Maximizing the Sharpe Ratio: A Genetic Programming Approach

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Working paper

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Contents

Introduction – Backgrounds

- Machine learning's applications in finance concentrate in estimating the cross-section expected stock returns.

- eg: Gu, Kelly and Xiu(2020); Kozak, Nagel and Santosh(2020)……

Introduction – Motivation

- Can we apply or extend existing learning tools to maximize directly our economic objective function?
- Eg Sharpe ratio

Introduction – Research Problem

- Whether genetic programming approach can outperform other ML methods in forming cross-section portfolio?
  - Yes
- Why our genetic programming approach performs better?
  - Mainly attributed in high-idiosyncratic volatility periods
  - Non-linearity

Introduction – Contribution

- The first to use the GP to maximize the Sharpe ratio, and the first to apply it for forecasting returns in the cross-section.

Research Design – Data

- 1945.01~2019.12
- CRSP & G7 markets
- Variable
  - Size and past return signals
  - 15 variables (mainly technical signals)
    - Size/ R_{-1}/ $R_{-12,-2}$/ $R_{-60,-13}$/ short term reversal/ mom/ long-term reversal/ 11 price moving average(3-, 5-, 10, 20, 50, 100, 200, 400, 600, 800, 1000-days)
  - 15 fundament variables (Lewellence, 2015)
    - logSize/logBM/ $R_{-12,-2}$/ $logIssues_{-1,-36}$/Accruals/ ROA/ logAG/ DY/ $logR_{-13,-36}$/ $logIssues_{-12,-1}$/β/ StdDev/Turnover/DebtPrice/SalesPrice

Genetic Programming Approach

- Start from biology gene and Darwin theory……
  - Survival of the fittest → the critique?
  - Gene recombination/ Gene mutation → how to evolve?

- Objective:
- Find a function G(·) to maximize the Sharpe ratio (SR) of the usual decile long-short spread portfolio

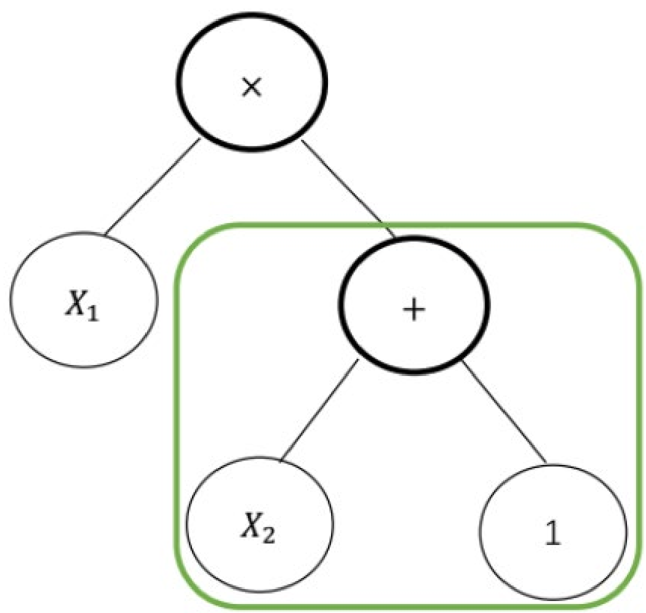
$$\max_{G(\cdot)\in\mathcal{M}} SR(Spread(G(\cdot))),$$

$$ER_G^{i,t} = G(X_{i,t-1}).$$

where M is the search space, G(·) is a function mapping from the stock characteristics to the expected return, and Spread(G(·)) is the resulting spread portfolio.

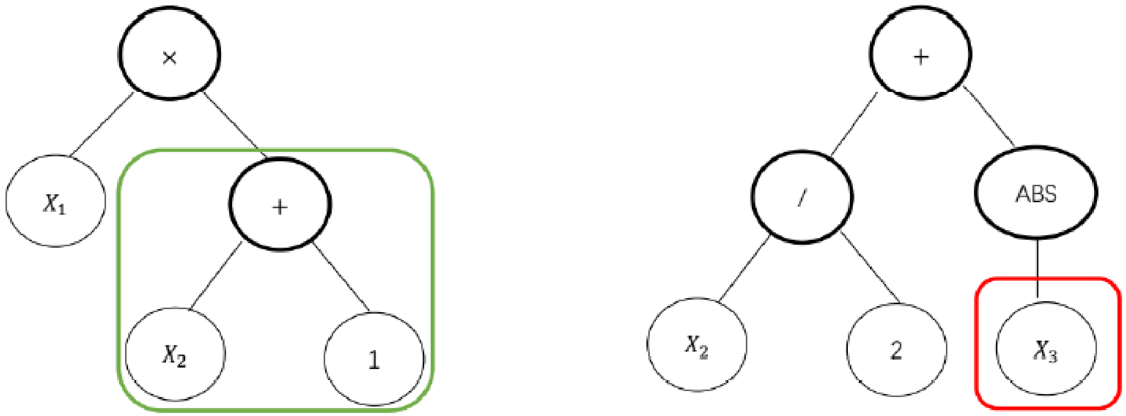
- Then, we can sort stocks by ER in each month into decile groups and construct a value-weighted spread portfolio, so weighted as all other portfolios in the paper, and denote it as Spread(G(·)).

