Quality_Control

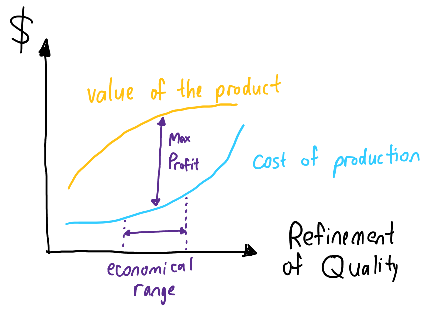
Tuesday, June 11, 2024 5:42 PM

☆ Manufacturers can afford the personnel, equipment, and costs to produce <u>good quality</u>.
(Reputation is in danger)

☆ Customers lack motivation & equipment to check everything they buy. Instead, they rely on "<u>Reputation</u>" and word of mouth

Quality Control
- → Inspection: Checking quality of what has been produced
- → Statistical Quality Control (SQC, or simply, QC)
  - > What we are producing in the future

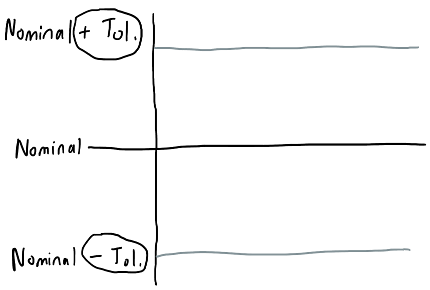
Economics of Quality Control



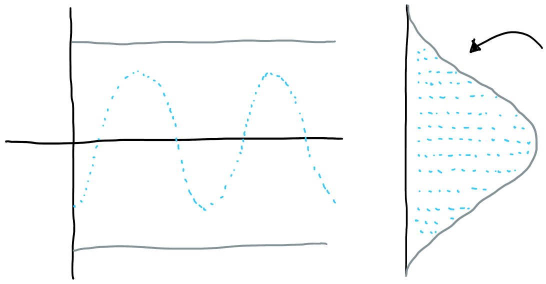
☆ It is not necessary to check the quality of every item produced

☆ There are sampling techniques which work adequately with amazing accuracy

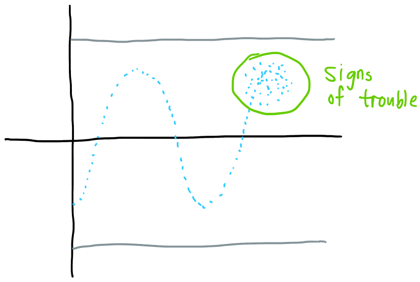
- Recognizing Change and Patterns

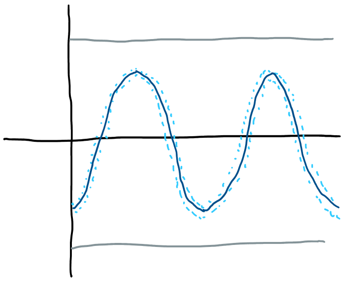


For now, we use "± Tol", but later we change it for practical purposes

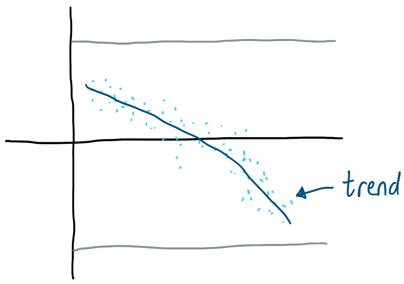


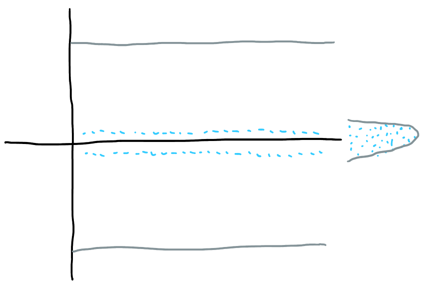
In the long run, all data have distributions like this (Normal)



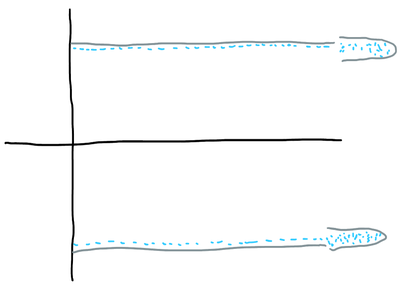


Periodic variation due to
> Shift changes
> Different machine operators
> etc.

