Solid Modeling

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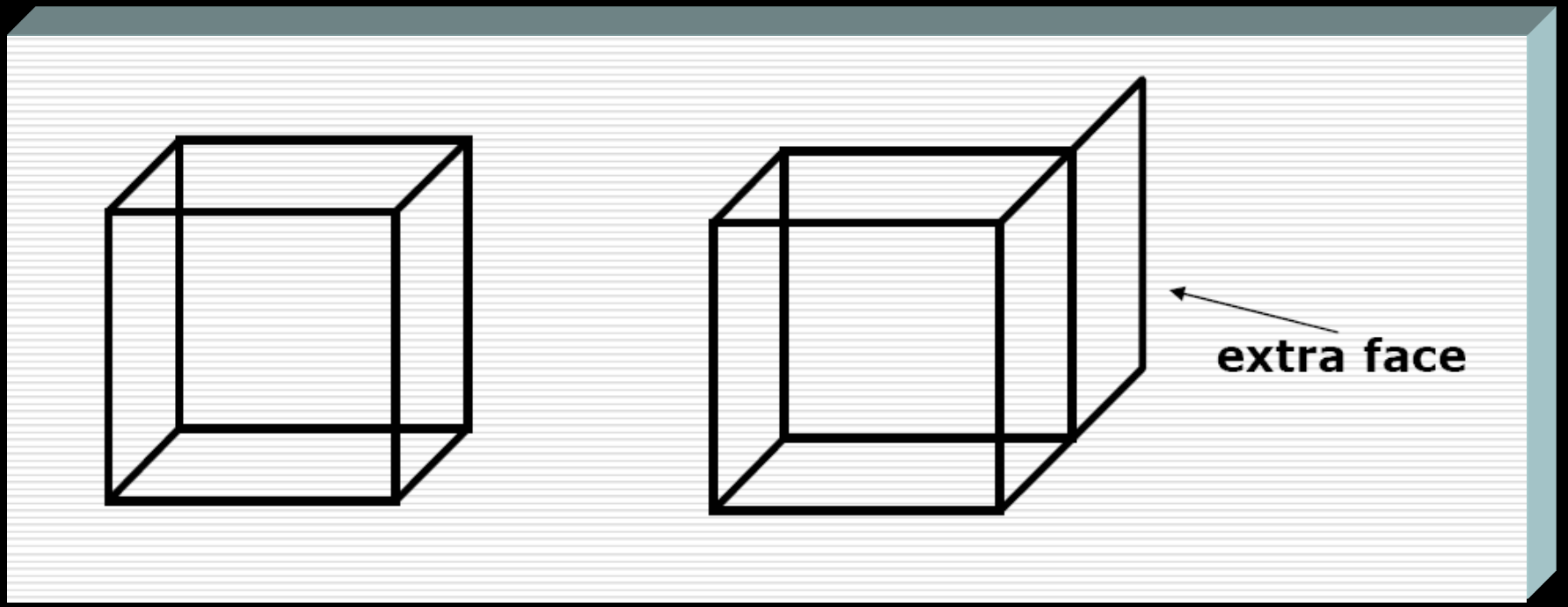
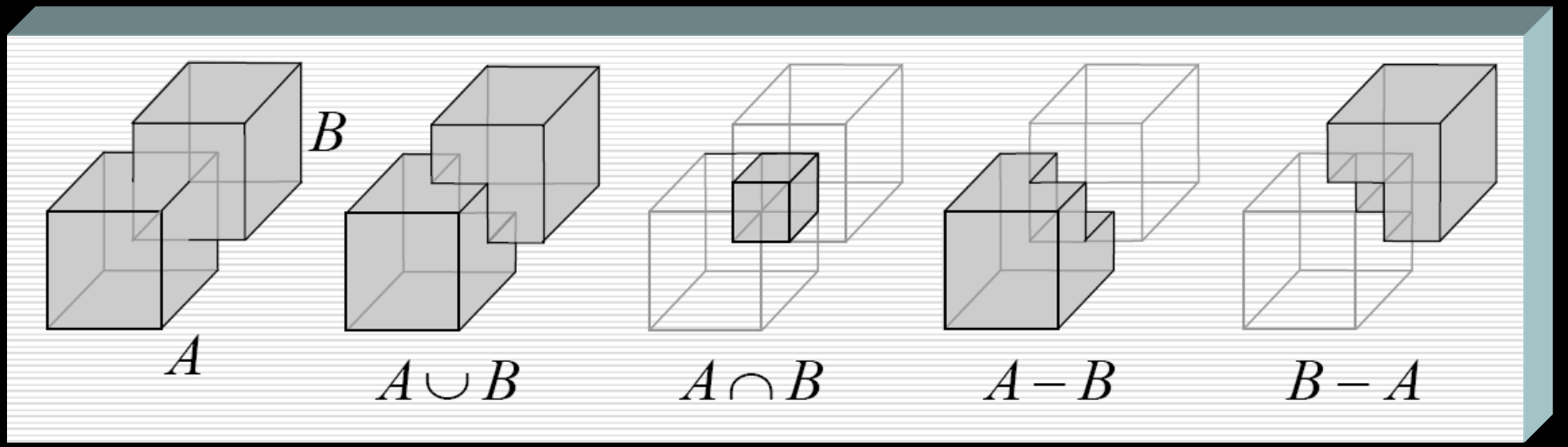
- Boundary Representations
- Spatial-Partitioning Representations
- Constructive Solid Geometry

What is Solid?



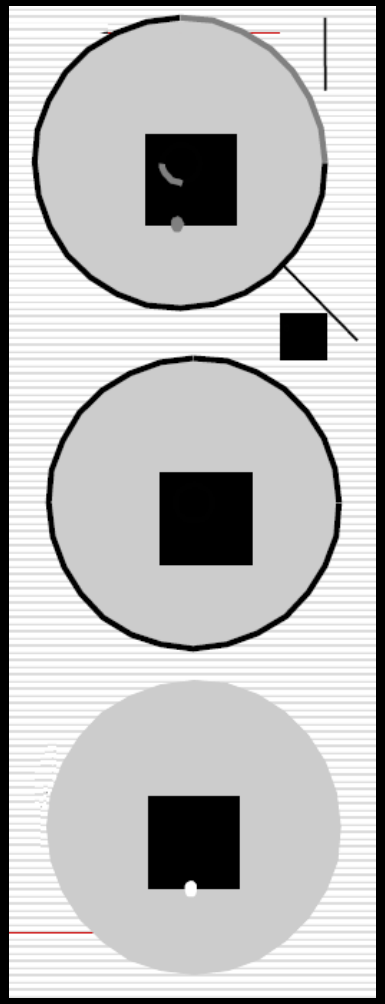
Ordinary Boolean Set Operations



- The result may not be a solid model!

