Quantitative Methods

Components of interest rates
Interest rate = real risk-free rate + inflation premium + default risk premium + liquidity premium + maturity premium
Nominal interest rate = real risk-free rate + inflation premium

Stated annual rate does not consider the effect of compounding.
Effective annual rate considers compounding

With periodic compounding, $EAR = \left(1 + \frac{\text{stated annual rate}}{m}\right)^m - 1$

With continuous compounding, $EAR = e^{\text{stated annual rate}} - 1$

Future value: value to which an investment will grow after one or more compounding periods. $FV = PV\,(1 + I/Y)^N$
Present value: current value of some future cash flow.
$PV = FV / (1 + I/Y)^N$

Annuity: series of equal cash flows at regular intervals.
- **Ordinary annuity:** cash flows occur at the end of time periods.
- **Annuity due**: cash flows occur at the start of time periods.

Perpetuity: annuity with never ending cash flows. $PV = \frac{PMT}{I/Y}$

Net present value (NPV): present value of a project's cash inflows minus the present value of its cash outflows.
$NPV = CF0 + \left[\frac{CF1}{(1+r)^1}\right] + \left[\frac{CF2}{(1+r)^2}\right] + \left[\frac{CF3}{(1+r)^3}\right]$

Internal rate of return (IRR): discount rate that makes the NPV equal to zero.
$CF0 = \left[\frac{CF1}{(1+IRR)^1}\right] + \left[\frac{CF2}{(1+IRR)^2}\right] + \left[\frac{CF3}{(1+IRR)^3}\right]$

Holding period return: total return for holding an investment over a given time period. $HPR = \frac{P_1 - P_0 + I_1}{P_0}$

Money weighted rate of return: the IRR of a project.

Time weighted rate of return: compound growth rate at which \$1 invested in a portfolio grows over a given measurement period.

Arithmetic mean: sum of all the observations divided by the total number of observations. $\mu = \frac{\sum_{i=1}^{N} X_i}{N}$
- Mode: Most frequently occurring value in a distribution.
- Median: Midpoint of a data set that has been sorted into ascending or descending order.

Geometric mean: used to calculate compound growth rate.
$R_G = [(1 + R1)\,(1 + R2)\ldots\ldots(1 + Rn)]^{1/n} - 1$

Weighted mean: different observations are given different weights as per their proportional influence on the mean. $\overline{X}_w = \sum_{i=1}^{n} w_i X_i$

Harmonic mean: used to find average purchase price for equal periodic investments. $X_H = n / \sum_{i=1}^{n}\left(\frac{1}{X_i}\right)$

Position of a percentile in a data set: $L_y = (n+1)\, y\,/100$

Range = maximum value – minimum value

Mean absolute deviation (MAD): average of the absolute values of deviations from the mean. $MAD = [\sum_{i=1}^{n}|X_i - \overline{X}|]/n$

Variance: mean of the squared deviations from the arithmetic mean.

Population variance $\sigma^2 = \sum_{i=0}^{N}(X_i - \mu)^2 / N$
Sample variance $s^2 = \sum_{i=0}^{n}(X_i - \overline{X})^2 / (n-1)$

Standard deviation: square root of variance.

Coefficient of variation: measures the risk per unit of return; lower value is better. $CV = \frac{s}{\overline{X}}$
- Downside Deviation: A measure of the risk of being below a given target.

$$S_{Target} = \sqrt{\sum_{\substack{\text{for all } X_i \le B}}^{n} \frac{(X_i - B)^2}{n - 1}}$$

- Leptokurtic distribution: Fatter tails than a normal distribution and an excess kurtosis > 0.
- Platykurtic distribution: Thinner tails than a normal distribution and an excess kurtosis < 0.
- Mesokurtic distribution: identical to a normal distribution and has an excess kurtosis = 0.

Sharpe ratio: measures excess return per unit of risk; higher value is better. $S_p = \frac{\overline{R}_p - \overline{R}_F}{s_p}$

Odds for an event = P (E) / [1 – P (E)]
Odds against an event = [1 – P (E)] / P(E)

Multiplication rule: used to determine the joint probability of two events. $P\,(AB) = P\,(A\,|\,B)\,P\,(B)$
Addition rule: used to determine the probability that at least one of the events will occur. P (A or B) = P(A) + P(B) – P(AB)

Total probability rule: used to calculate the unconditional probability of an event, given conditional probabilities.
$P(A) = P(A|B_1)P(B_1) + P(A|B_2)P(B_2) + \ldots + P(A|B_n)P(B_n)$

Covariance: measure of how two variables move together.
Cov (X,Y) = E[X - E(X)] [Y - E(Y)]

Correlation: standardized measure of the linear relationship between two variables; covariance divided by product of two standard deviations.
Corr (X,Y) = Cov (X,Y) / σ (X) σ (Y)

Expected value of a random variable: probability-weighted average of the possible outcomes of the random variable.
$E(X) = X_1 P(X_1) + X_2 P(X_2) + \ldots + X_n P(X_n)$

Expected returns and the variance of a 2-asset portfolio
E (R_P) = w_1 E (R_1) + w_2 E (R_2)
$\sigma^2 (R_P) = w_1^2 \sigma_1^2 (R_1) + w_2^2 \sigma_2^2 (R_2) + 2w_1 w_2\, \rho\,(R_1, R_2)\,\sigma\,(R_1)\,\sigma\,(R_2)$

Bayes' formula: used to update the probability of an event based on new information. $P(E|I) = \frac{P(I|E)}{P(I)} \times P(E)$

Combination Formula:

$$_nC_r = \frac{n!}{(n-r)!\,r!}$$

Permutation Formula:

$$_nP_r = \frac{n!}{(n-r)!}$$

Expected value of a binomial variable = np
Variance of a binomial variable = np(1-p)

Probabilities for a binomial distribution $P(x) = {_nC_x}\, p^x (1-p)^{n-x}$

Probabilities for a continuous uniform distribution
$P(x_1 \leq X \leq x_2) = \frac{x_2 - x_1}{b - a}$

Normal distribution: completely described by mean (μ) and variance (σ^2). Has a skewness of 0 and a kurtosis of 3.
Confidence intervals for a normal distribution are:
- 90% of all observations are in the interval x ± 1.65s.
- 95% of all observations are in the interval x ± 1.96s.
- 99% of all observations are in the interval x ± 2.58s.

Computing Z-scores (std normal distribution): $Z = \frac{(X-\mu)}{\sigma}$

Safety first ratio: used to measure shortfall risk; higher number is preferred. $SF_{ratio} = \frac{E(R_P) - R_T}{\sigma_P}$

Continuously compounded rate of return=
$r_{t,t+1} = \ln(S_{t+1}/S_t) = \ln(1 + R_{t,t+1})$

Value at risk (VaR): Minimum value of losses expected over a specified time period.

Sampling error: difference between a sample statistic and the corresponding population parameter.
Sampling error of the mean $= \bar{x} - \mu$

Central limit theorem: if we draw a sample from a population with mean μ and variance σ^2, the sampling distribution of the sample mean:
- will be normally distributed.
- will have a mean of μ.
- will have a variance of σ^2/n.

Standard error of the sample mean: standard deviation of the distribution of the sample means.
- if population variance is known, $\sigma_{\bar{X}} = \frac{\sigma}{\sqrt{n}}$
- if population variance is unknown, $s_{\bar{X}} = \frac{s}{\sqrt{n}}$

Confidence intervals: range of values, within which the actual value of the parameter will lie with a given probability.
- if population variance is known, $CI = \bar{X} \pm z_{\alpha/2}\, \frac{\sigma}{\sqrt{n}}$
- if population variance is unknown, $CI = \bar{X} \pm t_{\alpha/2}\, \frac{s}{\sqrt{n}}$

Bootstrap: Resampling method that uses computer simulation for statistical inference by repeatedly drawing samples with replacement.

