The bilinear quadrilateral element is also isoparametric since the shape functions are the same as the functions $\tau_i(\xi, \eta)$ of the geometrical transformation.

7.6 SHAPE FUNCTIONS OF SOME CLASSICAL ELEMENTS FOR C^0 PROBLEMS

7.6.1 One-Dimensional Elements

7.6.1.1 Two-Nodded Linear Element (Figure 7.10)

$$\begin{Bmatrix} N_1(\xi) \\ N_2(\xi) \end{Bmatrix} = \begin{Bmatrix} \frac{1}{2}(1-\xi) \\ \frac{1}{2}(1+\xi) \end{Bmatrix} \tag{7.79}$$

7.6.1.2 Three-Nodded Quadratic Element

$$\begin{Bmatrix} N_1(\xi) \\ N_2(\xi) \\ N_3(\xi) \end{Bmatrix} = \begin{Bmatrix} \frac{1}{2}\xi(1-\xi) \\ (1-\xi^2) \\ \frac{1}{2}\xi(1+\xi) \end{Bmatrix} \tag{7.80}$$

7.6.2 Two-Dimensional Elements

7.6.2.1 Four-Nodded Bilinear Quadrilateral (Figure 7.11)

$$\begin{Bmatrix} N_1(\xi,\eta) \\ N_2(\xi,\eta) \\ N_3(\xi,\eta) \\ N_4(\xi,\eta) \end{Bmatrix} = \begin{Bmatrix} 0.25(1-\xi-\eta+\xi\eta) \\ 0.25(1+\xi-\eta-\xi\eta) \\ 0.25(1+\xi+\eta+\xi\eta) \\ 0.25(1-\xi+\eta-\xi\eta) \end{Bmatrix} \tag{7.81}$$

FIGURE 7.10 One-dimensional elements.

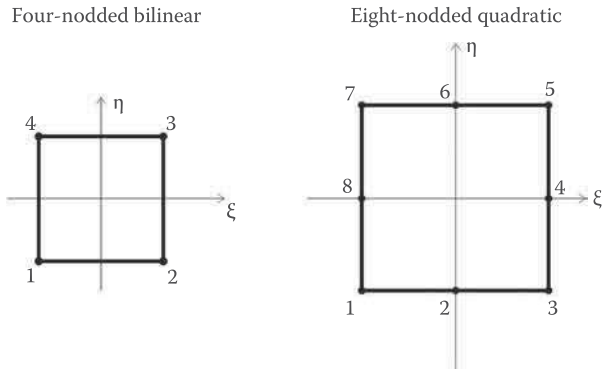

FIGURE 7.11 Two-dimensional quadrilateral elements.

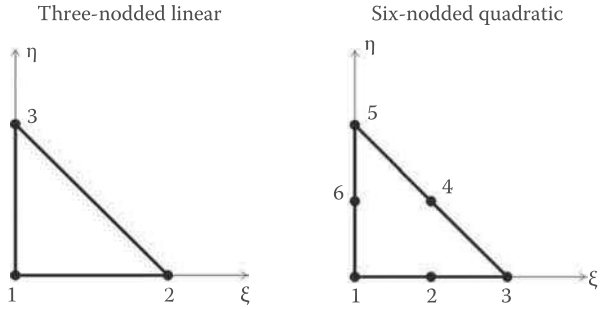

FIGURE 7.12 Two-dimensional triangular elements.

7.6.2.2 Eight-Nodded Quadratic Quadrilateral

$$\begin{Bmatrix} N_1(\xi,\eta) \\ N_2(\xi,\eta) \\ N_3(\xi,\eta) \\ N_4(\xi,\eta) \\ N_5(\xi,\eta) \\ N_6(\xi,\eta) \\ N_7(\xi,\eta) \\ N_8(\xi,\eta) \end{Bmatrix} = \begin{Bmatrix} -0.25(1-\xi)(1-\eta)(1+\xi+\eta) \\ 0.50(1-\xi^2)(1-\eta) \\ -0.25(1+\xi)(1-\eta)(1-\xi+\eta) \\ 0.50(1+\xi)(1-\eta^2) \\ -0.25(1+\xi)(1+\eta)(1-\xi-\eta) \\ 0.50(1-\xi^2)(1+\eta) \\ -0.25(1-\xi)(1+\eta)(1+\xi-\eta) \\ 0.50(1-\xi)(1-\eta^2) \end{Bmatrix} \tag{7.82}$$