- Evolution:
- Search space $\mathcal{M}$
  - spanned by a large set of functions combining an indicator set and an function set.
  - Indicator set: 15 X and some random constants
  - Function set: +/-/negative/x/ "/" /sin/cos/abs/cmp

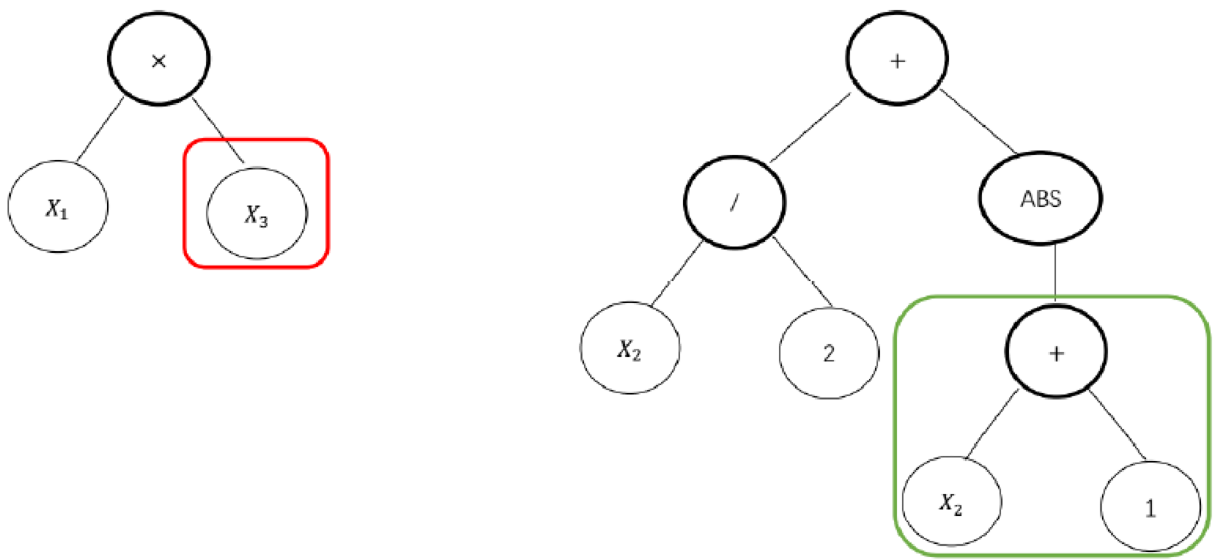


(A) $G(X) = X_1 * (X_2 + 1)$

- Randomly generates initial population of a certain number of individuals.
- Evaluate performance according to objective function.
- The individuals are randomly selected as parents individuals, in favor of the relatively fitter members.
- The parents individuals are combined by crossover and mutation, to creates offspring individuals.