Jackknife: Samples are selected by leaving out one observation at a time from the set (and not replacing it).

Sampling distribution: If we draw samples of the same size several times and calculate the sample statistic. The sample statistic will be different each time. The distribution of values of the sample statistic is called a sampling distribution.

Null hypothesis (H$_0$): hypothesis that the researcher wants to reject. It should always include the 'equal to' condition.
Alternative hypothesis (H$_a$): hypothesis that the researcher wants to prove.

One-tailed tests: we are assessing if the value of a population parameter is greater than or less than a hypothesized value.
Two-tailed tests: we are assessing if the value of a population parameter is different from a hypothesized value.

Test statistic calculated from sample data and is compared to a critical value to decide whether or not we can reject the null hypothesis.
$z - \text{statistic} = \frac{\bar{X} - \mu_0}{\sigma/\sqrt{n}}$
$t - \text{statistic} = \frac{\bar{X} - \mu_0}{s/\sqrt{n}}$

Type I error: reject a true null hypothesis.
Type II error: fail to reject a false null hypothesis.

Level of significance (α) = (1 – level of confidence) = P(Type I error)
Power of a test = 1 – P(Type II error)

Types of test statistics
One population mean: use t-statistic or z-statistic
Two population mean: use t-statistic
One population variance: use Chi-square statistic
Two-population variance: use F-statistic

Regression equation: Y_i=b_0+b_1 X_i+ε_i
Simple linear regression model assumptions:
1. Linearity: The relationship between the dependent variable, Y, and the independent variable, X, is linear.
2. Homoskedasticity: The varian ce of the regression residuals is the same for all observations.
3. Independence: The observations, pairs of Ys and Xs, are independent of one another. This implies the regression residuals are uncorrelated across observations.
4. Normality: The regression residuals are normally distributed.

Slope coefficient:
$$\hat{b}_1 = \frac{\text{Covairance of Y and X}}{\text{Variance of X}} = \frac{\sum_{i=1}^{N}(Y_i - \bar{Y})(X_i - \bar{X})}{\sum_{i=1}^{N}(X_i - \bar{X})^2}$$
Coefficient of determination:
$$R^2 = \frac{\text{Explained variation}}{\text{Total variation}} = \frac{\text{Regression sum of squares (RSS)}}{\text{Sum of squares total (SST)}}$$

ANOVA table

Source of variation	Degrees of freedom	Sum of squares	Mean sum of squares	F-statistic
Regression (explained variation)	k	RSS	$MSR = \dfrac{RSS}{k}$	$F = \dfrac{MSR}{MSE}$
Error (unexplained variation)	n - 2	SSE	$MSE = \dfrac{SSE}{n-k-1}$	
Total variation	n – 1	SST		

Standard error of estimate (SEE) = $\sqrt{MSE}$

Test statistic to test whether an estimated slope coefficient is statistically significant:

$$t = \frac{\widehat{b_1} - B_1}{s_{\widehat{b_1}}} = \frac{\text{Estimated value} - \text{Hypothesized value}}{\text{standard error}}$$

where $s_{\widehat{b_1}} = \dfrac{SEE}{\sqrt{\sum_{i=1}^{n}(X_i - \overline{X})^2}}$

Estimated variance of the prediction error:

$$s_f^2 = s^2 * \left[1 + \frac{1}{n} + \frac{(X - \overline{X})^2}{(n-1)s_x^2}\right]$$

Steps to determine the confidence interval around the prediction:
1. Make the prediction.
2. Compute the variance of the prediction error.
3. Determine tc at the chosen significance level α.
4. Compute the (1-α) prediction interval using the formula below: Ŷ ±t_c*s_f

<u>Log-lin model</u>: The dependent variable is logarithmic but the independent variable is linear.
<u>Lin-log model</u>: The dependent variable is linear but the independent variable is logarithmic.
<u>Log-log model</u>: Both the dependent and independent variables are in logarithmic form.

Economics

Own price elasticity $= \dfrac{\text{\% change in quanitity demanded}}{\text{\% change in own price}}$
If |own price elasticity| > 1, then demand is elastic.
If |own price elasticity| < 1, then demand is inelastic.

Income elasticity $= \dfrac{\text{\% change in quanitity demanded}}{\text{\% change in income}}$
If income elasticity > 0, then good is a normal good.
If income elasticity < 0, then good is an inferior good.

Cross price elasticity $= \dfrac{\text{\% change in quanitity demanded}}{\text{\% change in price of related good}}$
If cross price elasticity > 0, then related good is a substitute.
If cross price elasticity < 0, then related good is a complement.

Giffen good: highly inferior good; upward sloping demand curve.
Veblen good: high status good; upward sloping demand curve.

Breakeven & shutdown points of production
Breakeven quantity is the quantity for which TR = TC.
If TR < TC, then the firm should shut down in the long run.
If TR< TVC, then the firm should shut down in the short run.

Market structures
<u>Perfect competition</u>: many firms, very low barriers to entry, homogenous products, no pricing power.
<u>Monopolistic competition</u>: many firms, low barriers to entry, differentiated products, some pricing power, heavy advertising.
<u>Oligopoly</u>: few firms, high barriers to entry, products may be homogeneous or differentiated, significant pricing power.
<u>Monopoly</u>: single firm, very high barriers to entry, significant pricing power, advertising used to compete with substitutes.
For all market structures, profit is maximized when MR = MC.

Concentration ratio
<u>N-firm</u>: sum of percentage market shares of industry's N largest firms.
<u>HHI</u>: sum of squared market shares of industry's N largest firms.

Gross domestic product (GDP)
<u>Expenditure approach</u>: GDP = $(C + G^C) + (I + G^I) + (X - M)$
<u>Income approach</u>: GDP = Gross domestic income (GDI) = Net domestic income + Consumption of fixed capital (CFC) + Statistical discrepancy
where:
Compensation of employees = wages and salaries including direct compensation in cash or in kind + employers' social contributions.
Gross operating surplus represents corporate profits of businesses.
Businesses includes private corporations, non-profit corporations, and government corporations.
Gross mixed income = farm income + non-farm income (excluding rent) + rental income.
Gross domestic income = Compensation of employees + Gross operating surplus + Gross mixed income + Taxes less subsidies on production + Taxes less subsidies on products and imports

Personal income = Compensation of employees + Net mixed income from unincorporated businesses + Net property income

Personal Income = National Income - Indirect business taxes - Corporate income taxes - Undistributed corporate profits (retained earnings) + Transfer payments (ex: unemployment benefits paid by governments to households)

Household disposable income (HDI) = Household primary income - Net current transfers paid.
Household net saving = HDI - Household final consumption expenditures + Net change in pension entitlements.

Nominal GDP includes inflation.
Real GDP removes the impact of inflation.
GDP deflator is a price index that can be used to convert nominal GDP into real GDP.
Relationship between saving, investment, the fiscal balance, and the trade balance: $(S - I) = (G - T) + (X - M)$

Quantity theory of money:
money supply ∗ velocity = price ∗ real output

Business cycle phases: expansion, peak, contraction, trough.

Theories of business cycle:
<u>Keynesian</u>: shifts in AD cause business cycles; downward sticky wages prevent recovery; monetary/ fiscal policy should be used to influence AD.
<u>New Keynesian</u>: in addition to wages, other production factors are also downward sticky.

Monetarist: inappropriate changes in money supply cause business cycle; money supply should be steady and predictable.
Austrian: government interventions cause business cycles; markets should be allowed to self-correct.
New classical: changes in technology and external shock cause business cycles; no policy action is necessary.

