Fundamental Theories and Applications of Neural Networks

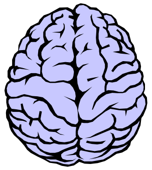
Lecture 2: Neuron models and basic learning rules

Contents of this lecture

- After this lecture, you should know
  - How a neuron works?
  - Some basic neuron models.
  - Basic steps for using a neural network.
  - General learning rule for one neuron.
  - Learning of discrete neuron.
  - Learning of continuous neuron.
  - Learning of single layer NNs with discrete neurons.
  - Learning of single layer NNs with continuous neurons.

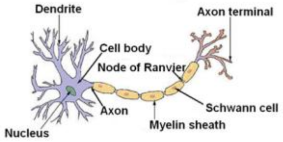
How “large” is a human brain ?

- A neuron is the basic element in a biological brain.
- There are approximately 100,000,000,000 neurons in a human brain.
- One neuron is connectedly with approximately 10,000 other neurons.
- The human brain is very large and very complex system.
- Although each neuron is slow, un-reliable, and non-intelligent, the whole brain can make decisions very quickly, in a relatively reliable and intelligent way.



What is a bio-neuron?

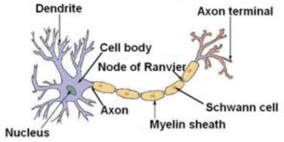
- A B-neuron contains
  - a cell body for signal processing,
  - many dendrites to receive signals,
  - an axon for outputting the result, and
  - a synapse between the axon and each dendrite.



From Wikipedia

A neuron works as follows

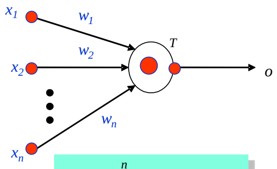
- Signals (impulses) come into the dendrites through the synapses.
- All signals from all dendrites are summed up in the cell body.
- When the sum is larger than a threshold, the neuron fires, and sends out an impulse signal to other neurons through the axon.



From Wikipedia

The McCulloch-Pitts neuron model

- Proposed by McCulloch and Pitts in 1943.
- A processor (system) with multiple input and a single output.
- Effective input: weighted sum of all inputs.
- Bias or threshold: if the effective input is larger than the bias, the neuron outputs a one, otherwise, it outputs a zero.



$$o = f\left(\sum_{i=1}^{n} w_i x_i - T\right)$$

$$f(u) = \begin{cases} 1 & \text{if } u \ge 0 \\ 0 & \text{otherwise} \end{cases}$$

Some terminologies

- The parameters used to scale the inputs are called the **weights**.
- The **effective input** is the weighted sum of the inputs.
- The parameter to measure the switching level is the **threshold** or bias.
- The function for producing the final output is called the **activation function**, which is the step function in the McCulloch-Pitts model.

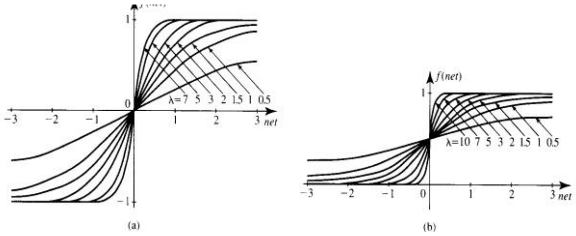
$$o = f\left(\sum_{i=1}^{n} w_i x_i - T\right)$$

$$f(u) = \begin{cases} 1 & \text{if } u \ge 0 \\ 0 & \text{otherwise} \end{cases}$$

Generalization of the neuron model

- In general, there are many different kinds of activation functions.
- The step function used in the McCulloch-Pitts model is simply one of them.
- Because the activation function takes only two values, this model is called discrete neuron.
- To make the neuron learnable, some kind of continuous function is often used as the activation function. This kind of neurons are called continuous neurons.
- Typical functions used in an artificial neuron are sigmoid functions, radial basis function, sinusoidal functions, etc.