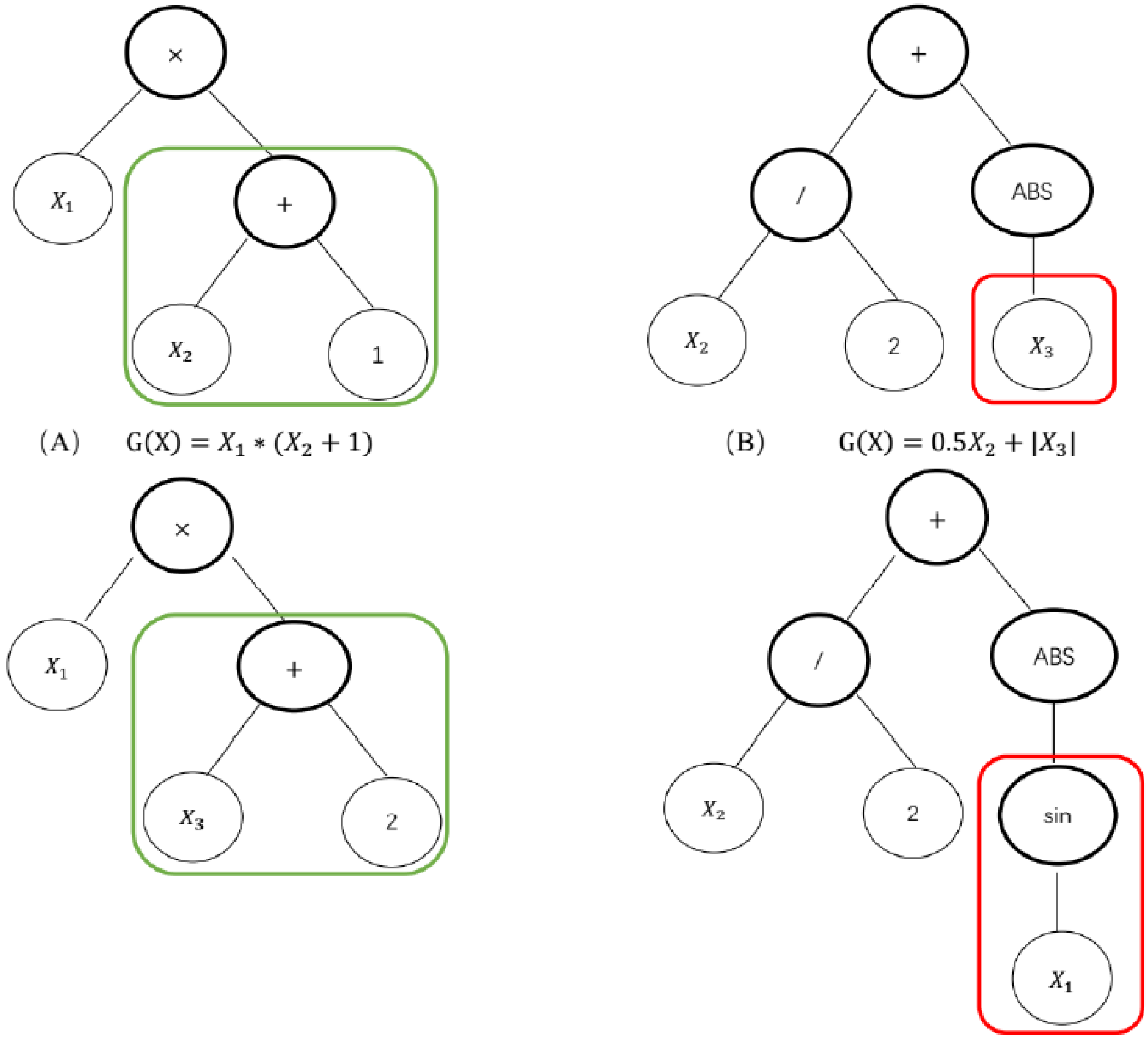


(A) $G(X) = X_1 * (X_2 + 1)$ (B) $G(X) = 0.5X_2 + |X_3|$



(C) $G(X) = X_1 * X_3$ (D) $G(X) = 0.5X_2 + |X_2 + 1|$

crossover operator



(A) $G(X) = X_1 * (X_2 + 1)$

(B) $G(X) = 0.5X_2 + |X_3|$

(E) $G(X) = X_1 * (X_3 + 2)$

(F) $G(X) = 0.5X_2 + |\sin(X_1)|$

mutation operator

- Hyperparameter tuning
  - Population (Pop): the number of individuals that GP will generate in each generation
  - Generation (Gen): the maximum generation that the evolution will iterate

- Other methods:
  - Ridge
  - Lasso
  - Enet
  - PCR
  - PLS
  - Neural Networks (1,2,3,4,5)

- Outstanding performance

Table 1

Spread portfolios

The table reports the summary statistics for the decile spread portfolios generated by the GP and other models. For each model, we report the average monthly return in percentage points, the Newey-west (1987) robust t-statistic, the annualized Sharpe ratio ($Sharpe$) and the skewness ($Skew$). The sample period is from 1991:01 to 2019:12.

