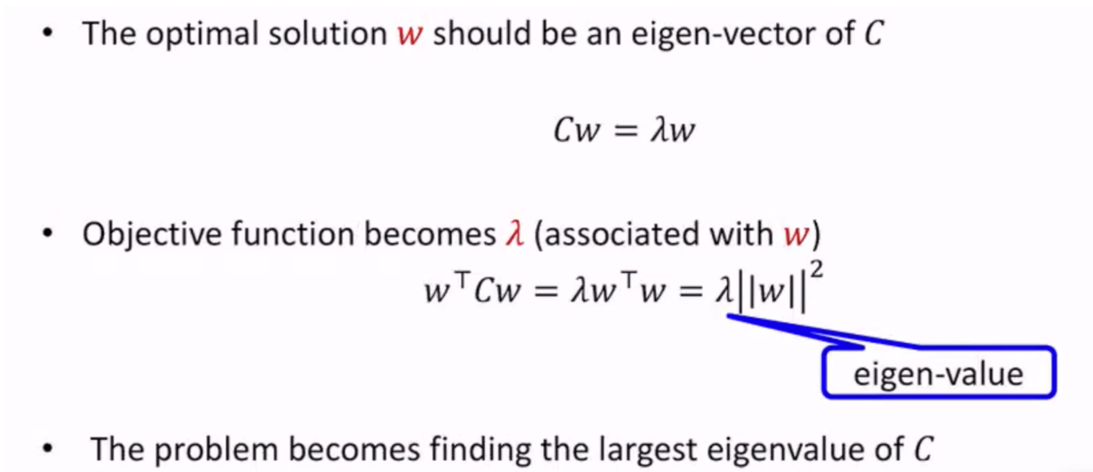
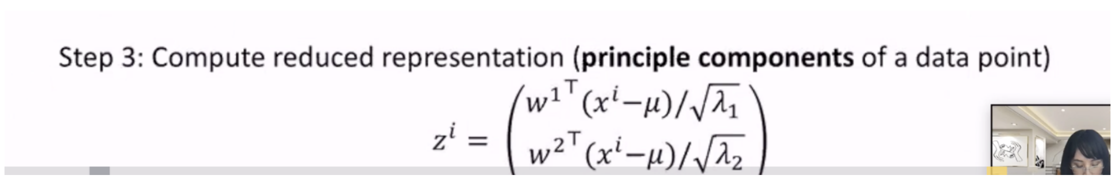
Homework 2

1. Conceptual questions
   1.1. In PCA, we maximize the objective function of w^TCw = (lambda)|| w ||^2. In maximizing the objective function, you find the w that also identifies the largest eigenvalue lambda. Thus, the largest weight vector w corresponds to the largest eigenvalue of your covariance matrix. Depending on how many principle directions you choose, each successive eigenvalue corresponds to a smaller Principle Component.





   1.2. SVD takes a matrix and provides the U, Sigma and V transpose (M = U*Sigma*V^T).

Taking another matrix C = M*M^T
Substitute M in

C = M*M^T
   = (U*(Sigma)*V^T)   *   (V(Sigma)^T*U^T)
*V*V^T cancels out as identity matrix*
   = U*Sigma* Sigma^T*U^T
   = U*U^T * Sigma*Sigma^T
*U*U^T also cancels out as identity matrix*
   = Sigma*Sigma^T
*Sigma*Sigma^T is an eigen decomposition*

U corresponds to the eigenvectors of C. Sigma^2 corresponds to eigenvalues of C, which are both used in an eigendecomposition.

1.3) From the lectures, ISOMAP can

1) Find the nearest neighbors of each datapoint within a specific distance (epsilon). So if a datapoint is within epsilon distance, it is considered a nearest neighbor, otherwise it is not
2) Find the shortest path distance matrix D between each pair of points. It will find the shortest path along the data cloud, which is a little different than finding just the shortest Euclidean distance, depending on the shape of the data.
3) Find low dimensional representation which preserves the distance information. This helps us unfold the data cloud and find the direct distances.

downsampling: reduce image by factor of 4, eg from 16x16 to 4x4

vectorize images to matrixes and perform PCA to find the eigenvectors U and largest eigen values lambda for k number of eigen values

find different "eigenfaces" through principle components: z = W^T(x-u)/(lambda)^1/2

Part 2a

1. import all the 10 images of Subject 1 (S1)
2. downsample by a factor of 4
3. Reshape (vectorize the data matrix)
4. The dimension of the S1 is ~(10,4800)
5. take the mean (M) of all the 10 images. ~(1,4800).
6. Subtract the mean from all the 10 images. ie. centering X1=S1-M
7. Perform SVD or find the covariance. (You may need to transpose for SVD)

if using SVD:

1. Find the eigenvectors corresponding to the 6 largest eigenvalues
2. These are the 6 eigenfaces. Reshape to get the images.

if using covariance:

1. Find the eigenvectors corresponding to the 6 largest eigenvalues
2. Find Z for the 6 eigenvectors(as described in lectures). These are the 6 eigenfaces
3. Reshape to get the images

Repeat the steps for Subject 2

You will get 6 eigenfaces for Subject 1 and 6 eigenfaces for Subject 2

In [1]:
```python
#Load an image
from PIL import Image
import math
import numpy as np

dir = r"yalefaces/Subject_1/"
fileName = dir + "subject01.happy.gif"
img = Image.open(fileName)

display(img)
```

In [2]:

```
# utility functions:

def proceed():
    import sys
    x = input('to continue enter 3: ')
    if(x == "3"):
        pass
    else:
        print('\nProcess terminated by user\n')
        sys.exit(0)

def image_grid(D,H,W,cols=10,scale=1):
    n = np.shape(D)[0]
    rows = int(math.ceil((n+0.0)/cols))
    fig = plt.figure(1,figsize=[scale*20.0/H*W,scale*20.0/cols*rows],dpi=300)
    for i in range(n):
        plt.subplot(rows,cols,i+1)
        fig = plt.imshow(np.reshape(D[i,:],[H,W]), cmap = plt.get_cmap("gray"))
        #display(fig = plt.imshow(np.reshape(D[i,:],[H,W]), cmap = plt.get_cmap(
        plt.axis('off')
        plt.show()
    plt.close()

def create_filenames(data_dir):
    dir_list = os.listdir(data_dir)
    return(dir_list)
```

In [3]:

```
#initializing/setting up data
import os
import matplotlib.pyplot as plt

view_list = ['P00A+000E+00', 'P00A+005E+10' , 'P00A+005E-10' , 'P00A+010E+00']

fileList = create_filenames(dir);

print('len(file_list): ', len(fileList), '\n')

print('display file names:')
for file in fileList:
     print(file)
print()
proceed()
```

```
im = Image.open(dir + fileList[2]).convert("L")

H,W = np.shape(im)
print ('shape of file ', dir + fileList[2], '= ', (H, W), '\n')

arr = np.zeros([len(fileList), H*W])
print("arr.shape\n",arr.shape)

for i in range(len(fileList)):
    im = Image.open(dir + fileList[i]).convert("L")
    arr[i,:] = np.reshape(np.asarray(im),[1,H*W])

image_grid(arr,H,W)

proceed()

img = Image.open(dir + fileList[0]).convert('L')
arrSingleImage = np.array(img)

# record the original shape
originalShape = arrSingleImage.shape
print("originalShape", originalShape)
print("arrSingleImage: \n", arrSingleImage, '\n')
proceed()

dsvector = arrSingleImage[::4,::4]

print("dsvector.shape", dsvector.shape)
print("dsvector\n",dsvector, "\n")
proceed()

arr3 = np.asarray(dsvector)
# make a PIL image
#img2 = Image.fromarray(arr3, 'RGBA')
img2 = Image.fromarray(arr3, 'L')
#img2 = Image.fromarray(arr2, 'L')
display(img2)

H,W = np.shape(img2)
print ('shape of downsampled image ', (H, W), '\n')

arr = np.zeros([len(fileList), H*W])

for i in range(len(fileList)):
    print('i: ', i)
    im = Image.open(dir + fileList[i]).convert("L")
    arrSingleImage = np.array(im)
    dsvector = arrSingleImage[::4,::4]
    H,W = np.shape(dsvector)
    arr[i,:] = np.reshape(np.asarray(dsvector),[1,H*W])
```

```
len(file_list):  10

display file names:
subject01.happy.gif
subject01.rightlight.gif
subject01.noglasses.gif
```

```
subject01.sad.gif
subject01.surprised.gif
subject01.glasses.gif
subject01.normal.gif
subject01.wink.gif
subject01.sleepy.gif
subject01.leftlight.gif

to continue enter 3: 3
shape of file  yalefaces/Subject_1/subject01.noglasses.gif =  (243, 320)

arr.shape
 (10, 77760)
```

```
to continue enter 3: 3
originalShape (243, 320)
arrSingleImage:
 [[130 130 130 ... 255 255 255]
 [255 255 255 ... 255 255 255]
 [255 255 255 ... 255 255 255]
 ...
 [255 255 255 ... 249 255 255]
 [255 255 255 ... 255 255 255]
 [ 68  68  68 ...  68  68  68]]

to continue enter 3: 3
dsvector.shape (61, 80)
dsvector
 [[130 130 130 ... 255 255 255]
 [255 255 255 ... 255 255 255]
 [255 255 255 ... 255 255 255]
 ...
 [255 255 255 ... 255 255 255]
 [255 255 255 ... 255 255 255]
 [255 255 255 ... 255 244 253]]

to continue enter 3: 3
```

```
shape of downsampled image  (61, 80)

i:  0
i:  1
i:  2
i:  3
i:  4
i:  5
i:  6
i:  7
i:  8
i:  9
```

Subject 1

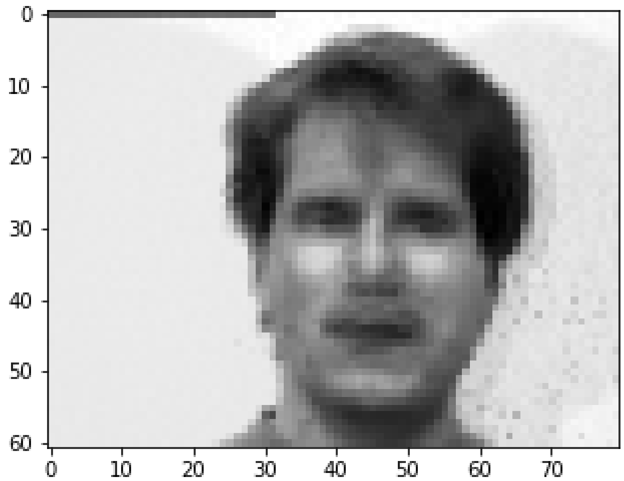
Subject 1 Mean Image

In [4]:

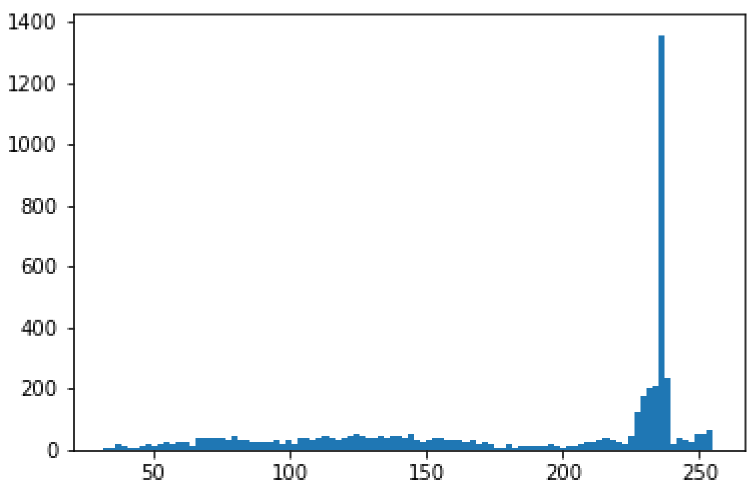
```
mean_image = np.mean(arr, axis=0)
#display(mean_image)
print("mean_image.shape \n", mean_image.shape)

plt.imshow(np.reshape(mean_image,[H,W]), cmap = plt.get_cmap("gray"))
plt.show()
plt.close()

plt.figure()
```

```
plt.hist(mean_image,bins=100);  #  heavily skewed to the right; is this correct

print('\n\ncell process complete\n\n')
```

```
mean_image.shape
 (4880,)
```




```
cell process complete
```

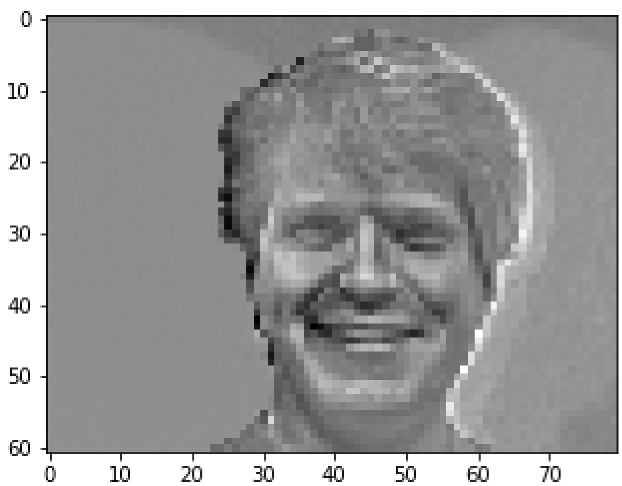


Subject 1 Normalize

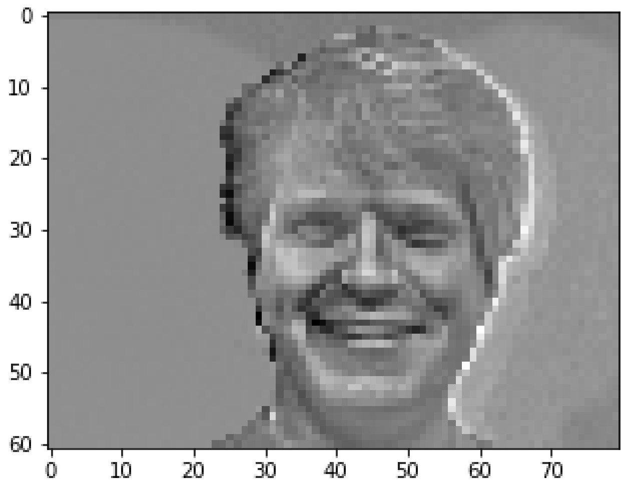
In [5]:
```
#SUBJECT 1 NORMALIZE
print(len(fileList))
number = 10
# arr_normalize = np.zeros([len(fileList), H*W])
arr_normalize = np.zeros([number, H*W])
#arr_normalize = arr[0:10,:] - mean_image
print("arr.shape \n", arr.shape)
print("mean_image.shape \n", mean_image.shape)
arr_normalize = arr - mean_image
print("arr_normalize.shape = ", arr_normalize.shape)

plt.imshow(np.reshape(arr_normalize[0],[H,W]), cmap = plt.get_cmap("gray")) #
plt.show()
```

```
print("Normalized Faces:")
for i in range(number):
    plt.imshow(np.reshape(arr_normalize[i],[H,W]), cmap = plt.get_cmap("gray"))
    plt.show()

#plt.hist(np.reshape(mean_image,[H,W]),bins=100);

plt.close()
plt.figure()
plt.hist(arr_normalize[0], bins=100);
#plt.hist(np.reshape(arr_normalize[0],[H,W]), bins=100);

#plt.hist(np.reshape(arr_normalize[0],[H,W]),bins=100);

for i in range(number):
    plt.subplot(1,10,i+1)
    #fig, ax = plt.subplots(1,10,i+1))
    #ax.set_xlim(left[0], right[-1])
    plt.hist(arr_normalize[i], bins=100 );
print('\n\n')

#image_grid(arr_normalize[:10,:],H,W)

print('\n\ncell process complete\n\n')
```

```
10
arr.shape
 (10, 4880)
mean_image.shape
 (4880,)
arr_normalize.shape =  (10, 4880)
```




```
Normalized Faces:
```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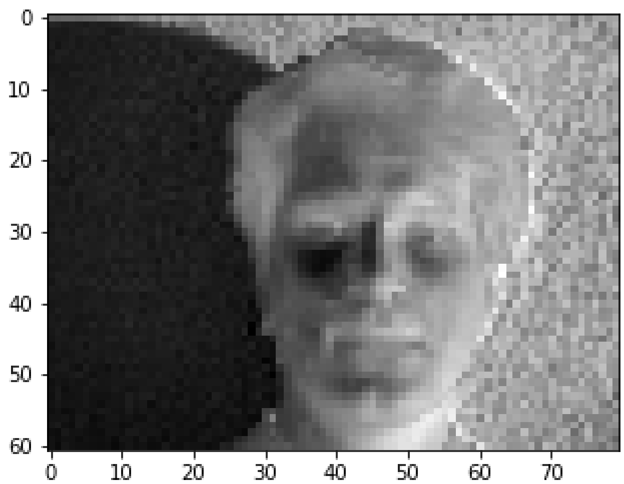
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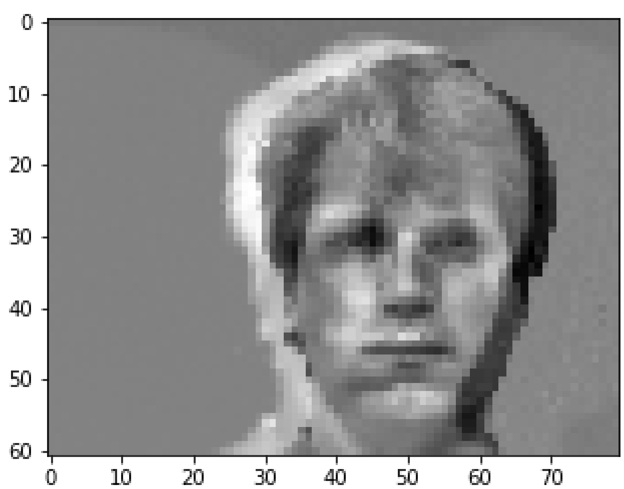
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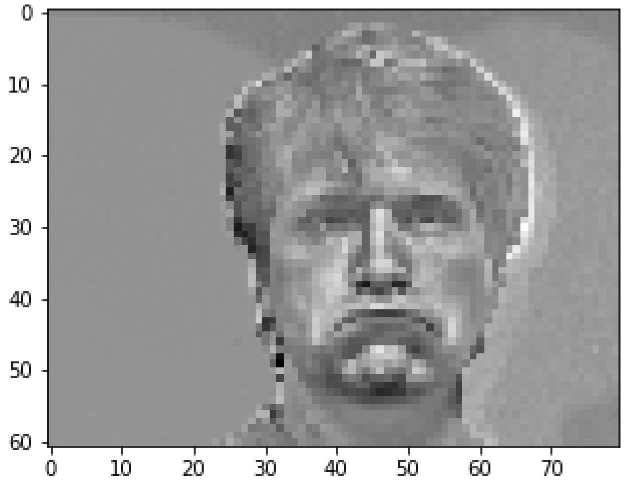
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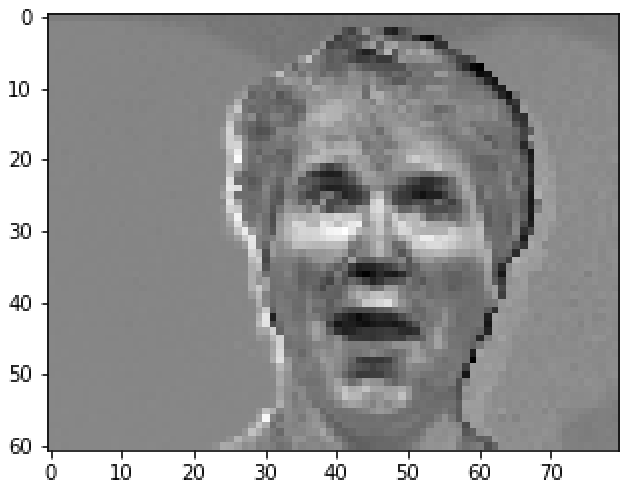
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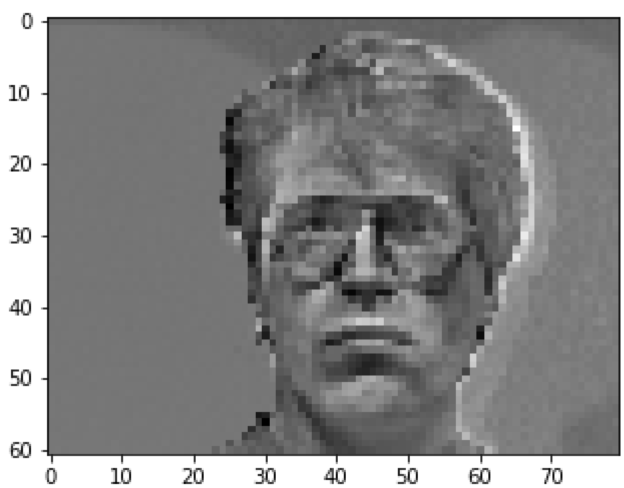
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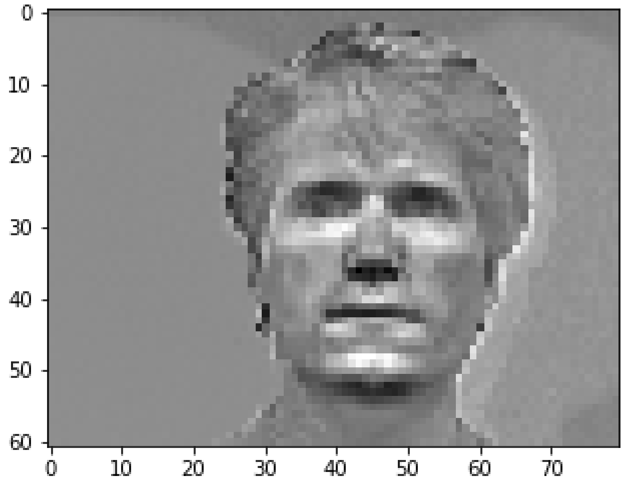
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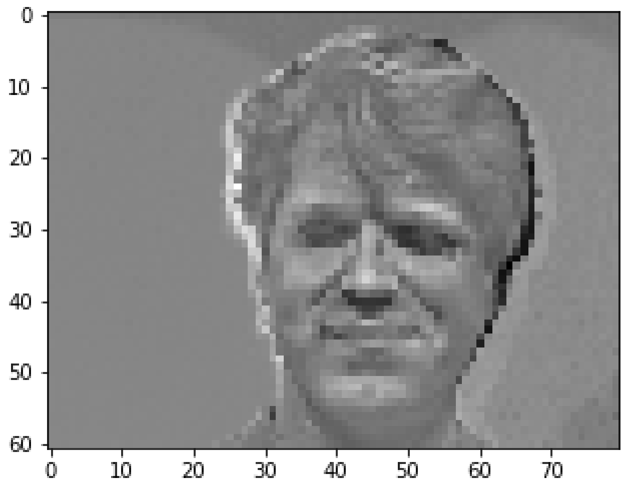
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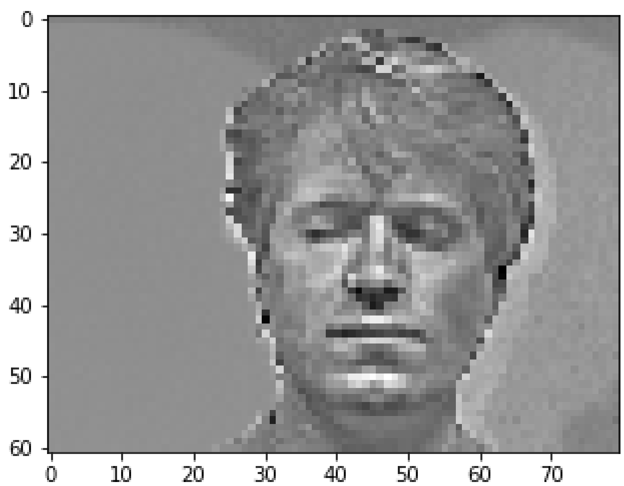
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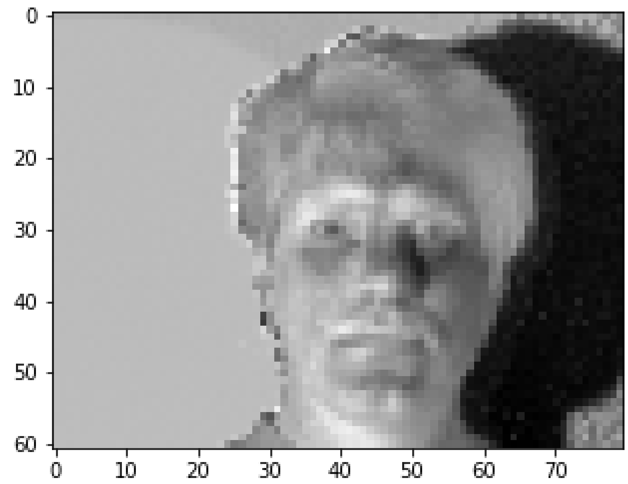
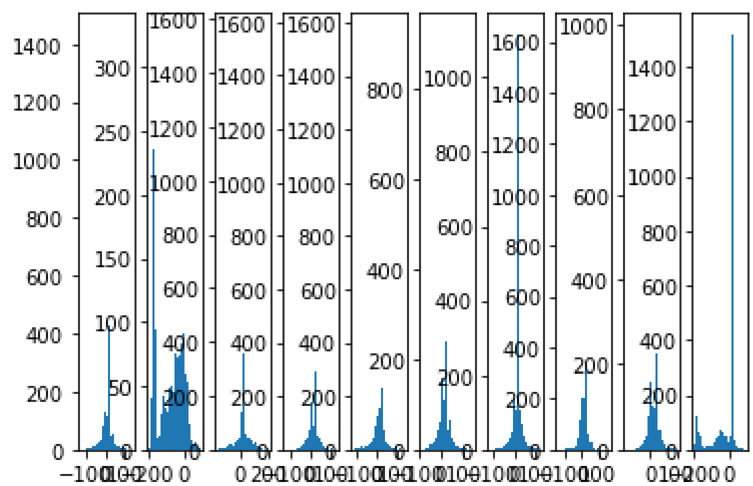


```
cell process complete
```



