Heuristics

This problem has many similarities with the Travelling Salesman Problem (TSP), which is known not to be computationally efficient to solve. To avoid this issue, it can be useful to solve the problem using a greedy heuristic.

1 Cheap Heuristic

The cheap heuristic attempts to find the cheapest 15 day trip with 15 possible cities. For each city, the heuristic calculates the cost of travelling from Melbourne to that city and remaining in that city for maxDays. The algorithm then chooses the city that has the minimum cost. In the same way, cities are added recursively until there are no days left. The traveller then flies back to Melbourne from their final city.

Algorithm 1 Cheap Heuristic

```
Begin in Melbourne
for each city i do
    cost(i) = costFromMelb(i) + maxDays × costDaily(i)
end for
Go to city i with the minimum cost and stay for maxDays
daysLeft = days − maxDays
while daysLeft > 0 do
    step = min(maxDays, daysLeft)
    for each city i not yet visited do
        cost(i) = costTravel(currentCity, i) + step × costDaily(i)
    end for
    Go to city i with the minimum cost and stay for step days
    Decrement daysLeft by step
end while
Return to Melbourne from final city
```

Using maxDays = 4 and the medium daily costs defined in Section DATA, the output of this heuristic is shown in Table 1. The heuristic solution is to spend the first 4 days in Istanbul, then fly to Moscow for 4 days, followed by Prague for 4 days and finally Venice for 3 days. The cost of this trip is $\$4,060$ including Melbourne flights, flights within Europe and daily costs. The utility of this trip is $1,102$.

Number of Days	City
4	Istanbul
4	Moscow
4	Prague
3	Venice

Table 1: Cheap Heuristic Output

2 Maximum Utility Heuristic

The maximum utility heuristic attempts to find the 15 day trip with the maximum utility regardless of cost. At each iteration, the algorithm chooses the city with the maximum utility from the set of cities containing the unvisited cities and the current city. After spending a day in any city, the utility of staying another day in that city is multiplied by a decay factor.

Algorithm 2 Maximum Utility Heuristic

Begin in Melbourne
for $j = 1, \ldots,$ days **do**
  Find city i with the maximum utility in the set {unvisited cities}$\bigcup${current city}
  Go to city i
  Reduce the utility of the current city by the decay factor
end for
Return to Melbourne from final city

Using a decay factor of 0.98 and the medium daily costs defined in Section DATA, the output of this heuristic is shown in Table 2. The utility of this trip is $1,330$, which is 21% greater than the utility of the trip found using the cheap heuristic. The cost of this trip is $\$5,450$, which is 34% ($\$1,390$) greater than the cheap heuristic.

Number of Days	City
4	London
4	Paris
2	Berlin
4	Rome
1	Barcelona

Table 2: Maximum Utility Heuristic Output