UNITS

A unit is a standard chosen to measure a physical quantity

PHYSICAL QUANTITY

The quantities which can be measured by an instrument and by means of which we can describe the laws of physics are called physical quantities.

TYPES OF PHYSICAL QUANTITIES

FUNDAMENTAL

Certain physical quantities have been chosen arbitrarily and their units are used for expressing all the physical quantities, such quantities are known as Fundamental, Absolute or Base Quantities.

DERIVED

Physical quantities which can be expressed as a combination of base quantities are called derived quantities.

e.g: Velocity $\left[\frac{m}{s}\right] = \frac{\text{Length [m]}}{\text{Time [s]}}$

SUPPLEMENTRY

Besides the seven fundamental physical quantities, two supplementary quantities are also defined, they are:

- Plane angle
- Solid angle

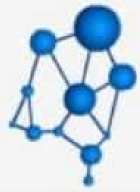
NOTE : The supplementary quantities have only units but no dimensions.

MAGNITUDE

Magnitude of physical quantity = (numerical value) x (unit)

Magnitude of a physical quantity is always constant. It is independent of the type of unit.

$$n_1 u_1 = n_2 u_2 = \text{constant}$$

FUNDAMENTAL UNITS							
QUANTITY	Length	Mass	Luminous intensity	Amount of substance	Time	Electric current	Temperature
UNITS	Metre	Kilogram	Candela	Mole	Second	Ampere	Kelvin

DIMENSIONS

Dimensions of a physical quantity are the power to which the fundamental quantities must be raised to represent the given physical quantity.

1 USE OF DIMENSIONS

CONVERSION OF UNITS

$$n_1 [u_1] = n_2 [u_2]$$

Suppose the dimensions of a physical quantity are 'a' in mass, 'b' in length and 'c' in time. If the fundamental units in one system are M_1, L_1 and T_1 and in the other system are M_2, L_2 and T_2 respectively. Then we can write.

$$n_1 [M_1^a L_1^b T_1^c] = n_2 [M_2^a L_2^b T_2^c]$$

ANALYZING DIMENSIONAL CORRECTNESS OF A PHYSICAL EQUATION

Every physical equation should be dimensionally balanced. This is called the **'Principle of Homogeneity'.** The dimensions of each term on both sides of an equation must be the same.

Note: A dimensionally correct equation may or may not be physically correct.

PRINCIPLE OF HOMOGENEITY OF DIMENSIONS

This principle states that the dimensions of all the terms in a physical expression should be same.

For e.g, in the physical expression $s = ut + \frac{1}{2}at^2$, the dimensions of **s, ut** and $\frac{1}{2}at^2$ all are same.

Note: Physical quantities separated by the symbols +, –, =, >, < etc., have the same dimensions.

2 LIMITATIONS OF DIMENSIONAL ANALYSIS

- By this method, the value of dimensionless constant can not be calculated.
- By this method, the equation containing trigonometrical, exponential and logarithmic terms cannot be analysed.
- If a physical quantity depends on more than three factors, then relation among them cannot be established because we can have only three equations by equating the powers of **M, L** and **T.**

Part-I

ERROR

Difference between the result of the measurement and the true value of what you were measuring

Types of Error

RANDOM ERROR

Random errors appear randomly because of the operator, fluctuations in the external conditions and variability of the measuring instruments. The effect of random error can be some what reduced by taking the average of measured values. Random errors have no fixed sign or size.

Thus they are represented in the form **A ± a**

SYSTEMATIC ERROR

Systematic error occurs due to an error in the procedure or miscalibration of the instrument etc. Such errors have same size and sign for all measurements. Such errors can be determined.

The systematic error is removed before beginning calculations. Bench error and zero error are examples of systematic error.

ABSOLUTE ERROR

Error may be expressed as absolute measures, giving the size of the error in a quantity in the same units as the quantity itself.

Least Count Error :- If the instrument has known least count, the absolute error is taken to be **half** of the least count unless otherwise stated.

RELATIVE (OR FRACTIONAL) ERROR

Error may be expressed as relative measures, giving the ratio of the quantity's error to the quantity itself

$$\text{Relative Error} = \frac{\text{Absolute error in a measurement}}{\text{Size of the measurement}}$$

RULES OF ERROR MEASUREMENT

01 ADDITION & SUBTRACTION RULE

The absolute random errors **add**

If $R = A + B$, or $R = A - B$, then $r = a + b$

02 PRODUCT & QUOTIENT RULE

The relative random errors **add**

If $R = AB$, or $R = \frac{A}{B}$, then $\frac{r}{R} = \frac{a}{A} + \frac{b}{B}$

03 POWER RULE

When a quantity Q is raised to a power P, the relative error in the result is P times the relative error in Q. This also holds for negative powers.

IF $R = Q^P$, then $\frac{r}{R} = P \times \frac{q}{Q}$

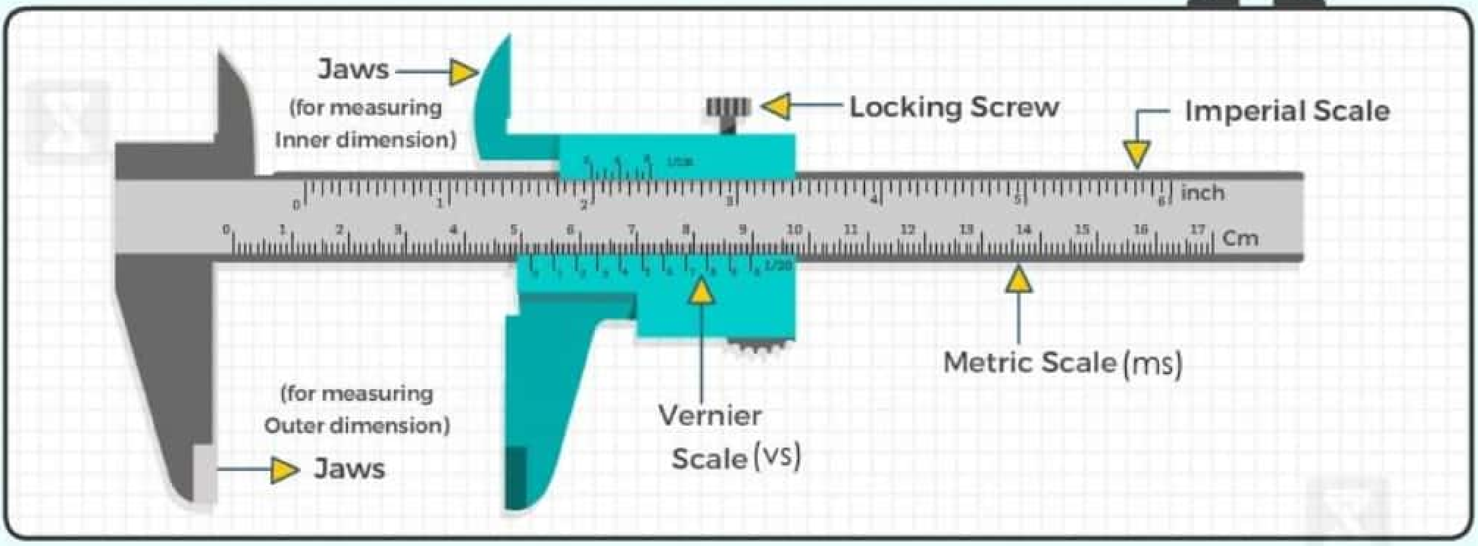
VERNIER CALLIPERS

Least count of Vernier Callipers

The least count of Vernier Callipers **(v.c)** is the minimum value of correct estimation of length without eye estimation. If N^{th} division of vernier calliper coincides with **(N–1)** division of main scale, then

$$N(vs) = (N - 1)\ ms \Rightarrow 1\ vs = \frac{N-1}{N} ms$$

vs = Vernier Scale Reading : ms = Main Scale Reading

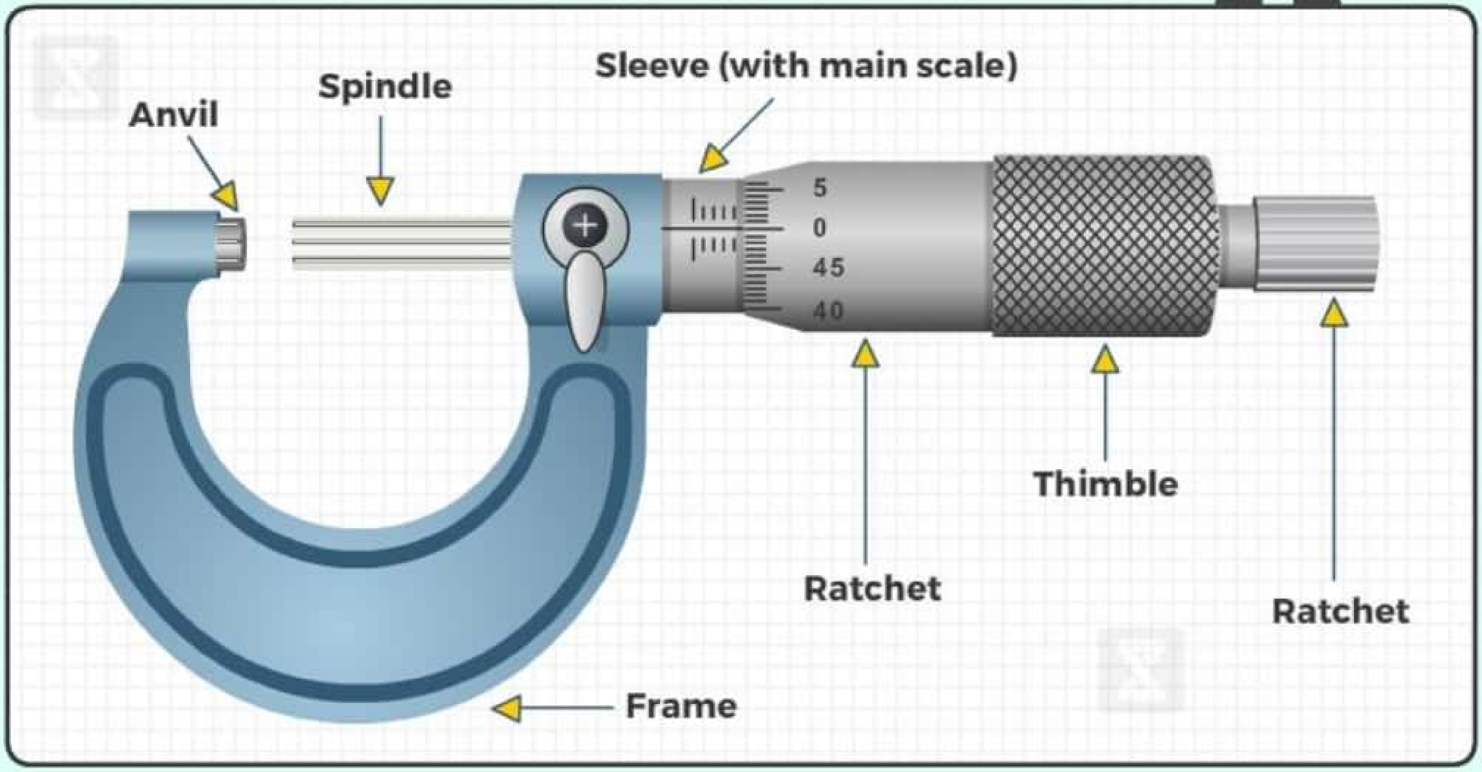


Vernier Constant $= 1\ ms - 1\ vs = \left(1 - \frac{N-1}{N}\right) ms = \frac{1}{N} ms$, which is equal to the value of the smallest division on the main scale divided by total number of divisions on the vernier scale.

SCREW GAUGE (OR MICROMETER SCREW)

The instrument is provided with two scales

- The main scale or pitch scale is (M) graduated along the axis of screw.
- The cap-scale or head scale (H) around the edge of the screw head.



Pitch :- The pitch of the instrument is distance between two consecutive threads of the screw which is equal to the distance moved by the screw due to one complete rotation of the cap. Thus for,

10 rotation of cap = 5 mm, then pitch = 0.5 mm.

Least count :- The minimum (or least) measurement (or count) of length is equal to one division on the head scale which is equal to pitch divided by the total cap divisions.

$$\text{Least count} = \frac{\text{Pitch}}{\text{Total cap divisions}}$$

Measurement of length by screw gauge

Length, L = n × pitch + f × least count,

where **n = main scale reading & f = caps scale reading**

Zero Error

In a perfect instrument the zero of the main scale coincides with the line of gradiation along the screw axis with no zero-error, otherwise the instrument is said to have zero-error which is equal to the cap reading with the gap closed. This error is positive when zero line of reference line of the cap lies **below** the line of graduation and vice-versa. The corresponding corrections will be just opposite.

REST AND MOTION

DISTANCE

- The length of the actual path traversed by the particle is termed as its distance.
- Distance = S = length of path ACB.
- Scalar quantity and is measured in meter. It can never decrease with time.

DISPLACEMENT

- The change in position vector of the particle for a given time interval is known as its displacement.
- Displacement = B – A
- It can decrease with time. Vector quantity and is measured in meter.

AVERAGE VELOCITY

$$\text{Average Velocity } (\vec{v}_{av}) = \frac{\text{Total Displacement}}{\text{Total Time Taken}} = \frac{\vec{B}-\vec{A}}{t}$$

AVERAGE SPEED

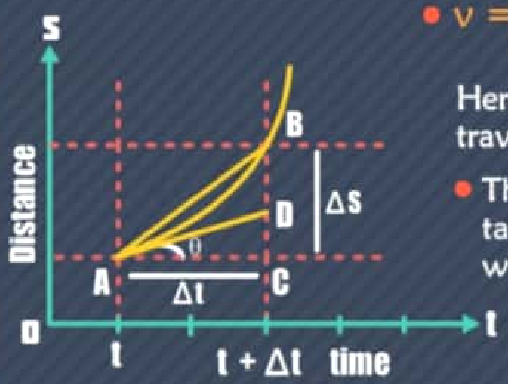
$$\text{Average Speed}(v_{av}) = \frac{\text{Total Distance Travelled}}{\text{Total Time Taken}} = \frac{S}{t}$$

INSTANTANEOUS SPEED

- The instantaneous speed is the speed at a particular instant of time.
- $v = \lim_{\Delta t \to 0} \frac{\Delta s}{\Delta t} = \frac{ds}{dt}$

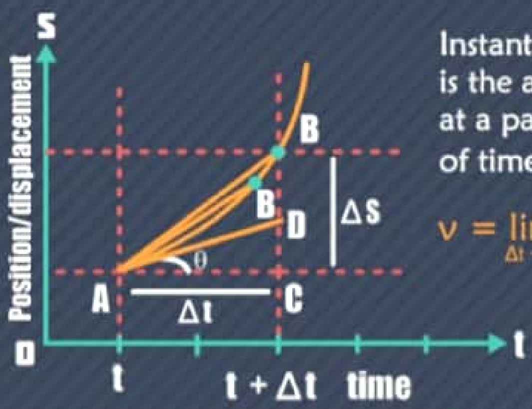
Here Δs is the distance travelled in time Δt.

- The slope of the tangent equal ds/dt, which is equal to the instantaneous speed at 't'.



$$v = \tan(\theta) = \frac{DC}{AC} = \frac{ds}{dt}$$

INSTANTANEOUS VELOCITY



Instantaneous velocity is the average velocity at a particular instant of time.

$$v = \lim_{\Delta t \to 0} \frac{\Delta r}{\Delta t} = \frac{dr}{dt}$$

EQUATIONS OF MOTION

1. $v = u + at$
2. $v^2 - u^2 = 2as$
3. $s = ut + \frac{1}{2}at^2$
4. $s_{nth} = u + \frac{a}{2}(2n - 1)$

ACCELERATION

When the velocity of a moving object/particle changes with time, we can say that it is accelerated.

Average Acceleration

$$a_{av} = \frac{\vec{v_2} - \vec{v_1}}{t_2 - t_1} = \frac{\Delta\vec{v}}{\Delta t}$$

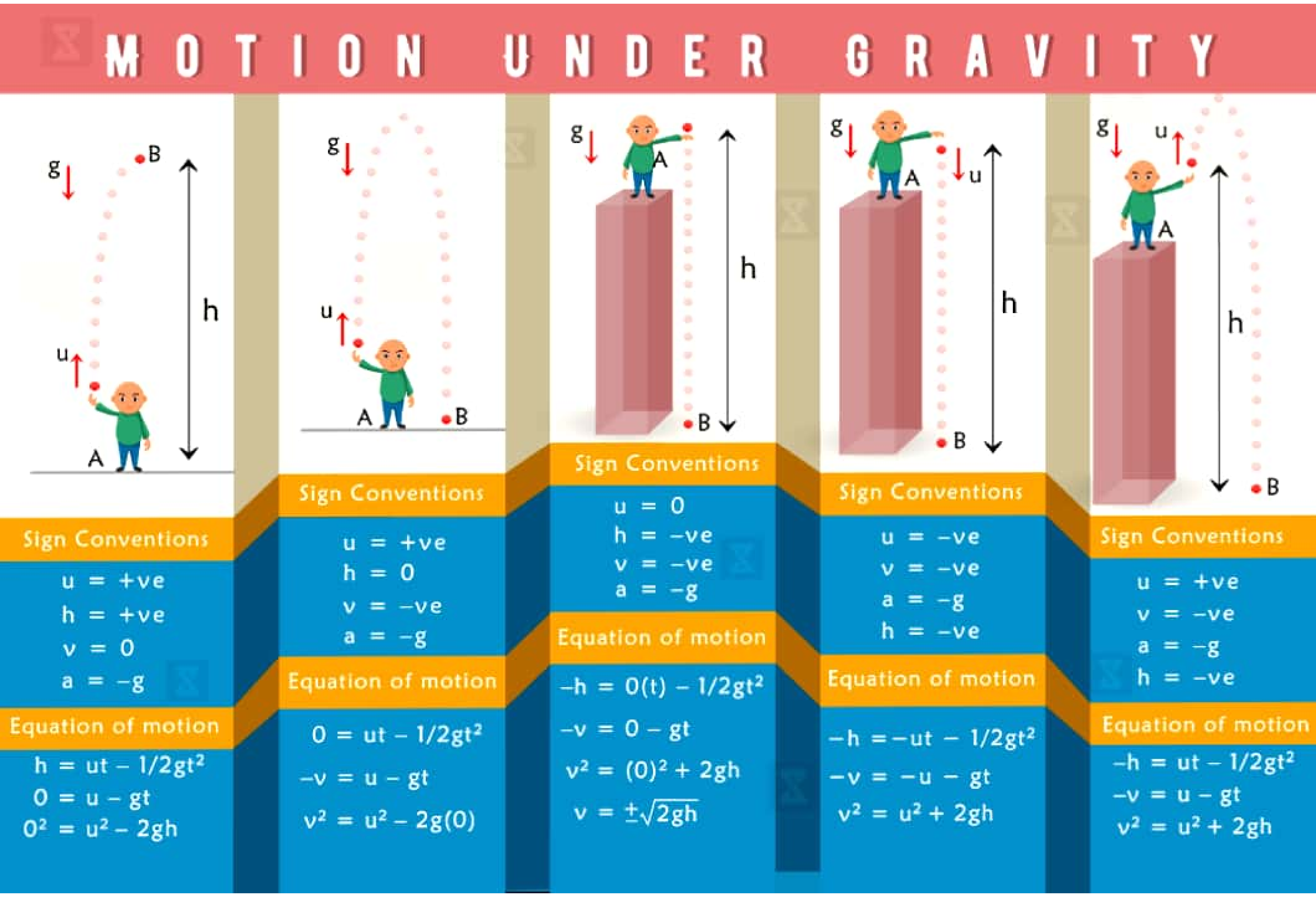
Instantaneous Acceleration

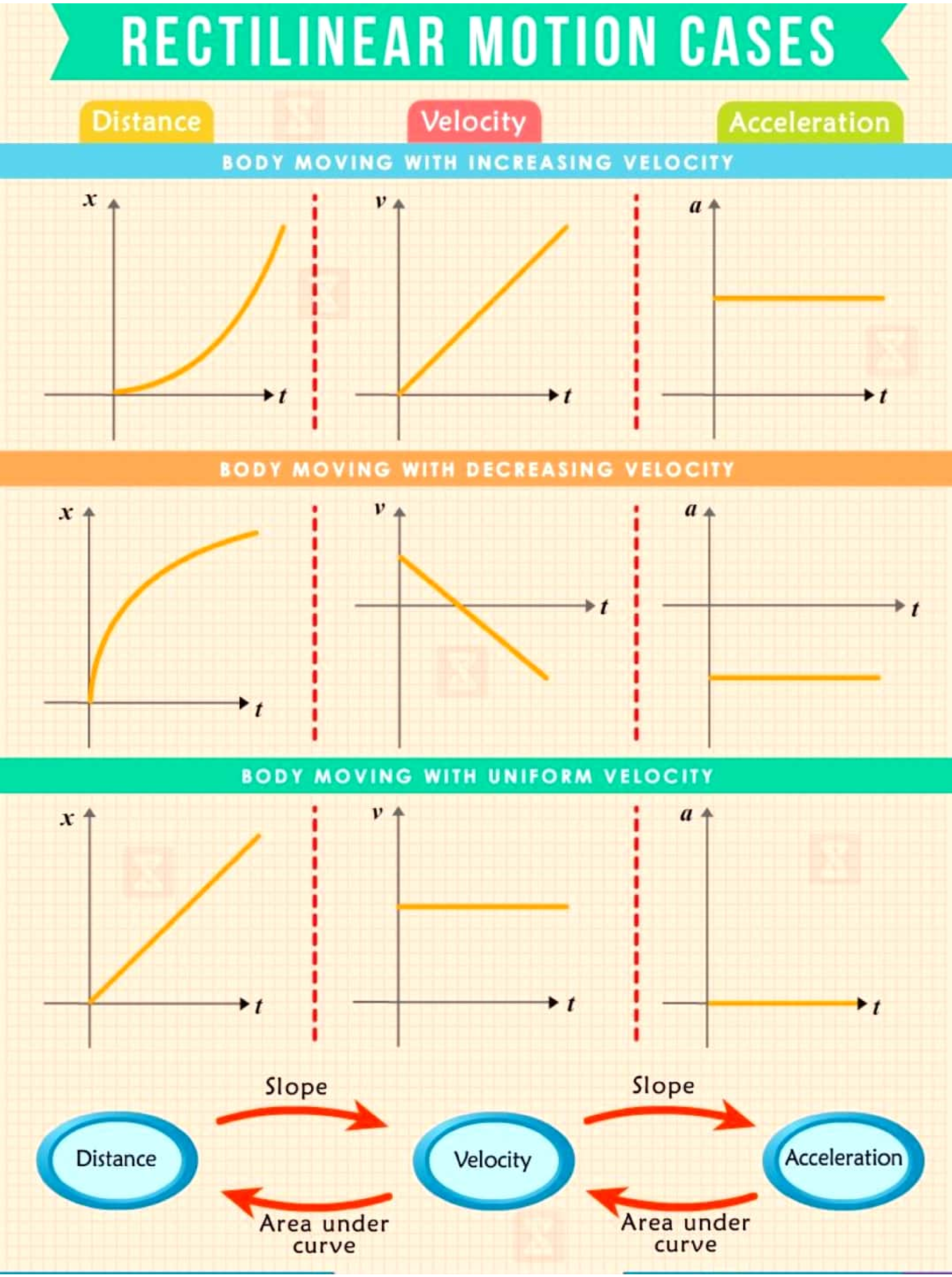
$$\vec{a} = \lim_{\Delta t \to 0} \vec{a}_{av} = \frac{d\vec{v}}{dt}$$

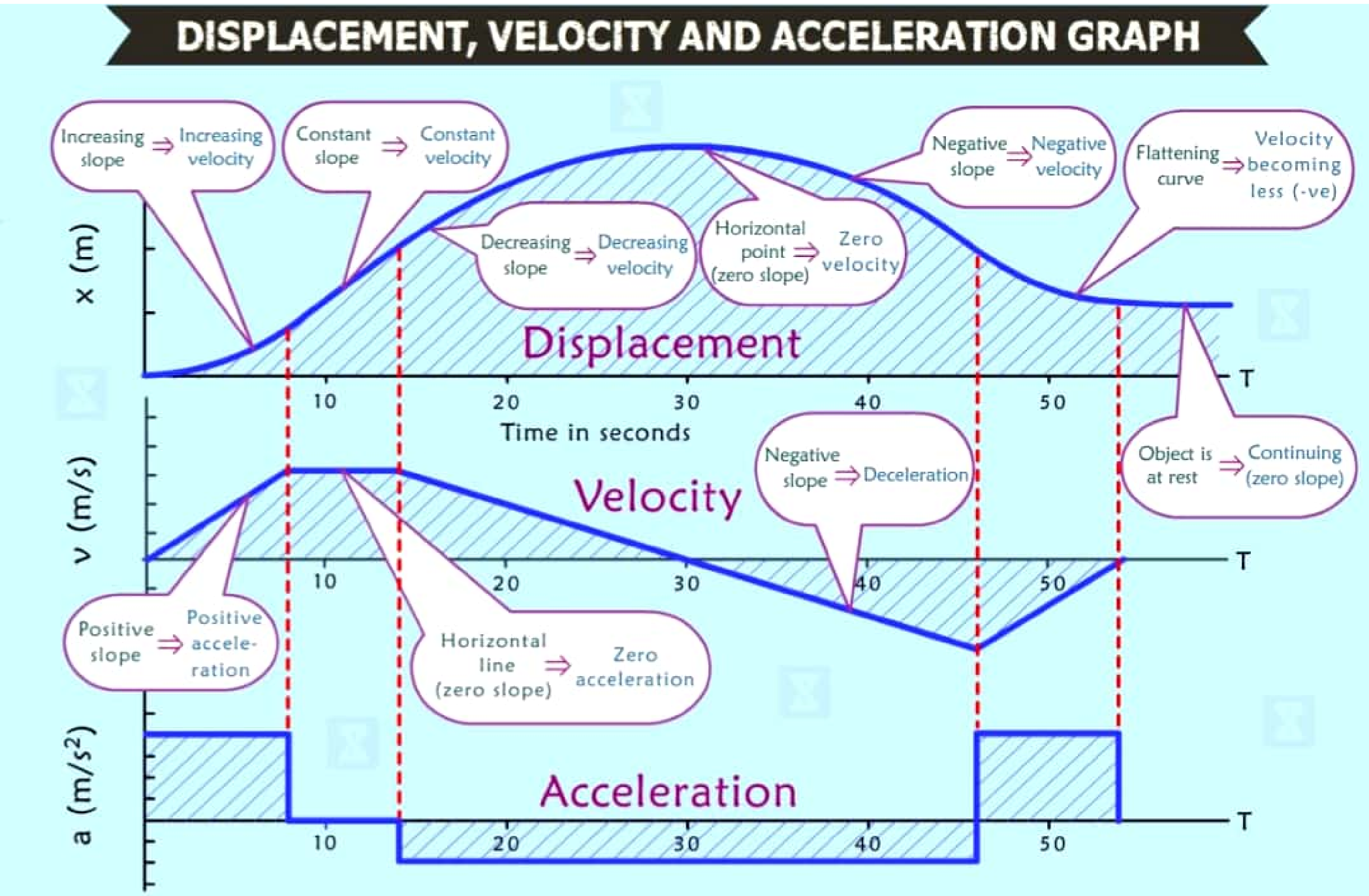
REACTION TIME

It's the difference between the time when one see a situation to the time when one acts.

Reaction Time $\Delta t = t_1 - t_0$


MOTION UNDER GRAVITY
g
B
u
A
h
Sign Conventions
u = +ve
h = +ve
v = 0
a = −g
Equation of motion
h = ut − 1/2gt²
0 = u − gt
0² = u² − 2gh
g
u
A
B
Sign Conventions
u = +ve
h = 0
v = −ve
a = −g
Equation of motion
0 = ut − 1/2gt²
−v = u − gt
v² = u² − 2g(0)
g
A
h
B
Sign Conventions
u = 0
h = −ve
v = −ve
a = −g
Equation of motion
−h = 0(t) − 1/2gt²
−v = 0 − gt
v² = (0)² + 2gh
v = ±√2gh
g
A
u
h
B
Sign Conventions
u = −ve
v = −ve
a = −g
h = −ve
Equation of motion
−h = −ut − 1/2gt²
−v = −u − gt
v² = u² + 2gh
g
u
A
h
B
Sign Conventions
u = +ve
v = −ve
a = −g
h = −ve
Equation of motion
−h = ut − 1/2gt²
−v = u − gt
v² = u² + 2gh


RECTILINEAR MOTION CASES
Distance
Velocity
Acceleration
BODY MOVING WITH INCREASING VELOCITY
x
t
v
t
a
t
BODY MOVING WITH DECREASING VELOCITY
x
t
v
t
a
t
BODY MOVING WITH UNIFORM VELOCITY
x
t
v
t
a
t
Slope
Slope
Distance
Velocity
Acceleration
Area under curve
Area under curve



DISPLACEMENT, VELOCITY AND ACCELERATION GRAPH
Increasing slope ⇒ Increasing velocity
Constant slope ⇒ Constant velocity
Decreasing slope ⇒ Decreasing velocity
Horizontal point (zero slope) ⇒ Zero velocity
Negative slope ⇒ Negative velocity
Flattening curve ⇒ Velocity becoming less (-ve)
x (m)
Displacement
T
10
20
30
40
50
Time in seconds
Object is at rest ⇒ Continuing (zero slope)
Negative slope ⇒ Deceleration
v (m/s)
Velocity
Positive slope ⇒ Positive acceleration
Horizontal line (zero slope) ⇒ Zero acceleration
a (m/s²)
Acceleration

RELATIVE VELOCITY

Relative velocity of A wrt B $\vec{V}_{AB} = \vec{V}_A - \vec{V}_B$

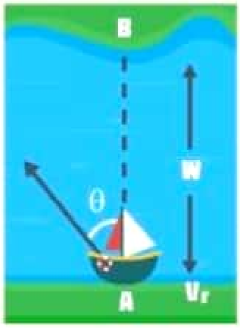
Relative acceleration of A wrt B $\vec{a}_{AB} = \vec{a}_A - \vec{a}_B$

RIVER–BOAT PROBLEM

$\vec{V}_r$ = absolute velocity of river

$\vec{V}_{br}$ = velocity of boatman with respect to river or velocity of boatman in still water

$\vec{V}_b$ = absolute velocity of boatman.



$$\vec{V}_b = \vec{V}_{br} + \vec{V}_r$$

Time taken by boatman to cross the river:

$$t = \frac{w}{V_{br}\cos\theta}$$

Displacement along x-axis when he reaches on the other bank:

$$x = (V_r - V_{br}\sin\theta)\frac{w}{V_{br}\cos\theta}$$

1. Condition when the boatman crosses the river in shortest interval of time-

$$t_{min} = \frac{w}{V_{br}}$$

2. Condition when the boatman wants to reach point B, i.e., at a point just opposite from where he started

$$\theta = \sin^{-1}\left(\frac{V_r}{V_{br}}\right)$$

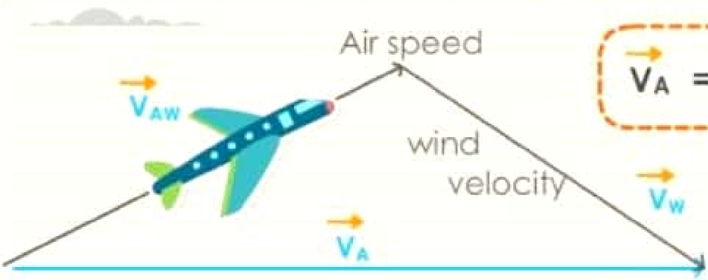
3. Shortest Path

when $V_r < V_{br} \rightarrow S_{min} = w$

when $V_r > V_{br} \rightarrow$

$$S_{min} = w\left(\frac{V_r}{V_{br}}\right)$$

AIRCRAFT WIND PROBLEM



$$\vec{V}_A = \vec{V}_{AW} + \vec{V}_W$$

$\vec{V}_{AW}$ = Velocity of aircraft wrt wind

$\vec{V}_W$ = Velocity of wind

$\vec{V}_A$ = Absolute Velocity of aircraft

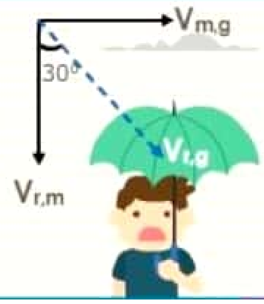
RAIN PROBLEM

$\vec{V}_{r,g}$ = Velocity of river wrt ground

$\vec{V}_{r,m}$ = Velocity of river wrt man

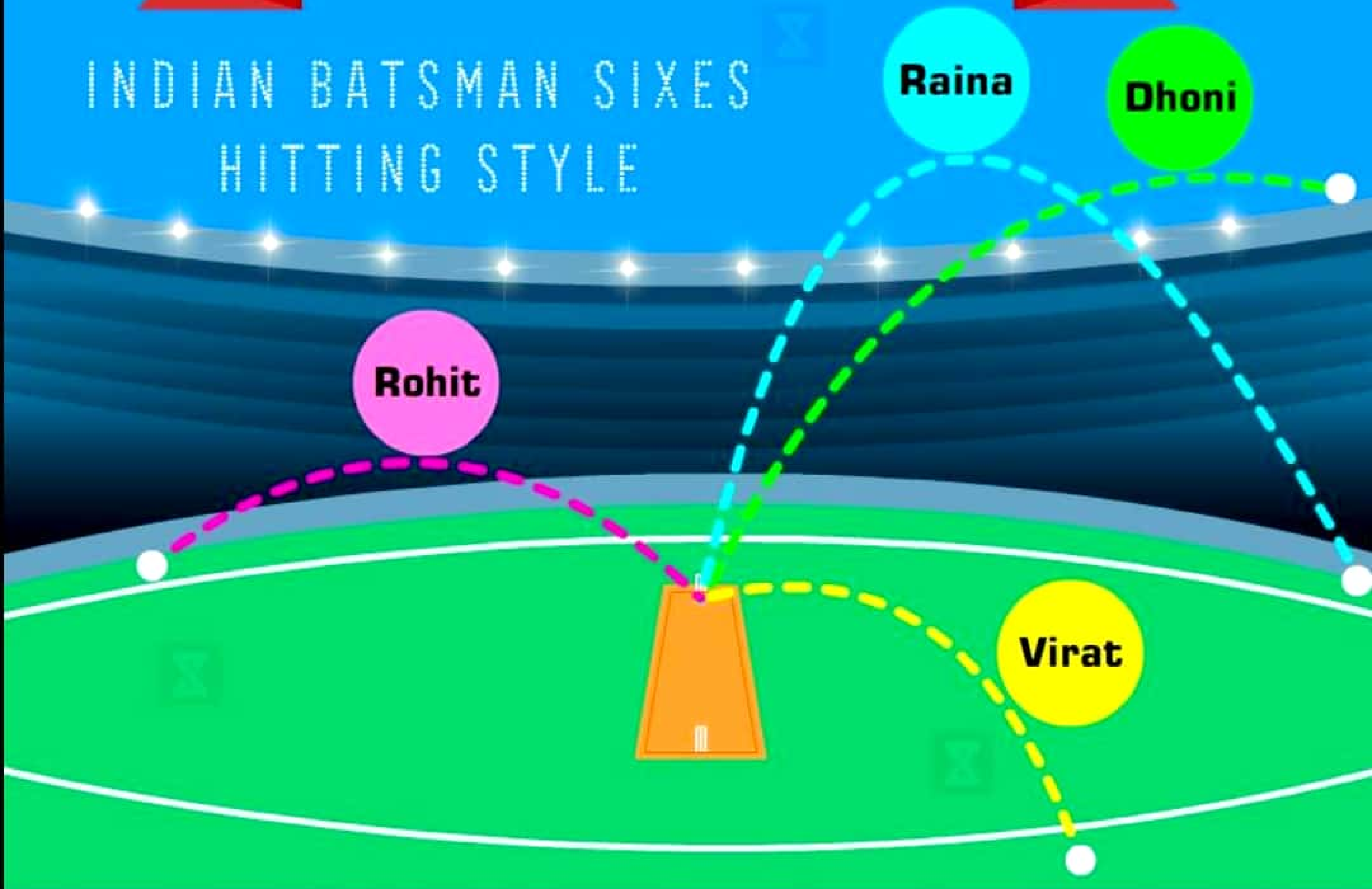
$\vec{V}_{m,g}$ = Velocity of man wrt ground

