IIT Madras

ONLINE DEGREE

Quadratic Functions

Graphing

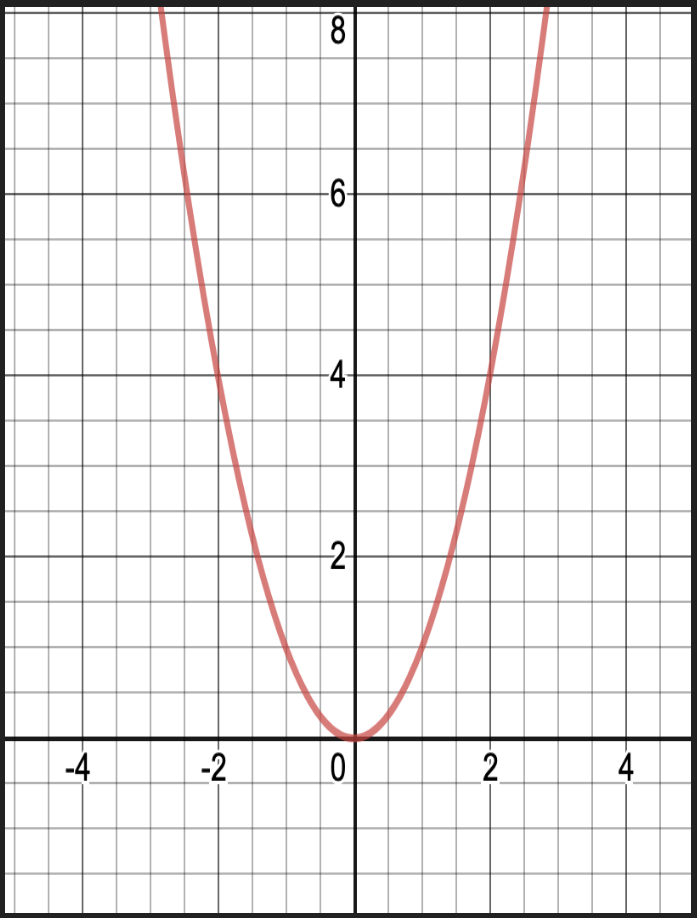
Quadratic Function (Definition)

- A quadratic function is described by an equation of the form
  - $f(x) = ax^2 + bx + c$, where $a \neq 0$.

Quadratic term   Linear term   Constant term

The graph of any quadratic function is called parabola.

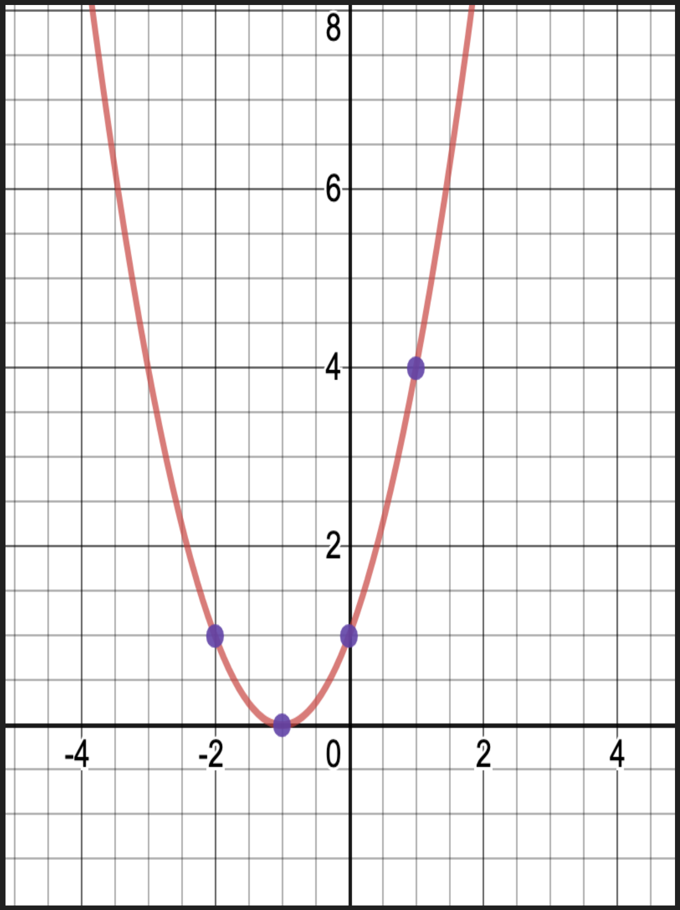
To graph a quadratic function, plot the ordered pairs on the coordinate plane that satisfy the function.



