Lest We Forget: Learn from Out-of-Sample Forecast Errors When Optimizing Portfolios

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Outline

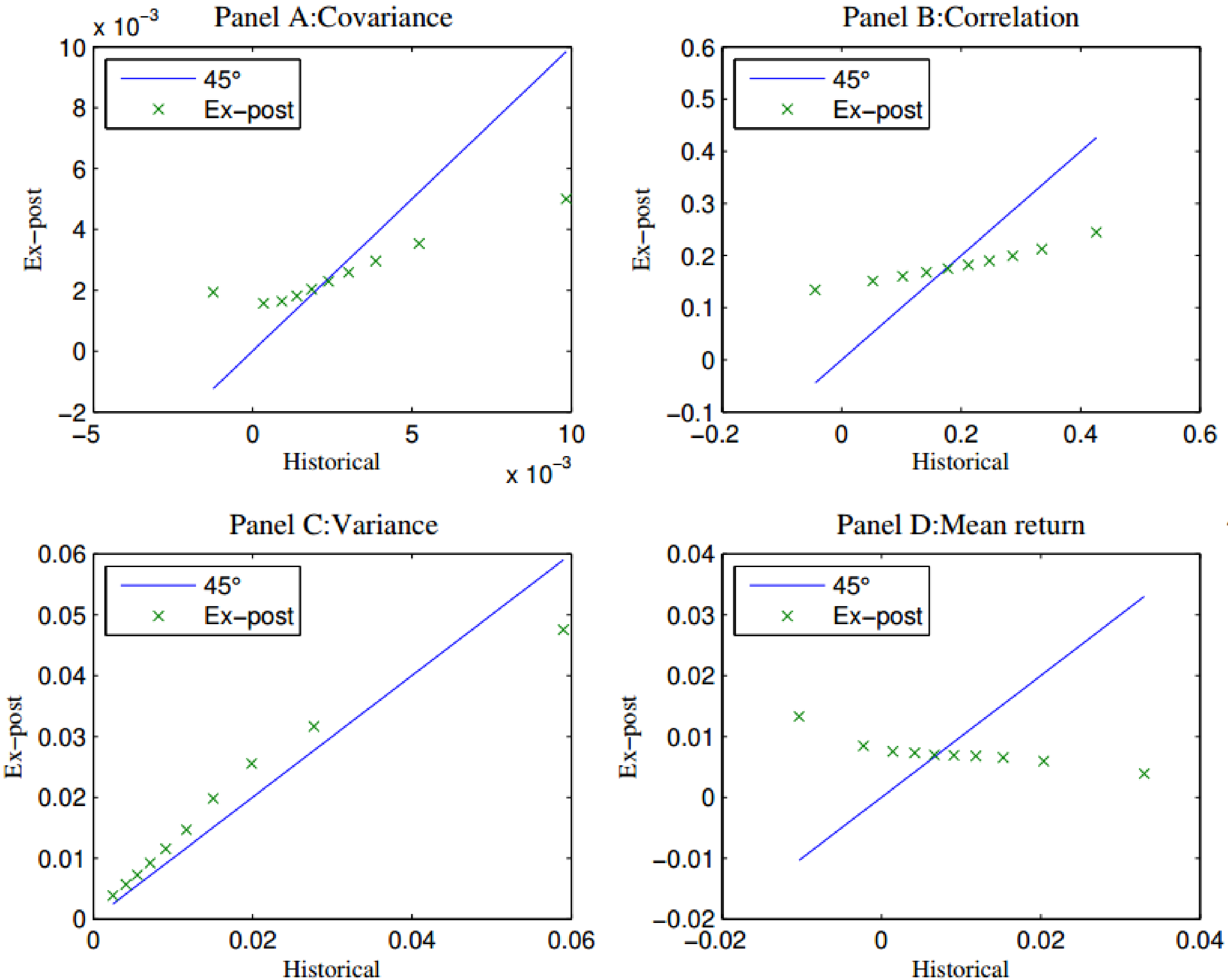
- Introduction
- Research design
  - Regression to the Mean in Optimization Inputs
  - Correcting out-of-sample forecast errors
- Empirical study
  - Constructing Galton Portfolios
  - The OOS Performance
- Conclusion

