$$Y = X\beta + Z\gamma + \varepsilon$$

$$\gamma \sim N(0, G)$$

$$\varepsilon \sim N(0, R)$$

$$Y | \gamma \sim N(X\beta + Z\gamma, R)$$

$$E(Y | \gamma) = E(X\beta + Z\gamma + \varepsilon | \gamma)$$

given $= X\beta + Z\gamma + E(\varepsilon | \gamma)$

$$= X\beta + Z\gamma + 0 = \boxed{X\beta + Z\gamma}$$

$$V(y | \gamma) = V(X\beta + Z\gamma + \varepsilon | \gamma)$$

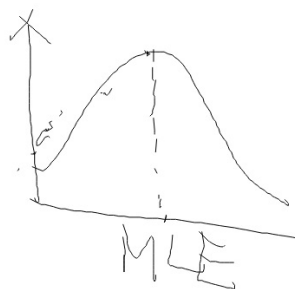given $= V(\varepsilon | \gamma) = \boxed{R}$

Recall:

$$f(y) = \frac{1}{\sqrt{2\pi}\sigma} \exp\left\{ -\frac{(y-\mu_y)^2}{2\sigma^2} \right\}$$

$$y \sim N(\mu_y, \sigma^2) \propto \exp\left\{ -\frac{(y-\mu_y)^2}{2\sigma^2} \right\}$$

$$f(y|r) = \exp\left\{ -\frac{1}{2}(y-x\beta-zr)' R^{-1}(y-x\beta-zr) \right\}$$

一個もつと ↙

likelihood

MLE   未知
       B, r

$$f(r) = \exp\left\{ -\frac{1}{2}(r-0)' G^{-1}(r-0) \right\}$$

$$= \exp\left\{ -\frac{1}{2} r' G^{-1} r \right\}$$

Recall:

$$P(A|B) = \frac{P(A,B)}{P(B)}$$

$$f(y, \gamma) = f(y|\gamma) \cdot f(\gamma)$$

probability density function

$$\propto \exp\left\{-\frac{1}{2}(y - X\beta - Z\gamma)' R^{-1}(y - X\beta - Z\gamma)\right\} \times$$

$$\exp\left\{-\frac{1}{2}(\gamma - 0)' G^{-1}(\gamma - 0)\right\}$$

$$\log f(y, r) \propto -\frac{1}{2}(y - X\beta - Zr)' R^{-1}(y - X\beta - Zr)$$

$$-\frac{1}{2} r' G^{-1} r$$

$$\boxed{\begin{array}{l} a < b \\ \log a < \log b \end{array}}$$

$$Q(\beta, r) = -(y - X\beta - Zr)' R^{-1}(y - X\beta - Zr)$$

$$- r' G^{-1} r$$

Recall: $y = \beta' x + \varepsilon$, $\varepsilon \sim N(0, \sigma^2)$

$$\hat{\beta} = (X'X)^{-1} X' y, \quad Var(\hat{\beta}) = (X'X)^{-1} \underset{\underset{MSE}{\uparrow}}{\sigma^2}$$

Weighted linear reg   (Recall)

weighted least squares

$$\hat{\beta} = (X'V^{-1}X)^{-1} X'V^{-1}Y$$

Inverse variance weighting (IVW)

$$Var(\hat{\beta}) = (X'V^{-1}X)^{-1}$$
$$(X'V^{-1}X)^{-1}$$