INDIAN INSTITUTE OF TECHNOLOGY MADRAS

सिद्धिर्भवति कर्मजा

# IIT Madras

ONLINE DEGREE

# Graphing a Polynomial Function

1. Find the x- and y- intercepts, if possible.
2. Check for symmetry. If the function is an even function, its graph is symmetrical about the y-axis, that is, $f(-x)=f(x)$. If a function is an odd function, its graph is symmetrical about the origin, that is, $f(-x)=-f(x)$.
3. Use the multiplicities of the zeros to determine the behavior of the polynomial at the x-intercepts.
4. Determine the end behavior by examining the leading term.
5. Use the end behavior and the behavior at the intercepts to sketch a graph.
6. Ensure that the number of turning points does not exceed one less than the degree of the polynomial.
7. Optionally, use graphing tools to check the graph.

# Example

Sketch a graph of $f(x)=-(x+2)^2(x-5)$.

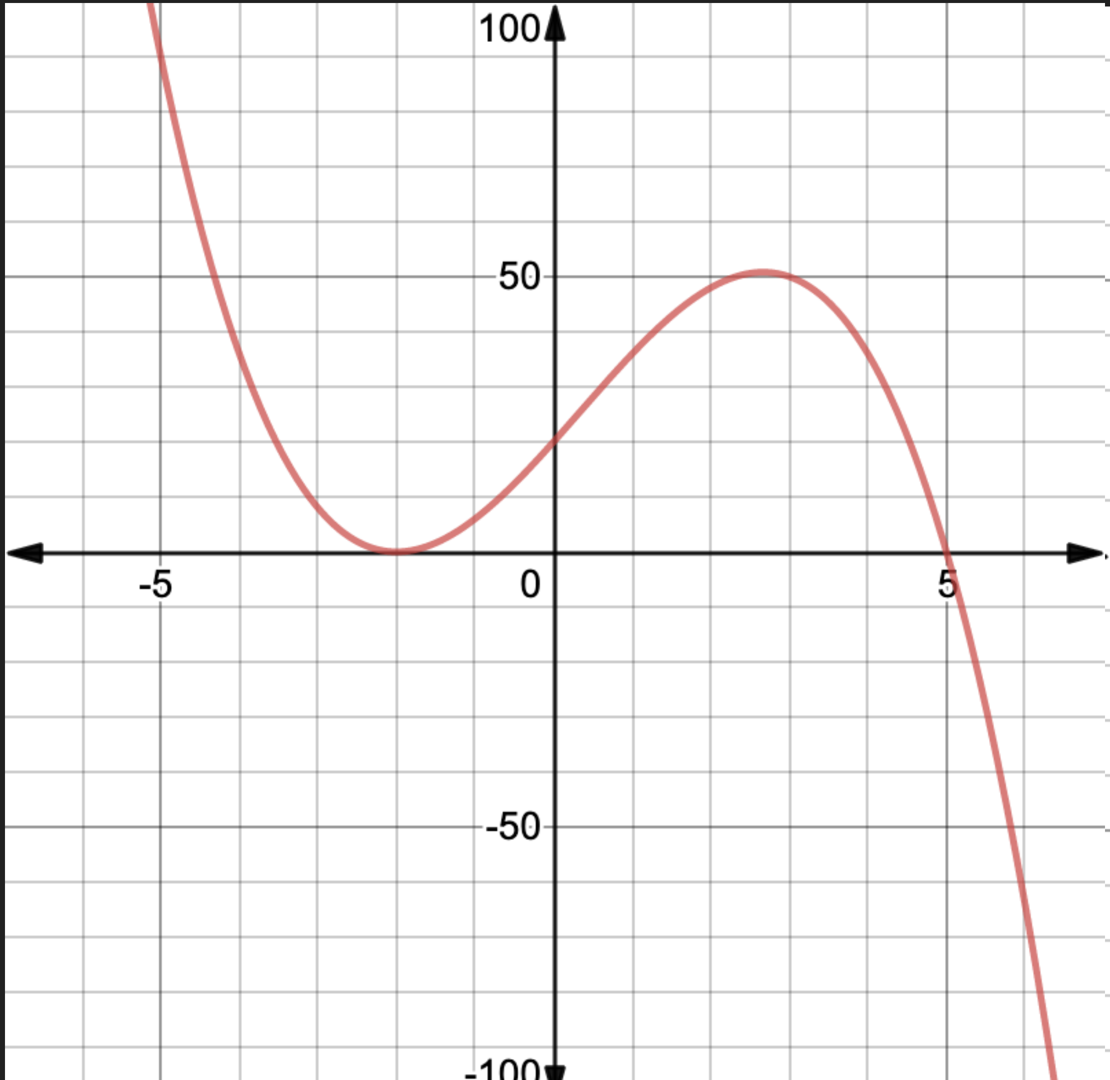
x-intercepts are x= -2, 5

x= -2 has multiplicity 2, quadratic
x=5 has multiplicity 1, linear

y-intercept f(0)=20.

The leading term is $-x^3$. Therefore, the odd degree polynomial with negative leading coefficient has the following end-behavior
$x \to \infty, \ f(x) \to -\infty$
$x \to -\infty, \ f(x) \to \infty$

f can have at most 3-1=2 turning points.