	GP	Ridge	Lasso	Enet	PCR	PLS	NN1	NN2	NN3	NN4	NN5
Low	0.08	0.58	0.54	0.51	0.61	0.57	0.71	0.68	0.67	0.43	0.37
2	0.57	0.67	0.69	0.77	0.75	0.70	0.80	0.76	0.91	0.68	0.67
3	0.57	0.90	0.90	0.81	0.81	0.89	0.86	0.92	0.87	0.78	0.76
4	0.50	0.93	1.00	1.00	0.98	0.95	0.92	0.96	1.04	0.86	0.87
5	0.74	1.14	1.12	1.10	1.18	1.13	1.16	1.09	1.17	0.85	0.91
6	1.06	1.12	1.14	1.16	1.18	1.13	1.08	1.30	1.03	0.92	0.85
7	1.09	1.30	1.41	1.36	1.21	1.29	1.12	1.29	1.16	1.17	0.94
8	1.53	1.42	1.38	1.40	1.51	1.41	1.16	1.38	1.40	1.18	1.08
9	1.49	1.58	1.46	1.52	1.46	1.61	1.31	1.41	1.36	1.33	1.42
High	1.79	1.64	1.67	1.61	1.53	1.61	1.66	1.90	1.80	1.56	1.47
H-L	**1.71*****	1.06***	1.13***	1.10***	0.92***	1.04***	0.95***	1.22***	1.13***	1.12***	1.10***
t-stat	**7.12**	3.99	4.35	4.27	3.66	3.92	4.07	4.93	4.22	5.07	5.15
Sharpe	**1.32**	0.74	0.81	0.79	0.68	0.73	0.76	0.92	0.78	0.94	0.96
Skew	**1.17**	0.45	0.24	0.34	0.45	0.48	0.10	1.50	0.99	0.38	0.58

Table 2

Subperiod performance

The table reports the summary statistics for the decile spread portfolios generated by the GP and other models over two subperiods. For each model, we report the average monthly return in percentage points, the Newey-west (1987) robust t-statistic, the annualized Sharpe ratio (*Sharpe*) and the skewness (*Skew*). The sample period in Panel A is from 1991:01 to 2003:12, and in Panel B is from 2004:01 to 2019:12.

	GP	Ridge	Lasso	Enet	PCR	PLS	NN1	NN2	NN3	NN4	NN5
Panel A: 1991:01-2003:12											
H-L	**2.93*** **	2.03***	2.10***	1.93***	1.74***	2.01***	1.60***	2.25***	1.88***	1.85***	2.04***
t-stat	**6.78**	4.59	4.95	4.49	4.28	4.56	4.10	5.45	4.01	4.79	5.80
Sharpe	**1.89**	1.28	1.38	1.25	1.19	1.27	1.14	1.52	1.12	1.33	1.61
Skew	**1.08**	0.49	0.24	0.34	0.43	0.55	0.25	1.71	0.94	0.12	0.42
Panel B: 2004:01-2019:12											
H-L	**0.72*** **	0.27	0.34	0.42	0.25	0.24	0.42	0.38	0.53*	0.53**	0.33
t-stat	**3.06**	0.87	1.10	1.40	0.82	0.80	1.53	1.34	1.77	2.19	1.34
Sharpe	**0.77**	0.22	0.28	0.35	0.21	0.20	0.38	0.34	0.44	0.55	0.34
Skew	**-0.02**	0.02	-0.04	0.04	0.28	0.01	-0.41	0.78	0.64	0.42	0.48

Table 3

Spread portfolios controlling for other models

This table reports the summary statistics for the decile spread portfolios of each model controlling for one of the other models. Panel A provides the results for the GP controlling for one of the other models, and Panel B provides the results for other models controlling for the GP. The sample period is from 1991:01 to 2019:12.

	Ridge	Lasso	Enet	PCR	PLS	NN1	NN2	NN3	NN4	NN5
Panel A: GP, controlling for other models										
Low	0.66	0.65	0.64	0.60	0.66	0.55	0.61	0.65	0.58	0.55
2	0.96	1.01	0.99	0.95	0.95	0.84	0.83	0.88	0.92	0.86
3	1.10	1.06	1.14	1.07	1.13	0.98	1.09	0.96	0.87	0.88
4	1.11	1.21	0.98	1.41	1.07	0.99	0.99	0.98	1.01	1.08
5	1.15	1.19	1.26	1.09	1.17	1.22	1.16	1.17	1.16	1.10
6	1.04	1.13	1.11	1.13	0.93	1.18	1.11	1.23	1.10	1.07
7	1.55	1.27	1.42	1.32	1.57	1.13	1.29	1.35	1.03	1.23
8	1.20	1.35	1.35	1.12	1.25	1.29	1.37	1.25	1.33	1.29
9	1.43	1.74	1.54	1.53	1.38	1.38	1.35	1.31	1.38	1.28
High	1.28	1.26	1.30	1.30	1.29	1.29	1.23	1.25	1.39	1.40
H-L	0.62***	0.62***	0.65***	0.70***	0.62***	0.74***	0.62***	0.60***	0.81***	0.85***
t-stat	4.27	4.13	4.43	4.47	4.30	4.90	4.09	4.20	5.58	5.70

