$$Y = X\beta + Z\gamma + \varepsilon$$

$$E(Y) = X\beta + Z\gamma$$

↓ Y是 binary outcome
count data

$$g[E(Y)] = X\beta + Z\gamma \quad (GLMM)$$

g: link function
$X\beta$: fixed effect
$Z\gamma$: random effect
GLMM → mixed

$$g[E(Y)] = X\beta \quad (GLM)$$

LMM

$E(Y) = X\beta + Z\gamma$

$Y = X\beta + Z\gamma + \varepsilon$

$P = X\beta + Z\gamma + \varepsilon$

↓ pseudo response

$\gamma \sim N(0, G)$

$\varepsilon \sim N(0, R)$

$\text{var}(Y) = \text{var}(X\beta + Z\gamma + \varepsilon)$

$= Z G Z' + R$

G-side random intercept model

||

R-side compound symmetry

GLMM

$$g(E(Y)) = X\beta + Z\gamma = \eta \quad \text{linear predictor}$$

$$\mu = E(Y) = g^{-1}(X\beta + Z\gamma) = g^{-1}(\eta)$$

Recall 泰勒展式

$f(x)$ 在 $x=a$ 那点展開

$$f(x) = f(a) + f'(x)\Big|_{x=a} \cdot (x-a) + \cdots$$

$$\mu = g^{-1}(\eta) = g^{-1}(\tilde{\eta}) + \left(g^{-1}(\eta)\right)'_{\eta=\tilde{\eta}} X \cdot (\beta - \tilde{\beta}) + \left(g^{-1}(\eta)\right)'_{\eta=\tilde{\eta}} Z(\gamma - \tilde{\gamma}) + \cdots$$

$$\tilde{\eta} = \begin{bmatrix} \tilde{\beta} \\ \tilde{\gamma} \end{bmatrix}$$

$$\mu - \bar{g}^{-1}(\tilde{\eta}) \doteq \left(g^{-1}(\eta)'\right)_{\eta=\tilde{\eta}} \left[X\beta - X\tilde{\beta} + Z\gamma - Z\tilde{\gamma} \right]$$

$$\left[\left(\bar{g}^{-1}(\eta)'\right)_{\eta=\tilde{\eta}}\right]^{-1} \left(\mu - \bar{g}^{-1}(\tilde{\eta})\right) \doteq X\beta - X\tilde{\beta} + Z\gamma - Z\tilde{\gamma}$$

$$\left[\left(\bar{g}^{-1}(\eta)'\right)_{\eta=\tilde{\eta}}\right]^{-1} \left(\mu - \bar{g}^{-1}(\tilde{\eta})\right) + X\tilde{\beta} + Z\tilde{\gamma} \doteq X\beta + Z\gamma$$

$$\left[\left(\bar{g}^{-1}(\eta)'\right)_{\eta=\tilde{\eta}}\right]^{-1} \left(Y - \bar{g}^{-1}(\tilde{\eta})\right) + X\tilde{\beta} + Z\tilde{\gamma} \doteq X\beta + Z\gamma$$

$\tilde{\beta}$ $\tilde{\gamma}$

0 0

1.0 0.5

Pseudo-response

假反应

$$P \doteq X\beta + Z\gamma$$

$$P = X\beta + Z\gamma + \varepsilon$$

1.7 0.8

比較 marginal model	GEE	R-side GLMM
SAS 語法	Proc genmod repeated	Proc glimmix random residual
var($\hat{\beta}$)	預設即是 empirical (三明治)	在第一行加 empirical
likelihood	quasi likelihood $V(\mu)=1, \mu, \mu(1-\mu)$ 只定義前2階動差	pseudo likelihood 把 Y 轉成 P 再求 β, γ
估計 correlation matrix	動差法 (method of moments)	最大概似法 (maximum likelihood)

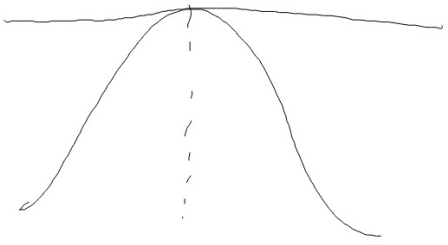
確法

$R=$ 

4×4

	1st residual	2nd	3rd	4th
1 subject	0.5	−0.2	0.7	−0.8
2 subject	−0.1	1.2	−1.0	0.7
3 subject				
⋮				
50 subject				

最大概似法



MLE

偏微分令它為0

牛頓法

迭代

無法收斂

unstructure 底下有 6 individual 相關係數要估

did not converge

	G-side	R-side
研究目的	Subject-specific effect γ_i $(i=1, \ldots, n)$ 体質など	population-averaged effect β_1 age + β_2 sex
	proc glimmix β, γ	V GEE Proc genmod + repeated Proc glimmix + random residual