In [6]:

```python
#PCA FUNCTION USING SVD

def pca(X, n_pc):
    n_samples, n_features = X.shape
    mean = np.mean(X, axis=0)
    centered_data = X-mean
    U, S, V = np.linalg.svd(centered_data)
    components = V[:n_pc]
    projected = U[:,:n_pc]*S[:n_pc]

    return projected, components, mean, centered_data
```

In [7]:

```python
#SVD APPLICATION

pca(arr, 6)
```

Out[7]:

```
(array([[-7.70911518e+02, -8.39781323e+02,  7.13698909e+02,
          8.96652496e+02, -3.86861172e+02,  6.91960202e+02],
        [ 7.58939515e+03, -1.16744642e+03, -2.72661499e+02,
```

```
         -5.64630883e+01, -1.10175700e-01, -2.00712825e+01],
        [-1.37270080e+03,  3.02214457e+02, -3.25908232e+03,
          3.07004178e+02,  2.96544088e+02,  7.14458225e+01],
        [-9.76598046e+02, -8.28768735e+02,  5.95968939e+02,
          3.49863202e+02,  5.46654324e+02, -7.71350020e+02],
        [-1.12411206e+03, -5.16630860e+02,  8.27123425e+01,
         -1.77604950e+03, -8.04660225e+01,  4.44868589e+02],
        [-9.07277609e+02, -8.77218248e+02,  6.45117864e+02,
          7.41321909e+02,  1.36113796e+02,  4.88281355e+02],
        [-1.03214679e+03, -8.64902037e+02,  7.01364973e+02,
         -3.71523595e+02,  1.06769036e+03, -2.24234570e+01],
        [-1.08154348e+03, -4.36353757e+02, -1.47300324e+02,
         -1.76091175e+02, -1.30304576e+03, -3.84555936e+02],
        [-1.05923599e+03, -6.87206447e+02,  4.01515297e+02,
          4.32721328e+01, -3.19086898e+02, -5.02006450e+02],
        [ 7.35131151e+02,  5.91609337e+03,  5.38665822e+02,
          4.20134387e+01,  4.25674568e+01,  3.85117849e+00]]),
 array([[-1.05320943e-02, -1.08718393e-02, -1.06453427e-02, ...,
         -9.71417584e-03, -5.91453060e-03, -8.67027516e-03],
        [ 2.69453184e-03,  2.78145222e-03,  2.72350530e-03, ...,
         -3.35033043e-03, -7.49511064e-03, -6.32307079e-03],
        [ 1.95833478e-03,  2.02150687e-03,  1.97939214e-03, ...,
          1.04280396e-04, -1.83021986e-03, -9.38685972e-04],
        [ 1.07155560e-03,  1.10612190e-03,  1.08307770e-03, ...,
          6.10448110e-04, -1.24346154e-03,  1.81985597e-04],
        [ 2.92505862e-06,  3.01941535e-06,  2.95651086e-06, ...,
         -4.71344276e-04,  3.13547262e-03,  1.75024292e-03],
        [ 9.74214465e-04,  1.00564074e-03,  9.84689889e-04, ...,
          7.80595863e-04,  3.73824377e-05,  1.11959275e-03]]),
 array([120.7, 120.4, 120.6, ..., 242.9, 235.6, 241.2]),
 array([[  9.3,   9.6,   9.4, ...,  12.1,   8.4,  11.8],
        [-83.7, -86.4, -84.6, ..., -69.9, -35.6, -58.2],
        [  9.3,   9.6,   9.4, ...,  12.1,  12.4,  13.8],
        ...,
        [  9.3,   9.6,   9.4, ...,  12.1,   5.4,   7.8],
        [  9.3,   9.6,   9.4, ...,  12.1,   8.4,  13.8],
        [  9.3,   9.6,   9.4, ..., -26.9, -49.6, -44.2]]))
```

In [8]:

```python
from sklearn.decomposition import PCA

pca = PCA()
pca.fit(arr_normalize)
```

Out[8]:

```
PCA()
```

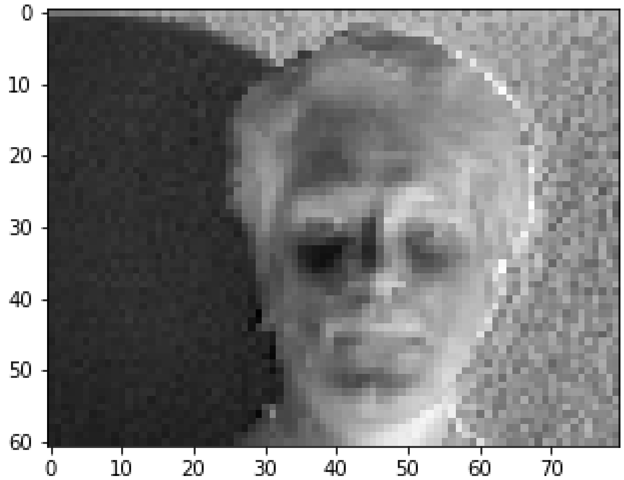
Subject 1 Eigenfaces

In [9]:

```python
#EIGENFACES
eigenfaces = pca.components_

for i in range(6):
    plt.imshow(np.reshape(eigenfaces[i],[H,W]), cmap = plt.get_cmap("gray")) #
    plt.show()

print("eigenfaces.shape= ", eigenfaces.shape)
```