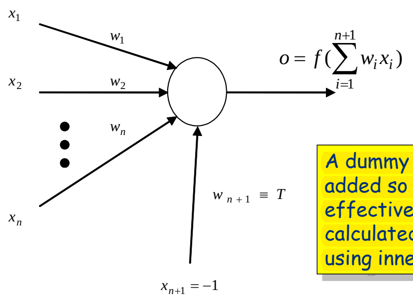
Activation function of continuous neuron: sigmoid function



$$f(u)=\frac{2}{1+\exp(-\lambda u)}-1 \qquad f(u)=\frac{1}{1+\exp(-\lambda u)}$$

(Fig. 2.5 in the textbook written by Prof. Zurada)

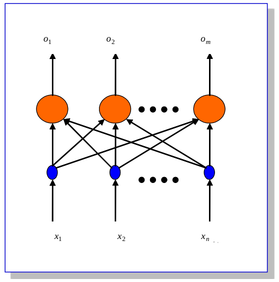
A neuron model with augmented input

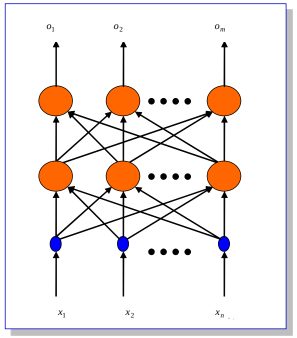


$$o = f\left(\sum_{i=1}^{n+1} w_i x_i\right)$$

A dummy input is added so that the effective input is calculated simply using inner product

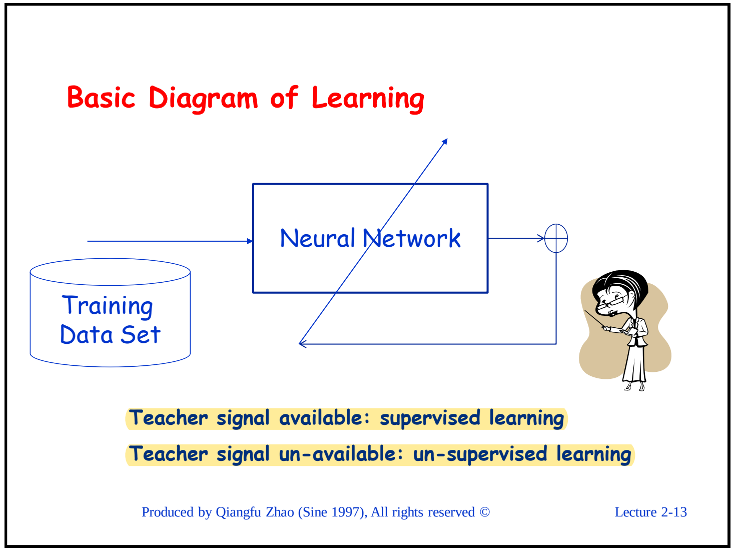
Single layer neural network and multi-layer neural network





Basic steps for using a neural network

- **Learning**: to store the information into the network.
  - Supervised and unsupervised learning.
  - On-line learning and off-line learning.
- **Recall**: to retrieve information stored in the network.
  - Auto-association and hetero-association.
  - Classification and/or recognition.



