# Financial statement errors: evidence from the distributional properties of financial statement numbers

Dan Amiram, Zahn Bozanic, Ethan Rouen

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# Background

- Financial statement data that are free of error—whether in the form of misestimations, mistakes, biases, or manipulation—are crucial for well-functioning capital markets.

- Current measures of financial statement errors has many limitations, such as their correlation with underlying firm characteristics and their reliance on time-series, cross-sectional, or forward-looking data.

- Benford's Law states that the first digits of all numbers in an **empirical dataset** will appear with decreasing frequency.

- Methods based on the law have been used to detect errors in published scientific studies, questionable election data in Iran and USA, suspicious macroeconomic data, internal accounts receivables data, and misreported tax returns.

# A Introduction: Benford's Law

- $N = \alpha * 10^{\beta}$ , $\alpha \in [1, 10)$, $\beta \in Z$
- $\mathrm{n} = \log_{10} N = \beta + \log_{10} \alpha$ , $\log_{10} \alpha \in [0, 1)$
- $\log_{10} \alpha \in \begin{cases} 0 \sim 0.301, \ \alpha \in [1, 2), \ N \ will \ start \ with \ 1 \\ 0.301 \sim 0.477, \ \alpha \in [2, 3), \ N \ will \ start \ with \ 2 \\ \cdots\cdots \end{cases}$
- Because distributions in **nature** tend to be smooth and symmetric due to the Central Limit Theorem, datasets tend to follow Benford's Law.
- $N = 11100 = \alpha * 2^{\beta} = 1.11 * 2^{100}$, $\log_2 N = 100 + \log_2 1.11$
- Given that we will stop at a random point each time we cumulate, the process will reach lower first digits (e.g., 1's and 2's) more often than higher leading or first digits (e.g., 8's and 9's).

| *d* | *p* |
|---|---|
| 1 | 30.1% |
| 2 | 17.6% |
| 3 | 12.5% |
| 4 | 9.7% |
| 5 | 7.9% |
| 6 | 6.7% |
| 7 | 5.8% |
| 8 | 5.1% |
| 9 | 4.6% |

# Motivation

- There is no research applying the Benford's Law to the **entire** population of numbers in a firm's annual financial statements in order to ascertain whether it can be used as a firm-year measure of the degree of errors in financial reporting.

- The financial statements' line items are estimates of **the realizations of cash flows** from unknown random distributions. The mixture distribution of the cash flows realization of these data may be distributed according to Benford's Law (Hill's theorem, 1995).

- The Benford's Law does not require timeseries or cross-sectional data to estimate, does not require forward-looking information, does not require returns or price information.

# Research Framework

- Whether the realized empirical distribution of the first digits of firms' financial statement numbers follows Benford's Law?
  - Whether in aggregate, by year, by industry, or by firm-year.
  - Introduce errors for a typical firm in our sample by randomly manipulating its revenue.

- Whether the FSD Score can reflect financial statement errors or not?
  - Examining the relation between FSD Score and commonly used measures of earnings manipulation (F-Score, M-Score).
  - Compare the FSD Score for the restated and unrestated numbers of the same restated firms.
  - Examining the relation between the level of conformity to the law and earnings persistence.

- Whether Benford's Law is predictive of material misstatements?

# Research Conclusion

- We show that at the aggregate level, financial statement numbers conform to Benford's Law in all industries and years, with the income statement  the greatest divergence from Benford's Law.

- We find that overestimating revenue, underestimating expenses are likely to introduce deviation from the law by a  simple simulation.

- We show that the FSD Score is significantly positively related with the Dechow–Dichev measure, discretionary accruals measures, and Beneish's M-Score.

- The restated numbers are significantly closer to Benford's Law relative to the misstated numbers. Furthermore, as firms' financial statements diverge from the law, their earnings persistence decreases.

- FSD score is leading predictive to material misstatements.