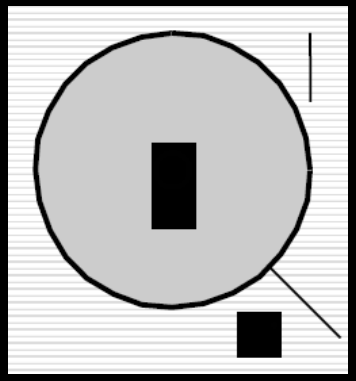
Regularized Boolean Set Operations

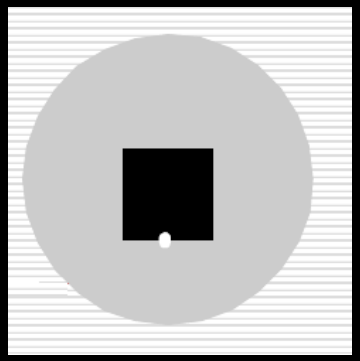
- Boundary / interior points
  - Points whose distance from the object and the object's complement is zero / other points
- Closed set
  - A set contains all its boundary points
- Open set
  - A set contains none of its boundary points



Regularized Boolean Set Operations

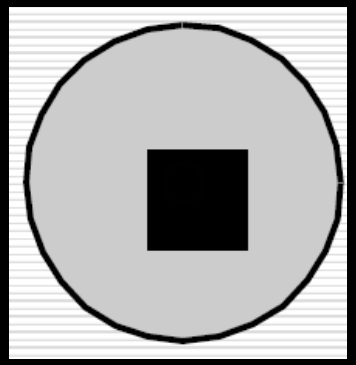
- Closure
  - The union of a set with the set of its boundary points
  - Is a closed set
- Boundary
  - The set of closed set's boundary points
- Interior
  - The complement of the boundary with respect to the object