Unemployment types:
Frictional: caused by the time lag necessary to match employees seeking work with employers seeking their skills.
Long-term: People who have been out of work for a long time (more than three to four months in many countries) but are still looking for a job.
Indexes used to measure inflation:
Laspeyres: uses base year consumption basket.
Paasche: uses current year consumption basket.
Fisher: geometric mean of Laspeyres and Paasche.

Economic indicators: leading, coincident, lagging.

Money multiplier $= \frac{1}{\text{reserve requirement}}$

Fisher effect: $R_{\text{Nominal}} = R_{\text{Real}} + E[I]$

Expansionary or contractionary monetary policy
Neutral interest rate = real trend rate of economic growth + inflation target
If policy rate > neutral interest rate → contractionary monetary policy.
If policy rate < neutral interest rate → expansionary monetary policy.

Fiscal multiplier $= \frac{1}{1 - \text{MPC}(1 - \text{tax rate})}$

Expansionary or contractionary fiscal policy
If budget deficit increases → expansionary fiscal policy.
If budget deficit decreases → contractionary fiscal policy.

Types of trade restrictions
Tariffs: taxes imposed on imported goods by the government.
Quotas: restrictions on the amount of imports allowed in a country over some period.
Export subsidies: government incentives to exporting firms which artificially reduce cost of production.
Voluntary export restraint: agreements by exporting countries to voluntarily restrict exported amount to avoid tariffs or quotas imposed by trading partners.
Minimum domestic content: restrictions imposed to ensure certain portion of the product content is produced in the country.

Types of trading blocs and regional trading agreements
Free-trade area: all barriers to import and export of goods and services are removed.
Customs union: free-trade area + all member countries adopt a common set of trade restrictions with non-members.
Common market: customs union + all barriers to the movement of labor and capital goods are removed.
Economic union: common market + member countries establish common institutions and economic policy.
Monetary union: economic union + member countries adopt a single currency.

Balance of payments accounts
Current account: represents the flows related to goods and services.
Capital account: represents acquisition and disposal of non-produced, non-financial assets.
Financial account: represents investment flows.

Real exchange rate = nominal exchange rate x (base currency CPI / price currency CPI).

Forward rate = spot rate $(1 + \text{interest rate }_{\text{Price currency}}) / (1 + \text{interest rate }_{\text{Base currency}})$

Exchange rate regimes
Formal dollarization: country uses the currency of another currency.
Monetary union: several countries use a common currency.
Currency board system: an explicit commitment to exchange domestic currency for a specified foreign currency at a fixed exchange rate.
Fixed parity: a country pegs its currency within margins of ± 1% vs. another currency or basket of currencies.
Target zone: similar to a fixed parity but with wider bands (± 2%).
Crawling peg: allows for periodical adjustments in pegged exchange rate.
Crawling bands: width of bands used in fixed peg is increased over time to make the gradual transition from fixed parity to a floating rate.
Managed float: monetary authority attempts to influence the exchange rate but does not set any specific target.
Independently float: exchange rate is entirely market driven.

Financial Statement Analysis

Financial statement analysis framework
1. Define the purpose and context of the analysis.
2. Collect data.
3. Process the data.
4. Analyze and interpret the data.
5. Develop and communicate conclusions.
6. Follow up.

Inventory methods
First in first out (FIFO) assumes that the earliest items purchased are sold first.
Last in first out (LIFO) assumes that the most recent items purchased are sold first.
Weighted average cost averages total cost over total units available.
Specific identification identifies each item in the inventory and uses its historical cost for calculating COGS, when the item is sold.

Non-recurring items
Discontinued operations: An operation that the company has disposed in the current period or is planning to dispose in future.

Unusual or infrequent items: either unusual in nature or infrequent in occurrence, but not both.

Basic EPS $= \frac{\text{Net income} - \text{Preferred dividends}}{\text{Weighted average number of shares outstanding}}$

Diluted EPS
$$= \frac{\text{NI} + \text{Conv debt int }(1 - t) - \text{Pref div} + \text{Conv pref div}}{\text{Weighted average shares} + \text{New shares issued}}$$

Other comprehensive income: includes transactions that are not included in net income. Four types of items are:
- Unrealized gain/losses from available for sale securities.
- Foreign currency translation adjustments.
- Unrealized gains/losses on derivative contracts used for hedging.
- Adjustments for minimum pension liability.

Financial assets
<u>Measured at Fair value through profit or loss (FVTPL) under IFRS or Held-for-Trading under US GAAP:</u> measured at fair value; unrealized gains shown on Income Statement.
<u>Measured at Fair value through other comprehensive income (FVTOCI) under IFRS or available-for-sale under US GAAP:</u> measured at fair value; unrealized gains/losses shown in OCI.
<u>Measured at Cost or Amortised Cost:</u> measured at cost or amortized cost; unrealized gains not recorded anywhere.

Direct method of computing CFO: take each item from the income statement and convert to cash equivalent by removing the impact of accrual accounting. The rules to adjust are:
- Increase in assets is use of cash (-ve adjustment).
- Decrease in asset is source of cash (+ve adjustment).
- Increase in liability is source of cash (+ve adjustment).
- Decrease in liability is use of cash (-ve adjustment).

Indirect method of computing CFO: CFO is obtained from reported net income through a series of adjustments.
- Begin with net income.
- Add back all noncash charges to income and subtract all noncash components of revenue.
- Subtract any gains that resulted from financing or investing cashflows.
- Add or subtract changes to related balance sheet operating accounts.

Free cash flow to the firm (FCFF)
FCFF = NI + NCC + Int (1 - Tax rate) – FCInv – WCInv
FCFF = CFO + Int (1 - Tax rate) – FCInv

Free cash flow to equity (FCFE)
FCFE = CFO – FCInv + Net borrowing

Common size analysis
- Common-size balance sheet expresses each balance sheet account as a percentage of total assets.
- Common-size income statement expresses each item as a percentage of sales.
- Common size cash flow statement expresses each item as a percentage of total cash inflows/outflows or as a percentage of sales.

Activity ratios: measure the efficiency of a company's operations
Inventory turnover = COGS / average inventory
Receivables turnover = revenue / average receivables
Payables turnover = purchases / average trade payables

Days of inventory on hand = 365 / inventory turnover
Days of sales outstanding = 365 / receivables turnover
Number of days of payable = 365 / payables turnover

Cash conversion cycle = days of inventory on hand + days of sales outstanding – number of days of payables

Liquidity ratios: measure a company's ability to meet short-term obligations.
Current ratio = current assets / current liabilities
Quick ratio = (cash + short term marketable investments + receivables) / current liabilities
Cash ratio = (cash + short term marketable investments) / current liabilities
Defensive interval ratio = (cash + short term marketable investments + receivables) / daily cash expenditures

Solvency ratios: measure a company's ability to meet long term obligations.
Debt to assets ratio = total debt / total assets
Debt to equity ratio = total debt / total shareholder's equity
Financial leverage ratio = average total assets / average total equity
Profitability ratios: measure the ability of a company to generate profits.
Gross profit margin = gross profit / revenue
Operating profit margin = operating profit / revenue
Net profit margin = net profit / revenue
Return on assets (ROA) = net income / average total assets
Return on equity (ROE) = net income / average total equity
Return on total capital = EBIT/(Average short term and long term debt Debt+equity)

Credit Analysis Ratio:
EBITDA interest coverage = EBITDA / interest payments
FFO (Funds from operations) to debt = FFO / Total debt
Free operating cash flow to debt = CFO (adjusted) minus capital expenditures / Total debt
EBIT margin = EBIT / Total revenue
EBITDA margin = EBITDA / Total revenue
Debt-to-EBITDA Ratio = Total debt / EBITDA
Return on capital = EBIT / Average beginning-of-year and end-of-year capital
Valuation ratios: express the relation between the market value of a company or its equity.
P/E = price per share / earnings per share
P/CF = price per share / cash flow per share
P/S = price per share / sales per share
P/BV = price per share / book value per share

DuPont analysis decomposes a firm's ROE to better analyze a firm's performance.

$$ROE = \left(\frac{net\ income}{sales}\right)\left(\frac{sales}{assets}\right)\left(\frac{assets}{equity}\right)$$

ROE = (net profit margin)(asset turnover)(leverage ratio)
ROE = ROA x Leverage

LIFO v/s FIFO: When prices are rising, and inventory levels are stable or increasing, as compared to FIFO, LIFO results in higher COGS, lower taxes, lower net income, lower ending inventory.