$$\vec{V}_{r,g} = \vec{V}_{r,m} + \vec{V}_{m,g}$$

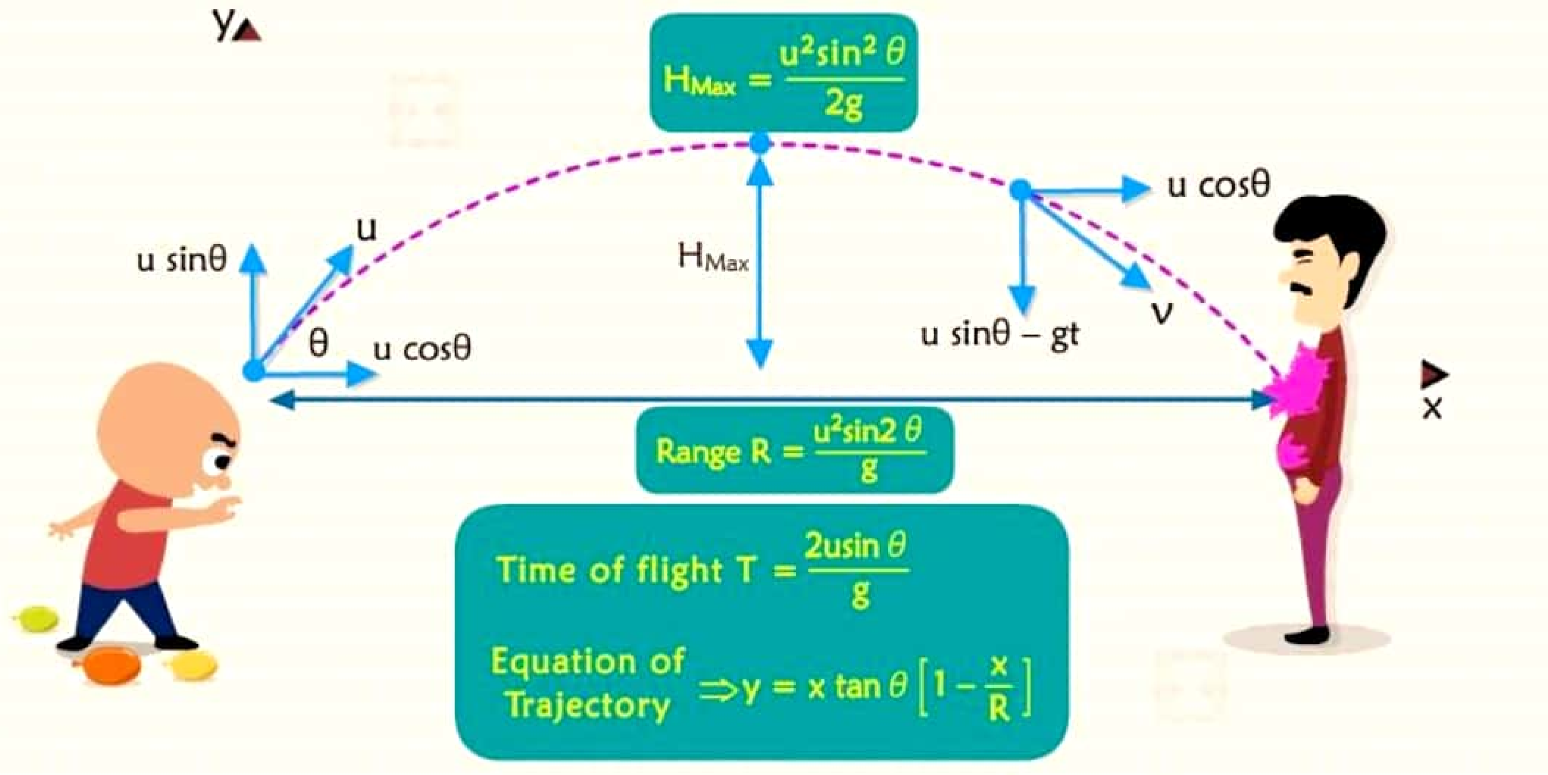


PROJECTILE MOTION

Part I

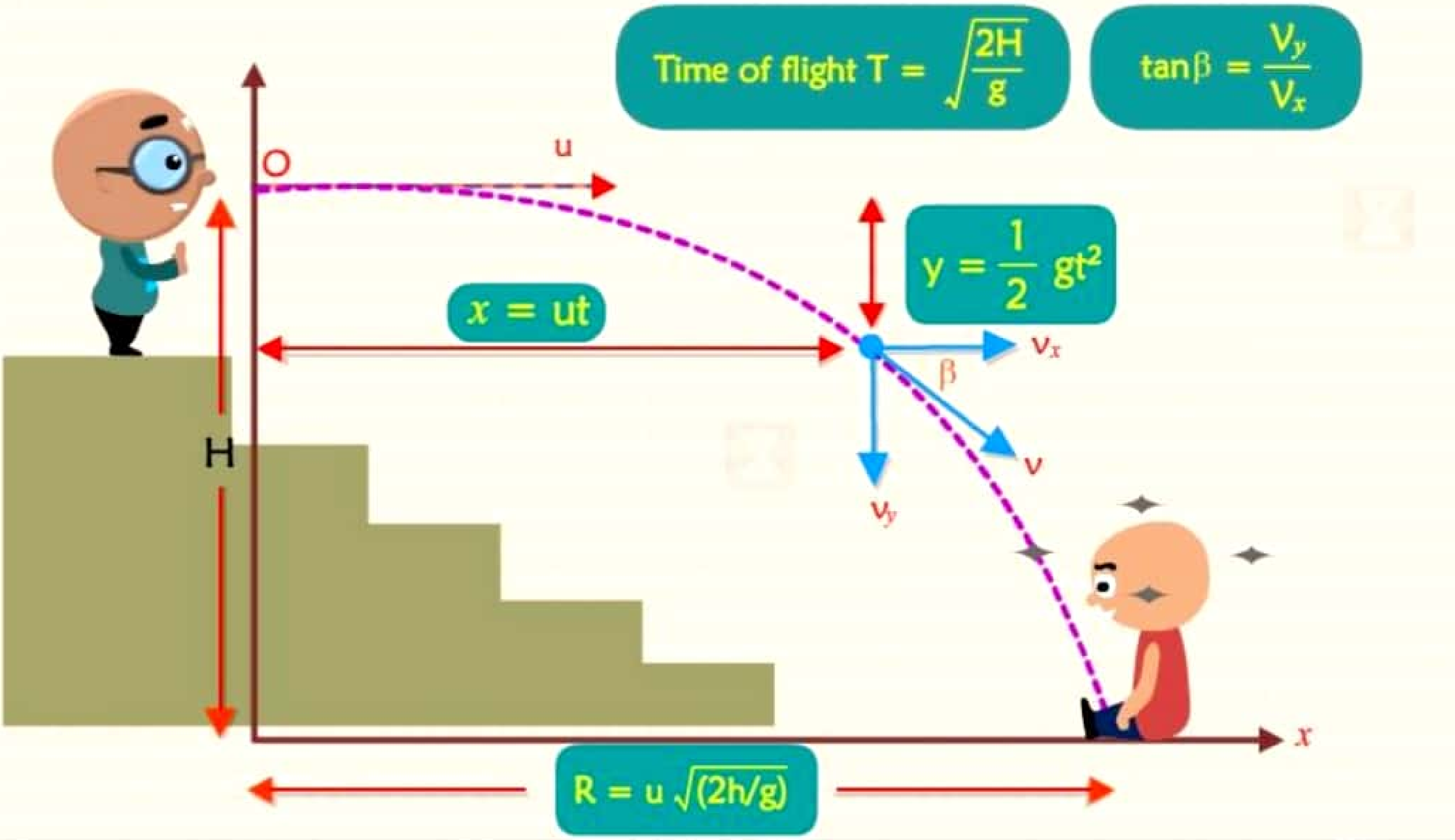


1 BASIC PROJECTILE MOTION

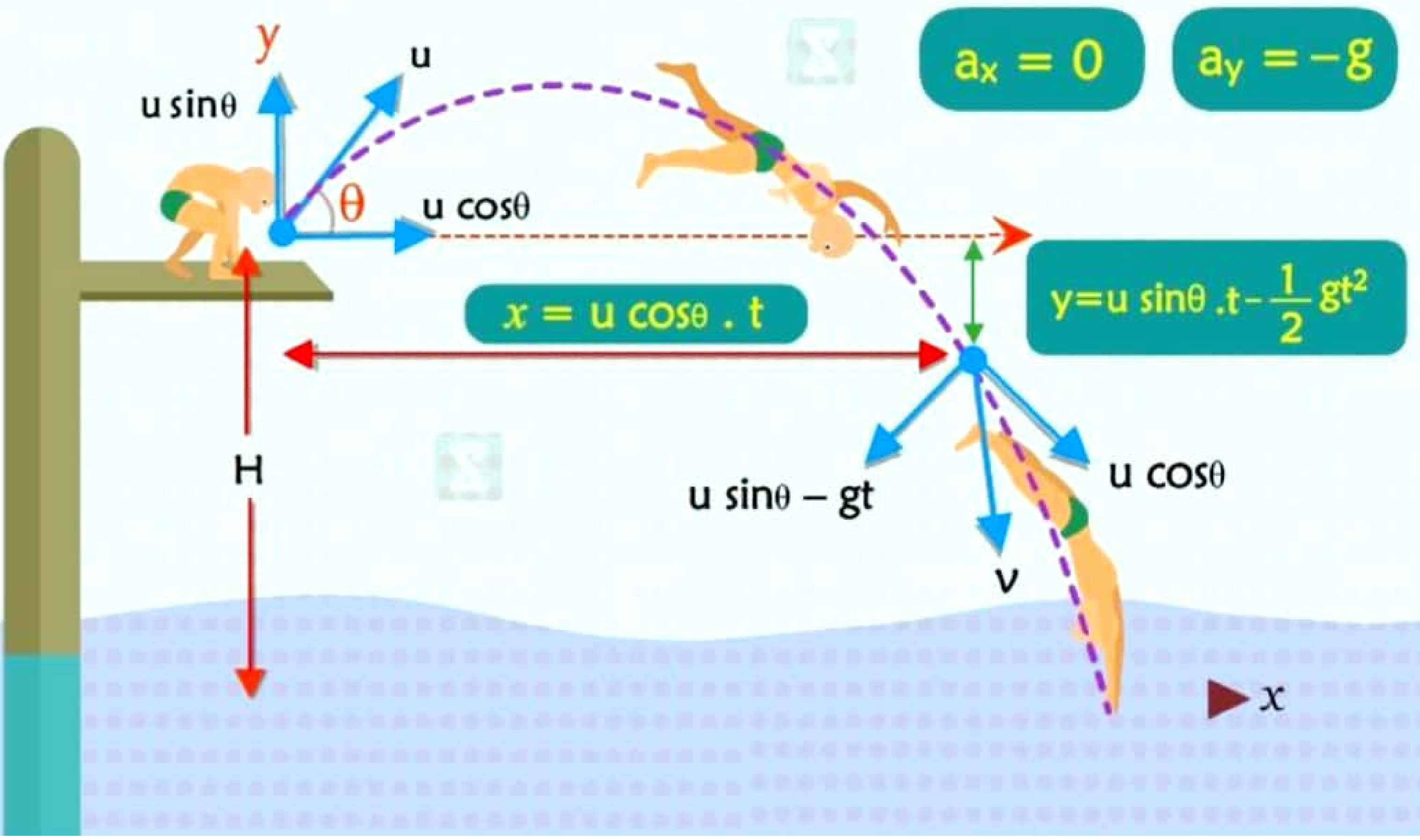


Part II

2 PROJECTILE FIRED PARALLEL TO HORIZONTAL

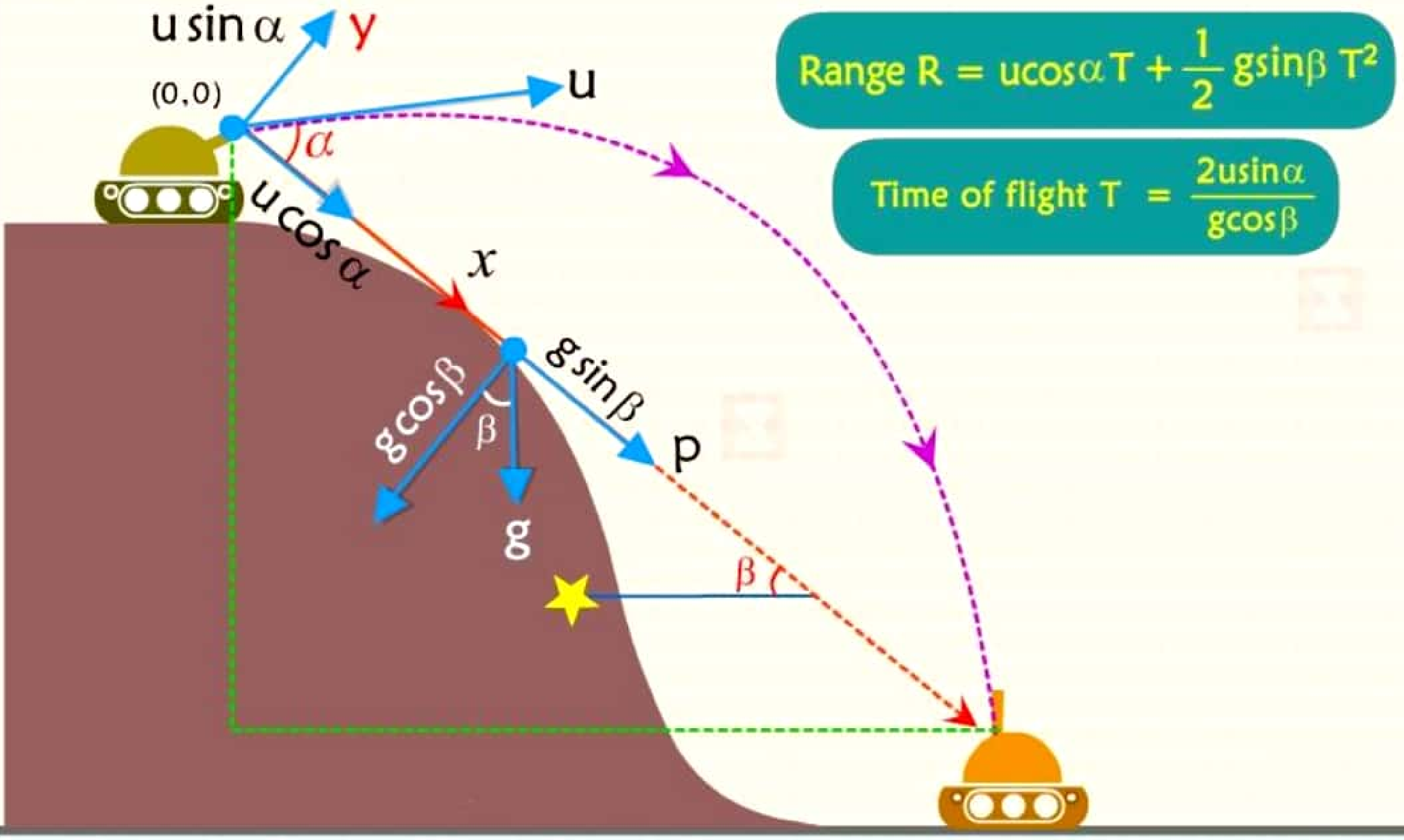


3 PROJECTILE AT AN ANGLEθ FROM HEIGHT 'H'



Part III

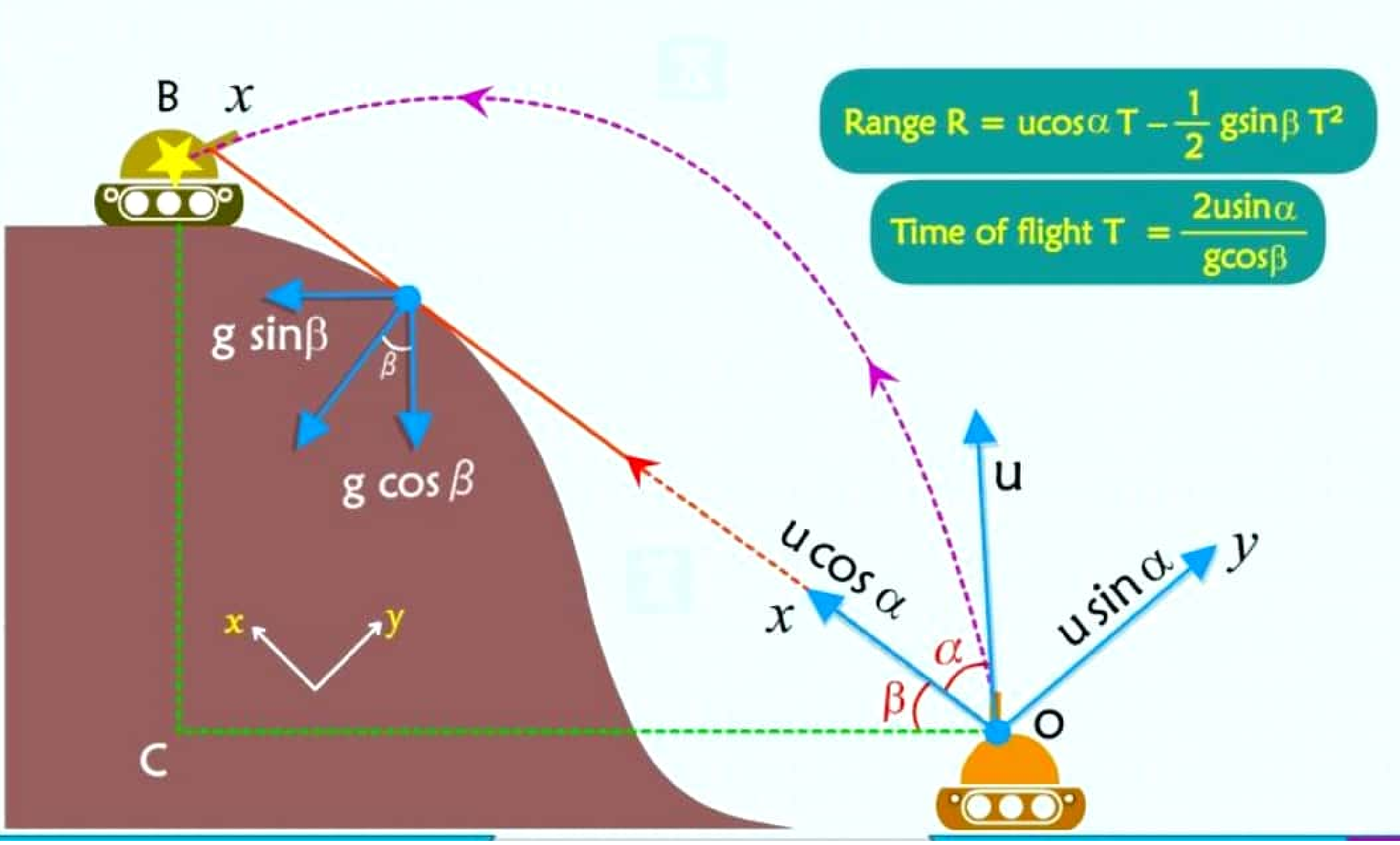
4 PROJECTILE MOTION DOWN THE INCLINED PLANE



$$\text{Range } R = u\cos\alpha\, T + \frac{1}{2} g\sin\beta\, T^2$$

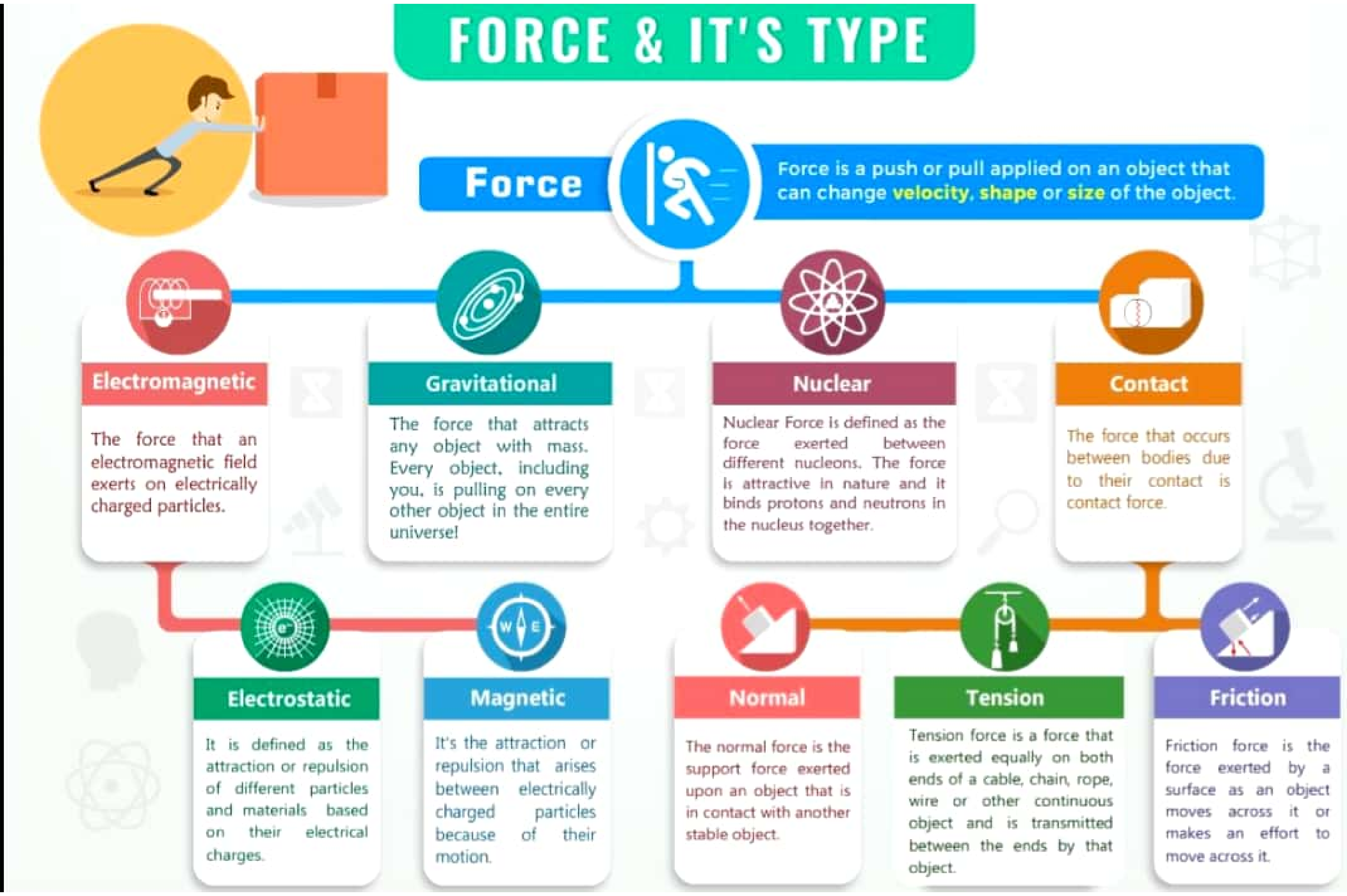
$$\text{Time of flight } T = \frac{2u\sin\alpha}{g\cos\beta}$$

5 PROJECTILE MOTION UP THE INCLINED PLANE



$$\text{Range } R = u\cos\alpha\, T - \frac{1}{2} g\sin\beta\, T^2$$

$$\text{Time of flight } T = \frac{2u\sin\alpha}{g\cos\beta}$$


FORCE & IT'S TYPE
Force
Force is a push or pull applied on an object that can change velocity, shape or size of the object.
Electromagnetic
The force that an electromagnetic field exerts on electrically charged particles.
Gravitational
The force that attracts any object with mass. Every object, including you, is pulling on every other object in the entire universe!
Nuclear
Nuclear Force is defined as the force exerted between different nucleons. The force is attractive in nature and it binds protons and neutrons in the nucleus together.
Contact
The force that occurs between bodies due to their contact is contact force.
Electrostatic
It is defined as the attraction or repulsion of different particles and materials based on their electrical charges.
Magnetic
It's the attraction or repulsion that arises between electrically charged particles because of their motion.
Normal
The normal force is the support force exerted upon an object that is in contact with another stable object.
Tension
Tension force is a force that is exerted equally on both ends of a cable, chain, rope, wire or other continuous object and is transmitted between the ends by that object.
Friction
Friction force is the force exerted by a surface as an object moves across it or makes an effort to move across it.

Part I

LAW'S OF MOTION

First Law

Every body remains in a state of rest or uniform motion unless acted upon by a **net external force**.

Second Law

The amount of acceleration of a body is proportional to the acting force and inversely proportional to the mass of the body.

$F = ma$

Third Law

For every action there is an equal but opposite reaction. If an object A exerts a force on object B, then object B will exert an equal but opposite force on object A.

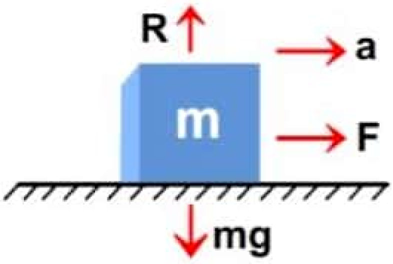
APPLICATION OF N.L.M

Part II

1 Motion of a Block on a Horizontal Smooth Surface

Case (i) **Horizontal pull**

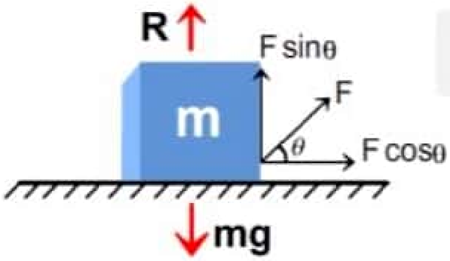
$F = ma$ or $a = \dfrac{F}{m}$



Case (ii) **Pull acting at an angle (θ)**

$R + F\sin\theta = mg$

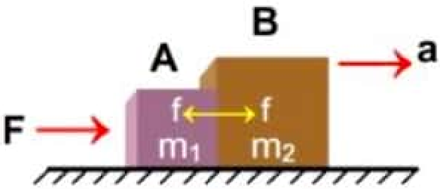
$a = \dfrac{F\cos\theta}{m}$



2 Motion of Bodies in Contact

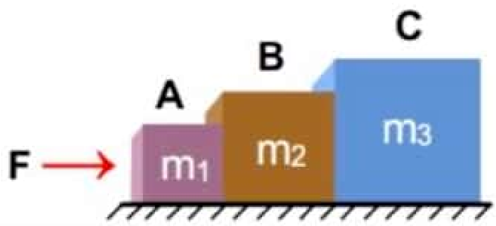
Case (i) **Two Body System**

$\Rightarrow a = \dfrac{F}{m_1 + m_2}$ & $f = \dfrac{m_2 F}{m_1 + m_2}$



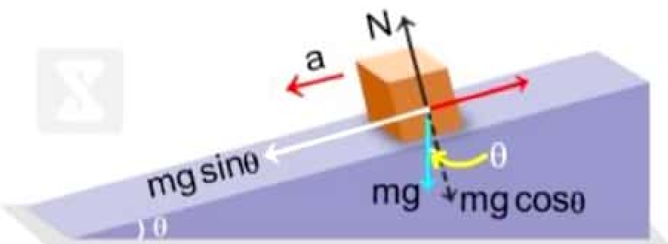
Case (ii) **Three Body System**

$\Rightarrow a = \dfrac{F}{m_1 + m_2 + m_3}$



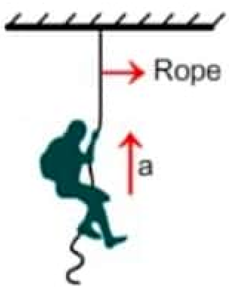
3 Motion of a Body on a Smooth Inclined Plane

$a = g\sin\theta$ $N = mg\cos\theta$



4 Climbing on the Rope

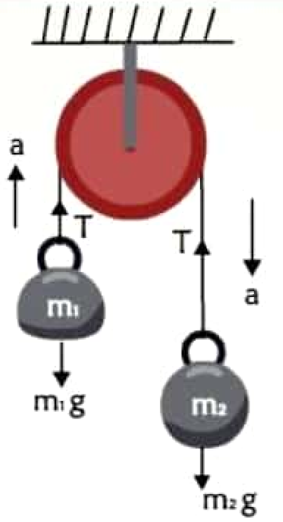
- $T > mg$, man accelerates in upward direction
- $T < mg$, man accelerates in downward direction



F.B.D of man

PULLEY BLOCK SYSTEM

1

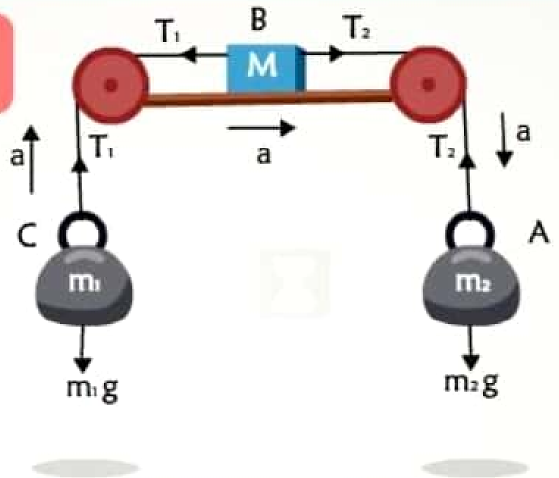


$m_2 > m_1$

$m_2 g - T = m_2 a$

$T - m_1 g = m_1 a$

2

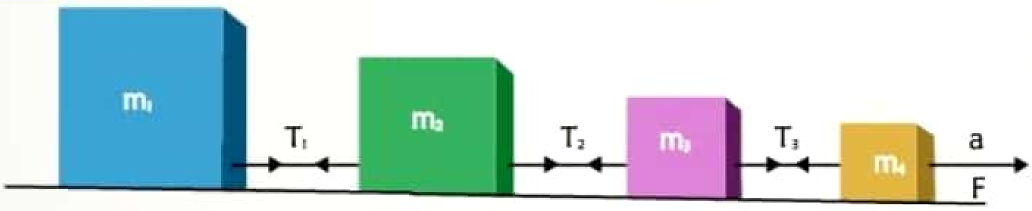


For body A, $m_2 g - T = m_2 a$

For body B, $T_2 - T_1 = Ma$

For body C, $T_1 - m_1 g = m_1 a$

3



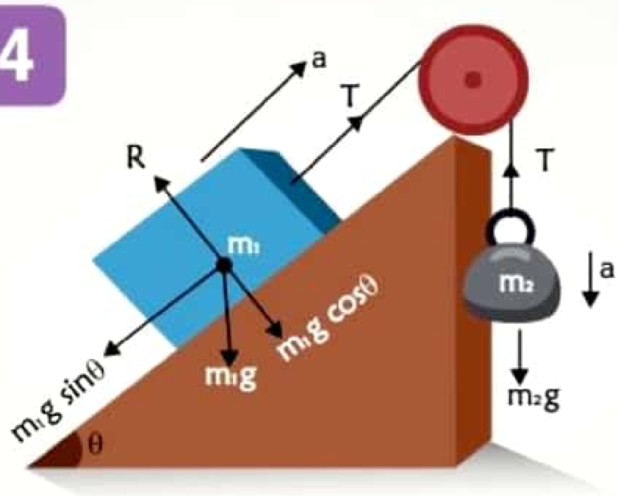
$$a = \frac{F}{(m_1 + m_2 + m_3 + m_4)}$$

$T_3 = (m_1 + m_2 + m_3)a$

$T_2 = (m_1 + m_2)a$

$T_1 = m_1 a$

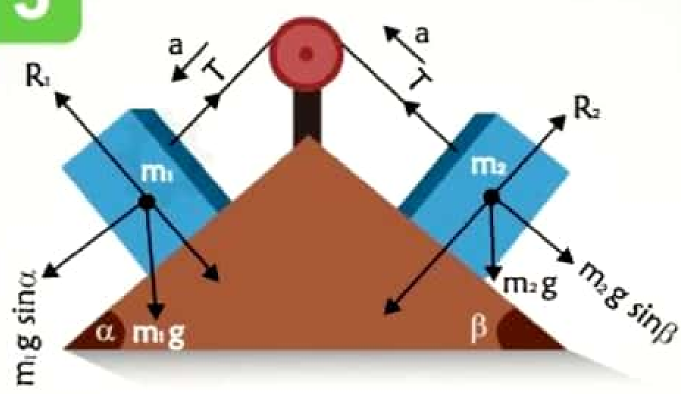
4



$m_2 g - T = m_2 a$

$T - m_1 g \sin\theta = m_1 a$

5



$m_1 g \sin\alpha - T = m_1 a$

$T - m_2 g \sin\beta = m_2 a$

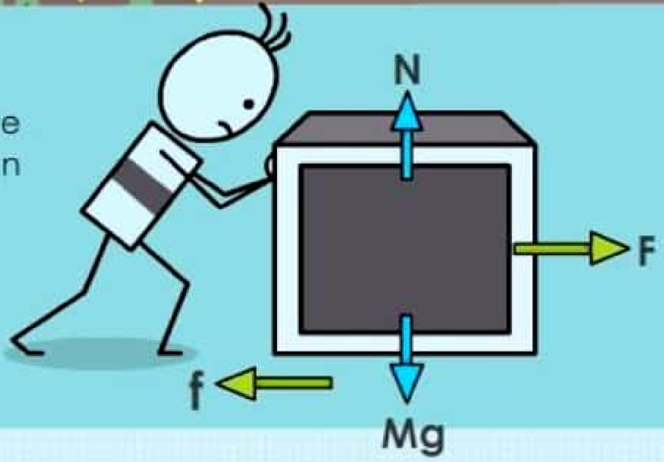
FRICTION

Part I

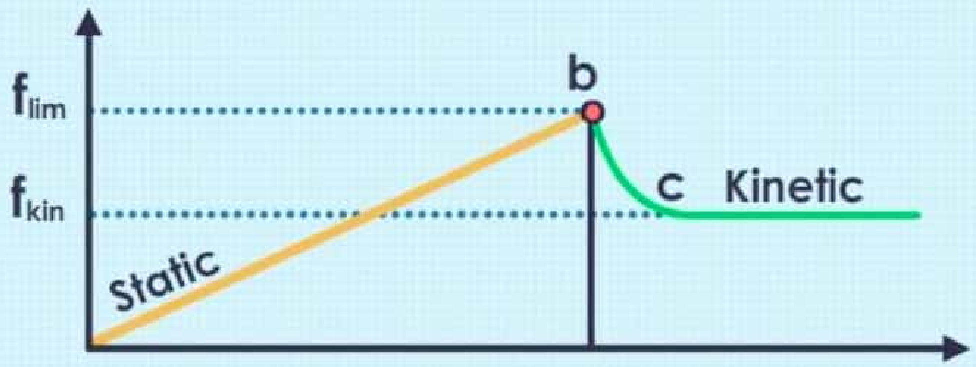
FRICTION

Friction is a contact force that opposes the relative motion or tendency of relative motion between two bodies.

$$f = \mu N = \mu mg$$



TYPES OF FRICTION FORCES



1. STATIC FRICTIONAL FORCE

The opposing force due to which there is no relative motion between the bodies in contact is called static friction force. It's a self-adjusting force.
Coefficient of static friction is μ_s.

2. LIMITING FRICTIONAL FORCE

The maximum frictional force that acts when the body is about to move is called limiting frictional force.

3. KINETIC FRICTIONAL FORCE

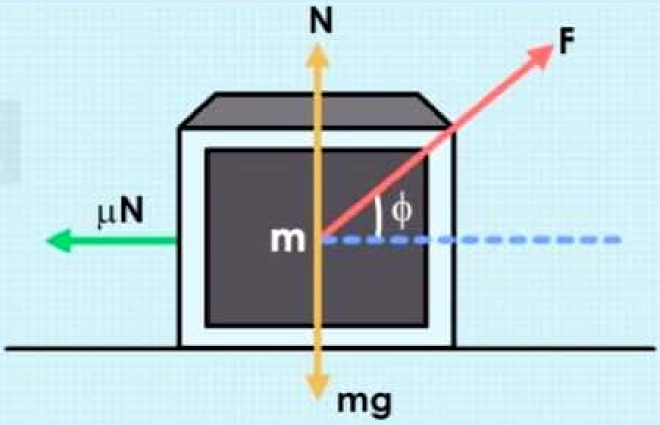
The frictional force between the surfaces in contact when relative motion starts between them is called Kinetic Frictional Force. Coefficient of kinetic friction is μ_k.

$$\boxed{\mu_k < \mu_s}$$

FRICTION

Part II

MINIMUM FORCE REQUIRED TO MOVE THE BODY



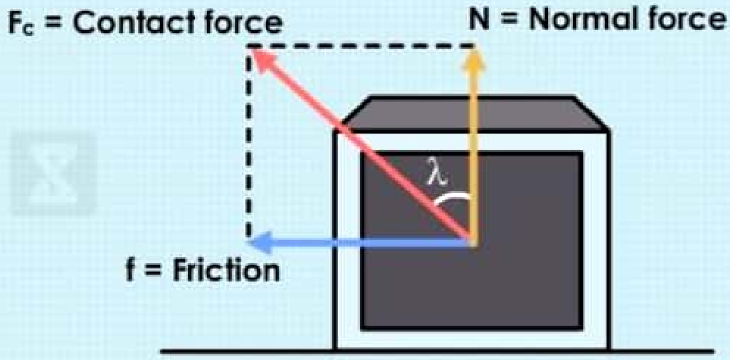
$$\boxed{F_{min} = \frac{\mu mg}{1 + \mu^2}}$$

N = Normal force

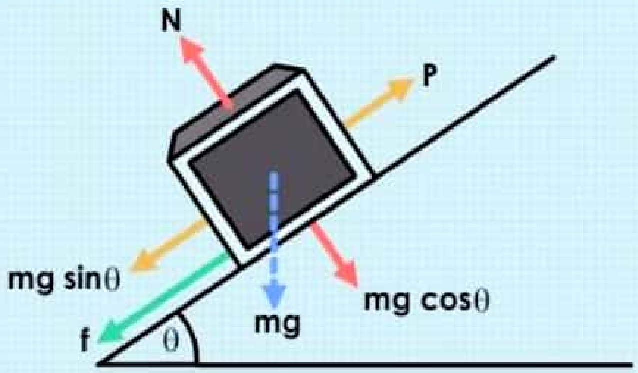
FRICTION AS A COMPONENT OF CONTACT FORCE

$$F_{c\,max} = \sqrt{\mu^2 N^2 + N^2} \quad \{\because f_{max} = \mu N\}$$

$$\boxed{F_{c\,max} = N\sqrt{\mu^2 + 1}}$$



MOTION ON A ROUGH INCLINED PLANE



Balancing Vertical Forces

$$\boxed{N = mg\cos\theta}$$

Balancing Horizontal Forces

$$\boxed{f = \mu N = \mu\, mg\cos\theta}$$

When sliding with acceleration 'a'

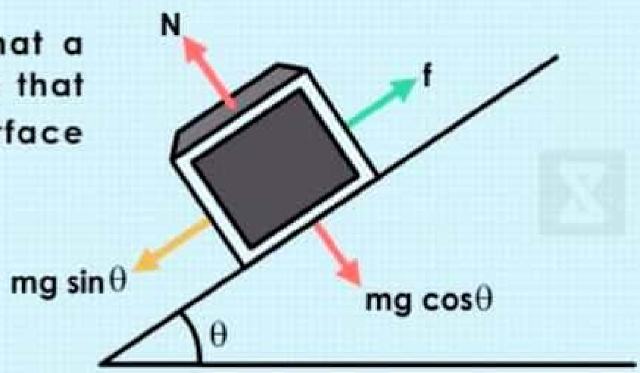
$$\boxed{mg\sin\theta - \mu mg\cos\theta = ma}$$

ANGLE OF REPOSE

The angle of repose is the maximum angle that a surface can be tilted from the horizontal, such that an object on it is just able to stay on the surface without moving.

or $\boxed{\tan\theta_c = \mu}$

where θ_c is called angle of repose.



Part I

CIRCULAR MOTION

When a particle moves in a plane such that its distance from a fixed (or moving) point remains constant then its motion is called a circular motion with respect to that fixed (or moving) point.

ANGULAR VELOCTIY (ω)

Average Angular Velocity

$$\omega_{av} = \frac{\text{Total Angle of Rotation}}{\text{Total time taken}} \; ; \quad \omega_{av} = \frac{\theta_2 - \theta_1}{t_2 - t_1} = \frac{\Delta\theta}{\Delta t}$$

where θ_1 and θ_2 are angular position of the particle at time t_1 and t_2 respectively.

Instantaneous Angular Velocity

The rate at which the position vector of a particle with respect to the centre rotates, is called as instantaneous angular velocity with respect to the centre.

$$\omega = \lim_{\Delta t \to 0} \frac{\Delta\theta}{\Delta t} = \frac{d\theta}{dt}$$

Relative Angular Velocity

$$\omega_{AB} = \frac{(V_{AB})_{\perp}}{r_{AB}}$$

here $V_{AB_{\perp}}$ = Relative velocity perpendicular to position vector AB

Relation between speed and angular Velocity : $v = r\omega$ is a scalar quantity ($\vec{\omega} \neq \frac{\vec{v}}{r}$)

ANGULAR ACCELERATION (α)

Average Angular Acceleration

Let ω_1 and ω_2 be the instantaneous angular speed at time t_1 and t_2 respectively, then the average angular acceleration α_{av} is defined as

$$\alpha_{av} = \frac{\omega_1 - \omega_2}{t_2 - t_1} = \frac{\Delta\omega}{\Delta t}$$

Instantaneous Angular Acceleration

It is the limit of average angular acceleration as Δt approaches zero, that is

$$\alpha = \lim_{\Delta t \to 0} \frac{\Delta\omega}{\Delta t} = \frac{d\omega}{dt} = \omega \frac{d\omega}{d\theta}$$

Part II

RADIAL AND TANGENTIAL ACCELERATION

$a_t = \dfrac{dv}{dt}$ = rate of change of speed and $a_r = \omega^2 r = r\left(\dfrac{v}{r}\right)^2 = \dfrac{v^2}{r}$

Angular and Tangential Acceleration Relation $a_t = \dfrac{dv}{dt} = r\dfrac{d\omega}{dt}$ or $a_t = r\alpha$

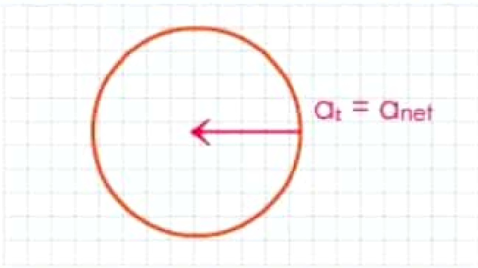
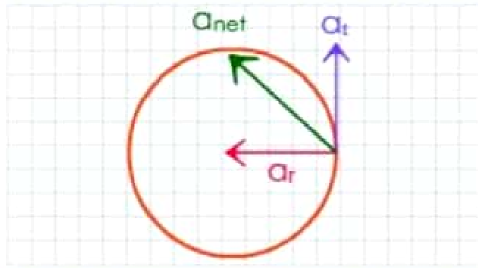
RELATIONS AMONG ANGULAR VARIABLES

Equations of Rotational Motion

$$\omega = \omega_0 + \alpha t$$

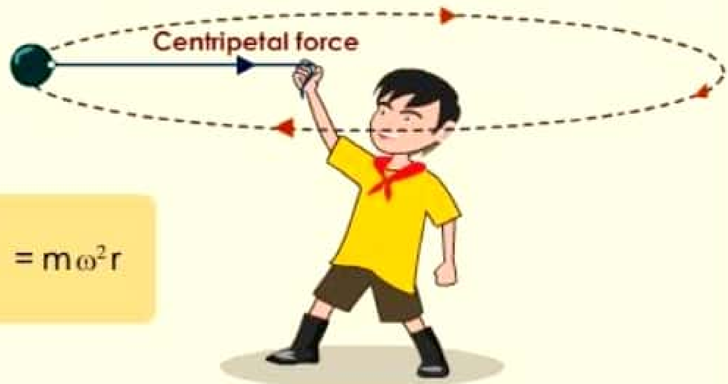
$$\theta = \omega_0 t + \frac{1}{2}\alpha t^2$$

$$\omega^2 - \omega_0^2 = 2\alpha\theta$$

Uniform Circular Motion	Non-Uniform Circular Motion
Speed of the particle is constant i.e., ω = constant	Speed of the particle is not constant i.e., $\omega \neq$ constant
$a_t = \dfrac{d\lvert\vec{v}\rvert}{dt} = 0$; $a_r = \dfrac{v^2}{r} \neq 0$ $\therefore$ $a_{net} = a_r$	$a_t = \dfrac{d\lvert\vec{v}\rvert}{dt} \neq 0$; $a_r \neq 0$ $\vec{a}_{net} = \vec{a}_r + \vec{a}_t$
	

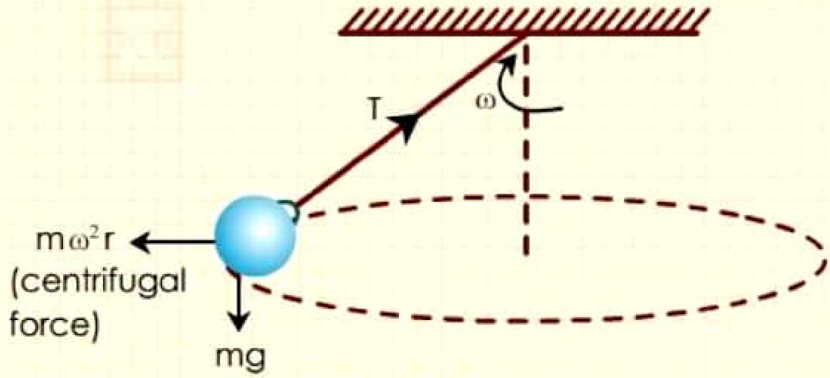
CENTRIPETAL FORCE

Centripetal force is the necessary resultant force towards the centre.



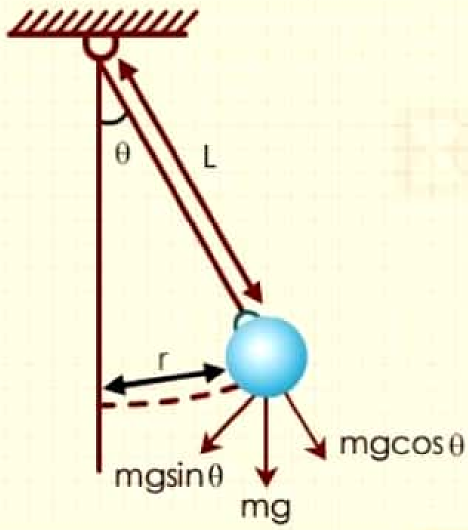
$$F = \frac{mv^2}{r} = m\omega^2 r$$

CENTRIFUGAL FORCE



- Centrifugal force is a fictitious force which has to be applied as a concept only in a rotating frame of reference to apply Newton's law of motion (in that frame)

$$F_c = m\omega^2 r$$



SIMPLE PENDULUM

Balancing Horizontal Forces:

$$T\sin\theta = m\omega^2 r$$

Balancing Vertical Forces:

$$T - mg\cos\theta = mv^2/L \Longrightarrow T = m(g\cos\theta + v^2/L)$$

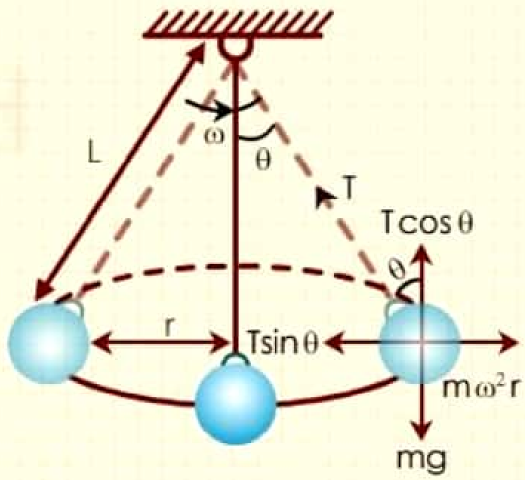
$$|\vec{F}_{net}| = \sqrt{(mg\sin\theta)^2 + \left(\frac{mv^2}{L}\right)^2} = m\sqrt{g^2\sin^2\theta + \frac{v^4}{L^2}}$$

CONICAL PENDULUM

FBD of ball shows:

$T\sin\theta = m\omega^2 r$ = centripetal force

$T\cos\theta = mg$



FBD of ball w.r.t ground

speed $v = \frac{r\sqrt{g}}{(L^2 - r^2)^{1/4}}$ and Tension $T = \frac{mgL}{(L^2 - r^2)^{1/2}}$

CIRCULAR TURNING ON ROADS

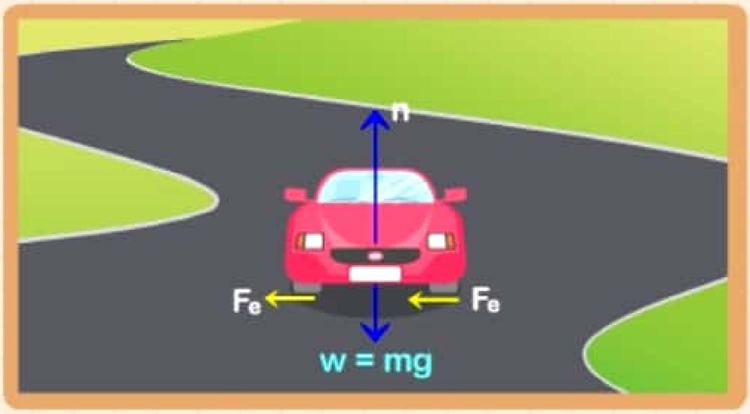
Centripital force required for turning is provided in following ways.