Example: Graph a function $f(x) = x^2 + 2x + 1$

1. Generate a table of ordered pairs satisfying the function.
2. Plot the points on the coordinate plane.
3. Connect a smooth curve joining the points.

x	y
-2	1
-1	0
0	1
1	4



Important Observations

- All parabolas have an axis of symmetry. That is, if the graph paper containing the graph of parabola is folded along the axis of symmetry the portion of parabola on either sides will exactly match each other.
- The point at which the axis of symmetry intersects the parabola is called the vertex.
- The y-intercept of a quadratic function is c

Let $f(x) = ax^2 + bx+c$, where $a \neq 0$.

- The y-intercept: $y = a(0)^2+b(0) + c = c$
- The equation of axis of symmetry: $x = -b/(2a)$. (to be derived later).
- The x-coordinate of the vertex: $-b/(2a)$.

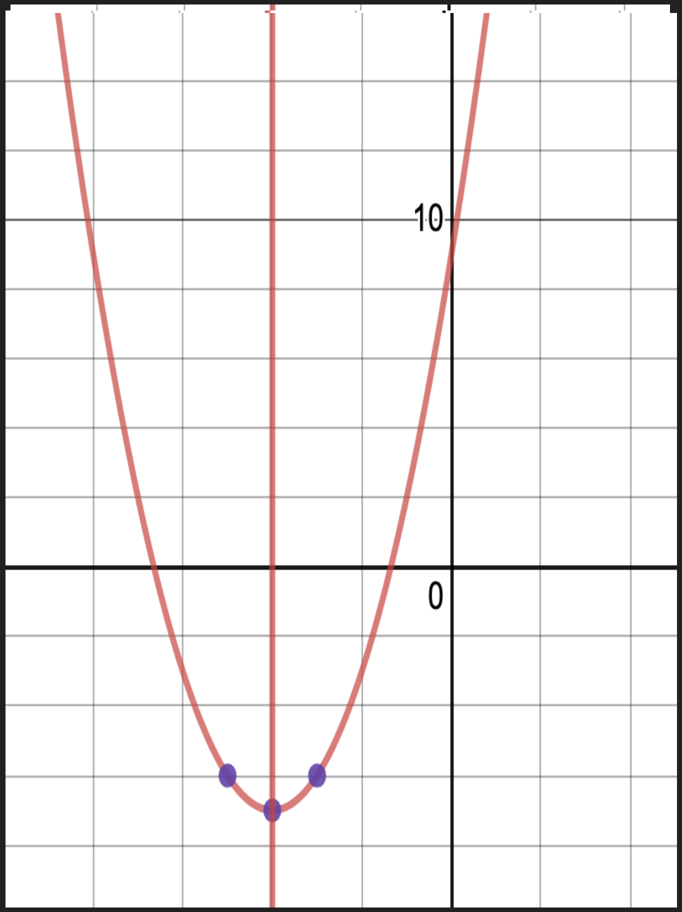
Example: Graph a function $f(x) = x^2+8x+9$