0
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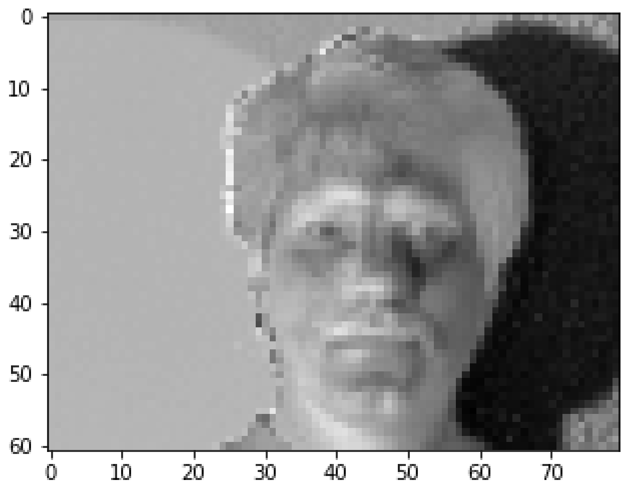
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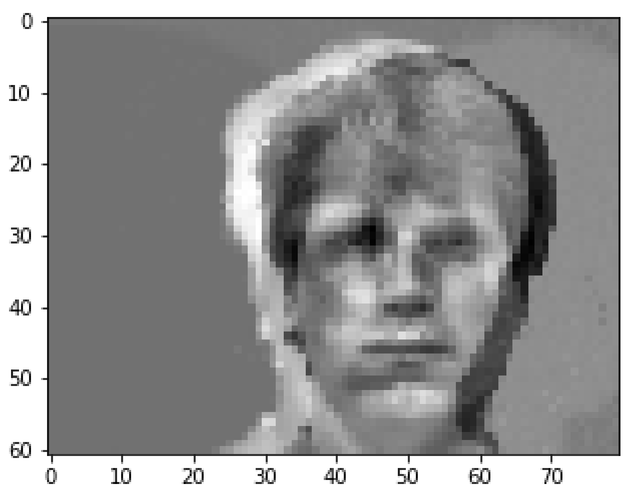
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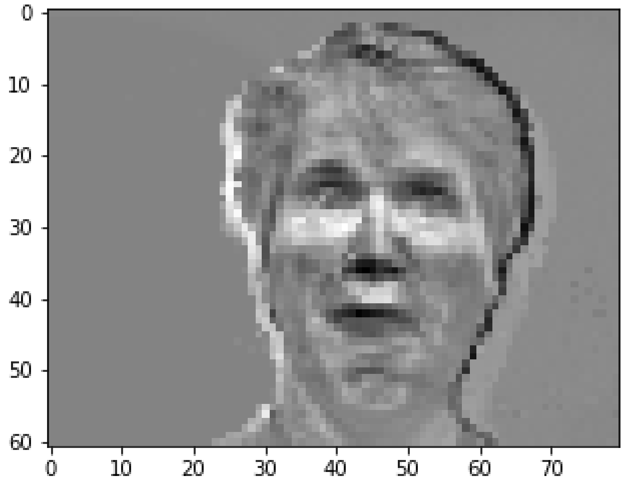
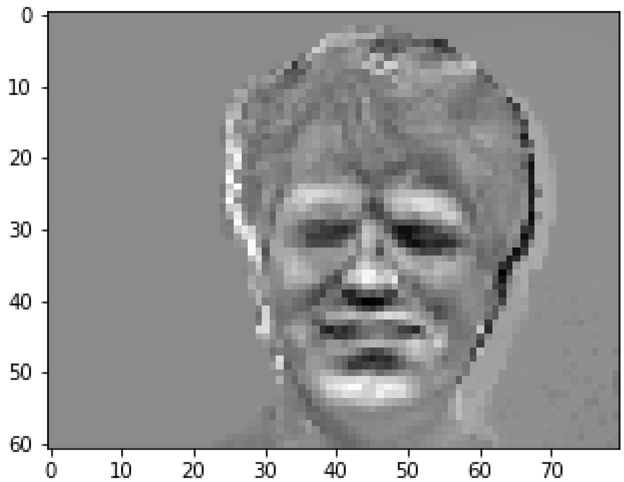
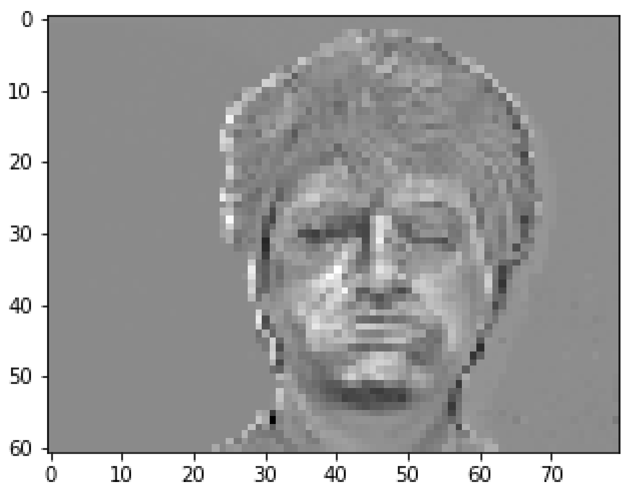







```
eigenfaces.shape=  (10, 4880)
```

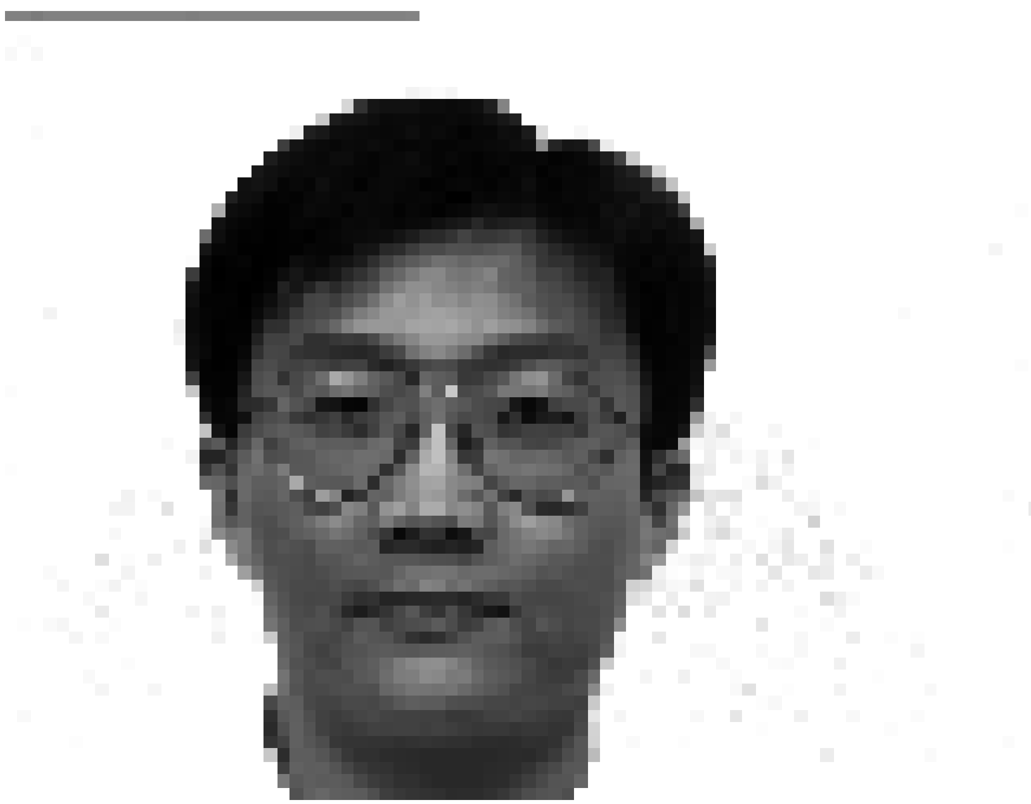
Subject 2

In [10]:

```
#SUBJECT 2 LOAD AND DOWNSIZE
dir2 = r"/Users/Sean/Downloads/PCA_demo_code_v2/yalefaces/Subject_2//"
fileList2 = create_filenames(dir2)
print('len(file_list): ', len(fileList2), '\n')
print('display file names:')
for file in fileList2:
     print(file)
```

```
print()
proceed()

arr2 = np.zeros([len(fileList2), H*W])    # array of all images of subject 1 exce

for i in range(len(fileList2)):
    im = Image.open(dir2 + fileList2[i]).convert("L")
    arrSingleImage = np.array(im)
    dsvector2 = arrSingleImage[::4,::4]
    H,W = np.shape(dsvector2)
    arr2[i,:] = np.reshape(np.asarray(dsvector2),[1,H*W])

image_grid(arr2,H,W)
```

```
len(file_list):  9

display file names:
subject02.glasses.gif
subject02.noglasses.gif
subject02.wink.gif
subject02.sad.gif
subject02.sleepy.gif
subject02.rightlight.gif
subject02.normal.gif
subject02.happy.gif
subject02.leftlight.gif

to continue enter 3: 3
```

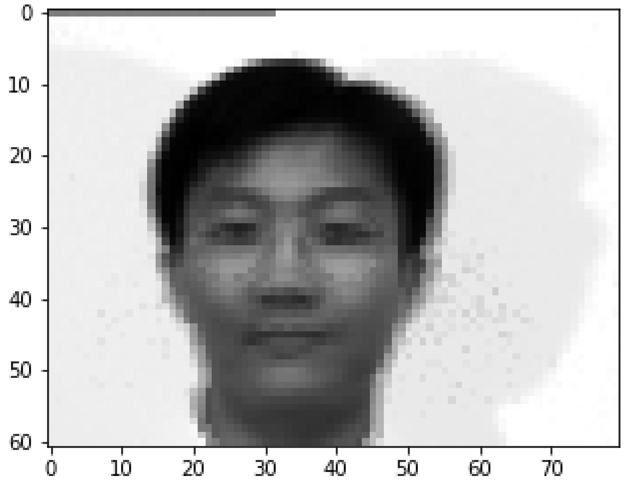


Subject 2 Mean and Normalized

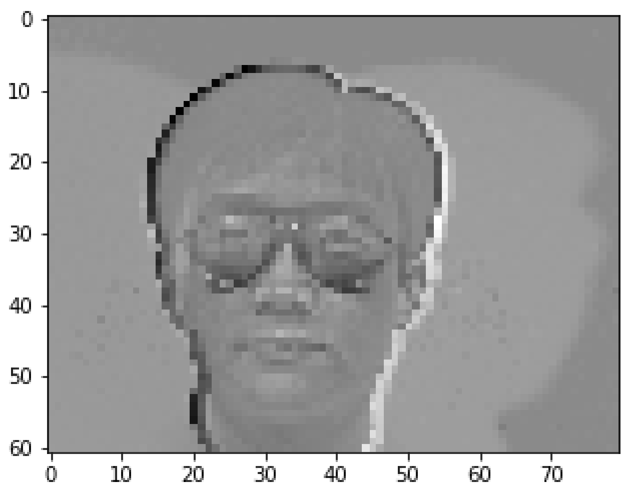
In [11]:

```
#SUBJECT 2 MEAN IMAGE AND NORMALIZING
mean_image2 = np.mean(arr2, axis=0)
print("mean_image2.shape= ", mean_image2.shape)
plt.imshow(np.reshape(mean_image2,[H,W]), cmap = plt.get_cmap("gray"))
plt.show()
plt.close()

print("len(fileList2) = ", len(fileList2))
# arr_normalize = np.zeros([len(fileList), H*W])
arr2_normalize = np.zeros([len(fileList2), H*W])
#arr_normalize = arr[0:10,:] - mean_image
print("arr.shape \n", arr2.shape)
print("mean_image.shape \n", mean_image2.shape)
arr2_normalize = arr2 - mean_image2
print("arr_normalize.shape = ", arr2_normalize.shape)

plt.imshow(np.reshape(arr2_normalize[0],[H,W]), cmap = plt.get_cmap("gray")) #
plt.show()

print("Normalized Faces:")
for i in range(len(fileList2)):
    plt.imshow(np.reshape(arr2_normalize[i],[H,W]), cmap = plt.get_cmap("gray"))
    plt.show()
```