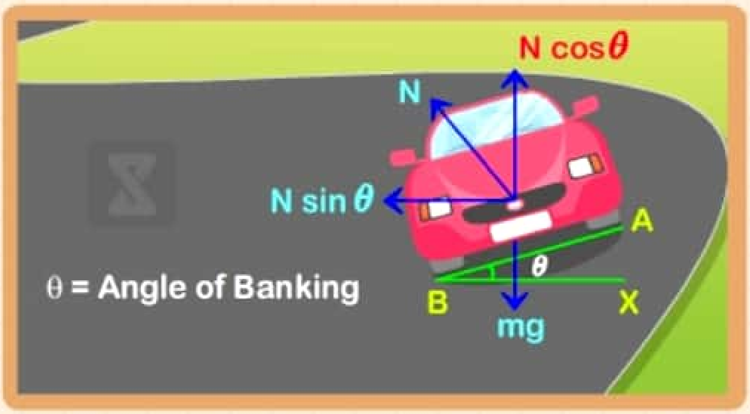
BY FRICTION ONLY

For a safe turn without sliding:

Safe Speed $v \leq \sqrt{\mu r g}$

- The safe speed of the vehicle should be less than $\sqrt{\mu r g}$
- The coefficient of friction should be more than v^2/rg.





BY BANKING OF ROADS ONLY

From FBD of car:

$N\sin\theta = \dfrac{mv^2}{r}$ & $N\cos\theta = mg$

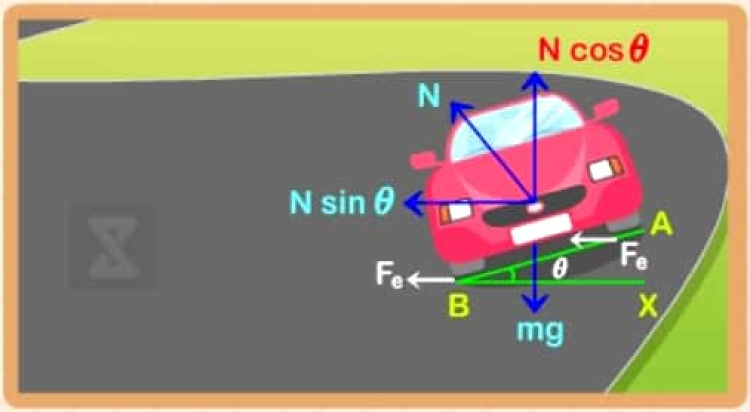
From these two equations, we get

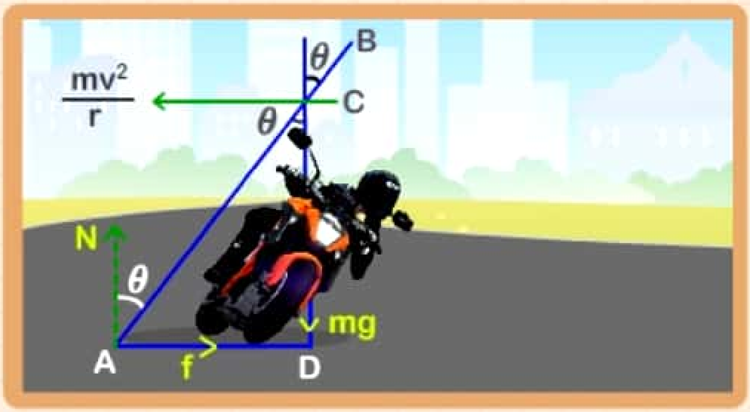
$\tan\theta = \dfrac{v^2}{rg}$ & $v = \sqrt{rg\tan\theta}$

BOTH FRICTION AND BANKING OF ROADS

Maximum safe speed $v_{max} = \sqrt{\dfrac{rg(\mu + \tan\theta)}{(1 - \mu\tan\theta)}}$

Minimum safe speed $v_{max} = \sqrt{\dfrac{rg(\mu - \tan\theta)}{(1 + \mu\tan\theta)}}$



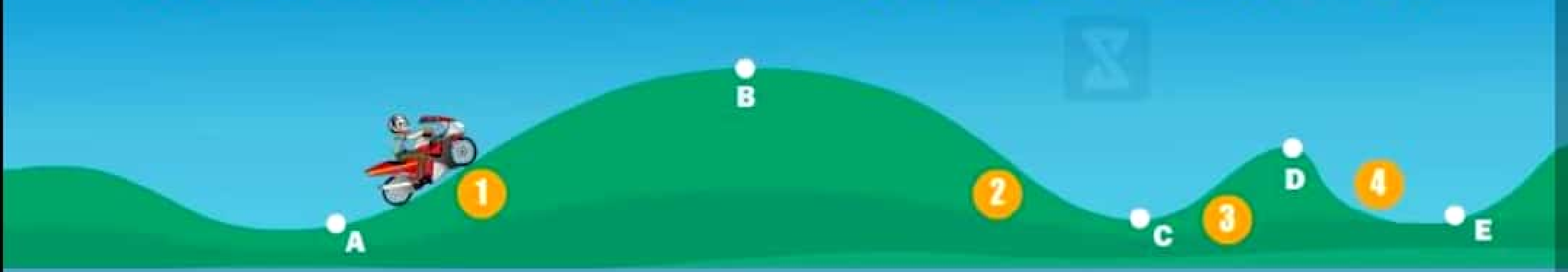


BIKE ON A CIRCULAR PATH

$$\frac{AD}{CD} = \frac{v^2}{rg} \Rightarrow \tan\theta = \frac{v^2}{rg}$$

Thus, the cyclist bends at an angle $\tan^{-1}[v^2/rg]$ with the vertical.

MOTORCYCLIST ON A CURVED PATH



A cyclist having mass m moving with constant speed v on a curved path

We divide the motion of the cyclist in four parts :

1. From A to B
2. From B to C
3. From C to D
4. From D to E

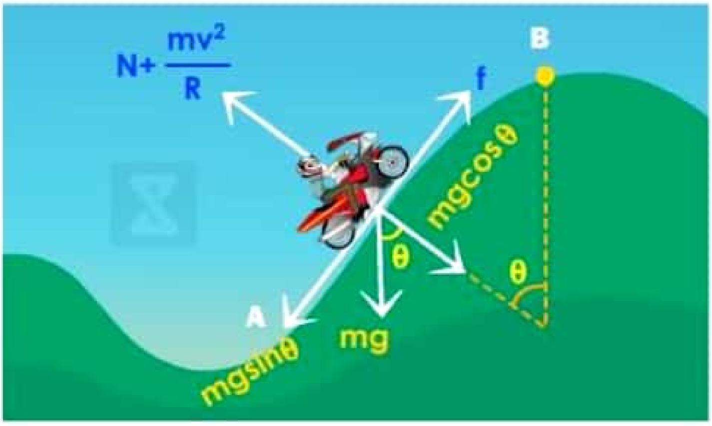
MOTION OF CYCLIST FROM A TO B

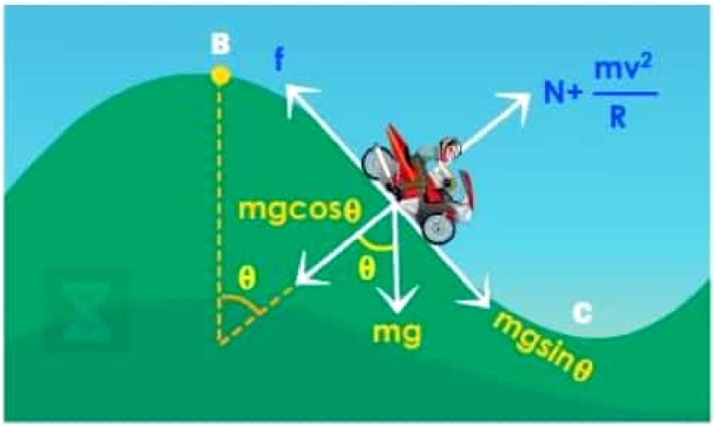
(1 and 3 are same type of motion)

$$N + \frac{mv^2}{R} = mg\cos\theta \quad : \quad f = mg\sin\theta$$

AS CYCLIST MOVE UPWARD

In 1 and 3 normal force increases but frictional force decreases because θ decreases.





MOTION OF CYCLIST FROM B TO C

$$N + \frac{mv^2}{R} = mg\cos\theta \Longrightarrow N = mg\cos\theta - \frac{mv^2}{R}$$

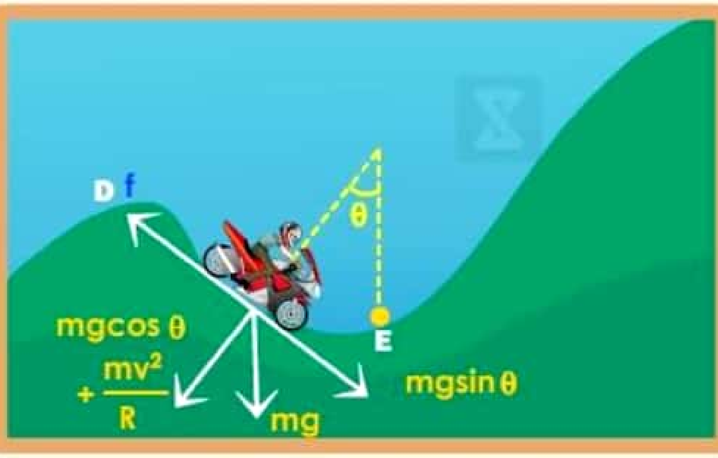
$$f = mg\sin\theta$$

From B to C, Normal force decreases but friction force increases because θ increases.

MOTION OF CYCLIST FROM D TO E

$$N = \frac{mv^2}{R} + mg\cos\theta \quad : \quad f = mg\sin\theta$$

From D to E, 'θ' decreases therefore mg cosθ increases whereas Normal force increases but frictional force decreases.



WORK, POWER, ENERGY

WORK

$W = \vec{F}.\vec{ds} = Fs\cos\theta$

F = Force Applied

$\vec{ds}$ = Displacement

θ = Angle Between Force and Displacement

$W = \tau\theta$

τ = Torque

θ = Angle of Rotation

POWER

$P = \frac{dW}{dt} = \frac{\vec{F}.\vec{ds}}{dt} = \vec{F}.\vec{v}$

ENERGY

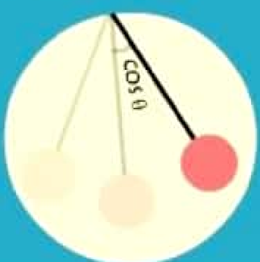
KINETIC ENERGY

$K.E._{Trans} = \frac{1}{2}mv^2$

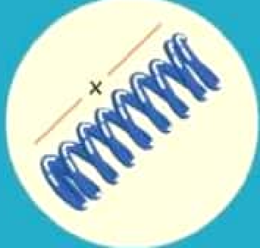
$K.E._{Rot} = \frac{1}{2}I\omega^2$

$K.E._{Rolling} = mv^2 + \frac{1}{2}I\omega^2$

POTENTIAL ENERGY



$PE_{Pendulum} = mgl(1-\cos\theta)$

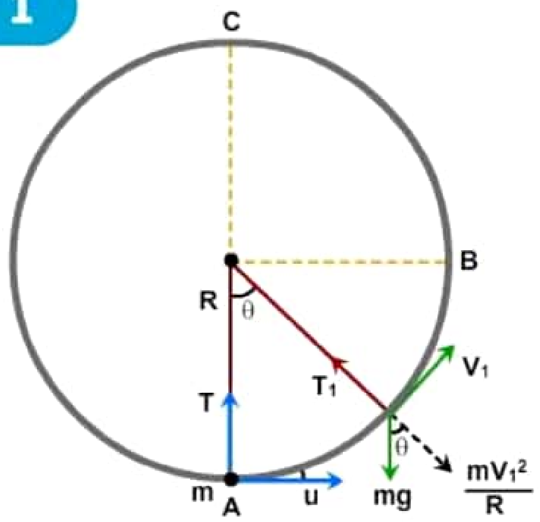


$PE_{spring} = \frac{1}{2}Kx^2$

$PE_{grav} = mgh$

VERTICAL CIRCULAR MOTION

1

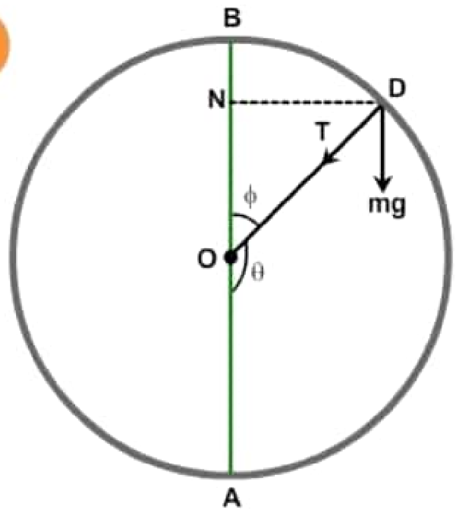


Ball will complete the circle

Condition: Initial velocity, $u > \sqrt{5gR}$

- Tension at A : $T_A = 6mg$
- Tension at B : $T_B = 3mg$
- If $u = \sqrt{5gR}$ ball will just complete the circle and velocity at topmost point is $v = \sqrt{gR}$

2

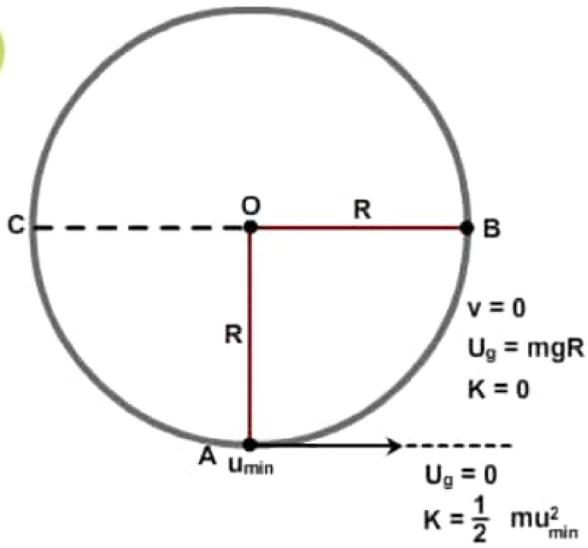


Ball will slack in between

Condition: $\sqrt{2gR} < u < \sqrt{5gR}$

- $\cos\phi = \dfrac{u^2 - 2gR}{3gR}$. v

3

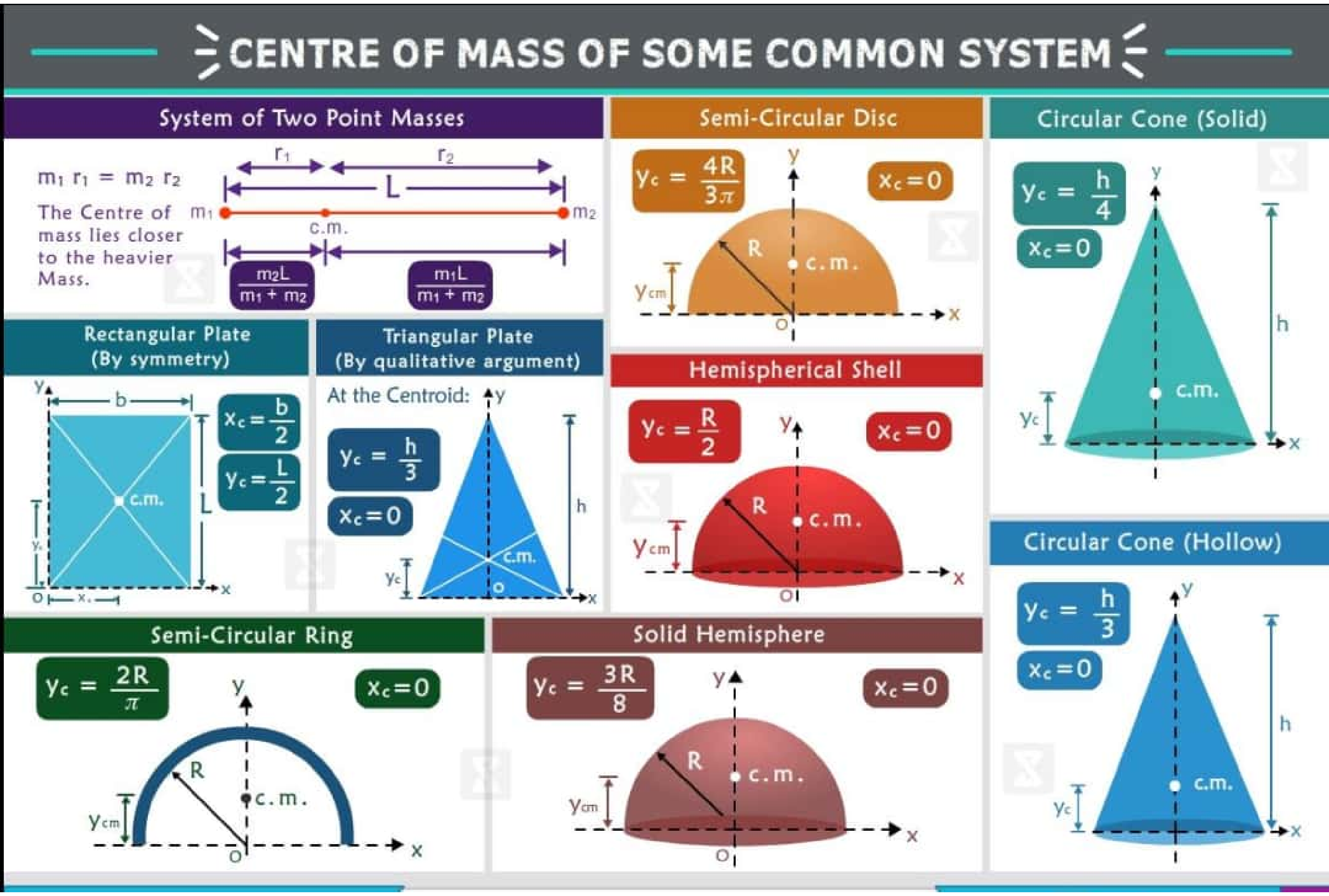


Ball will reach B

Condition: $u \leq \sqrt{2gR}$

- Ball will oscillate between CAB
- Velocity $v = 0$ but $T \neq 0$

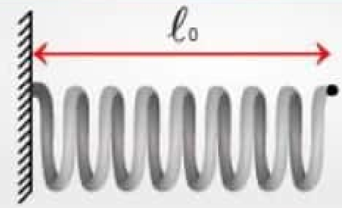
Note: At height **h** from bottom of ball velocity will be, $v = \sqrt{u^2 - 2gh}$


CENTRE OF MASS OF SOME COMMON SYSTEM
System of Two Point Masses
m1 r1 = m2 r2
The Centre of mass lies closer to the heavier Mass.
r1
r2
L
m1
m2
c.m.
m2L / (m1 + m2)
m1L / (m1 + m2)
Semi-Circular Disc
yc = 4R / 3π
xc = 0
R
c.m.
ycm
Circular Cone (Solid)
yc = h/4
xc = 0
h
c.m.
yc
Rectangular Plate (By symmetry)
xc = b/2
yc = L/2
b
L
c.m.
Triangular Plate (By qualitative argument)
At the Centroid:
yc = h/3
xc = 0
h
c.m.
Hemispherical Shell
yc = R/2
xc = 0
R
c.m.
ycm
Circular Cone (Hollow)
yc = h/3
xc = 0
h
c.m.
yc
Semi-Circular Ring
yc = 2R / π
xc = 0
R
c.m.
ycm
Solid Hemisphere
yc = 3R / 8
xc = 0
R
c.m.
ycm

SPRING FORCE

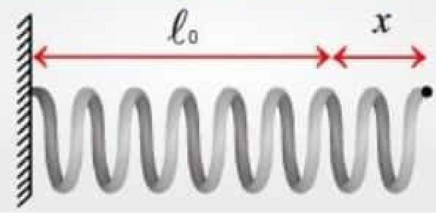
1 STRETCHED SPRING

Initial length (ℓ_0)



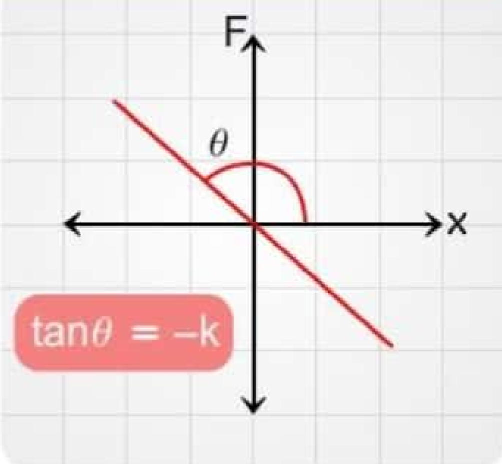
$F = 0$

Stretched by x



$F = -kx$

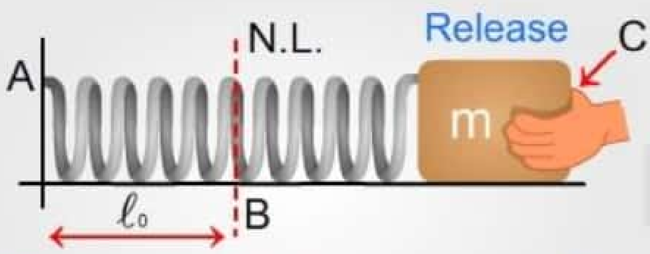
Spring Force v/s Displacement



$\tan\theta = -k$

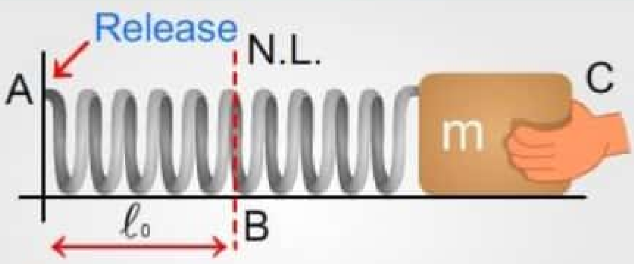
2 SPRING ATTACHED TO A BLOCK

Released at C



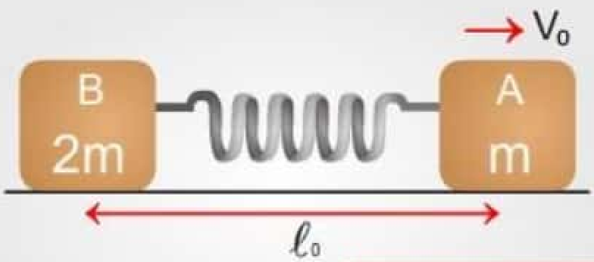
When the block is released at point C then spring force doesn't change instantaneously because of friction at mass m.

Released at A

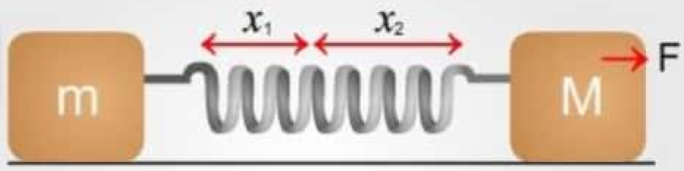


When point A is released then the spring force changes instantaneously to become zero.

3 SPRING BLOCK SYSTEM



Maximum Extension $x_{max} = V_0\sqrt{\frac{2}{3k}m}$



$$x_{max} = x_1 + x_2 = \frac{2mF}{k(m+M)}$$

IMPULSE AND MOMENTUM

IMPULSE

Impulse of a force 'F' acting on a body for a time interval $t = t_1$ to $t = t_2$ is defined as

$$\boxed{\vec{I} = \int_{t_1}^{t_2} \vec{F}\, dt} \qquad \boxed{\vec{I}_{Re} = \int_{t_1}^{t_2} \vec{F}_{Res}\, dt = \Delta\vec{P}}$$

(Impulse - Momentum Theorem)

COEFFICIENT OF RESTITUTION (e)

The coefficient of restitution is defined as the ratio of the impulses of reformation and deformation of either body.

$$e = \frac{\text{Impulse of reformation}}{\text{Impulse of deformation}} = \frac{\int F_r\, dt}{\int F_d\, dt} \qquad e = \frac{\text{Velocity of separation of point of contact}}{\text{Velocity of approach of point of contact}}$$

LINEAR MOMENTUM

Linear momentum is a vector quantity defined as the product of an object's mass m, and its velocity v. Linear momentum is denoted by the letter p and is called "momentum" in short:

$$p = mv$$

Note that a body's momentum is always in the same direction as its velocity vector. The units of momentum are kg.m/s.

CONSERVATION OF LINEAR MOMENTUM

For a single mass or single body, If net force acting on the body is zero. Then,

$$\vec{p} = \text{constant} \quad \text{or} \quad \vec{v} = \text{constant}$$

(if mass = constant)

If net external force acting on a system of particles or system of rigid bodies is zero. Then,

$$\vec{P}_{CM} = \text{constant} \quad \text{or} \quad \vec{V}_{CM} = \text{constant}$$

COLLISION

Note :- In every type of collision, only linear momentum remains constant.

HEAD ON ELASTIC COLLISION

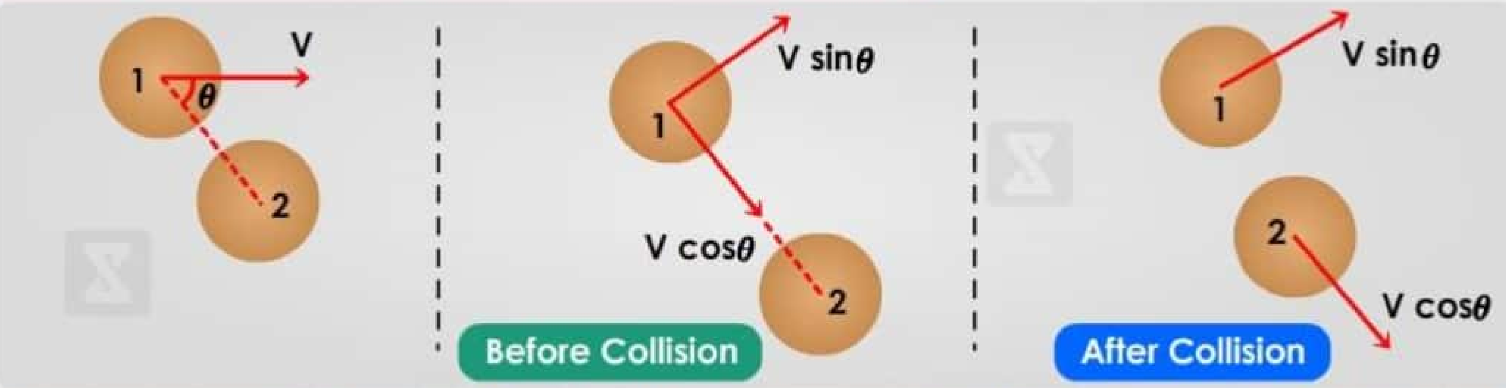
In this case, linear momentum and kinetic energy both are conserved. After solving two conservation equations. We get,

$$V'_1 = \left(\frac{m_1 - m_2}{m_1 + m_2}\right)V_1 + \left(\frac{2m_2}{m_1 + m_2}\right)V_2$$ and $$V'_2 = \left(\frac{m_2 - m_1}{m_1 + m_2}\right)V_2 + \left(\frac{2m_2}{m_1 + m_2}\right)V_1$$

HEAD ON INELASTIC COLLISION

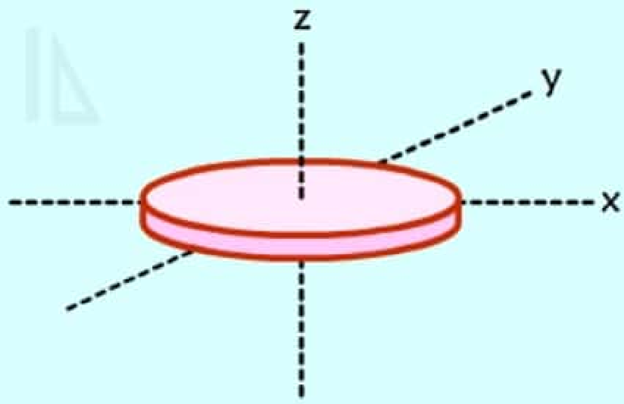
- In an inelastic collision, the colliding particles do not regain their shape and size completely after the collision.
- Some fraction of mechanical energy is retained by the colliding particles in the form of deformation potential energy. Thus, the kinetic energy of the particles no longer remains conserved.
- $(\text{Energy loss})_{\text{Perfectly Inelastic}} > (\text{Energy loss})_{\text{Partial Inelastic}}$
- $0 < e < 1$: e = coefficient of restitution

OBLIQUE COLLISION (BOTH ELASTIC IN ELASTIC)



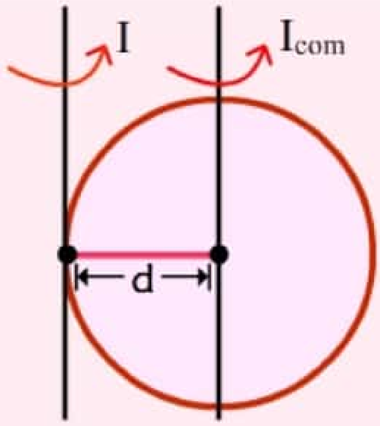
BALL	COMPONENT ALONG COMMON TANGENT DIRECTION		COMPONENT ALONG COMMON NORMAL DIRECTION	
	Before Collision	After Collision	Before Collision	After Collision
1	V $\sin\theta$	V $\sin\theta$	V $\cos\theta$	0
2	0	0	0	V $\cos\theta$

Perpendicular Axis Theorm



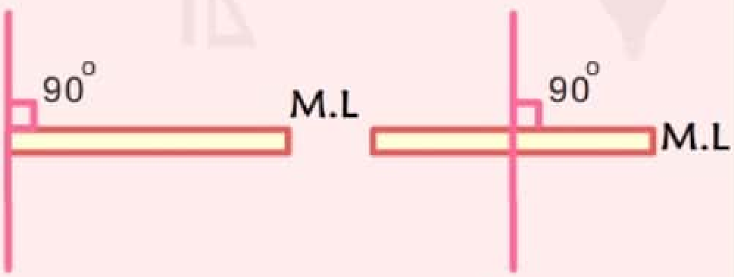
$$I_z = I_x + I_y$$

Parellel Axis Theorm



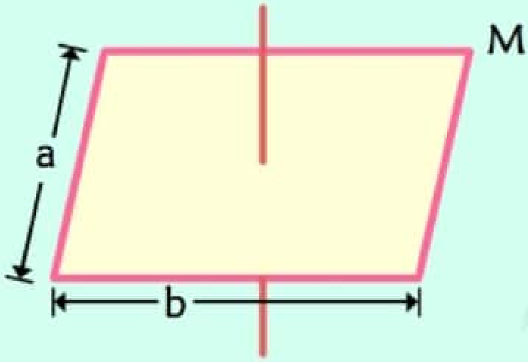
$$I = I_{com} + Md^2$$

Rod



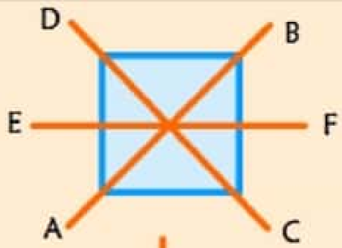
$\frac{ML^2}{3}$ (Uniform) $\frac{ML^2}{12}$ (Uniform)

Rectangular Plate



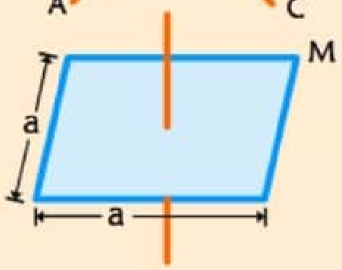
$$I = \frac{M(a^2+b^2)}{12} \text{ (Uniform)}$$

Square Plate



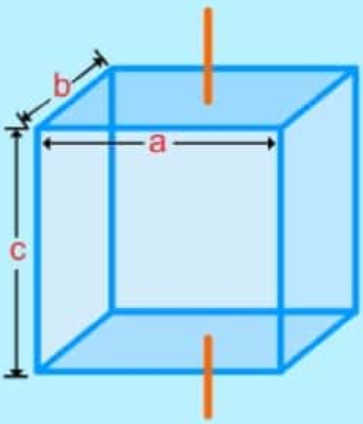
$$I_{AB} = I_{CD} = I_{EF}$$

$$= \frac{Ma^2}{12} \text{ (Uniform)}$$



$$\frac{Ma^2}{6} \text{ (Uniform)}$$

Cuboid



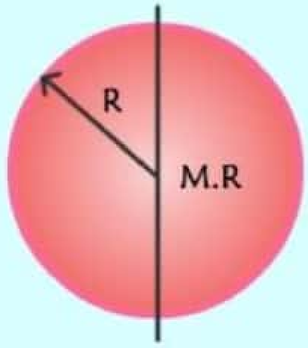
$$\frac{M(a^2+b^2)}{12}$$

(Uniform)

MOMENT OF INERTIA

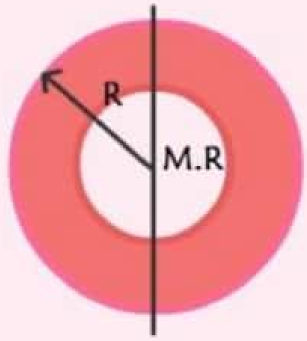
Part II

Solid Sphere



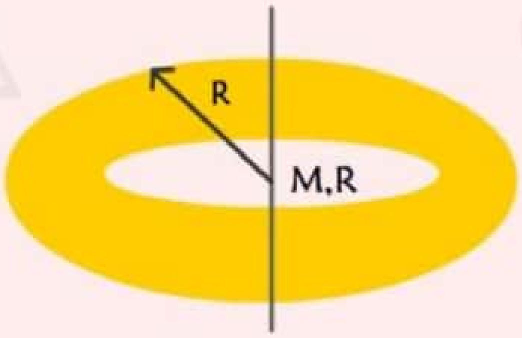
$I = \frac{2}{5} MR^2$ (Uniform)

Hollow Sphere



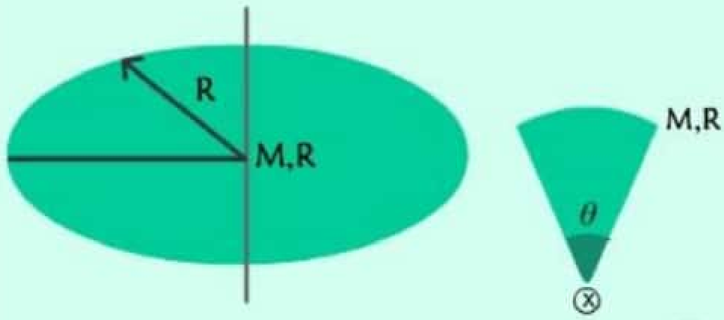
$I = \frac{2}{3} MR^2$ (Uniform)

Ring



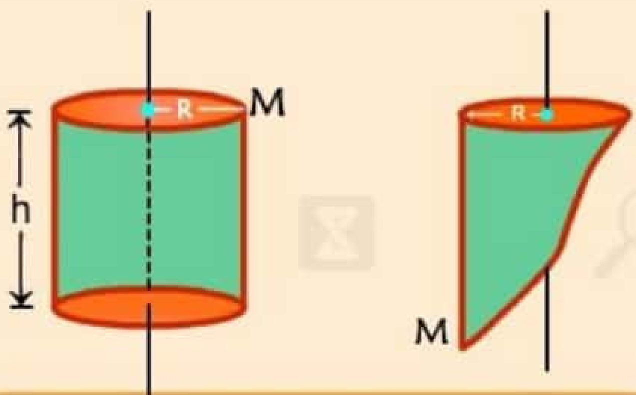
$I = MR^2$ (Uniform or Non Uniform)

Disc



$I = \frac{MR^2}{2}$ (Uniform)

Hollow cylinder



$I = MR^2$ (Uniform or Non Uniform)

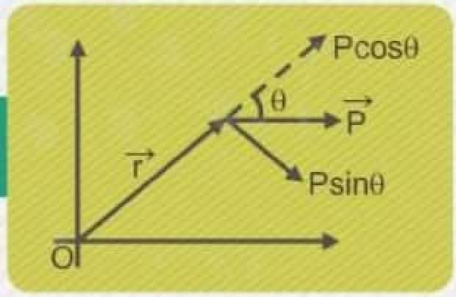
Solid cylinder

$I = \frac{MR^2}{2}$ (Uniform)

ANGULAR MOMENTUM

1 ANGULAR MOMENTUM OF A PARTICLE ABOUT A POINT

$$\vec{L} = \vec{r} \times \vec{P} \Rightarrow L = rP\sin\theta$$



2 ANGULAR MOMENTUM OF A RIGID BODY ROTATING ABOUT A FIXED AXIS

$$L = I\omega$$

Here, I is the moment of inertia of the rigid body about axis.

3 CONSERVATION OF ANGULAR MOMENTUM

The law of conservation of angular momentum states that when no external torque acts on an object, no change of angular momentum will occur.

Since $\vec{\tau}_{net} = \frac{d\vec{L}}{dt}$. Now if, $\vec{\tau}_{net} = 0$, then $\frac{d\vec{L}}{dt} = 0$, so that $\vec{L}$ = constant.

4 ANGULAR IMPULSE

The angular impulse of a torque in a given time interval is defined as $\int_{t_1}^{t_2} \vec{\tau}\, dt$

$$\vec{\tau} = \frac{d\vec{L}}{dt}$$

UNIFORM PURE ROLLING

Pure rolling means no relative motion (or no slipping at point of contact between two bodies.)

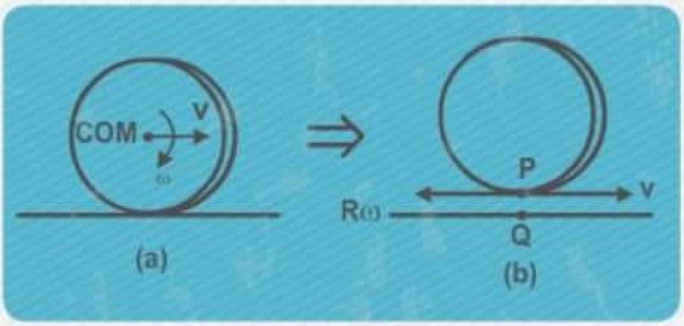
$V_P = V_Q$ or $V - R\omega = 0$ or $V = R\omega$

If $V_P > V_Q$ or $V > R\omega$, the motion is said to be forward slipping and if

$V_P < V_Q < R\omega$, the motion is said to be backward slipping.

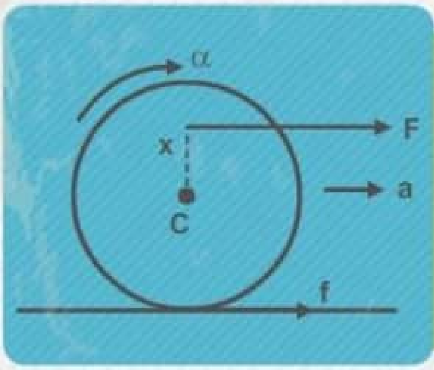
The condition of pure rolling on a stationary ground is,

$a = R\alpha$



1 PURE ROLLING WHEN FORCE F ACT ON A BODY

Suppose a force F is applied at a distance x above the centre of a rigid body of radius R, mass M and moment of inertia CMR^2 about an axis passing through the centre of mass. Applied force F can produces by itself a linear acceleration a and an angular acceleration α.



a = liner acceleration , α = angular acceleration from linear motion

$$F + f = Ma$$

From rotational motion : $Fx - fR = I\alpha$

$$a = \frac{F(R+X)}{MR(C+1)}, \quad f = \frac{F(x-RC)}{R(C+1)}$$

2 PURE ROLLING ON A INCLINED PLANS

A rigid body of radius R, and mass m is released at rest from height h on the incline whose inclination with horizontal is θ and assume that friciton is sufficient for pure rolling then,

$a = \alpha R$ and $v = R\omega$

ω = Angular Velocity

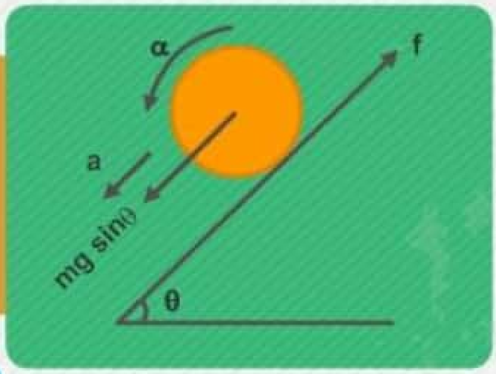
α = Angular Acceleration

Linear Acceleration.

$$a = \frac{g\sin\theta}{1+C}$$

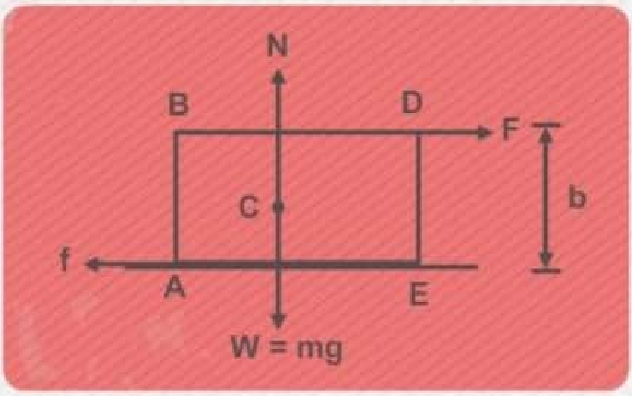
C = Center of Mass

So, body which have low value of C have greater acceleration.