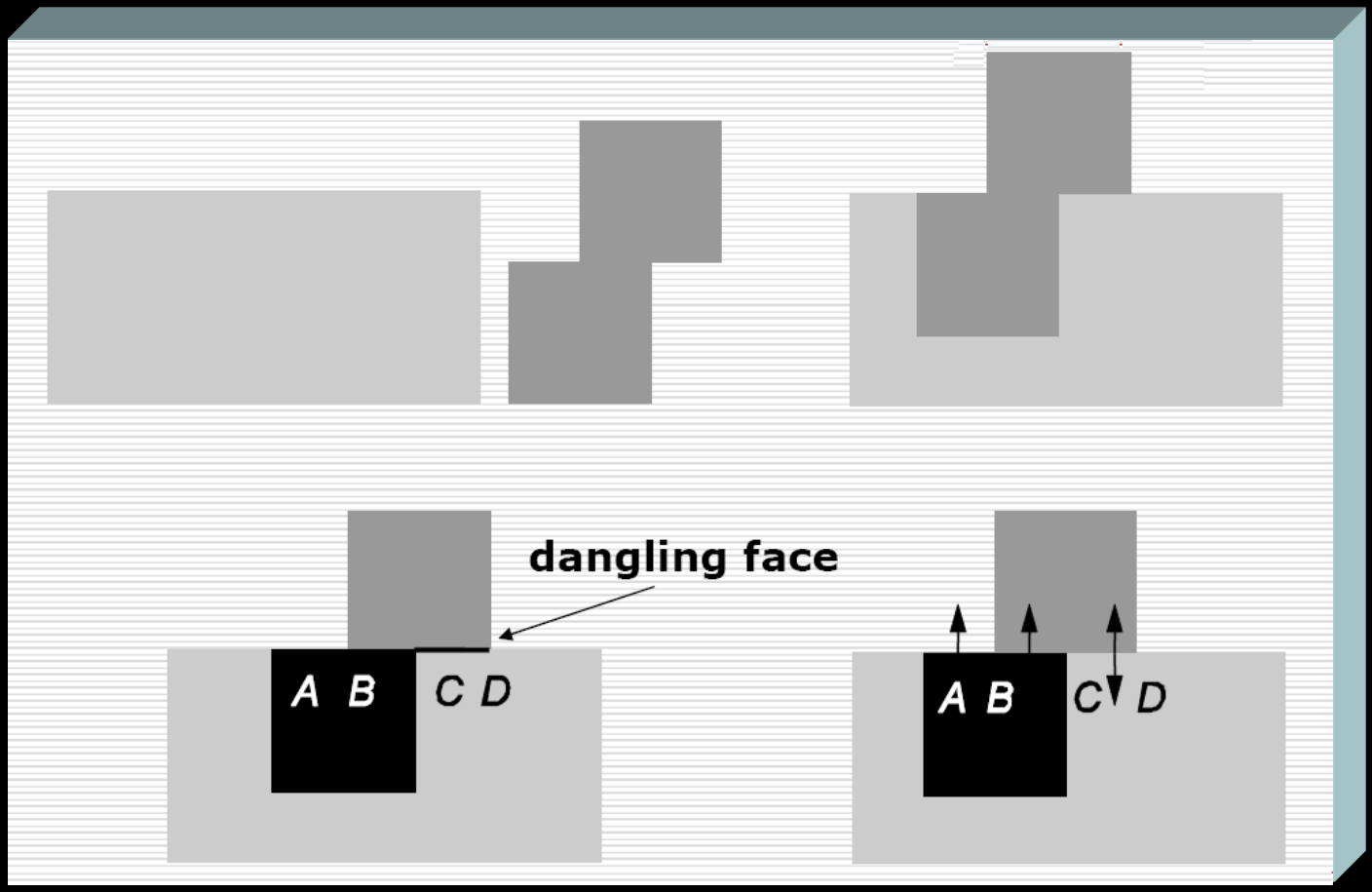


Regularized Boolean Set Operations

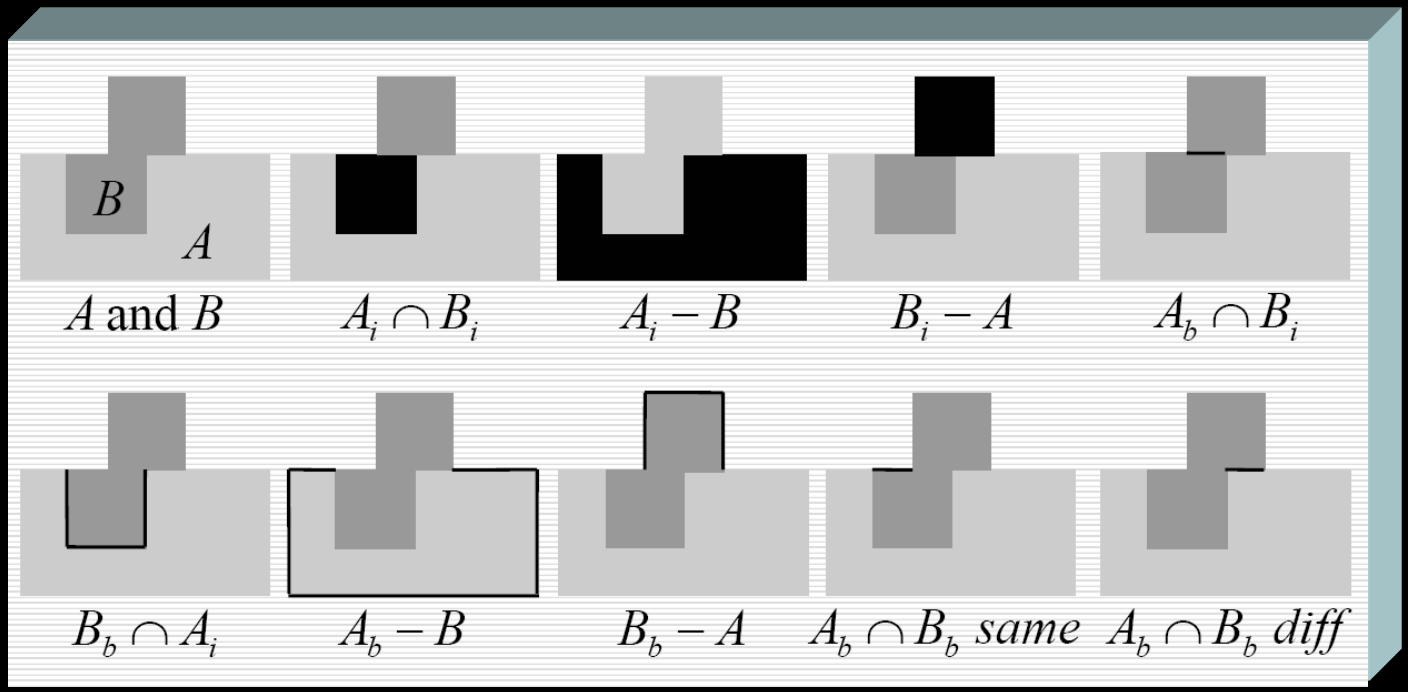
- Regularization
  - The closure of a set's interior points
- Regular set
  - A set is equal to its own regularization
- Regularized Boolean set operator
  - *A op* B*= closure (interior (*A op B*))
  - Only produce the regular set when applied to regular sets



Ordinary vs. Regularized Boolean Set Operations



Ordinary Boolean Operations on Subsets of Two Objects



Regularized Boolean Set Operations

set	$A \cup^* B$	$A \cap^* B$	$A -^* B$
$A_i \cap B_i$	X	X	
$A_i - B$	X		X
$B_i - A$	X		
$A_b \cap B_i$		X	
$B_b \cap A_i$		X	X
$A_b - B$	X		X
$B_b - A$	X		
$A_b \cap B_b$ *same*	X	X	
$A_b \cap B_b$ *diff*			X

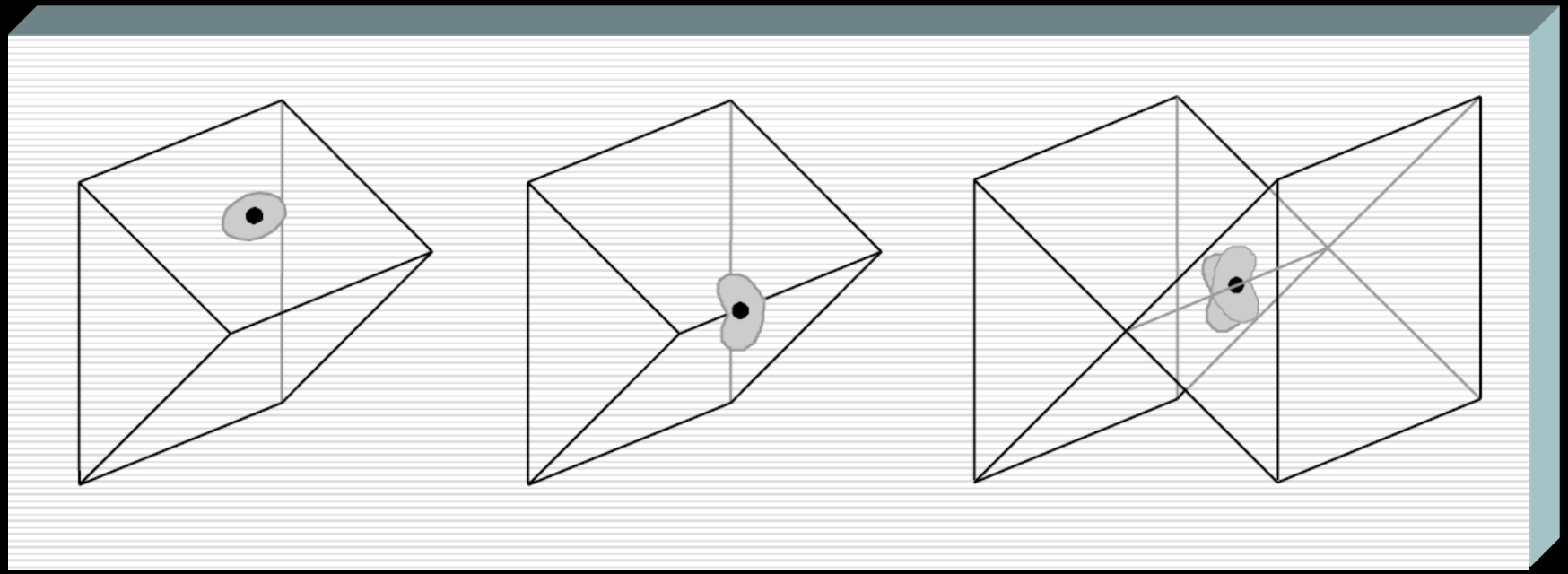
Boundary Representations = b-reps

- Describe an object in terms of its surface boundaries
  - Vertices / edges / faces
  - Some b-reps are restricted to planar, polygonal boundaries, and may require faces to be convex polygons or triangles
  - Some systems support only solids whose boundaries are **2-manifolds**

2-Manifolds

- What is 2-manifolds ?
  - Every point on a 2-manifold has some arbitrarily small neighborhood of points around it that can be considered topologically the same as a disk in the plane
  - Every edge is shared by exactly two triangles and every triangle shares an edge with exactly three neighboring triangles