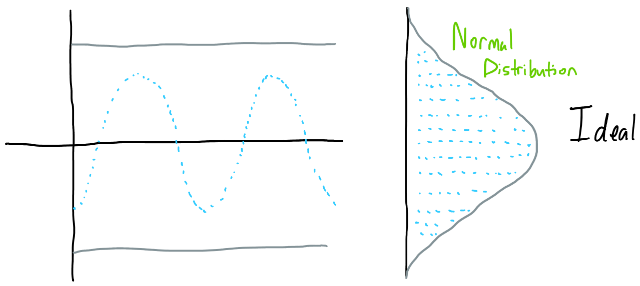




Too good! operator may be spending too much time to produce perfect items

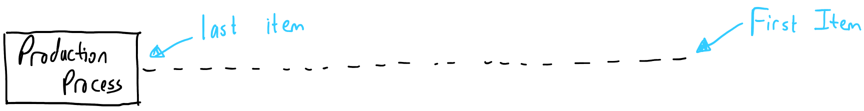


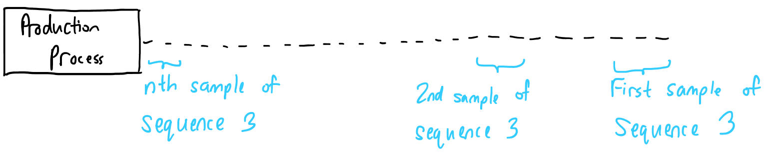
Too near the limits!



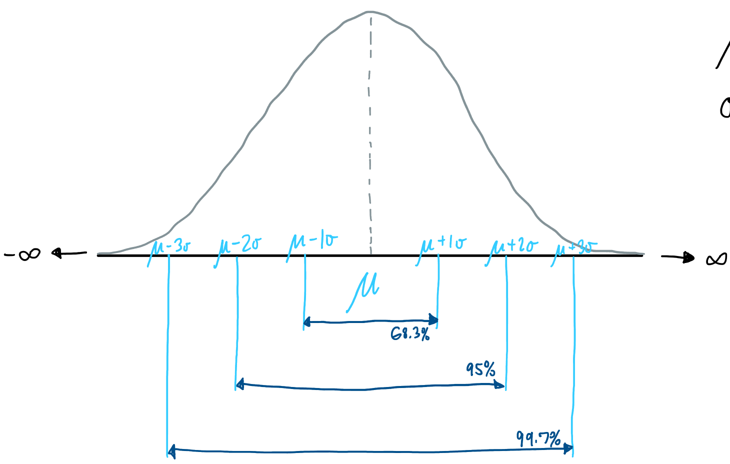
- If "one" characteristic is influenced by several external factors, its distribution turns out to be normal
  - e.g. food, gene, water, environment, etc.

☆ Because there are some sampling techniques that work amazingly well, we don't have to check every item that comes out of the production line.





Properties of Normal Distribution



μ = mean

σ = standard deviation

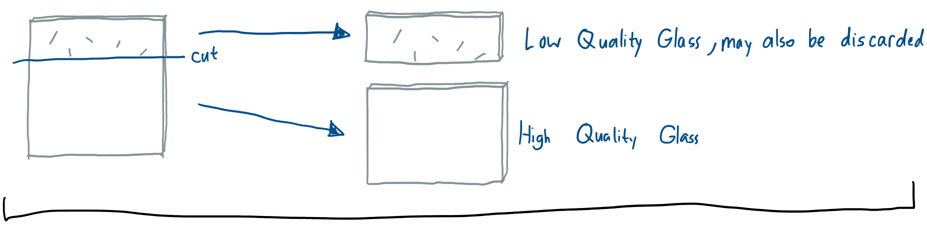
$$f(x) = \frac{1}{\sqrt{2\pi}} e^{\frac{-(x-\mu)^2}{2\sigma^2}} \; ; \quad \int_{-\infty}^{+\infty} f(x)\,dx = 1 = 100\%$$

• With 3-sigma limit [i.e. $\mu \pm 3\sigma$], 99.7% of data fall within $\mu \pm 3\sigma$

• But in practice, such as the diameter of a shaft, it does not extend to $-\infty$ or $+\infty$! (In fact, never becomes a diametric negative!)