TOPPLING

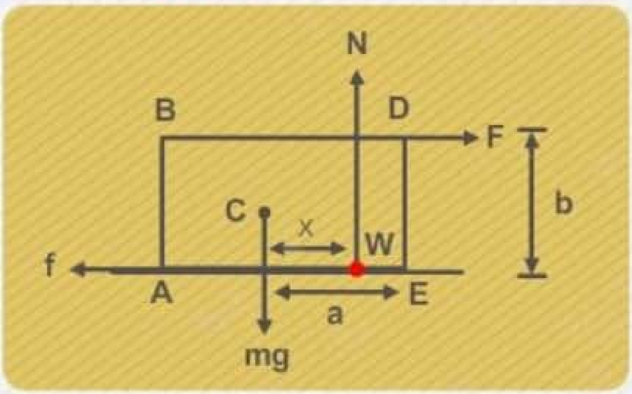
Torque about E



Balancing Torque at E

$$Fb = (mg)\,a \implies a = \frac{Fb}{mg}$$

Torque about W



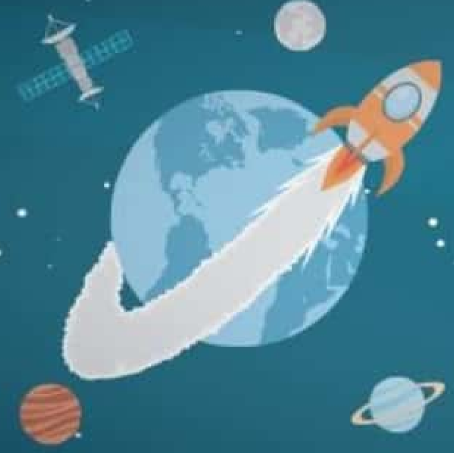
Balancing Torque at W

$Fb + N(a - x) = mg\,a$

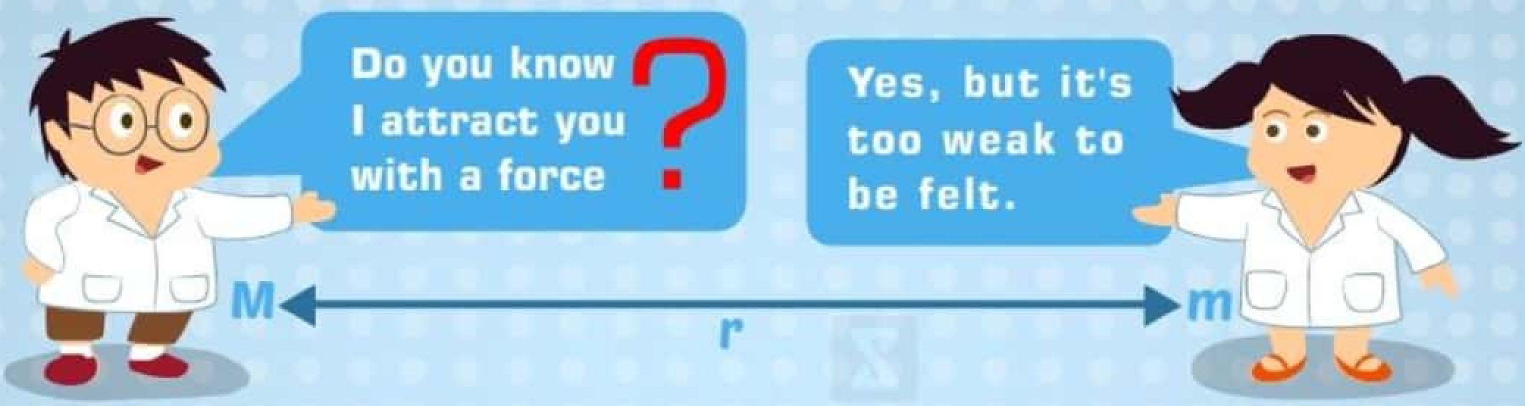
if $x = a$

$$F_{max}\,b = mga \implies F_{max} = \frac{mga}{b}$$

Part I



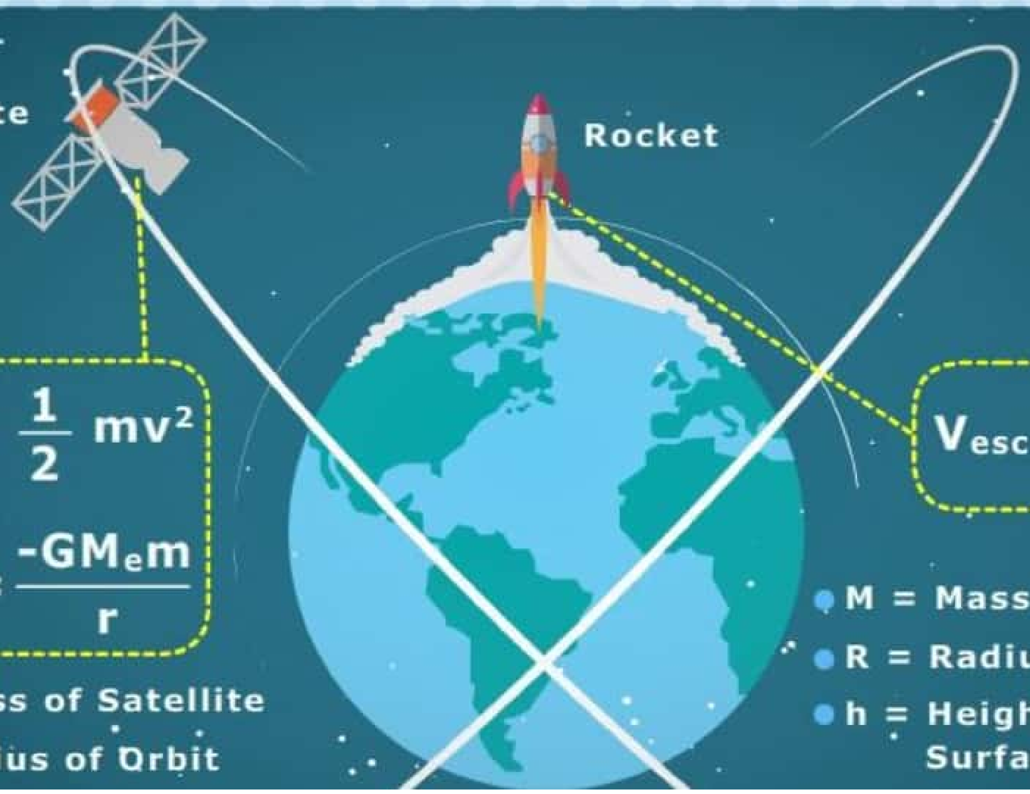
GRAVITATION



Force of attraction between them is gravitation and it is given by:

$$F = G\frac{Mm}{r^2}$$

G = Gravitational Constant



$$K.E = \frac{1}{2}mv^2$$

$$P.E = \frac{-GM_e m}{r}$$

- m = Mass of Satellite
- r = Radius of Orbit

$$V_{esc} = \frac{GM_e}{R + h}$$

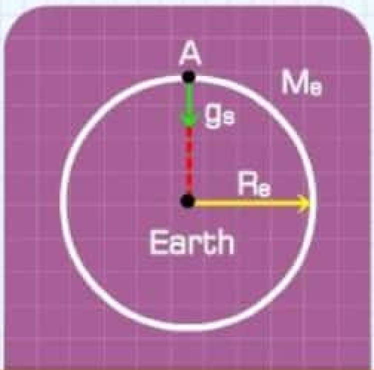
- M = Mass of Earth
- R = Radius of Earth
- h = Height from Earth Surface

Part II

GRAVITATIONAL FORCE

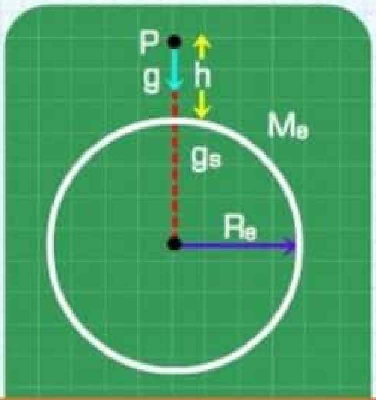
Acceleration Due to Gravity

On the surface of earth



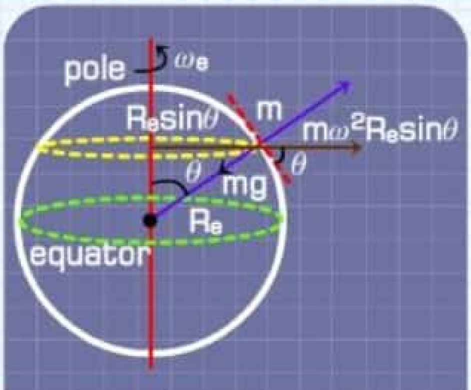
$$g = \frac{GM}{R^2} = 9.81\,ms^{-2}$$

At height h from the surface of earth



$$g' = \frac{g}{\left(1+\frac{h}{R}\right)^2} = g\left(1-\frac{2h}{R}\right) \text{ if } h << R$$

Effect of rotation of earth at latitude

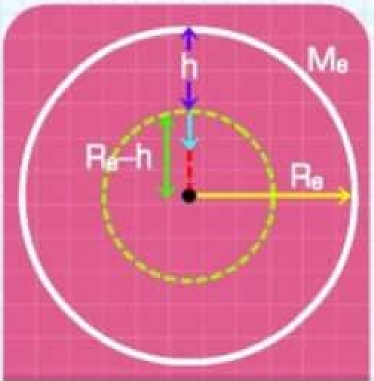


$$g' = g - R\omega^2\sin^2\phi$$

At equator, $\phi = 90^\circ$, $g' - R\omega^2 = 9.78\,m/s^2$

At poles, $\phi = 0$, $g' = g = 9.83\,m/s^2$

At depth d from the surface of earth



$$g' = g\left(1-\frac{d}{R}\right)$$

$g' = 0$ if $d = R$ i.e., at centre of earth

- At equator, effect of rotation of earth is maximum and value of g is minimum.
- At poles, effect of rotation of earth is zero and value of g is maximum.

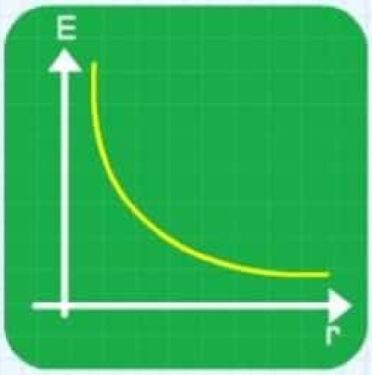
Gravitation Field Strength

Part III

Gravitation field strength at a point in gravitational field is defined as:

$\vec{E} = \frac{\vec{F}}{m}$ = Gravitational force per unit mass.

Due to a point mass

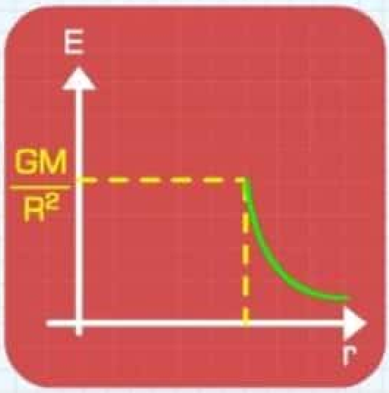


$E = \frac{GM}{r^2}$ (towards the mass)

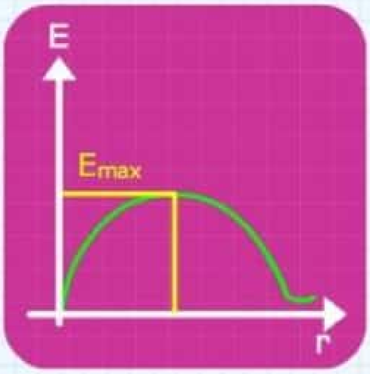
or $E \propto \frac{1}{r^2}$

Due to spherical shell

- Inside points, $E_i = 0$
- Just outside the surface, $E = \frac{GM}{R^2}$; **R - Radius of Sphere**
- Outside Point, $E_o = \frac{GM}{r^3}$; **r - Distance of centre from an external point**
- On the surface E-r graph is discontinuous.



On the axis of a ring



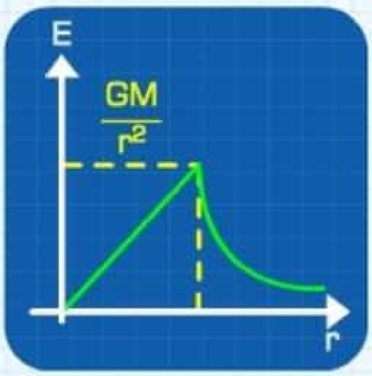
$E_{ix} = \frac{GMx}{(R^2+x^2)^{3/2}}$; At $x = 0$, $E = 0$ i.e., at centre

If $x >> R$, $E = \frac{GM}{x^2}$ i.e., ring behaves as a point mass

At $x \rightarrow \infty, E \rightarrow 0$; $E_{max} = \frac{2GM}{3\sqrt{3}R^2}$ at $x = \frac{R}{\sqrt{2}}$

Due to a solid sphere

- Inside points $E_i = \frac{GM}{R^3} r$
- At $r = 0$, $E = 0$ i.e. at centre
- At $r = R$, $E = \frac{GM}{R^2}$ i.e., on surface
- Outside points $E_o = \frac{GM}{R^2}$ or $E_o \propto \frac{1}{r^2}$
- At $r \rightarrow \infty, E \rightarrow 0$

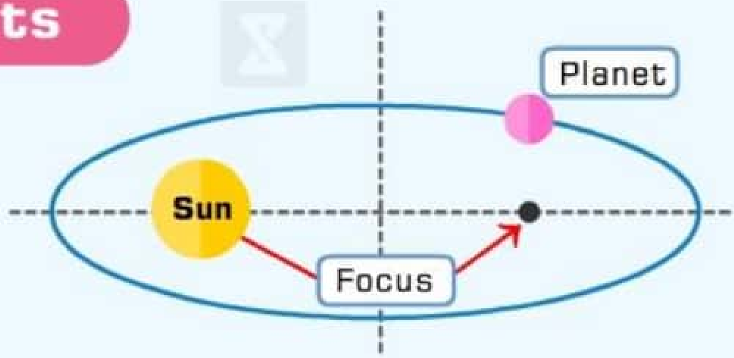


Kepler's law of Planetary Motion

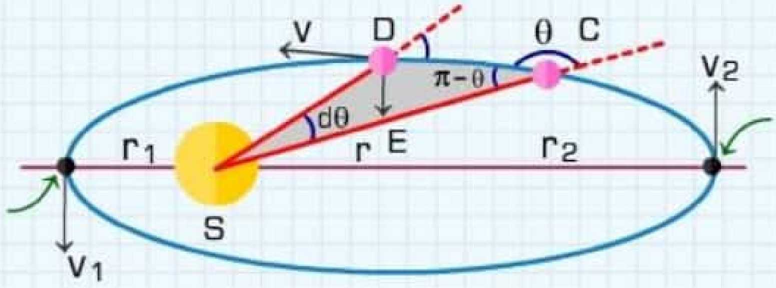
Part IV

1st Law The Law of Orbits

All the planets move around the sun in elliptical orbits with sun at one of the focus, not at centre of orbit.



The Law of Areas 2nd Law



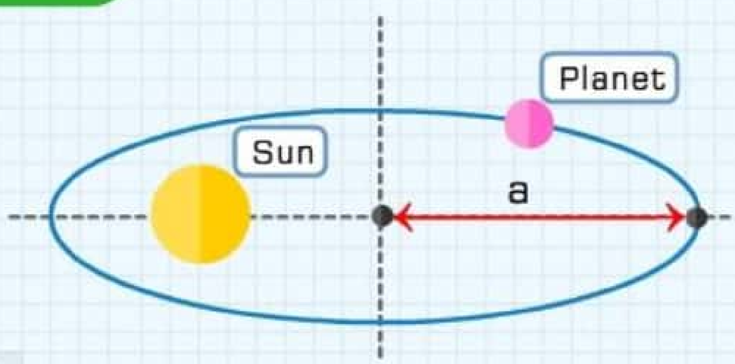
The line joining the sun and planet sweeps out equal areas in equal time.

$$\frac{dA}{dt} = \frac{L}{2m} = \text{Constant}$$

3rd Law The Law of Periods

The time period of revolution of a planet in its orbit around the sun is directly proportionally to the cube of semi - major axis of the elliptical path around the sun.

$$T^2 \propto a^3$$



Part I

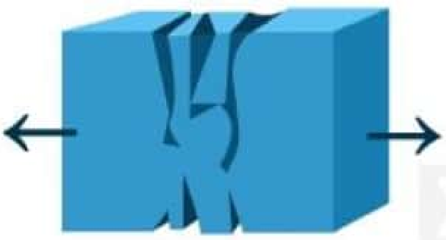
Elasticity

STRESS

The reaction force per unit area of the body due to the action of the applied force is called stress

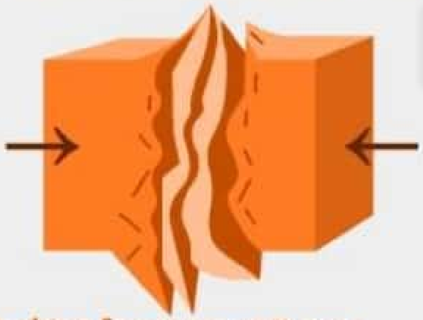
$$\text{Stress} = \frac{\text{Force}}{\text{Area}} \ \text{N/m}^2$$

TENSILE STRESS



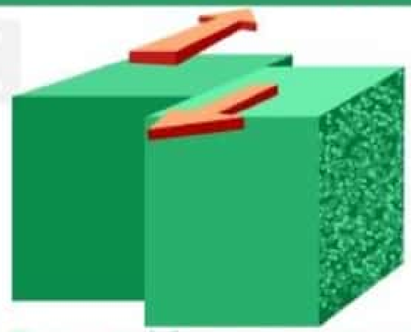
- Pulling force per unit area
- Increase in length or volume

COMPRESSIVE STRESS



- Pushing force per unit area
- Decrease in length or volume

TANGENTIAL STRESS



- Tangential force per unit area
- It causes shearing of bodies

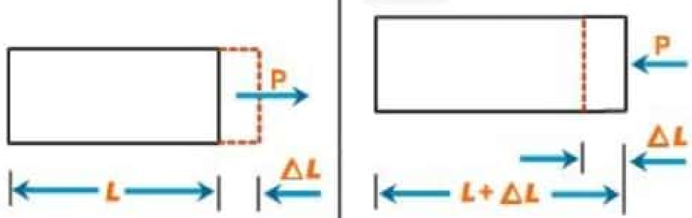
STRAIN

The ratio of the change in size or shape to the original size or shape of the body

$$\text{Strain} = \frac{\text{Change in size or shape}}{\text{Original size or shape}}$$

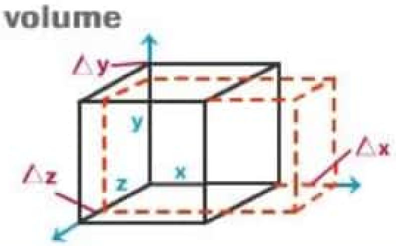
LINEAR STRAIN: Change in length per unit length

$$\text{Linear Strain} = \frac{\text{Change in length}}{\text{Original length}} = \frac{\Delta L}{L}$$



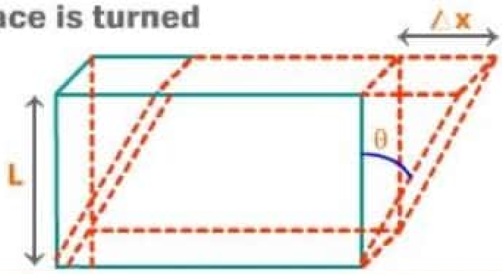
VOLUME STRAIN: Change in volume per unit volume

$$\text{Volume Strain} = \frac{\text{Change in volume}}{\text{Original volume}} = \frac{\Delta V}{V}$$



SHEAR STRAIN: Angle through which a line originally normal to fixed surface is turned

$$\text{Shear Strain} = \frac{\text{Deformation}}{\text{Original Dimension}} = \frac{\Delta X}{L}$$



Part II

THERMAL STRESS

Thermal Stress = $Y\alpha\Delta t$

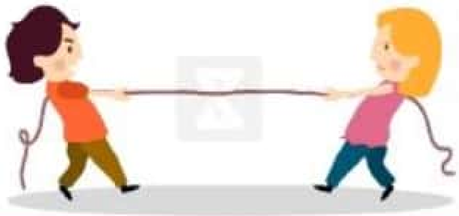
Y → Modulus of Elasticity

α → Coefficient of Linear Expansion

Δt → Change in Temperature

WORK DONE IN STRETCHING A WIRE

$$W = \frac{1}{2} F \times \Delta L = \frac{1}{2}\ \text{load} \times \text{elongation}$$



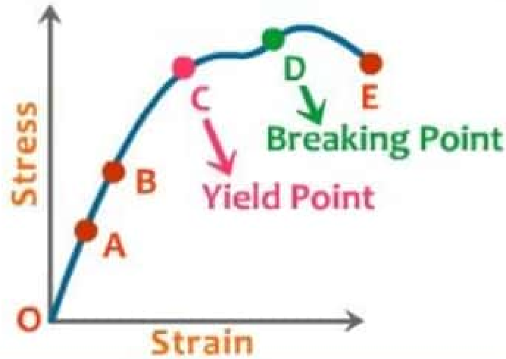
HOOKE'S LAW

$$\text{Modulus Of Elasticity} = \frac{\text{Stress}}{\text{Strain}}$$

Within the elastic limit, the stress developed in a body is proportional to the strain produced in it, thus the ratio of stress to strain is a constant.This constant is called the modulus of elasticity

STRESS STRAIN CURVE

If we increase the load gradually on a vertically suspended metal wire:



IN REGION OA	Strain is small (<2%) Stress $\propto$ Strain → Hook's law is valid
IN REGION AB	Stress is not proportional to strain, but wire will still regain its original length after removal of stretching force
IN REGION BC	Wire yields → strain increases rapidly with small change in stress. This behavior is shown up to point C known as yield point
IN REGION CD	Point D corresponds to maximum stress, which is called point of breaking or tensile strength.
IN REGION DE	The wire literally flows. The maximum stress corresponding to D, after which wire begins to flow. In this region, strain increase even if wire is unloaded and ruptures at E.

YOUNG'S MODULUS

Young's modulus is defined as the ratio of the linear stress to linear strain, provided the elastic limit is not exceeded.

$$Y = \frac{\text{Stress}}{\text{Strain}} = \frac{F}{A} \cdot \frac{L}{\Delta L}$$

BULK MODULUS

$$\beta = \frac{\text{Volume Stress}}{\text{Volume Strain}} = -\frac{V \Delta P}{\Delta V}$$

MODULUS OF RIGIDITY

$$\eta = \frac{\text{Tangential Stress}}{\text{Tangential Strain}} = -\frac{\frac{F}{A}}{\phi}$$

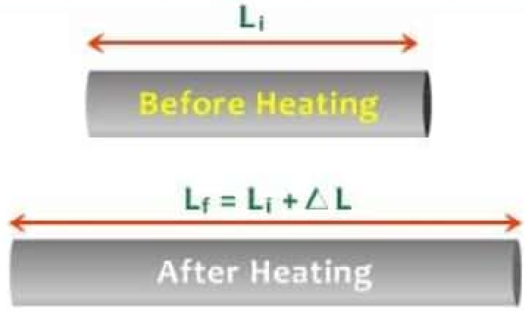
THERMAL EXPANSION

LINEAR EXPANSION

$$L_f = L_i (1 + \alpha \Delta T)$$

α = coefficient of linear expansion

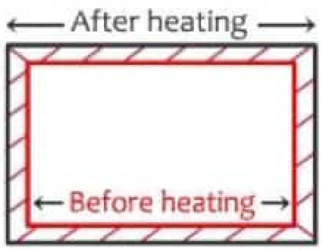
ΔT = Change in temperature



SUPERFICIAL OR AREAL EXPANSION

$$A_f = A_i (1 + \beta \Delta T)$$

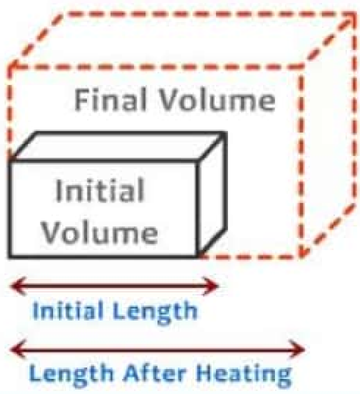
β = coefficient of Areal Expansion



VOLUME OR CUBICAL EXPANSION

$$V_f = V_i (1 + \gamma \Delta T)$$

γ = coefficient of Volume Expansion

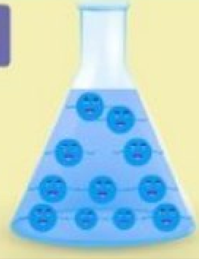


$$\alpha : \beta : \gamma = 1 : 2 : 3$$

FLUID MECHANICS

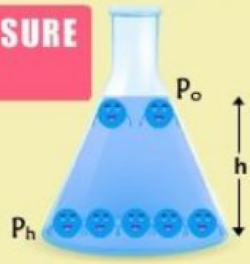
1 PRESSURE IN A FLUID

$$P = \frac{F}{A} = \frac{\text{Force}}{\text{Area}}$$



2 VARIATION IN PRESSURE WITH DEPTH

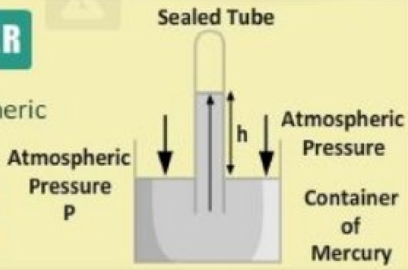
$$P_h = P_o + \rho g h$$



3 BAROMETER

Measures atmospheric pressure

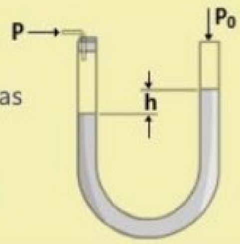
$$P_o = \rho g h$$



4 MANOMETER

Measures the Pressure of gas inside a container

$$P - P_o = \rho g h$$



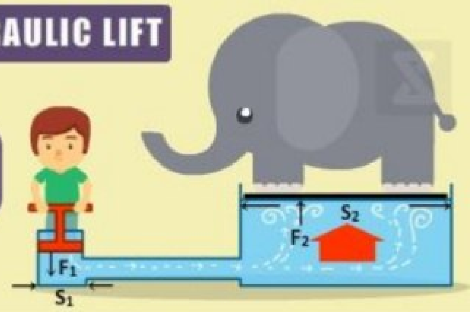
5 PASCAL'S LAW

The pressure applied at one point in an enclosed fluid is transmitted uniformly to every part of the fluid and to the walls of the container.

$$\frac{F_1}{S_1} = \frac{F_2}{S_2}$$

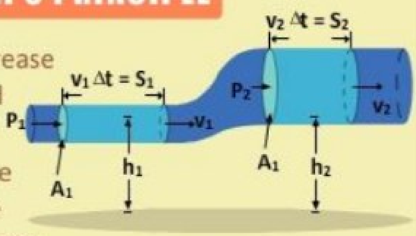
6 HYDRAULIC LIFT

$$P_2 = P_1$$
$$F_2 = P_1 S_2$$



7 BERNOULLI'S PRINCIPLE

A simultaneous increase in the speed of fluid occurs with a decrease in pressure or a decrease in the fluid's potential energy.



$$P + \rho g h + \frac{1}{2}\rho v^2 = \text{constant}$$

8 EQUATION OF CONTINUITY

In steady flow, the mass of fluid entering per second at one end is equal to the mass of fluid leaving per second at the other end

$$A_1 v_1 = A_2 v_2 = \text{Constant}$$

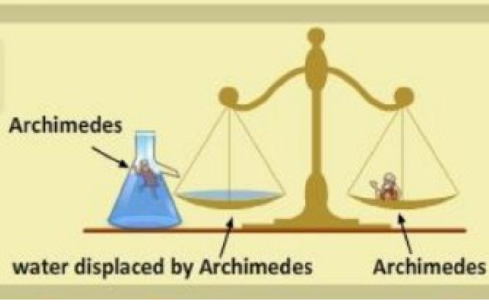
Meaning that in steady flow the product of cross-section and the speed of fluid remains constant everywhere .

9 ARCHIMEDE'S PRINICIPLE

A body totally or partially submerged in a fluid is subjected to an upward force equal in magnitude to the weight of fluid it displaces.

$$F_2 = V_i \rho_L g$$

V_i : submerged volume of solid



RADIATION CONDUCTION

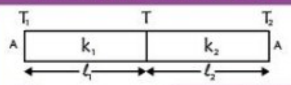
Law of Heat Transfer

The rate at which heat is transfered or conducted through a substance is directly proportional to the

(i) Area of the surface (A) perpendicular to the flow of heat.

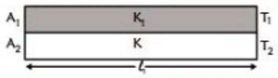
(ii) Temperature gradient $\frac{\Delta T}{x}$ along the path of heat transfer.

Slabs in Parallel and Series



$\frac{dQ}{dt}$ = constant $\quad k_{eq} = \frac{l_1 + l_2}{\frac{l_1}{K_1} + \frac{l_2}{K_2}}$

T = veries

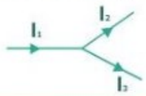


$\frac{dQ}{dt}$ = different $\quad K_{eq} = \frac{K_1A_1 + K_2A_2}{A_1 + A_2}$

T = same

Junction Law

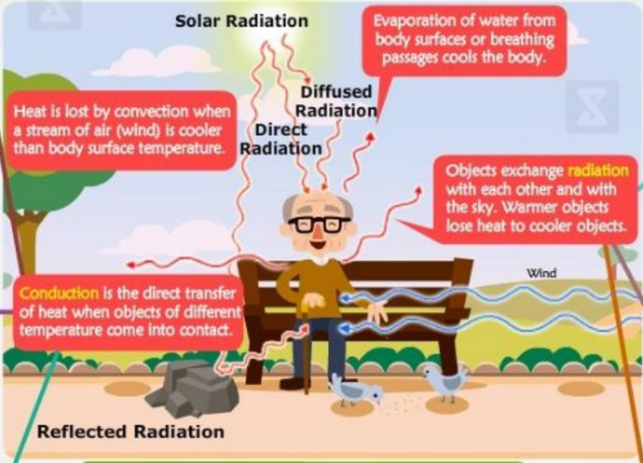
Rate of heat flow entering = Rate of heat flow exiting



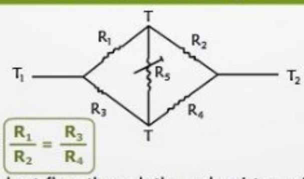
$\boxed{I_1 = I_2 + I_3}$

Kirchoff's Law

Emissive power of black body $= \frac{\text{Emissive power of body}}{\text{Absorptive power of body}}$ = Constant



Wheatstone Ridge



if $\frac{R_1}{R_2} = \frac{R_3}{R_4}$

$\Rightarrow$No heat flow through thermal resistance (R_5)

Stefan's Law

(i) Emissive power of a black body is proportional to fourth power of Absolute temperature.

$$\boxed{E = \sigma T^4}$$

σ = Stefan- Boltzmann Constant

(ii) Emissive power of body due to heat transfer from body to surrounding.

$$\boxed{E = e\sigma(T^4 - T_s^4)}$$

e = Emissivity

Newton's Law of Cooling

For small temperature difference, rate of cooling due to radiation is proportional to temperature difference.

$$\boxed{\frac{-dT}{dt} \propto \Delta T}$$

Wein's Displacement Law

Wavelength corresponding to maximum intensity of emission decreases with increase in temperature of black body.

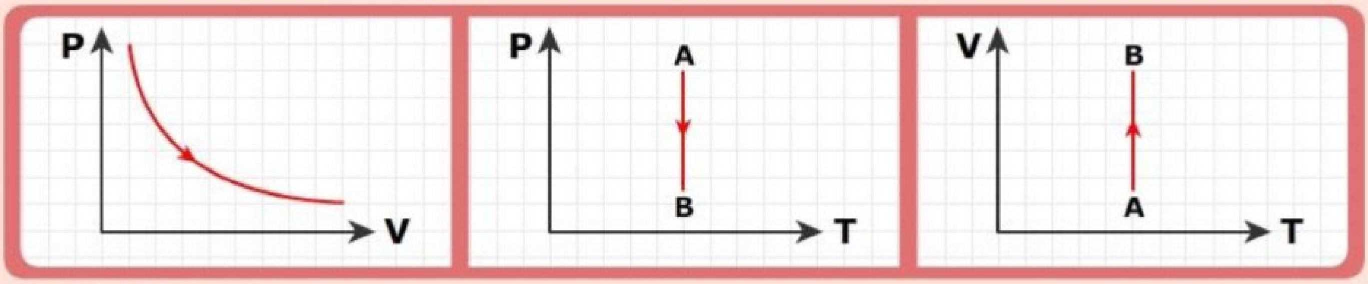
$$\boxed{\lambda_m \propto \frac{1}{T} \text{ or } \lambda_m T = \text{Constant}}$$

THE GAS LAWS

BOYLE'S LAW

According to this law, for a given mass of a gas, the volume of a gas at constant temperature (called **isothermal** process) is inversely proportional to its pressure, that is

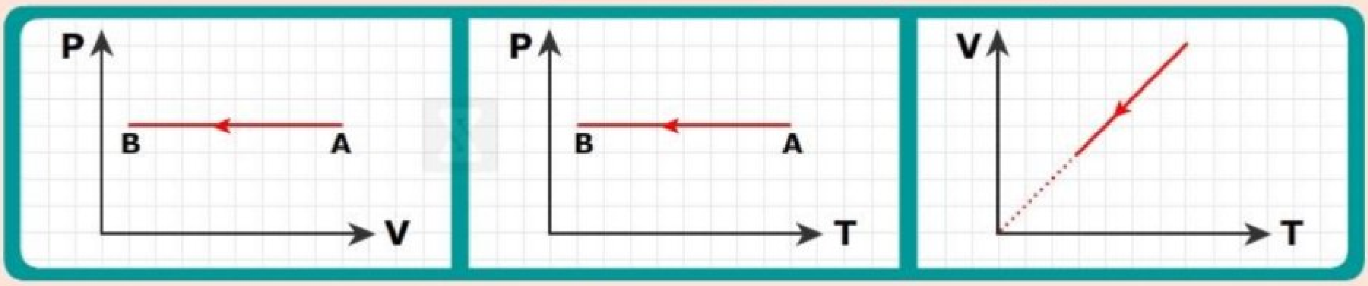
$$V \propto \frac{1}{P} \Longrightarrow PV = \text{Constant} \Longrightarrow P_i V_i = P_f V_f$$



CHARLE'S LAW

According to this law, for a given mass of a gas, the volume of a gas at constant pressure (called **isobaric** process) is directly proportional to its absolute temperature, that is

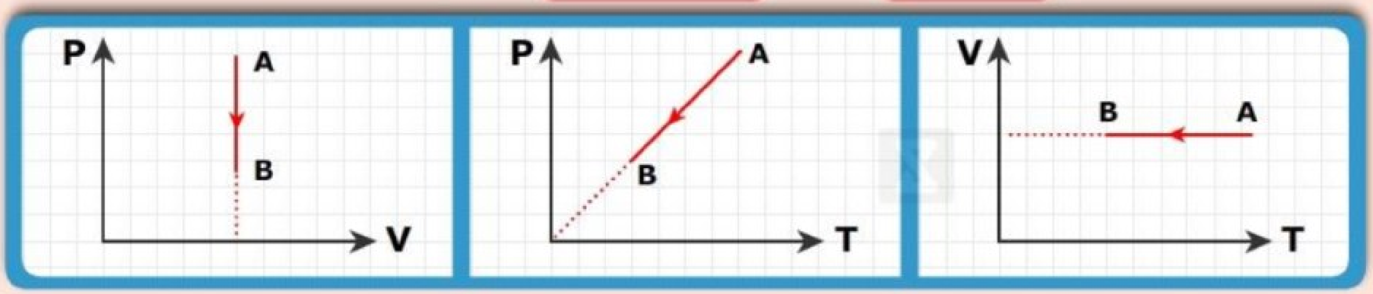
$$V \propto T \Longrightarrow \frac{V}{T} = \text{Constant} \Longrightarrow \frac{V_i}{T_i} = \frac{V_f}{T_f}$$



GAY LUSSAC'S LAW OR PRESSURE LAW

According to this law, for a given mass of a gas, the pressure of a gas at constant volume (called **isochoric** process) is directly proportional to its absolute temperature, that is

$$P \propto T \Longrightarrow \frac{P}{T} = \text{Constant} \Longrightarrow \frac{P_i}{T_i} = \frac{P_f}{T_f}$$



SIMPLE HARMONIC MOTION

Time Period

(i) Simple pendulum: $T = 2\pi\sqrt{\frac{l}{g}}$

(ii) Physical pendulum: $T = 2\pi\sqrt{\frac{I}{mgl}}$

(iii) Torsional pendulum: $T = 2\pi\sqrt{\frac{I}{c}}$

Equation of SHM

(i) Linear : $a = -\omega^2 x$

(ii) Angular : $\alpha = -\omega^2\theta$

Mass Spring system

(i) $T = 2\pi\sqrt{\frac{m}{k}}$

(ii) Two bodies system $T = 2\pi\sqrt{\frac{\mu}{K}}$

Where $(\mu) = \frac{m_1 m_2}{m_1 + m_2}$

Combination of Springs

(i) Series : $\frac{1}{k_{eff}} = \frac{1}{k_1} + \frac{1}{k_2}$

(ii) Parallel : $k_{eff} = k_1 + k_2$

(iii) Spring cut into two parts in ratio m:n

$k_1 \quad k_2$

$m \quad n$

$k_1 = \frac{(m+n)k}{m}$, $k_2 = \frac{(m+n)k}{n}$

SHM

Composition of 2 SHM

$x_1 = A_1 \sin \omega t$

$x_2 = A_2 \sin (\omega t + \phi)$

$x = A \sin (\omega t + \delta)$ Where

$A = \sqrt{A_1^2 + A_2^2 + 2A_1A_2\cos\phi}$

and $\tan\delta = \frac{A_2\sin\phi}{A_1 + A_2\cos\phi}$

Linear SHM

(i) Displacement : $x = A \sin (\omega t + \phi)$

(ii) Velocity : $\frac{dx}{dt} = A\omega \cos(\omega t + \phi)$

$= \omega\sqrt{A^2 - x^2}$

(iii) Acceleration : $\frac{d^2x}{dt^2} = -A\omega^2 \sin(\omega t + \phi)$

$= -\omega^2 x$

(iv) Phase : $\omega t + \phi$

(v) Phase Constant : ϕ

Energy in SHM

(i) K.E. $= \frac{1}{2} m\omega^2(A^2 - x^2)$

(ii) $U = \frac{1}{2} m\omega^2 x^2$

(iii) $E = K + U = \frac{1}{2} m\omega^2 A^2$

$=$ Constant.

Angular SHM

(i) Displacement : $\theta = \theta_o \sin (\omega t + \phi)$

(ii) Angular Velocity : $\frac{d\theta}{dt} = \theta_o \omega \cos(\omega t + \phi)$

(iii) Acceleration : $\frac{d^2\theta}{dt^2} = -\theta_o \omega^2 \sin(\omega t + \phi)$

$= -\omega^2\theta$

(iv) Phase : $\omega t + \phi$

(v) Phase Constant : ϕ

Part I

WAVE

Wave is distributed energy or distributed "disturbance".

MECHANICAL WAVES

Mechanical waves orginate from a disturbance in the medium (such as a stone dropping in a pond) and the disturbance propagates through the medium.

Mechnical waves are further classified in two categories such that:

1. Transverse waves (waves on a string)

If the disturbance travels in the x direction but the particles move in a direction, perpendicular to the x axis as the wave passes, it is called transverse waves.

2. Longitudinal waves (sound waves)

Longitudinal waves are characterized by the direction of vibration (disturbance) and wave motion. They are along the same direction.

NON-MECHANICAL WAVES

These are electromagnetic waves. The motion of the electromagnetic waves in a medium depends on the electromagnetic properties of the medium.

PARTICLE VELOCITY AND ACCELERATION

$$V_P = \frac{\partial}{\partial t}\, y(x, t) = \frac{\partial}{\partial t}\, A\sin(kx - \omega t) = -\omega A\cos(kx - \omega t)$$

$$a_p = \frac{\partial}{\partial t}\, V_P = \frac{\partial}{\partial t}\{-\omega A\cos(kx - \omega t)\} = -\omega^2 A\sin(kx - \omega t) = -\omega^2 y$$

ENERGY CALCULATION IN WAVES

Part II

1. KINETIC ENERGY PER UNIT LENGTH

The velocity of string element in transverse direction is greatest at one mean position and zero at the extreme positions of waveform.

$$K_L = \frac{dK}{dx} = \frac{1}{2}\mu\omega^2 A^2 \cos^2(kx-\omega t)$$

- **RATE OF TRANSMISSION OF KINETIC ENERGY**

$$\frac{dK}{dx} = \frac{1}{2}\mu v\omega^2 A^2 \cos^2(kx-\omega t)$$

2. ELASTIC POTENTIAL ENERGY

The Elastic potential energy of the string element results as string element is stretched during its oscillation.