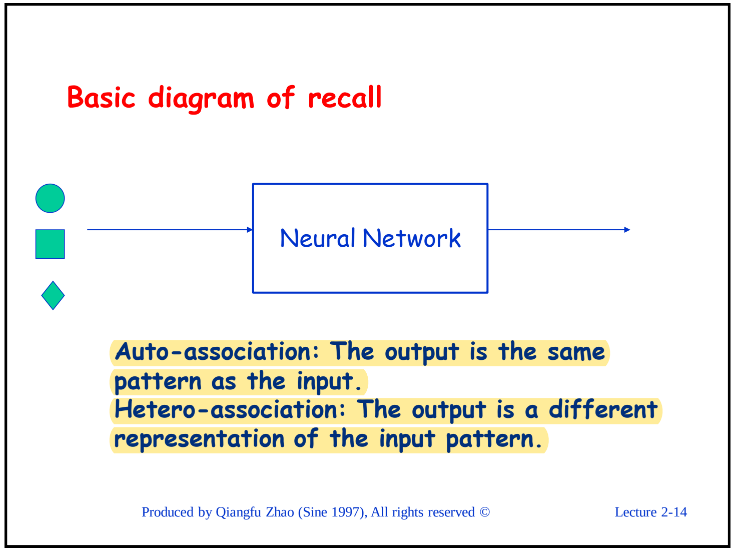
Basic Diagram of Learning

Neural Network

Training Data Set

Teacher signal available: supervised learning

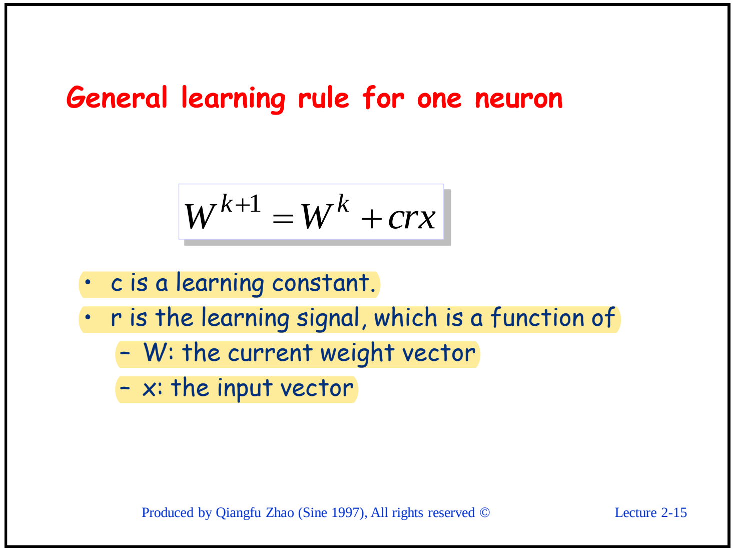
Teacher signal un-available: un-supervised learning



Basic diagram of recall

Neural Network

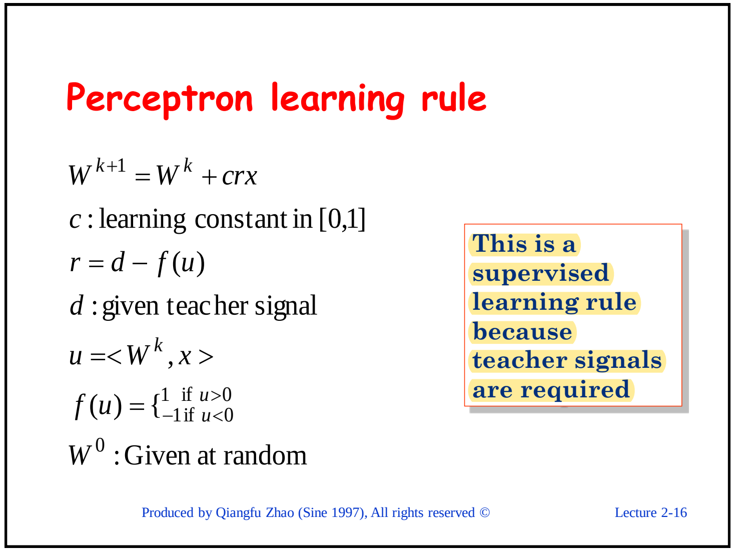
Auto-association: The output is the same pattern as the input.
Hetero-association: The output is a different representation of the input pattern.



