Things Need to Remember (Midterm 1)

$$\langle x|p\rangle = \frac{1}{\sqrt{2\pi\hbar}} e^{\frac{ipx}{\hbar}} \quad , \quad \langle p|x\rangle = \frac{1}{\sqrt{2\pi\hbar}} e^{\frac{-ipx}{\hbar}}$$

$$\hat{x} = \sqrt{\frac{\hbar}{2m\omega}}\,(\hat{a}^{+} + \hat{a})$$

$$\hat{p} = i\sqrt{\frac{m\omega\hbar}{2}}\,(\hat{a}^{+} - \hat{a})$$

$$\hat{H} = \hbar\omega\left(\hat{N} + \frac{1}{2}\right) = \frac{\hat{p}^2}{2m} + V(\vec{r}) \quad , \quad (\hat{N} = \hat{a}^{+}\hat{a})$$

$$\hat{a}^{+}|n\rangle = \sqrt{n+1}\,|n+1\rangle \quad ; \quad \hat{a}|n\rangle = \sqrt{n}\,|n-1\rangle$$

Schrödinger Picture : "state" depends on time, while operator not.

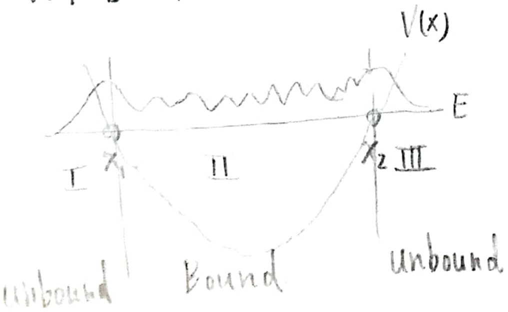
Heisenberg Picture : "Operator" depends on time, while state not.

Interacting Picture : "Propagator" depends on time, while state & operator not.

Variational Method:
$$E_\lambda = \frac{\int dx\, \psi^*(x)\left[\frac{-\hbar^2}{2m}\frac{\partial^2}{\partial x^2} + V(x)\right]\psi(x)}{\psi(x)\,\psi^*(x)}$$

WKB Method:

$$\psi(x) = \frac{A}{\sqrt{P(x)}}\, e^{\pm\frac{i}{\hbar}\int_{x_0}^{x} P(x')\,dx'} \quad , \quad P(x') = \sqrt{2m(E - V(x'))}$$



$$\Downarrow$$

$$\begin{cases} \psi_{III}(x) \sim \frac{A}{\sqrt{P(x)}}\, e^{\frac{-1}{\hbar}\int^{x}\sqrt{2m(V(x')-E)}\,dx} \\ \psi_{I}(x) \sim \frac{A}{\sqrt{P(x)}}\, e^{\frac{1}{\hbar}\int^{x}\sqrt{2m(V(x')-E)}\,dx} \\ \psi_{II}(x) \begin{cases} x \to x_1 : \frac{A}{\sqrt{P(x)}} \cos\left(\frac{1}{\hbar}\int_{x_1}^{x} P(x')\,dx - \frac{\pi}{4}\right) \\ x \to x_2 : \frac{A}{\sqrt{P(x)}} \cos\left(\frac{1}{\hbar}\int_{x_2}^{x} P(x')\,dx + \frac{\pi}{4}\right) \end{cases} \end{cases}$$

$\Rightarrow$ Quantization condition:

$$\int_{x_1}^{x_2} \sqrt{2m(E - V(x'))}\,dx' = \left(n + \frac{1}{2}\right)\hbar\pi$$