$$f(y) = \exp\left\{\frac{y\theta - b(\theta)}{a(\phi)} + c(y, \phi)\right\}$$

$$\mu = E(Y) = b'(\theta) \qquad Var(Y) = b''(\theta)\, a(\phi)$$

$$\frac{\partial \mu}{\partial \theta} = \frac{\partial b'(\theta)}{\partial \theta} = b''(\theta) \qquad = \underline{b''(\theta)}\ \phi$$

canonical parameter

$$= V(\mu) \cdot \phi$$

variance function

$Y \sim Normal(\mu, \sigma^2)$ $\qquad \sigma^2 = 1 \times \sigma^2$

$Y \sim Bernoulli(\pi)$ $\qquad \pi(1-\pi) = \pi(1-\pi) \times 1$

$Y \sim Poisson(\lambda)$ $\qquad \lambda = \lambda \times 1$

scale parameter

quasi-likelihood

$$\frac{\partial l}{\partial \mu} = V^{-1}(\mu) \cdot (Y-\mu)$$

$$\frac{\partial l^{\leftarrow \text{logLik}}}{\partial \mu} = \frac{\partial l}{\partial \theta} \times \frac{\partial \theta}{\partial \mu}$$

$$= \frac{\partial l}{\partial \theta} \times \left(\frac{\partial \mu}{\partial \theta}\right)^{-1}$$

$$= \frac{y-b'(\theta)}{a(\phi)} \times \left[b''(\theta)\right]^{-1}$$

$$= \frac{y-\mu}{a(\phi)} \times V^{-1}(\mu)$$

quasi-likelihood

GEE不管ϕ $\leftarrow$

$$f(y) = \exp\left\{\frac{y\theta - b(\theta)}{a(\phi)} + c(y, \phi)\right\}$$

$$\text{logLik} = l = \frac{y\theta - b(\theta)}{a(\phi)} + c(y, \phi)$$

$$\frac{\partial l}{\partial \theta} = \frac{y - b'(\theta)}{a(\phi)}$$

(1) $V(\mu)=1$

$$\frac{\partial l}{\partial \mu} = V^{-1}(\mu)\,(y-\mu)$$

$$l = \int \frac{\partial l}{\partial \mu}\, d\mu = \int \underbrace{V^{-1}(\mu)}_{=1}\,(y-\mu)\, d\mu$$

log quasiLik

$$= \int (y-\mu)\, d\mu$$ 不定積分

$$= -\frac{(y-\mu)^2}{2} + C$$ → constant

quasiLik

$$e^{l} = e^{-\frac{(y-\mu)^2}{2} + C}$$

Normal Dist. → $V(\mu)=1$ 的分布

(2) $V(\mu) = \mu$

$$l = \int \frac{\partial l}{\partial \mu} d\mu = \int \underbrace{V^{-1}(\mu)}_{\mu} (y - \mu) \, d\mu$$

↓
log quasiLik

$$= y \int \frac{1}{\mu} d\mu - \int 1 \, d\mu$$

$$= y \cdot \log\mu - \mu + C$$ → 與μ無關的常數

$$\text{quasiLik} = e^{l} = e^{y \log\mu - \mu + C}$$

$$= \mu^{y} e^{-\mu} \times e^{C}$$

$$\frac{\mu^{y} e^{-\mu}}{y!}$$

Poisson

$V(\mu) = \mu$

(3) $V(\mu) = \mu(1-\mu)$

$$l = \int \frac{\partial l}{\partial \mu} d\mu = \int \underbrace{V^{-1}(\mu)}_{\mu(1-\mu)} \cdot (y-\mu)\, d\mu$$

log quasi-Lik

$$= y \int \frac{1}{\mu(1-\mu)} d\mu - \int \frac{1}{1-\mu} d\mu$$

Bernoulli

$V(\mu) = \mu(1-\mu)$

$$= y \int \left[\frac{1}{\mu} + \frac{1}{1-\mu}\right] d\mu + \log(1-\mu) + C$$

$$= y \log\mu - y \cdot \log(1-\mu) + \log(1-\mu) + C$$

$$= y \cdot \log\mu + (1-y) \cdot \log(1-\mu) + C$$

Bernoulli

quasi-Lik $= e^{l} = \mu^{y} \cdot (1-\mu)^{1-y} \times e^{C}$

Likelihood $E(Y)$ 一階動量(moment)