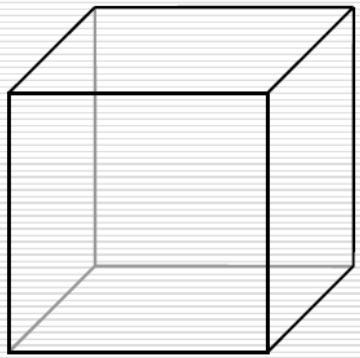
2-Manifolds Example



Euler's Formula

- Polyhedron
  - A solid that is bounded by a set of polygons whose edges are each a member of an even number of polygons
- Simple Polyhedron
  - A polyhedron that can be deformed into a sphere
- Euler's Formula
  - A simple polyhedron satisfies $V-E+F=2$

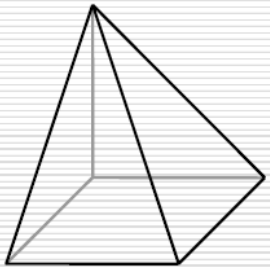
Simple Polyhedron Example



$V = 8$

$E = 12$

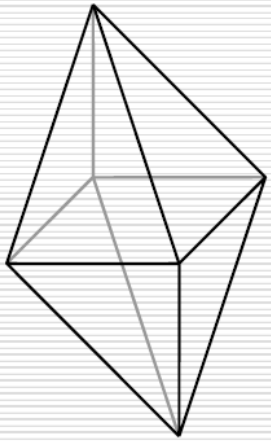
$F = 6$



$V = 5$

$E = 8$

$F = 5$



$V = 6$

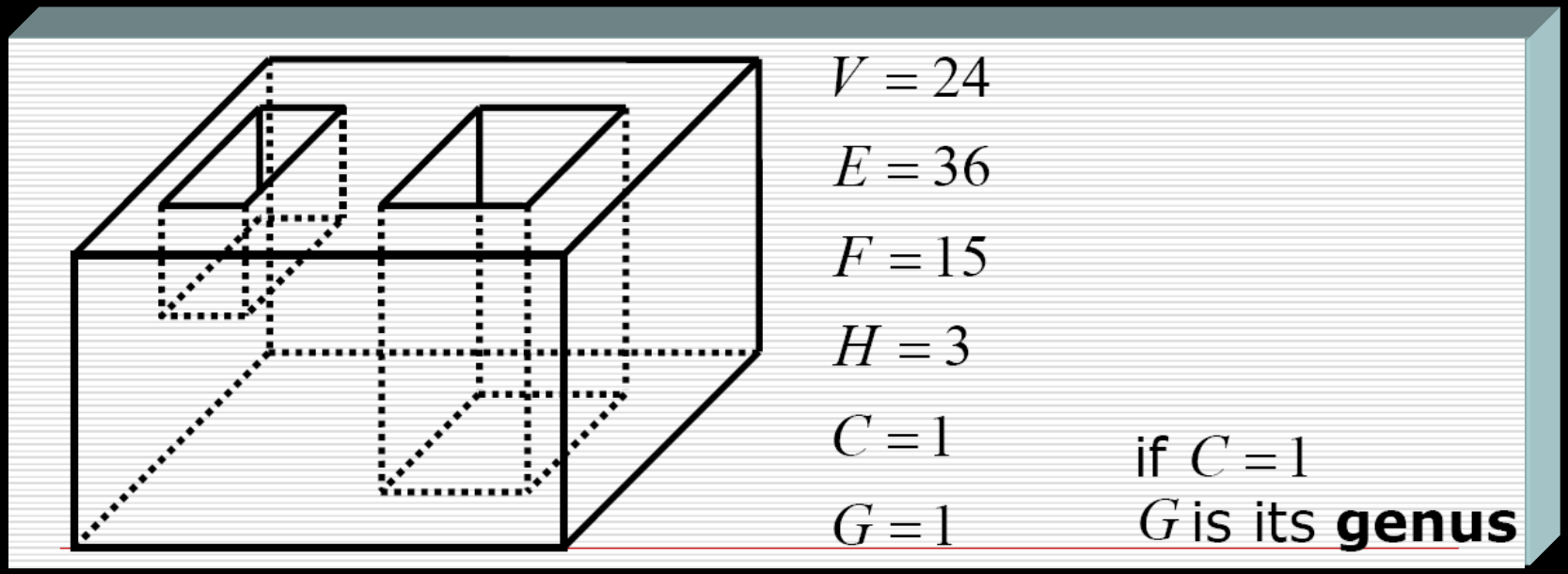
$E = 12$

$F = 8$

Euler's Formula

- States necessary but not sufficient conditions for a simple polyhedron
- Additional constraints are needed
  - 1 edge must connect 2 vertices
  - 1 edge must be shared by exactly 2 faces
  - At least 3 edges must meet at 1 vertex
  - faces must not interpenetrate

Euler's Formula Applies to 2-Manifolds with Holes

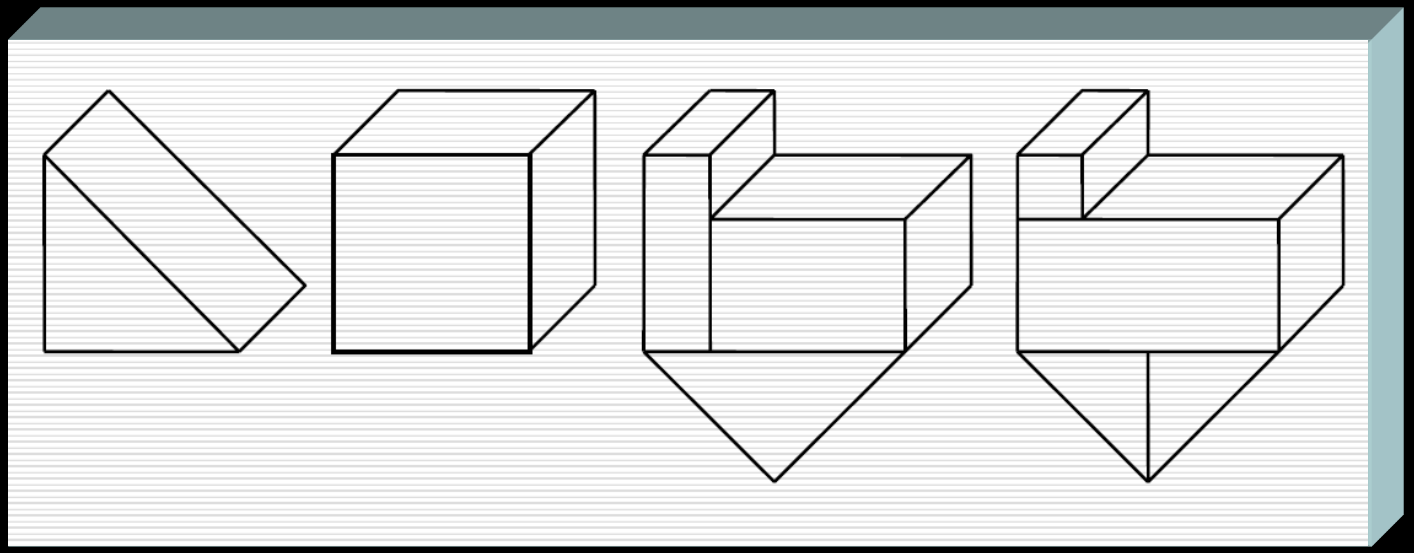


- $V\text{-}E\text{+}F\text{-}H\text{=}2(C\text{-}G)$
- H: the number of holes in the faces
- G: the number of holes that pass through the object
- C: the number of separate components