7.6.2.3 Three-Nodded Linear Triangle (Figure 7.12)

$$\begin{Bmatrix} N_1(\xi,\eta) \\ N_2(\xi,\eta) \\ N_3(\xi,\eta) \end{Bmatrix} = \begin{Bmatrix} 1-\xi-\eta \\ \xi \\ \eta \end{Bmatrix} \tag{7.83}$$

7.6.2.4 Six-Nodded Quadratic Triangle

$$\begin{Bmatrix} N_1(\xi,\eta) \\ N_2(\xi,\eta) \\ N_3(\xi,\eta) \\ N_4(\xi,\eta) \\ N_5(\xi,\eta) \\ N_6(\xi,\eta) \end{Bmatrix} = \begin{Bmatrix} -(1-\xi-\eta)(1-2(1-\xi-\eta)) \\ 4\xi(1-\xi-\eta) \\ -\xi(1-2\xi) \\ 4\xi\eta \\ -\eta(1-2\eta) \\ 4\eta(1-\xi-\eta) \end{Bmatrix} \tag{7.84}$$

7.6.3 Three-Dimensional Elements

7.6.3.1 Four-Nodded Linear Tetrahedra

$$\begin{Bmatrix} N_1(\xi,\eta,\zeta) \\ N_2(\xi,\eta,\zeta) \\ N_3(\xi,\eta,\zeta) \\ N_4(\xi,\eta,\zeta) \end{Bmatrix} = \begin{Bmatrix} 1-\xi-\eta-\zeta \\ \xi \\ -\eta \\ \zeta \end{Bmatrix} \tag{7.85}$$

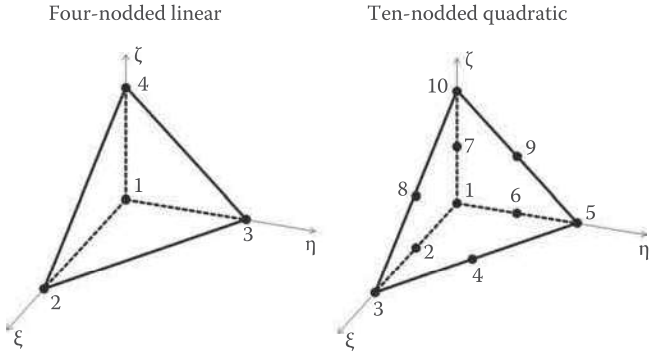

FIGURE 7.13 Three-dimensional tetrahedric elements.

7.6.3.2 Ten-Nodded Quadratic Tetrahedra (Figure 7.13)

$$\begin{Bmatrix} N_1(\xi,\eta,\zeta) \\ N_2(\xi,\eta,\zeta) \\ N_3(\xi,\eta,\zeta) \\ N_4(\xi,\eta,\zeta) \\ N_5(\xi,\eta,\zeta) \\ N_6(\xi,\eta,\zeta) \\ N_7(\xi,\eta,\zeta) \\ N_8(\xi,\eta,\zeta) \\ N_9(\xi,\eta,\zeta) \\ N_{10}(\xi,\eta,\zeta) \end{Bmatrix} = \begin{Bmatrix} -(1-\xi-\eta-\zeta)(1-2(1-\xi-\eta-\zeta)) \\ 4\xi(1-\xi-\eta-\zeta) \\ -\xi(1-2\xi) \\ 4\xi\eta \\ -\eta(1-2\eta) \\ 4\eta(1-\xi-\eta-\zeta) \\ 4\zeta(1-\xi-\eta-\zeta) \\ 4\xi\zeta \\ 4\eta\zeta \\ -\zeta(1-2\zeta) \end{Bmatrix} \tag{7.86}$$

7.6.3.3 Eight-Nodded Linear Brick Element

$$\begin{Bmatrix} N_1(\xi,\eta,\zeta) \\ N_2(\xi,\eta,\zeta) \\ N_3(\xi,\eta,\zeta) \\ N_4(\xi,\eta,\zeta) \\ N_5(\xi,\eta,\zeta) \\ N_6(\xi,\eta,\zeta) \\ N_7(\xi,\eta,\zeta) \\ N_8(\xi,\eta,\zeta) \end{Bmatrix} = \frac{1}{8} \begin{Bmatrix} (1-\xi)(1-\eta)(1-\zeta) \\ (1+\xi)(1-\eta)(1-\zeta) \\ (1+\xi)(1+\eta)(1-\zeta) \\ (1-\xi)(1+\eta)(1-\zeta) \\ (1-\xi)(1-\eta)(1+\zeta) \\ (1+\xi)(1-\eta)(1+\zeta) \\ (1+\xi)(1+\eta)(1+\zeta) \\ (1-\xi)(1+\eta)(1+\zeta) \end{Bmatrix} \tag{7.87}$$