1. Introduction-- Motivation

- Historical OOS forecast errors of portfolio optimization inputs, such as future means, variances, and correlations, are typically **not used** in subsequent estimations.
- A long history of (usually large) OOS errors **should be of some use** in correcting the estimates obtained from a historical sample.
- It is based on a simple **intuition**: learn from past OOS forecast errors to improve subsequent forecasts.

$$\boldsymbol{w}_t = \frac{\boldsymbol{\Sigma}_t^{-1}\boldsymbol{\mu}}{\boldsymbol{1}\boldsymbol{\Sigma}_t^{-1}\boldsymbol{\mu}},$$

1. Introduction-- Motivation

- 

$$\boldsymbol{w}_t = \frac{\boldsymbol{\Sigma}_t^{-1}\boldsymbol{\mu}}{\mathbf{1}\boldsymbol{\Sigma}_t^{-1}\boldsymbol{\mu}},$$

1. Introduction-- Framework

Galton method and OLS(**Fama-MacBeth**)

Empirical Bayesian interpretation and **Shrinkage**

Research design

Empirical study

Constructing Galton Portfolios

The OOS Performance

Analyzing the Sources of Improvement

1. Introduction-- Contribution

- This is the first paper that makes explicit **use of OOS errors** to improve the portfolio allocation decision.
- Compute the shrinkage parameters by **minimizing the sum of squared OOS errors**, instead of using closed form expressions for the posteriors.
- Use the entire historical sample as the **estimation universe**, including stocks that are no longer traded or in the current **investment universe**.

2. Regression to the Mean in Optimization Inputs

- The vector of relative weights of the mean-variance (MV) optimal risky portfolio is

$$\boldsymbol{w}_t = \frac{\boldsymbol{\Sigma}_t^{-1}\boldsymbol{\mu}}{\mathbf{1}\boldsymbol{\Sigma}_t^{-1}\boldsymbol{\mu}}, \tag{1}$$

- where μ is a N-by-1 vector of mean returns, 1 is a N-by-1 vectors of ones,N is the number of assets, and Σ is the covariance matrix

- The historical estimates

$$\boldsymbol{\mu}_{\mathsf{H},t} = \sum_{s=t-\mathsf{H}+1}^{t} \boldsymbol{r}_{\mathsf{H},s}/\mathsf{H}$$

$$\boldsymbol{\Sigma}_{\mathsf{H},t} = (\boldsymbol{r}_{\mathsf{H},t} - \boldsymbol{\mu}_{\mathsf{H},t})(\boldsymbol{r}_{\mathsf{H},t} - \boldsymbol{\mu}_{\mathsf{H},t})'/(\mathsf{H}-1)$$

2.Correcting out-of-sample forecast errors

- Consider the following reversion to the mean (or “Galton ") corrected forecast for mean returns:

$$\mu_{i,t+1|t} = g_{0,m,t} + g_{1,m,t}\mu_{i,\mathrm{H},t}. \qquad (2)$$

- $\mu_{i,t+1|t}$ in Equation (2) is expected to be a better predictor of ex post realizations compared to $\mu_{i,H,t}$. ---OLS

- Similarly, using linear “Galton" corrections,$g_{.,v,t}$ $g_{.,c,t}$ and $g_{.,\rho,t}$,for other optimization inputs (variances, covariances, and correlations)

2. Normal approximation of the prior and posterior.

- Assuming the central limit theorem applies, we approximate the prior distribution by a normal distribution, a vector $\boldsymbol{X_t}$ generated from an (approximate) two-stage Gaussian model:

$$\boldsymbol{X}_t|\boldsymbol{\mu}_x,\boldsymbol{\Sigma}_x \sim \mathcal{N}_N(\boldsymbol{\mu}_x,\boldsymbol{\Sigma}_x), \tag{3}$$

$$\boldsymbol{\mu}_x \sim \mathcal{N}(\mu_{0,x}\mathbf{1}_N,\tau_x^2\boldsymbol{\Sigma}_x), \tag{4}$$