General learning rule for one neuron

$$W^{k+1} = W^k + crx$$

- c is a learning constant.
- r is the learning signal, which is a function of
  - W: the current weight vector
  - x: the input vector



Perceptron learning rule

$W^{k+1} = W^k + crx$

c : learning constant in [0,1]

$r = d - f(u)$

d : given teacher signal

$u = < W^k, x >$

$f(u) = \{ {}^{1 \text{ if } u>0}_{-1 \text{if } u<0}$

W^0 : Given at random

This is a supervised learning rule because teacher signals are required

Delta learning rule

$W^{k+1} = W^k + crx$

c : learning constant in [0,1]

$r = [d - f(u)]f^{'}(u)$

d : given teacher signal

$u = < W^k, x >$

$f(u)$: *sigmoid* function

W^0 : Given at random

This is also a supervised learning rule because teacher signals are required

Program for perceptron learning

```
Initialization();

while(Error>desired_error){
    for(Error=0,p=0; p<n_sample; p++){
    o=FindOutput(p);
    Error+=0.5*pow(d[p]-o,2.0);
    LearningSignal=eta*(d[p]-o);
    for(i=0;i<I;i++){
        w[i]+=LearningSignal*x[p][i];
    }
  }
}
PrintResult();
```

Initialize the weights at random

For the p-th example, find the actual output

Update the total error

Update the weights

Program for delta rule

```
Initialization();
  while(Error>desired_error){
    for(Error=0,p=0; p<n_sample; p++){
    FindOutput(p);
    Error+=0.5*pow(d[p]-o,2.0);
    for(i=0;i<I;i++){
        delta=(d[p]-o)*(1-o*o)/2;
        w[i]=w[i]+c*delta*x[p][i];
    }
  }
}
PrintResult();
```

Initialize the weights at random

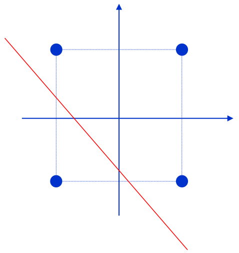
For the p-th example, find the actual output

Update the total error

Update the weights

Example 1

- There are four training examples shown in the left figure:
  - (1,1),(1,-1),(-1,1) and (-1,-1)
- The teacher signals are
  - 1, 1, 1, and -1
- That is, we want to divide the data into two groups using a line

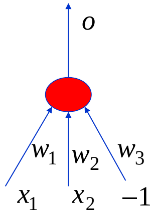


$w_1x_1 + w_2x_2 - w_3 = 0$

How to classify the data using one neuron?

$\forall x = [x_1, x_2, -1]^t$

$if\ (w_1 x_1 + w_2 x_2 - w_3 \geq 0)$ o = 1
else o = -1



1. The input is augmented with an extra element fixed to -1.
2. If effective input is larger than or equal to zero, the input belongs to group 1.
3. Otherwise, the input is in group 2.

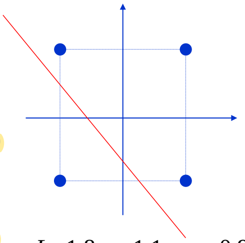
Results of perceptron learning

The initial weights:
(0.811319 0.102490 0.100490)

The error in the 1st learning cycle is 2.000000
The connection weights of the neurons are
(-0.188681 1.102490 -0.899510)

The error in the 2nd learning cycle is 4.000000
The connection weights of the neurons are
(1.811319 1.102490 -0.899510)

The error in the 3rd learning cycle is 0.000000
The connection weights of the neurons are
(1.811319 1.102490 -0.899510)

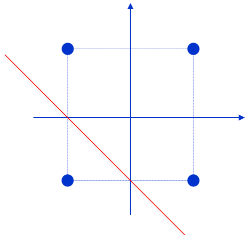


$L: 1.8x_1 + 1.1x_2 = -0.9$

Results of delta learning

Error in the 161-th learning cycle=0.010610
Error in the 162-th learning cycle=0.010541
Error in the 163-th learning cycle=0.010472
Error in the 164-th learning cycle=0.010405
Error in the 165-th learning cycle=0.010338
Error in the 166-th learning cycle=0.010273
Error in the 167-th learning cycle=0.010208
Error in the 168-th learning cycle=0.010144
Error in the 169-th learning cycle=0.010081
Error in the 170-th learning cycle=0.010018
Error in the 171-th learning cycle=0.009956