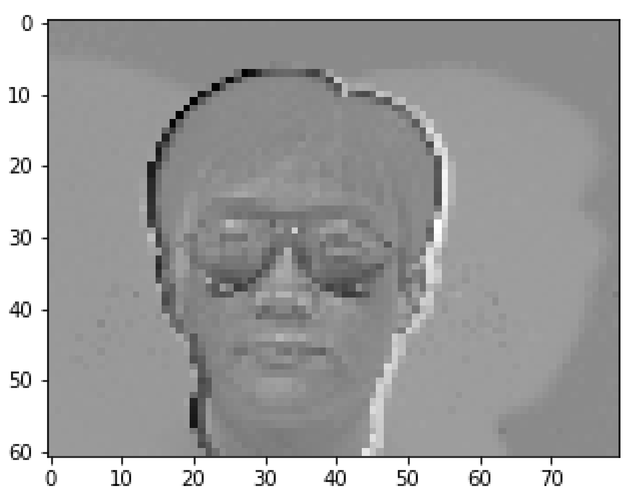
```
mean_image2.shape=  (4880,)
```

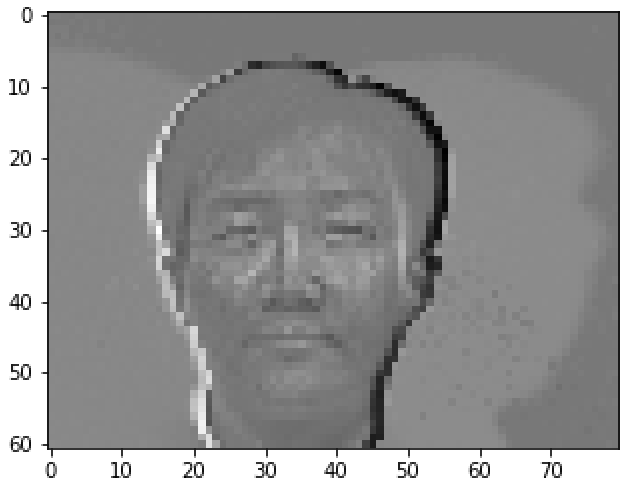


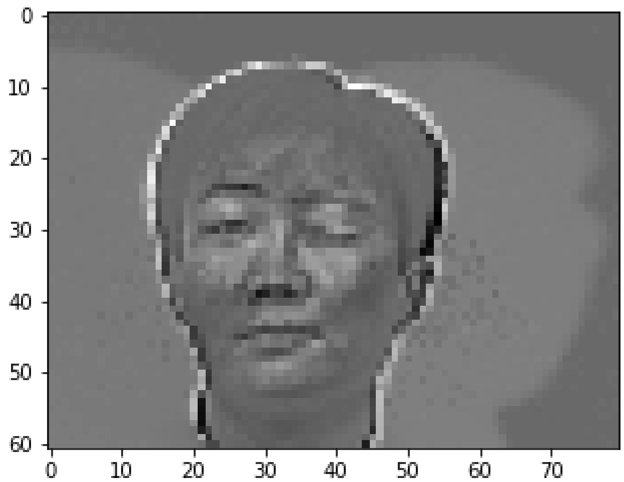

```
len(fileList2) =  9
arr.shape
 (9, 4880)
mean_image.shape
 (4880,)
arr_normalize.shape =  (9, 4880)
```