- Equation (4) is the Bayesian informative prior normally distributed with mean $\mu_{0,x}$ and covariance matrix $\tau_x^2\Sigma_x$. If $\mu_{0,x}$and τ_x are known, then we obtain Stein shrinkage(Efron and Morris ,1972)

$$\boldsymbol{\mu}_x|\boldsymbol{X}_t = \mu_{0,x}\mathbf{1} + g_{1,x}(\boldsymbol{X}_t - \mu_{0,x}\mathbf{1}), \tag{5}$$

- Where $g_{1,x} = \tau_x^2/(1+\tau_x^2),\ g_{1,x} \in [0,1].$

2.Shrinkage versus Galton forecasts

- Empirical Bayesian approaches first estimate **hyperparameters** from historical data and then apply analytically **derived expressions** to infer the shrinkage intensity from these estimated hyperparameters.

- It is likely to **underperform** in environments in which the assumptions of these competing models are consistent with the data.

Empirical Bayesian restriction on shrinkage coefficients.

- difference: the empirical Bayesian interpretation **restricts** the shrinkage coefficients to be **between 0 and 1**

3. Constructing Galton Portfolios

$$\mu_{i,t+1|t} = g_{0,m,t} + g_{1,m,t}\mu_{i,\mathsf{H},t}. \qquad (2)$$

$\longrightarrow$ $\mu_{i,\mathsf{G},t}$, $\sigma_{i,\mathsf{G},t}$, $\sigma_{i,j,\mathsf{G},t}$, or $\rho_{i,j,\mathsf{G},t}$

- Guarantee of being positive semidefinite

$$\boldsymbol{\Sigma}_{\mathsf{G},t} = diag(\boldsymbol{\sigma}_{\mathsf{G},t})\boldsymbol{\rho}_{\mathsf{G},t}diag(\boldsymbol{\sigma}_{\mathsf{G},t}), \qquad (6)$$

- The Galton mean-variance (MV) portfolio is

$$\boldsymbol{w}_{\mathsf{G},t}^{MV} = \frac{\boldsymbol{\Sigma}_{\mathsf{G},t}^{-1}\boldsymbol{\mu}_{\mathsf{G},t}}{\mathbf{1}\boldsymbol{\Sigma}_{\mathsf{G},t}^{-1}\boldsymbol{\mu}_{\mathsf{G},t}}. \qquad (7)$$

- The Galton global minimum variance (GMV) portfolio is

$$\boldsymbol{w}_{\mathsf{G},t}^{GMV} = \frac{\boldsymbol{\Sigma}_{\mathsf{G},t}^{-1}\mathbf{1}}{\mathbf{1}\boldsymbol{\Sigma}_{\mathsf{G},t}^{-1}\mathbf{1}}. \qquad (8)$$

3.Special cases of the Galton forecast optimization

$$\mu_{i,t+1|t} = g_{0,m,t} + g_{1,m,t}\mu_{i,\mathrm{H},t}. \tag{2}$$

- The equally weighted ("1 over N") Talmud portfolio

$$g_{1,m} = g_{1,v} = g_{1,\rho} = 0.$$

- The Markowitz ex post tangency portfolio(MV)

$$g_{1,m} = g_{1,v} = g_{1,\rho} = 1 \qquad \boldsymbol{w}_t = \frac{\boldsymbol{\Sigma}_t^{-1}\boldsymbol{\mu}}{\mathbf{1}\boldsymbol{\Sigma}_t^{-1}\boldsymbol{\mu}},$$

- The sample global minimum variance portfolio(GMV)

$$g_{1,m} = 0, g_{1,v} = g_{1,\rho} = 1 \qquad w_t = \frac{\Sigma_t^{-1}\mathbf{1}}{\mathbf{1}\Sigma_t^{-1}\mathbf{1}}$$