LIFO reserve is the difference between reported LIFO inventory and the amount that would have been reported in inventory if the FIFO method had been used.
LIFO liquidation occurs when the number of units in ending inventory is less than the number of units in the beginning inventory.

Conversion from LIFO to FIFO
FIFO inventory = LIFO inventory + LIFO reserve
FIFO COGS = LIFO COGS – (ending LIFO reserve – beginning LIFO reserve)

FIFO NI = LIFO NI + change in LIFO reserve (1 - T)
FIFO retained earnings = LIFO retained earnings + LIFO reserve (1 – T)

Capitalizing v/s expensing an asset
As compared to expensing, capitalizing an asset results in higher total assets, higher equity, lower income variability, higher CFO, lower CFI, lower debt/equity.

Depreciation methods:
Straight line depreciation expense = depreciable cost / estimated useful life
DDB depreciation expense = 2 x straight-line rate x beginning book value
Units of production depreciation expense per unit = depreciable cost / useful life in units

Financial reporting of leases from a lessee's (entity using the asset) perspective:
Under IFRS: Single accounting model for both finance and operating leases for lessees.
- Recognize a lease liability and corresponding right-of-use asset on the balance sheet, both equal to the present value of lease payments.
- The liability is subsequently reduced using the effective interest method
- The right-of-use asset is amortized, often on a straight-line basis over the lease term.
- Interest and amortization expenses are shown separately on the income statement.
- The principal repayment component is reported as cash outflow under financing activities. The interest expense can be reported under either operating or financing activities.

Under US GAAP: Two accounting models for lessees: one for finance leases and another for operating leases.
The finance lease accounting model is the same as the lease accounting model for IFRS.
The operating lease accounting model is different:
- Recognize a lease liability and corresponding right-of-use asset on the balance sheet, both equal to the present value of lease payments.
- The liability is subsequently reduced using the effective interest method
- But the amortization of the right-of-use asset is the lease payment less the interest expense.
- Interest expense and amortization expense are shown together as a single operating expense on the income statement.
- The entire lease payment is reported as cash outflow under operating activities.

Financial reporting of leases from a lessor's perspective.
Finance lease lessors (IFRS and US GAAP)
- Recognize a lease receivable asset equal to the present value of future lease payments and de-recognize the leased asset, simultaneously recognizing any difference as a gain or loss.
- The lease receivable is subsequently reduced by each lease payment using the effective interest method.
- Interest income is reported on the income statement, typically as revenue.
- The entire cash receipt is reported under operating activities on the statement of cash flows.

Operating lease lessors (IFRS and US GAAP)
- The balance sheet is not affected: the lessor continues to recognize the underlying asset and depreciate it.
- Lease revenue is recognized on a straight-line basis on the income statement.
- The entire cash receipt is reported under operating activities on the statement of cash flows.

Deferred tax assets are created when income tax payable is greater than income tax expense. If DTA is not expected to reverse, increase valuation allowance.

Deferred tax liabilities are created when income tax expense is greater than income tax payable. If DTL is not expected to reverse, treat it as equity.

Under the **effective interest rate method**, interest expense = book value of the bond liability at the beginning of the period x market rate of interest at issuance. The interest expense includes amortization of any discount or premium at issuance.

Pension plans
Defined contribution plan: cash payment made into the plan is recognized as pension expense on the income statement.
Defined benefits plan: companies must report the difference between the defined benefit pension obligations and the pension assets as an asset or liability on the balance sheet.

Corporate Issuers

Corporate governance refers to the system of controls and procedures by which individual companies are managed.

A **board of directors** is the central pillar of corporate governance. It is elected by shareholders to act in their interests. A board can have several committees that are responsible for specific functions. For example, audit committee, governance committee, remuneration committee, nomination committee, risk committee, investment committee.

ESG Investment Style	Description
Negative screening	Excluding certain sectors or companies or practices from a fund or portfolio based on specific ESG criteria.
Positive screening	Including certain sectors, companies, or practices in a fund or portfolio based on specific ESG criteria.
ESG integration	Refers to the practice of including material ESG factors in the investment process.
Thematic investing	This strategy picks investments based on a theme or single factor, such as energy efficiency or climate change.
Engagement/ active ownership	This strategy involves achieving targeted social or environmental objectives along with measurable financial returns by using shareholder power to influence corporate behavior.
Impact investing	Investments made with the intention to generate positive, measurable social and environmental impact alongside a financial return.

Capital allocation is the process that companies use for decision-making on capital investments i.e., investments with a life of a year or more. Basic principles of capital allocation are:
1. Decisions are based on cash flows.
2. Cash flows are not accounting net income or operating income.
3. Cash flows are based on opportunity cost
4. Cash flows are analyzed on an after-tax basis
5. Timing of cash flows is vital
6. Financial costs are ignored

$$\textbf{WACC} = w_d r_d (1 - t) + w_p r_p + w_e r_e$$

$$NPV = CF0 + \left[\frac{CF1}{(1+r)^1}\right] + \left[\frac{CF2}{(1+r)^2}\right] + \cdots + \left[\frac{CF(t)}{(1+r)^t}\right]$$

Decision rule:
For independent projects:
If NPV > 0, accept.
If NPV < 0, reject.
For mutually exclusive projects: Accept the project with higher and positive NPV.

IRR is the discount rate which makes NPV equal to 0.
Decision rule:
For independent projects:
If IRR > required rate of return (usually firms cost of capital adjusted for projects riskiness), accept the project.
If IRR < required rate of return, reject the project.
For mutually exclusive projects: Accept the project with higher IRR (as long as IRR > cost of capital).
Types of real options include:
1. Timing options
2. Sizing options - abandonment options or growth options
3. Flexibility options - price-setting options or production-flexibility options
4. Fundamental options
If NPV is positive without considering options, go ahead and make the investment.
If NPV is negative without considering options, calculate NPV (based on DCF alone) – Cost of options + Value of options.

Calculating cost of debt
The yield to maturity (YTM) approach: annualized return an investor earns for holding a bond till maturity.
Debt rating approach: use matrix pricing on comparable bonds.