• So for practical purposes the normal distribution is used and acceptable (because 99.7% of the data is confined within 3-sigma limits)

SQC
- Variables : For such variables like diameter, temperature, anything that gives measurable quantity
- Attributes
  - Defectives: A diode or two in a batch of 100 diodes
  - Defect: # of scratches here and there on sheet of glass, otherwise useable.



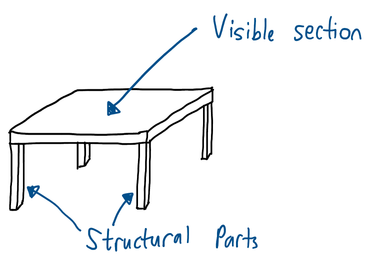
Quality Control for Defects (not Defectives!)

> could be used for "grading" (scoring)

• With wood (timber, or panels)

High - Quality Grade → Used for visible sections

Low - Quality Grade → Used as structural parts and not visible sections



• Average (mean) is not good enough to describe a distribution

• Some measure of spread is also required

- Some measure of spread is also required
- For example Normal distribution is explained by its mean (μ) and standard deviation

$$f(x) = \frac{1}{\sigma\sqrt{2\pi}} \exp\left[\frac{-(x-\mu)^2}{2\sigma^2}\right] = f(\mu, \sigma)$$

Sampling Techniques

- For example: Sample size $n = 5$ Sample average $\bar{X} = \frac{\sum_{i=1}^{5} X_i}{n}$; $\sigma = \sqrt{\frac{\sum_{i=1}^{5}(X_i-\mu)^2}{n}}$

- Average (mean) is easy to calculate
- Standard deviation is not so easy!
- But instead an alternative measure is used in SQC, called "Range"

$$R = X_{max} - X_{min} \geq 0$$

- Range is shown to have great utility in SQC as it relates to σ and σ can be determined through Range (R)

Sample Number	Measurements	Range
1	$X_1, X_2, X_3 = \bar{X}_1$	$X_{max} - X_{min} = R_1$
2	$X_1, X_2, X_3 = \bar{X}_2$	$X_{max} - X_{min} = R_2$
3	$X_1, X_2, X_3 = \bar{X}_3$	$X_{max} - X_{min} = R_3$
⋮		
N (25)	$X_1, X_2, X_3 = \bar{X}_N$	$X_{max} - X_{min} = R_N$
	$\bar{\bar{X}} = \frac{\sum \bar{X}}{25}$	$\bar{R} = \frac{\sum R_i}{N}$

Sample size for example: $n = 3$

$$\bar{\bar{X}} = \frac{\sum_{i=1}^{N} \bar{X}_i}{N} = \text{taken as} = \mu$$

$$\bar{R} = \frac{\sum_{i=1}^{N} R_i}{N}$$

- It has been shown that

$$\bar{R} = d_2\sigma \quad ; \quad \sigma_R = d_3\sigma$$

standard deviation of sample range

> d_2 and d_3 depend on sample size n, and determined through standard statistical table

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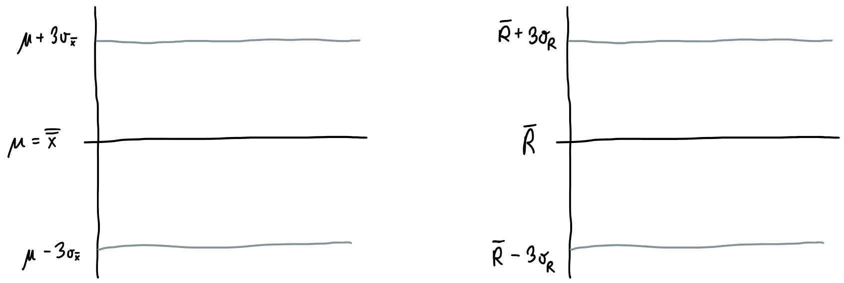
- We decide on a sample size, n



> Set up these two charts next to the production line. We take periodic samples & plot the findings on the charts

