TCSS543A – Autumn 2020
Homework 3 (30 points)

This is a group assignment. Each group may consist of 1, 2, or 3 students. In other words, you can choose to work by yourself, or with one colleague, or with two. *However, each student must submit their own copy of this homework to Canvas, and you are responsible for any interpersonal issues that might arise in the group.*

Details: In this assignment, you will implement three different algorithmic techniques in **Java**.

Your homework should be electronically submitted via Canvas by midnight on the due date. You are expected to submit the following files:

- Report. The submitted report MUST be typeset using any common software and submitted as a PDF. We recommend using LaTeX to prepare your solution (you could use any LaTeX tools such as Overleaf, TexShop, TeXnicCenter, TeXWorks, etc). Describe in it what you did, what is working fine and what might be improved, and the data required as specified below.
- Java source code. The main class must be called `TCSS543.java` and your program must run a standalone program that prints to the standard output, to be invoked from the command line as:

  ```
  java TCSS543 > output.txt
  ```

- The output file(s) obtained as the result of the above command, containing the data you document in your report.

Remember to cite *all* sources you use other than the text, course material or your notes.

This project involves implementing Brélaz's Dsatur heuristic algorithm to obtain an approximate solution to the vertex coloring problem on graphs, which is known to be NP-hard. Reference: Daniel Brélaz, "New Methods to Color the Vertices of a Graph," Communications of the ACM, vol. 22, no. 4, pp. 251-256, 1979.

In the worst case, a naïve implementation of Brélaz's `Dsatur` algorithm has $O(|V|^2 \cdot |E|)$ running time. Describe how to improve this by using a careful choice of data structures and supporting algorithms and analyze the resulting running time.

Generate random graphs using the strategy described in Section 3.1 of Brélaz's paper, with $|V| = 10, 20, 30, \ldots$ vertices (try to go as far as possible). Apply Brélaz's `Dsatur` algorithm to color them and measure the running time (for each value of $|V|$, measure the time needed to color 100 random graphs of that size at once, then divide the time by 100). Plot your results and compare them against your asymptotic worst-case analysis above. What can you say about the observed (i.e. average) running time?

Bonus (for up to 5 extra points): Plot the estimated (i.e. observed on average) minimum number of colors needed to color the vertices of a random graph as a function of $|V|$. What can you say about the dependence between $|V|$ and that number?