- **POTENTIAL ENERGY PER UNIT LENGTH**

$$\frac{dU}{dx} = \frac{1}{2}\mu\omega^2 A^2 \cos^2(kx-\omega t)$$

- **RATE OF TRANSMISSION OF ELASTIC POTENTIAL ENERGY**

$$\frac{dU}{dt}\Big|_{avg} = \frac{1}{2}\times\frac{1}{2}\mu v\omega^2 A^2 \frac{1}{4}\mu v\omega^2 A^2$$

3. MECHANICAL ENERGY PER UNIT LENGTH

$$E_L = \frac{dE}{dx} = 2x\frac{1}{2}\mu\omega^2 A^2 \cos^2(kx-\omega t) = \mu\omega^2 A^2 \cos^2(kx-\omega t)$$

4. AVERAGE POWER TRANSMITTED

The average power transmitted by wave is equal to time rate of transmission of mechanical energy over integral wavelengths.

$$P_{avg} = \frac{1}{2}\rho s v\omega^2 A^2$$

5. ENERGY DENSITY

$$u = \frac{1}{2}\rho v w^2 A^2$$

6. INTENSITY

Intensity of wave (I) is defined as power transmitted per unit cross section area of the medium.

$$I = \rho s v\omega^2 \frac{A^2}{2s} = \frac{1}{2}\rho v w^2 A^2$$

PHASE DIFFERENCE BETWEEN TWO PARTICLES IN THE SAME WAVE:

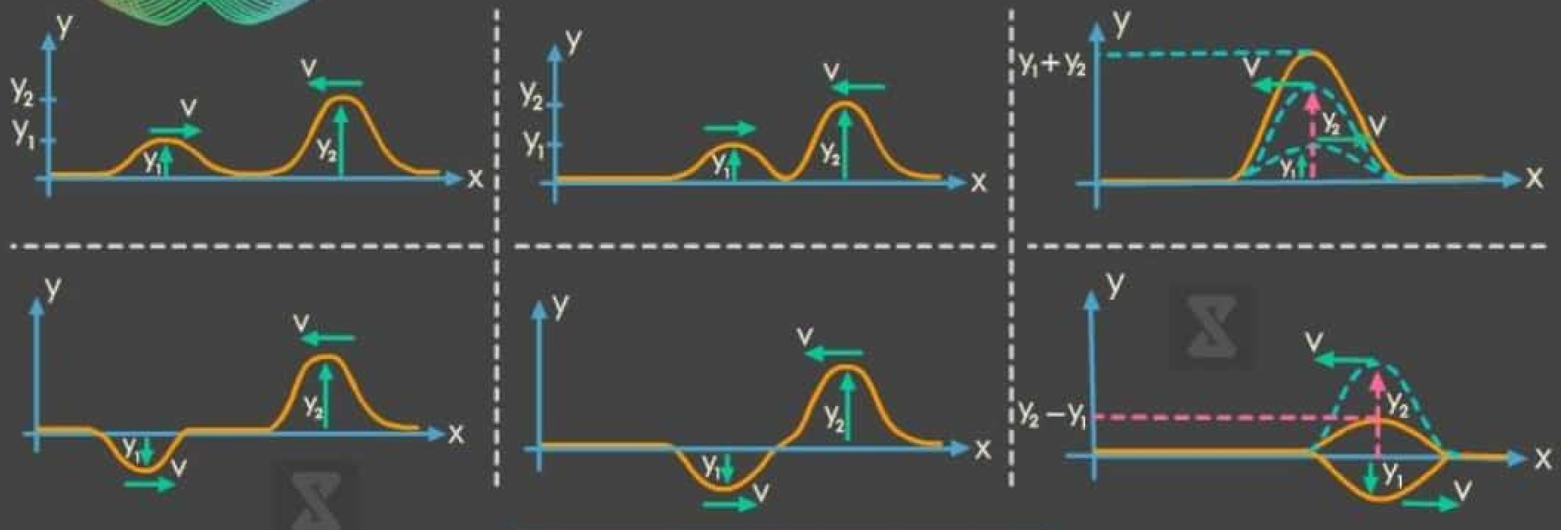
$$\Delta x \Rightarrow \frac{\Delta\phi}{k}$$

SUPERPOSITION AND STANDING WAVES

Part I

PRINCIPLE OF SUPERPOSITION

When two or more waves superpose on a medium particle then the resultant displacement of that medium particle is given by the vector sum of the individual displacements produced by the component waves at that medium particle independently.



INTERFERENCE OF WAVES

- If the two waves are exactly in same phase, that is the shape of one wave exactly fits on to the other wave then they combine to double the displacement of every medium particle **as shown in figure (a)**. This phenomenon is called as constructive interference.

- If the superposing waves are exactly out of phase or in opposite phase then they combine to cancel all the displacements at every medium particle and medium remains in the form of a straight line **as shown in figure (b).** This phenomenon is called as destructive interference.

Figure (a) ⟶ Constructive Interference

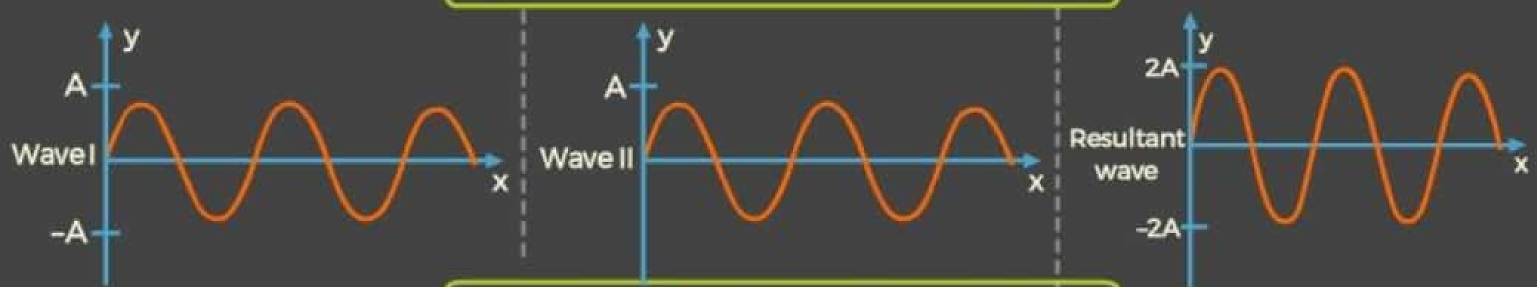
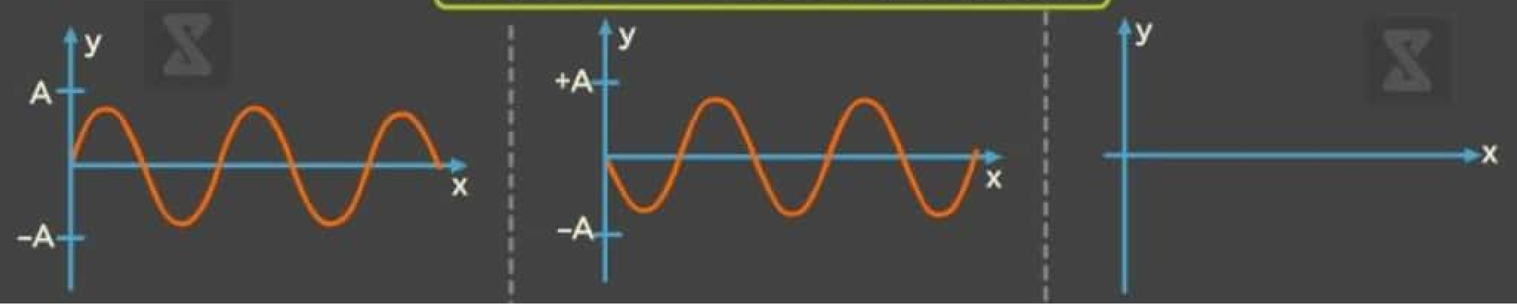


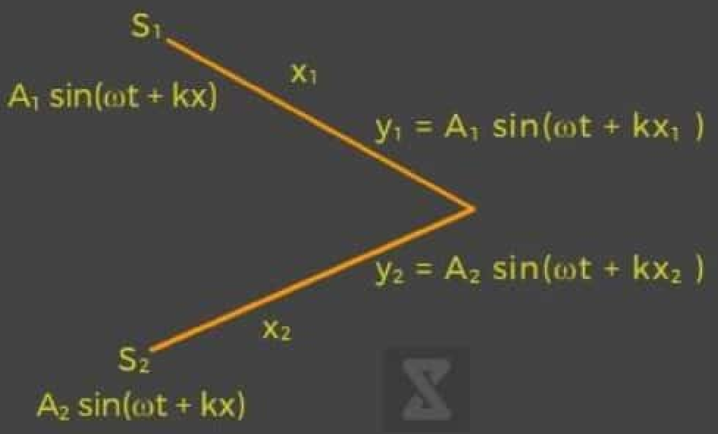
Figure (b) ⟶ Destructive Interference



SUPERPOSITION AND STANDING WAVES

Part II

ANALYTICAL TREATMENT OF INTERFERENCE OF WAVES



Whenever two or more than two waves superimpose each other, they give sum of their individual displacement.

$$y_1 = A_1 \sin(\omega t + kx_1)$$

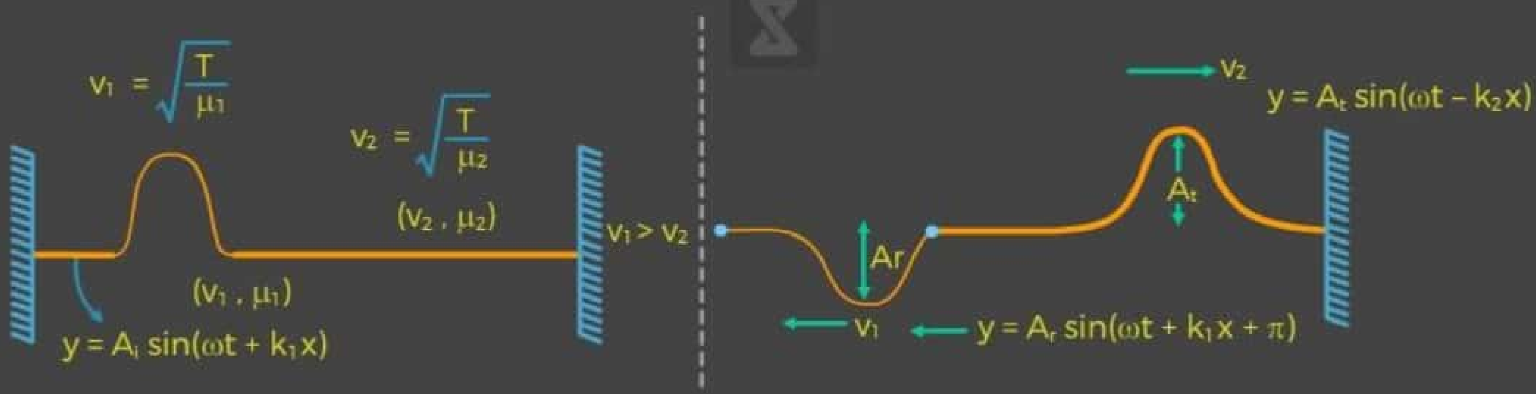
$$y_2 = A_2 \sin(\omega t + kx_2)$$

Due to superposition

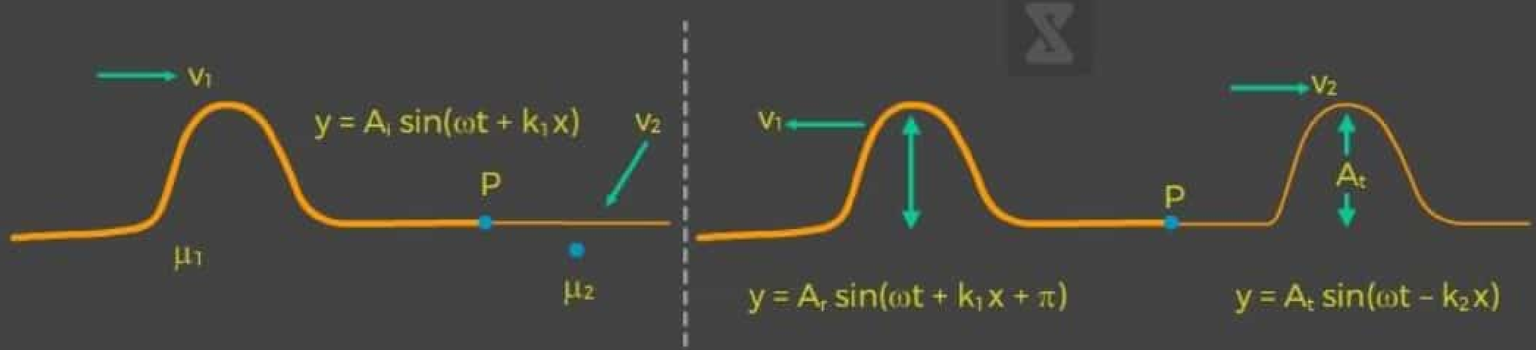
$$y_{net} = y_1 + y_2$$

REFLECTION AND TRANSMISSION BETWEEN TWO STRING

If a wave pulse is produced on a lighter string moving towards the friction, a part of the wave is reflected and a part is transmitted on the heavier string. The reflected wave is inverted with respect to the original one.



On the other hand if the wave is produced on the heavier string which moves toward the junction, a part will be reflected and a part transmitted, no inversion in waves shape will take place.



SUPERPOSITION AND STANDING WAVES

Part III

STANDING WAVES

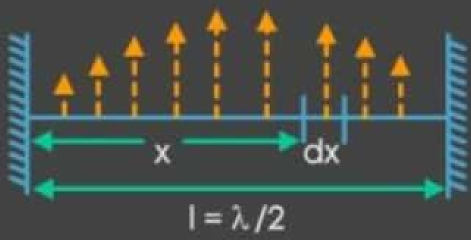
When two coherent waves travelling in opposite directions superpose then simultaneous interference of all the medium particles takes place. These waves interfere to produce a pattern of all the medium particles is what we call, a stationary wave.

ENERGY OF STANDING WAVE IN ONE LOOP

When all the particles of one loop are at extreme position then total energy in the loop is in the form of potential energy only. When the particles reaches its mean position then total potential energy converts into kinetic energy of the particles, so we can say that total energy of the loop remains constant.

Total kinetic energy at mean position is equal to total energy of the loop because potential energy at mean position is zero.

$$\text{Total K.E} = \frac{1}{2}\lambda A^2 \omega^2 \mu$$



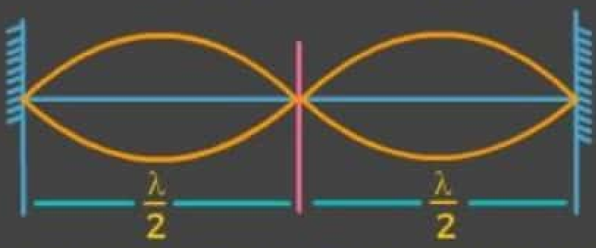
STATIONARY WAVES IN STRINGS

WHEN BOTH ENDS OF A STRING ARE FIXED

Fundamental Mode

The string vibrates in one loop in which the ends are the nodes and the centre is the antinode. This mode of vibration is known as the fundamental mode and frequency of vibration is known as the fundamental frequency or first harmonic.

$$f_1 = \frac{V}{2L}$$



First Overtone

The frequency f_2 is known as second harmonic or first overtone.

$$f_2 = \frac{V}{L}$$

Second Overtone

The frequency f_3 is known as third harmonic or second overtone.

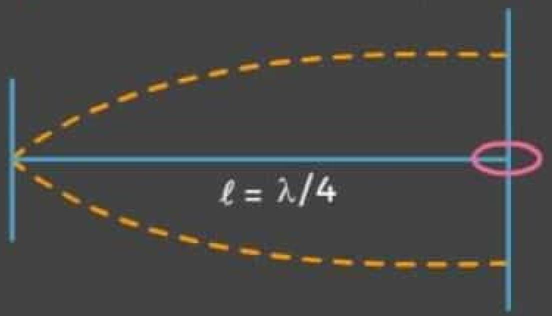
$$f_3 = \frac{3v}{2L}$$

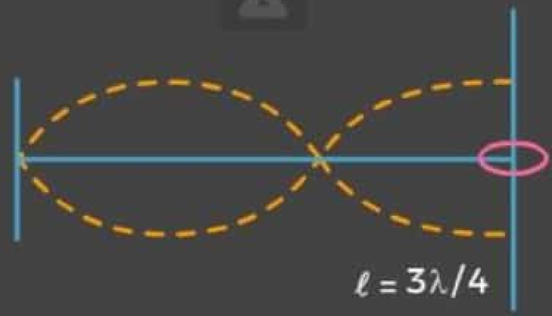
SUPERPOSITION AND STANDING WAVES

Part IV

- **When one end of the string is fixed and other is free**

Note- Free end acts as antinode





$f = \frac{1}{4\ell}\sqrt{\frac{T}{\mu}}$ fundamental or Ist harmonic

$f = \frac{3}{4\ell}\sqrt{\frac{T}{\mu}}$ IIIrd harmonic or Ist overtone

In general : $f = \frac{(2n+1)}{4\ell}\sqrt{\frac{T}{\mu}}$ ((2n+1)thharmonic, nth overtone)

S.No.	Travelling waves	Stationary waves
1.	These waves advance in a medium with a definite velocity	These waves remain stationary between two boundaries in the medium.
2.	In these waves, all particles of the medium oscillate with same frequency and amplitude.	In these waves, all particles except nodes oscillate with same frequency but different amplitudes. Amplitude is zero at nodes and maximum at antinodes.
3.	At any instant, phase of vibration varies continuosly from one particle to the other i.e. phase difference between two particles can have any value between 0 and $2x$	At any instant, the phase of all particles between two successive nodes is the same, but phase of particles on one side of a node is opposite to the phase of particles on the other side of the node, i.e, phase difference between any two particles can be either 0 or π
4.	In these waves, at no instant all the particles of the medium pass through their mean positions simultaneously.	In these waves, all particles of the medium pass through their mean position simultaneously twice in each time period.
5.	These waves transmit energy in the medium.	These waves do not transmit energy in the medium.

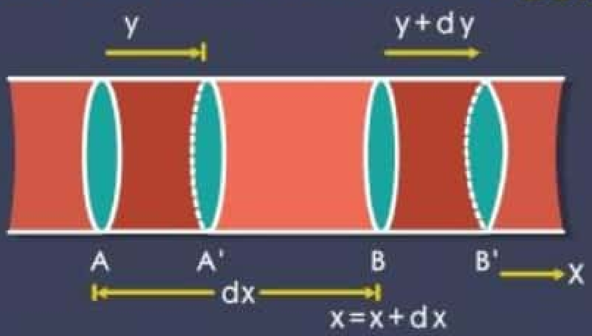
Part I

SOUND WAVE

PROPAGATION OF SOUND WAVES

Sound waves propagate in any medium through a series of periodic compressions and rarefactions of pressure, which is produced by the vibrating source.

COMPRESSION WAVES



When a longitudinal wave is propagated in a gaseous medium, it produces compression and rarefaction in the medium periodically.

Velocity and Acceleration of particle :

General equation of wave is given by

$$y = A\sin(\omega t - kx)$$

$$V_P = \frac{\partial y}{\partial t} = A\,\omega\cos(\omega t - kx)$$

VELOCITY OF SOUND/LONGITUDINAL WAVES IN SOLIDS

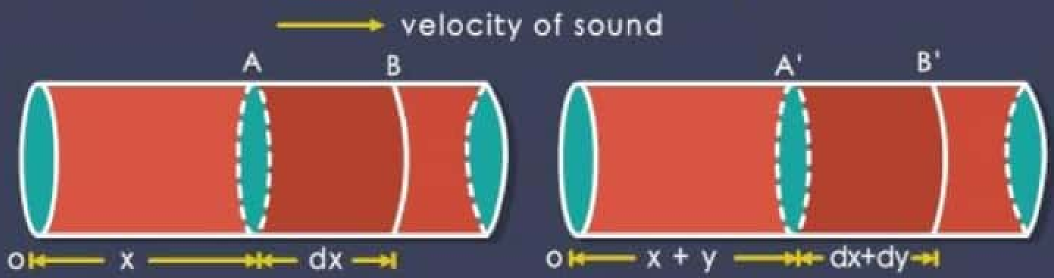
- **In Solid** $v = \sqrt{\frac{Y}{\rho}}$

  Y = Young Modulus

- **In Fluid** $v = \sqrt{\frac{B}{\rho}}$

  B = Bulk Modulus

- **In Gas** $B = -V\frac{dP}{dV}$



Newton's Formula for velocity of Sound in Gases, $v = \sqrt{\frac{P}{\rho}}$

Laplace Correction, $v = \sqrt{\frac{B}{\rho}} = \sqrt{\frac{\gamma P}{\rho}}$

Effect of Temperature on Velocity of Sound, $\frac{V_1}{V_2} = \sqrt{\frac{T_1}{T_2}}$

Where,
P = Pressure
ρ = Density
V = Volume
T = Temperature

Part II

LONGITUDINAL STANDING WAVES

Two longitudinal waves of same frequency and amplitude, travelling in opposite directions interfere to produce a standing wave.

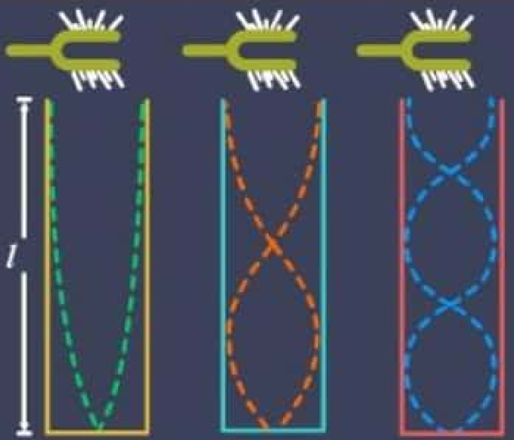
If the two interfering waves are given by:

$$p_1 = p_0 \sin(\omega t - kx) \text{ and } p_2 = p_0 \sin(\omega t + kx + \phi)$$

$$p = p_0 \sin\left(\omega t + \frac{\phi}{2}\right)$$

WAVES IN A VIBRATING AIR COLUMN

Vibration of Air in a Closed Organ Pipe



Fundamental frequency of oscillations of closed organ pipe of length l is given as

$$n_1 = \frac{v}{\lambda} = \frac{v}{4l}$$

- n_1 → Fundamental Frequency
- v → Velocity
- λ → Wavelength
- l → Length of organ pipe

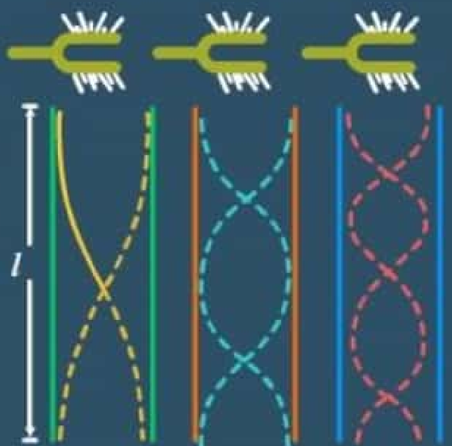
Vibration of Air in Open Organ Pipe

$$\lambda = 2l$$

The fundamental frequency of organ pipe can be given as

$$n_1 = \frac{v}{\lambda} = \frac{v}{2l}$$

$$f = \frac{nv}{2\ell}$$



End Correction

The displacment antinode at an open end of an organ pipe lies slightly outside the open end. The distance of the antinode from the open end is called end correction and its value is given by

$$e = 0.6\ r$$

where r = radius of the organ pipe, and

$$f_{closed} = \frac{v}{4(\ell + 0.6r)} \qquad f_{open} = \frac{v}{2(\ell + 1.2r)}$$

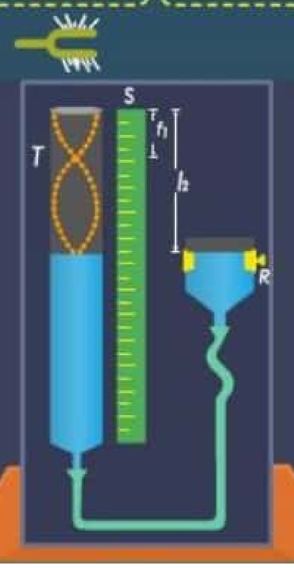
Resonance Tube

This is an apparatus used to determine the velocity of sound in air experimentally and also to compare frequencies of two tuning forks.

$$\lambda = 2(l_2 - l_1)$$

Thus, sound velocity in air can be given as

$$v = n_0 \lambda = 2n_0(l_2 - l_1)$$

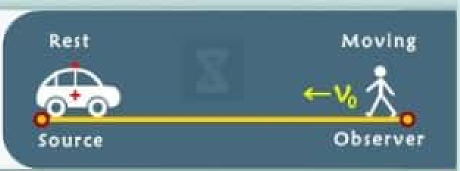


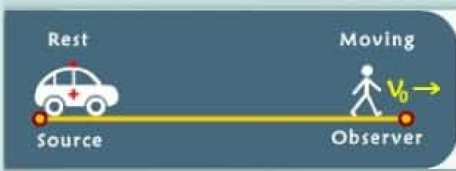
DOPPLER EFFECT

DOPPLER EFFECT: The shift in frequency of a wave emitted by a source moving relative to an observer as perceived by the observer: the shift is to higher frequencies when the source approaches and to lower frequencies when it recedes.

v_s = Speed of source   f = Original frequency   f' = Apparent frequency v_0 = Speed of observer   v = Speed of sound in air

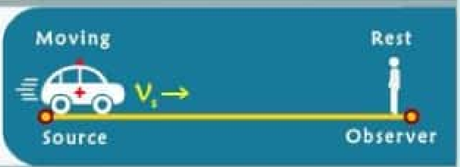
$$f' = \left(\frac{v + v_0}{v}\right) f$$

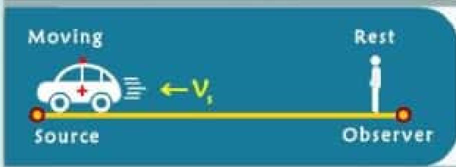




$$f' = \left(\frac{v - v_0}{v}\right) f$$

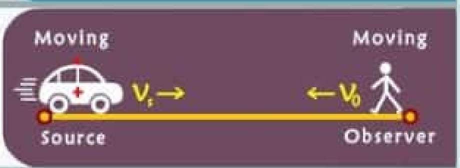
$$f' = \left(\frac{v}{v - v_s}\right) f$$

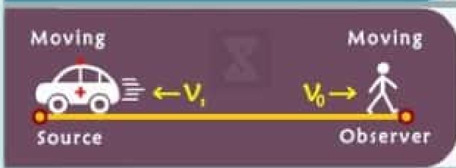




$$f' = \left(\frac{v}{v + v_s}\right) f$$

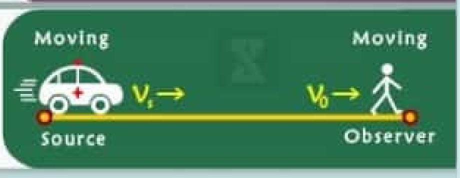
$$f' = \left(\frac{v + v_0}{v - v_s}\right) f$$

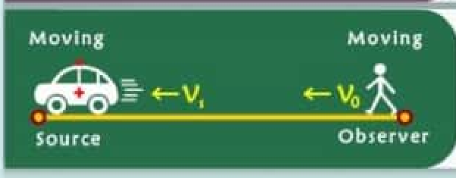




$$f' = \left(\frac{v - v_0}{v + v_s}\right) f$$

$$f' = \left(\frac{v - v_0}{v - v_s}\right) f$$





$$f' = \left(\frac{v + v_0}{v + v_s}\right) f$$

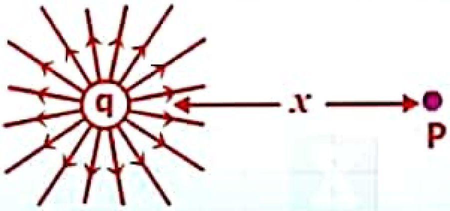
Shortcut Trick

Whenever source moves towards observer, then do substraction in denominator and vice-versa.

Whenever observer moves towards source, then do addition in numerator and vice-versa.

ELECTRIC FIELD

Electric Field due to Point Charge



$$E = \frac{kq}{x^2}$$

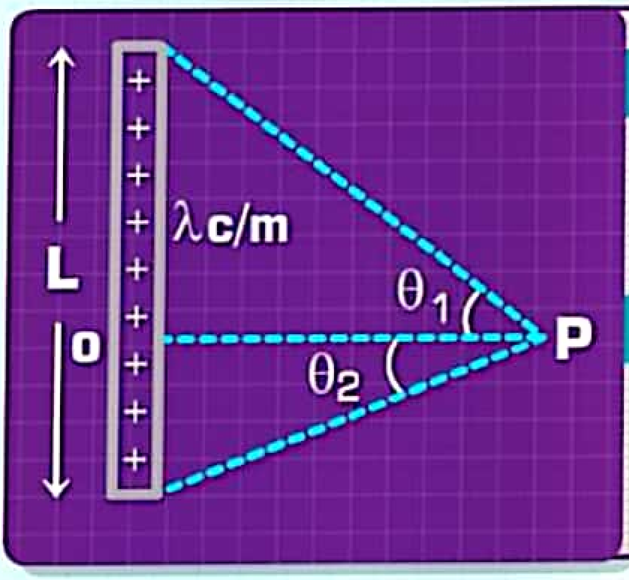
Vector Form $\vec{E} = \frac{kq}{x^3} \cdot \vec{x}$

$$k = \frac{1}{4\pi\varepsilon_0}$$

q = Charge ; x = Distance

If a charge q_0 is placed at a point in electric field, it experiences a net force $\vec{F}$ on it, then electric field strength at that point can be $\vec{E} = \frac{\vec{F}}{q_0}$

ELECTRIC FIELD DUE TO A UNIFORMLY CHARGED ROD



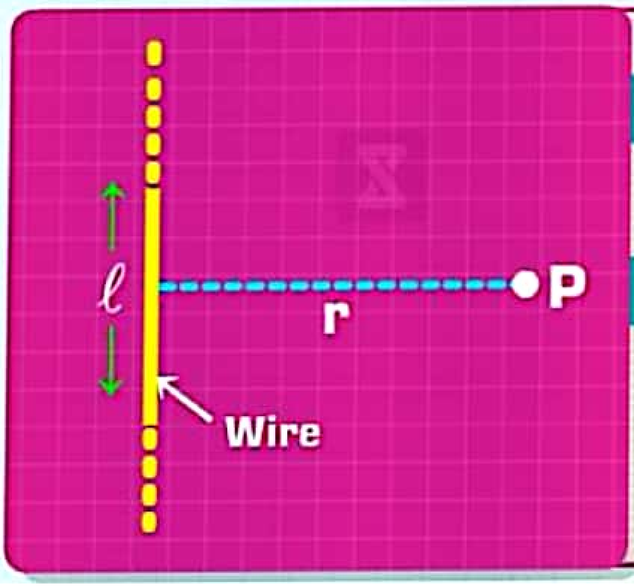
PARALLEL

$$E_{\parallel} = \frac{k\lambda}{r}(\cos\theta_2 - \cos\theta_1)$$

PERPENDICULAR

$$E_{\perp} = \frac{k\lambda}{r}(\sin\theta_2 - \sin\theta_1)$$

ELECTRIC FIELD DUE TO INFINITE WIRE (ℓ>>r)



Since $\ell >> r \Rightarrow \theta_1 = \theta_2 = 90^\circ$

PERPENDICULAR

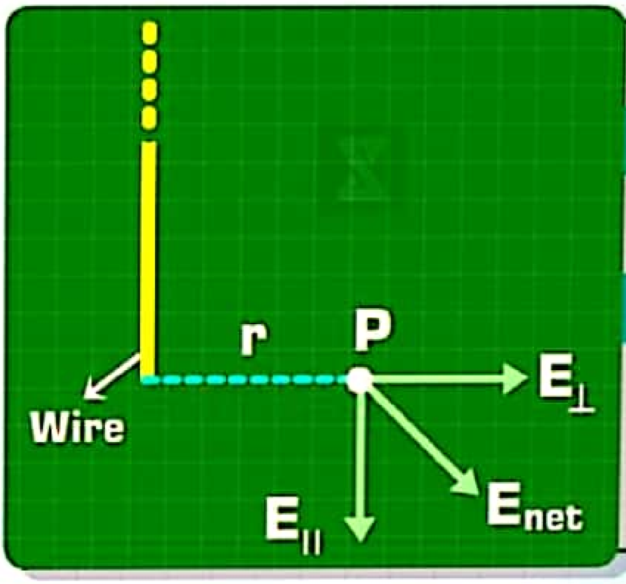
$$E_{\perp} = \frac{k\lambda}{r}(\sin 90^\circ + \sin 90^\circ) \Rightarrow \boxed{E_{\perp} = \frac{2k\lambda}{r}}$$

PARALLEL

$$E_{\parallel} = \frac{k\lambda}{r}(\cos 90^\circ - \cos 90^\circ) \Rightarrow \boxed{E_{\parallel} = 0}$$

At P, $E_{net} = E_{\perp} + E_{\parallel}$ $\quad E_{net} = \frac{2k\lambda}{r}$

ELECTRIC FIELD DUE TO SEMI INFINITE WIRE



$\theta_1 = 90^\circ$, $\theta_2 = 0^\circ$

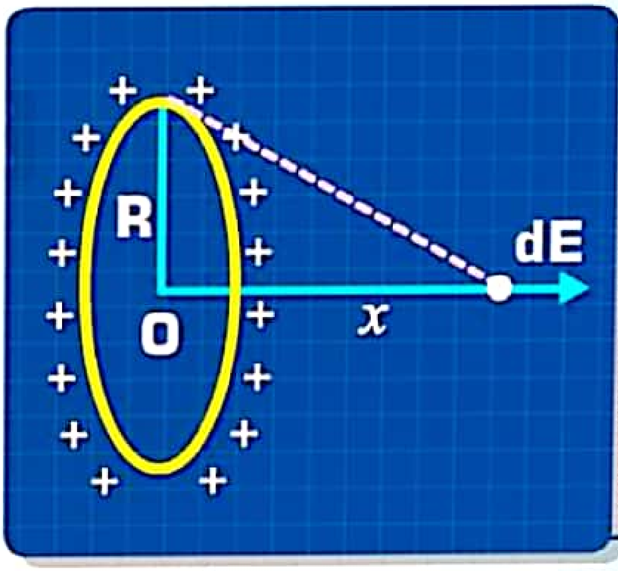
PERPENDICULAR

$$E_\perp = \frac{k\lambda}{r}(\sin 90^\circ + \sin 0^\circ) = \frac{k\lambda}{r}$$

PARALLEL

$$E_{\parallel} = \frac{k\lambda}{r}(\cos 0^\circ - \cos 90^\circ) = \frac{k\lambda}{r}$$

ELECTRIC FIELD DUE TO UNIFORMLY CHARGED RING

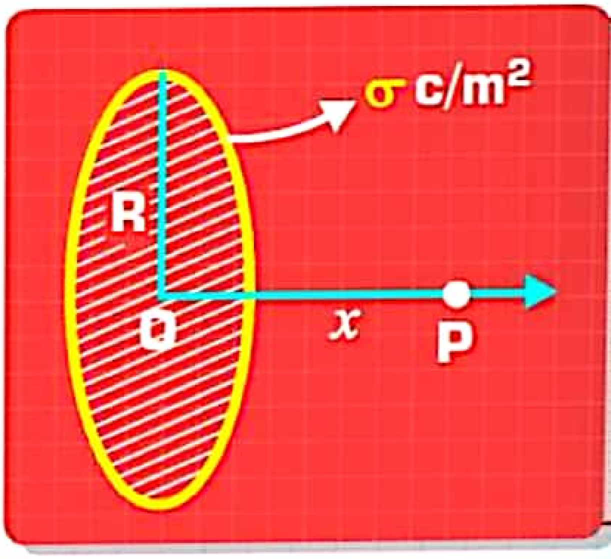


$$E = \frac{kQx}{(R^2 + x^2)^{3/2}}$$

For maxima, $x = \pm\frac{R}{\sqrt{2}}$

$$E_{max} = \pm \frac{2}{3\sqrt{3}} \frac{kQ}{R^2}$$

ELECTRIC FIELD ON THE AXIS OF DISC



$$E = \frac{\sigma}{2\varepsilon_0}\left[1 - \frac{x}{\sqrt{x^2 + R^2}}\right]$$ [along the axis]

If $x >> R$

$$E = 0$$

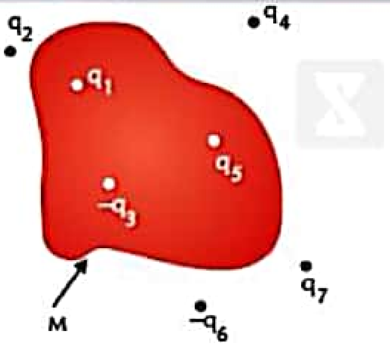
If $x << R$

$$E = \frac{\sigma}{2\varepsilon_0}(1 - 0) = \frac{\sigma}{2\varepsilon_0}$$

ELECTRIC FIELD STRENGTH

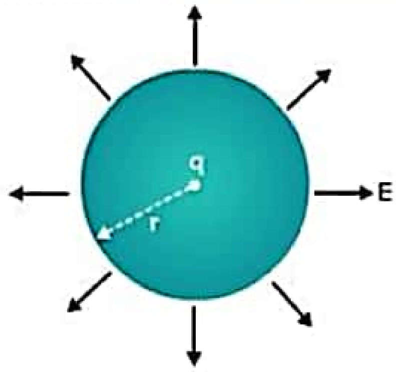
Part III

Gauss's Law



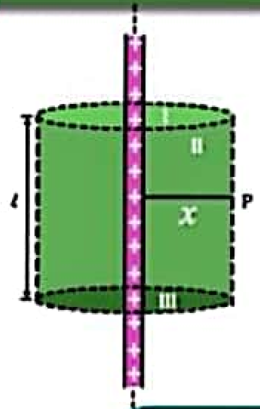
$$\oint_M \vec{E} \cdot d\vec{S} = \frac{q_1 + q_5 - q_3}{\varepsilon_0}$$

Electric Field due to a Point Charge



$$E = \frac{1}{4\pi\varepsilon_0} \cdot \frac{q}{r^2}$$

Electric Field Strength due to a Long Charged Wire

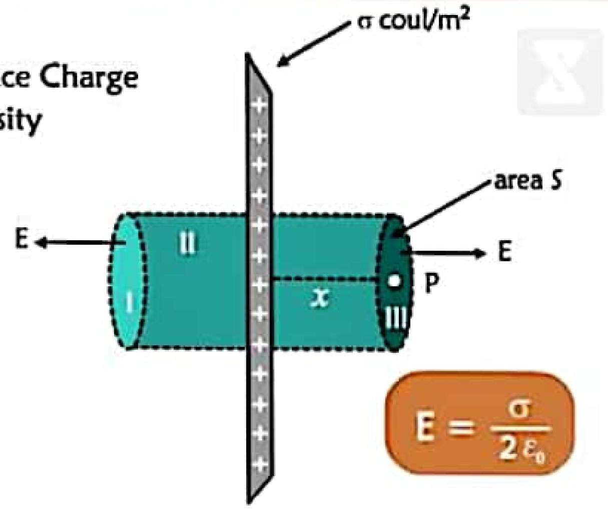


λ = Linear Charge Density

$$E = \frac{\lambda}{2\pi\varepsilon_0 x}$$

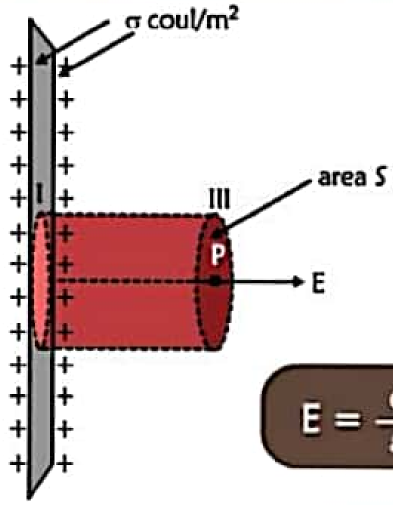
Electric Field Strength due to Non-Conducting Uniformly Charged Sheet

σ = Surface Charge Density



$$E = \frac{\sigma}{2\varepsilon_0}$$

Electric Field Strength due to Charged Conducting Sheet

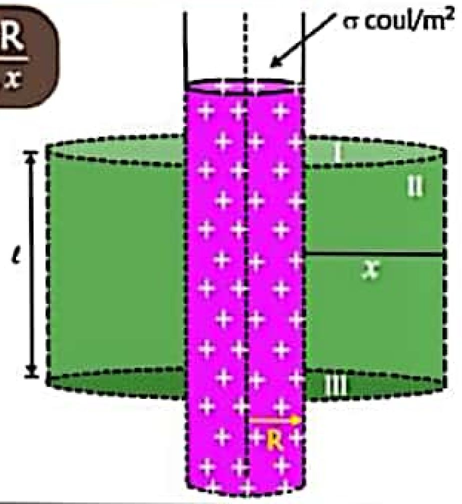


$$E = \frac{\sigma}{\varepsilon_0}$$

Electric Field Strength due to a Long Uniformly Charged Cylinder

Conducting Cylinder

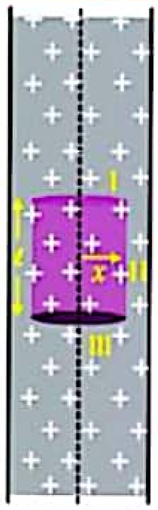
$$E = \frac{\sigma R}{\varepsilon_0 x}$$



