Strain component ε_{xx} for an edge dislocation

The displacement field is

$$u_x = \frac{b}{2\pi}\left[\tan^{-1}\left(\frac{y}{x}\right) + \frac{xy}{2(1-\nu)(x^2+y^2)}\right].$$

Thus,

$$\varepsilon_{xx} = \frac{\partial u_x}{\partial x} = \frac{b}{2\pi}\left[\frac{\partial}{\partial x}\tan^{-1}\left(\frac{y}{x}\right) + \frac{\partial}{\partial x}\left(\frac{xy}{2(1-\nu)(x^2+y^2)}\right)\right].$$

Derivative of the first term

$$\frac{\partial}{\partial x}\tan^{-1}\left(\frac{y}{x}\right) = \frac{1}{1+(y/x)^2}\left(-\frac{y}{x^2}\right) = -\frac{y}{x^2+y^2}.$$

Derivative of the second term

$$\frac{\partial}{\partial x}\left(\frac{xy}{x^2+y^2}\right) = \frac{y(x^2+y^2) - xy(2x)}{(x^2+y^2)^2} = \frac{y(y^2-x^2)}{(x^2+y^2)^2}.$$

Including the constant factor:

$$\frac{\partial}{\partial x}\left(\frac{xy}{2(1-\nu)(x^2+y^2)}\right) = \frac{y(y^2-x^2)}{2(1-\nu)(x^2+y^2)^2}.$$

Combine both parts

$$\varepsilon_{xx} = \frac{b}{2\pi}\left[-\frac{y}{x^2+y^2} + \frac{y(y^2-x^2)}{2(1-\nu)(x^2+y^2)^2}\right].$$

Convert to polar coordinates

Using

$$x = r\cos\theta, \quad y = r\sin\theta, \quad x^2+y^2 = r^2, \quad y^2 - x^2 = -r^2\cos 2\theta,$$

we obtain

$$-\frac{y}{x^2+y^2} = -\frac{r\sin\theta}{r^2} = -\frac{\sin\theta}{r},$$

$$\frac{y(y^2-x^2)}{2(1-\nu)(x^2+y^2)^2} = \frac{r\sin\theta(-r^2\cos 2\theta)}{2(1-\nu)r^4} = -\frac{\sin\theta\cos 2\theta}{2(1-\nu)r}.$$

Final expression

$$\boxed{\varepsilon_{xx} = -\frac{b}{2\pi r}\left[\sin\theta + \frac{\sin\theta\cos 2\theta}{2(1-\nu)}\right].}$$

Strain component ε_{yy} for an edge dislocation

The displacement field is

$$u_y = -\frac{b}{2\pi}\left[\frac{1-2\nu}{4(1-\nu)}\ln(x^2+y^2) + \frac{x^2-y^2}{4(1-\nu)(x^2+y^2)}\right].$$

Hence,

$$\varepsilon_{yy} = \frac{\partial u_y}{\partial y} = -\frac{b}{2\pi}\left[\frac{1-2\nu}{4(1-\nu)}\frac{\partial}{\partial y}\ln(x^2+y^2) + \frac{\partial}{\partial y}\left(\frac{x^2-y^2}{4(1-\nu)(x^2+y^2)}\right)\right].$$

Derivative of the first term

$$\frac{\partial}{\partial y}\ln(x^2+y^2) = \frac{2y}{x^2+y^2}.$$

Derivative of the second term

Using the quotient rule,

$$\frac{\partial}{\partial y}\left(\frac{x^2-y^2}{x^2+y^2}\right) = \frac{-2y(x^2+y^2)-(x^2-y^2)(2y)}{(x^2+y^2)^2} = \frac{-4x^2y}{(x^2+y^2)^2}.$$

Including the constant,

$$\frac{\partial}{\partial y}\left(\frac{x^2-y^2}{4(1-\nu)(x^2+y^2)}\right) = \frac{-x^2y}{(1-\nu)(x^2+y^2)^2}.$$

Combine both contributions

$$\varepsilon_{yy} = -\frac{b}{2\pi}\left[\frac{1-2\nu}{4(1-\nu)}\frac{2y}{x^2+y^2} - \frac{x^2y}{(1-\nu)(x^2+y^2)^2}\right].$$

