Exam of MATHEMATICAL METHODS B
February 4, 2020

TTPU

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Exercise 1. Consider the complex variable function

$$f(z) = z^2 + z + 1 - i$$

(a) Find the set of all $z \in \Omega$ where $f(z) = 0$.

(b) Using the Cauchy-Riemann conditions, find the set of all $z \in \Omega$ where f is holomorphic.

(c) Compute $f'(z)$ whenever it exists.

Exercise 2. Consider the curve

$$\gamma\colon [0, 2\pi] \to \mathbb{C}, \quad \gamma(t) = -1 + 2e^{it}$$

(a) Check that γ is closed and that the trace of γ is $\partial\Omega$ for some regular set $\Omega \subset \mathbb{C}$.

(b) Decide how γ is oriented (counterclockwise or clockwise).

(c) Find $k \in \mathbb{N}$ such that the trace of γ is traversed k times.

(d) Say if γ is a Jordan curve.

(e) Compute

$$\int_\gamma \frac{2e^z}{z^2 - 4}\, dz.$$

Exercise 3. Let

$$f(z) = \frac{1}{z^2 - iz^3}, \quad z_0 = 0, \quad \Omega = B_2(0)$$

(a) Find the Laurent expansion of f on an appropriate $B_r(z_0) \setminus \{z_0\}$ putting in evidence the principal part of f at z_0.

(b) Classify the isolated singularity z_0 and compute $\operatorname{Res}(f, z_0)$.

(c) Compute

$$\int_{\partial\Omega} \frac{1}{z^2 - iz^3} + \frac{\cos z}{(z-1)^2}\, dz.$$

Exercise 4. Let $u \colon \mathbb{R} \to \mathbb{C}$ be given by

$$u(x) = (x^2 + 3x)\mathbf{1}_{(-\infty,0)}(x)$$

(a) Show that T_u is a tempered distribution.

(b) Compute T_u'.

(c) Is T_u' a regular distribution? Justify your answer.

Exercise 5. Let

$$u(t) = H(t+1)e^{-2t}, \quad v(t) = \cos t H(2t), \quad F(s) = \frac{3s}{s+1}.$$

(a) Compute the Fourier transform of u.

(b) Compute the Laplace transform of v.

(c) Compute the inverse Laplace transform of F.