Uniformly Charged Non – Conducting Cylinder

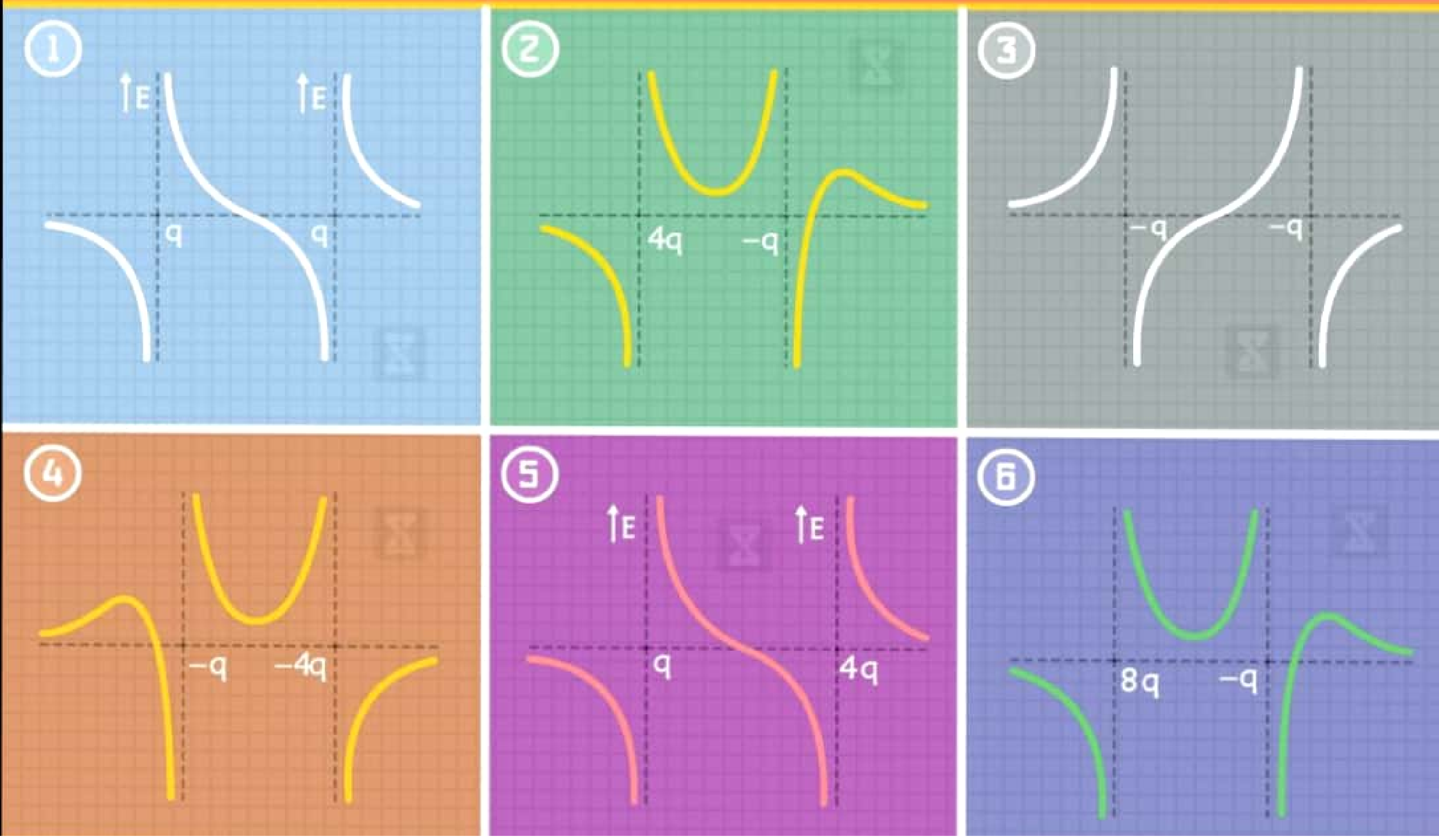
$$E_{inside} = \frac{\rho x}{2\varepsilon_0}$$

ρ = Volume Charge Density



GRAPH OF ELECTRIC FIELD DUE TO BINARY CHARGES

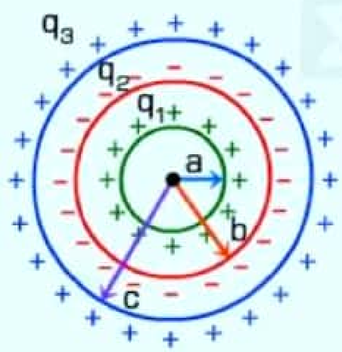
Part IV



ELECTRIC POTENTIAL

Part V

POTENTIAL DUE TO CONCENTRIC SPHERES



At a point $r > c$

$$V = \frac{1}{4\pi\varepsilon_0}\frac{q_1 - q_2 + q_3}{r}$$

At a point $a < r < b$

$$V = \frac{1}{4\pi\varepsilon_0}\frac{q_1}{r} - \frac{1}{4\pi\varepsilon_0}\frac{q_2}{b} + \frac{1}{4\pi\varepsilon_0}\frac{q_3}{c}$$

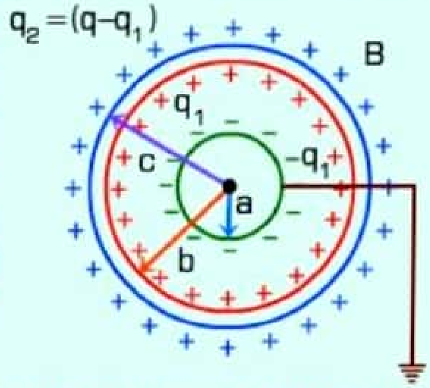
At a point $b < r < c$

$$V = \frac{1}{4\pi\varepsilon_0}\frac{q_1 - q_2}{r} + \frac{1}{4\pi\varepsilon_0}\frac{q_3}{c}$$

At a point $r < a$

$$V = \frac{1}{4\pi\varepsilon_0}\left[\frac{q_1}{a} - \frac{q_2}{b} + \frac{q_3}{c}\right]$$

DIFFERENCE BETWEEN TWO CONCENTRIC SPHERES WHEN ONE OF THEM IS EARTHED



$$V_{in} = \frac{1}{4\pi\varepsilon_0}\left[-\frac{q_1}{a} + \frac{q_2}{b}\right]$$

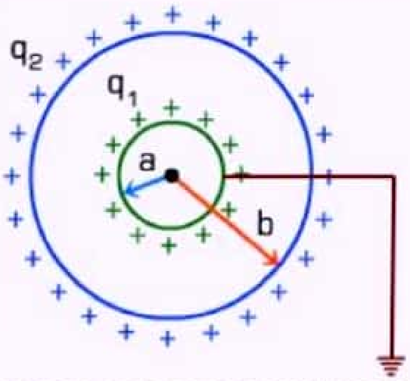
$$V_{out} = \frac{1}{4\pi\varepsilon_0}\left[-\frac{q_1}{b} + \frac{q_2}{b}\right]$$

$$\frac{q_2}{c} = q_1\left(\frac{1}{a} - \frac{1}{b}\right) \quad .....(i)$$

$$q_1 + q_2 = q \quad .....(ii)$$

Solving (i) and (ii) we can get q_1 and q_2

DIFFERENCE BETWEEN TWO CONCENTRIC UNIFORMLY CHARGED METALLIC SPHERES

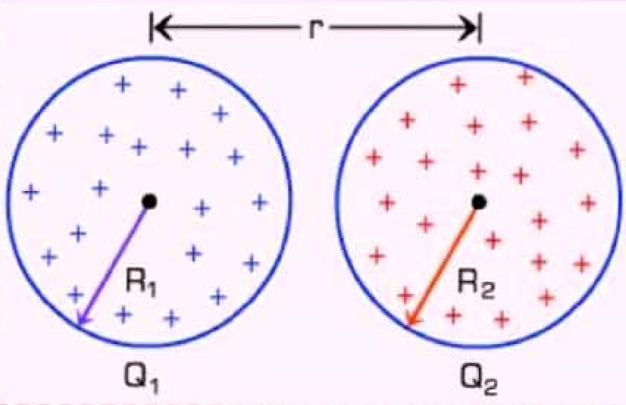


$$V_{in} = \frac{1}{4\pi\varepsilon_0}\frac{q_1}{a} + \frac{1}{4\pi\varepsilon_0}\frac{q_2}{b}$$

$$V_{out} = \frac{1}{4\pi\varepsilon_0}\frac{q_1}{b} + \frac{1}{4\pi\varepsilon_0}\frac{q_2}{b}$$

$$\Delta V = V_{in} - V_{out} \Rightarrow \Delta V = \frac{q_1}{4\pi\varepsilon_0}\left(\frac{1}{a} - \frac{1}{b}\right)$$

TOTAL ELECTROSTATIC ENERGY OF A SYSTEM OF CHARGES



$$U = U_{self} + U_{interaction}$$

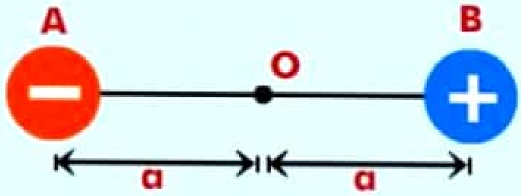
$$U = \frac{3KQ_1^2}{5R_1} + \frac{3KQ_2^2}{5R_2} + \frac{KQ_1Q_2}{r}$$

Part VI

ELECTRIC DIPOLE

ELECTRIC DIPOLE

$$\vec{p} = q.2\vec{a}$$



SI unit : Coulomb - meter

It is a vector quantity

Direction of dipole moments $(\vec{p})$ is from negative charge to positive charge

ELECTRIC FIELD ON AXIAL LINE OF AN ELECTRIC DIPOLE

$$E = \frac{1}{4\pi\varepsilon_0} \frac{2qa}{(r^2-a^2)^2}$$

For a<<r

$$\vec{E}_{axial} = \frac{1}{4\pi\varepsilon_0} \frac{2\vec{p}}{r^3}$$

E_{axial} is along the direction of dipole moment

ELECTRIC FIELD ON EQUATORIAL LINE OF AN ELECTRIC DIPOLE

$$E = -\frac{1}{4\pi\varepsilon_0} \cdot \frac{q.2a}{(r^2-a^2)^{3/2}}$$

For a<<r

$$\vec{E}_{equiatoral} = -\frac{1}{4\pi\varepsilon_0} \cdot \frac{\vec{p}}{r^3}$$

$E_{equatorial}$ is along the opposite direction of dipole moment

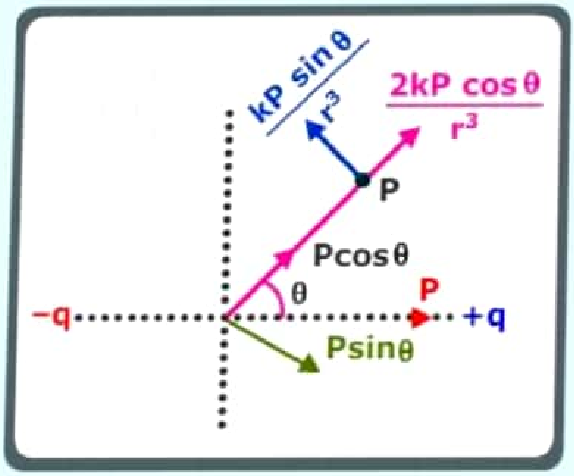
DIPOLE IN A UNIFORM EXTERNAL ELECTRIC FIELD

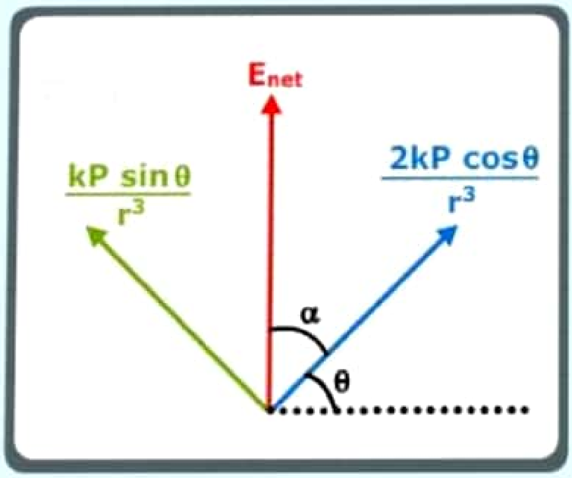
VECTOR FORM

$$\vec{\tau} = \vec{p} . \vec{E}$$

$$U = -pE\cos\theta$$

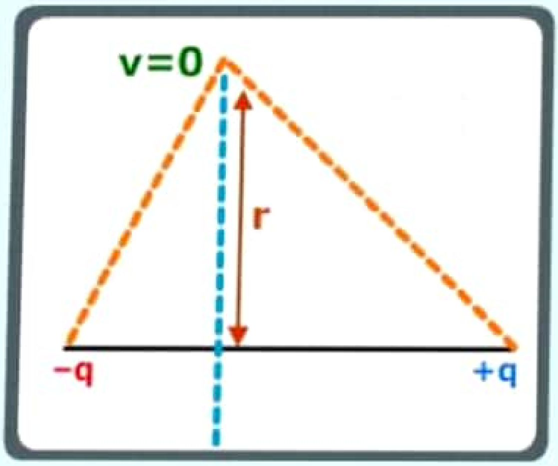
ELECTRIC FIELD AT A GENERAL POINT DUE TO A DIPOLE

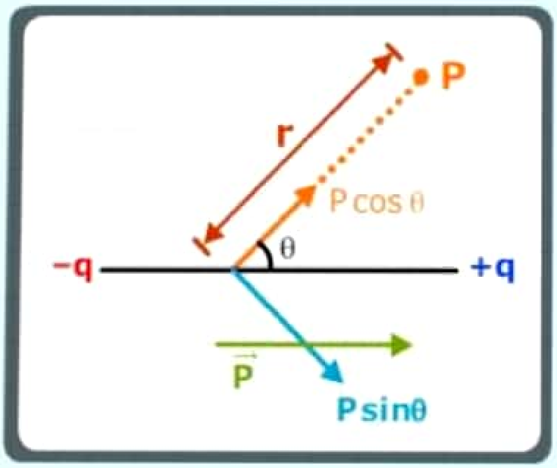




$$E_{net} = \frac{kP}{r^3}\sqrt{1+3\cos^2\theta}\ ,\ \tan\alpha = \frac{\tan\theta}{2}\ ;\ k = \frac{1}{4\pi\varepsilon_0}$$

ELECTRIC POTENTIAL DUE TO A DIPOLE





POTENTIAL AT 'P' DUE TO DIPOLE, $V_p = \frac{2kP\cos\theta}{r^2}$

AT AN AXIAL POINT, $V_{net} = \frac{kp}{r^2}$ (As $P = q.2a$)

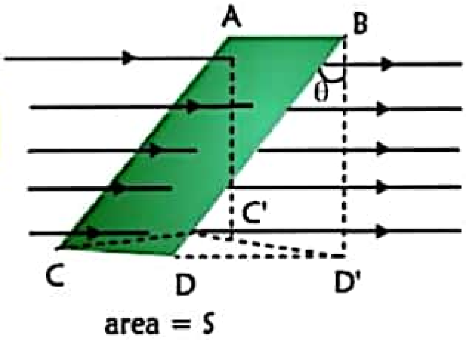
AT PERPENDICULAR BI-SECTOR, $V_{net} = 0$

ELECTRIC FLUX

Electric Field Strength in terms of Electric Flux

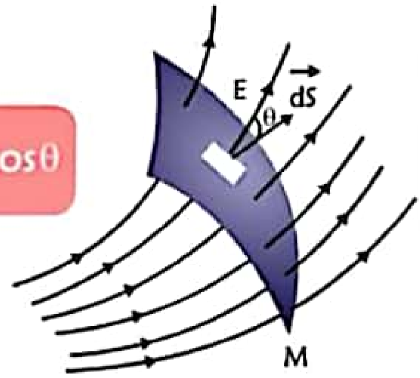
$\phi = \vec{E}.\vec{S}$

$\phi = E.S.\cos\theta$



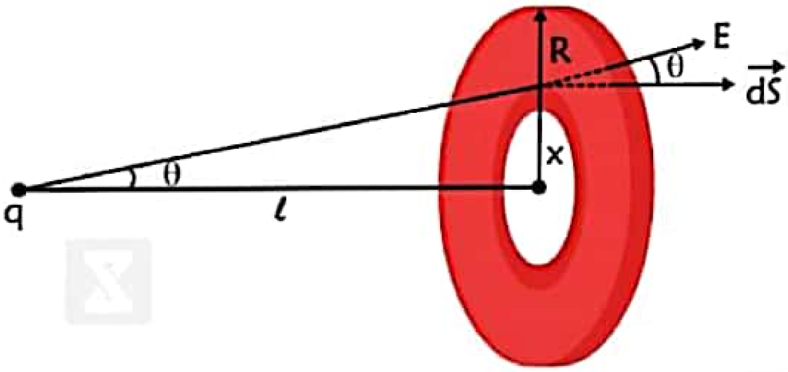
Electric Flux in Non-uniform Electric Field

$\phi = \int d\phi = \int_M EdS \cos\theta$



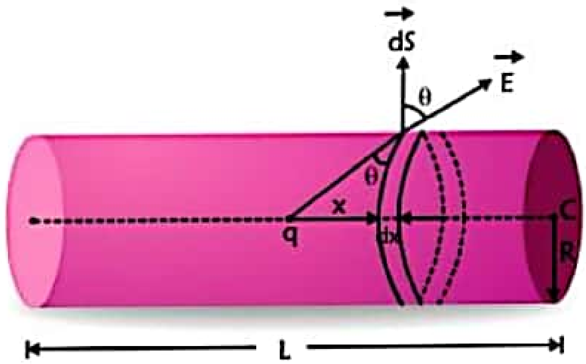
Electric Flux through a Circular Disc

$$\phi = \frac{q}{\epsilon_0}\left[1 - \frac{\ell}{\sqrt{R^2 + x^2}}\right]$$



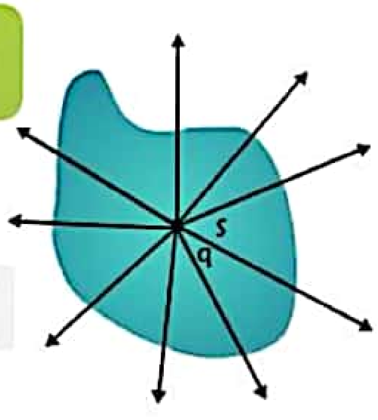
Electric Flux through the Lateral Surface of a Cylinder due to a Point Charge

$$\phi = \frac{q}{\epsilon_0} \cdot \frac{\ell}{\sqrt{R^2 + x^2}}$$



Electric Flux produced by a Point Charge

$$\phi_s = \frac{q}{\epsilon_0}$$

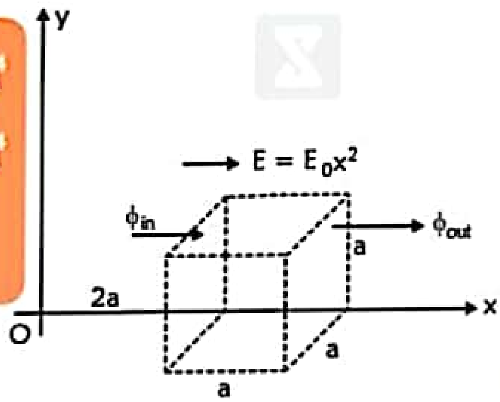


Flux Calculation in the Region of Varying Electric Field

$\phi_{in} = E_0\,(2a)^2 . a^2 = 4E_0a^4$

$\phi_{out} = E_0\,(3a)^2 . a^2 = 9E_0a^4$

$\phi_{net} = 5E_0a^4$



CAPACITOR

1 Capacitor

Capacitor is a passive device of the circuit which stores electrical energy or charge. It is also known as **condenser**.

$$C = \frac{Q}{V} \quad \text{or} \quad C = \frac{\epsilon_0 A}{d}$$

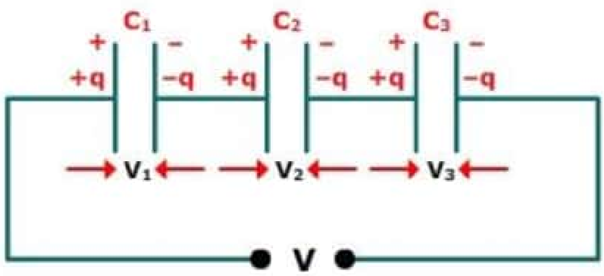
Capacitance is measured in **Farad (F)**

Q = Charge **A** = Area

V = Voltage **d** = Diameter

2 Combination

i Series



- Charge stored on each capacitor is same and equal to the magnitude of the charge, which comes from the battery..

$$Q = q_1 = q_2 = q_3$$

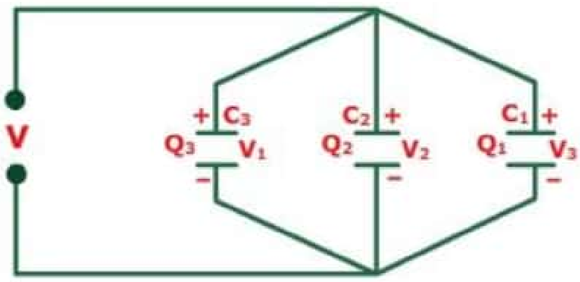
- The sum of voltage across the individual capacitor is equal to the voltage of the battery.

$$V = V_1 + V_2 + V_3$$

- $\frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3}$

- Equivalent capacitance of the capacitor is always less than the smallest value of the capacitance of the capacitor in the circuit.

ii Parallel



- The Voltage across each capacitor is the same,and it is equal to the voltage of the battery.

$$V = V_1 = V_2 = V_3$$

- The sum of the charge stored on an individual capacitor is equal to the magnitude of the charge, which comes from the battery.

$$Q = q_1 + q_2 + q_3$$

- $C_{eq} = C_1 + C_2 + C_3$

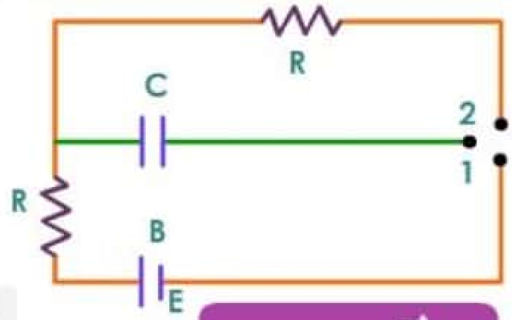
- Equivalent capacitance of the capacitor is always greater then the largest value of the capacitance of the capacitor in the circuit.

Part I

CIRCUIT SOLUTION

CHARGING AND DISCHARGING OF A CAPACITOR

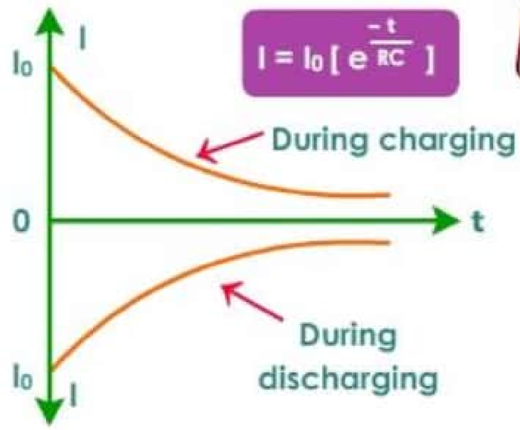
CHARGING OF A CAPACITOR



$$q = q_0\left[1 - e^{\frac{-t}{RC}}\right]$$

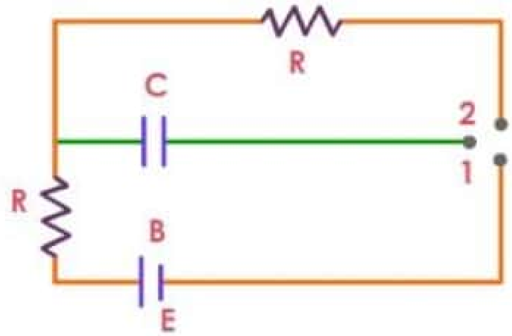
Where q_0 = maximum final value of charge at $t = \infty$.
Time $t = RC$ is known as Time Constant.

$$I = I_0\left[e^{\frac{-t}{RC}}\right]$$

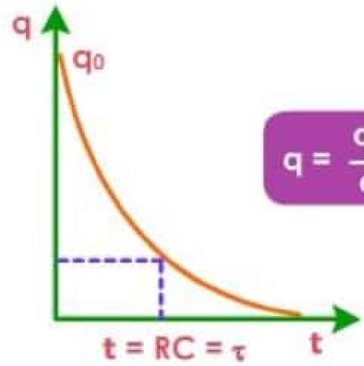


If $t = RC = \tau$ = Time constant
Then, $I = 0.37\ I_0$

DISCHARGING OF A CAPACITOR



$$q = q_0\left[e^{\frac{-t}{RC}}\right]$$



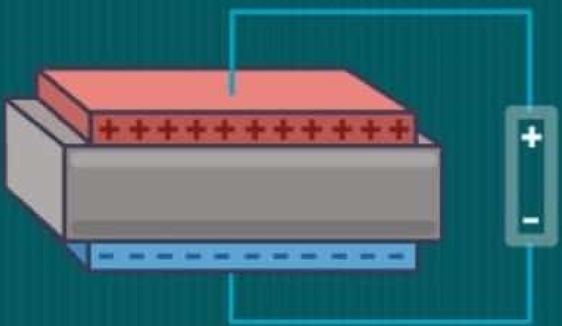
$$q = \frac{q_0}{e} = 0.37q_0$$

If $t = RC = \tau$ = time constant,
Then, $q = 0.37\ q_0$

FORCE BETWEEN THE PLATES OF A CAPACITOR

$$F = -\frac{d}{dx}\left[\frac{q^2}{2\varepsilon_0 A}x\right] = \frac{-1}{2}\frac{q^2}{\varepsilon_0 A}$$

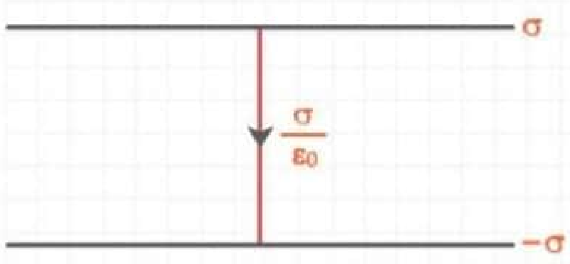
The negative sign implies that the force is attractive.



Part II

CAPACITOR WITH DIELECTRIC

1. Without Dielectric



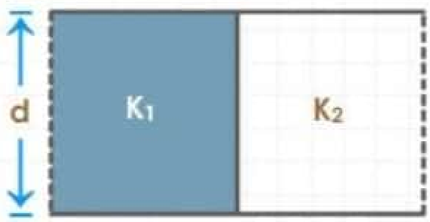
$$E = \frac{\sigma}{\varepsilon_0}$$

2. With Dielectric

$$C = \frac{AK\varepsilon_0}{d}$$

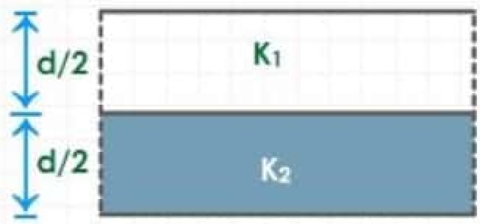
A = Area of Dielectric Slab

3. Dielectric Placed Vertically



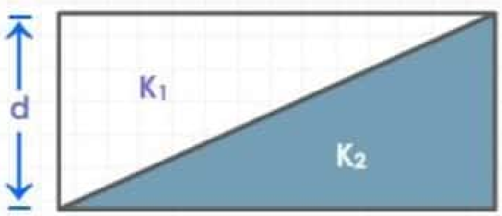
$$C = C_1 + C_2 \longrightarrow C = \frac{\varepsilon_0(K_1+K_2)A}{2d}$$

4. Dielectric Placed Horizontally



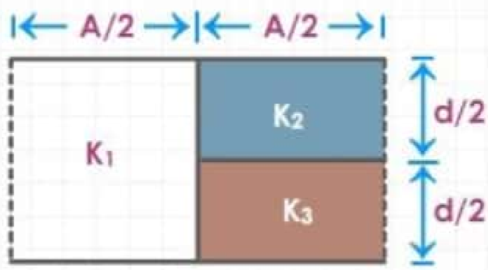
$$\frac{1}{C} = \frac{1}{C_1} + \frac{1}{C_2} \longrightarrow C = \frac{2\varepsilon_0 A K_1 K_2}{(K_1+K_2)d}$$

5. Dielectric Placed Diagonally



$$C = \frac{\varepsilon_0 A K_1 K_2}{(K_2 - K_1)} \log_e \frac{K_1}{K_2}$$

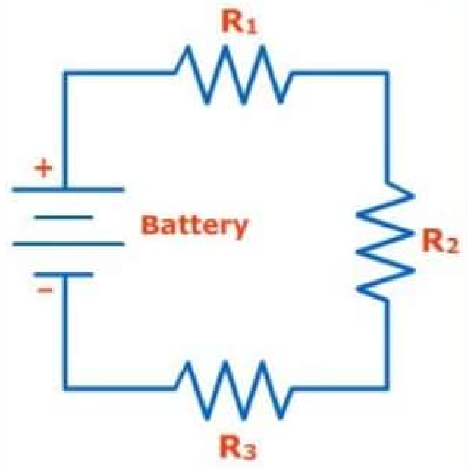
6. Capacitor With 3 Dielectrics



$$C = \frac{\varepsilon_0 A}{d}\left[\frac{K_1}{2} + \frac{K_2 K_3}{K_2+K_3}\right]$$

RESISTANCE

1 Resistance



The opposing effect to the flow of current is known as Resistance of the conductor. It is denoted by "R".

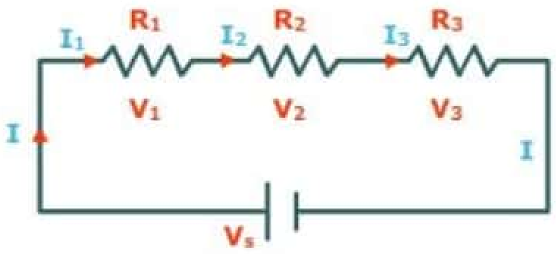
$$R = \frac{\rho l}{A}$$

ρ = Resistivity
l = Length
A = Area

Resistance (R) is measured in **Ohm** (Ω).

2 Combination

i Series



- The current passing through the individual resistance is same and its equal to magnitude of current that comes from the battery.

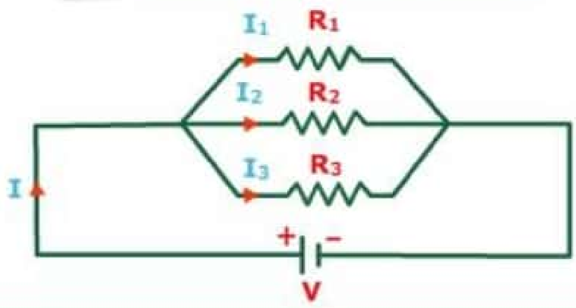
$$I = I_1 = I_2 = I_3$$

- The sum of the voltage across the individual resistance is equal to the voltage of the battery.

$$V = V_1 + V_2 + V_3$$

- $R_{eq} = R_1 + R_2 + R_3$
- The equivalent resistance of the circuit is always greater than the value of resistance in the circuit.

ii Parallel



- The sum of current passing through each resistance is equal to the total current coming from the battery.

$$I = I_1 + I_2 + I_3$$

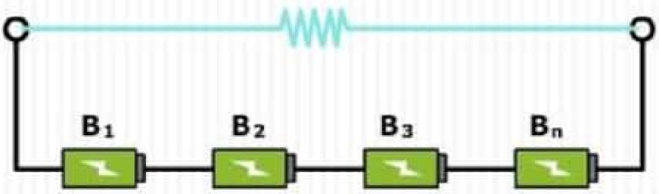
- The voltage across the individual resistance is same and is equal to the voltage of the battery.

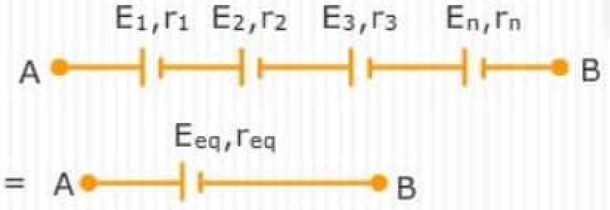
$$V = V_1 = V_2 = V_3$$

- $\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}$
- The equivalent resistance of the circuit is always less than the smallest value of resistance in the circuit.

GROUPING OF CELLS

1 CELLS IN SERIES





Equivalent EMF

$$E_{eq} = E_1 + E_2 + ... + E_n$$

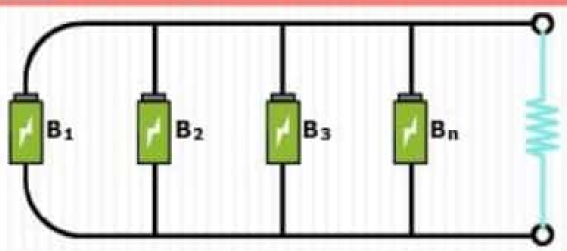
Equivalent internal resistance

$$r_{eq} = r_1 + r_2 + r_3 + r_4 + ...r_n$$

In n cells each of emf E are arranged in series and if r is internal resistance of each cell, then the total emf is equal to nE

and, current in the circuit, $I = \dfrac{nE}{R + nr}$

2 CELLS IN PARALLEL



$$E_{eq} = \frac{E_1/r_1 + E_2/E_2 + ...E_n/r_n}{1/r_1 + 1/r_2 + ... + 1/r_n}$$

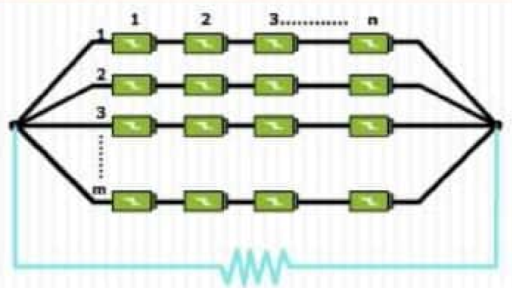
$$\frac{1}{r_{eq}} = \frac{1}{r_1} + \frac{1}{r_2} + ...+ \frac{1}{r_n}$$

If m cells, each of emf E and internal resistance r be connected in parallel and if this combination is connected to an external resistance (R) then the emf of the circuit $= E$.

internal resistance of the circuit $= \dfrac{r}{m}$

and $I = \dfrac{E}{R + \frac{r}{m}} = \dfrac{mE}{mR+r}$

3 CELLS IN MULTIPLE ARC



n = number of rows

m = number of cells in each row

Current $I = \dfrac{mE}{R + \frac{mr}{n}}$

for maximum current $nR = mr$

4 ELECTRICAL POWER

$$\text{Power, } P = \frac{V.dq}{dt} = VI = I^2 R = \frac{V^2}{R}$$

$$\text{Work, } W = VIt = I^2 Rt = \frac{V^2}{R} t$$

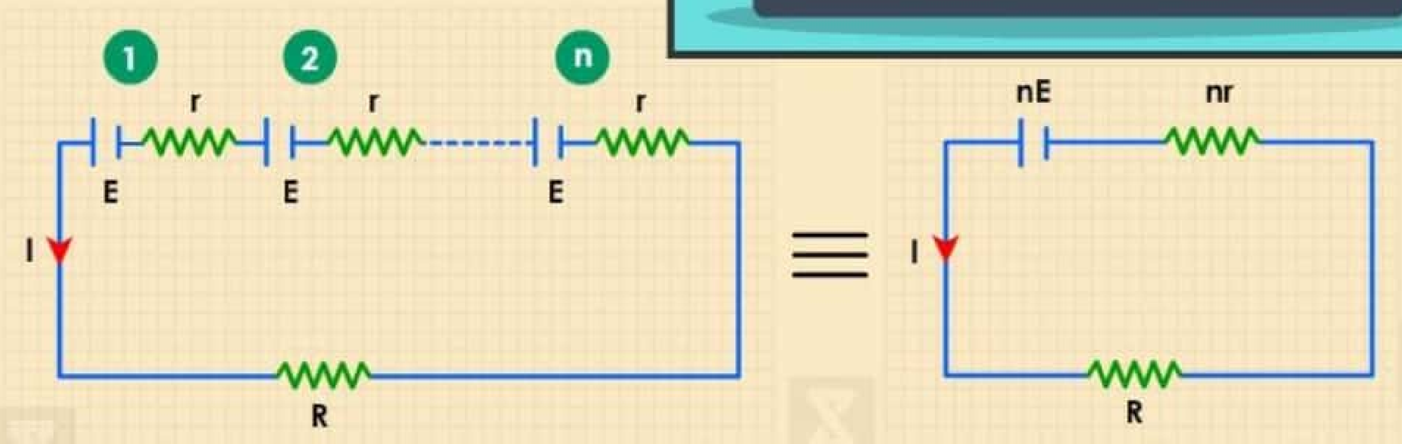
$$\text{Heat, } H = I^2 Rt \text{ Joule} = \frac{I^2Rt}{4.2} \text{ calorie}$$

Part I

CELLS AND ELECTRIC POWER

COMBINATIONS OF CELLS

1 CELL IN SERIES



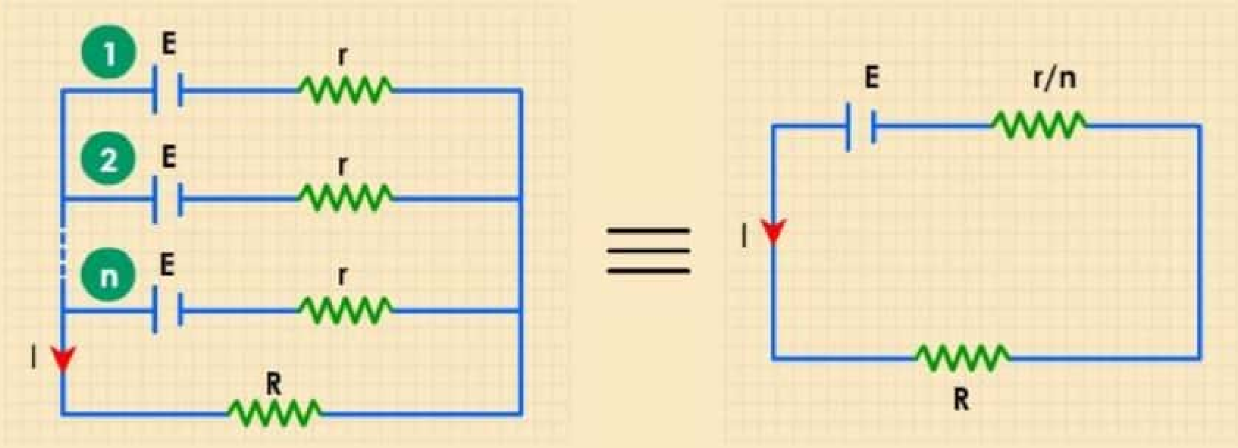
- Net EMF of the cells = nE,
- Total internal resistance = nr,
- Hence total resistance of the circuit = nr + R,

$$I = \frac{\text{net EMF}}{\text{Total Resistance}} = \frac{nE}{nr + R}$$

Case I If nr << R, then I ≅ n E/R i.e. current obtained from the cells is approximately equal to n times the current obtained from a single cell.

Case II If nr >> R, then I ≅ nE/nr = E/r i.e. current obtained from the combination of n cells is nearly the same as obtained from a single cell.

2 CELL IN PARALLEL



When E.M.F's and internal resistance of all the cells are equal

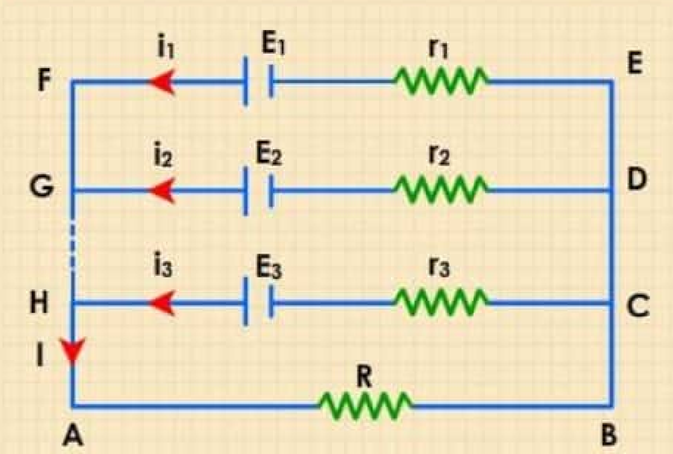
- E.M.F of battery = E.
- Total internal resistance of the combination of n cells = r/n
- Total resistance of the circuit = (r/n) + R

$$I = \frac{\text{net E.M.F}}{\text{Total Resistance}} = \frac{E}{(r/n)+R} = \frac{nE}{r+nR}$$

Case I If $r << R$, the $I \cong nE/nR = E/R$; then total current obtained from combination is approximately equal to current given by one cells only.

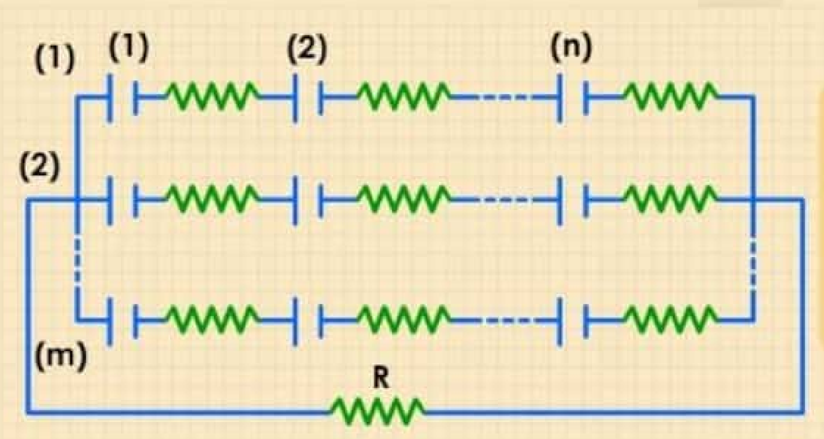
Case II If $r >> R$, then $I \cong nE/r$; then total current is approximately equal to n times the current given by one cell.