7.6.3.4 Twenty-Nodded Quadratic Brick Element (Figure 7.14)

$$\begin{Bmatrix} N_1(\xi,\eta,\zeta) \\ N_2(\xi,\eta,\zeta) \\ N_3(\xi,\eta,\zeta) \\ N_4(\xi,\eta,\zeta) \\ N_5(\xi,\eta,\zeta) \\ N_6(\xi,\eta,\zeta) \\ N_7(\xi,\eta,\zeta) \\ N_8(\xi,\eta,\zeta) \\ N_9(\xi,\eta,\zeta) \\ N_{10}(\xi,\eta,\zeta) \\ N_{11}(\xi,\eta,\zeta) \\ N_{12}(\xi,\eta,\zeta) \\ N_{13}(\xi,\eta,\zeta) \\ N_{14}(\xi,\eta,\zeta) \\ N_{15}(\xi,\eta,\zeta) \\ N_{16}(\xi,\eta,\zeta) \\ N_{17}(\xi,\eta,\zeta) \\ N_{18}(\xi,\eta,\zeta) \\ N_{19}(\xi,\eta,\zeta) \\ N_{20}(\xi,\eta,\zeta) \end{Bmatrix} = \begin{Bmatrix} \frac{1}{8}(1-\xi)(1-\eta)(1-\zeta)(-2-\xi-\eta-\zeta) \\ \frac{1}{4}(1-\xi^2)(1-\eta)(1-\zeta) \\ \frac{1}{8}(1+\xi)(1-\eta)(1-\zeta)(-2+\xi-\eta-\zeta) \\ \frac{1}{4}(1+\xi)(1-\eta^2)(1-\zeta) \\ \frac{1}{8}(1+\xi)(1+\eta)(1-\zeta)(-2+\xi+\eta-\zeta) \\ \frac{1}{4}(1-\xi^2)(1+\eta)(1-\zeta) \\ \frac{1}{8}(1-\xi)(1+\eta)(1-\zeta)(-2-\xi+\eta-\zeta) \\ \frac{1}{4}(1-\xi)(1-\eta^2)(1-\zeta) \\ \frac{1}{4}(1-\xi)(1-\eta)(1-\zeta^2) \\ \frac{1}{4}(1+\xi)(1-\eta)(1-\zeta^2) \\ \frac{1}{4}(1+\xi)(1+\eta)(1-\zeta^2) \\ \frac{1}{4}(1-\xi)(1+\eta)(1-\zeta^2) \\ \frac{1}{8}(1-\xi)(1-\eta)(1+\zeta)(-2-\xi-\eta+\zeta) \\ \frac{1}{4}(1-\xi^2)(1-\eta)(1+\zeta) \\ \frac{1}{8}(1+\xi)(1-\eta)(1+\zeta)(-2+\xi-\eta+\zeta) \\ \frac{1}{4}(1+\xi)(1-\eta^2)(1+\zeta) \\ \frac{1}{8}(1+\xi)(1+\eta)(1+\zeta)(-2+\xi+\eta+\zeta) \\ \frac{1}{4}(1-\xi^2)(1+\eta)(1+\zeta) \\ \frac{1}{8}(1-\xi)(1+\eta)(1+\zeta)(-2-\xi+\eta+\zeta) \\ \frac{1}{4}(1-\xi)(1-\eta^2)(1+\zeta) \end{Bmatrix} \tag{7.88}$$

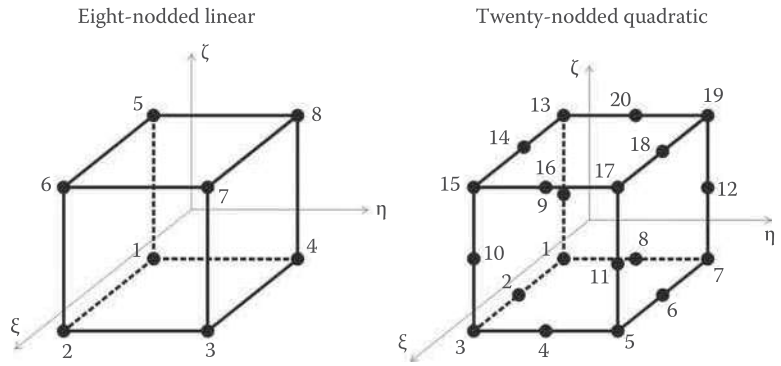

FIGURE 7.14 Three-dimensional brick elements.