Introduction to Geometry

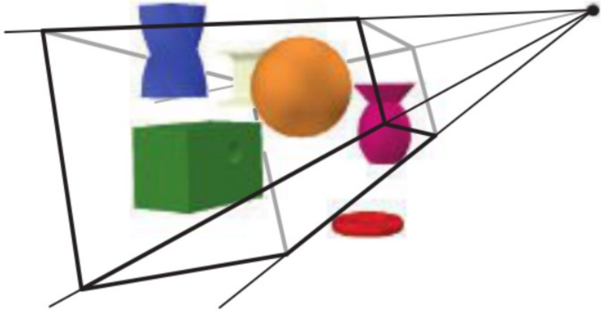
Topic

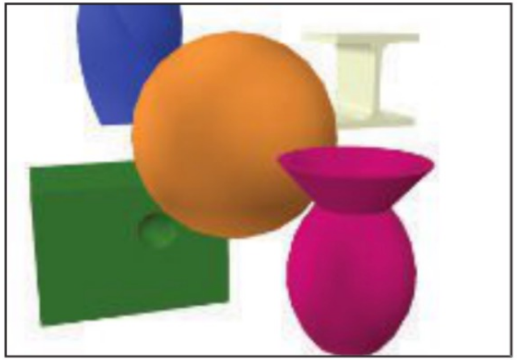
- Introduction to geometry
  - Examples of geometry
  - Representations of geometry
    - Best?
    - Implicit Representations of geometry
    - Explicit Representations of geometry

Reference

- 闫令琪，UCSB，GAMES101: 现代计算机图形学入门
  - （https://sites.cs.ucsb.edu/~lingqi/teaching/games101.html）
- 胡事民，清华大学，高等计算机图形学

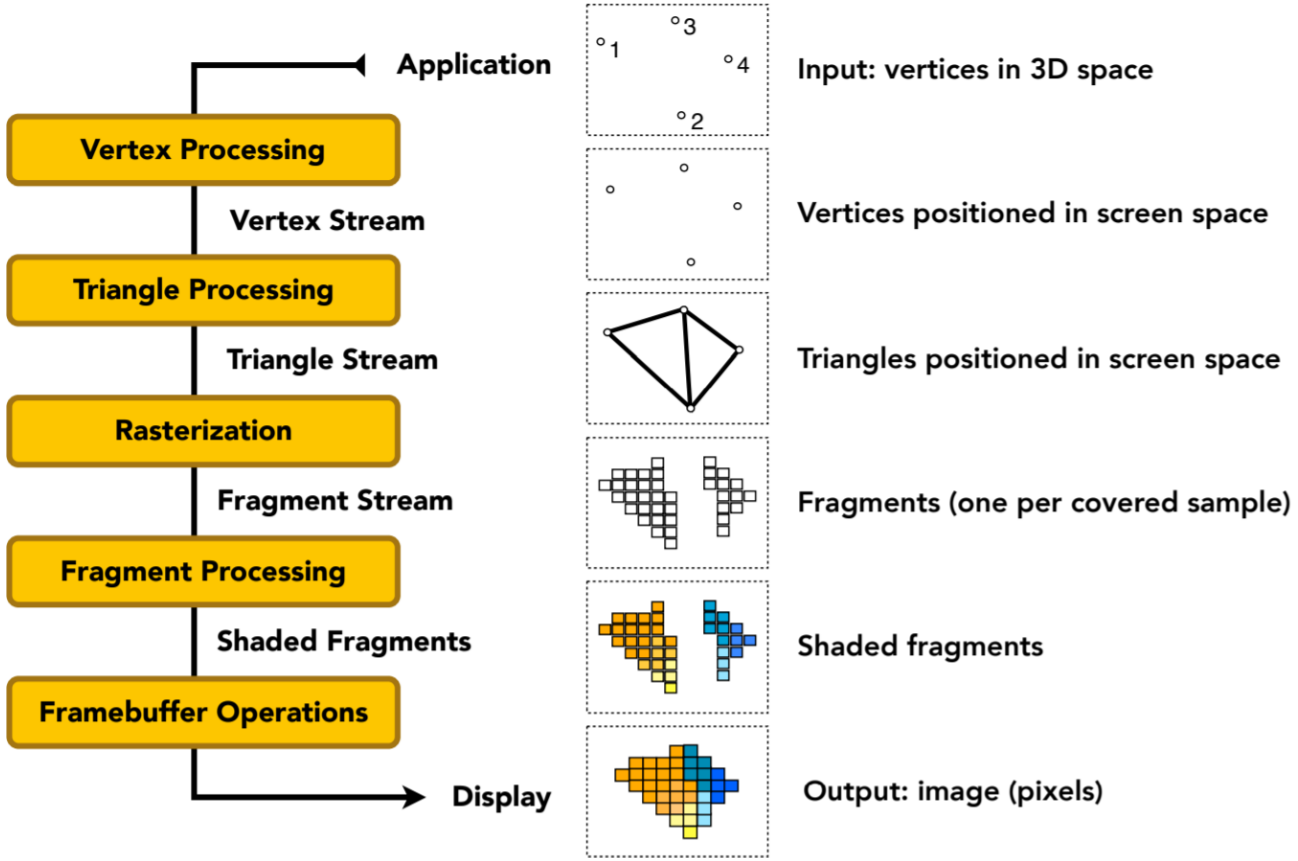
Last week: Graphics rendering pipeline





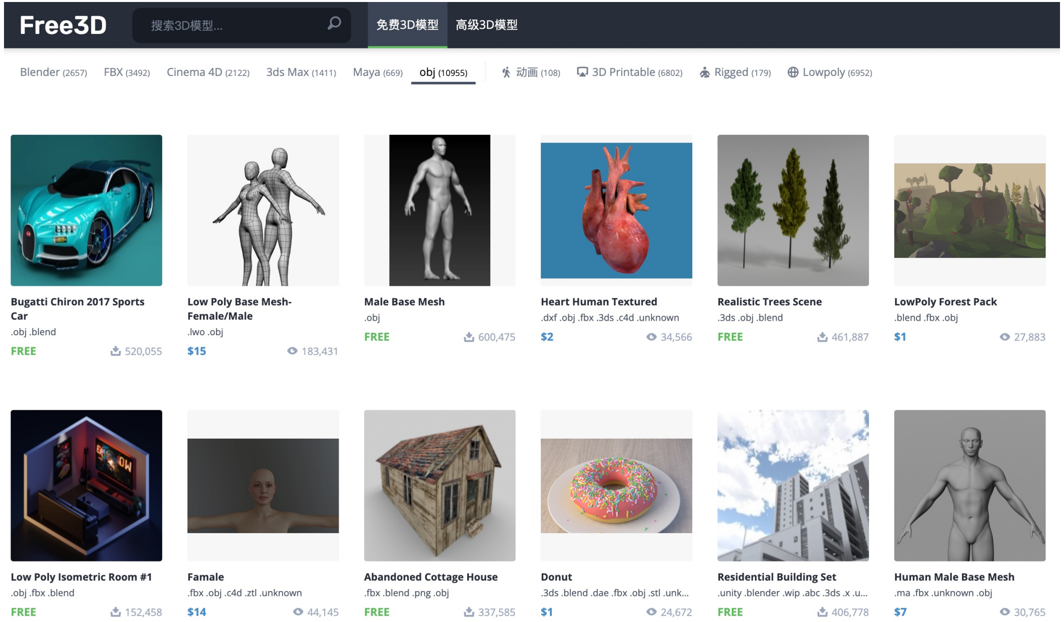
- The pipeline renders a 2D image, given a virtual camera, 3D objects, light sources, etc.
- The locations and shapes of the objects in the image are determined by: their geometry, the environment, and the placement of camera in the environment.
- The appearance of the objects is affected by: material, light sources, textures, and shading equations

Last week: Graphics rendering pipeline



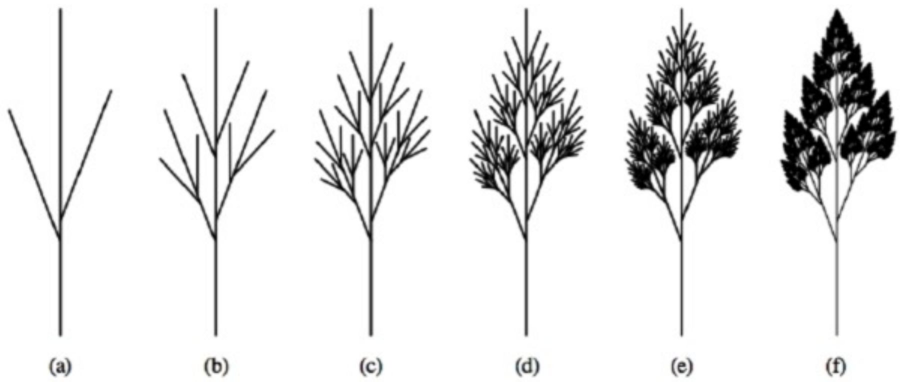
Origin of 3D data

- **Method for Model Generation:**
  1. Input from geometry file



Origin of 3D data

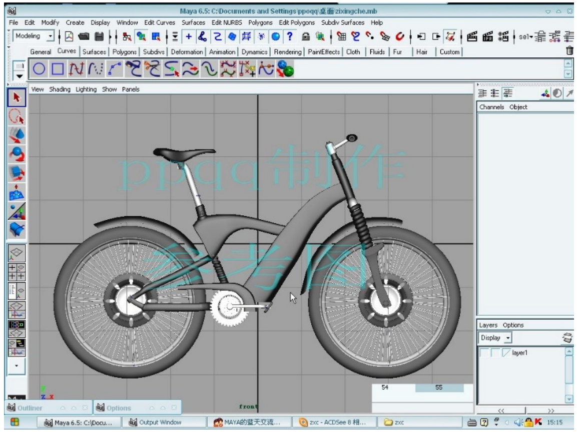
- **Method for Model Generation:**
  1. Input from geometry file
  2. Procedure Modeling:
     - Created through program code, such as L-systems and fractal geometry



Origin of 3D data

- **Method for Model Generation:**
  1. Input from geometry file
  2. Procedure Modeling
  3. Software modeling:

  e.g. 3DS MAX / MAYA

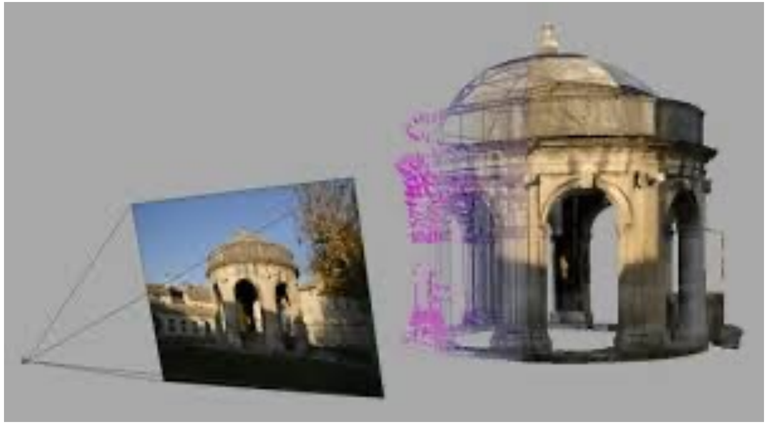


Origin of 3D data

- **Method for Model Generation:**
  1. Input from geometry file
  2. Procedure Modeling
  3. Software Modeling
  4. Using a 3D scanner to obtain a real model

Origin of 3D data

- **Method for Model Generation:**
  - Photogrammetry
    - a tradition vision method
    - reconstructing 3D data based on the photo.



Origin of 3D data

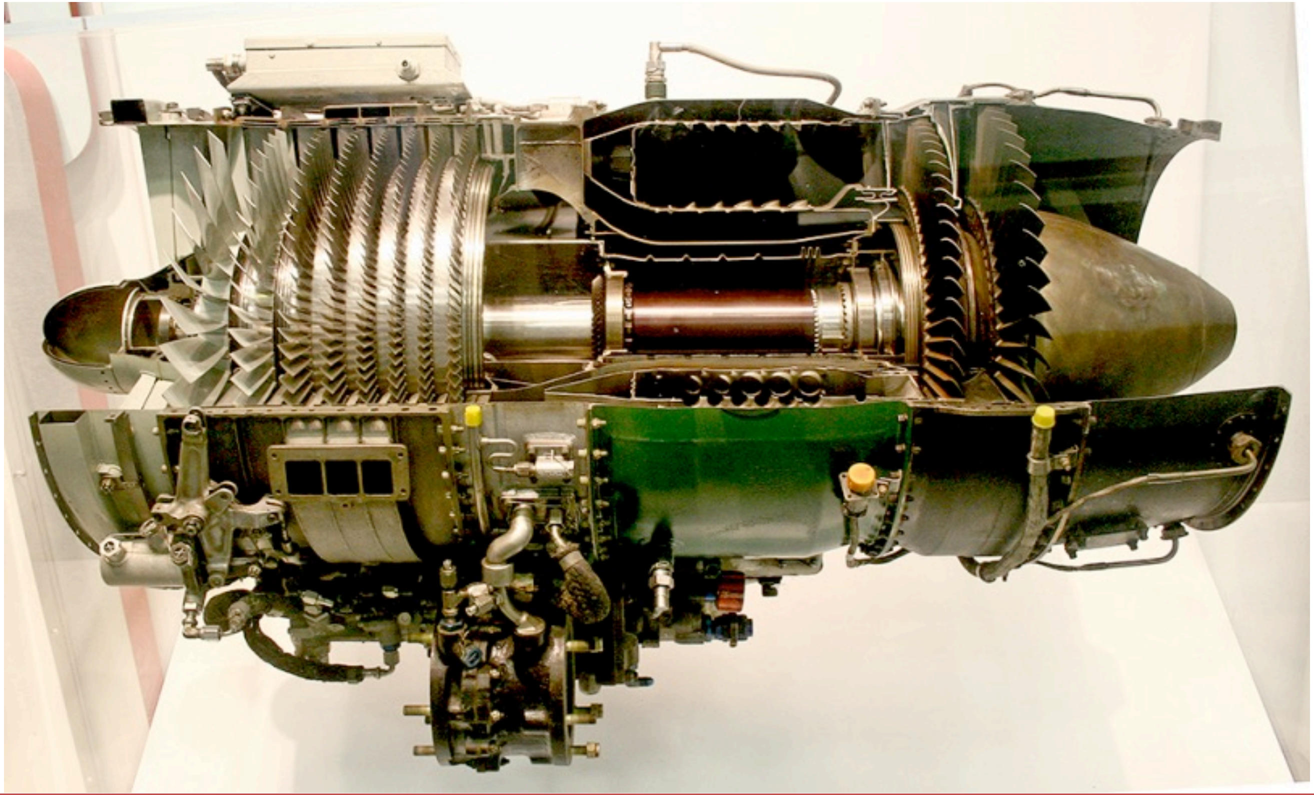
- **Method for Model Generation:**
  - Photogrammetry
  - Based on RGBD data： Microsoft Kinect
  - Combination Method

Examples of Geometry

Examples of Geometry

Examples of Geometry

Examples of Geometry



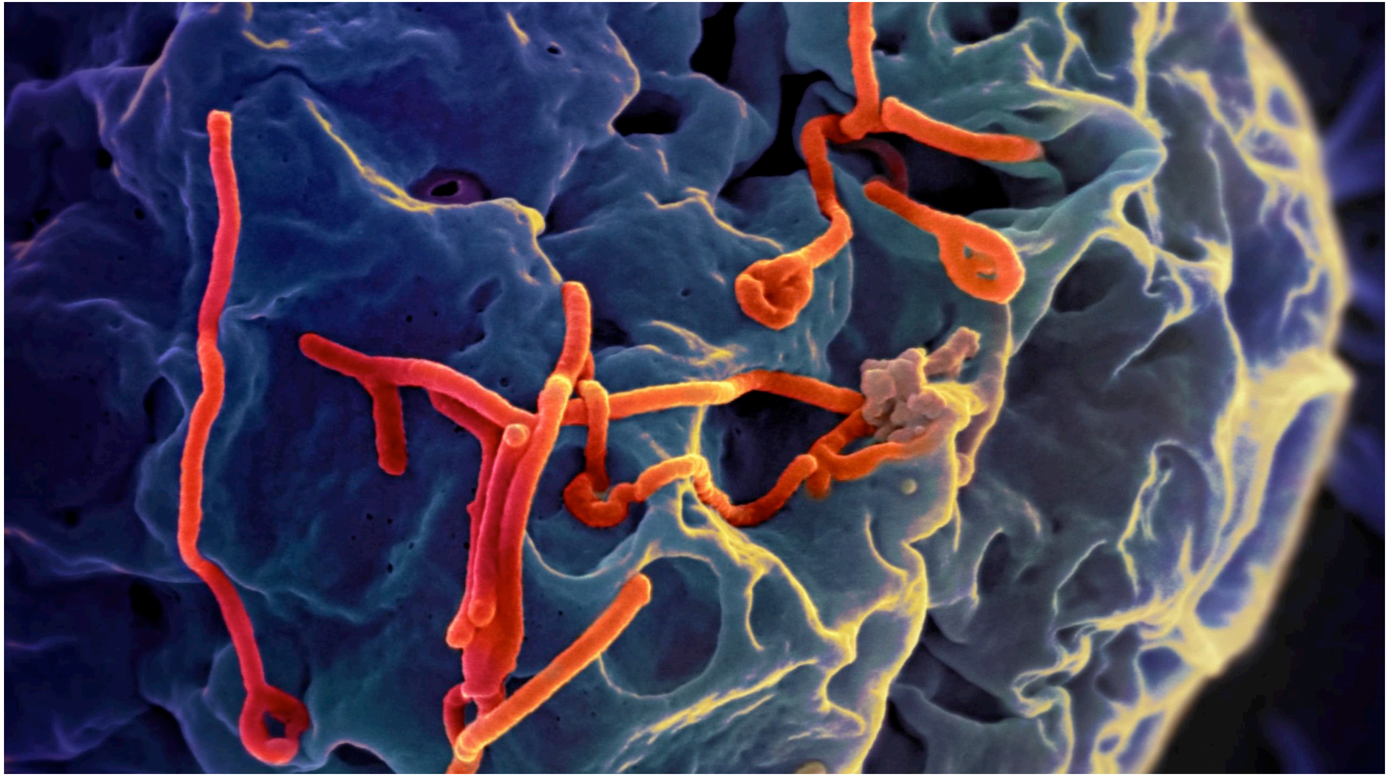
Examples of Geometry

Examples of Geometry

Examples of Geometry

Examples of Geometry

Examples of Geometry

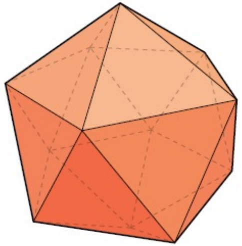
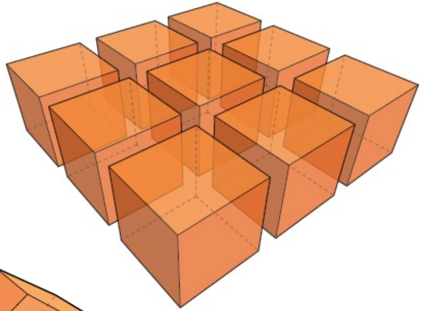
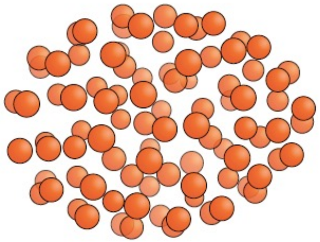
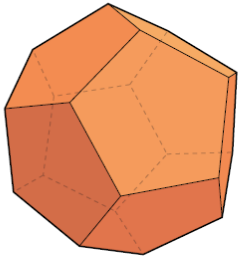
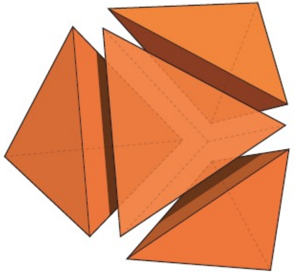


Examples of Geometry

Representations of Geometry

Many Ways to Represent Geometry

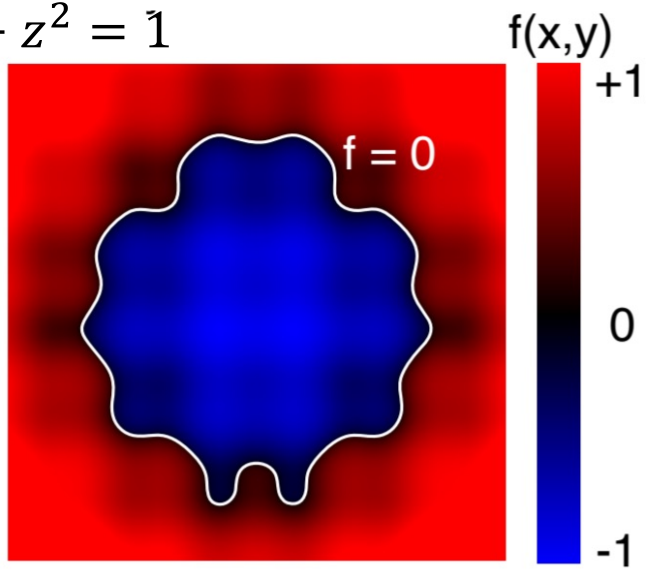
- Implicit (隐式)
  - Level sets
  - Algebraic surface
  - Distance functions
  - …
- Explicit(显式)
  - Point cloud
  - Polygon mesh
  - Subdivision, NURBS
  - …

- Each choice best suited to a different task/type of geometry

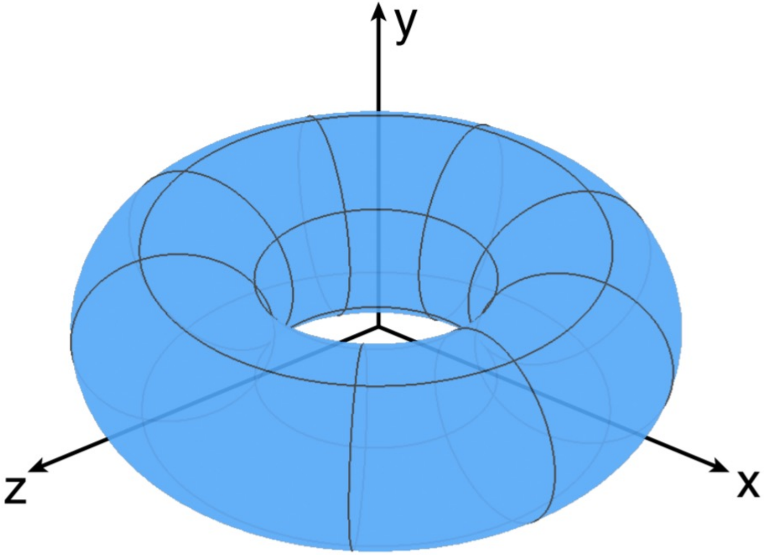
"Implicit" Representations of Geometry

- Based on classifying points
  - Points satisfy some specified relationship
- E.g. sphere: all points in 3D, where $x^2+y^2+z^2=1$
- More generally, f(x, y ,z)=0



Implicit Surface – Sampling Can Be Hard

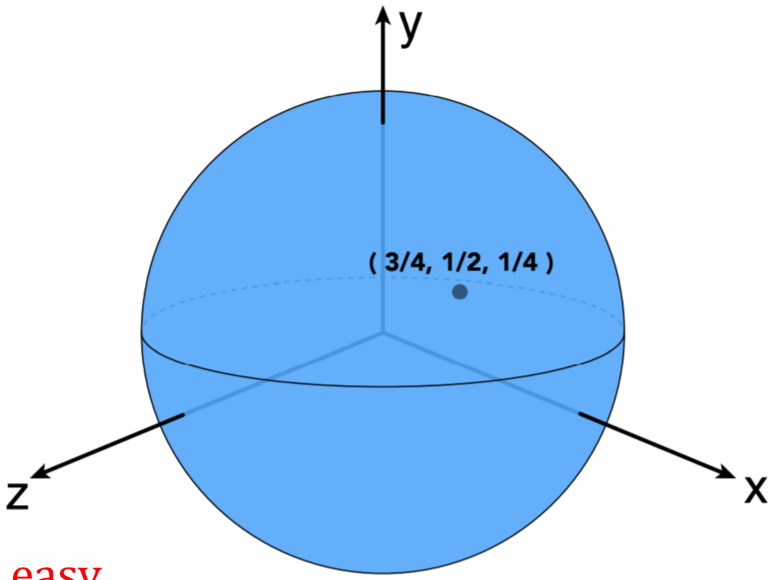
- $f(x, y, z) = (2 - \sqrt{x^2 + y^2})^2 + z^2 - 1$
- What points lie on $f(x, y, z) = 0$?



Some tasks are hard with implicit representations

Implicit Surface – Inside/Outside Tests Easy

- $f(x, y, z) = x^2 + y^2 + z^2 - 1$
- Is (3/4, 1/2, 1/4) inside?
- Just plug it in:
  - f(x, y, z)=-1/8<0
  - Yes, inside!

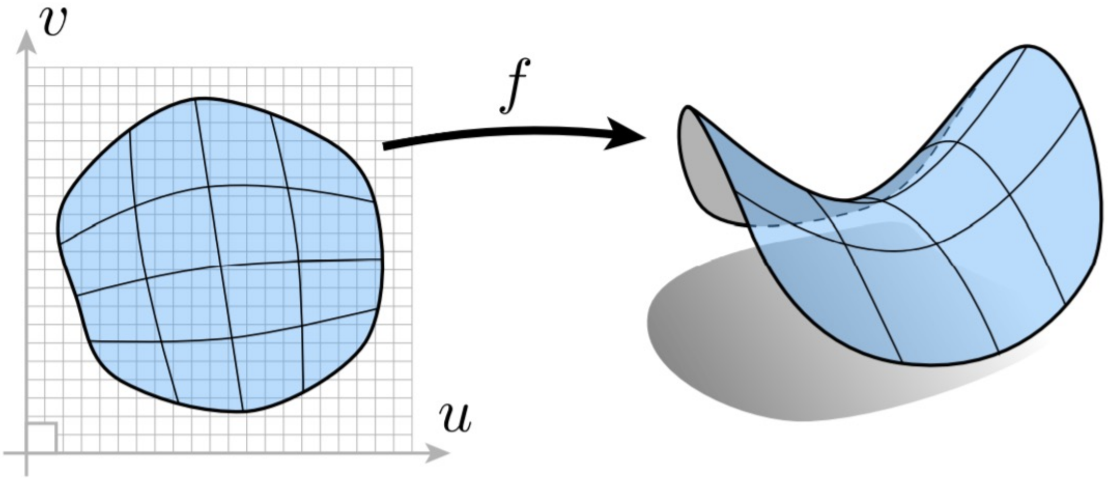


Implicit representations make some tasks easy

"Explicit" Representations of Geometry

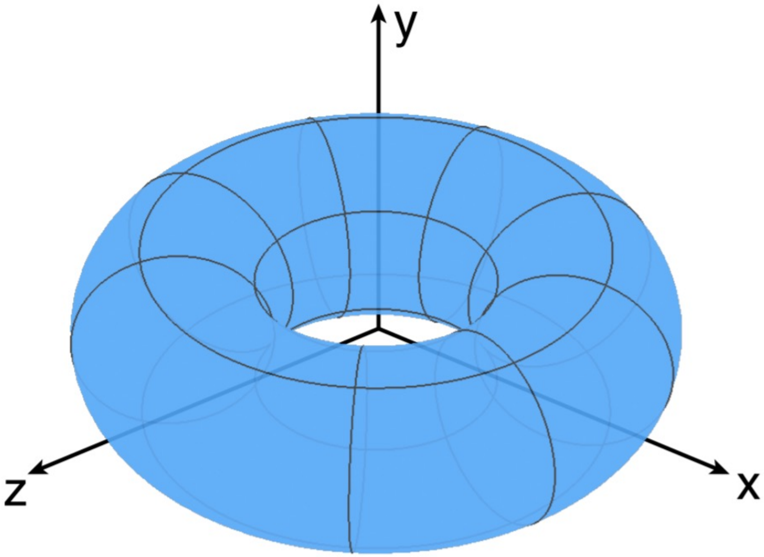
- All points are **given directly**
- or via **parameter mapping**

Generally: $f : \mathbb{R}^2 \to \mathbb{R}^3; (u, v) \mapsto (x, y, z)$



Explicit Surface – Sampling Is Easy

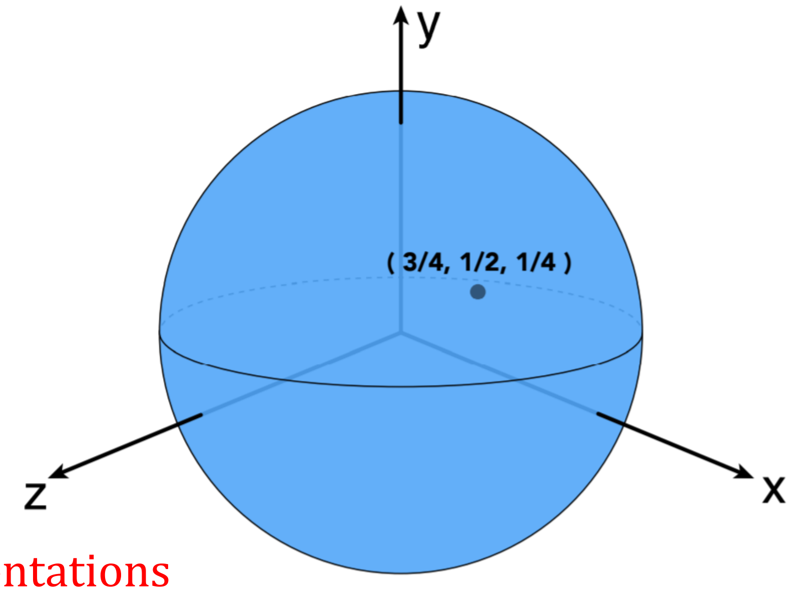
- $f(u, v) = ((2 + \cos u)\cos v, (2 + \cos u)\sin v, \sin u)$
- What points lie on this surface?
- Just plug in (u, v) values !



Explicit representations make some tasks easy

Explicit Surface – Inside/Outside Test Hard

- $f(u,v) = (\cos u \sin v, \sin u \sin v, \cos v)$
- Is $(3/4, 1/2, 1/4)$ inside?



Some tasks are hard with explicit representations

No “Best ” Representation – Geometry is Hard

“I hate meshes.
I cannot believe how hard this is.
Geometry is hard.”

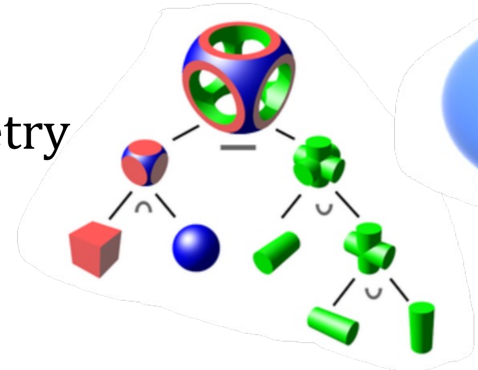
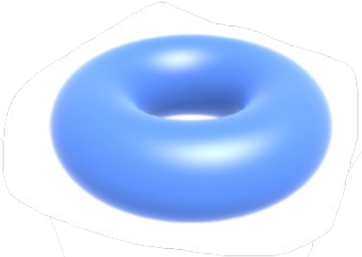
— David Baraff
Senior Research Scientist
Pixar Animation Studios

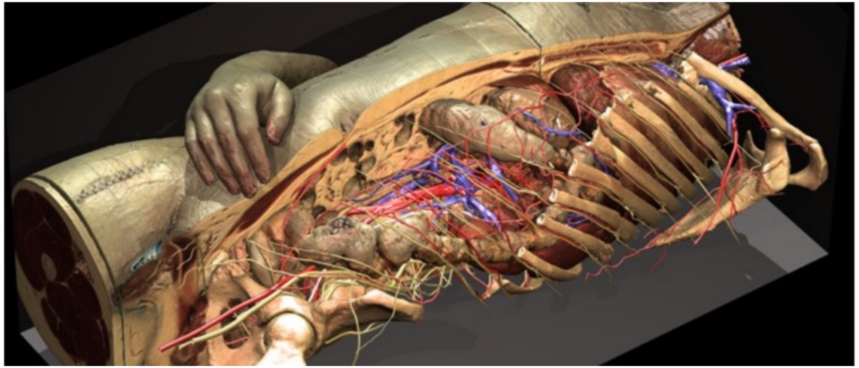
Best Representation Depends on the Task!

Implicit Representations

Many Implicit Representations in Graphics

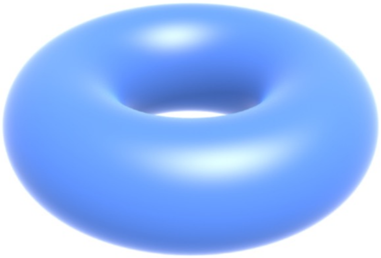
- Algebraic surfaces
- Constructive solid geometry
- Level set methods
- Fractals
- NeRF
- …

Algebraic Surfaces (Implicit)

- Surface is zero set of a polynomial in x, y, z



$$x^2+y^2+z^2=1$$

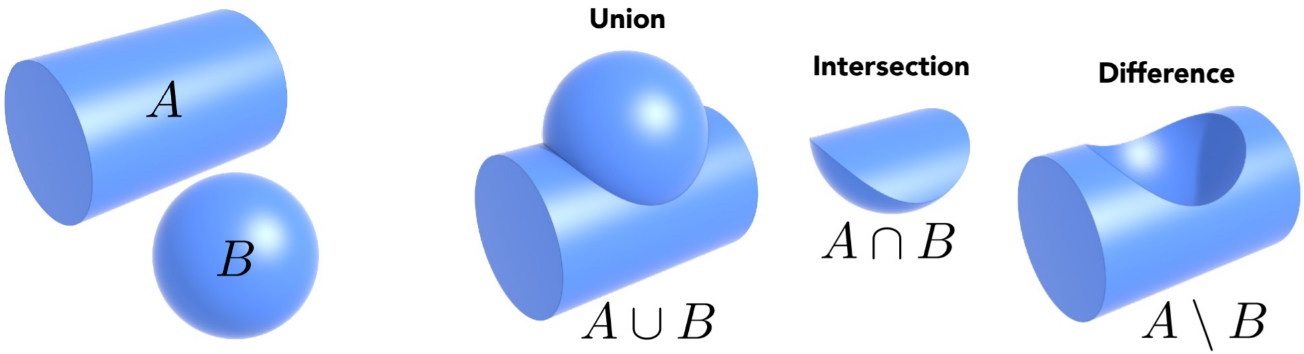
$$(R-\sqrt{x^2+y^2})^2+z^2=r^2$$

$$(x^2+\frac{9y^2}{4}+z^2-1)^3 = x^2z^3+\frac{9y^2z^3}{80}$$

More complex shapes?

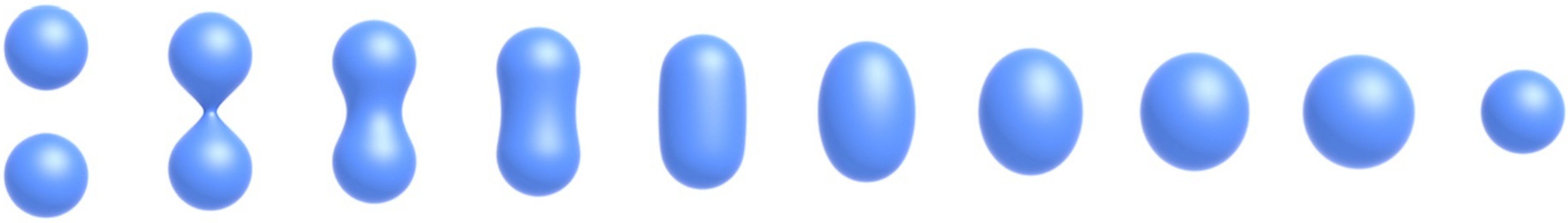
Constructive Solid Geometry(Implicit)

- Combine implicit geometry via Boolean operations



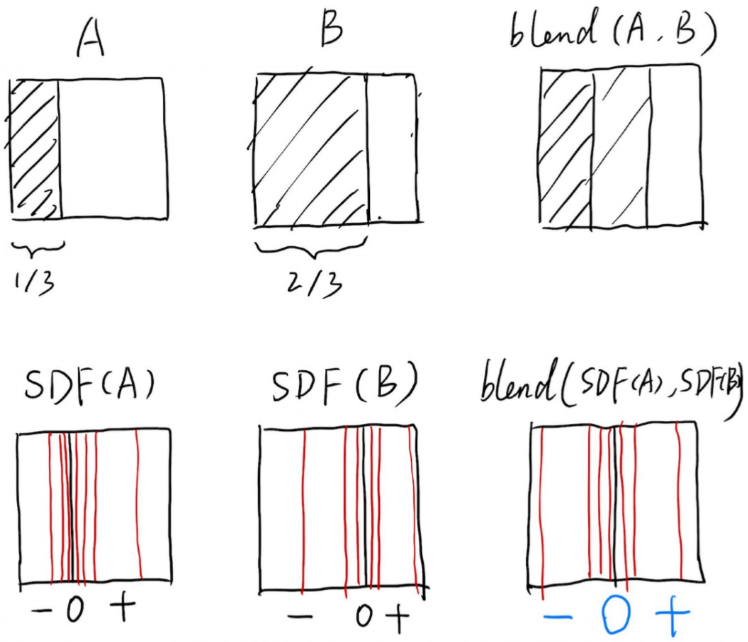
Distance Functions (Implicit)

- Instead of Booleans, gradually blend surfaces together using Distance functions:
- giving minimum distance (could be signed distance) from anywhere to object



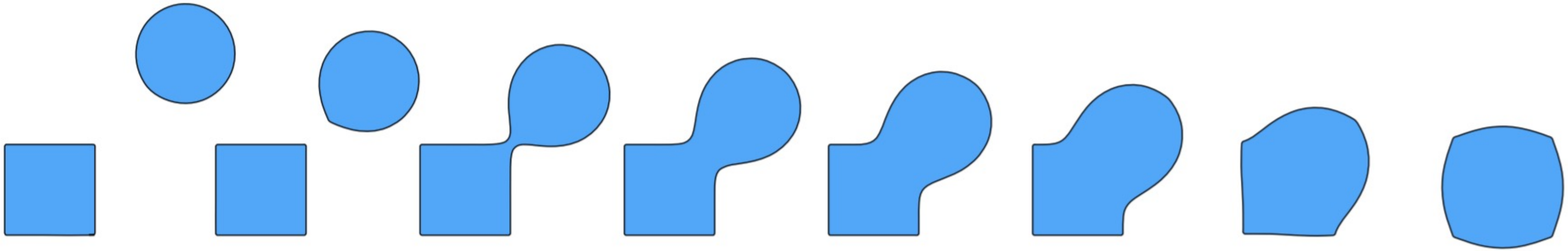
Distance Functions (Implicit)

- An Example: Blending a moving boundary



Blending Distance Functions (Implicit)

- Can blend any two distance functions d1, d2:

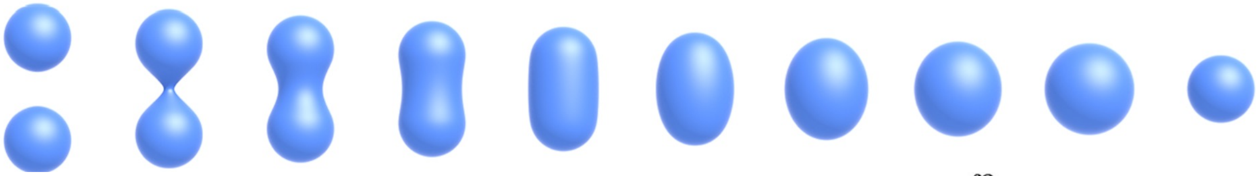


Similar strategy to points, though many possibilities. E.g.,

$$f(x) := e^{d_1(x)^2} + e^{d_2(x)^2} - \frac{1}{2}$$

Blending Distance Functions (Implicit)

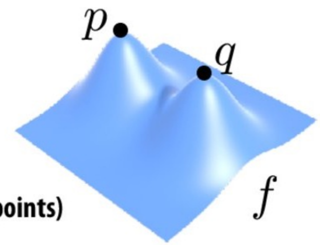
- Gradually blend two surfaces together by blending distance functions d1, d2

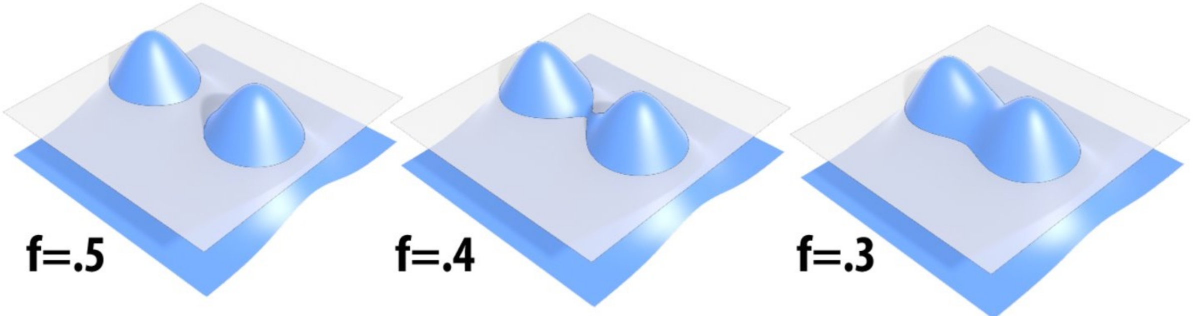


- **Easier to understand in 2D:**

$$\phi_p(x) := e^{-\underbrace{|x-p|}_{d}^{2}}$$ (Gaussian centered at p)

$$f := \phi_p + \phi_q$$ (Sum of Gaussians centered at different points)

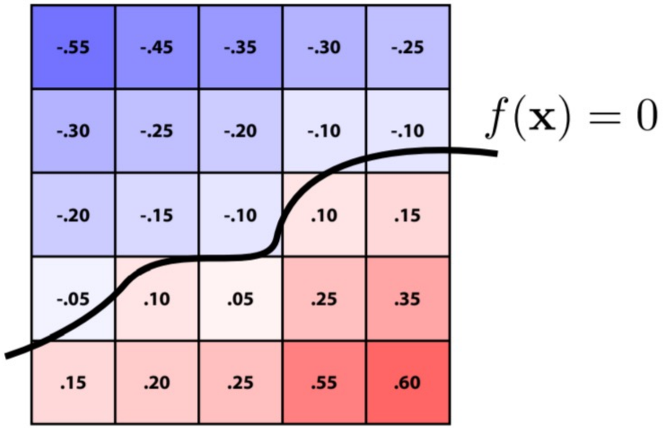




Scene of Pure Distance Functions (Demo)

Level Set Methods 水平集(Also implicit)

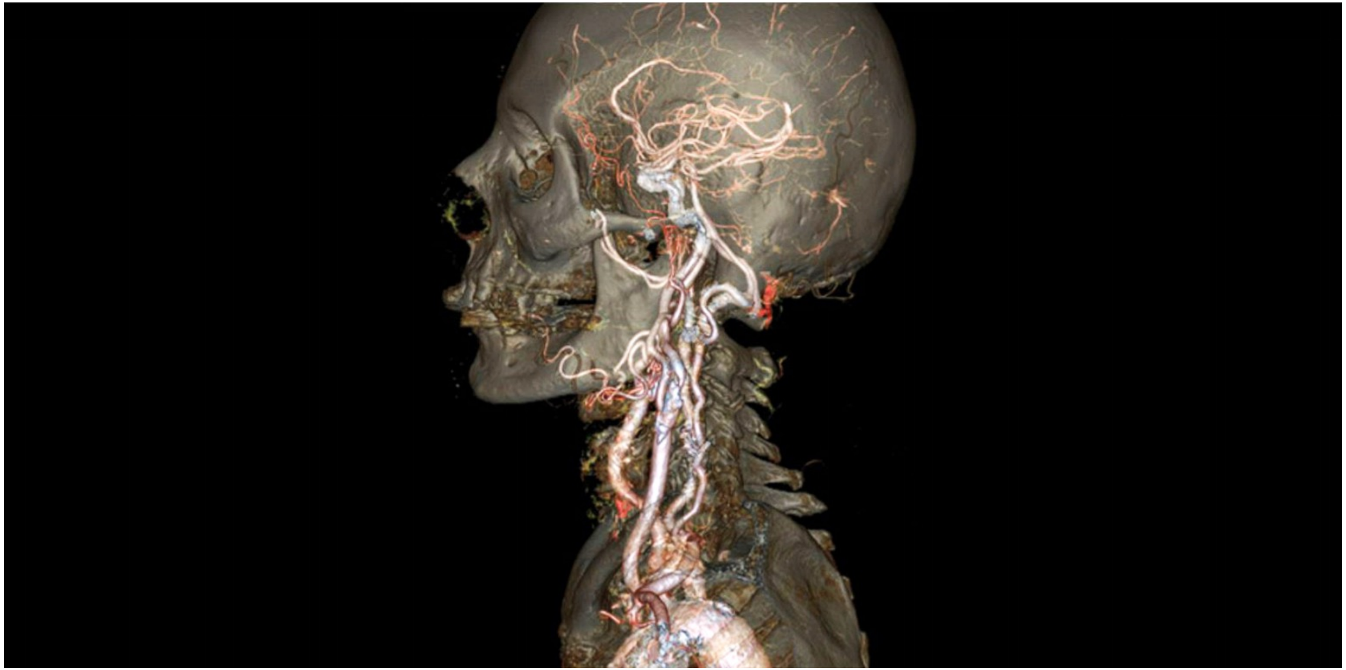
- Closed-form equations are hard to describe complex shapes
- Alternative: store a grid of values approximating function



- Surface is found where interpolated values equal zero Provides much more explicit control over shape (like a texture)

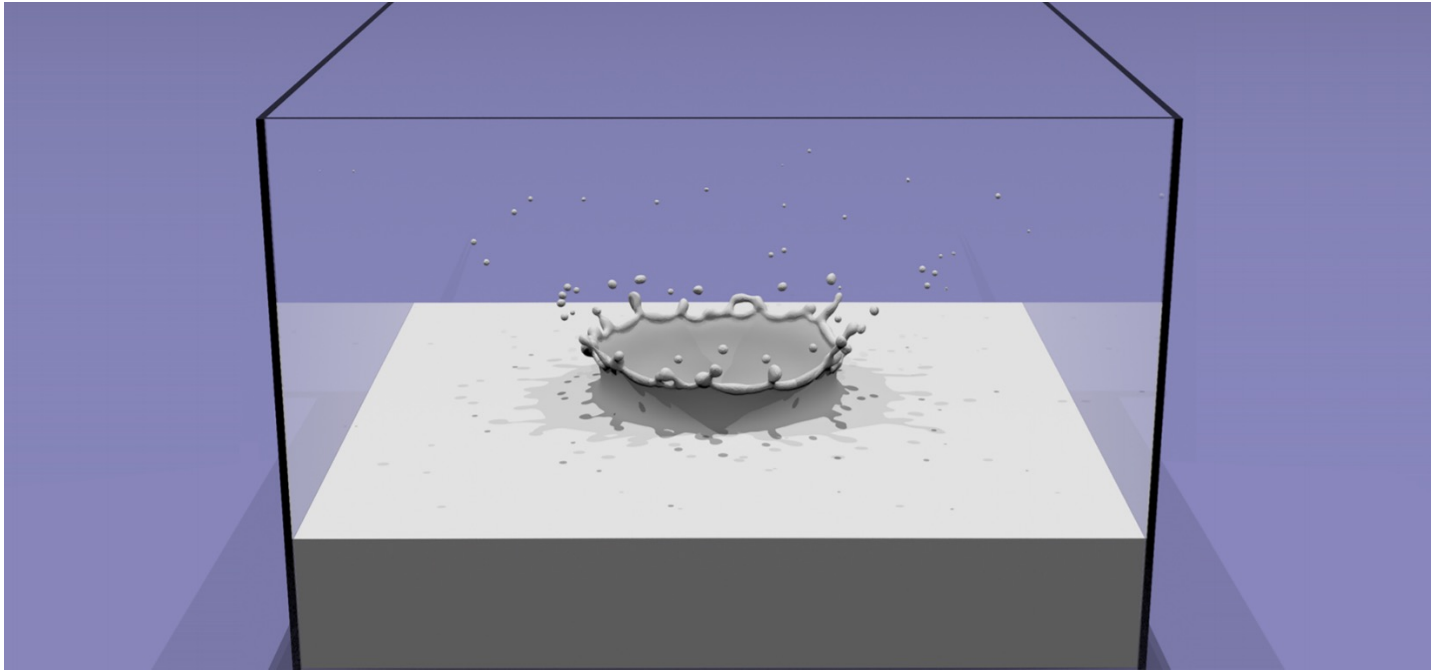
Level Sets from Medical Data (CT, MRI, etc.)

- Level sets encode, e.g., constant tissue density



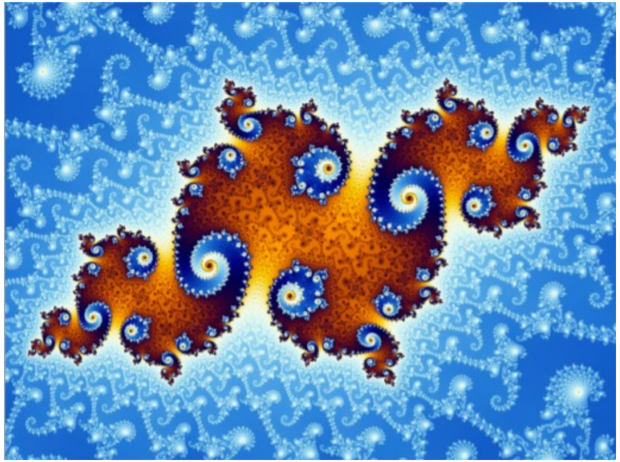
Level Sets in Physical Simulation

- Level set encodes distance to air-liquid boundary



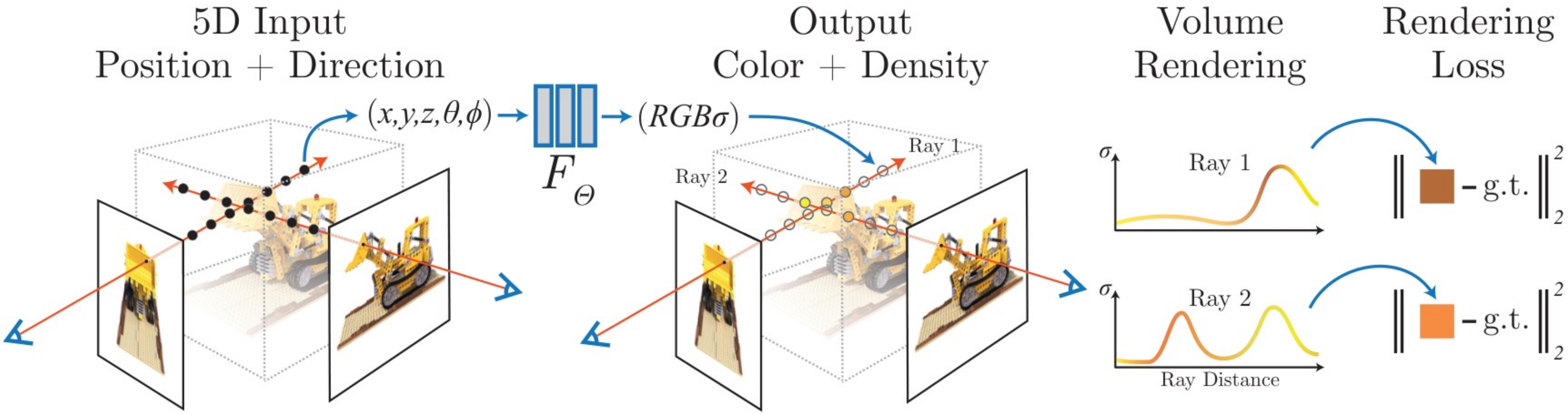
Fractals (Implicit)

- Exhibit self-similarity, detail at all scales
- "Language" for describing natural phenomena
- Hard to control shape!



NeRF (Neural Radiance Fields)

- Implicit representation of complex scenes using neural network
- Volumetric representation
- Learn from a sparse set of input views



NeRF (Neural Radiance Fields)

- Implicit representation of complex scenes using neural network
- Volumetric representation
- Learn from a sparse set of input views

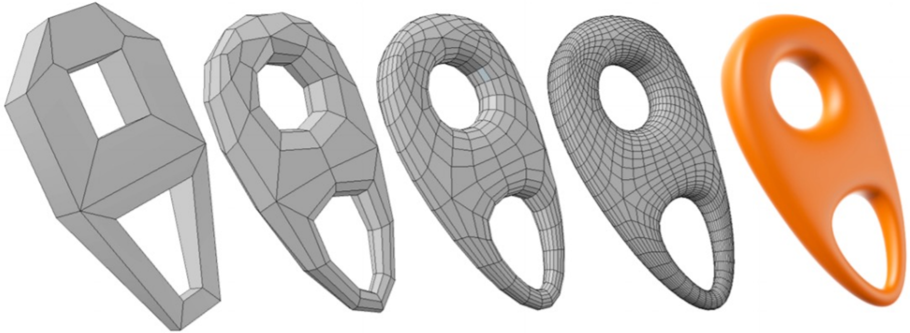
Implicit Representations - Pros & Cons

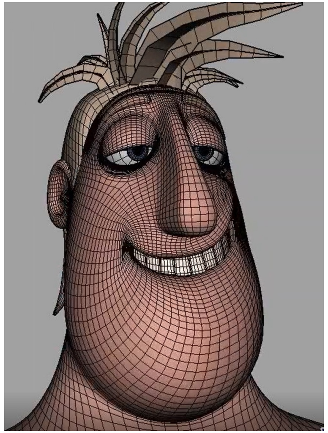
- Pros:
  - compact description (e.g., a function)
  - certain queries easy (inside object, distance to surface)
  - good for ray-to-surface intersection (more later)
  - for simple shapes, exact description / no sampling error
  - easy to handle changes in topology (e.g., fluid)
- Cons:
  - difficult to model complex shapes

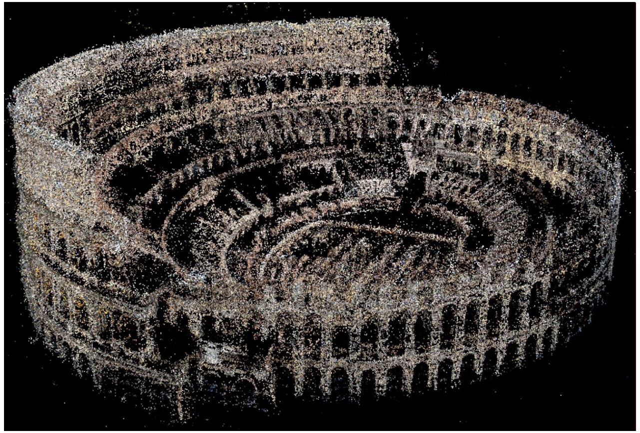
Explicit Representations

Many Explicit Representations in Graphics

- Triangle meshes
- Bezier surfaces
- Subdivision surfaces
- NURBS
- Point cloud
- …

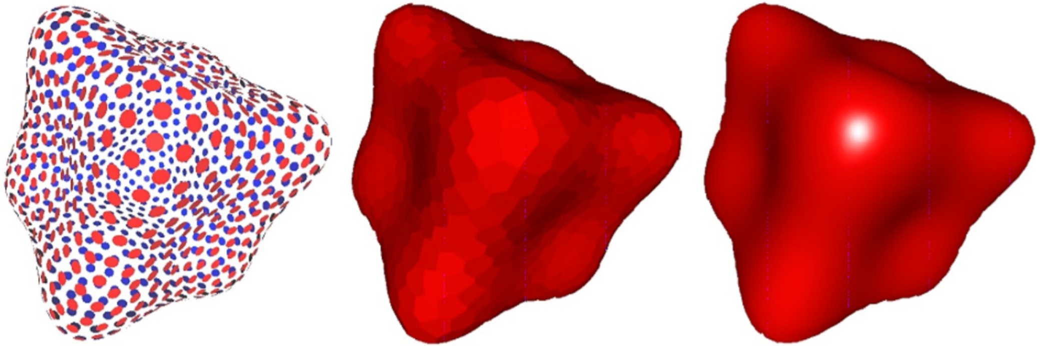






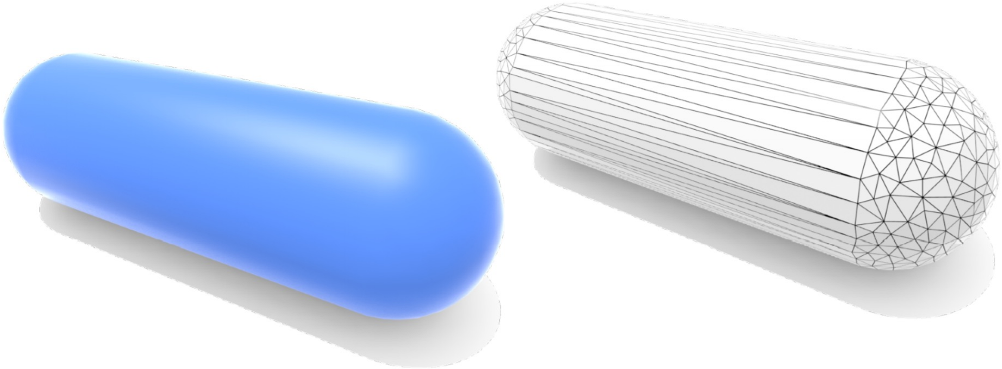
Point Cloud (Explicit)