# Variable Construction

| Digit | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
|---|---|---|---|---|---|---|---|---|---|
| Total occurrences | 8 | 5 | 3 | 3 | 2 | 2 | 1 | 2 | 2 |
| Empirical distribution (AD) | 0.2857 | 0.1786 | 0.1071 | 0.1071 | 0.0714 | 0.0714 | 0.0357 | 0.0714 | 0.0714 |
| Theoretical distribution (ED) | 0.3010 | 0.1761 | 0.1249 | 0.0969 | 0.0792 | 0.0669 | 0.0580 | 0.0512 | 0.0458 |

$$\text{MAD} = \frac{\left(\sum_{i=1}^{K} |\text{AD} - \text{ED}|\right)}{K} = (|0.2857 - 0.3010| + |0.1786 - 0.1761| + \cdots)/9 = 0.0140$$

The probability that the first digit will be $d$ for a given PDF

The probability that the first digit will be $d$ following Benford's Law

$$\text{FSD Score} = \frac{\sum_{d=1}^{9} ABS\left[\left(\sum_{n=-\infty}^{\infty} \int_{n+\log(d)}^{n+\log(d+1)} PDF\left(\log(N)\right)dN\right) - \left(Log(d+1) - Log(d)\right)\right]}{9}$$

# Sample selection

- Our sample consists of **all** annual financial statement data from Compustat for the period **2001–2011**.
- For simplicity and objectivity, we use **all Compustat variables** that appear in the balance sheet, income statement, and statement of cash flow to calculate the FSD Score.
- We remove firm-years where the total number is less than **100** and firms with **negative total assets** and **winsorize** variables at the 1 and 99 % levels to eliminate the influence of outliers.

| | Average number of line items | Number of firm-years | Average digit distributions | | | | | | | | | FSD_Score |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | | | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | |
| Panel C: FSD_Score by number of financial statement line items | | | | | | | | | | | | |
| Top 1 % of line items | 169 | 420 | 0.3019 | 0.1798 | 0.1249 | 0.0980 | 0.0776 | 0.0672 | 0.0558 | 0.0506 | 0.0442 | 0.0231 |
| Top tercile | 144 | 14,730 | 0.3037 | 0.1781 | 0.1247 | 0.0969 | 0.0789 | 0.0660 | 0.0572 | 0.0498 | 0.0448 | 0.0259 |
| Middle tercile | 124 | 14,941 | 0.3041 | 0.1769 | 0.1248 | 0.0958 | 0.0786 | 0.0661 | 0.0579 | 0.0506 | 0.0454 | 0.0292 |
| Bottom tercile | 108 | 13,661 | 0.3042 | 0.1771 | 0.1241 | 0.0966 | 0.0788 | 0.0661 | 0.0577 | 0.0502 | 0.0453 | 0.0335 |
| Bottom 1 % of line items | 100 | 736 | 0.3017 | 0.1764 | 0.1254 | 0.0957 | 0.0799 | 0.0666 | 0.0597 | 0.0499 | 0.0447 | 0.0362 |
| Overall | 125 | 43,332 | 0.3040 | 0.1773 | 0.1245 | 0.0964 | 0.0787 | 0.0661 | 0.0576 | 0.0502 | 0.0452 | 0.0296 |

# 1. Aggregate conformity to Benford's Law

- In the aggregate, the FSD Score is 0.0009, well below 0.006, which can be considered close conformity to the law in very large samples (Nigrini, 2012).
- Panels B and C of Table 2 show similar results when examining aggregate financial results by industry and by fiscal year.

Panel A: FSD_Score for all firm-years' financial statement numbers

| | |
|---|---|
| 43,332 | 0.0009 |

| FF Industry | Number of firm-years | Aggregate FSD_Score |
|---|---|---|
| Panel B: FSD_Score for all financial statement numbers by industry | | |
| 1 | 1410 | 0.0009 |
| 2 | 689 | 0.0018 |
| 3 | 1931 | 0.0009 |
| 4 | 888 | 0.0013 |
| Panel C: FSD_Score for all financial statement numbers by year | | |
| 2001 | 4418 | 0.0008 |
| 2002 | 4345 | 0.0011 |
| 2003 | 4177 | 0.0013 |
| 2004 | 4153 | 0.0010 |

# 1. Individual conformity to Benford's Law

- Overall, the result supports our conjecture that a significant majority of firm-year empirical distributions conform to Benford's Law.