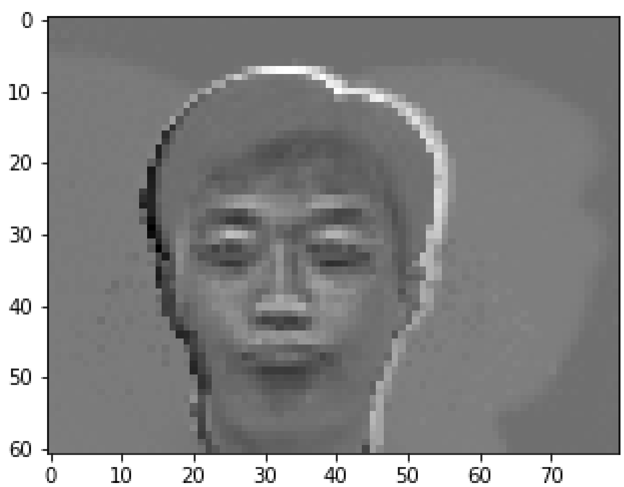


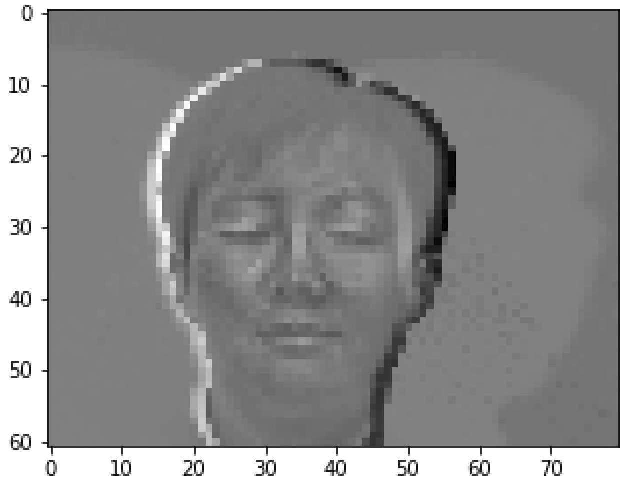

```
Normalized Faces:
```

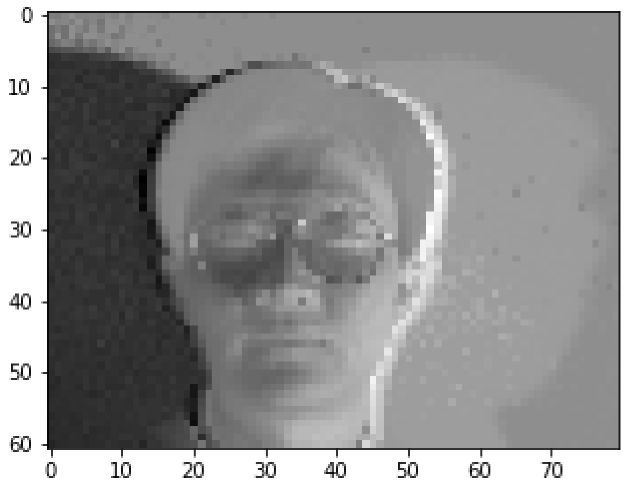


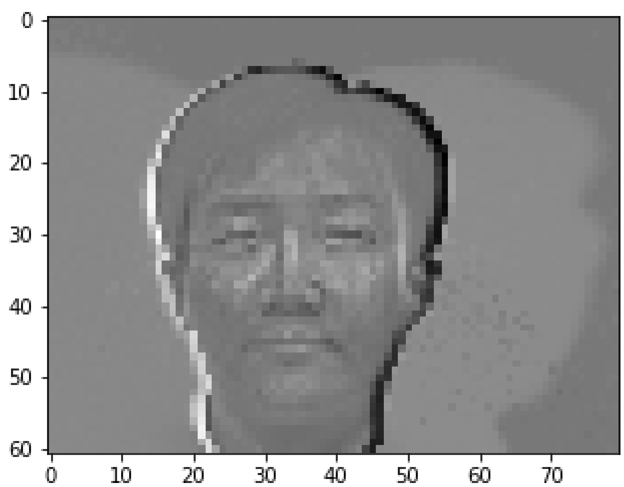

0
10
20
30
40
50
60
0 10 20 30 40 50 60 70

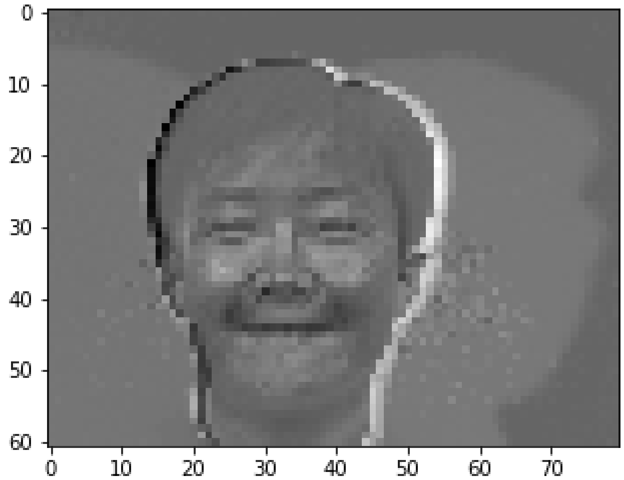

0
10
20
30
40
50
60
0 10 20 30 40 50 60 70

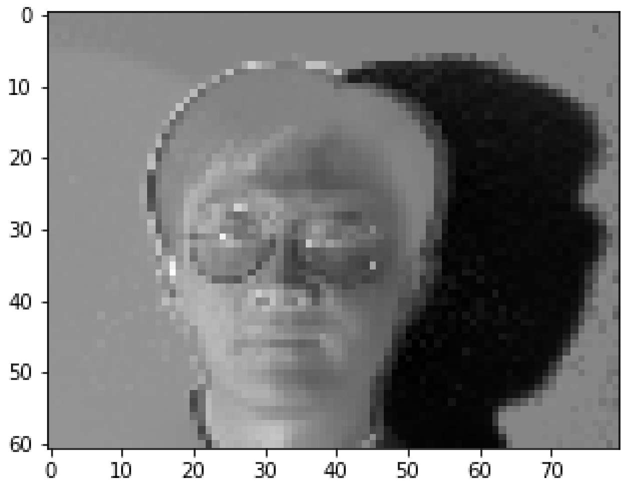

0
10
20
30
40
50
60
0 10 20 30 40 50 60 70


0
10
20
30
40
50
60
0 10 20 30 40 50 60 70


0
10
20
30
40
50
60
0 10 20 30 40 50 60 70


0
10
20
30
40
50
60
0 10 20 30 40 50 60 70

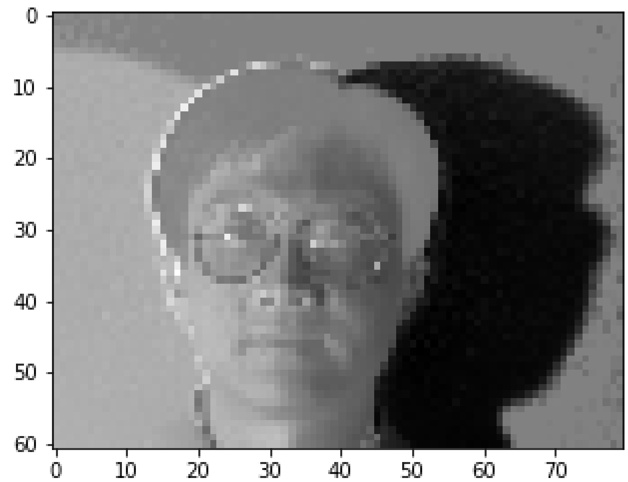


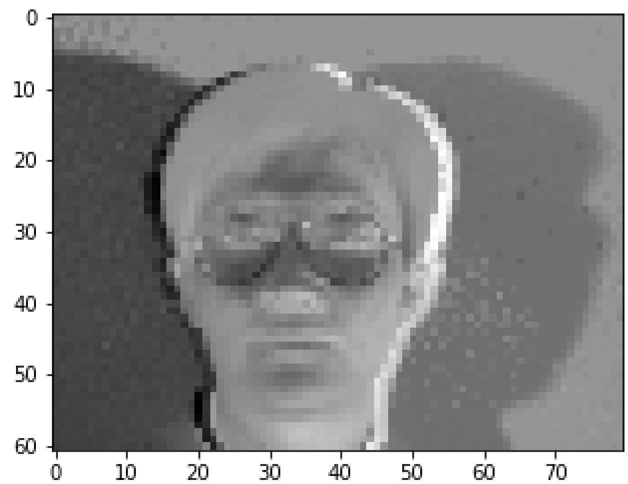


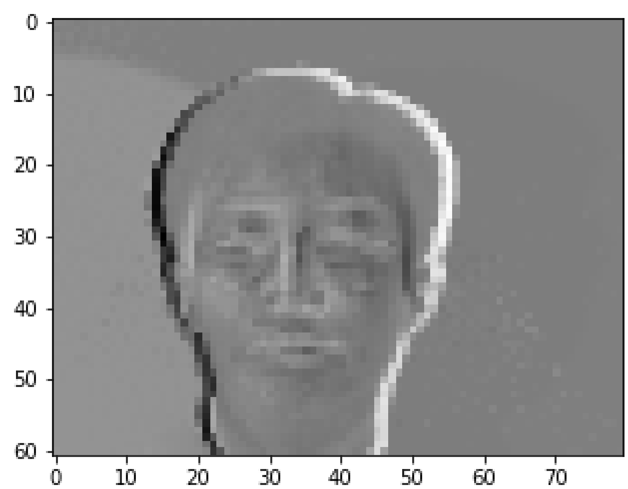
Subject 2 Eigenfaces

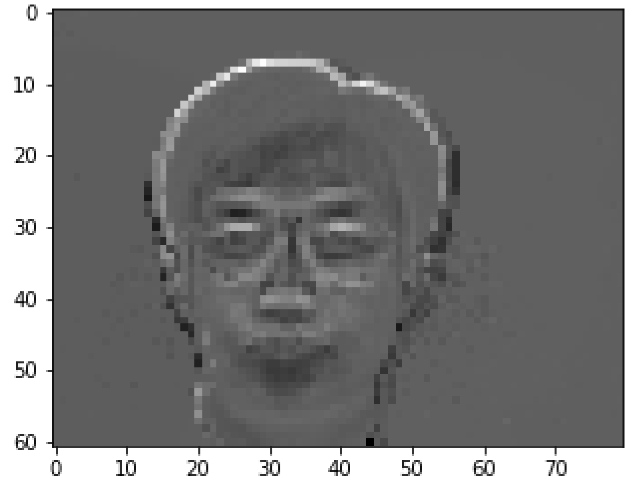
In [12]:

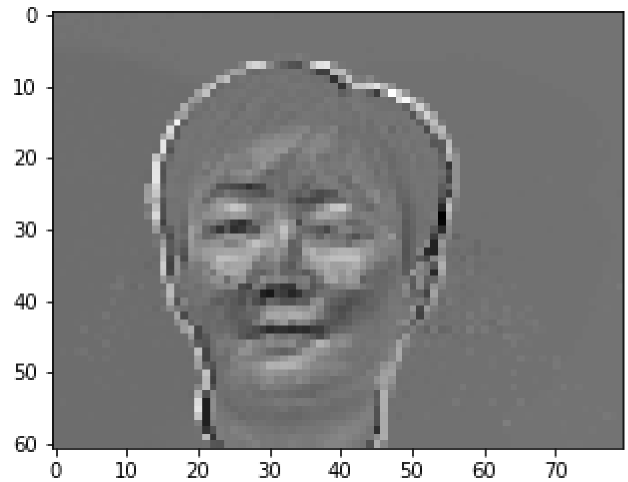
```
#SUBJECT 2 EIGENFACES
pca2 = PCA()
pca2.fit(arr2_normalize)
eigenfaces2 = pca2.components_

for i in range(6):
    plt.imshow(np.reshape(eigenfaces2[i],[H,W]), cmap = plt.get_cmap("gray")) #
    plt.show()

print("eigenfaces2.shape= ", eigenfaces2.shape)
```

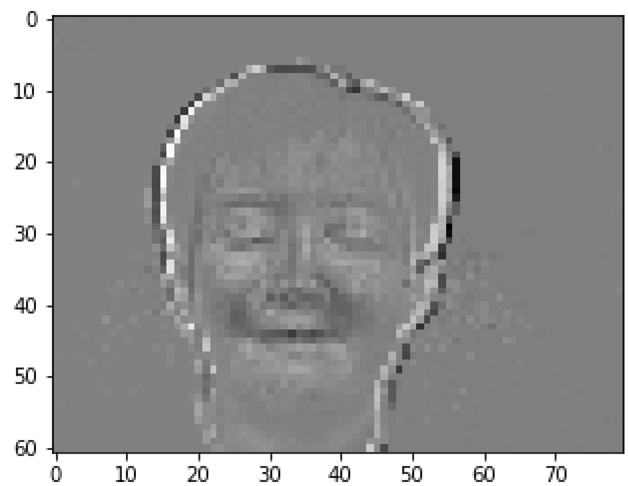













```
eigenfaces2.shape=  (9, 4880)
```

For both Subject 1 and Subject 2, the eigenfaces are ordered in descending image contrast, lighting, shape, and position. This is evident from the images with the left and right light as the first two eigenfaces for both subjects. This is the basis of feature recognition to be able to identify distinct characteristics of an individual's face/images.

In [ ]:

Part 2b

import the test images and downsample and reshape as above.

For Subject 1

Test image t1=Test image(subject1) -M (<-- this is normalized?)

For Subject 2:

Test image t2=Test image(Subject2) -M (<-- this is normalized?)

Residual Distance

S11= | (testimage(subject1) - eigenface(subject1) x eigenface(subject1).T x testimage(subhect 1)|^2 (some transposing may be needed to get the right matching dimensions)

S12= | (testimage(subject2) .T - eigenface(subejct1) x eigenface(subject1).T x testimage(subject 2)|^2

You may use any number of eigenfaces(1 to 6) for part 2.

You do the same for S21 and S22

Please note the residuals calculated from the covariance approach may be in range 10^15-10^17

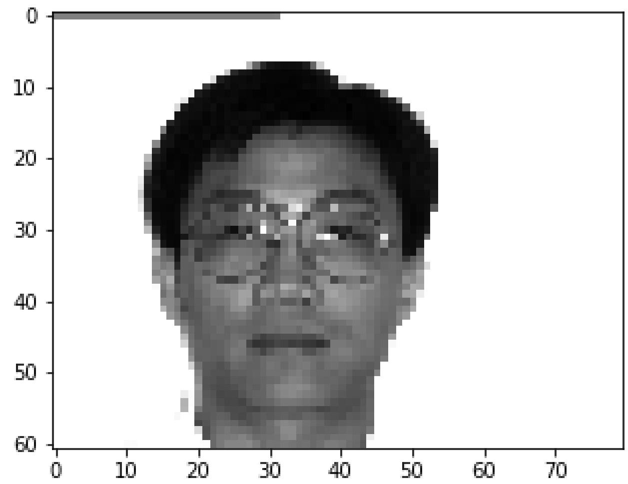
The residuals calculated from the SVD approach may be in the range 10^6-10^9

In [13]:

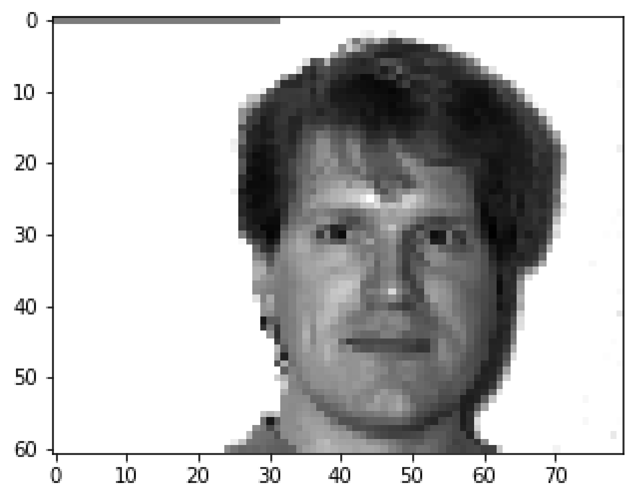
```
#Downsample Subject 1 and 2

Subject2 = Image.open("yalefaces/testfaces/subject02-test.gif").convert("L")
Subject1 = Image.open("yalefaces/testfaces/subject01-test.gif").convert("L")
display(Subject2)
display(Subject1)

dir3 = r"yalefaces/testfaces//"
fileList3 = create_filenames(dir3)

print('len(file_list): ', len(fileList3), '\n')

print('display file names:')
for file in fileList3:
     print(file)
print()
proceed()

arr3 = np.zeros([len(fileList3), H*W])
for i in range(len(fileList3)):
    print('i: ', i)
    im = Image.open(dir3 + fileList3[i]).convert("L")
    arrSingleImage3 = np.array(im)
    dsvector3 = arrSingleImage3[::4,::4]
```

```
    H,W = np.shape(dsvector3)
    #display(im)
    #arr[i,:] = np.reshape(np.asarray(im),[1,H*W]) # ValueError: cannot reshape
    arr3[i,:] = np.reshape(np.asarray(dsvector3),[1,H*W])
    print('arr[i].shape: ', arr3[1].shape)
    print("arr[i]=",arr3[i])
    plt.imshow(np.reshape(arr3[i],[H,W]), cmap = plt.get_cmap("gray")) #
    plt.show()
```

```
len(file_list):  2

display file names:
subject02-test.gif
subject01-test.gif

to continue enter 3: 3
i:  0
arr[i].shape:  (4880,)
arr[i]= [130. 130. 130. ... 255. 255. 255.]
```



```
i:  1
arr[i].shape:  (4880,)
arr[i]= [130. 130. 130. ... 255. 255. 255.]
```