Cost of preferred stock = preferred dividend / market price of preferred shares

Calculating cost of equity
Capital asset pricing model: $r_e = RFR + \beta\,[E\,(R_m) - RFR]$
Dividend discount model: $r_e = \frac{D_i}{P_o} + g$
Bond yield plus risk premium: r_e = bond yield + risk premium

Pure play method
Derive asset beta for comparable company
$$\beta_{asset} = \beta_{equity} * \frac{1}{1 + \frac{(1-t)D}{E}}$$
Derive the equity levered beta for the project
$$\beta_{equity} = \beta_{asset} * \left(1 + \frac{(1-t)D}{E}\right)$$

Degree of operating leverage (DOL) measures operating risk. It is the ratio of the percentage change in operating income to the percentage change in quantity sold.
$$DOL = \frac{Q(P - V)}{Q(P - V) - F} = \frac{S - TVC}{S - TVC - F}$$

Degree of financial leverage (DFL) measures financial risk. It is the ratio of percentage change in earnings per share to percentage change in operating income.
$$DFL = \frac{Q(P - V) - F}{Q(P - V) - F - I} = \frac{EBIT}{EBIT - interest}$$

Degree of total leverage (DTL) combines DOL and DFL. It is the ratio of percentage change in earnings per share to percentage change in units sold.

$$DTL = \frac{Q(P - V)}{Q(P - V) - F - I} = \frac{S - TVC}{S - TVC - F - I}$$

Breakeven quantity of sales is the quantity of units sold to earn revenue equal to the fixed and variable costs i.e. for net income to be 0.
$$Q(BE) = \frac{Fixed\ operating\ costs\ +\ fixed\ financing\ costs}{Price\ per\ unit\ -\ variable\ cost\ per\ unit}$$
Operating breakeven quantity of sales ignores the fixed financing costs i.e. quantity sold for operating income to be 0.
$$Q(OBE) = \frac{Fixed\ operating\ costs}{Price\ per\ unit\ -\ variable\ cost\ per\ unit}$$
Financing Options Available to a Company

Internal	External		
	Financial intermediaries	Capital markets	Other
▪ **After-tax operating cash flows** ▪ **Accounts payable** ▪ **Accounts receivable** ▪ **Inventory & marketable securities**	▪ Uncommitted lines of credit ▪ Committed lines of credit ▪ Revolving credit ▪ Secured loans ▪ Factoring	▪ Commercial paper ▪ Public & private debt ▪ Hybrid securities – preferred equity, convertibles ▪ Common equity	▪ Leasing

Primary sources: Cash sources used in day-to-day operations (e.g., cash balances, trade credit, lines of credit from bank).
Secondary sources: Impacts the day-to-day operations, alter the financial structure, and may indicate deteriorating financial condition (e.g., liquidating assets, filing for bankruptcy, negotiating debt agreements).
Drags on liquidity delay cash inflows (e.g., bad debts, obsolete inventory, uncollected receivables).
Pulls on liquidity accelerate cash outflows (e.g., earlier payment of vendor dues).

Equity

Types of financial intermediaries
1. Brokers, exchanges, and alternative trading systems
2. Securitizers
3. Depository institutions
4. Insurance companies
5. Clearinghouse
6. Depositories or custodians
7. Arbitrageurs

$$\textbf{Leverage ratio} = \frac{value\ of\ position}{value\ of\ equity\ investment\ in\ that\ position}$$

$$\textbf{Margin call price}\ =\ initial\ purchase\ price\ \times \frac{(1 - initial\ margin)}{(1 - maintenance\ margin)}$$

Execution instruction: specifies how the order will be filled. Types are: market orders, limit orders, all or nothing orders, hidden orders, iceberg orders.
Validity instruction: specifies when the orders may be filled. Types are: day orders, good-till-cancelled orders, immediate or cancel orders, good-on-close orders, stop-loss orders.
Clearing instructions convey who is responsible for clearing and settling the trade.

Types of markets
Quote driven markets: trade takes place at the price quoted by dealers who maintain an inventory of the security.
Order driven markets: trading rules match buyers to sellers, thus making them supply liquidity to each other.
Brokered markets: brokers arrange trades between counterparties.

Characteristics of a well-functioning financial system
Operationally efficient markets where trading costs like commissions, bid-ask spreads and price impacts are low, increases market pricing efficiencies.
Informational efficient markets allow for absorption of timely financial disclosures making the prices a close reflection of the fundamental values.
Allocationally efficient markets allow for better utilization of capital by allocating it to the most productive use.

A **security market index** serves as a benchmark that investors can use to track, measure and compare performance. Index returns can be calculated using two methods:
Price return = (index end value – index start value)/ index start value
Total return = (index end value – index start value + income earned over the holding period)/ index start value

Different weighting methods used in index construction
Price weighting: weights are the arithmetic averages of the prices of constituent securities.

$$\text{Price weighted index} = \frac{\text{Sum of Stock Prices}}{\text{No. of stocks in index adjusted for splits}}$$

Equal weighted index: weights are the arithmetic averages of the returns of constituent securities.
Market capitalization weighted index: weight of each security is determined by dividing its market capitalization with total market capitalization.
Fundamental weighing: weights are based on fundamental parameters such as earnings, book value, cash flow, revenue, and dividends.

Forms of market efficiency
Weak form: prices reflect only past market data.
Semi-strong form: prices reflect past market data + public information.
Strong form: prices reflect past market data + public information + private information.

Industry classification systems: The three main methods for classifying companies are,
- Products and/or services offered
- Business cycle sensitivities
  - Cyclical: earnings dependent on the stage of the business cycle
  - Non-cyclical: earnings relatively stable over the business cycle
- Statistical similarities

A **peer group** is a set of comparable companies engaged in similar business activities. They are influenced by the same set of factors.

Porter's five forces: the profitability of companies in an industry
is determined by: (1) threat of new entrants, (2) bargaining power of suppliers, (3) bargaining power of buyers, (4) threat of substitutes, (5) intensity of rivalry among existing competitors.

Industry life-cycle phases:
Embryonic: slow growth, high prices, requires significant investment, high risk.

Growth: rapidly increasing demand, profitability improves, prices fall, and competition is low.
Shakeout: growth starts slowing down, competition is intense, profitability declines.
Mature: little or no growth, industry consolidates, barriers to entry are high.
Decline: growth is negative, excess capacity, high competition.
External factors that influence an industry include: technology, demographics, government, social factors and macroeconomic influences.

Dividend discount models: value is estimated as the present value of expected future dividends plus the present value of a terminal value.

$$V_0 = \sum\nolimits_{t=1}^{n} \frac{D^t}{(1+r)^t} + \frac{P^n}{(1+r)^n}$$

Free cash flow to equity models: value is estimated as the present value of expected future free cash flow to equity.
FCFE = CFO – FCInv + net borrowing

$$V_0 = \sum\nolimits_{t=1}^{\infty} * \left(\frac{FCFE^t}{(1+r)^t} \right)$$

Gordon growth model: assumes that dividends will grow indefinitely at a constant growth rate.

$$V_0 = \frac{D_1}{r-g}$$

Multi-stage dividend discount model: used for companies with high growth rate over an initial few number of periods followed by a constant growth rate of dividends forever.

$$V_0 = \sum\nolimits_{t=1}^{n} \frac{D_0(1+g_s)^t}{(1+r)^t} + \frac{V_n}{(1+r)^n}$$

$$V_n = \frac{D_{n+1}}{r-g}$$

Multiples based on fundamentals: tell us what a multiple should be based on some valuation models.

$$\text{Forward P/E} = P_0/E_1 = \frac{\frac{D_1}{E_1}}{r-g} = \frac{\text{dividend payout ratio}}{r-g}$$

Multiples based on comparables: compares the stock's price multiple to a benchmark value based on an index or with a peer group. Commonly used price multiples are P/E, P/CF, P/S, P/BV.

Enterprise value = market value of debt +market value of equity – cash and short-term investments.

Fixed Income
Basic features of a fixed-income security
Issuer: Entity issuing the bond. Bonds can be issued by supranational organizations, sovereign governments, non-sovereign governments, quasi-government entities, corporate issuers.
Maturity date: Date when issuer will pay back principal (redeem bond)
- Money market securities: original maturity is one year or less.
- Capital market securities: original maturity is more than a year.
Par value: Principal amount that is repaid to bond holders at maturity.
- Premium bond: market price > par value
- Discount bond: market price < par value
- Par bond: market price = par value
Coupon rate: percentage of par value that the issuer agrees to pay to the bondholder annually as interest; coupon rate can be fixed or floating; coupon frequency may be annual, semi-annual, quarterly or monthly.