Convert to polar form

$$x = r\cos\theta, \qquad y = r\sin\theta, \qquad x^2+y^2 = r^2.$$

Thus,

$$\frac{2y}{x^2+y^2} = \frac{2r\sin\theta}{r^2} = \frac{2\sin\theta}{r},$$

$$\frac{-x^2y}{(x^2+y^2)^2} = \frac{-r^2\cos^2\theta\, r\sin\theta}{r^4} = -\frac{\cos^2\theta\,\sin\theta}{r}.$$

Final expression

$$\boxed{\varepsilon_{yy} = -\frac{b}{2\pi r}\left[\frac{1-2\nu}{2(1-\nu)}\sin\theta - \frac{\cos^2\theta\,\sin\theta}{1-\nu}\right]}$$

Strain component ε_{xy} for an edge dislocation

The displacement fields are

$$u_x = \frac{b}{2\pi}\left[\tan^{-1}\left(\frac{y}{x}\right) + \frac{xy}{2(1-\nu)(x^2+y^2)}\right],$$

$$u_y = -\frac{b}{2\pi}\left[\frac{1-2\nu}{4(1-\nu)}\ln(x^2+y^2) + \frac{x^2-y^2}{4(1-\nu)(x^2+y^2)}\right].$$

The engineering shear strain is

$$\varepsilon_{xy} = \frac{1}{2}\left(\frac{\partial u_x}{\partial y} + \frac{\partial u_y}{\partial x}\right).$$

Derivative of u_x with respect to y

$$\frac{\partial}{\partial y}\tan^{-1}\left(\frac{y}{x}\right) = \frac{x}{x^2+y^2},$$

$$\frac{\partial}{\partial y}\left(\frac{xy}{x^2+y^2}\right) = \frac{x(x^2-y^2)}{(x^2+y^2)^2}.$$

Including constants:

$$\frac{\partial}{\partial y}\left(\frac{xy}{2(1-\nu)(x^2+y^2)}\right) = \frac{x(x^2-y^2)}{2(1-\nu)(x^2+y^2)^2}.$$

Thus,

$$\frac{\partial u_x}{\partial y} = \frac{b}{2\pi}\left[\frac{x}{x^2+y^2} + \frac{x(x^2-y^2)}{2(1-\nu)(x^2+y^2)^2}\right].$$

Derivative of u_y with respect to x

$$\frac{\partial}{\partial x}\ln(x^2+y^2) = \frac{2x}{x^2+y^2},$$

$$\frac{\partial}{\partial x}\left(\frac{x^2-y^2}{x^2+y^2}\right) = \frac{4xy^2}{(x^2+y^2)^2}.$$

Including constants:

$$\frac{\partial}{\partial x}\left(\frac{x^2-y^2}{4(1-\nu)(x^2+y^2)}\right) = \frac{xy^2}{(1-\nu)(x^2+y^2)^2}.$$

Thus,

$$\frac{\partial u_y}{\partial x} = -\frac{b}{2\pi}\left[\frac{1-2\nu}{4(1-\nu)}\frac{2x}{x^2+y^2} + \frac{xy^2}{(1-\nu)(x^2+y^2)^2}\right].$$

Combine contributions

$$\varepsilon_{xy} = \frac{1}{2}\left(\frac{b}{2\pi}\left[\frac{x}{x^2+y^2} + \frac{x(x^2-y^2)}{2(1-\nu)(x^2+y^2)^2}\right] - \frac{b}{2\pi}\left[\frac{1-2\nu}{2(1-\nu)}\frac{x}{x^2+y^2} + \frac{xy^2}{(1-\nu)(x^2+y^2)^2}\right]\right).$$

Convert to polar form

$$x = r\cos\theta, \qquad x^2 - y^2 = r^2\cos 2\theta, \qquad xy^2 = r^3\cos\theta\,\sin^2\theta.$$

Then

$$\frac{x}{x^2+y^2} = \frac{\cos\theta}{r}, \qquad \frac{x(x^2-y^2)}{2(1-\nu)(x^2+y^2)^2} = \frac{\cos\theta\cos 2\theta}{2(1-\nu)r},$$

$$\frac{2x}{x^2+y^2} = \frac{2\cos\theta}{r}, \qquad \frac{xy^2}{(x^2+y^2)^2} = \frac{\cos\theta\sin^2\theta}{r}.$$