Panel B: Other models, controlling for GP										
Low	0.97	1.04	1.05	1.03	0.96	0.91	0.82	0.61	0.84	0.94
2	1.08	1.17	1.29	1.34	1.27	1.18	0.99	1.13	1.18	1.26
3	1.26	1.23	1.37	1.28	1.25	0.83	1.01	1.22	1.13	1.39
4	1.25	1.33	1.30	1.30	1.16	1.32	0.94	1.18	1.23	1.29
5	1.14	1.33	1.28	1.20	1.23	1.13	1.00	1.21	1.38	1.28
6	1.37	1.27	1.25	1.05	1.37	1.13	0.97	1.32	1.12	1.20
7	1.05	1.06	1.04	1.02	1.08	1.19	0.98	1.13	1.04	1.17
8	1.00	1.02	1.05	1.07	1.02	1.10	0.94	0.98	1.08	1.14
9	0.87	0.87	0.85	0.94	0.87	1.03	0.95	1.00	0.96	0.97
High	0.96	0.96	0.96	0.95	0.96	0.90	0.97	0.88	0.92	0.90
H-L	0.00	-0.08	-0.09	-0.07	0.00	-0.01	0.15	0.27	0.09	-0.04
t-stat	-0.01	-0.32	-0.38	-0.31	-0.02	-0.03	0.68	1.15	0.28	-0.11

- Time-varying outperformance

$$\Delta R_t = \beta_L Low_{t-1}^{Vol} + \beta_H High_{t-1}^{Vol} + \beta MKT_t + \epsilon_t,$$

dummy variables indicating low- and high-IVOL periods of previous month

	β_L	t-stat	β_H	t-stat
Ridge	0.26	0.94	1.05***	2.75
Lasso	0.15	0.50	1.02**	2.51
Enet	0.10	0.36	1.13***	2.67
PCR	0.30	1.08	1.28***	2.80
PLS	0.27	0.96	1.08***	2.80
NN1	0.23	0.98	1.29***	3.01
NN2	0.41	1.60	0.57	1.57
NN3	0.36	1.55	0.79*	1.70
NN4	0.35	1.41	0.83**	2.31
NN5	0.50**	2.18	0.73*	1.91
Average	0.31	1.20	1.02	2.43

- The improved performance of GP over other models is mainly attributed to the high-IVOL periods, during which the information uncertainty level is high.

- # Factor performance

based on a 2x3 double sorting on size and ER_{GP}

	GPF	Mkt	SMB	HML	RMW	CMA	IA	ROE	MGMT	PERF	PEAD	FIN
Panel A: Summary statistics												
Mean	1.20***	0.69**	0.21	0.30	0.34*	0.26*	0.28**	0.44***	0.53***	0.64**	0.51***	0.56**
t-stat	(6.65)	(2.56)	(1.30)	(1.38)	(1.78)	(1.86)	(2.25)	(2.67)	(2.80)	(2.19)	(4.04)	(2.07)
Std. dev.	2.37	4.24	3.25	3.04	2.71	2.08	1.99	2.80	2.96	4.47	2.06	4.44
Sharpe	1.75	0.56	0.22	0.34	0.44	0.44	0.49	0.54	0.62	0.50	0.86	0.44
Skew	0.85	-0.67	0.74	0.16	-0.41	0.60	0.32	-0.72	0.46	0.02	0.30	-0.03
Kurt	5.93	4.34	11.17	5.42	12.95	5.43	5.09	7.48	5.53	6.28	7.32	8.36
Panel B: Correlation matrix												
GPF	1.00	0.12	0.11	-0.15	-0.11	-0.10	-0.14	-0.06	-0.06	0.08	0.07	-0.13
Mkt	0.12	1.00	0.22	-0.16	-0.46	-0.36	-0.32	-0.45	-0.45	-0.45	-0.12	-0.54
SMB	0.11	0.22	1.00	-0.28	-0.55	-0.14	-0.25	-0.45	-0.42	-0.11	0.11	-0.57
HML	-0.15	-0.16	-0.28	1.00	0.38	0.66	0.68	0.14	0.67	-0.23	-0.25	0.64
RMW	-0.11	-0.46	-0.55	0.38	1.00	0.25	0.33	0.73	0.50	0.42	-0.08	0.76
CMA	-0.10	-0.36	-0.14	0.66	0.25	1.00	0.91	0.14	0.74	0.05	-0.10	0.59
IA	-0.14	-0.32	-0.25	0.68	0.33	0.91	1.00	0.20	0.76	0.00	-0.17	0.67
ROE	-0.06	-0.45	-0.45	0.14	0.73	0.14	0.20	1.00	0.34	0.66	0.21	0.55
MGMT	-0.06	-0.45	-0.42	0.67	0.50	0.74	0.76	0.34	1.00	0.13	-0.08	0.81
PERF	0.08	-0.45	-0.11	-0.23	0.42	0.05	0.00	0.66	0.13	1.00	0.43	0.24
PEAD	0.07	-0.12	0.11	-0.25	-0.08	-0.10	-0.17	0.21	-0.08	0.43	1.00	-0.11
FIN	-0.13	-0.54	-0.57	0.64	0.76	0.59	0.67	0.55	0.81	0.24	-0.11	1.00