Currency denomination: bonds can be issued in any currency. Dual currency bonds pay interest in one currency and principal in another currency.

Places where bonds are issued and traded

- Bonds issued in a particular country in local currency are domestic bonds if they are issued by entities incorporated in the country and foreign bonds if they are issued by entities incorporated in another country.
- Eurobonds are issued internationally, outside the jurisdiction of any single country and are denoted in currency other than that of the countries in which they trade.
- Global bonds are issued in the Eurobond market and at least one domestic market simultaneously.

Cash flows of fixed-income securities

Bullet structure: pays coupon periodically and entire payment of principal occurs at maturity.

Fully amortized bond: regular payments include both interest and principal; outstanding principal amount is reduced to zero by the maturity date.

Partially amortizing bond: regular payments include both interest and principal; balloon payment is required at maturity to repay the remaining principal as a lump sum.

Sinking fund agreements: issuer is required to retire a portion of the bond issue at specified times during the bond's life.

Floating rate notes (FRN): coupon is set based on some reference rate plus a spread.

Contingency provisions

Callable bond: gives the issuer the right to redeem the bond prior to maturity at a specified call price; call provision lowers price.

Putable bond: gives the bondholder the right to sell bonds back to the issuer prior to maturity at a specified put price; put provision increases price.

Convertible bond: gives the bondholder the right to convert the bond into common shares of the issuing company; increases price.

Mechanisms available for issuing bonds

Underwritten offerings: investment bank buys the entire issue and takes the risk of reselling it to investors or dealers.

Best effort offerings: investment bank serves only as a broker and sells the bond issue only if it is able to do so.

Shelf registrations: issuer files a single document with regulators that allows for additional future issuances.

Auction: price discovery through bidding.

Private placement: entire issue is sold to a qualified investor or to a group of investors.

Relationships among a bond's price, coupon rate, maturity, and market discount rate (yield-to-maturity)

A bond's price moves inversely with its YTM.

coupon rate > market discount rate → premium.

coupon rate < market discount rate → discount.

price of a longer-term bond is more volatile than the price of a shorter-term bond.

Internal credit enhancements: include

- Senior/junior structure
- Overcollateralization
- Excess spread

External credit enhancements: Relies on a third party called a guarantor, to provide a guarantee. These include

- Surety bonds/bank guarantees
- Letter of credit

Bond pricing

Using market discount rate: bond's price is the present value of its future cash flows, discounted at the bond's market discount rate, also called YTM.

Using spot rates:

$$PV = \frac{PMT}{(1+Z_1)^1} + \frac{PMT}{(1+Z_2)^2} + \cdots + \frac{PMT+FV}{(1+Z_N)^N}$$; where Z_N are the spot rates.

Full price or dirty price = flat price + accrued interest

$$\text{Accrued interest} = \frac{t}{T} * PMT$$

Matrix pricing: method used to value illiquid bonds by using prices and yields on comparable securities.

Stated Annual Rate: The formula for conversion based on periodicity is

$$\left(1 + \frac{APR_m}{m}\right)^m = \left(1 + \frac{APR_n}{n}\right)^n$$

Yield measures:

Street convention yield: assumes payments are made on scheduled dates, neglecting weekends and holidays for simplicity.

True yield: is the yield-to-maturity calculated using an actual calendar. Here we consider weekends and holidays.

Current yield is the annual coupon payment divided by the flat price.

Simple yield: adjusts the current yield by using straight-line amortization of the discount or premium.

Yield to call: assumes bond will be called.

Yield to worst: lower of YTM and YTC.

Money market yields: are quoted on a discount rate or add-on rate basis.

Bond equivalent yield: is an add-on rate based on a 365-day year.

Price of a money market instrument quoted on a discount basis

$$PV = FV \times \left(1 - \frac{\text{days to maturity}}{\text{year}} \times DR\right)$$

Money market discount rate

$$\text{Money market discount rate } DR = \left(\frac{\text{Year}}{\text{days to maturity}}\right) * \frac{FV-PV}{FV}$$

Present value or price of a money market instrument quoted on an add-on basis

$$PV = \frac{FV}{1 + \frac{\text{Days to maturity}}{\text{Year}} \times AOR}$$

Add-on rate

$$AOR = \left(\frac{\text{Year}}{\text{Days}}\right) * \frac{FV-PV}{PV}$$

Relationship between AOR and DR

$$AOR = \frac{DR}{1 - \frac{\text{Days to maturity}}{\text{Year}} * DR}$$

The factors that affect the repo rate include:

- The risk of the collateral
- Term of the repurchase agreement
- Delivery requirement
- Supply and demand
- Interest rates of alternative financing

A **forward rate** is a lending or borrowing rate for a short term loan to be made in the future. Implied **spot rates** can be calculated as geometric averages of forward rates.

Yield spread

G-spread: benchmark is yield-to-maturity on government bonds.

I-spread: benchmark is a swap rate.

Z-spread (zero-volatility spread): the constant spread that is added to each spot rate to make the present value of the bond equal to its price.

Option-adjusted spread (OAS): adjusts Z-spread by the value of the embedded option.

Securitization refers to a process in which financial assets such as mortgages, loans or receivables are pooled together. Securities are issued that are backed by this pool, called ABS.

Credit tranching: focus is on redistribution of credit risk.

Time tranching: focus is on redistribution of prepayment risk.

Prepayment risk has two components:

Contraction risk: faster prepayments.

Extension risk: slower prepayments.

Types of ABS

Agency RMBS: backed by conforming home mortgages.

Non-agency RMBS: backed by nonconforming home mortgages.

CMO: backed by RMBS, has multiple tranches.

CMBS: backed by commercial mortgages.

Auto loan ABS: backed by auto loans.

Credit card ABS: backed by credit card receivables.

CDO: backed by a diversified pool of one or more debt obligations.

Covered bonds have lower credit risks and offer lower yields than otherwise similar ABS. They differ from ABS because of their: dual recourse nature, balance sheet impact, dynamic cover pool, and redemption regimes in the event of sponsor default.

Changes in interest rate affects the realized rate of return for any bond investor in two ways:

Market price risk: bond price decreases when the interest rate goes up.

Coupon reinvestment risk: value of reinvested coupons increases when the interest rate goes up.

For short term horizon, market price risk dominates. For long term horizon, coupon reinvestment risk dominates.

Macaulay duration: Time horizon at which market price risk exactly offsets coupon reinvestment risk. Also interpreted as the weighted average of the time to receipt of coupon interest and principal payments.

Duration gap = Macaulay duration – Investment horizon

Modified duration: linear estimate of the percentage price change in a bond for a 100 basis points change in its yield-to-maturity.

Modified duration = macaulay duration / (1 + r)

$$\text{Approximate modified duration} = \frac{(PV_-) - (PV_+)}{2 * \Delta\, yield * PV_0}$$

Effective duration: linear estimate of the percentage change in a bond's price that would result from a 100 basis points change in the benchmark yield curve. Used for bonds with embedded options.

$$\text{Effective duration} = \frac{(PV_-) - (PV_+)}{2 * \Delta curve * PV_0}$$

Key rate duration is a measure of the price sensitivity of a bond to a change in the spot rate for a specific maturity.