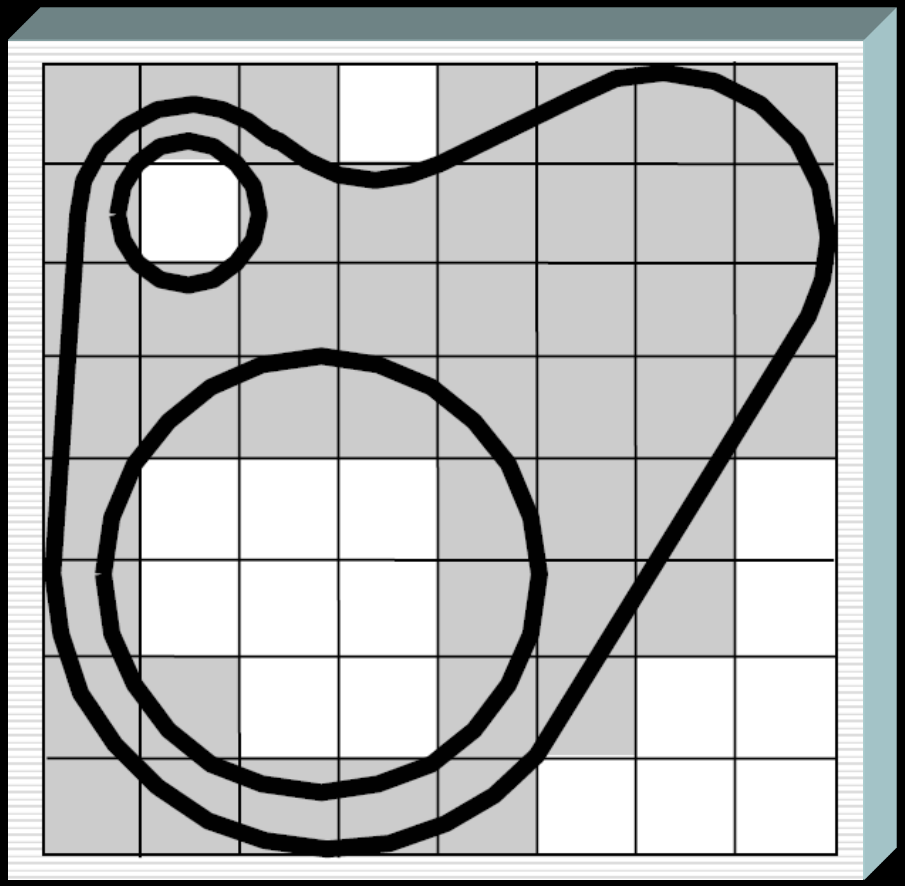
Spatial-Partitioning Representations

- Cell Decomposition
- Spatial-Occupancy Enumeration
- Octrees
- Binary Space-Partitioning Trees