When E.M.F's and internal resistance of all the cells connected in parallel are different



$$I = \frac{\sum_{i=0}^{n} \frac{E_i}{r_i}}{1+R\sum_{i} \frac{1}{r_i}} \text{ and } E_{eq.} = \frac{\sum \frac{E_i}{r_i}}{\sum \frac{1}{r_i}}, \; r_{eq.} = \frac{1}{\sum \frac{1}{r_i}}$$

3 CELL IN MIXED GROUPING



$$\text{Total resistance of the circuit} = \left[\left(\frac{nr}{m}\right) + R\right]$$

$$I = \frac{\text{net E.M.F}}{\text{Total Resistance}} = \frac{nE}{(nr/m)+R} = \frac{nmE}{nr+mR}$$

ELECTRICAL POWER

The energy liberated per second in a device is called its power. The electrical power P delivered by an electrical device is given by

$$P = \frac{dq}{dt}V \qquad P = VI \qquad P = I^2R \qquad P = \frac{V^2}{R} \text{ watt}$$

Part I

INSTRUMENTS MEASURING VARIOUS ELECTRICAL QUANTITIES

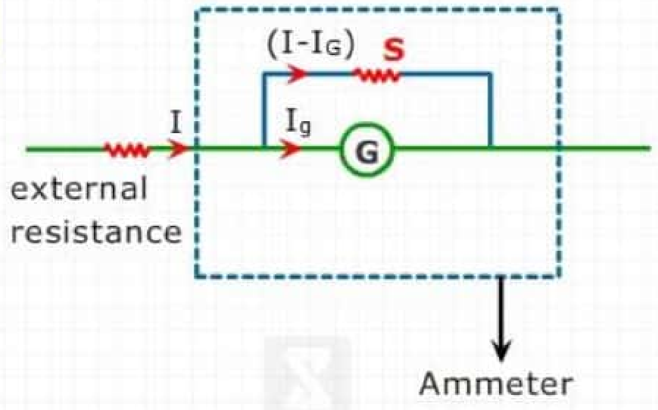
01 AMMETER

A shunt (small resistance) is connected is parallel with galvanometer to convert it into ammeter.

I_G = Current through galvanometer

R_G = Resistance of galvanometer

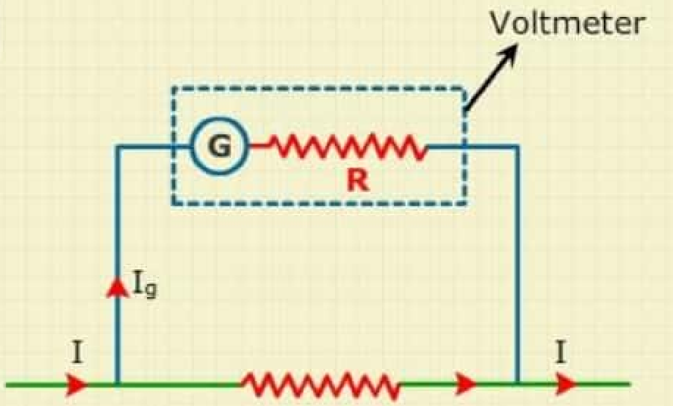
$$S = \frac{I_G R_G}{I - I_G}$$



02 VOLTMETER

A high resistance is put in series with galvanometer. It is used to measure potential difference across a resistor in a circuit.

$$I_G = \frac{V}{R_G + R}$$

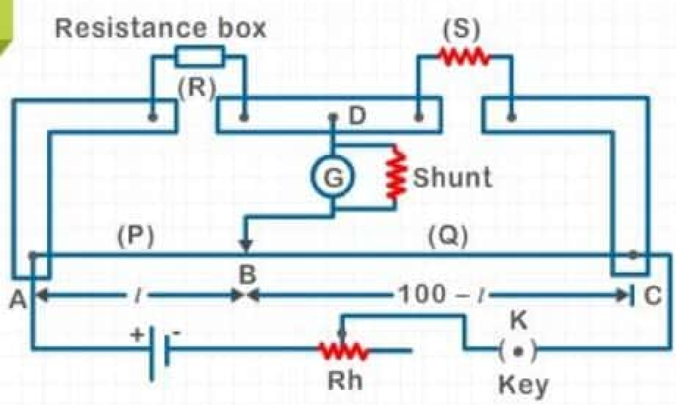


03 METRE-BRIDGE

$$S = \frac{R(100 - l)}{l}$$

R = Resistance taken in the resistance box

l = Length measured



POTENTIOMETER

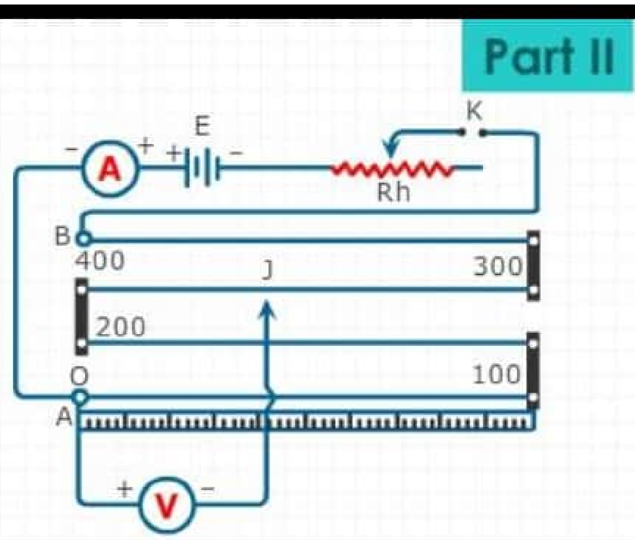
l = Length

A = Area of cross-section

ρ = Resistivity of material

I = Current

$$V = I\rho\frac{l}{A}$$



APPLICATION OF POTENTIOMETER

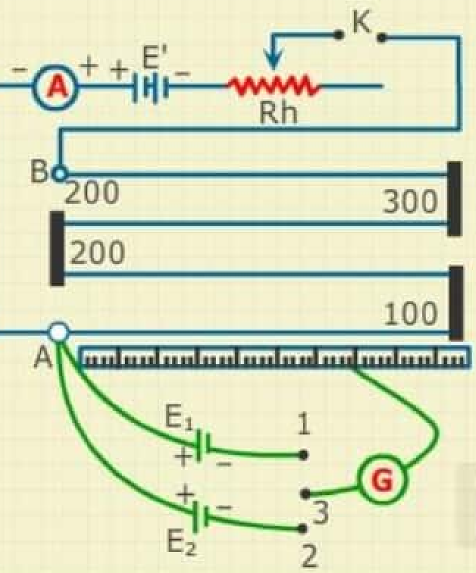
APPLICATION-01

To find EMF of an unknown cell and compare EMF of two cells

ℓ_1 = Balancing length when key is between gaps of terminal 1 and 2

ℓ_2 = Balancing length when key is between gaps of terminal 2 and 3

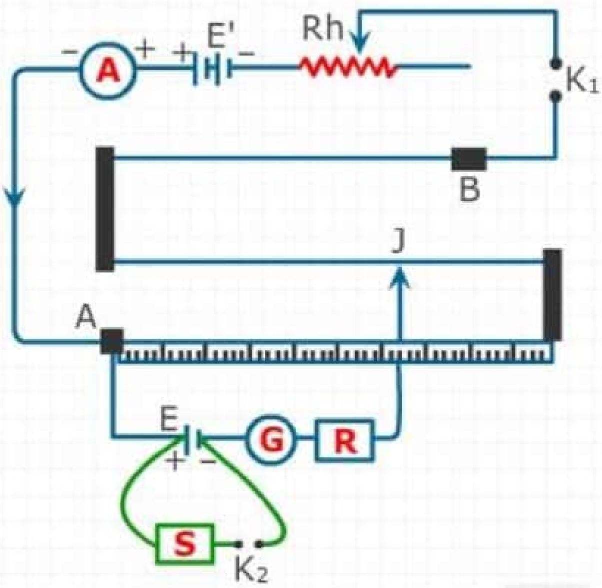
$$\frac{E_1}{E_2} = \frac{\ell_1}{\ell_2}$$



APPLICATION-02

To find the internal resistance of a cell

$$r' = \left[\frac{\ell_1 - \ell_2}{\ell_2}\right]$$

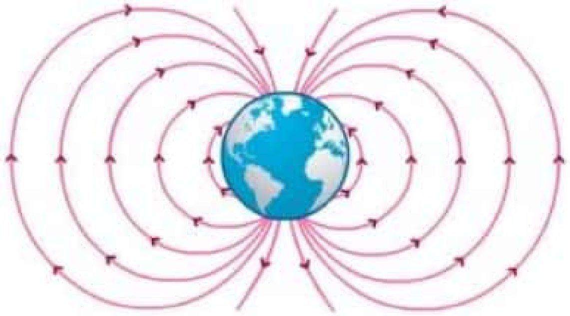


APPLICATION-03

To find current if resistance is known

$$I = \frac{X\ell_1}{R_1}$$

Part I



MAGNETIC FIELD

Magnetic field is the region surrounding a moving charge in which its magnetic effects are perceptible on another moving charge (electric current).

BIOT-SAVART LAW

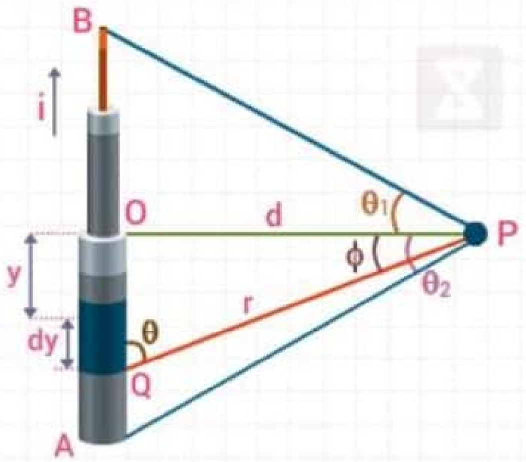
Biot-Savart law gives the magnetic induction due to an infinitesimal current element. According to 'Biot-Savart Law', the magnetic field induction dB at P due to the current element dl is given by,

$$\vec{dB} = k\,\frac{i(\vec{dl} \times \vec{r})}{r^3}$$

FIELD DUE TO A STRAIGHT CURRENT CARRYING WIRE

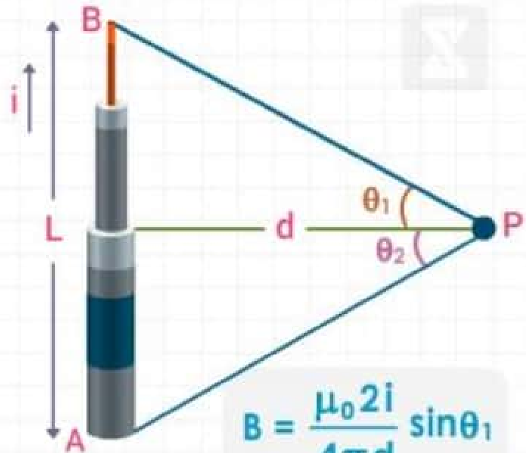
1 When the wire is of finite length

At any point P



$$B = \frac{\mu_0 i}{4\pi d}\,[\sin\theta_1 + \sin\theta_2]$$

P is on perpendicular Bi-sector

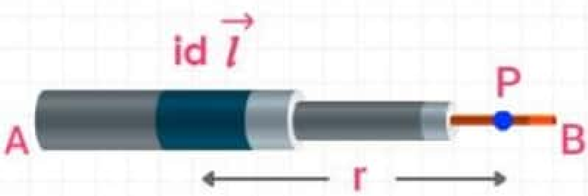


$$B = \frac{\mu_0 2i}{4\pi d}\sin\theta_1$$

$$\text{where } \sin\theta_1 = \frac{L}{L^2 + 4d^2}$$

When the point lies along the length of wire (but not on it)

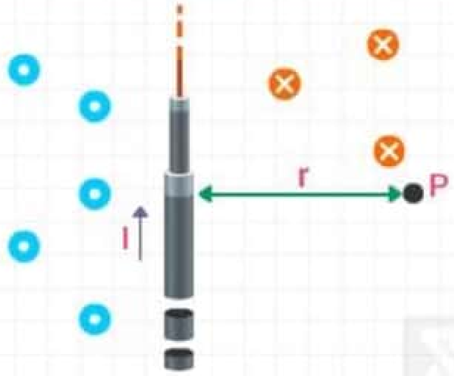
$$\vec{B} = \int_A^B \vec{dB} = 0$$



Part II

2 When the wire is of infinite length

Case-I



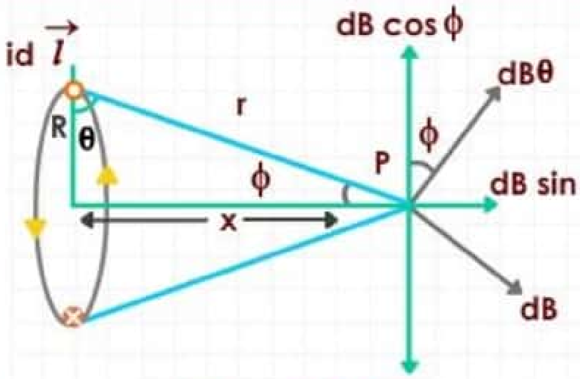
$$B = \frac{\mu_0 I}{2\pi r} \Rightarrow B \propto \frac{I}{r}$$

Case-II

$$B = \frac{\mu_0 I}{4\pi r}$$

MAGNETIC FIELD AT AN AXIAL POINT OF A CIRCULAR COIL

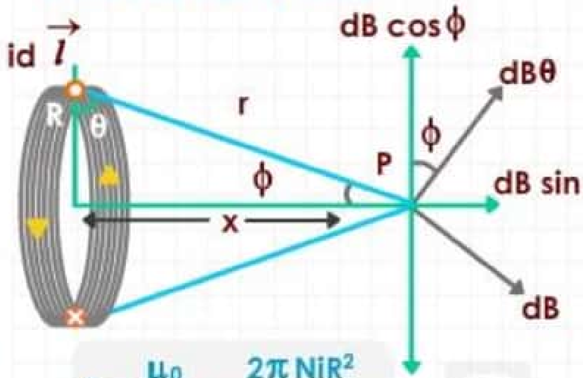
Case - I



$$B = \frac{\mu_0}{4\pi} \frac{2\pi i R^2}{(R^2 + x^2)^{3/2}}$$

Case - II

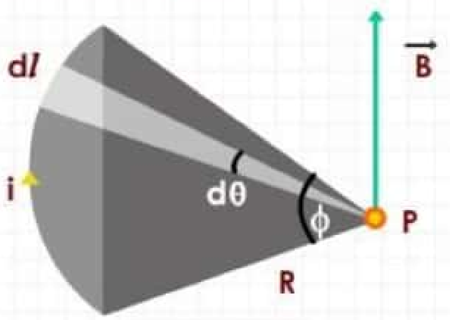
If the coil has 'N' turns-



$$B = \frac{\mu_0}{4\pi} \frac{2\pi N i R^2}{(R^2 + x^2)^{3/2}}$$

FIELD AT THE CENTRE OF A CURRENT ARC

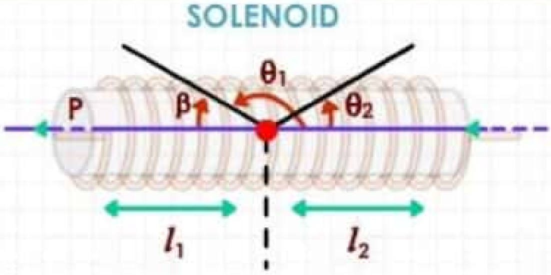
Case - I



$$B = \frac{\mu_0}{4\pi} \frac{i\phi}{R}$$

Case - II

SOLENOID



$$B = \frac{\mu_0 n i}{2} (\cos\theta_1 - \cos\theta_2)$$

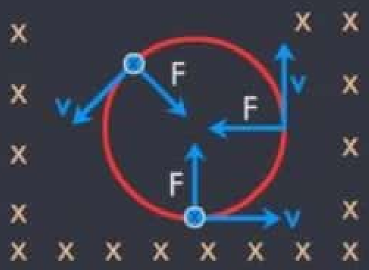
MAGNETIC FORCE
DUE TO CHARGE PARTICLES

Charge q moving with velocity $\vec{v}$, in a magnetic field has magnetic force $\vec{F} = q(\vec{v} \times \vec{B})$

MOTION OF A CHARGED PARTICLE IN A UNIFORM MAGNETIC FIELD

CHARGED PARTICLE GIVEN VELOCITY PERPENDICULAR TO THE FIELD

The particle will move on a circular path.



Time period

$$T = \frac{2\pi r}{v} = \frac{2\pi m}{qB}$$

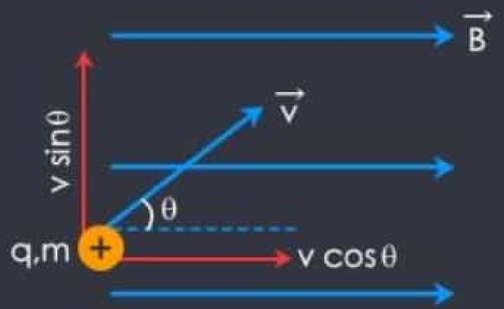
Frequency

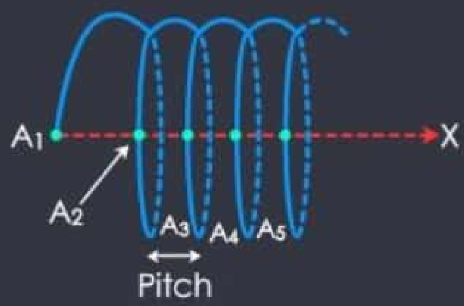
$$\nu = \frac{1}{T} = \frac{qB}{2\pi m}$$

$$\frac{mv^2}{r} = qvB \Rightarrow \boxed{r = \frac{mv}{qB}}$$

CHARGED PARTICLE IS MOVING AT AN ANGLE TO THE FIELD

$v_{\parallel} = v\cos\theta$ and $v_{\perp} = v\sin\theta$





The radius of path is, $r = \frac{mv_{\perp}}{qB} = \frac{mv\sin\theta}{qB}$, Time period $(T) = \frac{2\pi r}{v_{\perp}} = \frac{2\pi m}{qB}$

Frquency $(f) = \frac{qB}{2\pi m}$

MOTION OF CHARGED PARTICLE IN COMBINED ELECTRIC & MAGNETIC FIELD

When the moving charged particle is subjected simultaneously to both electric field **E** and magnetic field **B**, the moving charged particle will experience electric force $F_e = q\vec{E}$ and magnetic force $F_m = q(\vec{v} \times \vec{B})$

$$\boxed{\vec{F} = q\vec{E} + q(\vec{v} \times \vec{B})}$$

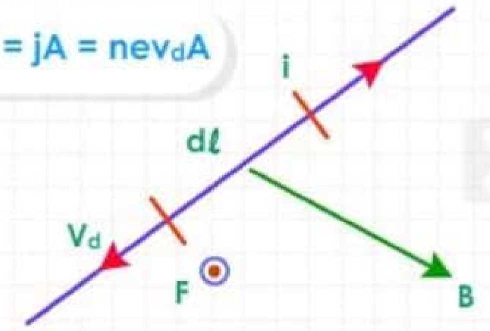
which is 'Lorentz force equation'.

Part I

MAGNETIC PROPERTY

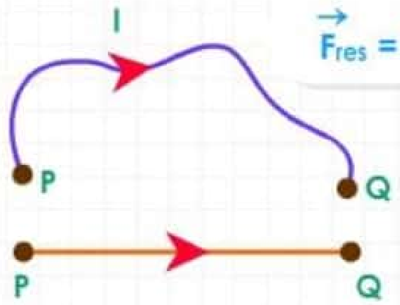
MAGNETIC FORCE ON A CURRENT CARRYING WIRE

$i = jA = nev_dA$



v_d = Drift speed
n = No. of free electrons per unit volume
j = Current density

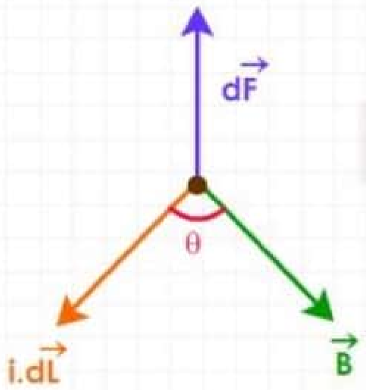
$\vec{F}_{res} = i\vec{L} \times \vec{B}$

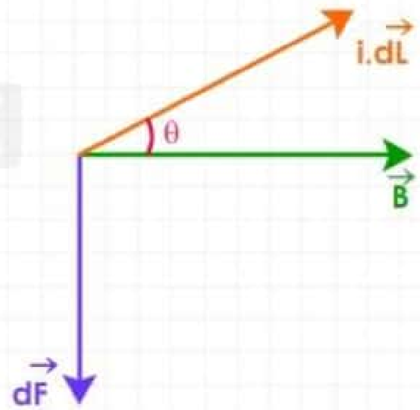


$\vec{L}$ = Vector length of the wire

DIRECTION OF FORCE

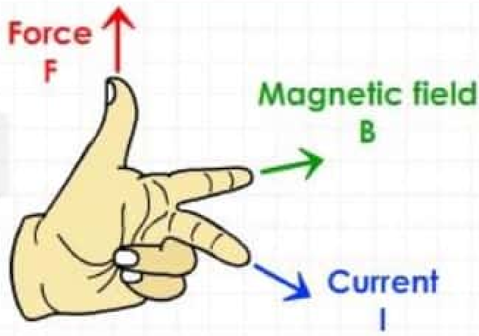
The direction of force is always perpendicular to the plane containing $i.\vec{dL}$ and $\vec{B}$ and is same as that of cross-product of two vectors $(\vec{a} \times \vec{b})$ with $a = i.\vec{dL}$ and $b = \vec{B}$



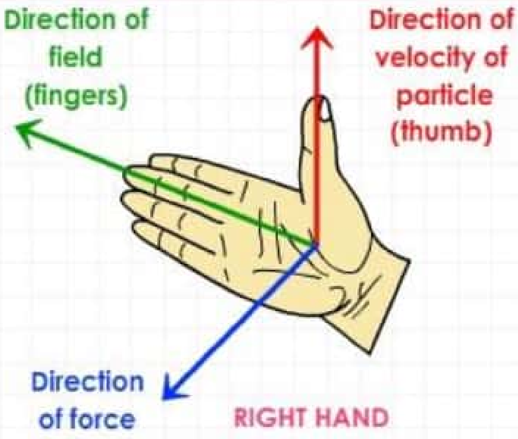


The direction of force when current element $i.\vec{dL}$ and $\vec{B}$ are perpendicular to each other can also be determined by applying either of the following rules:

1. **Fleming's Left-hand Rule** : Stretch the forefinger, central finger and thumb of the left hand mutually perpendicular. Then if the forefinger points in the direction of the field $(\vec{B})$ and the central finger is in the direction of current, the thumb will point in the direction of force (or motion).

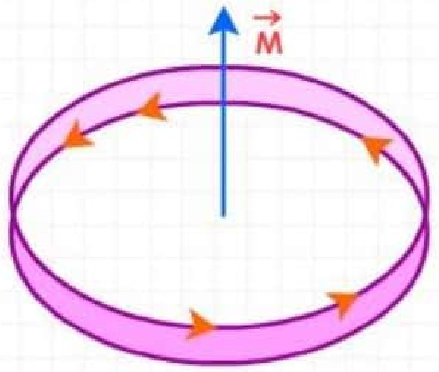


2. **Right-hand Palm rule** : Stretch the fingers and thumb of the right-hand at right angles to each other. To find the direction of the magnetic force on a positive moving charge, the thumb of the right hand ponts in the direction of velocity of particle v, the fingers in the direction of Magnetic Field B, then the Force F is directed perpendicular to the right hand palm



CURRENT LOOP IN A UNIFORM MAGNETIC FIELD

MAGNETIC MOMENT

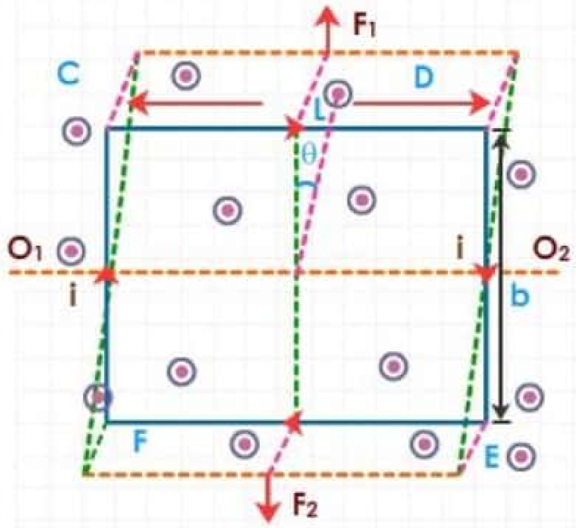


$$\vec{M} = Ni\pi R^2 = NiA$$

A = Area of loop | R = Radius of loop

N = No. of loops | I = Current

TORQUE ON A CURRENT LOOP

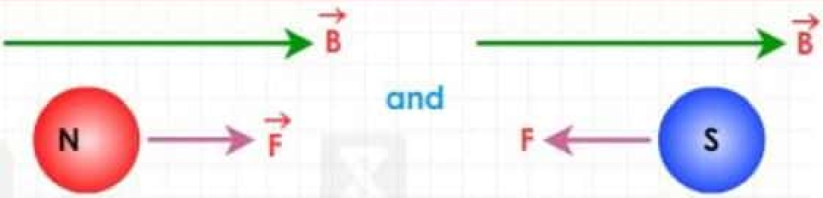


$$\vec{\tau} = \vec{M} \times \vec{B}$$

MAGNETIC FIELD AND STRENGTH OF MAGNETIC FIELD

$$\vec{B} = \frac{\vec{F}}{M}$$

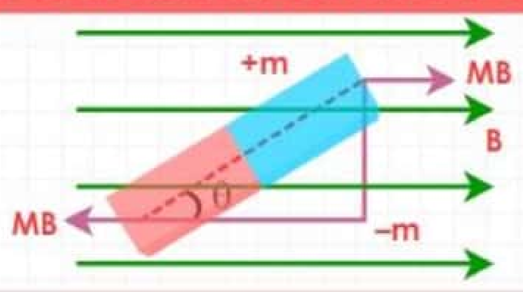
S.I. unit of $\vec{B}$ is Tesla or weber/m^2



MAGNETIC IN AN EXTERNAL UNIFORM MAGNETIC FIELD

$F_{res} = 0$ (for any angle)

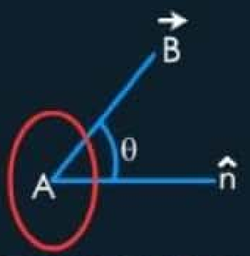
$\tau = MB \sin\theta$



ELECTROMAGNETIC FORCE

MAGNETIC FLUX

Magnetic Flux is the amount of magnetic field passing through a given area.



$$\phi = \int \vec{B}.d\vec{A} \Rightarrow \boxed{\phi = \vec{B}\,.\,\vec{A} = BA\cos\theta}$$

Unit ⟶ weber (Wb)

FARADAY'S LAW OF ELECTROMAGNETIC INDUCTION

Whenever the flux of a magnetic field through the area bounded by a closed conducting loop changes, an emf is produced in the loop. The emf is given by

$$\boxed{\varepsilon = -\frac{d\phi}{dt}}$$

LENZ'S LAW

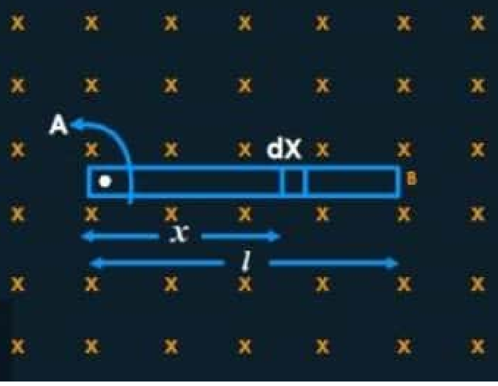
According to lenz's law, if the flux associated with any loop changes than the induced current flows in such a fashion that it tries to oppoose the cause which has produced it.

MOTIONAL EMF

$$\boxed{E = \int (\vec{v} \times \vec{B}).d\vec{l}}$$

EMF developed across the ends of the rod moving perpendicular to magnetic field v velocity perpendicular to the rod is $\boxed{\varepsilon = vB\,\ell}$

INDUCED EMF IN A ROTATING ROD



$$\int dE = \int_0^{\ell} B\omega x\,dx$$

$$V_A - V_B = \frac{B\omega l^2}{2}$$

INDUCED ELECTRIC FIELD

$$\text{EMF, } e = \oint \vec{E}\,.\,d\vec{l}$$

Using Faraday's law of induction

$$\varepsilon = -\frac{d\phi}{dt}$$

$$\text{or, } \oint \vec{E}\,.\,d\vec{l} = -\frac{d\phi}{dt}$$

Part I

SELF INDUCTION

1 SELF INDUCTION

If current in the coil changes by Δi in a time interval Δt, the average emf induced in the coil is given as

$\varepsilon = -\dfrac{\Delta(N\phi)}{\Delta t} = -\dfrac{\Delta(Li)}{\Delta t} = -\dfrac{L\Delta i}{\Delta t}$, S.I unit of inductance is wb/amp or Henry (H)

SELF INDUCTANCE OF SOLENOID

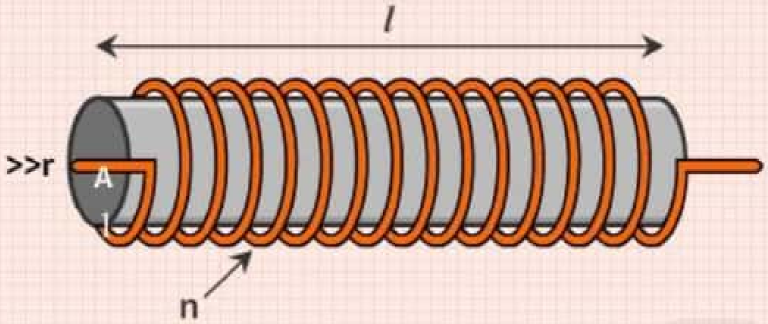
$L = \mu_o n^2 \pi r^2 l$

n = no. of turns/length

r = radius ; μ_o = Permeability

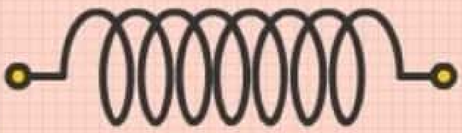
l = length

Inductance/Volume = $\mu_o n^2$



2 INDUCTOR

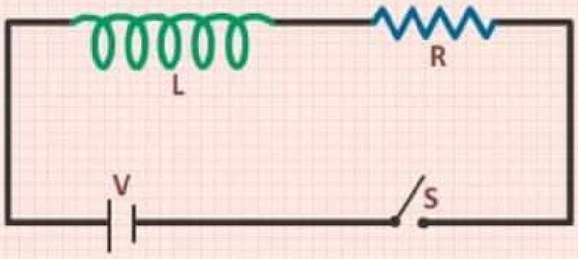
$V_A - L\dfrac{di}{dt} = V_B$, Energy stored in inductor , $U = \dfrac{1}{2} Li^2$



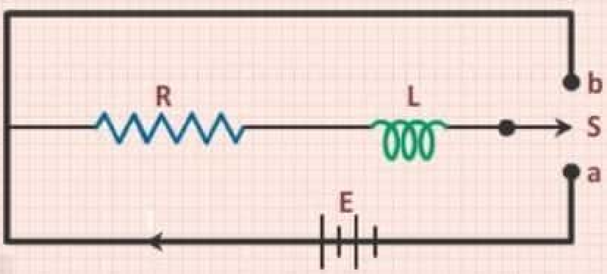
3 L – R CIRCUIT

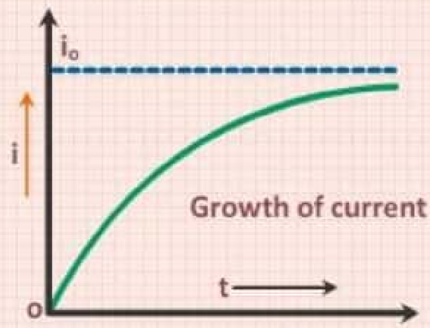
At t = 0, inductor behaves as an open switch.

At t = ∞, inductor behaves as plane wire.



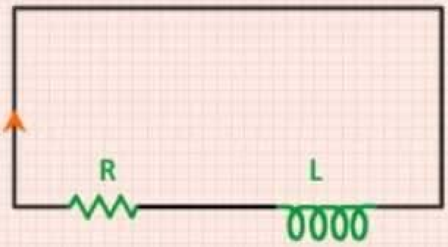
GROWTH OF CURRENT

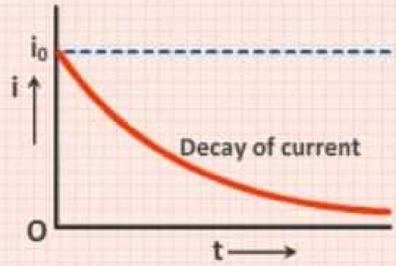




The maximum current in the circuit $i_0 = E/R$. So $i = i_0\left\{1 - e^{-\frac{R}{L}t}\right\}$

4 DECAY OF CURRENT

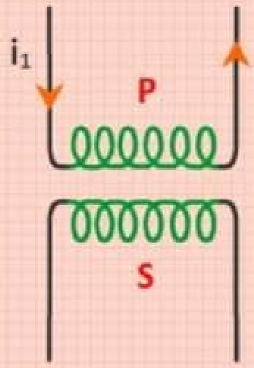




$$i = i_0\, e^{-\frac{R}{L}t} = i_0\, e^{-\frac{t}{\tau}}$$

5 MUTUAL INDUCTANCE

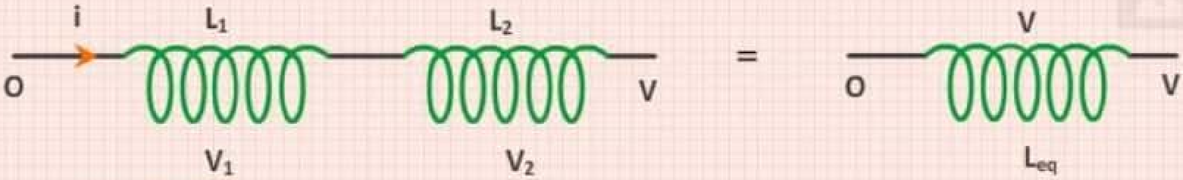
$$\mathcal{E} = -M\frac{di_1}{dt} \Longrightarrow \phi_2 = Mi_1$$



M = Mutual inductance

Unit of Mutual inductance is Henry (H)

6 SERIES COMBINATION OF INDUCTORS



$$\therefore V = V_2 + V_1$$

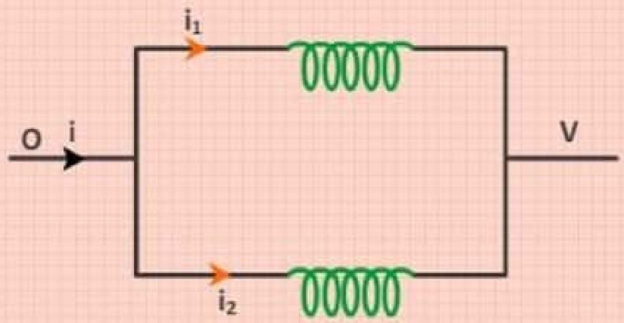
$$L_{eq}\frac{di}{dt} = L_1\frac{di}{dt} + L_2\frac{di}{dt} \Rightarrow L_{eq} = L_1 + L_2 + \ldots\ldots.$$

7 PARALLEL COMBINATION OF INDUCTOR

$$i = i_1 + i_2 \Rightarrow \frac{di}{dt} = \frac{di_1}{dt} + \frac{di_2}{dt}$$

$$\frac{V}{L_{eq}} = \frac{V}{L_1} + \frac{V}{L_2}$$

$$\frac{1}{L_{eq}} = \frac{1}{L_1} + \frac{1}{L_2} + \ldots\ldots..$$

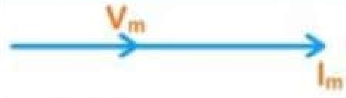
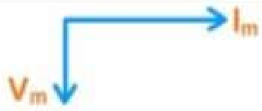


Part I

ALTERNATING CURRENT

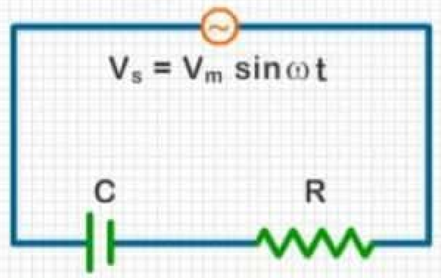
It is the movement of electrical charge through a medium that changes direction periodically

1 SUMMARY

AC SOURCE CONNECTED WITH	PHASE ϕ	PHASE DIFFERENCE	IMPEDANCE Z	PHASOR DIAGRAM
Pure Resistor	0	V_R is in same phase with i_R	R	
Pure Inductor	$\frac{\pi}{2}$	V_L leads i_L by 90^0	X_L	
Pure Capacitor	$-\frac{\pi}{2}$	V_C lags i_C by 90^0	X_C	

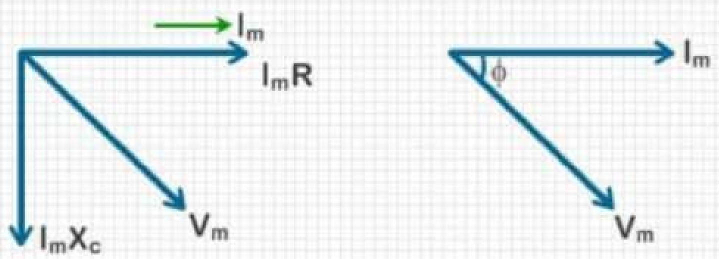
2 RC SERIES CIRCUIT WITH AN AC SOURCE

Circuit Diagram



$$I_m = -\frac{V_m}{\sqrt{R^2 + X_c^2}} \Rightarrow Z = \sqrt{R^2 + X_c^2}$$

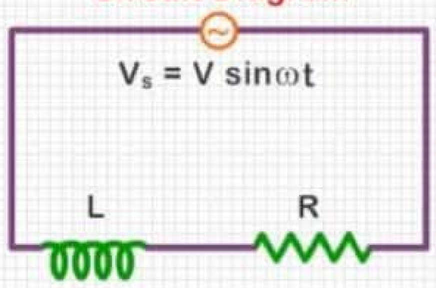
Phasor Diagram



$$\tan\phi = \frac{I_m X_c}{I_m R} = \frac{X_c}{R}$$

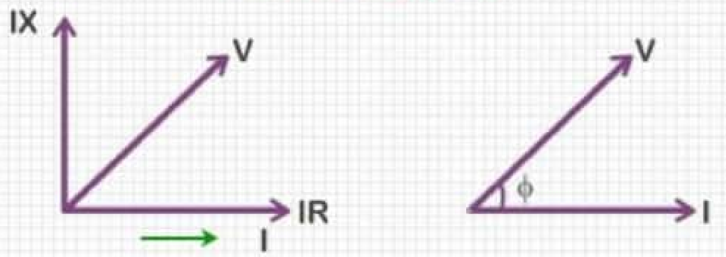
3 LR SERIES CIRCUIT WITH AN AC SOURCE

Circuit Diagram



$$V = I\sqrt{R^2 + X_L^2}$$

Phasor Diagram

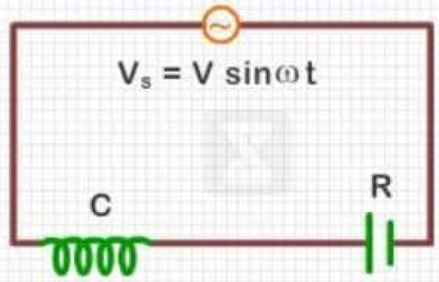


$$\tan\phi = \frac{IX_L}{IR} = \frac{X_L}{R}$$

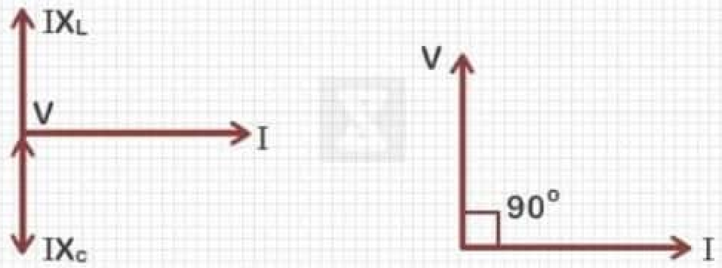
Part II

4 LC SERIES CIRCUIT WITH AN AC SOURCE

Circuit Diagram



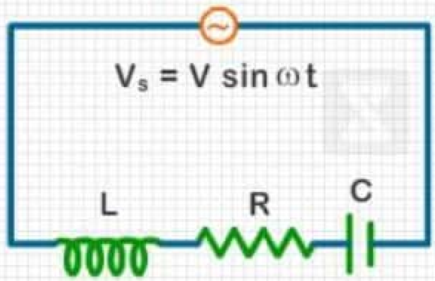
Phasor Diagram



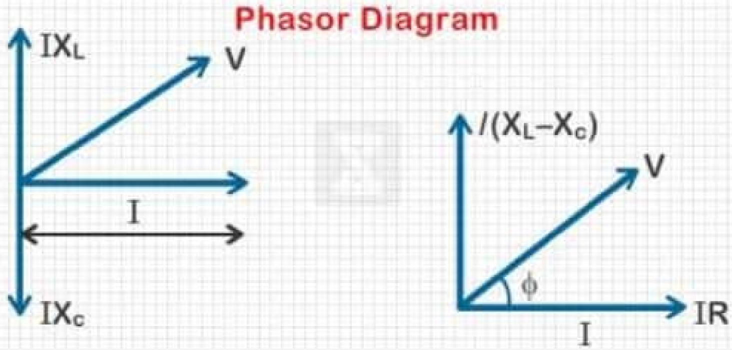
From the phasor diagram $V = I\left|(X_L - X_c)\right| = IZ$, $\varphi = 90^\circ$

5 RLC SERIES CIRCUIT WITH AN AC SOURCE

Circuit Diagram



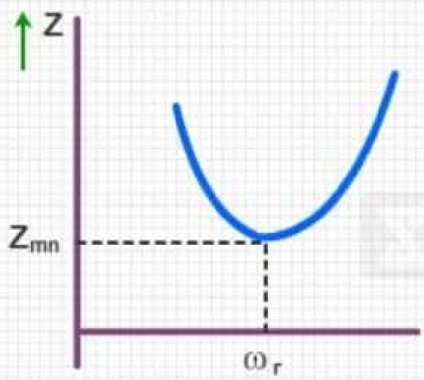
Phasor Diagram

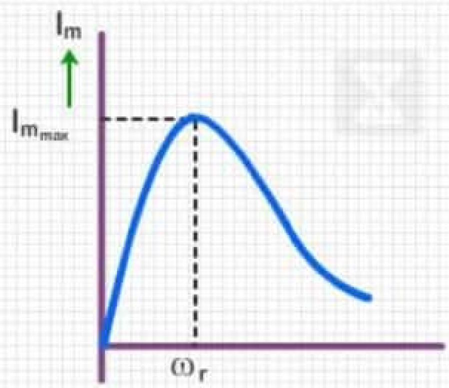


From the phasor diagram $V = \sqrt{(IR)^2 + (IX_L - IX_c)^2}$, $Z = \sqrt{R^2 + (X_L - X_c)^2}$

$$\tan\phi = \frac{I(X_L - X_c)}{IR} = \frac{(X_L - X_c)}{R}$$

6 RESONANCE





Amplitude of current (and therefore I_{rms} also) in an RLC series circuit is maximum for a given value of V_m and R, if the impedance of the circuit is minimum, which will be when $X_L - X_C = 0$. This condition is called resonance.