Price value of a basis point (PVBP) is an estimate of the change in the price of a bond given a 1 basis point change in the yield-to-maturity. $PVBP = \frac{PV_- - PV_+}{2}$

Convexity refers to the curvature of a bond's price-yield relationship. $\text{Approximate convexity} = \frac{PV_- + PV_+ - 2*PV_0}{(\Delta yield)^2 * PV_0}$

Effective convexity, like effective duration, is useful for bonds with embedded options. $\text{Effective convexity} = \frac{PV_- + PV_+ - 2*PV_0}{(\Delta curve)^2 * PV_0}$

Duration + convexity effect:

$\% \Delta\, PV^{FULL} = (-AnnModDur * \Delta yield) + [\frac{1}{2} * AnnConvexity * (\Delta yield)^2]$

Credit risk has two components:

Risk of default: probability that the borrower will default.

Loss severity: if the borrower does default, how severe is the loss.

Expected loss = default probability x loss severity given default

Notching refers to the practice of adjusting an issue credit rating upward or downward from the issuer credit rating, to reflect the seniority or other provisions in that specific issue.

Four Cs of traditional credit analysis: capacity, collateral, covenants, and character.

Derivatives

Exchange-traded derivatives: standardized, highly regulated, transparent and free of default.

Over-the-counter derivatives: customized, flexible, less regulated than exchange-traded derivatives, but are not free of default risk.

Forward commitment: obligation to buy or sell an asset or make a payment in the future.

Contingent claim: has a future payoff only if some future event takes place.

Types of derivatives: forward contracts, future contracts, options, swaps, credit derivatives.

Call option payoff, profit at expiration

- Call buyer payoff: $c_T = Max(0, S_T - X)$
- Call seller payoff: $-c_T = -Max(0, S_T - X)$
- Call buyer profit: $Max(0, S_T - X) - c_0$
- Call seller profit: $\Pi = -Max(0, S_T - X) + c_0$

Put option payoff, profit at expiration

- Put buyer payoff: $p_T = Max(0, X - S_T)$
- Put seller payoff: $-p_T = -Max(0, X - S_T)$
- Put buyer profit: $\Pi = Max(0, X - S_T) - p_0$
- Put seller profit: $\Pi = -Max(0, X - S_T) + p_0$

Arbitrage-free pricing: a derivative must be priced such that no arbitrage opportunities exist, and there can only be one price for the derivative that earns the risk-free return.

asset + derivative = risk-free asset

Price v/s Value

The price of a forward or futures contract is the forward price that is specified in the contract.

$F_0(T) = (S_0) \times (1 + r)^T - (\gamma - \theta) \times (1 + r)^T$

The value of a forward or futures contract is zero at initiation. Its value may increase or decrease during its life according to changes in the spot price.

$$V_t(T) = S_t - (\gamma - \theta)(1 + r)t - \frac{F_0(T)}{(1+r)^{T-t}}$$

Moneyness refers to whether an option is in the money or out of the money.

Exercise value of an option is the maximum of zero and the amount that the option is in the money.

Time value of an option is the amount by which the option premium exceeds the exercise value.

Factors that determine the value of an option

Increase in	Value of call	Value of put
Value of underlying	Increase	Decrease
Exercise price	Decrease	Increase
Risk-free rate	Increase	Decrease
Time to expiration	Increase	Increase (except for deep in the money European options)
Volatility	Increase	Increase
Holding costs	Increase	Decrease
Holding benefits	Decrease	Increase

The lowest price of the call option is given by:

$$c_0 >= \max (0, S_0 - \frac{X}{(1+r)^T})$$

The lowest price for a put option is given by:

$$p_0 >= \max (0, \frac{X}{(1+r)^T} - S_0)$$

Put–call parity for European options
fiduciary call = protective put

$$c_0 + \frac{X}{(1 + r)^T} = p_0 + S_0$$

Alternative Investments

The three methods of investing in alternative investments are:
- Fund investing: The investor contributes capital to a fund, and the fund makes investments on the investors' behalf, e.g., investments in a PE fund.
- Co-investing: The investor can make investments alongside a fund, e.g., investments in a portfolio company of a fund.
- Direct investing: The investor makes a direct investment in a company or project without the use of an intermediary, e.g., direct investments in infrastructure or real estate assets.

Hedge funds
Types
Event-driven**:** includes merger arbitrage, distressed/restructuring, activist shareholder and special situation.
Relative value: strategies that seek to profit from pricing discrepancies.
Macro: strategies based on top-down analysis of global economic trends.
Equity hedge: strategies based on bottom-up analysis. Includes market neutral, fundamental growth, fundamental value, quantitative directional, and short bias.

Hedge fund fees
Common fee structure is 2 and 20 which means 2% management fee and 20% incentive fee. Sometimes, the incentive fee is paid only if the returns exceed a hurdle rate. In some cases, the incentive fee

is paid only if the fund has crossed the high watermark. High watermark is the highest value net of fees reported by the fund so far.

Private equity categories include leveraged buyouts and venture capital.
Leveraged buyouts (LBOs) include:
- Management buyouts: existing management team is involved in the purchase.
- Management buy-ins: external management team replaces the current management.
Stages in venture capital include:
- Formative stage: consists of angel investing, seed and early stages.
- Later stage: company is in expansion phase.
- Mezzanine stage: company is preparing for an IPO.

Exit strategies include:
- Trade sale: company is sold to a competitor or another strategic buyer.
- IPO: company is sold to the public.
- Recapitalization: Increases leverage or introduces it to the company. Re-leverages itself when interests are low and pays itself a dividend.
- Secondary sale: company is sold to another private equity firm or another investor.
- Write off/ liquidation: worst case scenario, company is sold at a loss.

Real estate
Includes private as well as public investments and equity as well as debt investments.

Investment characteristics of real estate are as follows:
- Indivisibility – requires large capital investments
- Illiquidity
- Unique characteristics (no two properties are identical).
- Fixed location.
- Requires professional operational management.
- Local markets can be very different from national or global markets.

Natural resources include commodities, farmland, and timberland.
Commodity: Investments take place through derivative instruments. The return on commodity investment is based mainly on price changes rather than an income stream such as dividends.
Timberland provides an income stream through the sale of trees, wood, and other timber products. Itis considered a sustainable investment that mitigates climate-related risks.
Farmland also provides an income component related to harvest quantities and agricultural commodity prices. It does not provide production flexibility, as farm products must be harvested when ripe.

Infrastructure
Includes real assets that are planned for public use and to provide essential services. They are typically capital intensive and long-lived.

Performance appraisal of alternative investments.
- Traditional risk and return measures (such as the Sharpe ratio) are not always appropriate for alternative investments.
- Many metrics are used to evaluate the performance of alternative investments such as: the Sharpe ratio, Sortino ratio, Treynor ratio, Calmar ratio, MAR ratio, batting average, and slugging performance.

- The IRR calculation is frequently used to evaluate private equity investments.
- The cap rate is frequently used to evaluate real estate investments.
- Leverage, illiquidity, and redemption pressure pose special challenges while evaluating hedge funds' performance.

Portfolio Management

Portfolio management process has three phases:
1. Planning
2. Execution
3. Feedback

Asset managers are usually referred to as a **buy-side firm** since it uses (buys) the services of sell-side firms.

The three key trends in the asset management industry include growth of passive investing, "Big Data" in the investment process, and robo-advisers (use of automation and investment algorithms) in the wealth management industry.