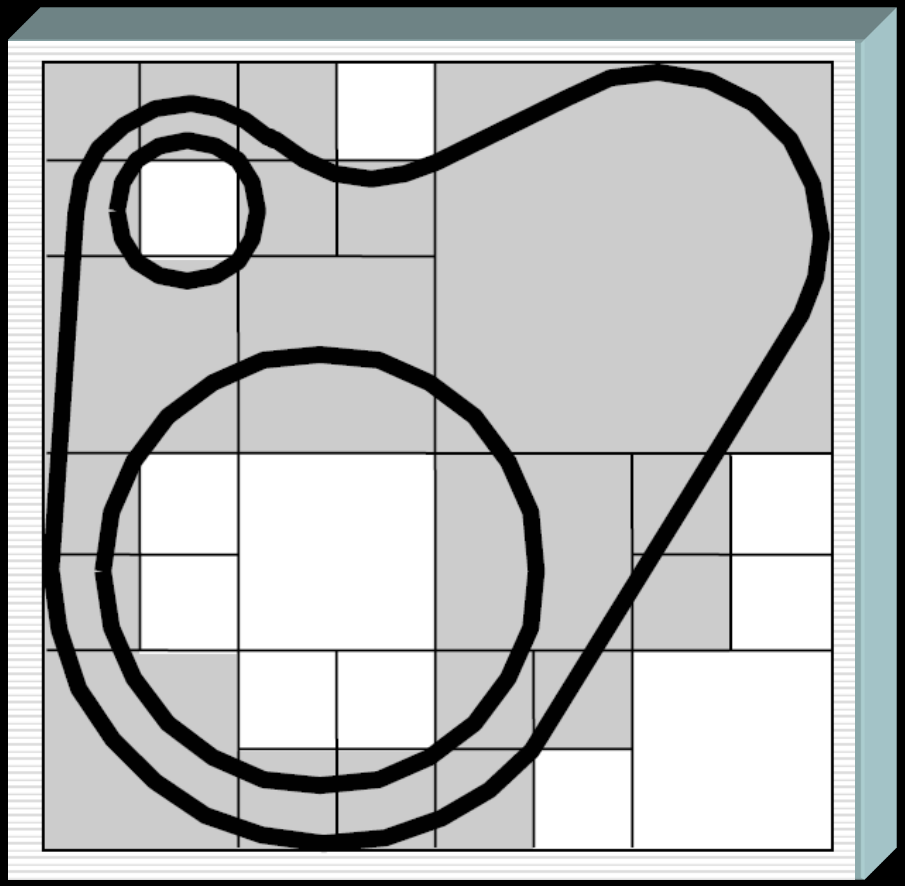
Cell Decomposition



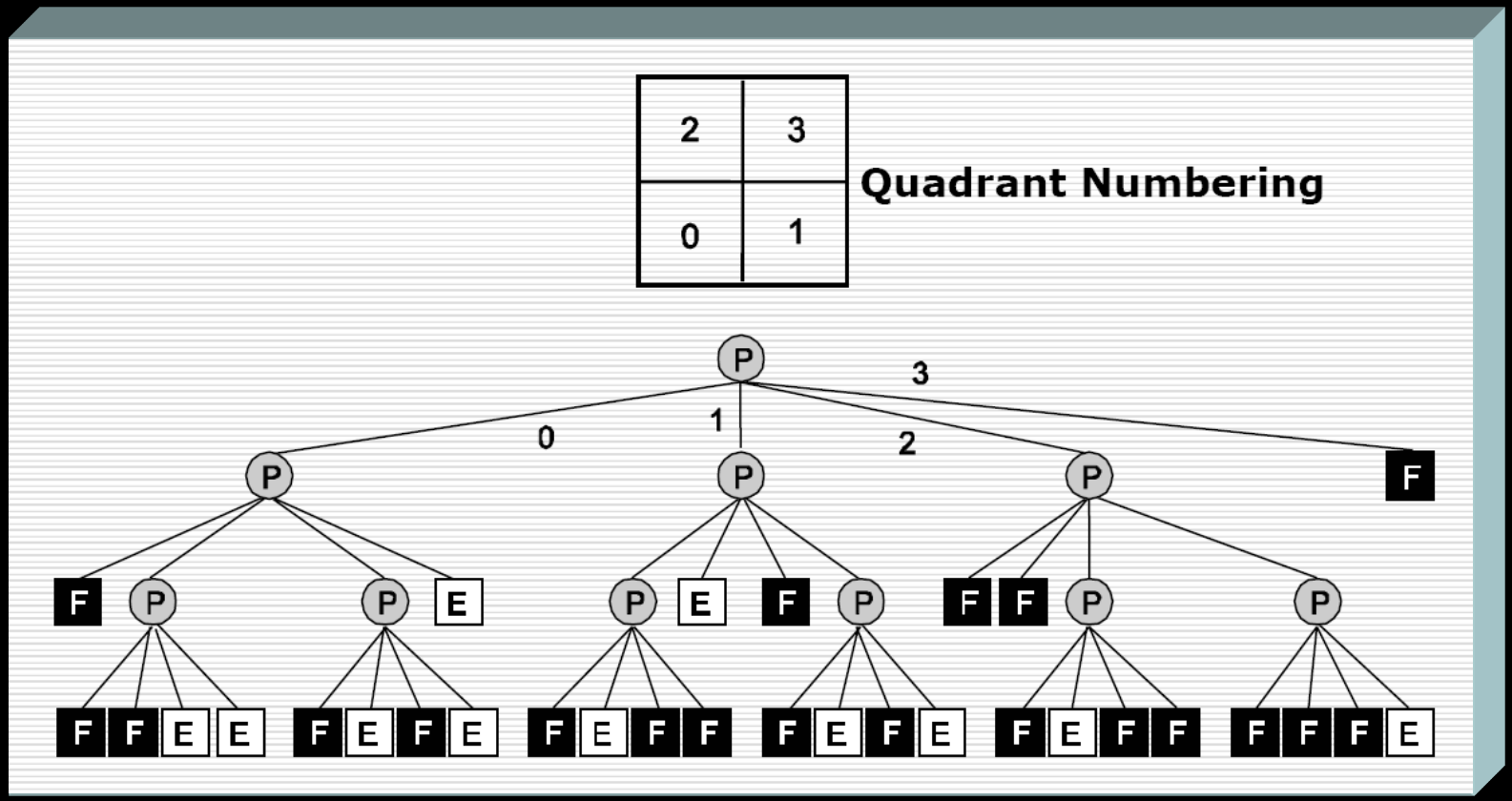
Spatial-Occupancy Enumeration



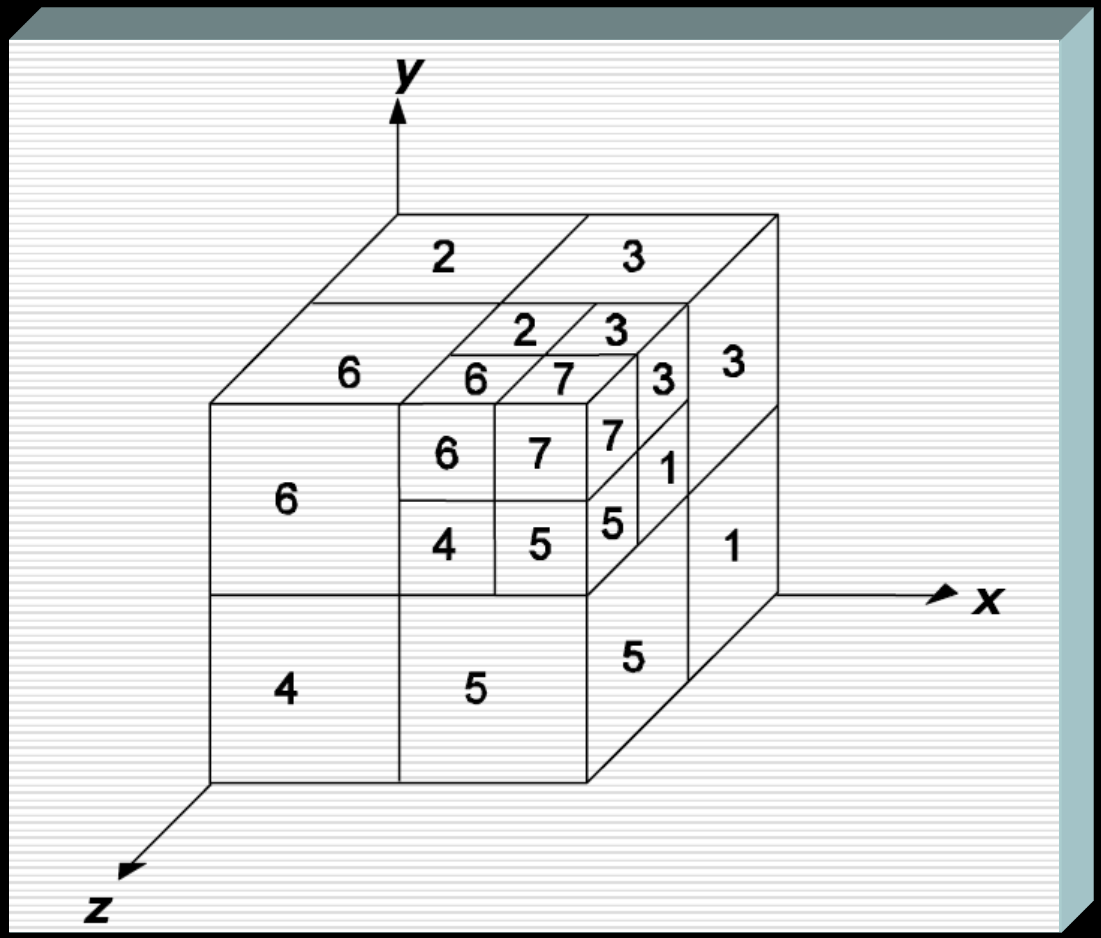
Quadtree(Octrees)



