# 1 Base model

**Variables**
$x_{id}$: [binary] equal to 1 if the traveller is in city $i$ on day $d$.
$y_i$: [binary] equal to 1 if the traveller ever visits city $i$.
$m_{ijd}$: [binary] equal to 1 if the traveller moves from city $i$ to city $j$ on day $d$.

**Parameters**
$D$: the total number of days.
$\mathcal{C}$: the set of cities available for visiting.
$\mu_{ij}$: the cost to move from city $i$ to city $j$.
$\delta_i$: the daily cost of city $i$.
$\alpha$: the minimum number of days allowed in a city.
$\omega$: the maximum number of days allowed in a city.

$$\text{Min} \sum_{d=1}^{D}\sum_{i\in\mathcal{C}} \delta_i \cdot x_{id} + \sum_{d=1}^{D-1}\sum_{i\in\mathcal{C}}\sum_{j\in\mathcal{C}} \mu_{ij}\cdot m_{ijd}$$

$$\text{s.t.} \qquad \sum_{d=1}^{D} x_{id} \geq \alpha \cdot y_i \qquad \forall i \in \mathcal{C} \tag{1}$$

$$\sum_{d=1}^{D} x_{id} \leq \omega \cdot y_i \qquad \forall i \in \mathcal{C} \tag{2}$$

$$x_{id} + x_{j(d+1)} \leq 1 + m_{ijd} \qquad \forall i, j \in \mathcal{C}, d = 1..D-1 \tag{3}$$

$$\sum_{i\in\mathcal{C}} x_{id} = 1 \qquad d = 1..D \tag{4}$$

$$\sum_{i\in\mathcal{C}}\sum_{j\in\mathcal{C}} m_{ijd} = 1 \qquad d = 1..D-1 \tag{5}$$

$$\sum_{d=1}^{D}\sum_{j\in\mathcal{C}} m_{ijd} \leq 1 + \sum_{d=1}^{D} m_{iid} \qquad \forall i \in \mathcal{C} \tag{6}$$

$$x_{j1} + \sum_{d=1}^{D}\sum_{i\in\mathcal{C}} m_{ijd} \leq 1 + \sum_{d=1}^{D} m_{jjd} \qquad \forall j \in \mathcal{C} \tag{7}$$

$$x_{id}, y_i, m_{ijd} \in \{0,1\} \qquad \forall i, j \in \mathcal{C}, d = 1..D \tag{8}$$

$$\tag{9}$$

## 2 Decaying Enjoyment

Assuming that the enjoyment of staying in a city decays with the number of days stayed. We add the following to the model.

**Variables**
$s_{ij}$: [binary] equal to 1 if traveller stays in city i for j days.

**Parameters**
$u_{ij}$: The total utility of staying in city i for j days.

Then we no longer need (1) and (2) since we hope that the traveller naturally leaves the city once the marginal enjoyment is low enough.

Then the new model is

$$\text{Max} \sum_{i\in\mathcal{C}} \sum_{j=1}^{D} s_{ij} * u_{ij}$$

$$\begin{aligned}
\text{s.t.} \quad & \sum_{j=1}^{D} s_{ij} = y_i && \forall i \in \mathcal{C} && (1)\\
& s_{ij} \cdot j \leq \sum_{d=1}^{D} x_{id} && \forall i \in \mathcal{C}, j = 1...D && (2)\\
& x_{id} + x_{j(d+1)} \leq 1 + m_{ijd} && \forall i, j \in \mathcal{C}, d = 1..D-1 && (3)\\
& \sum_{i\in\mathcal{C}} x_{id} = 1 && d = 1..D && (4)\\
& \sum_{i\in\mathcal{C}} \sum_{j\in\mathcal{C}} m_{ijd} = 1 && d = 1..D-1 && (5)\\
& \sum_{d=1}^{D} \sum_{j\in\mathcal{C}} m_{ijd} \leq 1 + \sum_{d=1}^{D} m_{iid} && \forall i \in \mathcal{C} && (6)\\
& x_{j1} + \sum_{d=1}^{D} \sum_{i\in\mathcal{C}} m_{ijd} \leq 1 + \sum_{d=1}^{D} m_{jjd} && \forall j \in \mathcal{C} && (7)\\
& x_{id}, y_i, m_{ijd} \in \{0,1\} && \forall i, j \in \mathcal{C}, d = 1..D && (8)\\
& && && (9)
\end{aligned}$$

With this we find the optimal enjoyment possible for the traveller irrespective of cost, then we optimise again to minimise cost at the optimal enjoyment.

$$\text{Min} \sum_{d=1}^{D} \sum_{i\in\mathcal{C}} \delta_i \cdot x_{id} + \sum_{d=1}^{D-1} \sum_{i\in\mathcal{C}} \sum_{j\in\mathcal{C}} \mu_{ij} \cdot m_{ijd}$$

s.t.

(1)-(9) from previous model is statisfied and

$$\sum_{i\in\mathcal{C}} \sum_{j=1}^{D} s_{ij} * u_{ij} = (\sum_{i\in\mathcal{C}} \sum_{j=1}^{D} s_{ij} * u_{ij})_{optimal}$$