MA 1101 : Mathematics I

Problem 1.
Let $X := \{1, 2, 3, 4, 5\}$.

  (i) Give three examples of reflexive relations on X that are neither symmetric nor transitive.

 (ii) Give three examples of symmetric relations on X that are neither reflexive nor transitive.

(iii) Give three examples of transitive relations on X that are neither reflexive nor symmetric.

(iv) Give three examples of relations on X that are reflexive, symmetric, but not transitive.

 (v) Give three examples of relations on X that are symmetric, transitive, but not reflexive.

(vi) Give three examples of relations on X that are reflexive, transitive, but not symmetric.

(vii) Give three examples of relations on X that neither reflexive, nor symmetric, nor transitive.

(viii) Give three examples of relations on X that reflexive, symmetric and transitive.

Problem 2.
Let $X \neq \emptyset$ and let R be a relation on X with the property that, for every $x \in X$, there exists an $a \in X$ (a depends on X) such that xRa. Show that, R is reflexive if R is symmetric and transitive.

Problem 3.
Define a relation $\sim$ on $\mathbb{R} \times \mathbb{R}$ as
$$(x_1, x_2) \sim (y_1, y_2) \text{ if } x_1 = y_1.$$

  (i) Check that $\sim$ is an equivalence relation.

 (ii) Identify and draw the equivalence classes.

Problem 4.
Define a relation $\sim$ on $\mathbb{R} \times \mathbb{R}$ as

$$(x_1, x_2) \sim (y_1, y_2) \text{ if } x_1^2 + x_2^2 = y_1^2 + y_2^2.$$

  (i) Check that $\sim$ is an equivalence relation.

 (ii) Identify and draw the equivalence classes.

Problem 5.
Define a relation $\sim$ on $\mathbb{N} \times \mathbb{N}$ as
$$(m, n) \sim (p, q) \text{ if } m + q = n + p.$$

  (i) Check that $\sim$ is an equivalence relation.

 (ii) Identify and draw the equivalence classes.

Problem 6.
Define a relation $\sim$ on $\mathbb{R} \times \mathbb{R} \setminus \{(0, 0)\}$ as

$$(x_1, x_2) \sim (y_1, y_2) \text{ if } (y_1, y_2) = \alpha(x_1, x_2), \text{ for some } \alpha \neq 0.$$

  (i) Check that $\sim$ is an equivalence relation.

(ii) Identify and draw the equivalence classes.

Problem 7.
Let $n \in \mathbb{N}$ and let X be a set of n elements. Calculate the number of

   (i) relations on X.

  (ii) reflexive relations on X.

 (iii) symmetric relations on X.

 (iv) reflexive and symmetric relations on X.