| | | |
|---|---|---|
| Panel A: Number of firm-years conforming to Benford's Law | | |
| 37,104 | | 85.63 |
| Financial statement | Firm-years conforming | Percent conforming |
| Panel B: Number of firm-years conforming to Benford's Law by financial statement | | |
| Balance sheet | 39,274 | 90.64 |
| Income statement | 34,138 | 78.78 |
| Cash flow statement | 42,259 | 97.52 |
| FF Industry | Firm-years conforming | Percent conforming |
| Panel C: Number of firm-years conforming to Benford's Law by industry | | |
| 1 | 1218 | 86.38 |
| 2 | 580 | 84.18 |
| 3 | 1680 | 87.00 |
| 4 | 765 | 86.15 |
| Fiscal year | Firm-years conforming | Percent conforming |
| Panel D: Number of firm-years conforming to Benford's Law by fiscal year | | |
| 2001 | 3795 | 85.90 |
| 2002 | 3753 | 86.38 |
| 2003 | 3562 | 85.28 |
| 2004 | 3538 | 85.19 |

# 1. Conformity to Benford's Law: Statement partitions

- On average, the aggregate FSD Score for the income statement diverges the most from Benford's Law, suggesting more errors.

- On average, equity, liability accounts, income accounts contain a relatively higher level of errors.

| Financial statement | Number of accounts | Aggregate FSD_Score |
|---|---|---|
| Panel A: Aggregate FSD_Score by financial statement | | |
| Balance sheet | 111 | 0.0005 |
| Income statement | 101 | 0.0020 |
| Cash flow statement | 38 | 0.0005 |
| Panel B: Aggregate FSD_Score by financial statement subcategory | | |
| Balance sheet | | |
| Assets | 37 | 0.0004 |
| Liabilities | 43 | 0.0007 |
| Equity | 28 | 0.0010 |
| Income statement | | |
| Expenses | 11 | 0.0012 |
| Income | 32 | 0.0027 |

# 1. Simulation analysis

- We choose to manipulate sales for Alcoa's 2011 financial statements, which is a firm that generally, but not perfectly, conforms to Benford's Law.
- The results of this simulation, when run 1,000 times, show that the random revenue manipulation increased the FSD Score 95 % of the time.

| | |
|---|---|
| 1. Increase accounts receivables | Increase revenue |
| 2. Increase cost of goods sold | Decrease inventory |
| 3. Increase tax expense | Increase tax payable |

| Income statement | Balance sheet | Statement of cash flow |
|---|---|---|
| Sales | Receivables—Trade | Income before extraordinary items (cash flow) |
| Cost of goods sold | Receivables—Total | Accounts receivable—decrease(increase) |
| Gross profit (Loss) | Inventories—finished goods | Inventory—decrease (increase) |
| Operating income after depreciation | Inventories—total | Income taxes—accrued—increase/(decrease) |
| Operating income before depreciation | Current assets—total | |
| Pretax income | Assets—total | |
| Pretax income–domestic | Income taxes payable | |
| Income taxes—federal | Current liabilities—total | |
| Income taxes—total | Liabilities—total | |

# 2. The relation with existing measures

| Variable | Tercile by FSD_Score | | |
|---|---|---|---|
| | Top tercile | Middle tercile | Bottom tercile |
| Panel A: Univariate evidence | | | |
| Accrual Quality | | | |
| ABS_JONES_RESID | 0.2076 | 0.1787 | 0.1644*** |
| STD_DD_RESID | 0.1434 | 0.1195 | 0.1067*** |
| MANIPULATOR | 0.1646 | 0.1407 | 0.1223*** |
| F_SCORE | 0.3733 | 0.4071 | 0.4212*** |
| ABS_WCACC | 0.0611 | 0.0532 | 0.0479*** |
| ABS_RSST | 0.1579 | 0.1374 | 0.1198*** |
| Earnings Quality | | | |
| EARNINGS PERSISTENCE | 0.6094 | 0.6831 | 0.6921*** |
| LOSS | 0.4212 | 0.3483 | 0.3097*** |

Panel B: Multivariate evidence

$$FSD\_Score_{i,t} = \alpha + \beta_1 ABS\_JONES\_RESID_{i,t} + \beta_2 STD\_DD\_RESID_{i,t} + \beta_3 MANIPULATOR_{i,t} + \beta_4 F\_SCORE_{i,t} + \beta_5 ABS\_WCACC_{i,t} + \beta_6 ABS\_RSST_{i,t} + \beta_7 LOSS_{i,t} + \varepsilon_{i,t}$$

| Variable | FSD_Score |
|---|---|
| ABS_JONES_RESID | 0.0002 |
| | (1.45) |
| STD_DD_RESID | 0.0042*** |
| | (12.62) |
| MANIPULATOR | 0.0010*** |

# 2. Univariate evidence to detect accounting errors

- To understand whether Benford's Law captures firms with a higher propensity for errors in their accounting results ex post.