Final expression

$$\boxed{\varepsilon_{xy} = \frac{b}{4\pi r}\left[\cos\theta + \frac{\cos\theta\cos 2\theta}{2(1-\nu)} - \frac{1-2\nu}{2(1-\nu)}\cos\theta - \frac{\cos\theta\sin^2\theta}{1-\nu}\right]}.$$

$$\boxed{\varepsilon_{xy} = \frac{b}{4\pi r(1-\nu)}\cos\theta\cos 2\theta}$$

Stress Components from Strain Fields (Plane Strain)

Strain fields

$$\varepsilon_{xx} = -\frac{b}{2\pi r}\left[\sin\theta + \frac{\sin\theta\cos 2\theta}{2(1-\nu)}\right],$$
$$\varepsilon_{yy} = -\frac{b}{2\pi r}\left[\frac{1}{2(1-\nu)}\sin\theta - \frac{\sin\theta\cos 2\theta}{2(1-\nu)}\right],$$
$$\varepsilon_{xy} = \frac{b}{4\pi r(1-\nu)}\cos\theta\cos 2\theta.$$

Hooke's law (plane strain)

$$\sigma_{xx} = 2G\,\varepsilon_{xx} + \lambda(\varepsilon_{xx}+\varepsilon_{yy}), \qquad \sigma_{yy} = 2G\,\varepsilon_{yy} + \lambda(\varepsilon_{xx}+\varepsilon_{yy}),$$
$$\sigma_{xy} = 2G\,\varepsilon_{xy}.$$

$$G = \frac{E}{2(1+\nu)}, \qquad \lambda = \frac{2G\nu}{1-2\nu}.$$

Trace term

$$\varepsilon_{xx}+\varepsilon_{yy} = -\frac{b}{2\pi r}\frac{\sin\theta}{1-\nu}.$$

Stress component σ_{yy}

$$\sigma_{yy} = 2G\varepsilon_{yy} + \lambda(\varepsilon_{xx}+\varepsilon_{yy}),$$
$$\sigma_{yy} = -\frac{b}{2\pi r}\left[2G\left(\frac{\sin\theta - \sin\theta\cos 2\theta}{2(1-\nu)}\right) + \lambda\left(\frac{\sin\theta}{1-\nu}\right)\right].$$

Factor common terms:

$$\sigma_{yy} = -\frac{b}{2\pi r}\frac{\sin\theta}{1-\nu}\left[G(1-\cos 2\theta) + \lambda\right].$$

Use $1-\cos 2\theta = 2\sin^2\theta$:

$$\sigma_{yy} = -\frac{b}{2\pi r}\frac{\sin\theta}{1-\nu}\left[2G\sin^2\theta + \lambda\right].$$

Substitute $\lambda = \frac{2G\nu}{1-2\nu}$:

$$\boxed{\sigma_{yy} = -\frac{Gb}{2\pi r(1-\nu)}\sin\theta\left(2\sin^2\theta + \frac{2\nu}{1-2\nu}\right)}.$$

Stress component σ_{xy}

$$\sigma_{xy} = 2G\,\varepsilon_{xy} = 2G\left[\frac{b}{4\pi r(1-\nu)}\cos\theta\cos 2\theta\right],$$

Simplified:

$$\boxed{\sigma_{xy} = \frac{Gb}{2\pi r(1-\nu)}\cos\theta\cos 2\theta}.$$

Stress Field of an Edge Dislocation in Terms of Shear Modulus G

$$\sigma_{xx} = -\frac{Gb}{2\pi(1-\nu)}\,\frac{\sin\theta}{r}\left[1-\cos 2\theta\right],$$

$$\sigma_{yy} = -\frac{Gb}{2\pi(1-\nu)}\,\frac{\sin\theta}{r}\left[1+\cos 2\theta\right],$$

$$\sigma_{xy} = \frac{Gb}{2\pi(1-\nu)}\,\frac{\cos\theta\cos 2\theta}{r}.$$