The portfolio having the least risk (variance) among all the portfolios of risky assets is called the **global minimum-variance portfolio.**

The part of minimum-variance frontier above the global minimum-variance portfolio is called the **efficient frontier.**
Drawing a line tangent from the risk free asset to the efficient frontier will give the **capital allocation line (CAL).**
The point where this line intersects the efficient frontier is called the **optimal risky portfolio.**
The **optimal investor portfolio** is the point at which the investor's indifference curve is tangential to the CAL line.
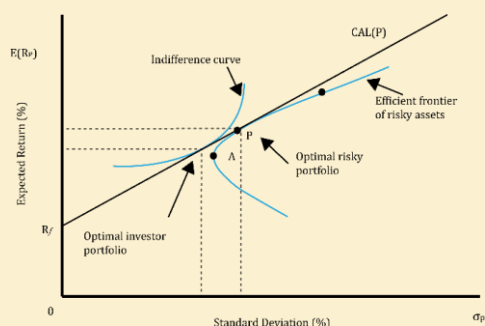
Under **homogeneity of expectations**, all investors have the same efficient frontier. Thus, all investors have the same optimal risky portfolio and CAL. This optimal CAL, using homogenous expectations, is the **capital market line (CML)**. The optimal risky portfolio is called the **market portfolio.**

Total risk (standard deviation) = systematic risk (non-diversifiable) + unsystematic risk (diversifiable)

Money-weighted return is the internal rate of return on money invested that considers all the cash inflows and cash outflows. It is similar to the internal rate of return (IRR).

Time-weighted rate of return is the compound growth rate at which $1 invested in a portfolio grows over a given measurement period.

Unlike time-weighted rate of return, money-weighted rate of return is impacted by the timing and amount of cash flows.

If the portfolio manager does not control the timing and amount of investment, then the time-weighted return should be used.

Beta is a standardized measure of covariance of an asset's return with the market returns.
$$\beta_i = \frac{Cov(i,M)}{\sigma_M^2} = \frac{\rho_{iM}*\sigma_i*\sigma_M}{\sigma_M^2} = \frac{\rho_{iM}*\sigma_i}{\sigma_M}$$
SML plots returns versus systematic risk i.e. beta on the x-axis. The equation of the line is given by CAPM.
- Securities on the SML line (CAPM) → fairly valued.
- Securities above the SML line → undervalued.
- Securities below the SML line → overvalued.

CML: $R_p = R_f + (\frac{R_m - R_f}{\sigma_m})* \sigma_p$

$r_e = R_f + \beta[E(R_{mkt}) - R_f]$

Slope of the CML is the Sharpe ratio

Slope of the SML is the market risk premium

The four measures commonly used in performance evaluation are:

Sharpe ratio $= \frac{\text{portfolio excess returns}}{\text{portfolio total risk}} = \frac{R_p - R_f}{\sigma_p}$

$M^2 = \frac{(R_p - R_f)\sigma_m}{\sigma_p} - (R_m - R_f)$

Treynor measure $= \frac{\text{Portfolio risk premium}}{\text{beta risk (systematic risk)}} = \frac{R_p - R_f}{\beta_p}$

Jenson's alpha = Actual portfolio return − SML expected return = $R_p - [R_f + \beta(R_m - R_f)]$

Investment policy statement (IPS)
Investment objectives
- Return objectives
- Risk tolerance
Investment constraints (LLTTU)
- Liquidity requirements
- Legal and regulatory
- Time horizon
- Tax
- Unique circumstances

IPS may also include policy regarding sustainable investing which takes into account environmental, social, and governance (ESG) factors.
The ESG implementation approaches may have a negative impact on expected risk and return of a portfolio as it may limit the manager's investment universe and the manner in which investment management firms operate.
Risk management is the process by which an organization or individual defines the level of risk to be taken (i.e. risk tolerance), measures the level of risk being taken (i.e. risk exposure), and modifies the risk exposure to match the risk tolerance.

Methods to estimate target capital structure weights.
1. Assume the current capital structure at market value weights for the components.
2. Examine trends in the capital structure or statements by management regarding capital structure policy.
3. Use averages of comparable companies' capital structures.

Modigliani–Miller propositions regarding capital structure.

	Without Taxes	With Taxes
Proposition I	$V_L = V_U$	$V_L = V_U + tD$
Proposition II	$r_e = r_0 + (r_0 - r_d)\,D/E$	$r_e = r_0 + (r_0 - r_d)(1 - t)\,D/E$

Technical Analysis

Charts: line, bar, candlestick, volume.

Reversal patterns: head & shoulders, inverse head & shoulders, double/triple tops & bottoms.

Continuation patterns: triangles, rectangles, flags, pennants.

Price based indicators: moving averages, Bollinger bands.

Momentum oscillators: ROC, RSI, Stochastic, MACD.

Sentiment indicators: put/call ratio, VIX, margin debt, short interest.

Head and shoulders pattern: price target = neckline – (head – neckline)

$$\text{Short interest ratio} = \frac{\text{short interest}}{\text{average daily trading volume}}$$

Uptrend: If prices are reaching higher highs and higher lows. An upward trendline can be drawn by connecting the increasing low points with a straight line.

Downtrend: If prices are reaching lower highs and lower lows. A downward trendline can be drawn by connecting the decreasing high points with a straight line.

Support: Price level at which there is sufficient buying pressure to stop a further price decline.

Resistance: Price level at which there is sufficient selling pressure to stop the further price hike.

Major **fintech** applications include:
- Text analytics and natural language processing
- Robo-advisory services
- Risk analysis
- Algorithmic trading

Major DLT applications include:
- Cryptocurrencies
- Tokenization
- Post-trade clearing and settlement
- Compliance

Ethical and Professional Standards

I(A) Knowledge of the law: comply with the strictest law; disassociate from violations.

I(B) Independence and objectivity: do not offer, solicit or accept gifts; but small token gifts are ok.

I(C) Misrepresentation: do not guarantee performance; avoid plagiarism.

I(D) Misconduct: do not behave in a manner that affects your professional reputation or integrity.

II(A) Material nonpublic information: do not act or help others to act on this information; but mosaic theory is not a violation.

II(B) Market manipulation: do not manipulate prices/trading volumes to mislead others; do not spread false rumors.

III(A) Loyalty, prudence, and care: place client's interest before employer's or your interests.

III(B) Fair dealing: treat all client's fairly; disseminate investment recommendations and changes simultaneously.

III(C) Suitability: in advisory relationships, understand client's risk profile, develop and update an IPS periodically; in fund/index

management, ensure investments are consistent with stated mandate.

III(D) Performance presentation: do not misstate performance; make detailed information available on request.

III(E) Preservation of confidentiality: maintain confidentiality of clients; unless disclosure is required by law, information concerns illegal activities, client permits the disclosure.

IV(A) Loyalty: do not harm your employer; obtain written consent before starting an independent practice; do not take confidential information when leaving.

IV(B) Additional compensation arrangements: do not accept compensation arrangements that will create a conflict of interest with your employer; but you may accept if written consent is obtained from all parties involved.

IV(C) Responsibilities of supervisors: prevent employees under your supervision from violating laws.

V(A) Diligence and reasonable basis: have a reasonable and adequate basis for any analysis, recommendation or action.

V(B) Communication with clients and prospective clients: distinguish between fact and opinion; make appropriate disclosures.

V(C) Record retention: maintain records to support your analysis.

VI(A) Disclosure of conflicts: disclose conflict of interest in plain language.

VI(B) Priority of transactions: client transactions come before employer transactions which come before personal transactions.

VI(C) Referral fees: disclose referral arrangements to clients and employers.

VII(A) Conduct as participants in CFA Institute programs: don't cheat on the exams; keep exam information confidential.

VII(B) Reference to CFA Institute, the CFA designation, and the CFA program: don't brag, references to partial designation not allowed.

GIPS
- The GIPS standards were created to avoid misrepresentation of performance.
- A composite is an aggregation of one or more portfolios managed according to a similar investment mandate, objective, or strategy.
- Verification is performed by an independent third party with respect to an entire firm. It is not done on composites, or individual departments.