Panel A: Spanning test	Wald test under			Bekaert-Urias test		
	W	W_e	W_a	J_1	J_2	J_3
CAPM	895.85***	451.07***	645.15***	69.84***	69.08***	452.33***
	[0.00]	[0.00]	[0.00]	[0.00]	[0.00]	[0.00]
FF-3	217.31***	114.03***	155.78***	63.64***	71.72***	137.74***
	[0.00]	[0.00]	[0.00]	[0.00]	[0.00]	[0.00]
FF-5	95.58***	59.19***	73.41 ***	64.19***	71.74***	90.15***
	[0.00]	[0.00]	[0.00]	[0.00]	[0.00]	[0.00]
HXZ-4	102.35***	61.80***	80.62***	62.86***	71.70***	84.06***
	[0.00]	[0.00]	[0.00]	[0.00]	[0.00]	[0.00]
SY-4	68.65***	46.09***	63.01***	42.58***	47.11***	53.81***
	[0.00]	[0.00]	[0.00]	[0.00]	[0.00]	[0.00]
DHS-3	98.73***	58.60***	73.41***	55.08***	60.63***	85.44***
	[0.00]	[0.00]	[0.00]	[0.00]	[0.00]	[0.00]

Panel B: Sh^2 in the Sharpe ratio test				
	Original	With GPF	$\Delta(Sh^2)$	p-value
CAPM	0.026	0.265	0.239***	[0.00]
FF-3	0.046	0.304	0.258***	[0.00]
FF-5	0.137	0.390	0.253***	[0.00]
HXZ-4	0.147	0.403	0.256***	[0.00]
SY-4	0.210	0.408	0.198***	[0.00]
DHS-3	0.200	0.438	0.238***	[0.00]

- the GPF can add substantial investment value to existing factor models.
- the GP factor can improve the pricing ability of existing models substantially

Table 7

Risk-adjusted returns

The table reports the risk-adjusted returns of the spread portfolios generated by the GP and other methods. Newey-west (1987) robust t-statistics are reported in parentheses. The sample period is from 1991:01 to 2016:12.

	GP	Ridge	Lasso	Enet	PCR	PLS	NN1	NN2	NN3	NN4	NN5
CAPM	1.69***	0.95***	1.00***	0.97***	0.86**	0.93***	1.02***	1.25***	0.98***	1.07***	1.07***
	(5.78)	(2.94)	(2.91)	(2.96)	(2.36)	(2.85)	(4.07)	(3.90)	(3.45)	(4.34)	(3.60)
FF-3	1.77***	0.84***	0.90***	0.86***	0.69***	0.82***	0.94***	1.12***	0.93***	1.03***	1.01***
	(5.39)	(3.50)	(3.37)	(3.54)	(2.77)	(3.36)	(4.62)	(4.49)	(4.23)	(4.55)	(3.86)
FF-5	1.86***	0.88***	0.95***	0.89***	0.67***	0.86***	0.81***	1.12***	1.14***	0.95***	0.97***
	(4.76)	(3.53)	(3.40)	(3.56)	(2.65)	(3.37)	(3.70)	(3.83)	(4.91)	(4.15)	(3.86)
HXZ-4	1.80***	0.75***	0.77***	0.73***	0.55**	0.72***	0.51**	0.96***	1.02***	0.81***	0.86***
	(4.97)	(3.01)	(2.73)	(2.89)	(2.13)	(2.84)	(2.06)	(3.84)	(4.23)	(3.62)	(3.08)
SY-4	1.56***	0.70***	0.79**	0.72***	0.55**	0.68**	0.42*	0.87***	0.98***	0.89***	0.70***
	(5.33)	(2.68)	(2.57)	(2.69)	(2.02)	(2.55)	(1.67)	(3.56)	(4.36)	(3.73)	(2.77)
DHS-3	1.67***	1.34***	1.33***	1.33***	1.12***	1.32***	1.00***	1.48***	1.50***	1.23***	1.13***
	(6.26)	(4.59)	(4.27)	(4.64)	(3.63)	(4.48)	(3.39)	(4.93)	(5.94)	(4.57)	(3.62)
CAPM+GPF	-0.01	-0.17	-0.05	-0.06	-0.06	-0.19	0.30	0.15	-0.07	0.04	-0.05
	(-0.06)	(-0.64)	(-0.16)	(-0.20)	(-0.20)	(-0.70)	(1.43)	(0.59)	(-0.33)	(0.20)	(-0.24)

