Ex03 Algorithm assignment

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1. Algorithm assignment

* **The Gaussian Kernel-based probability density estimation method**

In statistics, kernel density estimation (KDE) is a non-parametric way to estimate the probability density function of a random variable. Kernel density estimation is a fundamental data smoothing problem where inferences about the population are made, based on a finite data sample.

Definition

Let () be independent and identically distributed samples drawn from some univariate distribution with an unknown density ƒ at any given point x. We are interested in estimating the shape of this function ƒ. Its kernel density estimator is

where K is the kernel — a non-negative function — and h > 0 is a smoothing parameter called the bandwidth. A kernel with subscript h is called the scaled kernel and defined as (x) = 1/h K(x/h). Intuitively one wants to choose h as small as the data will allow; however, there is always a trade-off between the bias of the estimator and its variance. A range of kernel functions are commonly used: uniform, triangular, biweight, triweight, Epanechnikov, normal, and others. Due to its convenient mathematical properties, the normal kernel is often used, which means K(x) = ϕ(x), where ϕ is the standard normal density function (= Gaussian).

(Gaussian kernel)

* **The maximum likelihood approach when the conditional probability 𝑝(𝑥|), 𝑖 = 1,2, …, 𝑁, are given.**

The relationship is with Bayesian estimation. A maximum likelihood estimator coincides with the most probable Bayesian estimator given a uniform prior distribution on the parameters. Indeed, the maximum a posteriori estimate is the parameter θ that maximizes the probability of θ given the data, given by Bayes' theorem:

where is the prior distribution for the parameter θ and where is the probability of the data averaged over all parameters. Since the denominator is independent of θ, the Bayesian estimator is obtained by maximizing with respect to θ. If we further assume that the prior is a uniform distribution, the Bayesian estimator is obtained by maximizing the likelihood function . Thus, the Bayesian estimator coincides with the maximum likelihood estimator for a uniform prior distribution

1. Program assignment
2. Summary and discussion