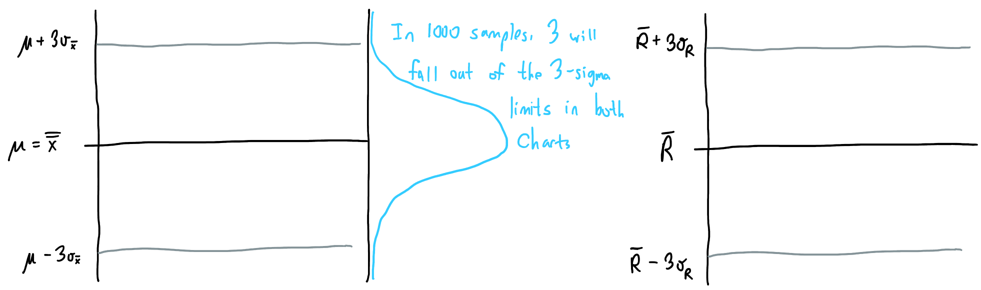
We have to determine $\bar{\bar{X}}$, $\sigma_{\bar{x}}$, $\bar{R}$, and $\sigma_{\bar{R}}$ → Standard deviation of sample range

$\bar{\bar{X}}$ → Average of the main population or average of sample average shown by μ

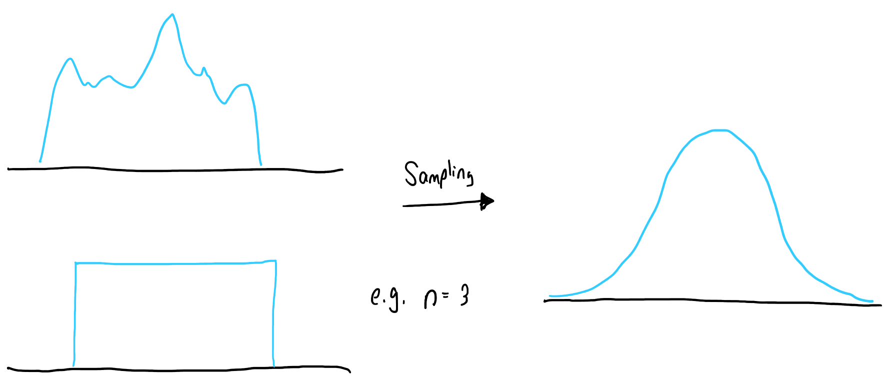
$\sigma_{\bar{x}}$ → Standard deviation of sample average

$\bar{R}$ → Average Range

We have to setup two charts:



$$\frac{X_1 + X_2 + X_3}{n} = X_i$$



* So we have to use Sample average (sample of e.g. 5)

$$\sigma = \sqrt{\frac{\sum_{i=1}^{5}(x_i - \mu)}{n}} \geq 0$$

$$R = X_{max} - X_{min}$$

$$\left\{ \begin{array}{l} \bar{R} = d_2 \sigma \\ \sigma_{\bar{R}} = d_3 \sigma \end{array} \right\}$$

d_2 and d_3 depend on Sample size n (table to be given)

* We also need $\sigma_{\bar{X}}$

$$\sigma = \frac{\bar{R}}{d_2} \quad \ldots \quad \sigma_{\bar{x}} = \frac{\sigma}{\sqrt{n}}$$

$$R = d_2 \sigma \qquad \sigma_{\bar{R}} = d_3 \sigma$$

The Aimed-At Values Method (p.83)

① Main population is normally distributed

$\boxed{\text{Production Line}}$ . . . .

② Tolerances must be known

③ An acceptable percentage defective also must be specified, e.g. 3%

① The main population may or may not be normally distributed

> Two population may have different properties and act similar

$$\begin{pmatrix} \mu_1 , & \sigma_1 \\ \mu_2 , & \sigma_2 \end{pmatrix}$$

The Estimated-Values Method

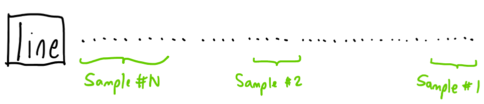
- Factors of Production:
  - Good machinery
  - Skilled operation

- Good machinery
- Skilled operation
- Good Tools
- Good raw materials
- ...

