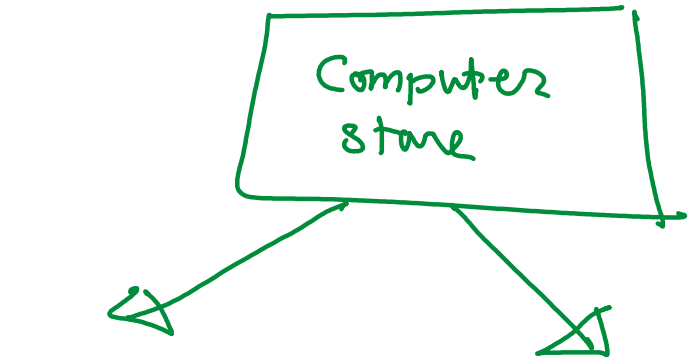
Composite Functions

Monday, 9 November 2020 10:06 AM



→ 85% of price − 3000

Let x denote the item price (MRP)

$f(x) = 0.85x$

$g(x) = x - 3000$

$h(x) = 0.85x - 3000$

$h(x) = f(x) - 3000$

$= g(f(x))$

$g(\square) = \square - 3000$

$h(x) = (g \circ f)(x)$

Evaluating $(g \circ f)(P) = h$

$x = 14000$ Evaluation of $(gof)(x)$

$$(gof)(x) = g(f(x))$$

$$= f(x) - 3000$$

$$= 0.85x - 3000$$

$$g(f(14000))$$

$$= 0.85 \times 14000 - 3000$$

$$= 11900 - 3000$$

$$= 8,900$$

The composition of Functions

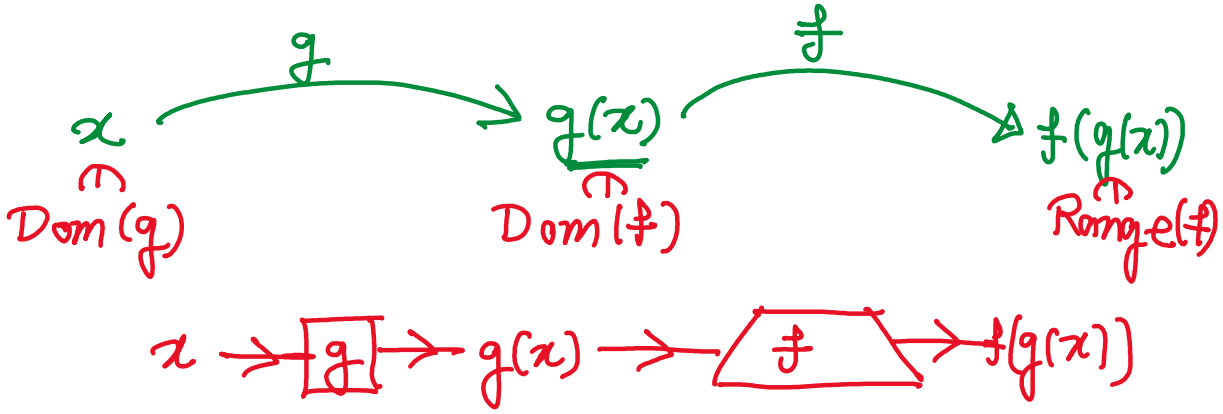
The composition of the functions f & g is denoted fog & is defined by

$$(fog)(x) = f(g(x))$$

The domain of the composite function fog is the set of all x such that

- (i) x is in the domain of g.
- (ii) $g(x)$ is in the domain of f.

$(f \circ g)(x) = f(g(x))$



Example. Given $f(x) = 3x - 4$, $g(x) = x^2$,

find (a) $(g \circ f)(x)$ (b) $(f \circ g)(x)$.

Soln.

$$(g \circ f)(x) = g(f(x))$$
$$= (f(x))^2$$
$$= (3x-4)^2$$

$g(\square) = \square^2$

$(g \circ f)(x) = g(f(x)) = g(3x-4)$ Replace $f(x) = 3x-4$

$= (3x-4)^2$

$(f \circ g)(x) = f(g(x))$
$= 3g(x) - 4$
$= 3x^2 - 4$

$f(\Delta) = 3\Delta - 4$

Exercise. $f(x) = x+1$ $g(x) = x^2 - 1$

find $(g \circ f)(x)$ $(f \circ g)(x)$.

Determination of the domain for composite f^n.

$$(f \circ g)(x) = f(g(x))$$

The following values must be excluded from input x.

- $x \notin \text{Dom}(g) \Rightarrow x \notin \text{Dom}(f \circ g)$
- $\{x \mid g(x) \notin \text{Dom}(f)\}$ must not be included in $\text{Dom}(f \circ g)$.

Example. $f(x) = \boxed{\frac{2}{x-1}}$ $g(x) = \boxed{\frac{3}{x}}$

$(f \circ g)(x)$ & $\text{Dom}(f \circ g)$

$$\boxed{(f\circ g)(x)} = f(g(x)) \qquad f(\square)=\frac{2}{\square-1}$$

$$= \frac{2}{g(x)-1}$$

$$= \frac{2}{3/x-1} = \boxed{\frac{2x}{3-x}}$$

- $(f\circ g)(x) = \boxed{\frac{2x}{3-x}} = \frac{0}{3} = 0$

Rule 1. $x \notin \text{Dom}(g) \Rightarrow x \notin \text{Dom}(f\circ g)$

$g(x) = \frac{3}{x}$, $x \neq 0$

$x = 0 \notin \text{Dom}(g) \Rightarrow x = 0 \notin \text{Dom}(f\circ g)$

Rule 2. $g(x) \notin \text{Dom}(f)$

$f(x) = \frac{2}{x-1}$ $\boxed{x \neq 1}$

$$\text{Dom}(f\circ g) = \{x \mid x \neq 0 \overset{\text{and}}{,} x \neq 3\}$$

Exercise. $f(x) = \frac{1}{x+1}$ $g(x) = \frac{1}{x}$

$(f \circ g)(x)$ and $\text{Dom}(f \circ g)$