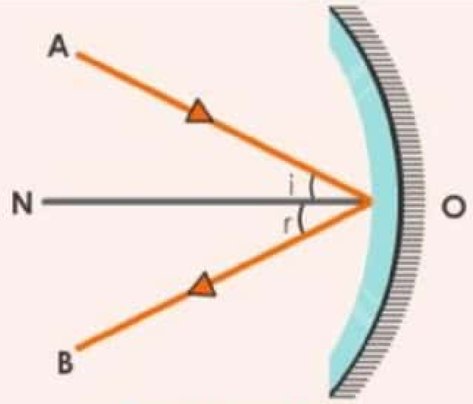
So at resonance: $X_L - X_C = 0 \Rightarrow \omega = \frac{1}{\sqrt{LC}}$

Part I

MIRRORS

1 REFLECTION

When a ray of light is incident at a point on the surface of a mirror, the surface throws **partly or wholly** the incident energy back into the **medium of incidence**. This phenomennon is called reflection.



Concave mirror

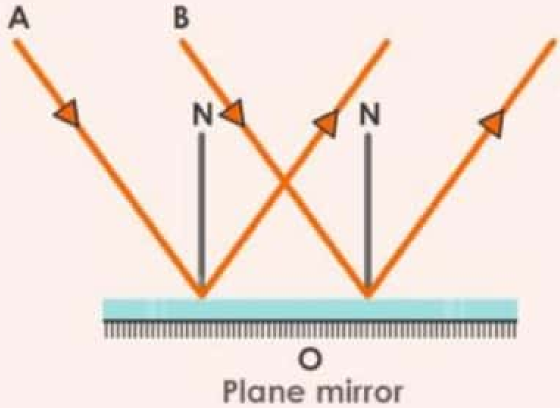
2 LAW OF REFLECTION

- The **incident ray**, the **reflected ray** and the **normal** to the reflecting surface at the point of incidence, **all lie in the same plane.**
- The angle of incidence is **equal to** the angle of reflection, i.e., **$\angle i = \angle r$**

Note: These laws hold good for all reflecting surfaces either plane or curved.

3 PLANE MIRROR

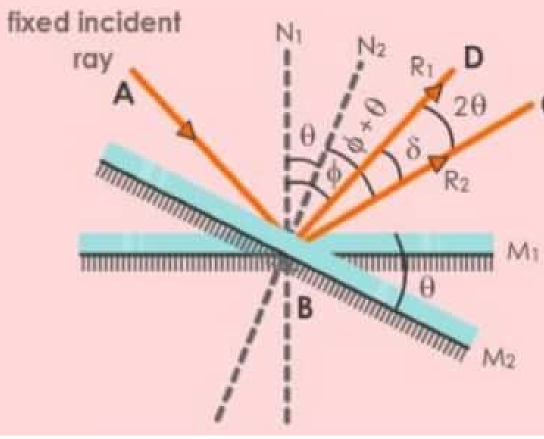
A beam of parallel rays of light, incident on a plane mirror will get reflected as a beam of parallel reflected rays.



Plane mirror

4 ROTATION OF MIRROR

For a **fixed incident light ray**, if the mirror be **rotated** by an **angleθ** (about an axis which lies in the plane of mirror and perpendicular to the plane of incidence), the **reflected ray turns through an angle of 2θ** in the same direction.



5 NUMBER OF IMAGES FORMED BY TWO INCLINED MIRRORS

θ = Angle between mirrors

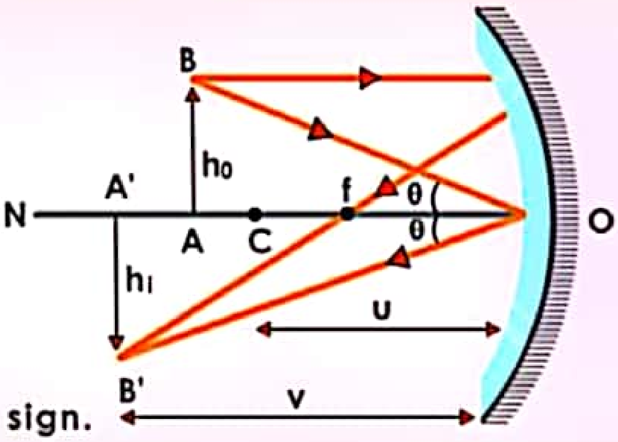
- If $\frac{360^\circ}{\theta}$ = **even number** ; number of images = $\frac{360^\circ}{\theta}$ **−1.**
- If $\frac{360^\circ}{\theta}$ = **odd number** ; number of images = $\frac{360^\circ}{\theta}$ **−1**, If the object is placed on the angle bisector.
- If $\frac{360^\circ}{\theta}$ = **odd number** ; number of images = $\frac{360^\circ}{\theta}$, If the object is not placed on the angle bisector.
- If $\frac{360^\circ}{\theta} \neq$ **integer**, then the number of images = **nearest even integer.**

6 TRANSVERSE MAGNIFICATION

$\Delta ABO \sim \Delta A'B'O$

$$x = \frac{h_i}{v} = \frac{h_o}{u} \Rightarrow m = \frac{h_i}{h_o} = -\frac{v}{u}$$

- The above formula is valid for **both concave and convex mirror.**
- **h_i, h_o, v** and **u** should be **put with appropriate sign.**



7 CONCAVE MIRROR

S.No	Position of object	Details of images			
		Location	Type	Orientation	Magnification
1.	At ∞	At F	real	inverted	$\lvert m \rvert << 1$
2.	Between C and ∞	Bet. F and C	real	inverted	$\lvert m \rvert < 1$
3.	At C	At C	real	inverted	$\lvert m \rvert = 1$
4.	Between F and C	Bet. C and ∞	real	inverted	$\lvert m \rvert > 1$
5.	At F	At infinity	real	inverted	$\lvert m \rvert >> 1$
6.	Between F and P	Behind the mirror	virtual	erect	$\lvert m \rvert > 1$

8 CONVEX MIRROR

Position of object	At infinity	In front of mirror
Details of images	At F, virtual, erect, $\lvert m \rvert << 1$	Between P and F, virtual, erect, $\lvert m \rvert < 1$

Part III

9 VELOCITY IN SPHERICAL MIRROR

Velocity of Image

- Object moving along the principal axis, $V_{IM} = -\frac{v^2}{u^2}(V_{OM})$
- Object moving perpendicular to the principal axis, $\frac{dh_I}{dt} = -\frac{v}{u}\frac{dh_o}{dt}$
- Object moving parallel to the Principal axis, $v_y = \frac{dh_I}{dt} = -h_0\left[\frac{dv}{dt}\cdot\frac{1}{u} - \frac{v}{u^2}\cdot\frac{du}{dt}\right]$

Refraction of Light

$$\mu = \frac{c}{v} = \frac{\text{speed of light in vacuum}}{\text{speed of light in medium}}$$

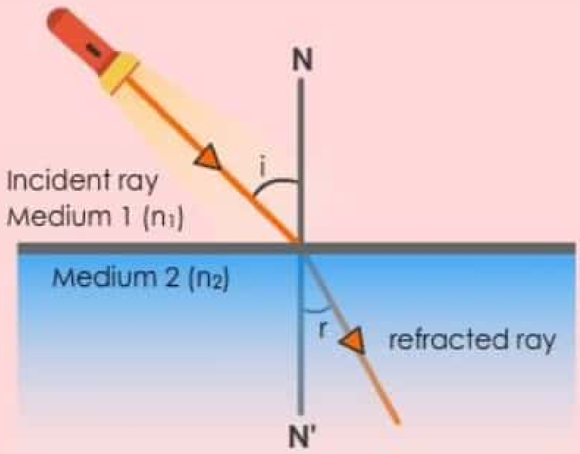
μ = Refractive Index

10 LAWS OF REFRACTION

- The **incident ray, the normal** to any refracting surface at the point of incidence and the **refracted ray,** all **lie in the same plane** called the plane of incidence or plane of refraction.
- $\frac{\sin i}{\sin r}$ = **Constant** for any pair of media and for light of a given wavelength.

This is known as **Snell's Law.**

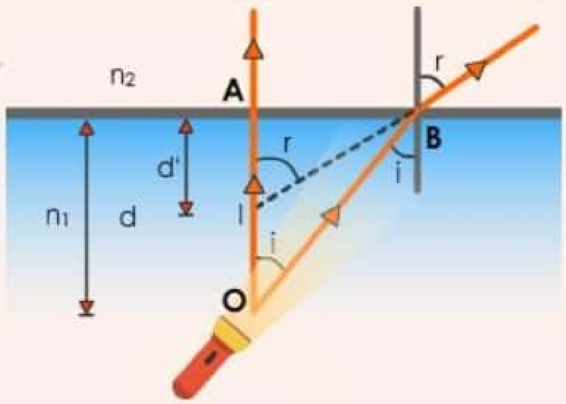
Also, $\frac{\sin i}{\sin r} = \frac{n_2}{n_1} = \frac{v_1}{v_2} = \frac{\lambda_1}{\lambda_2}$



11 APPARENT DEPTH AND NORMAL SHIFT

When the object is in denser medium and the observer is in rarer medium (near normal incidence)

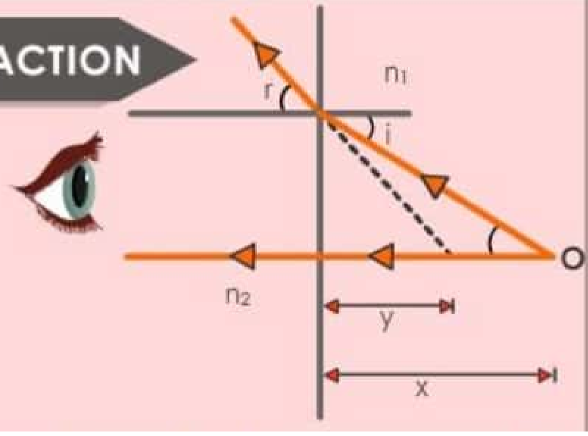
$$\frac{n_2}{n_1} = \frac{d'}{d} = \frac{\text{Apparent depth}}{\text{Real depth}}$$



12 IMAGE VELOCITY IN CASE OF PLANE REFRACTION

$$\frac{n_2}{n_1} = \frac{y}{x} \Rightarrow y = \frac{n_2}{n_1}.x$$

$$\frac{dy}{dt} = \frac{n_2}{n_1}\frac{dy}{dt} \Rightarrow V_{IS} = \frac{n_2}{n_1}V_{OS}$$



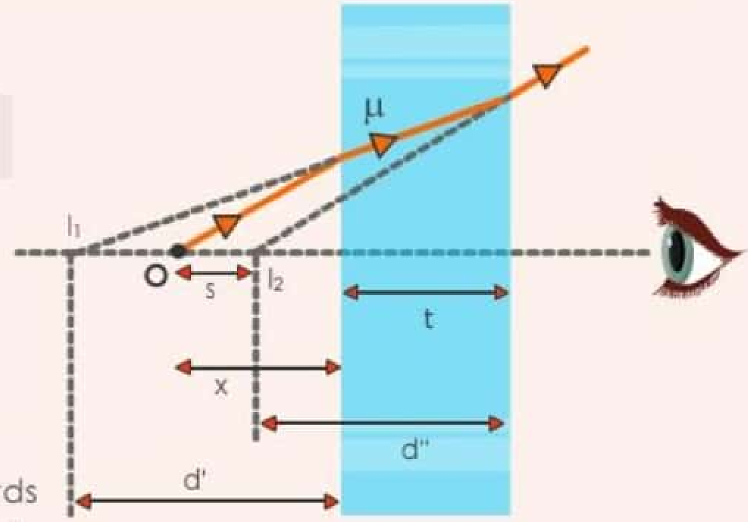
13 REFRACTION THROUGH A GLASS SLAB

Apparent shift due to the slab when object is seen normally through the slab

$$s = t\left[1 - \frac{\mu_{\text{surrounding}}}{\mu_{\text{slab}}}\right]$$

IMPORTANT POINTS

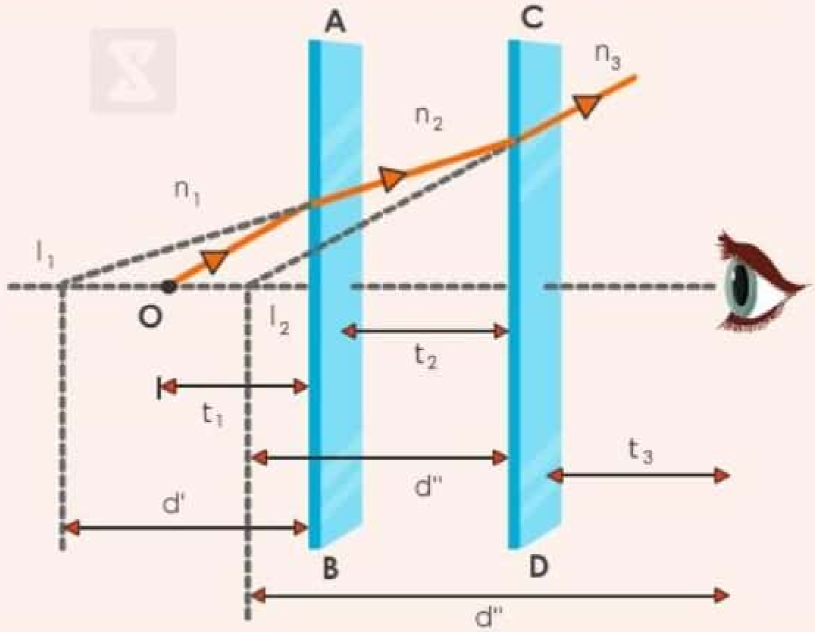
- Rays should be **paraxial.**
- **Medium** on both side of the slab **should be same.**
- Shift comes **out from** the object.
- Shift is **independent** of the **distance of the object** from the slab.
- If shift comes **out Positive** then shift is towards the **direction of incident rays** and vice versa.



Apparent distance between object and observer when both are in different medium

$$d'' = n_3\left[\frac{t_1}{n_1} + \frac{t_2}{n_2} + \frac{t_3}{n_3}\right]$$

If object and observer are in **same medium** then **shift formula** should be used and if both are in **different medium** then the **above formula** of apparent distance should be used.



14 CRITICAL ANGLE AND TOTAL INTERNAL REFLECTION

Critical angle is the angle made in a **denser medium** for which the **angle of refraction in rarer medium is 90°.**

$$\therefore C = \sin^{-1}\frac{n_r}{n_d}$$

Conditions of Total Internal Reflection

- Light is incident on the interface from denser medium.
- Angle of incidence should be **greater than** the critical angle ($i > c$).

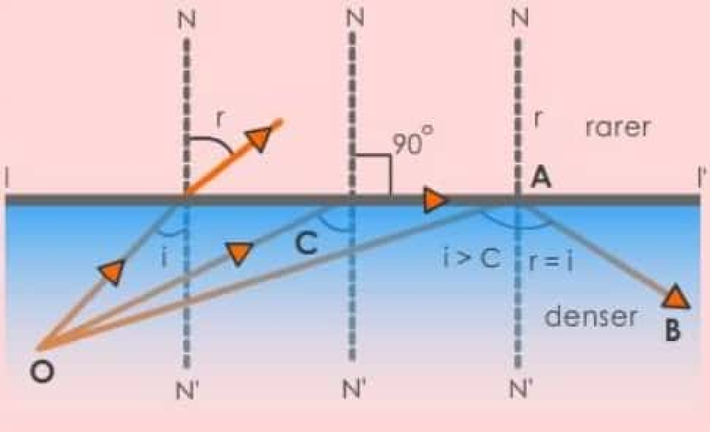
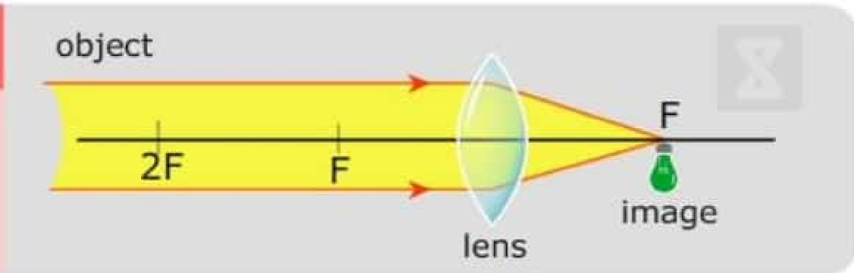


IMAGE FORMED BY LENSES

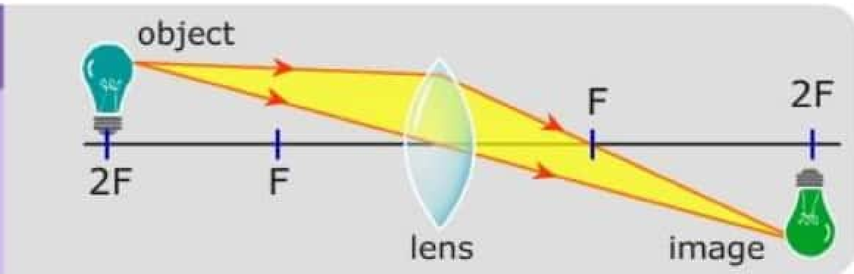
Distant Object

- Real
- Smaller than object
- Inverted
- At Focus



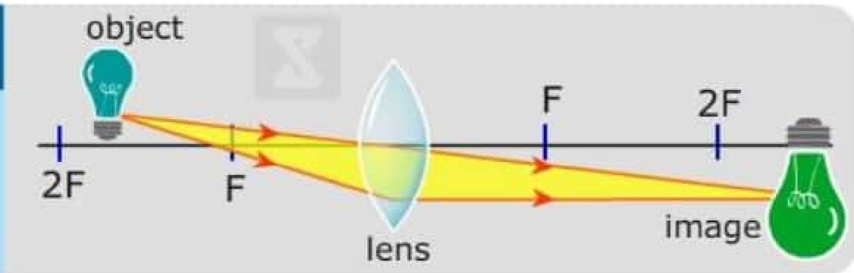
Object at 2F

- Real
- Same size as object
- Inverted
- At 2F



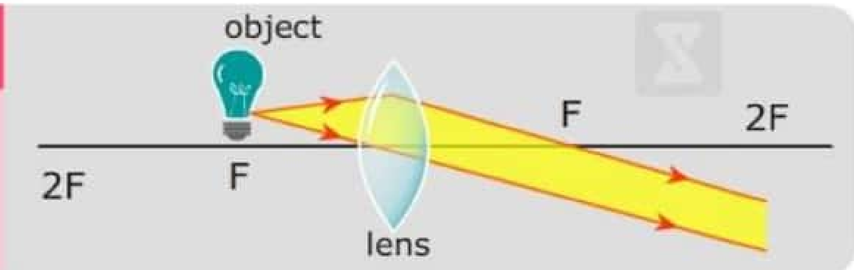
Object between 2F and F

- Real
- Larger than object
- Inverted
- Beyond 2F



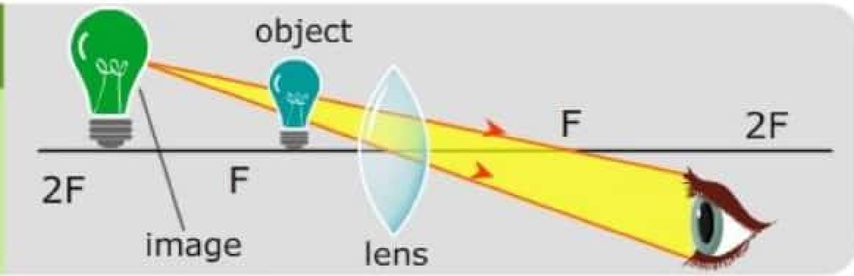
Object at F

- Real
- Highly magnified
- Inverted
- At infinity



Object between F and lens

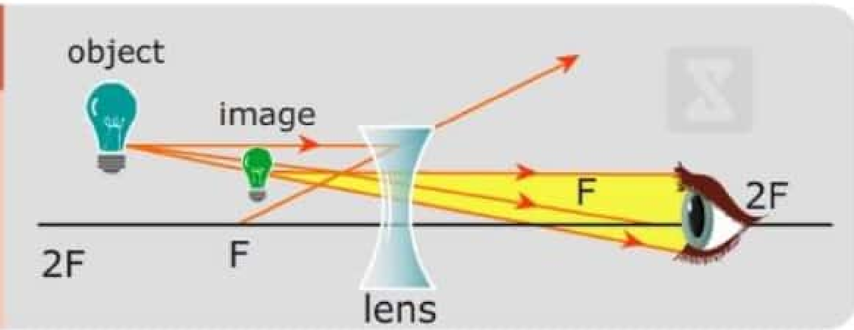
- Virtual
- Magnified
- Erect
- At same side as object



Images formed by a concave lens

Object is at F

- Virtual
- Smaller than object
- Upright
- Between object and the lens



WAVE OPTICS

WAVE FRONT

- Wave front is a locus of particles having same phase.
- Direction of propagation of wave is perpendicular to wave front.
- Every particle of a wave front acts as a new source and is known as secondary wavelet.

Coherent source

If the phase difference due to two source at a particle point remains constant with time, then the two sources are considered as coherent source

INTERFERENCE

$$A_{net}^2 = A_1^2 + A_2^2 + 2A_1A_2\cos\phi$$

$$I_{net} = I_1 + I_2 + 2\sqrt{I_1I_2}\cos\phi$$

For constructive interference

$$I_{net} = \left(\sqrt{I_1} + \sqrt{I_2}\right)^2$$

For destructive interference

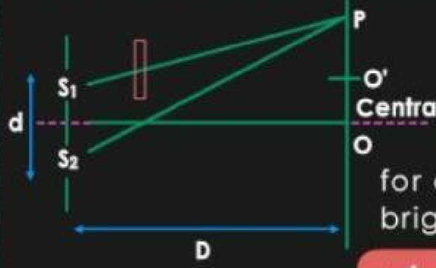
$$I_{net} = \left(\sqrt{I_1} - \sqrt{I_2}\right)^2$$

FRINGE WIDTH

It is the distance between two maxima of successive order on one side of the central maxima. This is also equal to the distance between two successive minima.

$$\beta = \frac{\lambda D}{d}$$

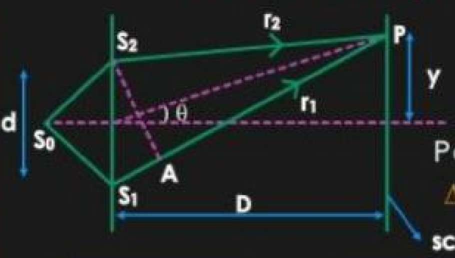
DISPLACEMENT OF FRINGE



for central bright fringe;

$$\Rightarrow \frac{yd}{D} = t(\mu - 1)$$

YOUNG'S DOUBLE SLIT EXPERIMENT (Y.D.S.E)



Path difference

$$\Delta p = S_1P - S_2P$$

$$\sqrt{\left(y+\frac{d}{2}\right)^2 + D^2} - \sqrt{\left(y-\frac{d}{2}\right)^2 + D^2} \quad ...(1)$$

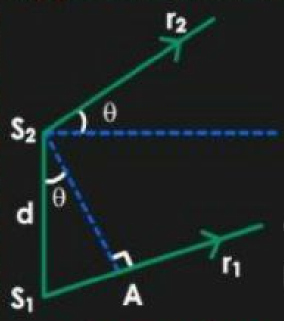
Approximation I :

For $D >> d$, We can approximate rays $\vec{r_1}$ and $\vec{r_2}$ as being approximately parallel, at angleθ to the principle axis.

Now, $S_1p - S_2P = S_1A = S_1S_2\sin\theta$

$\Rightarrow$ Path difference $= d\sin\theta$ ...(2)

Approximation II :



Further if θ is small, i.e.

$y << D$, $\sin\theta \simeq \tan\theta = \frac{y}{D}$

and hence,

path difference $= \frac{dy}{D}$ ..(3)

for maxima

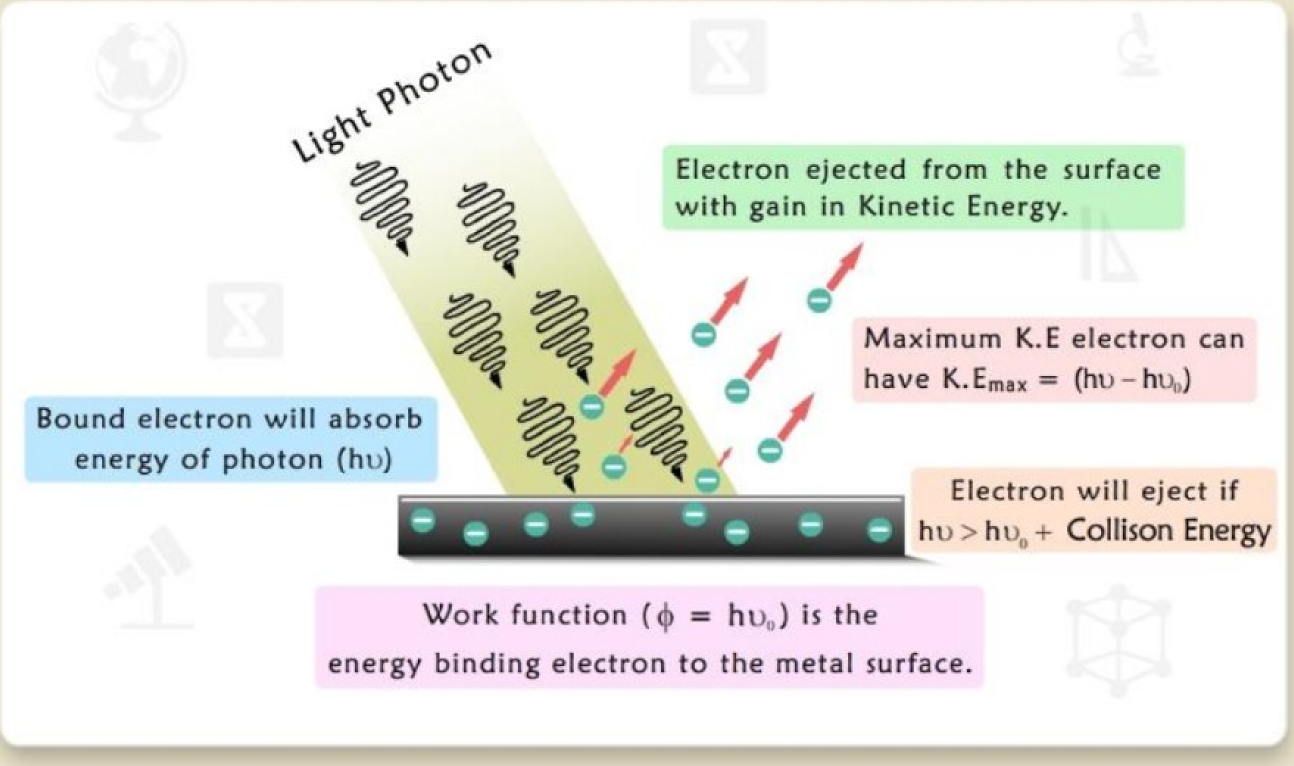
$$\Delta p = \frac{dy}{D} = n\lambda$$

for minima

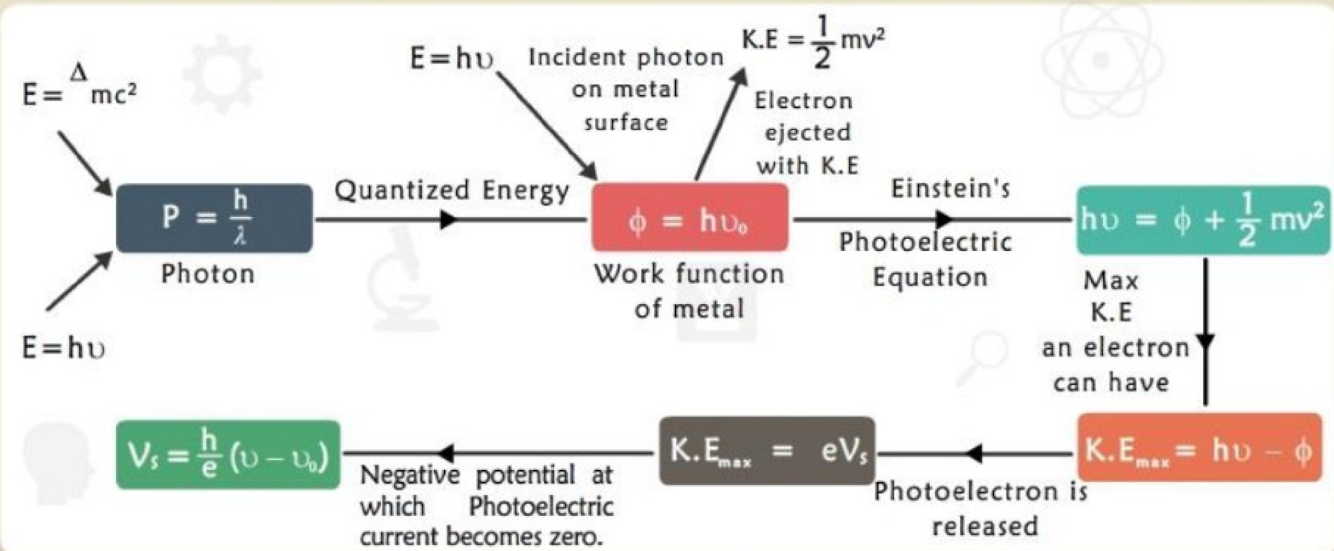
$$\Delta p = \pm\frac{\lambda}{2}, \pm\frac{3\lambda}{2} \pm\frac{5\lambda}{2}$$

PHOTOELECTRIC EFFECT

Photoelectric effect is the observation that many metals emit electrons when light shines upon them.



Only 0.01% of electrons are ejected from the surface.



HISTORY OF ATOMIC MODEL

1885

Johann Balmer derived a formula for mathematically predicting hydrogen spectrum.

1897

J J Thomson discovered Electron

1904

J J Thomson proposed plum pudding model

1911

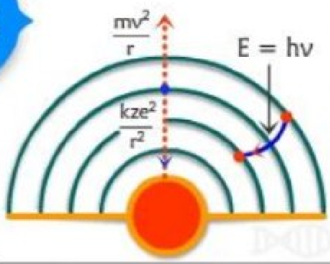
Rutherford proposed a model where positive charge is at the center, and electron moves around in a spiral path and losses energy.

1913

Bohr's Atomic Model

$r = 0.529 \times \frac{n^2}{z}$ Å

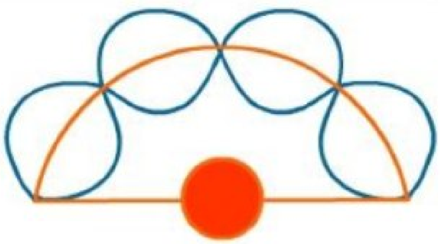
$\frac{kze^2}{r^2} = \frac{mv^2}{r}$



- Bohr worked with J J Thomson and found flaws in his theory
- He proposed electron revolves around nucleus in orbits.
- Electron is stabilized by centripetal and electrostatic forces.
- Electron don't lose energy in an orbit.
- Electron losses or gains energy by moving across orbits.
- He proved Balmer was right by derving his formula theoretically.
- Only applicable for one electron systems.
- Failed to predict dual nature of electron.

1923

De Broglie introduced the concept of dual nature in electrons. He used Einstein's $E = mc^2$ and proposed any moving particle or object has an associated wave.



Part I

NUCLEAR PHYSICS

MASS DEFECT

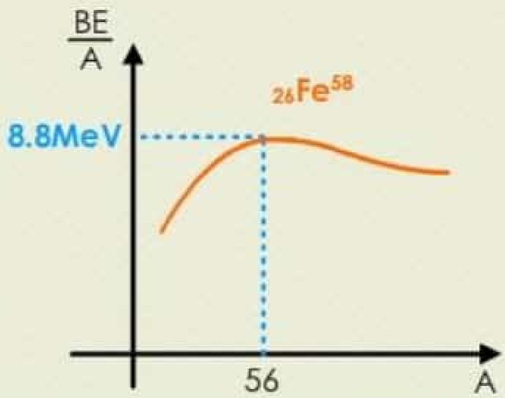
Mass Defect = $M_{expected} - M_{observed}$

$\Delta m = [Zm_p + (A - Z)m_n - [M_{atom} - Zm_e]$

BINDING ENERGY

It is the minimum energy required to break the nucleus into its constituent particles.

Binding Energy (B.E.) = Δmc^2 = $\Delta m \times 931$ MeV



- Binding energy per nucleon is more for medium nuclei than for heavy nuclei. Hence, medium nuclei are highly stable.
- *The heavier nuclei* being unstable have tendency to split into medium nuclei. This process is called **Fission**.
- *The Lighter nuclei* being unstable have tendency to fuse into a medium nucleus. This process is called **Fusion**.

RADIOACTIVITY

- It was discovered by **Henry Becquerel**.
- Spontaneous emission of radiations (α, β, γ) from unstable nucleus is called **radioactivity**. Substances which show radioactivity are known as **radioactive substance**.
- ***In radioactive decay,*** an unstable nucleus emits α particle or β particle. After emission of α or β particle the remaining nucleus may emit γ- particle, and convert into a more stable nucleus.

α- particle

It is a doubly charged helium nucleus. It contains two protons and two neutrons.

Mass of α - particle = Mass of $_2He^4$ atom $- 2m_e = 4\ m_p$

Charge of α - particle = + 2e

β- particle

β^- (electron)

Mass = m_e : Charge = − e

β^+ (positron)

Mass = m_e : Charge = + e

positron is an antiparticle of electron.

γ- particle

They are energetic photons of energy of the order of MeV and having zero rest mass.

RADIOACTIVE DECAY (DISPLACEMENT LAW)

1 α - DECAY

$${}_{Z}X^{A} \rightarrow {}_{Z-2}Y^{A-4} + {}_{2}He^{4} + Q$$

Q value is definied as energy released during the decay process.

Q value = rest mass energy of reactants − rest mass energy of products

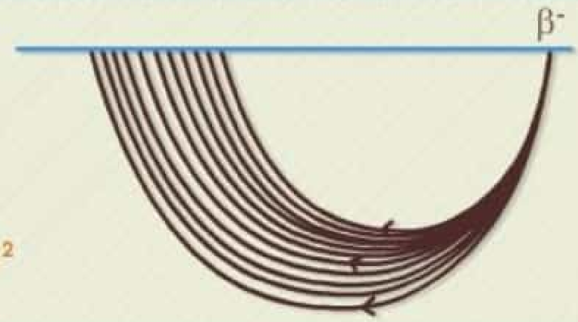
Let, M_x = mass of atom ${}_ZX^A$, M_y = mass of atom ${}_{Z-2}Y^{A-4}$, M_{He} = mass of atom ${}_2He^4$

$$\boxed{Q \text{ value} = [M_x - M_y - M_{He}]c^2}$$

2 β⁻ - DECAY

$${}_{Z}X^{A} \rightarrow {}_{Z+1}Y^{A} + {}_{-1}e^{0} + Q$$

$$\boxed{T_e = \frac{m_y}{m_e + m_y} Q,} \quad \boxed{T_y = \frac{m_e}{m_e + m_y} Q,}$$



Q value = $[M_x - \{(M_Y - m_e) + m_e\}]\, c^2 = [M_x - M_y\,]c^2$

3 β⁺ - DECAY

$${}_{Z}X^{A} \rightarrow {}_{Z-1}Y^{A} + {}_{+1}e^{0} + \nu + Q$$

Q value = $[M_x - \{(M_Y + m_e) + m_e\}]\, c^2 = [M_x - M_y - 2M_e]c^2$

RADIOACTIVE DECAY : STATISTICAL LAW

- Rate of radioactive decay is directly proportional to N
- where N = number of active nuclei.
- Rate of radioactive decay of $A = \frac{-dN}{dt} = \lambda N$
- where λ = decay constant of the radioactive substance.
- Number of nuclei decayed (i.e., the number of nuclei of B formed)

$$\boxed{N = N_0(1 - e^{-\lambda t})}$$

1 HALF LIFE ($T_{1/2}$)

$$\boxed{t_{\frac{1}{2}} = \frac{\ln 2}{\lambda} = \frac{0.693}{\lambda}}$$

2 ACTIVITY

Activity is defined as the rate of radioactive decay of nuclei

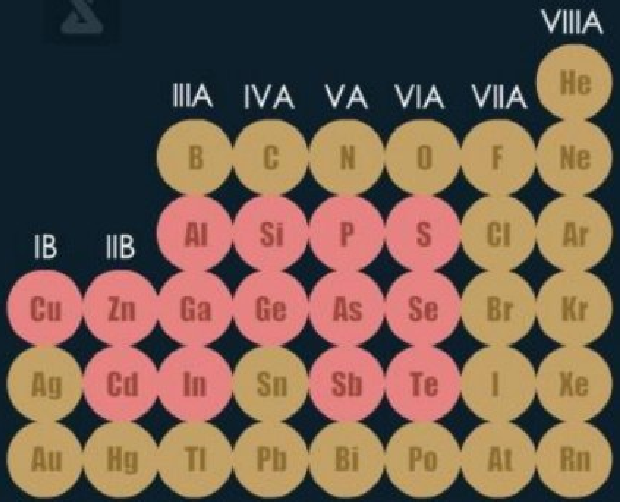
$$\boxed{A = A_0\, e^{-\lambda t}}$$

3 AVERAGE LIFE

$$T_{avg} = \frac{\text{sum of ages of all the nuclei}}{N_0} = \frac{\int_0^{\infty} \lambda N_0\, e^{-\lambda t}\, dt\,.t}{N_0} = \frac{1}{\lambda}$$

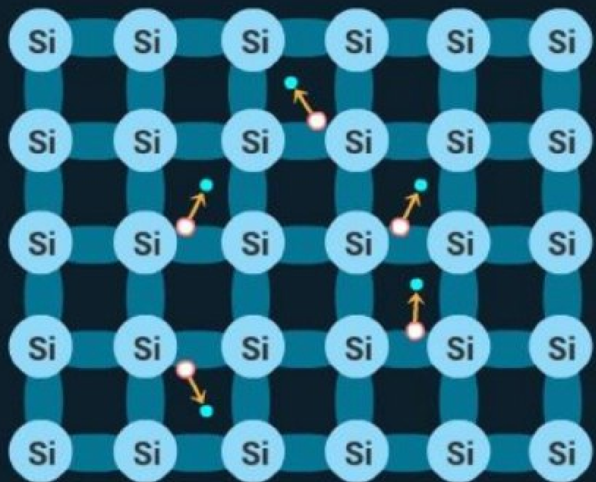
N-TYPE SEMICONDUCTOR P-TYPE

SEMICONDUCTOR



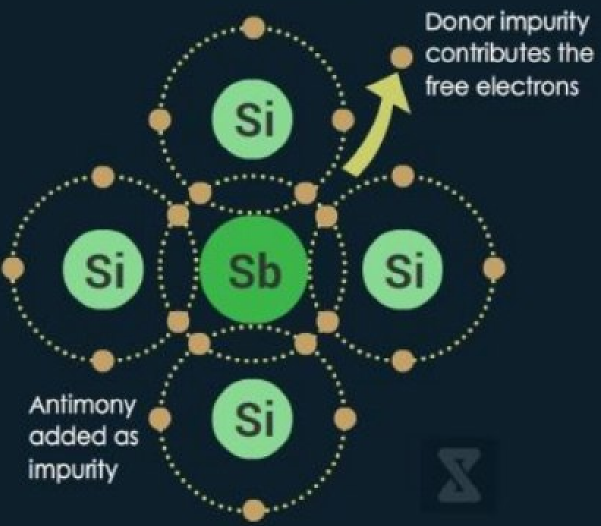
The elements of 4th group of the periodic table are called semiconductors.
Eg: Germanium, Silicon, etc.

INTRINSIC SEMICONDUCTOR



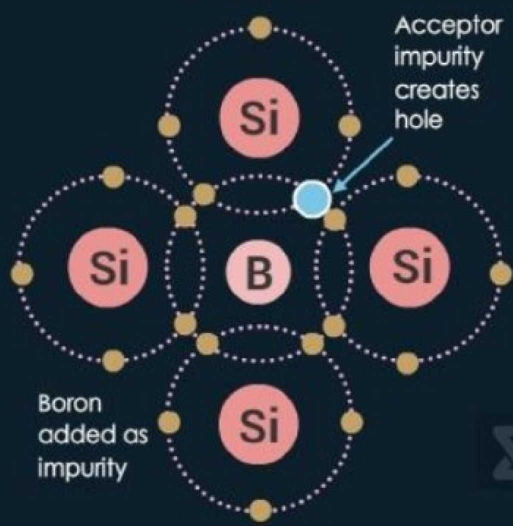
Pure semiconductor is called intrinsic semiconductor.

N-Type



When impurity of 5th group is added in an intrinsic semiconductor, then N-type semiconductor is formed.

P-Type



When impurity of 3rd group is added in an intrinsic semiconductor, then P-type semiconductor is formed.