Elastic Fields of an Edge Dislocation

1. Displacement Fields (Edge Dislocation)

$$u_x = \frac{b}{2\pi}\left[\tan^{-1}\left(\frac{y}{x}\right) + \frac{xy}{2(1-\nu)(x^2+y^2)}\right],$$

$$u_y = -\frac{b}{2\pi}\left[\frac{1-2\nu}{4(1-\nu)}\ln(x^2+y^2) + \frac{x^2-y^2}{4(1-\nu)(x^2+y^2)}\right].$$

2. Strain Field Derivations

$\varepsilon_{xx} = \partial u_x/\partial x$

$$\varepsilon_{xx} = -\frac{b}{2\pi r}\left[\sin\theta + \frac{\sin\theta\cos 2\theta}{2(1-\nu)}\right].$$

$\varepsilon_{yy} = \partial u_y/\partial y$

$$\varepsilon_{yy} = -\frac{b}{2\pi r}\left[\frac{1}{2(1-\nu)}\sin\theta - \frac{\sin\theta\cos 2\theta}{2(1-\nu)}\right].$$

$\varepsilon_{xy} = \frac{1}{2}(\partial_x u_y + \partial_y u_x)$

$$\varepsilon_{xy} = \frac{b}{4\pi r(1-\nu)}\cos\theta\cos 2\theta.$$

Trace term

$$\varepsilon_{xx} + \varepsilon_{yy} = -\frac{b}{2\pi r}\frac{\sin\theta}{1-\nu}.$$

3. Hooke's Law (Plane Strain)

$$\sigma_{xx} = 2G\varepsilon_{xx} + \lambda(\varepsilon_{xx} + \varepsilon_{yy}),$$
$$\sigma_{yy} = 2G\varepsilon_{yy} + \lambda(\varepsilon_{xx} + \varepsilon_{yy}),$$
$$\sigma_{xy} = 2G\varepsilon_{xy}.$$

Elastic constants:

$$G = \frac{E}{2(1+\nu)}, \qquad \lambda = \frac{2G\nu}{1-2\nu}.$$

4. Stress Field Derivations

σ_{xx}

$$\sigma_{xx} = 2G\varepsilon_{xx} + \lambda(\varepsilon_{xx} + \varepsilon_{yy}),$$
$$\sigma_{xx} = 2G\left[-\frac{b}{2\pi r}\left(\sin\theta + \frac{\sin\theta\cos 2\theta}{2(1-\nu)}\right)\right] + \lambda\left[-\frac{b}{2\pi r}\frac{\sin\theta}{1-\nu}\right].$$
$$\boxed{\sigma_{xx} = -\frac{Gb}{2\pi r(1-\nu)}\,\sin\theta\,(1-\cos 2\theta)}.$$

σ_{yy}

$$\sigma_{yy} = 2G\varepsilon_{yy} + \lambda(\varepsilon_{xx} + \varepsilon_{yy}),$$

$$\sigma_{yy} = -\frac{b}{2\pi r}\frac{\sin\theta}{1-\nu}\left[G(1-\cos 2\theta) + \lambda\right],$$

$$\sigma_{yy} = -\frac{b}{2\pi r}\frac{\sin\theta}{1-\nu}\left[2G\sin^2\theta + \frac{2G\nu}{1-2\nu}\right].$$

$$\boxed{\sigma_{yy} = -\frac{Gb}{2\pi r(1-\nu)}\,\sin\theta\left(2\sin^2\theta + \frac{2\nu}{1-2\nu}\right)}.$$

σ_{xy}

$$\sigma_{xy} = 2G\varepsilon_{xy},$$

$$\sigma_{xy} = 2G\left(\frac{b}{4\pi r(1-\nu)}\cos\theta\cos 2\theta\right),$$

$$\boxed{\sigma_{xy} = \frac{Gb}{2\pi r(1-\nu)}\cos\theta\cos 2\theta}.$$

5. Final Simplified Classical Stress Fields

In terms of (r, θ) only

$$\boxed{\sigma_{xx} = -\frac{Gb}{2\pi(1-\nu)} \frac{\sin\theta}{r} (1 - \cos 2\theta)}$$

$$\boxed{\sigma_{yy} = -\frac{Gb}{2\pi(1-\nu)} \frac{\sin\theta}{r} (1 + \cos 2\theta)}$$

$$\boxed{\sigma_{xy} = \frac{Gb}{2\pi(1-\nu)} \frac{\cos\theta \cos 2\theta}{r}}$$

These are the textbook elastic stress fields for an edge dislocation.