# Intermediate Value Theorem

Let $f$ be a polynomial function. The **Intermediate Value Theorem** states that if $f(a)$ and $f(b)$ have opposite signs, then there exists at least one value $c$ between $a$ and $b$ for which $f(c)=0$.

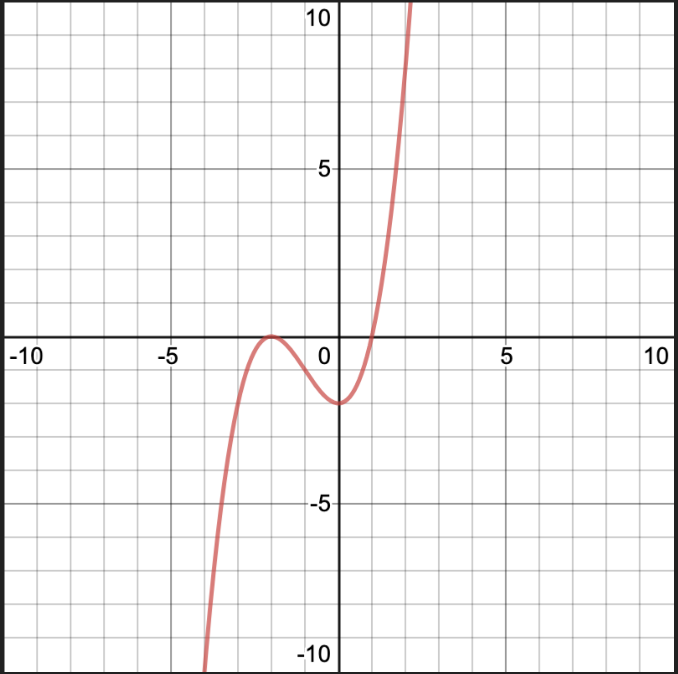
# Deriving Formula for Polynomial Functions

Given the graph, how to find the formula for polynomial function?

1. Find the x-intercepts of the graph to find the factors of the polynomial.
2. Understand the behavior of the graph at the x-intercepts to determine the multiplicity of each factor.
3. Find the polynomial of least degree containing all the factors found in the previous step.
4. Use any other point on the graph (the y-intercept may be easiest) to determine the stretch factor (?).

# Example

Write the formula for polynomial given in the graph.




x= -2,1 are the x-intercepts and the function has two turning points. The end behavior is similar to odd degree polynomial with positive leading term. That is, it may be be polynomial of degree 3.

The behavior at x =1 is linear and x=-2 is of even degree and hence quadratic. The resultant polynomial is of degree 3 with zeros -2 and 1 with multiplicities 2 and 1 respectively.

The polynomial has form $f(x) = a(x+2)^2(x-1)$.

To determine a, use y-intercept. From the graph, f(0) = -2
From the form f(0) = -4a. Therefore, a = ½.

Hence, the function must be $f(x) = \frac{1}{2}(x+2)^2(x-1)$.