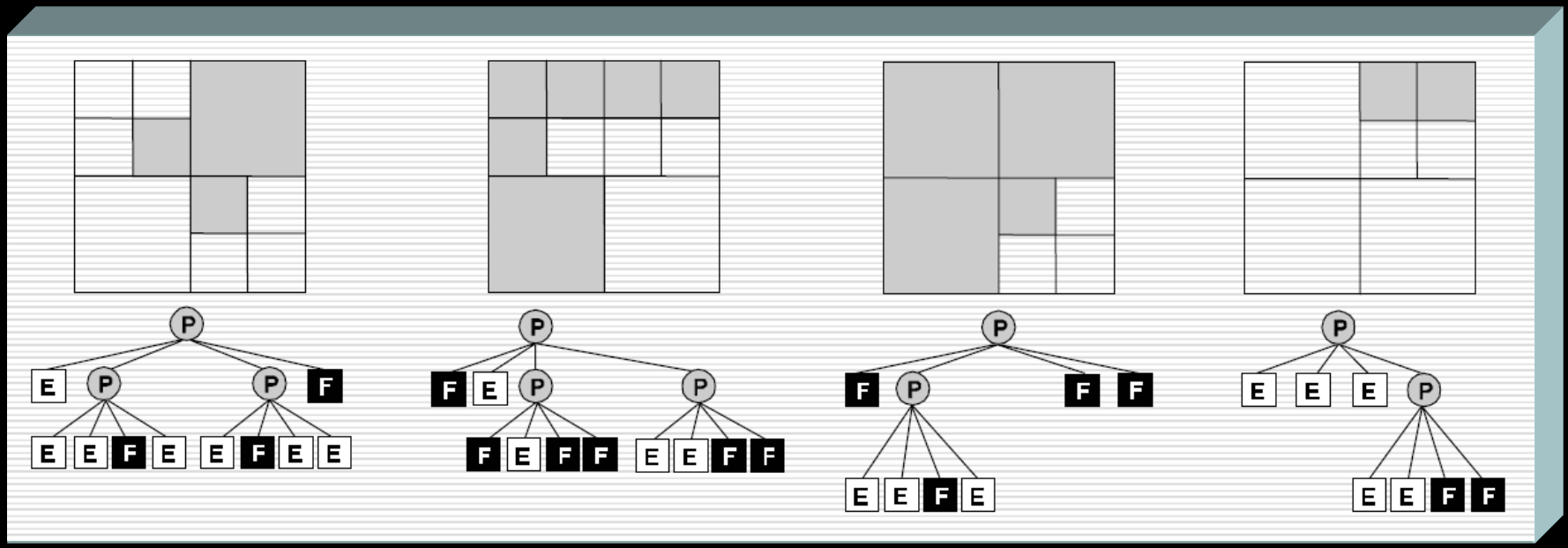
Quadtree Data Structure



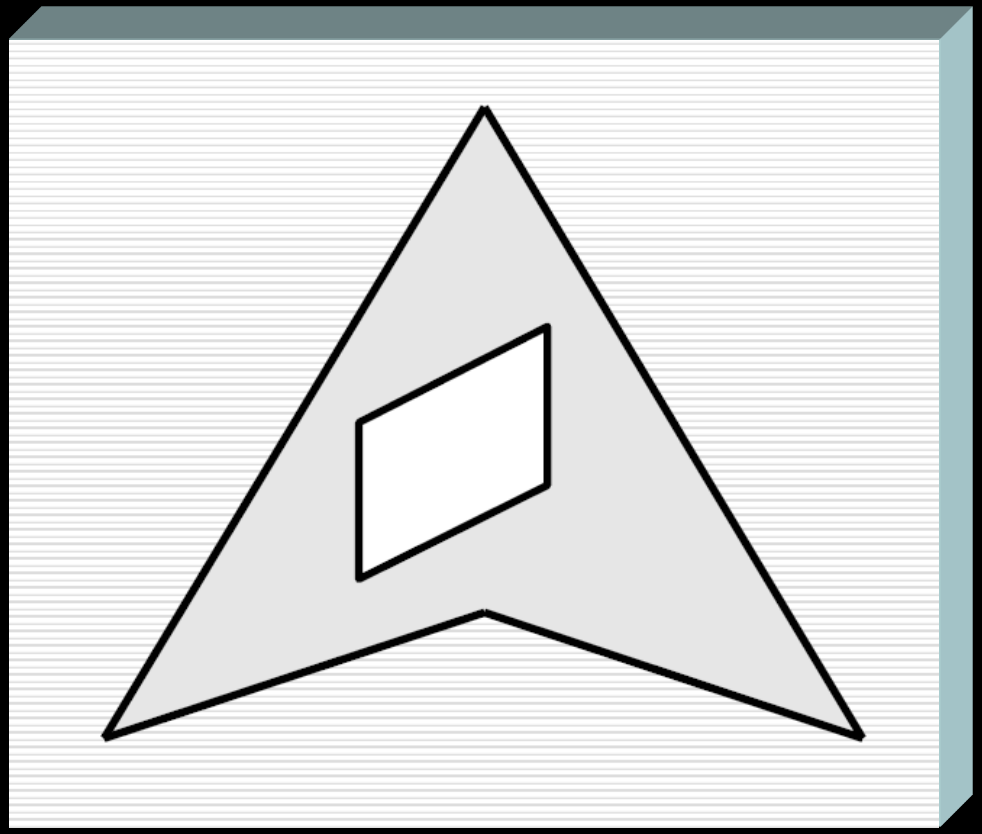
Octree Enumeration



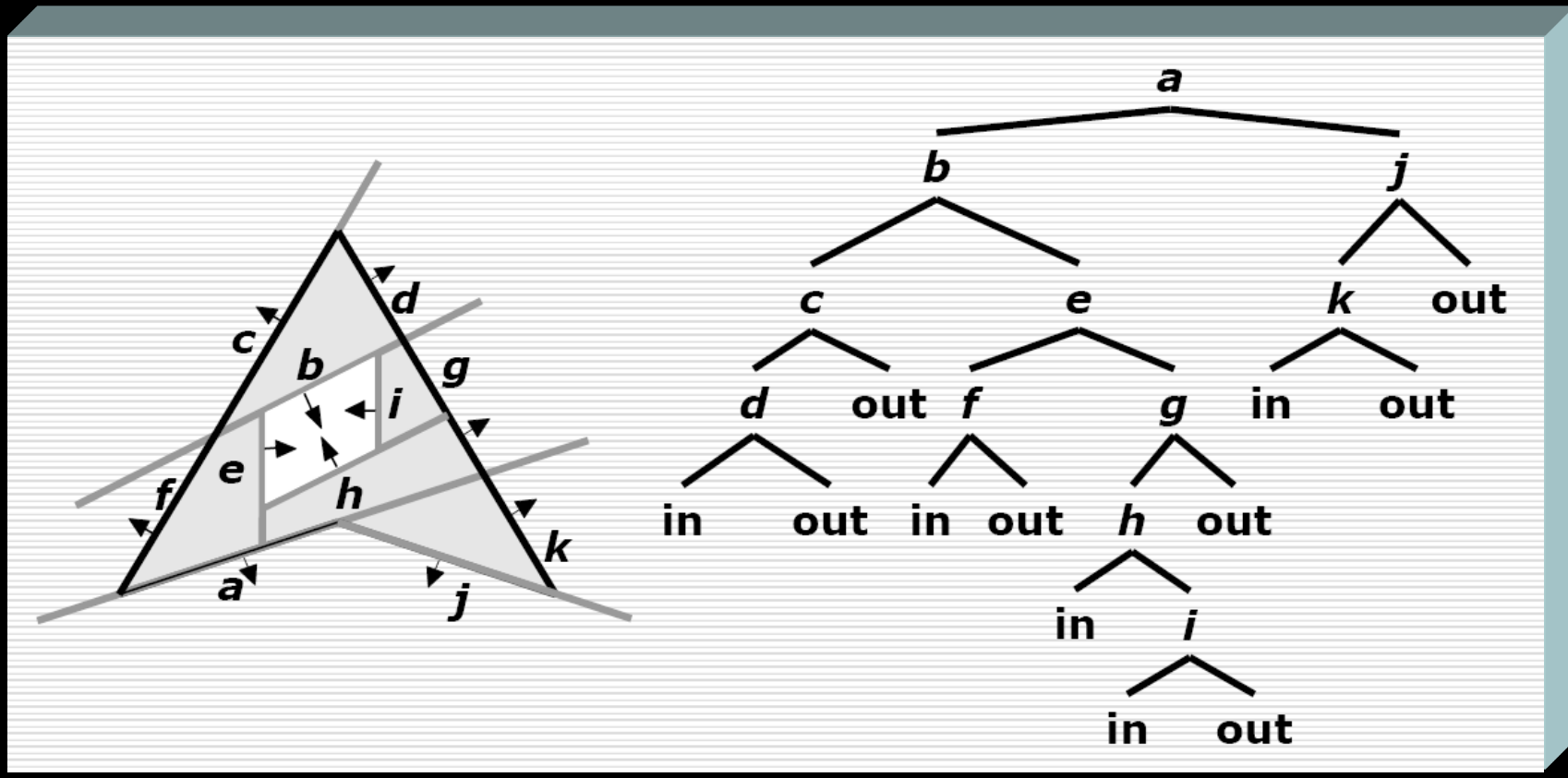
Boolean Set Operations on Quadtrees



