INDIAN INSTITUTE OF TECHNOLOGY MADRAS

सिद्धिर्भवति कर्मजा

IIT Madras

ONLINE DEGREE

Mathematics for Data Science 1
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Week 9 Tutorial

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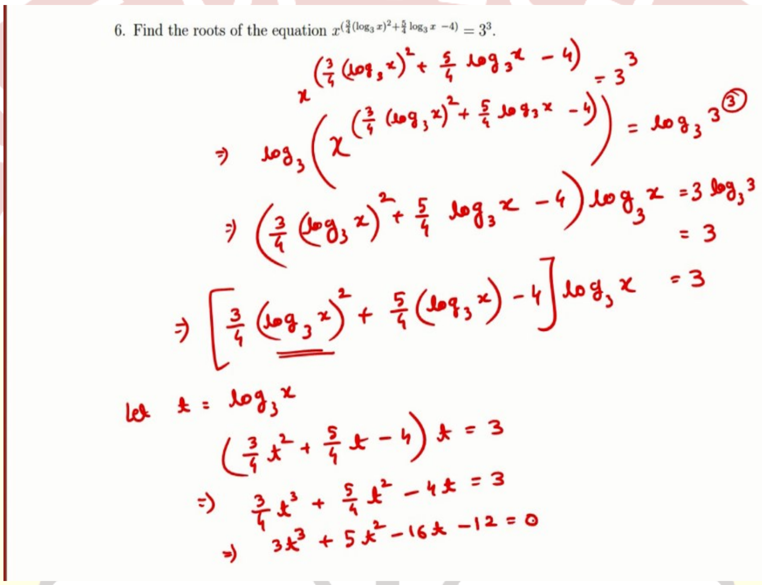

In the sixth question we have to find the roots of the equations given like this. So, let us try to solve this. So, $x^{\frac{3}{4}(\log_3 x)^2+\frac{5}{4}\log_3 x-4} = 3^3$. So, let us take log 3, log base 3 on both side. So, it will give us $\log_3 x^{\frac{3}{4}(\log_3 x)^2+\frac{5}{4}\log_3 x-4} = \log_3 3^3$.

So, this power will come in front, so we will get, $[\frac{3}{4}(\log_3 x)^2+\frac{5}{4}\log_3 x-4]\log_3 x = \log_3 3$ will also come in front. So, we will get $\log_3 3$ which is 1 and we will get 3. So, basically we are getting $\frac{3}{4}(\log_3 x)^2+\frac{5}{4}\log_3 x-4]\log_3 x = 3$. So, now let us assume $t = log\ 3x$.

So, let us substitute there in $\log_3 x$, so we will get $(\frac{3}{4}t^2+\frac{5}{4}t-4)t = 3$ So, if we simplify this, we will get $\frac{3}{4}t^3+\frac{5}{4}t^2-4t = 3$. So, if we multiply 4 on both side we will get, $3t^3+5t^2-16t$ and $3\times 4 = 12$ and if we take 12 on this side we will get - 12 that is equal to 0.

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$$\Rightarrow \left(\frac{3}{4}(\log_3 x)^2 + \frac{5}{4}\log_3 x - 4\right)\log_3 x = 3\log_3 3 = 3$$

$$\Rightarrow \left[\frac{3}{4}(\log_3 x)^2 + \frac{5}{4}(\log_3 x) - 4\right]\log_3 x = 3$$

let $t = \log_3 x$

$$\left(\frac{3}{4}t^2 + \frac{5}{4}t - 4\right)t = 3$$

$$\Rightarrow \frac{3}{4}t^3 + \frac{5}{4}t^2 - 4t = 3$$

$$\Rightarrow 3t^3 + 5t^2 - 16t - 12 = 0$$

verify

$t=2$

$$\Rightarrow (t-2)(3t^2+11t+6) = 0$$

$$\Rightarrow (t-2)(t+3)(3t+2) = 0$$

$t = 2, -3, -2/3$ $\quad \log_3 x = 2 \Rightarrow x = 3^2 \quad | \quad x = 3^{-3} \quad | \quad x = 3^{-2/3}$

Now if we use trial and error method I mean hit and trial method you can say. So, if we put t equal to 2 in this polynomial you will get 0. So, $t = 2$ will be a root, so $t - 2$ will be a factor of it. So, if we take $(t - 2)$ out, we will get the other factor to be $3t^2 + 11t + 6 = 0$. And again if you factorize this quadratic you will get $(t + 3) \times (3t + 2)$.

So, I want all of you to verify these two step at your own time. So, we will get three roots t equal to 2, -3 and $\frac{-2}{3}$. Now what is t? t is $\log_x 3$. So, this is 2, so our $x = 3^2$. So, for the other two cases we will get , $x = 3^{-3}$ and for the last case we will get $x = 3^{\frac{-2}{3}}$. So, the roots are $3^2, 3^{-3}, 3^{\frac{-2}{3}}$. Thank you.