3.Fama-MacBeth estimation of the Galton coefficients

- Rolling window: H =60;   ex post window of E =12

$$X_{i,\mathsf{E},t+\mathsf{E}} = g_0 + g_1 X_{i,\mathsf{H},t} + \epsilon_{i,t}$$

$$\mathrm{MSFE} = \mathrm{E}[(\boldsymbol{X}_{\mathsf{E},t+\mathsf{E}} - \boldsymbol{X}_{t+\mathsf{E}|t})^2].$$

- For each period $s \leq t$ in the sample, we run the following cross-sectional regression:

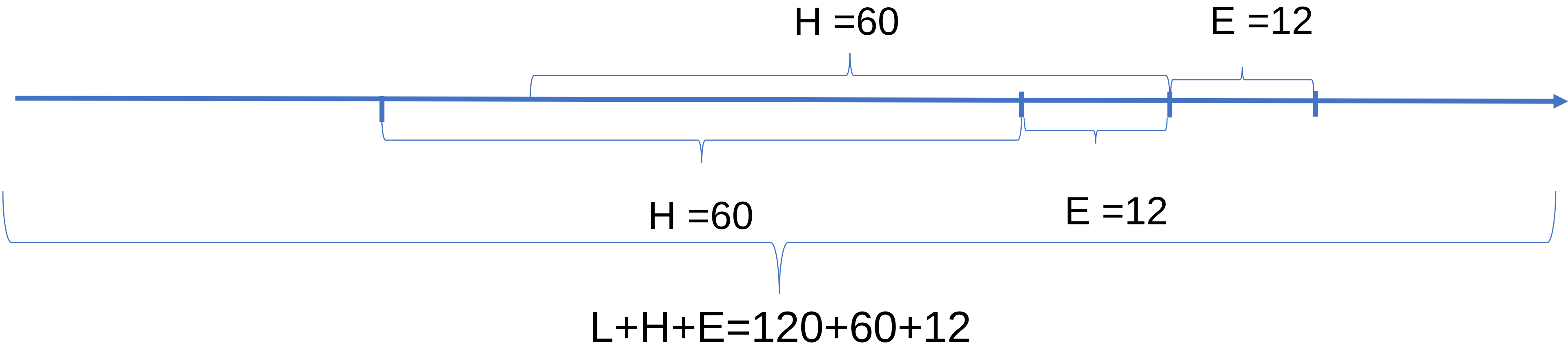
$$X_{i,\mathsf{E},s} = g_{0,s}^{fm} + g_{1,s}^{fm} X_{\mathsf{H},s-\mathsf{E}} + e_{s,i},$$

$$\hat{g}_{0,t} = \sum_{s=1}^{t} \hat{g}_{0,s}^{fm}/t \qquad \hat{g}_{1,t} = \sum_{s=1}^{t} \hat{g}_{1,s}^{fm}/t$$

- Require one additional learning period (L) to correct for past OOS errors

4. The OOS Performance

- Data
  - estimation universe: monthly returns of the entire universe of CRSP
  - investment universe : the 50 stocks with the largest market capitalization in December (Annual adjustment)
- Period： 1952.01 to 2016.12
  - OOS prediction : 1967:01
  - initial learning period (L):120 months



4. The OOS Performance-- Estimated Galton coefficients

$$X_{i,\mathsf{E},t+\mathsf{E}} = g_0 + g_1 X_{i,\mathsf{H},t} + \epsilon_{i,t}$$

Table 1
Regression to the mean

	Covariance	Correlation	Variance	Mean return
A. All stocks (including microcaps)				
Intercept	0.00	0.13	0.01	0.01
t-stat (=0)	8.63	11.20	8.63	5.87
Slope	0.36	0.28	0.55	-0.16
t-stat(=0)	10.30	17.48	11.43	-4.88
Greater than 0 (%)	1.00	1.00	0.98	0.21
t-stat(=1)	-17.94	-44.69	-9.46	-35.21
Smaller than 1 (%)	0.93	1.00	0.88	1.00
R-squared (%)	4.18	3.11	12.17	1.94
Min	392,941.00	392,941.00	887.00	887.00
Average	5,412,795.63	5,412,795.63	2,991.97	2991.97
Max	10,720,765.00	10,720,765.00	4,631.00	4631.00

- The best estimate is somewhere between the past values and the mean values.