$E(Y^2)$ 二

$E(Y^3)$ 三 〃

⋮

$E(Y^m)$ m 〃

GEE在分布上的假設比較寬鬆

$\underline{V(\mu)} = 1,\ \mu,\ \mu(1-\mu)$

Variance function 變異數

$\mu \downarrow$ 平均數

只定義了一，二階動量的關係

GEE(generalized estimating equation)

$$\frac{\partial l}{\partial \beta} = \frac{\partial l}{\partial \mu} \times \underbrace{\frac{\partial \mu}{\partial \beta}}_{p\times 1} \overset{set}{=} 0 \qquad (N\times 1)$$

$$= \underset{p\times N}{D'}\ \underset{N\times N}{V^{-1}(\mu)}\ \underset{N\times 1}{(Y-\mu)} \qquad p\times 1$$

$$\underset{N\times 1}{\mu} = E(Y) \qquad N: \text{no. of obs.}$$

$$D_{N\times p}$$

$$\begin{pmatrix} \beta_0 \\ \beta_1 \\ \vdots \\ \beta_{p-1} \end{pmatrix}_{p\times 1}$$

GEE(求解回歸係數)

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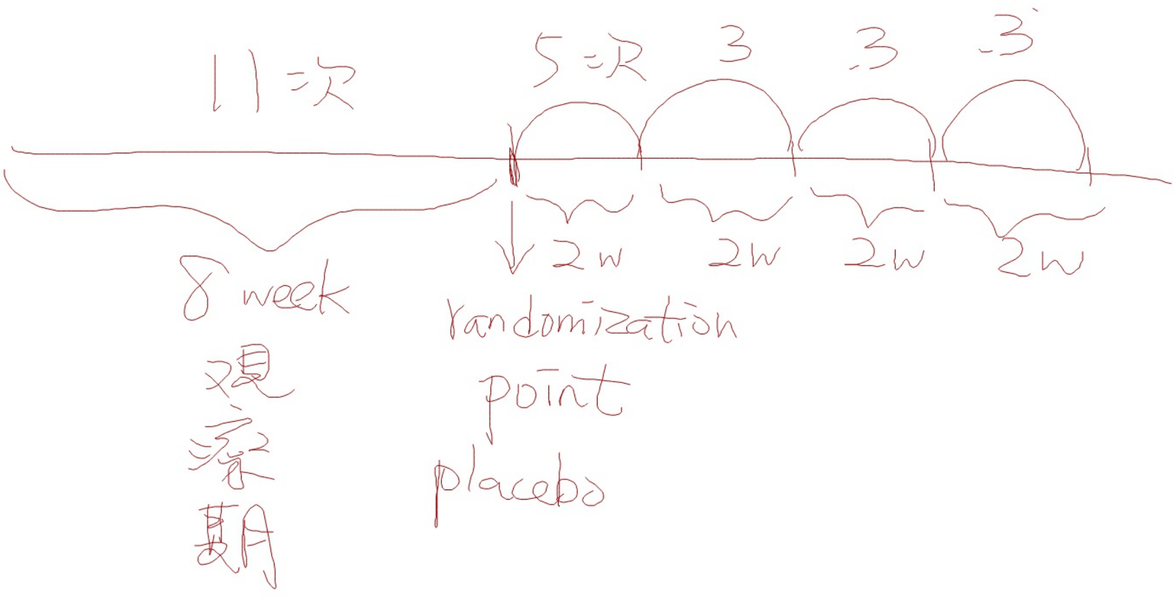

$$\mathrm{covb} = \mathrm{cov}(\hat{\beta})$$

$$= \begin{bmatrix} \mathrm{Var}(\hat{\beta}_0) & \mathrm{cov}(\hat{\beta}_0,\hat{\beta}_1) & \mathrm{cov}(\hat{\beta}_0,\hat{\beta}_2) \\ & \mathrm{Var}(\hat{\beta}_1) & \mathrm{cov}(\hat{\beta}_1,\hat{\beta}_2) \\ & & \mathrm{Var}(\hat{\beta}_2) \end{bmatrix}_{3\times 3}$$

對稱方陣