- existing factor models cannot explain the predicted returns of the machine learning methods.
- the GP factor improves substantially the pricing ability of existing models.

Robustness

Alternative characteristic set

This table reports the performance of the decile spread portfolios based on the alternative characteristic set. For each spread portfolio, we report the average monthly return in percentage points, the Newey-west (1987) robust t-statistic, the annualized Sharpe ratio ($Sharpe$), the skewness ($Skew$), and the maximum drawdown (MDD) in percentage. The sample period is from 2001:01 to 2019:12.

	GP	Ridge	Lasso	Enet	PCR	PLS	NN1	NN2	NN3	NN4	NN5
Low	0.12	0.35	0.17	0.23	0.21	0.15	0.11	0.27	0.24	0.24	0.22
2	0.61	0.29	0.37	0.34	0.28	0.30	0.60	0.38	0.51	0.39	0.72
3	0.57	0.78	0.60	0.70	0.76	0.71	0.61	0.83	0.71	0.79	0.64
4	0.72	0.65	0.82	0.77	0.73	0.79	0.86	0.75	0.86	0.78	0.65
5	0.88	0.77	0.88	0.77	0.78	0.73	0.83	0.77	0.77	0.81	0.73
6	1.00	0.95	0.78	0.98	0.78	0.59	0.80	0.92	0.68	0.76	0.75
7	0.80	0.76	0.83	0.78	0.88	1.03	0.71	0.62	0.95	0.67	0.81
8	0.95	0.81	0.65	0.69	0.71	0.80	0.94	0.76	0.91	0.71	1.05
9	1.00	0.83	0.90	0.86	0.87	0.85	0.77	0.83	0.65	0.94	0.71
High	1.11	0.61	0.67	0.62	0.61	0.68	0.65	0.87	0.85	0.80	0.81
H-L	0.99***	0.26	0.51	0.39	0.40	0.53	0.54	0.61*	0.61*	0.56*	0.58
t-stat	3.29	0.60	1.14	0.89	0.94	1.26	1.45	1.76	1.91	1.66	1.57
Sharpe	0.74	0.13	0.26	0.20	0.21	0.28	0.32	0.40	0.43	0.37	0.35
Skew	0.91	0.40	0.43	0.44	0.48	0.21	0.71	0.55	0.34	0.21	0.55

	GP	Ridge	Lasso	Enet	PCR	PLS	NN1	NN2	NN3	NN4	NN5
Panel A: UK											
Mean	1.69	1.29	1.34	1.43	1.31	1.29	1.33	1.66	1.30	1.01	1.13
t-stat	5.77	3.74	3.89	4.34	3.69	3.75	4.18	5.23	3.86	2.87	3.63
Sharpe	1.09	0.71	0.73	0.82	0.70	0.71	0.79	0.99	0.73	0.54	0.69
Panel B: Canada											
Mean	2.05	1.85	1.79	1.80	1.23	1.79	1.63	1.55	0.90	0.89	0.85
t-stat	4.62	3.43	3.35	3.34	2.39	3.32	3.11	3.32	1.83	1.72	1.65
Sharpe	0.86	0.64	0.63	0.62	0.45	0.62	0.58	0.62	0.34	0.32	0.31
Panel C: Germany											
Mean	2.11	1.36	1.60	1.41	1.40	1.39	0.82	0.75	0.65	0.80	1.40
t-stat	6.52	3.54	4.01	3.53	3.62	3.64	2.49	2.03	1.71	2.01	3.95
Sharpe	1.22	0.66	0.75	0.66	0.68	0.68	0.46	0.38	0.32	0.38	0.74

Table 10

Performance under alternative parameters

The table reports the annualized Sharpe ratio and average return of the spread portfolios generated by GP under alternative hyperparameters $< Pop, Gen >$ and M. Panel A, B, and C reports the results for M = 5, 3, and 10, respectively. The training sample is from 1945:01 to 1980:12. The validation sample is from 1981:01 to 1990:12. The OOS sample is from 1991:01 to 2019:12.