4. The OOS Performance

Table 2
OOS performance of the MVE portfolio

Mean variance portfolios: Performance statistics									
Strategies	Sharpe	Active share	Turnover	Bankruptcy rate	Statistics for portfolio weights (w_i) and their stability				
					[Min w_i	Max w_i	Mean[$\sigma_t(w_i)$]	SD[$\sigma_t(w_i)$]	$\sum w_i I(w_i < 0)$]
Standard strategies									
Value weighted	0.37	0.00	2.18	0.00	0.69	9.17	1.66	0.51	0.00
Talmud 1/N	0.38	26.51	6.59	0.00	2.00	2.00	0.00	0.00	0.00
Markowitz	-0.27	6,630.02	127,08.17	6.50	-1,001.13	927.99	364.13	2,332.89	-6,612.71
Galton strategies									
Galton	0.47	120.76	78.35	0.00	-13.42	15.63	6.14	1.66	-78.63
Galton (excl. microcaps)	0.52	105.09	65.72	0.00	-10.97	14.31	5.35	1.46	-63.67
Galton EB	0.61	64.47	34.26	0.00	-4.74	10.83	3.33	0.77	-26.38
Shrinkage strategies									
Jorion	-0.27	2,988.79	6,434.03	3.00	-443.28	418.04	163.30	1,059.82	-2,970.82
Markowitz (JS)	-0.02	554.08	686.50	0.33	-91.56	97.91	31.98	46.26	-508.30
Elton Gruber (JS)	-0.16	169.50	191.59	0.50	-13.29	31.82	9.19	36.89	-121.96
Ledoit Wolf (JS)	0.11	125.91	121.73	0.00	-9.97	28.61	7.28	5.08	-80.33
Expected utility Maximization strategies (with investment in the riskless rate)									
Kan Zhou	0.05		401.28	0.00	-52.33	43.82	15.60	23.52	-249.82
Tu Zhou (CKZ)	0.22		186.75	0.00	-23.23	21.57	7.24	12.22	-95.58

- Bankruptcy rate (percentage of months returns below -100%).
- Galton EB is the most likely to have attractive performance after transaction costs.

4. The OOS Performance

- **Shrinkage strategies and alternative empirical Bayesian methods**

- *Jorian* :Jorion (1986)
  - shrinks the **expected returns** toward the expected return on the sample GMV
- *Elton Gruber* :The Elton and Gruber (1973) (EG)
  - shrinks correlations to the global average correlation, but does not shrink volatility estimates

$$\boldsymbol{\Sigma}_{\mathsf{EG},t} = diag(\boldsymbol{\sigma}_{\mathsf{H},t})\boldsymbol{\rho}_{\mathsf{EG},t} diag(\boldsymbol{\sigma}_{\mathsf{H},t}),$$

- *Ledoit Wolf* : The Ledoit and Wolf (2004a) (LW)
  - the covariance matrix is partially shrunk toward the constant-correlation (EG) model.

$$\boldsymbol{\mu}_{\mathsf{JS}} = \mu_0 \mathbf{1} + g_{1,\mathsf{JS}}(\boldsymbol{\mu}_{\mathsf{H}} - \mu_0 \mathbf{1}),$$

- James-Stein shrinkage :

$$g_{1,\mathsf{JS}} = \max\left(1 - \frac{(N-3)\sigma_0^2/T}{\|\boldsymbol{\mu}_{\mathsf{H}} - \mu_0 \mathbf{1}\|^2}, 0\right), N \geq 4.$$

  - μ_0 and σ_0^2 are the grand mean and grand variance obtained by pooling all *N* time series of returns

