Statistics for Data Science -1

Lecture 7.5: Conditional Probability: Independent events-examples

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Learning objectives

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2. Distinguish between independent and dependent events.
3. Solve applications of probability.

Independent events: example

Rolling a dice

Deck of cards

Example: Roll a dice twice

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- Experiment: Roll a dice twice

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- Sample space:

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$$P(E_1 \cap E_2) = P(\{(3,5)\}) = \frac{1}{36}$$

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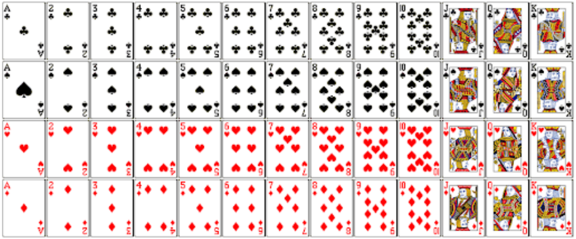
Example: deck of cards

Consider again the experiment of randomly selecting one card from a deck of 52 playing cards.



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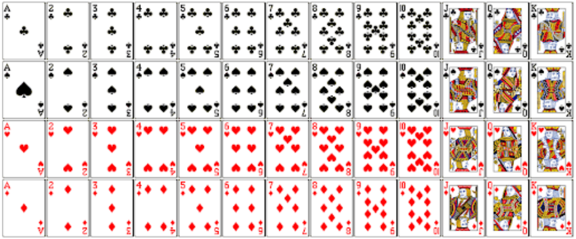

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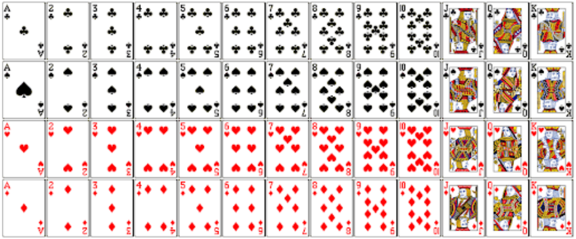

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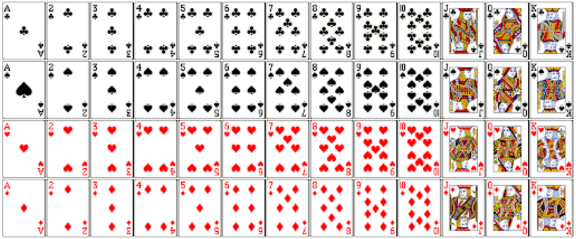

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- $E_1 \cap E_2$ is the event that a face card and a king is selected which is the event a king is selected.

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- $P(E_1) = P(\{JH, JC, JS, JD, KH, KC, KS, KD, QH, QC, QS, QD\}) = \frac{12}{52}$

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- $E_2 \cap E_3$ is the event that a king and a heart is selected which is the event a kingheart is selected.

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Section summary

- Examples of independent and dependent events.