- We find that the FSD Score is lower after firms restate their misstated data, higher for loss firms, higher for firms that just beat the zero-earnings benchmark, and lower for firms that receive an AAER.

| | FSD_Score | Number of firm-years | *t* statistic |
|---|---|---|---|
| RESTATED_NUMS = 0 | 0.0289 | 4935 | 5.36*** |
| RESTATED_NUMS = 1 | 0.0280 | 4935 | |
| LOSS = 0 | 0.0289 | 27,743 | 23.98*** |
| LOSS = 1 | 0.0310 | 15,589 | |
| $-0.005 \leq NI_t/MKT\_VAL_{t-1} < 0$ | 0.0283 | 588 | 2.32** |
| $0 \leq NI_t/MKT\_VAL_{t-1} \leq 0.005$ | 0.0296 | 426 | |
| AAER = 1 | 0.0270 | 82 | 2.75*** |
| AAER = 0 | 0.0296 | 42,963 | |

# 2. Multivariate evidence to detect accounting errors

- We investigate all firm-years in Compustat from 2001 to 2011 where both misstated and restated financial results are available.
- Since the regression compares the firm to itself, we do not include additional firm control variables in this specification.

$$FSD_{Score\,i,t} = \alpha + \beta_1 RESTATED\_NUMS_{i,t} + \beta_2 ABS\_JONES\_RESID_{i,t} + \beta_3 STD\_DD\_RESID_{i,t} + \beta_4 MANIPULATOR_{i,t} + \beta_5 F\_SCORE_{i,t} + \beta_6 ABS\_WCACC_{i,t} + \beta_7 ABS\_RSST_{i,t} + \varepsilon_{i,t}$$

| Variable | FSD_Score | |
|---|---|---|
| | (1) | (2) |
| RESTATED_NUMS | −0.0009*** | −0.0009*** |
| | (−5.26) | (−5.33) |
| ABS_JONES_RESID | | 0.0000 |
| | | (0.06) |
| STD_DD_RESID | | 0.0049*** |
| | | (7.88) |
| MANIPULATOR | | 0.0005* |
| | | (1.89) |
| F_SCORE | | −0.0033*** |
| | | (−10.11) |
| ABS_WCACC | | 0.0024* |
| | | (1.65) |
| ABS_RSST | | 0.0032*** |
| | | (5.66) |
| Constant | 0.0288*** | 0.0289*** |
| | (244.85) | (149.34) |
| Observations | 10,192 | 10,192 |
| R-squared | 0.003 | 0.030 |

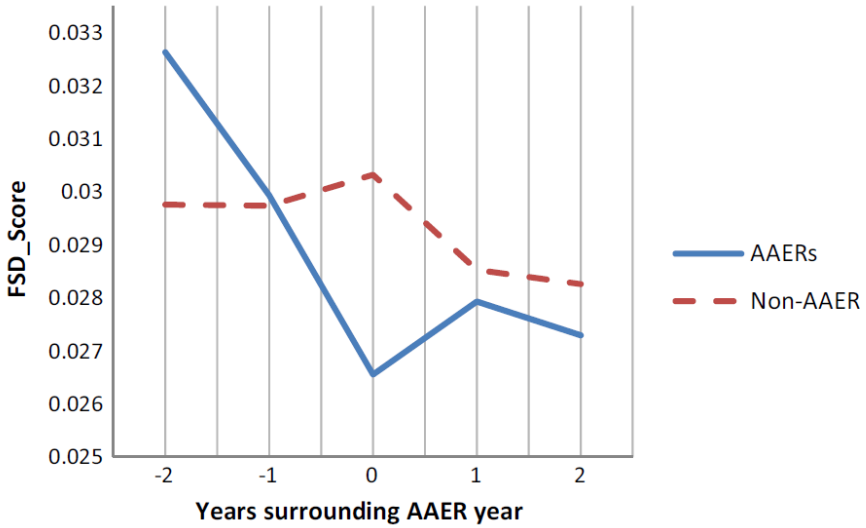
# 2. FSD Score and Earnings persistence

- The idea is based on the notion that it is less likely that current earnings will be as informative about future earnings in firms with lower accounting quality (Richardson, 2005).

$$NI_{i,t+1} = \alpha + \beta_1 NI_{i,t} + \beta_2 FSD\_Score_{i,t} + \beta_3 FSD\_Score \times NI_{i,t} + \beta_4 ABS\_JONES\_RESID_{i,t} + \beta_5 STD\_DD\_RESID_{i,t} + \beta_6 MANIPULATOR_{i,t} + \beta_7 F\_SCORE_{i,t} + \beta_8 ABS\_WCACC_{i,t} + \beta_9 ABS\_RSST_{i,t} + \beta_{10} LOSS_{i,t} + \beta_{11} SALES\_GROWTH_{i,t} + \beta_{12} DIV_{i,t} + \beta_{13} SIZE_{i,t} + \beta_{14} MTB_{i,t} + \beta_{15} SI_{i,t} + \beta_{16} AGE_{i,t} + \beta_{17} RET\_VOL_{i,t} + \beta_{18} NI\_VOL_{i,t} + \varepsilon_{i,t}$$

| Variable | $NI_{t+1}$ |
|---|---|
| NI | 0.3268*** |
| | (16.70) |
| FSD_Score | −1.0403*** |
| | (−5.58) |
| FSD × NI | −2.6277*** |
| | (−5.26) |
| ABS_JONES_RESID | 0.0005 |
| | (0.11) |
| STD_DD_RESID | −0.1625*** |
| | (−12.00) |
| MANIPULATOR | −0.0018 |
| Observations | 28,042 |
| R-squared | 0.225 |

# 2. FSD Score and AAER




- If the SEC does indeed detect and prosecute all firms that make material misstatements, then a positive coefficient is expected.

- However, prior research suggests that SEC AAERs reflect only firms that experience significant declines in their ability to hide the misstatements (Dechow et al. 2011), which may result in a nonpositive relation as FSD Scores decrease.

# 2. FSD Score and AAER

$$AAER_{i,t} = \alpha + \beta_1 FSD\_Score + \beta_2 ABS\_JONES\_RESID_{i,t} + \beta_3 STD\_DD\_RESID_{i,t} + \beta_4 MANIPULATOR_{i,t} + \beta_5 F\_SCORE_{i,t} + \beta_6 ABS\_WCACC_{i,t} + \beta_7 ABS\_RSST_{i,t} + \beta_8 CH\_CS_{i,t} + \beta_9 CH\_ROA_{i,t} + \beta_{10} SOFT\_ASSETS_{i,t} + \beta_{11} ISSUE_{i,t} + \beta_{12} MTB_{i,t} + \beta_{13} AT_{i,t} + \varepsilon_{i,t}$$

| Variable | AAER | | |
|---|---|---|---|
| | (1) | (2) | (3) |
| FSD_Score | −40.691*** | | |
| | (−3.87) | | |
| $FSD\_Score_{t-1}$ | | 21.963* | |
| | | (1.80) | |
| $FSD\_Score_{t-2}$ | | | 39.222*** |
| | | | (7.34) |
| ABS_JONES_RESID | −1.078 | −1.074 | −1.059 |
| | (−1.38) | (−1.33) | (−1.32) |
| F_SCORE | 1.980*** | 1.978*** | 1.994*** |
| | (5.88) | (5.80) | (5.58) |
| Constant | −5.686*** | −7.620*** | −8.198*** |
| | (−5.67) | (−9.15) | (−8.73) |
| Observations | 27,805 | 27,805 | 27,805 |

# Conclusion

- We use numerical methods to demonstrate that financial statements without error are distributed according to Benford's Law. Overestimating revenue, underestimating expenses are likely to introduce deviation from the law.

- We show that at the aggregate level, financial statement numbers conform to Benford's Law in all industries and years, with the income statement  the greatest divergence from Benford's Law.

- Firms that report just above the zero earnings threshold have weaker conformity than firms reporting just below zero.

- The restated numbers are significantly closer to Benford's Law relative to the misstated numbers. Furthermore, as firms' financial statements diverge from the law, their earnings persistence decreases.

- FSD score is leading predictive to material misstatements.

# Inspiration

- Evidence suggests that stock indices' returns conform to Benford's Law (Ley 1996).
- Select stocks with normal price changes, whose income changes are less affected by insider operations, and the fundamental information is more effective.