4. The OOS Performance

- **Expected utility maximization in the presence of estimation error.**

- *Kan and Zhou*: Kan and Zhou (2007) (KZ)

$$\tilde{U}(\hat{w}) = \tilde{\mu}_p - \frac{\gamma}{2}\tilde{\sigma}_p^2$$

  propose instead an optimized combination of three funds (the sample GMV, the sample MV, and the risk-less rate) that maximizes expected investor utility

- *Tu and Zhou (CKZ)*: Tu and Zhou (2011) (TZ)

  build on KZ and propose optimal combinations of naive 1/*N* portfolios with optimized strategies. We include the TZ combination of the 1=*N* with KZ, denoted as CKZ

4. The OOS Performance

Table 3
Risk statistics and VAR-style hit rates of MVE portfolios

A. Expectation versus realization statistics	Excess returns (r)			Standard deviation (σ) and corresponding hit rates					
	Expected r	Realized r	RMSFE	Expected σ	Realized σ	r<Qz(1%)	r<Qz(5%)	r>Qz(95%)	r>Qz(99%)
Value weighted		5.51			14.80				
p-value									
Talmud 1/N		5.92			15.37				
p-value									
Markowitz	398.28	-253.57	431.91	54.08	933.51	49.33	54.67	26.17	22.33
p-value		(.00)				(.00)	(.00)	(.00)	(.00)
					Galton strategies				
Galton	13.32	8.60	5.29	19.03	18.27	0.83	4.33	3.67	1.17
p-value		(.07)				(.68)	(.45)	(.13)	(.68)
Galton (excl. microcaps)	9.61	8.41	4.64	16.49	16.09	1.00	4.67	3.67	1.17
p-value		(.60)				(1.00)	(.71)	(.13)	(.68)
Galton EB	8.54	7.36	3.51	14.69	12.14	1.50	3.50	2.33	0.50
p-value		(.49)				(.22)	(.09)	(.00)	(.22)
					Shrinkage strategies				
Jorion	80.30	-111.70	151.47	23.26	415.56	40.17	44.33	33.33	28.67
p-value		(.01)				(.00)	(.00)	(.00)	(.00)
Markowitz (JS)	14.74	-2.02	28.64	4.46	95.83	34.33	39.50	36.83	32.00
p-value		(.23)				(.00)	(.00)	(.00)	(.00)
Elton Gruber (JS)	16.57	-26.24	50.24	12.76	162.61	10.83	16.00	12.33	5.67
p-value		(.08)				(.00)	(.00)	(.00)	(.00)
Ledoit Wolf (JS)	13.64	2.39	6.60	10.17	21.05	8.00	13.83	9.33	4.17
p-value		(.00)				(.00)	(.00)	(.00)	(.00)
					Expected utility Maximization strategies (with investment in the riskless rate)				
Kan Zhou		1.19			25.71				
p-value		(0.01)							
Tu Zhou (CKZ)		3.70			16.86				
p-value		(0.00)							

- The Galton methods achieve the most accurate performance.