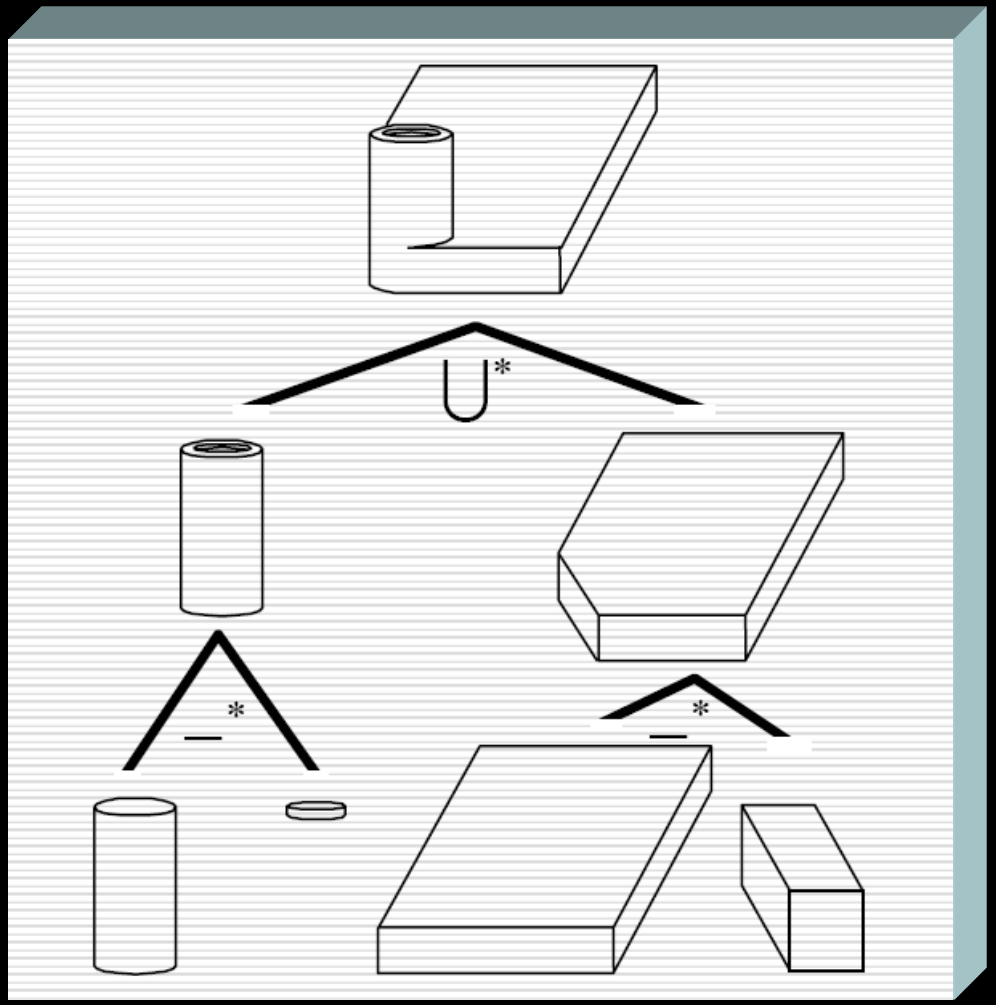
Binary Space-Partitioning (BSP) Trees



Binary Space-Partitioning (BSP) Trees



Constructive Solid Geometry (CSG)



Constructive Solid Geometry (CSG)