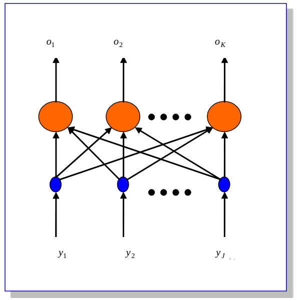
The connection weights of the neurons:
3.165432 3.167550 -3.163318



$L: 3x_1 + 3x_2 = -3$

Single layer neural network for solving multi-class problems

- There are J inputs and K outputs.
- The last input is fixed to -1 (dummy input).
- For a given input vector **y**
  - The effective input of the k-th neuron is net$_k$.
  - The actual output of the k-th neuron is o$_k$.
  - The desired output of the k-th neuron is d$_k$.
  - The error to be minimized is E.

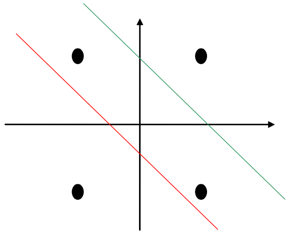


Learning of single layer network

- The learning of a single layer network can be performed by adopting the perceptron learning rule or the delta learning rule separately to each neuron.
- The only thing to do is to add one more LOOP in the program.

Example 2

- Find a single layer neural network with two discrete neurons.
- One is to realize the AND gate, and another is to realize the OR gate.

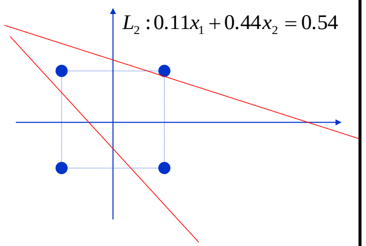


Numerical results for the example

The initial condition:
W[0]:0.248875 0.165883 0.093191
W[1]:0.111389 0.443656 0.543946

For the 1-th learning cycle:
The error is 6.000000
W[0]:1.248875 1.165883 -0.906809
W[1]:0.111389 0.443656 0.543946

For the 2-th learning cycle:
The error is 0.000000
W[0]:1.248875 1.165883 -0.906809
W[1]:0.111389 0.443656 0.543946



$L_2 : 0.11x_1 + 0.44x_2 = 0.54$

$L_1 : 1.25x_1 + 1.27x_2 = -0.9$

Team Project I: Part 1

- Write a computer program to realize the perceptron learning rule and the delta learning rule.
- Train a neuron using your program to realize the AND gate. The input pattern and their teacher signals are given as follows:
  - Data: (0,0,-1); (0,1,-1); (1,0,-1); (1,1,-1)
  - Teacher signals: -1, -1, -1, 1
- Program outputs:
  - Weights of the neuron, and
  - Neuron output for each input pattern.

Remarks

- The program given in the web page is for delta learning rule only. You should extend this program for this homework.
- The learning process is iterative. You should provide the data one by one, and start from the first datum again when all data are used once.
- One learning cycle is called an epoch.
- The total errors for all data is used as the terminating condition.
- From this experiment we can see that a neuron can be used to realize an AND gate.

Team Project I: Part 2

- Extend the program written in the first step to learning of single layer neural networks.
- The program should be able to design
  - Case 1: A single layer neural network with discrete neurons.
  - Case 2: A single layer neural network with continuous neurons.
- Test your program using the following data
  - Inputs: (10,2,-1), (2,-5,-1), (-5,5,-1).
  - Teacher signals: (1,-1,-1), (-1,1,-1), and (-1,-1,1)