The y-intercept: *9*

The axis of symmetry: $x = -8/(2(1)) = -4$

The vertex: *(-4, -7)*

x	y
-3	-6
-4	-7
-5	-6



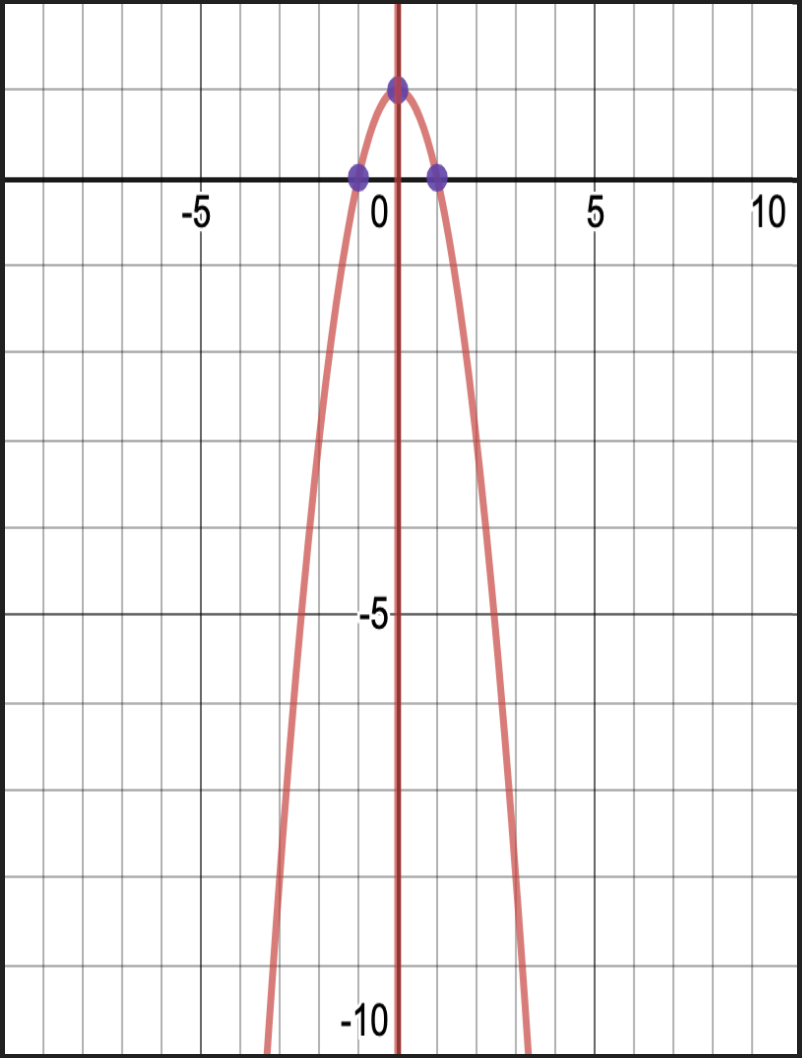
Example: Graph a function $f(x) = -x^2+1$.

The y-intercept: *1*

The axis of symmetry: $x = 0$

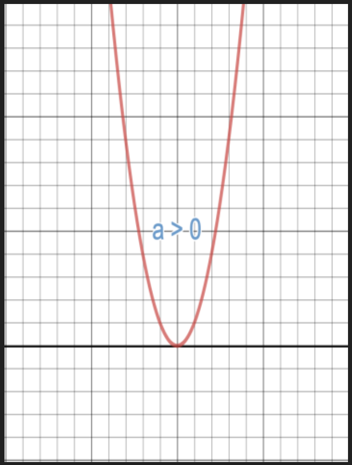
The vertex: *(0, 1)*

x	y
-1	0
0	1
1	0



Maximum and Minimum Values

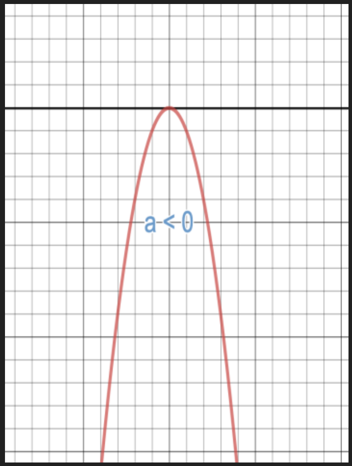
The y-coordinate of the vertex of a given quadratic function is the ***minimum or maximum*** value attained by the function.



The graph of a quadratic function $f(x)= ax^2+bx+c$, where $a \neq 0$ is:

- Opens up and has minimum value, if $a > 0$.
- Opens down and has maximum value if $a < 0$.
- The range of a quadratic function is

$$\mathbf{R} \cap \{f(x) | f(x) \geq f_{min}\} \text{ or } \mathbf{R} \cap \{f(x) | f(x) \leq f_{max}\}.$$



Example

Let $f(x) = x^2 - 6x + 9$.

1. Determine whether f has minimum or maximum value. If so, what is the value?
2. State the domain and the range of f.

Observe that $a=1$, $b=-6$, and $c=9$.

Since, $a>0$, the function opens up and has the minimum value.

The minimum value is given by y-coordinate of the vertex. The x-coordinate of the vertex is $-b/(2a) = 3$. Therefore, the minimum value is $f(3) = 0$.

Domain = $\mathbf{R}$ and Range = $\mathbf{R} \cap \{f(x) | f(x) \geq 0\}$.