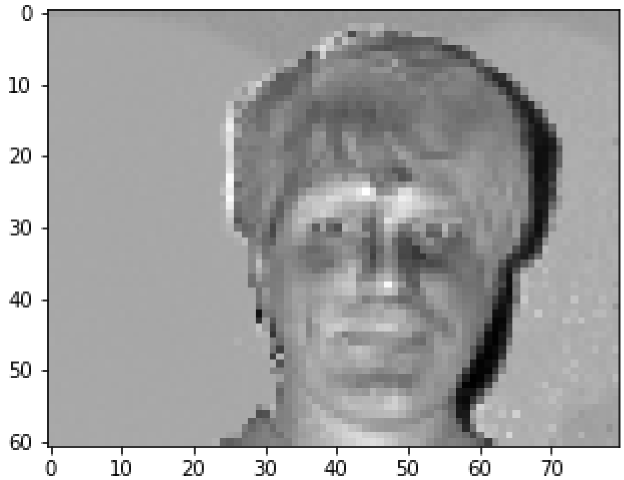
In [14]:

```
#NORMALIZE TESTIMAGES

#Test Image 1
print("H,W = ", H, W)
normalize_test_imaget1 = np.zeros([1, H*W])
print("normalize_test_imagett1.shape= ", normalize_test_imaget1.shape)
print("mean_image.shape =", mean_image.shape)
#arr_normalize = arr[0:10,:] - mean_image
normalize_test_imaget1 = arr3[1] - mean_image
print("normalize_test_imaget1.shape= ", normalize_test_imaget1.shape)

print("\n Test_Image_T1:")
plt.imshow(np.reshape(normalize_test_imaget1,[H,W]), cmap = plt.get_cmap("gray"))
plt.show()

#Test Image 2
normalize_test_imaget2 = np.zeros([1, H*W])
print("normalize_test_imaget2.shape= ", normalize_test_imaget2.shape)
print("mean_image2.shape =", mean_image2.shape)
#arr_normalize = arr[0:10,:] - mean_image
normalize_test_imaget2 = arr3[0] - mean_image2
print("normalize_test_imaget2.shape= ", normalize_test_imaget2.shape)

print("\n Test_Image_T2:")
```

```
plt.imshow(np.reshape(normalize_test_imaget2,[H,W]), cmap = plt.get_cmap("gray")
plt.show()
```

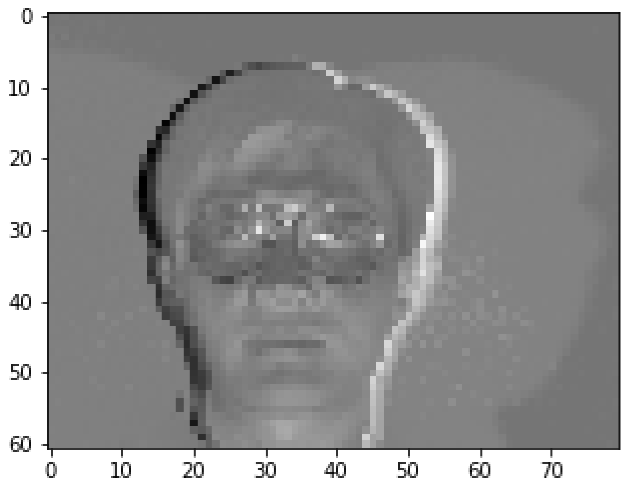
```
H,W =  61 80
normalize_test_imagett1.shape=  (1, 4880)
mean_image.shape = (4880,)
normalize_test_imaget1.shape=  (4880,)
```

Test_Image_T1:



```
normalize_test_imaget2.shape=  (1, 4880)
mean_image2.shape = (4880,)
normalize_test_imaget2.shape=  (4880,)
```

Test_Image_T2:



In [15]:

```
#RESIDUAL DISTANCE, S11

#S11= | ( testimage(subject1)  - eigenface(subject1) x eigenface(subject1).T x t
#S12= | ( testimage(subject2)  - eigenface(subejct1) x eigenface(subject1).T x t

normalize_test_imaget1 = np.reshape(normalize_test_imaget1,[4880,1])
eigenfaces1top = eigenfaces[0]
eigenfaces1top = np.reshape(eigenfaces1top,[4880,1])
print("normalize_test_image1t.shape= ", normalize_test_imaget1.shape)
print("eigenfaces1top.shape= ", eigenfaces1top.shape)
print("eigenfaces1top.T.shape=", eigenfaces1top.T.shape)

print("np.dot(eigenfaces1top, eigenfaces1top.T)", np.dot(eigenfaces1top, eigenfa
```

```
S11 = np.sum((normalize_test_imaget1 - np.dot((np.dot(eigenfaces1top, eigenfaces
#S11 = np.sum((normalize_test_imaget1 - eigenfaces1 @ eigenfaces1.T @ normalize_
print("S11: ", S11)

eigenfaces2top = eigenfaces2[0]
```

```
normalize_test_image1t.shape=  (4880, 1)
eigenfaces1top.shape=  (4880, 1)
eigenfaces1top.T.shape= (1, 4880)
np.dot(eigenfaces1top, eigenfaces1top.T) (4880, 4880)
S11:  7201612.087076735
```

In [16]:

```
#RESIDUAL DISTANCE FUNCTION, (NEED HELP EDITING FUNCTION)!!!!!!!!!!!!!!!!!!!!!!

def residual_distance(testimageS1, testimageS2, eigenfacesS1, eigenfacesS2):
    testimageS1 = np.reshape(testimageS1,[4880,1])
    testimageS1 = np.reshape(testimageS1,[4880,1])

    eigenfaces1top = eigenfacesS1[0]
    eigenfaces1top = np.reshape(eigenfaces1top,[4880,1])
    ResDist = np.sum((testimageS1 - np.dot((np.dot(eigenfaces1top, eigenfaces1to
    print("for S==1, J==1: Residual Distance =", ResDist, "\n")

    eigenfaces2top = eigenfacesS2[0]
    eigenfaces2top = np.reshape(eigenfaces2top,[4880,1])
    ResDist = np.sum((testimageS1 - np.dot((np.dot(eigenfaces2top, eigenfaces2to
    print("for S==2, J==1: Residual Distance =", ResDist, "\n")

    eigenfaces2top = eigenfacesS2[0]
    eigenfaces2top = np.reshape(eigenfaces2top,[4880,1])
    ResDist = np.sum((testimageS2 - np.dot((np.dot(eigenfaces2top, eigenfaces2to
    print("for S==2, J==2: Residual Distance =", ResDist, "\n")

    eigenfaces1top = eigenfacesS1[0]
    eigenfaces1top = np.reshape(eigenfaces1top,[4880,1])
    ResDist = np.sum((testimageS2 - np.dot((np.dot(eigenfaces1top, eigenfaces1to
    print("for S==1, J==2: Residual Distance =", ResDist, "\n")



residual_distance(normalize_test_imaget1, normalize_test_imaget2, eigenfaces, ei
```

```
for S==1, J==1: Residual Distance = 7201612.087076735

for S==2, J==1: Residual Distance = 6304902.427078757

for S==2, J==2: Residual Distance = 4242539.9084916385

for S==1, J==2: Residual Distance = 4845168.873500652
```

2b)

My residual distances are 10^6, which is on the lower end of what it should be for the SVD approach. The residual distance is finding the summation of the difference between the subject's test image and

the test image scaled by the eigenface identity matrix, which outputs a singular scalar value. Presumably, residual distances that use the same subjects for the test image and the eigenfaces should have smaller values than if there were different subjects used for the eigenfaces and test images, which is close to what I got. From my results, S11>S21 and S22<S12, though I think it should have been S11 < S21

2c)

Eigenface face recognition can work better if you have a larger pool of images with varying contrasting shades of that individual's face. This woud allow your mean to be a better representation of the subject and not have poor lighting or too much light/exposure impact the facial recognition. Based on this homework, it seemed like lighting had a big influence on the facial recognition. It is a little hard to tell based on just these two subjects how much skin tone, face shape, hair, and other features impact these results. I would be curious to see this homework also performed with more subjects to test those features.

In [ ]:

```

```

In [1]:

```
# from PCA_demo.py:
import numpy as np
import matplotlib.pyplot as plt
import scipy.io as spio
import scipy.sparse.linalg as ll
from scipy import linalg
from scipy.spatial.distance import cdist
from sklearn.utils.graph_shortest_path import graph_shortest_path

def proceed():
        import sys
        x = input('to continue enter 3: ')
        if(x == "3"):
            pass
        else:
            print('\nProcess terminated by user\n')
            sys.exit(0)
```

Adjacency Matrix

In [2]:

```
def adjacency_matrix(data, dist_func="epsilon", epsilon=1):
    N, M = data.shape
    distance = cdist(data.T, data.T, metric=dist_func)
    adjacency =  np.zeros((M, M)) + np.inf
    less_than_epsilon = distance < epsilon
    adjacency[less_than_epsilon] = distance[less_than_epsilon]
    short_dist = graph_shortest_path(adjacency)
    return short_dist
```

ISOMAP

In [3]:

```
def isomap(dist_matrix, dim=2):

    N, M = dist_matrix.shape
    diag_array = np.eye(M) - (1/M)*np.ones((M, M))
    dist_matrix = dist_matrix**2
    C = -1/(2*M) * diag_array.dot(dist_matrix).dot(diag_array)
    eigen_values, eigen_vects = linalg.eig(C)
    index = eigen_values.argsort()[::-1]
    eigen_values = eigen_values[index ]
    eigen_values = eigen_values[:dim]
    eigen_vects = eigen_vects[:, index ]
    eigen_vects = eigen_vects[:, :dim]
    Z = (eigen_vects.dot(np.diag(eigen_values**(-1/2)))).real
    return Z
```

Scatterplot and Images

In [4]:

```
def plot_graph(reduced_matrix, image_array, title="Face Distances", filename="fa
```

```
    N, M = image_array.shape
    print('N, M: ', N, ' ', M, '\n')
    proceed()
    figure1 = plt.figure()
    figure1.set_size_inches(10, 10)
    figure1 = figure1.add_subplot(111)
    figure1.set_title(title)
    figure1.set_xlabel('Component: 1')
    figure1.set_ylabel('Component: 2')


    rangex= (max(reduced_matrix[:, 0]) - min(reduced_matrix[:, 0])) * 0.1
    rangey = (max(reduced_matrix[:, 1]) - min(reduced_matrix[:, 1])) * 0.1

    for i in range(40):
        img_num = np.random.randint(0, M)
        x0 = reduced_matrix[img_num, 0] - (rangex/2)
        x1 = reduced_matrix[img_num, 0] + (rangex/2)
        y0 = reduced_matrix[img_num, 1] - (rangey/2)
        y1 = reduced_matrix[img_num, 1] + (rangey/2)
        img = image_array[:, img_num].reshape(64, 64).T
        figure1.imshow(img, aspect='auto', cmap=plt.cm.gray, interpolation='near


    figure1.scatter(reduced_matrix[:, 0], reduced_matrix[:, 1], marker='.',alpha
    figure1.set_ylabel('Direction 1')
    figure1.set_xlabel('Direction 2')
    print("SEAN HERE", figure1)
    display(figure1)
    plt.show(figure1)

    return None
```