Typically 5

Sample #	Sample size	Sample Average	Sample Range
1	5	$(x_1+x_2+x_3+x_4+x_5)/5$	$X_{max} - X_{min}$
2	5	$(X_1+X_2+X_3+X_4+X_5)/5$	$X_{max} - X_{min}$
⋮	⋮	⋮	⋮
N		$\bar{\bar{X}} = \mu = \frac{\sum^{25} \bar{x}}{25}$	$\bar{R} = \frac{\sum R_i}{25}$

Typically 25



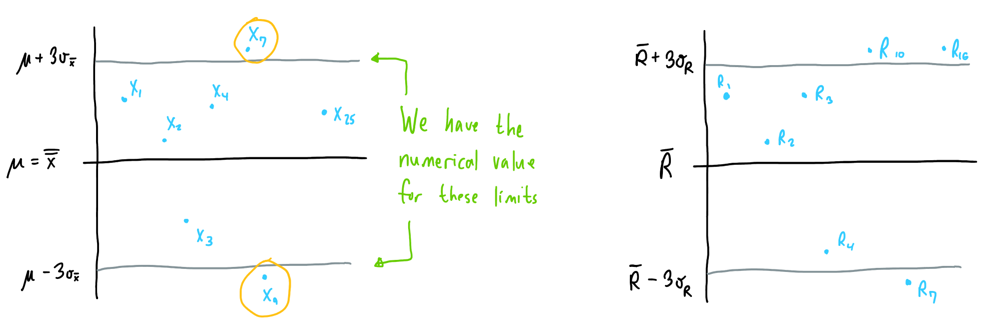
- $\mu, \bar{R}$ are calculated

- From the Relation $\bar{R} = d_2 \sigma$ we calculate σ

$$\sigma = \frac{\bar{R}}{d_2} \quad \& \quad \sigma_{\bar{R}} = d_3 \sigma \quad \& \quad \sigma_{\bar{x}} = \frac{\sigma}{\sqrt{n}}$$

$\rightarrow \mu, \sigma_{\bar{x}}, \bar{R} \; \& \; \sigma_{\bar{R}}$

- To verify credibility & stability of data we plot the data on the charts we have established



- In 1000 samples we expect 3 samples to fall outside the 3-sigma limits. For 25 samples we do not expect any data to fall outside limits!

- We recalculate the μ as follows

$$\mu_{new} = \frac{\sum_{i=1}^{25} \bar{X}_i - \bar{X}_7 - \bar{X}_9}{23}$$

- Recalculate the new $\bar{R}$

$$\bar{R}_{new} = \frac{\sum_{i=1}^{25} R_i - R_7 - R_{10} - R_{16}}{22}$$

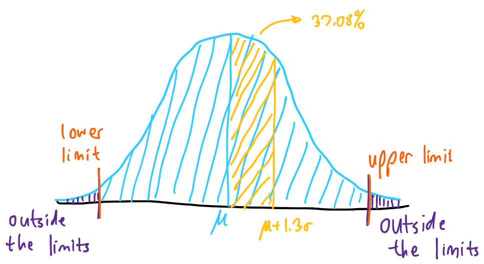
- Recalculate σ

$$\sigma_{new} = \frac{\bar{R}_{new}}{d_2}$$ — same as before

$$\sigma_{\bar{x}, new} = \frac{\sigma_{new}}{\sqrt{n}}$$

$$\sigma_{\bar{R}, new} = d_3 \, \sigma_{new}$$

- We want to know between UL & LL what percentage of data falls inside the limits



$$UL = \mu + Z_1 \sigma$$

$$LL = \mu - Z_2 \sigma$$

e.g. $Z = 1.3 \rightarrow 0.3708 = 37.08\%$

- For example we know that 99.7% of data is confined between $\pm$ 3-sigma limit

countable number of scratches (or defect)

$$P_i = \frac{r}{n}$$

$$np = r = C$$ ← count

$$\sigma_{np} = \sqrt{np(1-p)} = \sqrt{np(1-0)} = \sqrt{np}$$

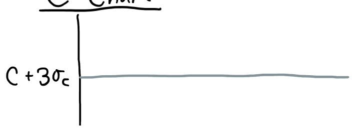
approaches

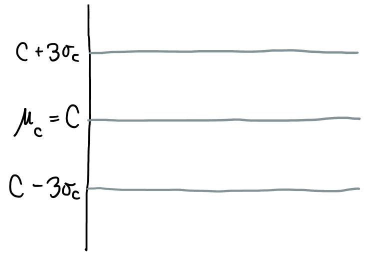
As $n \rightarrow \infty$

$P_i \rightarrow 0$

We call $\sigma_c = \sqrt{C}$

C-Chart





Conclusion to c-chart

- The underlying distribution that govern c-chart is "poisson distribution", a special case of Binomial distribution

- Whereas Binomial distribution has a "closed-interval", Poisson distribution extends on one side to infinity.