A motivating example for **random effect model**:

Suppose statistical grades can be modelled as $E\left(Y_{ij}\right)=100+1\cdot gender_i-1\cdot age_{ij}$, related to a student's gender and age. For two male students, if their grades and age are

	statistical grades	gender	age
Student A, 1st exam	90	1	30
Student A, 2nd exam	92	1	31
Student A, 3rd exam	88	1	32
Student A, 4th exam	90	1	33
Student B, 1st exam	10	1	30
Student B, 2nd exam	8	1	31
Student B, 3rd exam	11	1	32
Student B, 4th exam	9	1	33

Please calculate the residuals of these eight observations.

Please calculate the residuals of these eight observations.

$90=100+1\cdot1-1\cdot30+e_{11}$ $\quad e_{11}=90-100-1\cdot1+1\cdot30=19$

$92=100+1\cdot1-1\cdot31+e_{12}$ $\quad e_{12}=92-100-1\cdot1+1\cdot31=22$

$88=100+1\cdot1-1\cdot32+e_{13}$ $\quad e_{13}=88-100-1\cdot1+1\cdot32=19$

$90=100+1\cdot1-1\cdot33+e_{14}$ $\quad e_{14}=90-100-1\cdot1+1\cdot33=22$

Centered around 20

$10=100+1\cdot1-1\cdot30+e_{21}$ $\quad e_{21}=10-100-1\cdot1+1\cdot30=-61$

$8=100+1\cdot1-1\cdot31+e_{22}$ $\quad e_{22}=8-100-1\cdot1+1\cdot31=-62$

$11=100+1\cdot1-1\cdot32+e_{23}$ $\quad e_{23}=11-100-1\cdot1+1\cdot32=-58$

$9=100+1\cdot1-1\cdot33+e_{24}$ $\quad e_{24}=9-100-1\cdot1+1\cdot33=-59$

Centered around -60

These residuals are not centered around 0 and not independent to each other, which violates the assumption of conventional linear regression. If we let Student A have a subject-specific effect $\gamma_1=20$, and let Student B have a subject-specific effect $\gamma_2=-60$, the residuals will be

$$90 = 100 + 1\cdot 1 - 1\cdot 30 + 20 + e_{11} \qquad e_{11} = 90 - 100 - 1\cdot 1 + 1\cdot 30 - 20 = -1$$

$$92 = 100 + 1\cdot 1 - 1\cdot 31 + 20 + e_{12} \qquad e_{12} = 92 - 100 - 1\cdot 1 + 1\cdot 31 - 20 = 2$$

$$88 = 100 + 1\cdot 1 - 1\cdot 32 + 20 + e_{13} \qquad e_{13} = 88 - 100 - 1\cdot 1 + 1\cdot 32 - 20 = -1$$

$$90 = 100 + 1\cdot 1 - 1\cdot 33 + 20 + e_{14} \qquad e_{14} = 90 - 100 - 1\cdot 1 + 1\cdot 33 - 20 = 2$$

Centered around 0

$$10 = 100 + 1\cdot 1 - 1\cdot 30 - 60 + e_{21} \qquad e_{21} = 10 - 100 - 1\cdot 1 + 1\cdot 30 + 60 = -1$$

$$8 = 100 + 1\cdot 1 - 1\cdot 31 - 60 + e_{22} \qquad e_{22} = 8 - 100 - 1\cdot 1 + 1\cdot 31 + 60 = -2$$

$$11 = 100 + 1\cdot 1 - 1\cdot 32 - 60 + e_{23} \qquad e_{23} = 11 - 100 - 1\cdot 1 + 1\cdot 32 + 60 = 2$$

$$9 = 100 + 1\cdot 1 - 1\cdot 33 - 60 + e_{24} \qquad e_{24} = 9 - 100 - 1\cdot 1 + 1\cdot 33 + 60 = 1$$

Centered around 0