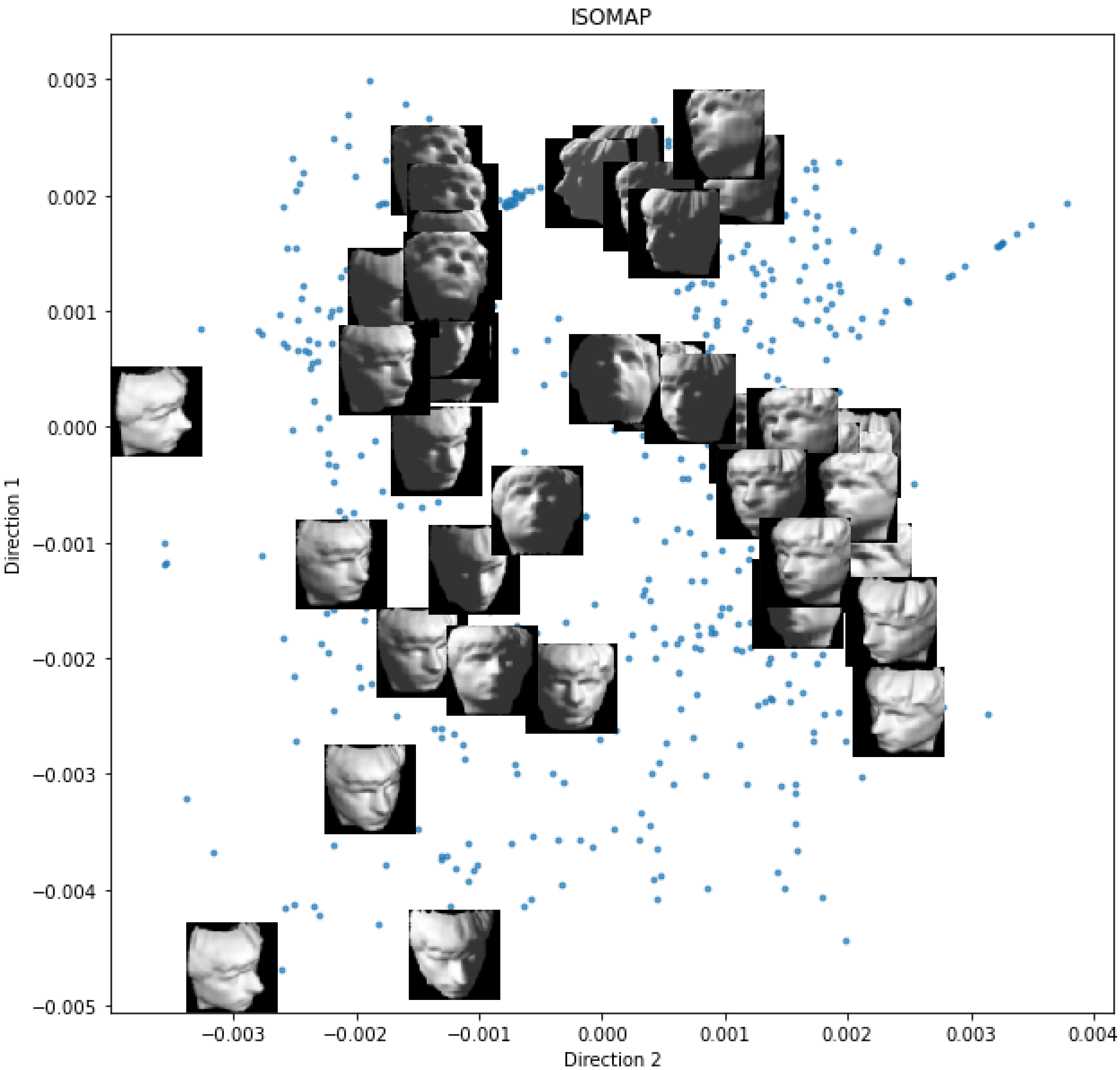
In [5]:

```
def main():

    print('\n\n\n3. Order of faces using ISOMAP\n\n')

    print('reading file as .mat: ')
    imageVectorsMat = spio.loadmat('isomap.mat')
    m = imageVectorsMat['images']
    print('type(m): ', type(m))
    print('m.shape: ', m.shape, '\n')
    proceed()

   # D = make_adjacency(m, epsilson=350, dist_func="cityblock")
    #D = make_adjacency(df, epsilon=350, dist_func="euclidean")
    print("--------------------------------------------------3A-----------------
    Adj_matrix = adjacency_matrix(m, epsilon=10, dist_func="euclidean")
    print('Adj_matrix:\n ', Adj_matrix.shape, '\n')
    print('3a) Adj_matrix:\n ', Adj_matrix, '\n\n')
    proceed()

    print("--------------------------------------------------3B-----------------
    z = isomap(Adj_matrix)
    print('z.shape: ', z.shape, '\n')
    print('z: ', z, '\n\n')
    proceed()
```

```
    print("--------------------------------------------------3C/B---------------
    plot_graph(z, image_array=m, title="ISOMAP")
    print("\n\n")

    #---
    ## PCA
    from sklearn.decomposition import PCA
    pca = PCA(n_components=2)
    z = pca.fit(m)
    print("PCA\n--------\n")
    print("\n")
    plot_graph(z.components_.T, image_array=m, title="PCA",
            filename="pca_faces.png")


main()
```

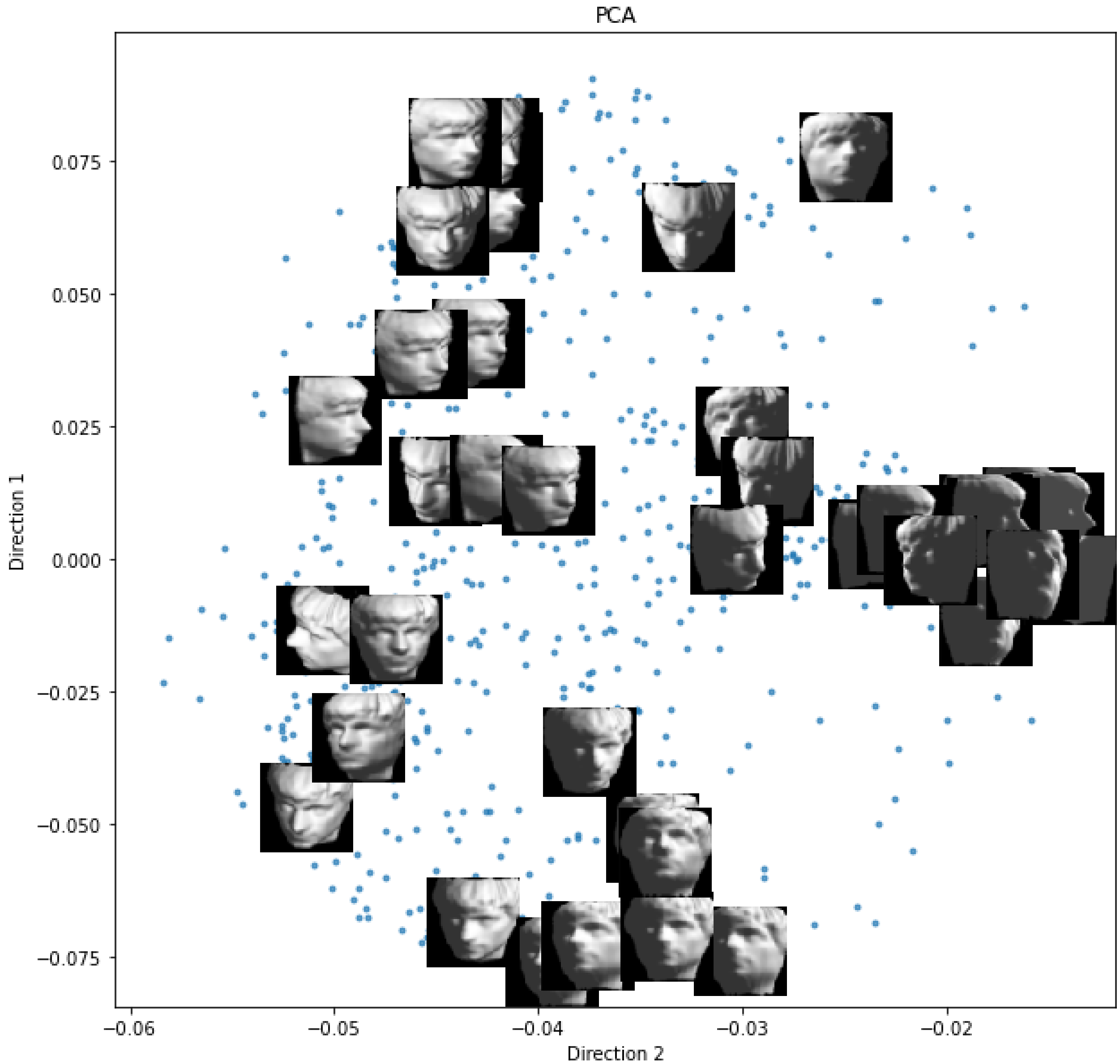
```
3. Order of faces using ISOMAP


reading file as .mat:
type(m):  <class 'numpy.ndarray'>
m.shape:  (4096, 698)

to continue enter 3: 3
--------------------------------------------------3A-----------------------------
----------------------
Adj_matrix:
  (698, 698)

3a) Adj_matrix:
  [[ 0.          61.03137046  6.74323967 ... 39.85343883 62.49785768
  26.2558373 ]
 [61.03137046  0.          64.36033515 ... 22.92304977 55.11152462
  54.99413899]
 [ 6.74323967 64.36033515  0.          ... 44.38349594 60.75417936
  25.00274763]
 ...
 [39.85343883 22.92304977 44.38349594 ...  0.          51.01502474
  34.72362778]
 [62.49785768 55.11152462 60.75417936 ... 51.01502474  0.
  48.5255165 ]
 [26.2558373  54.99413899 25.00274763 ... 34.72362778 48.5255165
   0.        ]]


to continue enter 3: 3
--------------------------------------------------3B-----------------------------
----------------------
z.shape:  (698, 2)

z:  [[ 0.00156036 -0.00017263]
 [-0.00144146  0.00094479]
 [ 0.001594   -0.0009043 ]
 ...
 [-0.00075922  0.00192043]
 [-0.00206722 -0.00085217]
 [ 0.00069384  0.0002459 ]]
```

```
to continue enter 3: 3
---------------------------------------------------3C/B--------------------------
------------------------
N, M:  4096   698

to continue enter 3: 3
SEAN HERE AxesSubplot(0.125,0.125;0.775x0.755)
<AxesSubplot:title={'center':'ISOMAP'}, xlabel='Direction 2', ylabel='Direction
1'>
```




```
PCA
--------



N, M:  4096   698

to continue enter 3: 3
SEAN HERE AxesSubplot(0.125,0.125;0.775x0.755)
```

```
<AxesSubplot:title={'center':'PCA'}, xlabel='Direction 2', ylabel='Direction 1'>
```



3a) The adjacency matrix comes out as a (698, 698) matrix with values < epsilon

3b) Similar to the paper and lectures, the ISOMAP organized the faces depending on the face direction. Faces looking to the right are on the left side of the graph, faces looking left are on the right side of the graph, and faces looking straight are in the middle of the graph.

3c/d) PCA ordered the faces based on contrast of white to dark (similar to how PCA did on question 2), as opposed to direction of the faces that ISOMAP used. Each method has its own uses, so it depends on which features you are looking to identify.

In [ ]: