

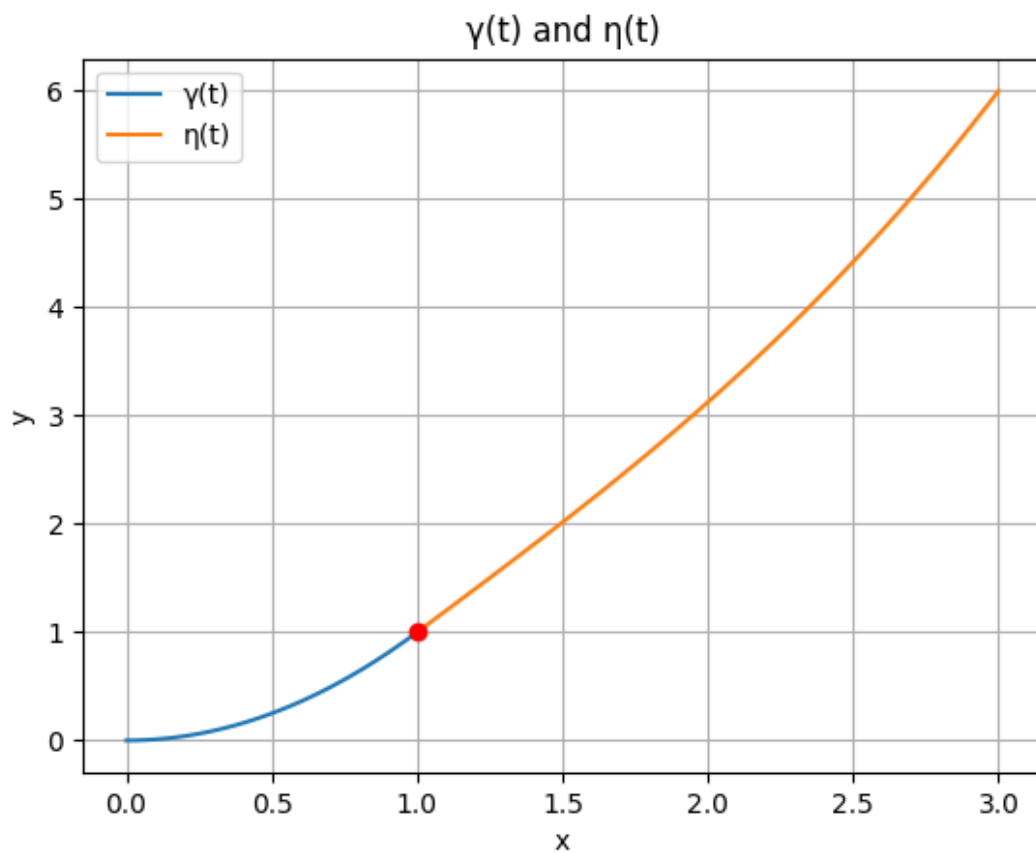
Homework 3

Weiji Xie @ 2025/04/12

Task 1

Plot

The result image is shown below:



Answer

$$\begin{aligned}\gamma(t) &= (t, t^2) \\ \frac{d\gamma}{dt} &= (1, 2t) \\ \frac{d\gamma}{dt}(1) &= (1, 2)\end{aligned}$$

(1)

$$\begin{aligned}\eta(t) &= (2t + 1, t^3 + 4t + 1) \\ \frac{d\eta}{dt} &= (2, 3t^2 + 4) \\ \frac{d\eta}{dt}(0) &= (2, 4)\end{aligned}$$

C1 continuity: Notice that the derivative of $\gamma'(t=1)$ and $\eta'(t=0)$ are not equal, so $\gamma(t)$ and $\eta(t)$ are not C1 continuous at $t=1$.

G1 continuity: Notice that the derivative of $\gamma'(t=1)$ and $\eta'(t=0)$ have the same direction, so $\gamma(t)$ and $\eta(t)$ are G1 continuous at $t=1$.

Task 2

Calculation

By definition:

$$\begin{aligned}
B_{0,1}(t) &= \mathbb{1}[0 \leq t < 1] \\
B_{1,1}(t) &= \mathbb{1}[1 \leq t < 3] \\
B_{2,1}(t) &= \mathbb{1}[3 \leq t < 4] \\
B_{3,1}(t) &= \mathbb{1}[4 \leq t < 5] \\
B_{0,2}(t) &= \frac{t}{1}B_{0,1}(t) + \frac{3-t}{2}B_{1,1}(t) \\
B_{1,2}(t) &= \frac{t-1}{2}B_{1,1}(t) + \frac{4-t}{1}B_{2,1}(t) \\
B_{2,2}(t) &= \frac{t-3}{1}B_{2,1}(t) + \frac{5-t}{1}B_{3,1}(t) \\
B_{0,3}(t) &= \frac{t}{3}B_{0,2}(t) + \frac{4-t}{3}B_{1,2}(t) \\
B_{1,3}(t) &= \frac{t-1}{3}B_{1,2}(t) + \frac{5-t}{2}B_{2,2}(t) \\
B_{0,4}(t) &= \frac{t}{4}B_{0,3}(t) + \frac{5-t}{4}B_{1,3}(t)
\end{aligned} \tag{2}$$

The result is shown below:

$$B_{0,2}(t) = \begin{cases} t, & 0 \leq t < 1 \\ \frac{3-t}{2}, & 1 \leq t < 3 \\ 0, & \text{其他} \end{cases} \tag{3}$$

$$B_{1,2}(t) = \begin{cases} \frac{t-1}{2}, & 1 \leq t < 3 \\ 4-t, & 3 \leq t < 4 \\ 0, & \text{其他} \end{cases} \tag{4}$$

$$B_{2,2}(t) = \begin{cases} t-3, & 3 \leq t < 4 \\ 5-t, & 4 \leq t < 5 \\ 0, & \text{其他} \end{cases} \tag{5}$$

$$B_{0,3}(t) = \begin{cases} \frac{t^2}{3}, & 0 \leq t < 1 \\ \frac{t(3-t)+(4-t)(t-1)}{6}, & 1 \leq t < 3 \\ \frac{(4-t)^2}{3}, & 3 \leq t < 4 \\ 0, & \text{其他} \end{cases} \tag{6}$$

$$B_{1,3}(t) = \begin{cases} \frac{(t-1)^2}{6}, & 1 \leq t < 3 \\ \frac{(t-1)(4-t)}{3} + \frac{(5-t)(t-3)}{2}, & 3 \leq t < 4 \\ \frac{(5-t)^2}{2}, & 4 \leq t < 5 \\ 0, & \text{其他} \end{cases} \quad (7)$$

$$B_{0,4}(t) = \begin{cases} \frac{t^3}{12}, & 0 \leq t < 1 \\ \frac{t}{4} \left(\frac{t(3-t)+(4-t)(t-1)}{6} \right) + \frac{5-t}{4} \left(\frac{(t-1)^2}{6} \right), & 1 \leq t < 3 \\ \frac{t}{4} \left(\frac{(4-t)^2}{3} \right) + \frac{5-t}{4} \left(\frac{(t-1)(4-t)}{3} + \frac{(5-t)(t-3)}{2} \right), & 3 \leq t < 4 \\ \frac{(5-t)^3}{8}, & 4 \leq t < 5 \\ 0, & \text{其他} \end{cases} \quad (8)$$

Plot