- Easiest representation: list of points (x,y,z)
- Easily represent any kind of geometry
- Useful for LARGE datasets (>>1 point/pixel)
- Often converted into polygon mesh
- Difficult to draw in undersampled regions



Polygon Mesh (Explicit)

- Store vertices & polygons (often triangles or quads)
- Easier to do processing / simulation, adaptive sampling
- More complicated data structures
- Perhaps most common representation in graphics



Polygon Mesh (Explicit)

- Basic Concepts of Triangle mesh
- Descriptions
  - A list of faces $F = (f_1, f_2, \ldots, f_n)$
    - Each face is triangle.
  - A list of vertices $V = (v_1, v_2, \ldots, v_n)$
  - A sequence of vertices from V forms a face in F

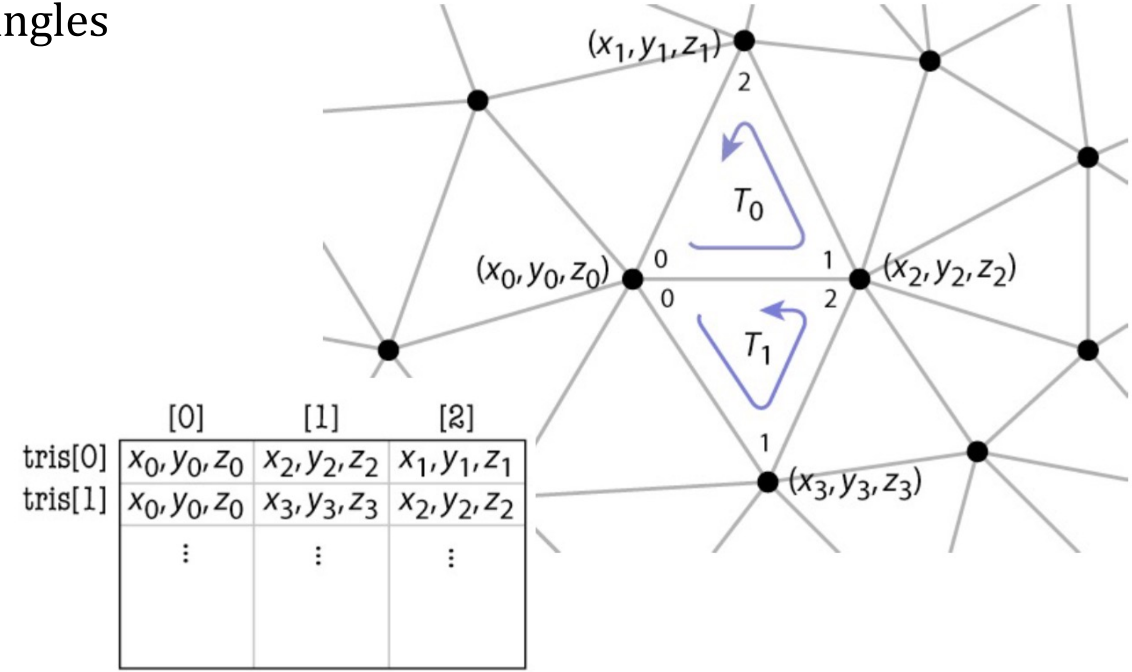
    e.g.

    $f_1 : (v_1, v_2, v_3)$, $f_2 : (v_4, v_5, v_6)$,

    $f_3 : (v_7, v_8, v_9)$, …

Polygon Mesh (Explicit)

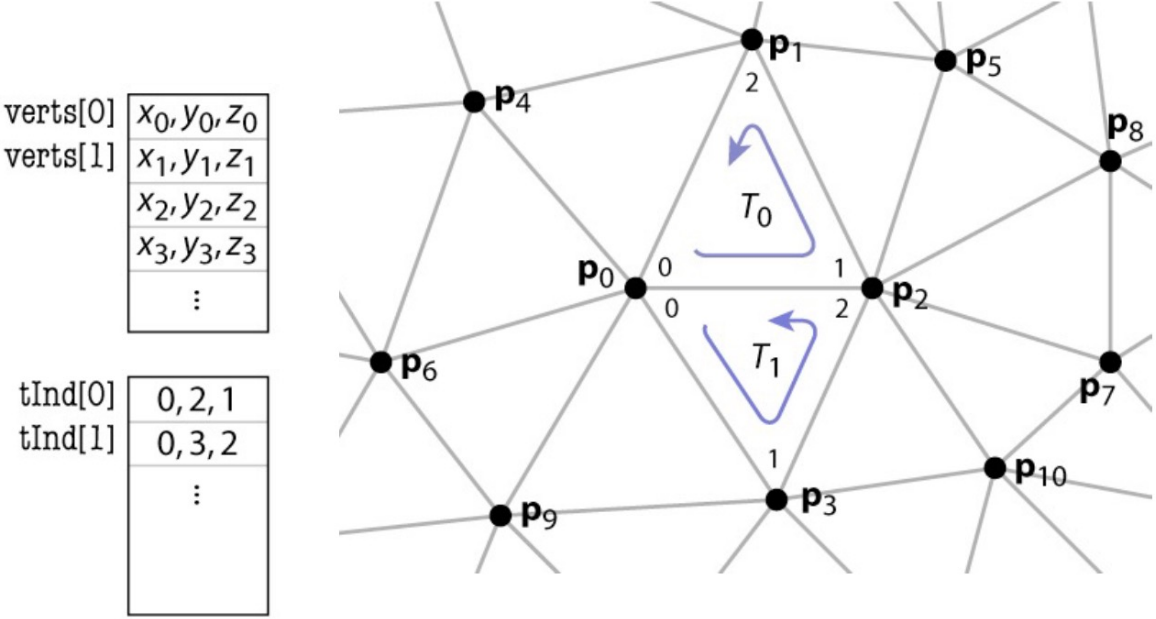
List of Triangles



	[0]	[1]	[2]
tris[0]	x_0, y_0, z_0	x_2, y_2, z_2	x_1, y_1, z_1
tris[1]	x_0, y_0, z_0	x_3, y_3, z_3	x_2, y_2, z_2
	⋮	⋮	⋮

Polygon Mesh (Explicit)

Lists of Points / Indexed Triangle



The Wavefront Object File Format (.obj)

- Commonly used in Graphics research
- Just a text file that specifies vertices, normals, texture coordinates **and their connectivities**

```
# This is a comment

v 1.000000 -1.000000 -1.000000
v 1.000000 -1.000000 1.000000
v -1.000000 -1.000000 1.000000
v -1.000000 -1.000000 -1.000000
v 1.000000 1.000000 -1.000000
v 0.999999 1.000000 1.000001
v -1.000000 1.000000 1.000000
v -1.000000 1.000000 -1.000000

vt 0.748573 0.750412
vt 0.749279 0.501284
vt 0.999110 0.501077
vt 0.999455 0.750380
vt 0.250471 0.500702
vt 0.249682 0.749677
vt 0.001085 0.750380
vt 0.001517 0.499994
vt 0.499422 0.500239
vt 0.500149 0.750166
vt 0.748355 0.998230
vt 0.500193 0.998728
vt 0.498993 0.250415
vt 0.748953 0.250920

vn 0.000000 0.000000 -1.000000
vn -1.000000 -0.000000 -0.000000
vn -0.000000 -0.000000 1.000000
vn -0.000001 0.000000 1.000000
vn 1.000000 -0.000000 0.000000
vn 1.000000 0.000000 0.000001
vn 0.000000 1.000000 -0.000000
vn -0.000000 -1.000000 0.000000

f 5/1/1 1/2/1 4/3/1
f 5/1/1 4/3/1 8/4/1
f 3/5/2 7/6/2 8/7/2
f 3/5/2 8/7/2 4/8/2
f 2/9/3 6/10/3 3/5/3
f 6/10/4 7/6/4 3/5/4
f 1/2/5 5/1/5 2/9/5
f 5/1/6 6/10/6 2/9/6
f 5/1/7 8/11/7 6/10/7
f 8/11/7 7/12/7 6/10/7
f 1/2/8 2/9/8 3/13/8
f 1/2/8 3/13/8 4/14/8
```

Geometry Exercise 1

- 1. List **Implicit** Representations in Graphics

- 2. List **Explicit** Representations in Graphics

Thank you!