-\sin \beta z = \frac{e^{-j\beta z} - e^{j\beta z}}{2j}$$

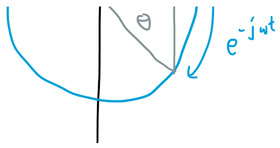
$$\downarrow$$

$$(\cos \omega t + j \sin \omega t)$$

$$Re(\;) = D$$



$$\cos \theta = \frac{e^{j\omega t} + e^{-j\omega t}}{2}$$


θ
e^{-jωt}