4. The OOS Performance

B. Value-at-risk-style hit rates using various moment estimators

	Covariance and mean estimation methods							
	Galton	Galton (EB)	Markowitz (JS)	Ledoit Wolf (JS)	Galton	Galton (EB)	Markowitz (JS)	Ledoit Wolf (JS)
Portfolios	First percentile hit rates (r<Qz(1%))				Fifth percentile hit rates (r<Qz(5%))			
Standard Strategies								
Value weighted	1.17	1.50	3.50	3.83	4.00	4.67	7.50	7.67
Talmud 1/N	1.33	1.50	3.67	3.83	4.67	5.33	7.67	8.17
Markowitz	0.00	0.00	36.17	1.67	0.67	2.33	41.50	6.17
Galton strategies								
Galton	0.83	1.00	4.17	4.67	4.33	4.50	8.33	9.00
Galton (excl. microcaps)	0.67	1.00	4.17	4.00	4.50	4.67	9.50	9.00
Galton EB	1.17	1.50	5.17	4.00	3.17	3.50	11.50	9.50
Shrinkage Strategies								
Jorion	0.00	0.00	35.33	2.33	1.33	2.50	40.83	6.33
Markowitz (JS)	0.00	0.00	34.33	3.50	1.50	2.17	39.50	6.00
Elton Gruber (JS)	1.33	2.17	4.17	6.67	4.17	5.67	9.83	13.67
Ledoit Wolf (JS)	0.50	0.83	9.67	8.00	3.17	3.83	15.50	13.83
Expected utility Maximization strategies (with investment in the riskless rate)								
Kan Zhou	0.00	0.00	34.83	2.83	1.67	2.33	40.00	6.00
Tu Zhou (CKZ)	0.67	1.00	6.67	3.33	3.00	3.17	12.50	6.00
% not rejected (p-value$\geq$.05)	66.67	58.33	0.00	8.33	41.67	58.33	0.00	41.67
% not rejected (p-value$\geq$.01)	100.00	91.67	0.00	8.33	66.67	66.67	0.00	41.67
High (% $>$ target)	33.33	33.33	100.00	100.00	0.00	16.67	100.00	100.00
Low (% $\leq$ target)	66.67	66.67	0.00	0.00	100.00	83.33	0.00	0.00

- Galton moments provide effective risk management measure for a variety of strategies.

4. Robustness Check

- Stock universes
  - 50 largest firms →→
  - we simulate 1,000 horse races of sampling 50 randomly from the stock universe each year

- Galton estimation window choices
  - H =60/120/180
  - E=12/60
  - L=60/108/180
  - N=30/50/75/100

5. Analyzing the Sources of Improvement

Table 6
Dissecting the Galton strategy: OOS performance of hybrid strategies

	Mean shrinkage estimators							
							Galton strategies	
	Markowitz	Jorion	James-Stein	Cross-validation	(50 largest)	(all stocks)	(excl. microcaps)	(EB, empirical bayes) [equivalent to GMV]
A. Sharpe ratios with hybrid shrinking of moments								
Covariance matrix shrinkage estimators								
Markowitz	-0.27	-0.27	-0.02	-0.17	0.20	0.24	0.23	0.19
Jorion	-0.27	-0.27	-0.02	-0.17	0.20	0.24	0.23	0.19
Elton Gruber (JS)	-0.12	0.13	-0.16	0.15	0.47	0.44	0.47	0.52
Ledoit Wolf (JS)	0.12	-0.13	0.11	0.16	0.56	0.53	0.55	0.57
Cross-validation	-0.08	0.03	0.32	-0.13	0.55	0.56	0.56	0.55
Galton (50 largest)	-0.00	0.11	0.06	-0.18	0.58	0.52	0.56	0.60
Galton	-0.21	-0.18	0.05	-0.16	0.56	0.47	0.52	0.60
Galton (covariance)	-0.26	-0.06	0.10	-0.26	0.52	0.43	0.48	0.57
Galton (excl. microcaps)	-0.28	-0.22	0.05	-0.03	0.57	0.47	0.52	0.61
Galton EB	-0.28	-0.22	0.05	-0.03	0.57	0.47	0.52	0.61

The main sources of this improvement is the estimation to mean.

5. Analyzing the Sources of Improvement

Table 7
Galton coefficients of characteristic-sorted portfolios with persistent means

	Covariance	Correlation	Variance	Mean return
A. Size and value				
Intercept	0.00	-0.03	0.00	0.01
t-stat (=0)	1.29	-0.31	1.34	2.88
Slope	0.89	1.00	0.91	0.37
t-stat(=0)	5.11	8.79	4.59	2.47
t-stat(=1)	-0.66	0.01	-0.46	-4.15
R-squared (%)	43.16	40.87	42.00	16.60
B. Operating profitability and investment				
Intercept	0.00	0.35	0.00	0.01
t-stat (=0)	2.54	5.82	2.00	2.85
Slope	0.81	0.51	0.87	0.30
t-stat(=0)	6.87	8.71	6.75	3.42
t-stat(=1)	-1.66	-8.37	-1.01	-8.04
R-squared (%)	25.40	5.21	32.61	8.44

- The coefficient of the mean is positive.