	Sharpe ratio									Mean return								
	Train			Validation			OOS			Train			Validation			OOS		
Gen\Pop	100	200	400	100	200	400	100	200	400	100	200	400	100	200	400	100	200	400
Panel A: Average of Top 5 Models																		
10	2.04	2.30	2.28	1.69	1.68	1.59	1.07	0.93	1.06	1.77	2.15	2.16	1.46	1.67	1.63	1.38	1.37	1.59
20	2.38	2.39	2.44	1.85	1.49	1.68	1.22	0.92	1.14	1.91	2.19	2.24	1.58	1.46	1.62	1.45	1.33	1.66
40	2.85	2.96	2.84	1.86	2.66	2.07	1.11	1.32	1.01	2.29	2.22	2.38	1.68	2.28	1.94	1.56	1.71	1.41
Panel B: Average of Top 3 Models																		
10	2.07	2.34	2.30	1.81	1.71	1.62	1.15	0.92	1.00	1.72	2.20	2.15	1.49	1.69	1.68	1.37	1.36	1.54
20	2.39	2.40	2.45	1.87	1.48	1.64	1.21	0.90	1.13	1.93	2.19	2.25	1.61	1.44	1.59	1.43	1.29	1.66
40	2.87	2.96	2.85	1.76	2.63	2.05	1.14	1.27	1.02	2.31	2.22	2.37	1.63	2.26	1.92	1.62	1.69	1.41
Panel C: Average of Top 10 Models																		
10	2.00	2.26	2.24	1.79	1.54	1.55	1.03	0.95	1.06	1.71	2.13	2.13	1.48	1.56	1.59	1.34	1.39	1.54
20	2.36	2.37	2.43	1.85	1.49	1.67	1.21	0.93	1.13	1.90	2.17	2.22	1.58	1.49	1.63	1.44	1.34	1.63
40	2.82	2.96	2.82	1.85	2.68	2.05	1.11	1.27	0.99	2.30	2.21	2.38	1.69	2.28	1.94	1.58	1.68	1.39

- What drives GP's performance?
  - ✓ Objective function

regress the expected returns generated by the two GP models on each other, and then examine the performance of the resulting spread portfolio sorted by the residuals.

	Original		Controlling for each other	
	GP_{MSE}	GP_{SR}	GP^{ω}_{MSE}	GP^{ω}_{SR}
Low	0.07	0.08	0.66	0.64
2	0.40	0.57	1.31	0.87
3	0.58	0.57	1.29	1.11
4	0.79	0.50	1.26	1.14
5	0.80	0.74	1.22	1.15
6	0.95	1.06	1.25	1.19
7	1.20	1.09	1.05	1.11
8	1.22	1.53	0.97	1.38
9	1.53	1.49	0.92	1.32
High	1.50	1.79	0.92	1.55
H-L	1.44***	1.71***	0.27	0.91***
t-stat	5.59	7.12	0.83	5.69
Std. dev.	4.79	4.47	5.72	2.84
Sharpe	1.04	1.32	0.16	1.11
Skew	0.37	1.17	-0.69	0.52

✓ Linearity vs nonlinearity

Table 12

Simulation: Linear vs nonlinear

This table reports the OOS performances of various models in the linear and nonlinear simulations. $\hat{R}_{i,t}$ is the fitted return from step 2 in the simulation procedure. The simulation procedure is discussed in section 5.2.

$$\tilde{R}_{i,t} = \hat{R}_{i,t} + \tilde{\epsilon}_{i,t}$$

	Annual SR		Mean Rt	
	Linear	Nonlinear	Linear	Nonlinear
$\hat{R}_{i,t}$	1.33	3.36	1.26	3.49
GP	1.24	3.08	1.08	2.50
Ridge	1.30	2.03	1.24	1.97
Lasso	1.32	2.05	1.27	2.03
Enet	1.32	2.07	1.25	2.01
PCR	1.30	2.03	1.24	1.97
PLS	1.30	2.03	1.25	1.97

Conclusion

- We find that the performance of the GP spread portfolio in the cross-section outperforms substantially the usual MSE-based models