5. Analyzing the Sources of Improvement

Table 8
Optimizing assets with persistent means: OOS performance and risk summary

A. Size (ME), Book-to-market (BTM), Beta, Investments (IN), Operating Profitability (OP), and Momentum (MOM) portfolios

Strategies	Sharpe ratio				Ratio of realized σ to average expected σ			
	ME-BTM	OP-IN	ME-BETA	ME-MOM	ME-BTM	OP-IN	ME-BETA	ME-MOM
Value weighted	0.56	0.56	0.57	0.56				
Talmud 1/N	0.57	0.59	0.59	0.54				
Markowitz	1.41	0.71	-0.08	1.30	1.42	1.32	7.67	1.67
Galton strategies								
Galton	1.49	0.96	0.76	1.59	0.95	1.11	1.12	1.25
Galton EB	1.44	0.96	0.79	1.59	0.95	1.11	1.10	1.26
Galton GMV	1.08	0.91	0.71	0.87	1.02	1.08	1.17	0.94
Shrinkage strategies								
Jorion	1.51	0.81	-0.06	1.42	1.39	1.35	7.05	1.67
Markowitz (JS)	1.17	0.77	0.69	1.11	1.38	1.36	1.40	1.47
Elton Gruber (JS)	0.63	0.87	0.56	0.76	2.40	1.61	2.41	1.96
Ledoit Wolf (JS)	0.81	0.83	0.59	1.02	1.67	1.53	1.64	1.27
Expected utility maximization strategies (with investment in the riskless rate)								
Kan Zhou	1.426	0.77	0.59	1.434				
Tu Zhou (CKZ)	1.435	0.75	0.50	1.432				
Global minimum variance strategies								
Markowitz GMV	1.17	0.77	0.69	0.81	1.38	1.36	1.40	1.45
Elton Gruber GMV	0.63	0.88	0.56	0.76	2.40	1.61	2.41	1.72
Ledoit Wolf GMV	0.81	0.85	0.59	0.77	1.67	1.53	1.64	1.30

- GMV portfolios do not necessarily dominate their MV counterparts

6. Managing the Transaction Costs of Optimized Portfolios

Table 9
OOS performance with transaction costs

Strategies	Sharpe	Turnover	Net Sharpe	Turnover	Net Sharpe	Turnover	Net Sharpe	Turnover
			10 bps		30 bps		50 bps	
A. Transaction cost agnostic strategies								
Standard strategies								
Value weighted	0.37	2.18	0.37		0.36		0.36	
Talmud 1/N	0.38	6.59	0.38		0.37		0.36	
Markowitz	-0.27	12708.17	-0.30		-0.38		-0.41	
Galton strategies								
Galton	0.47	78.35	0.42		0.31		0.21	
Galton (excl. microcaps)	0.52	65.72	0.47		0.37		0.27	
Galton EB	0.61	34.26	0.57		0.50		0.43	
Shrinkage strategies								
Jorion	-0.27	6434.03	-0.33		-0.42		-0.46	
Markowitz (JS)	-0.02	686.50	-0.11		-0.25		-0.38	
Elton Gruber (JS)	-0.16	191.59	-0.16		-0.18		-0.19	
Ledoit Wolf (JS)	0.11	121.73	0.04		-0.09		-0.20	
Expected utility maximization strategies (with investment in the riskless rate)								
Kan Zhou	0.05	401.28	-0.15		-0.53		-0.88	
Tu Zhou (CKZ)	0.22	186.75	0.08		-0.19		-0.45	

- Galton EB strategy outperforms for all three levels of costs

7. Conclusion

- Firstly, Galton-optimized portfolios perform quite well OOS in **Sharp ratio**.
- Secondly, the method also performs better at **estimating the risk** of portfolios constructed using different approaches.
- Thirdly, the performance of the Galton optimizing portfolios shows robustness to **trading costs**.