IIT Madras

ONLINE DEGREE

Mathematics for Data Science 1
Prof. Neelesh S Upadhye
Department of Mathematics
Indian Institute of Technology, Madras

Lecture - 14
Section formula

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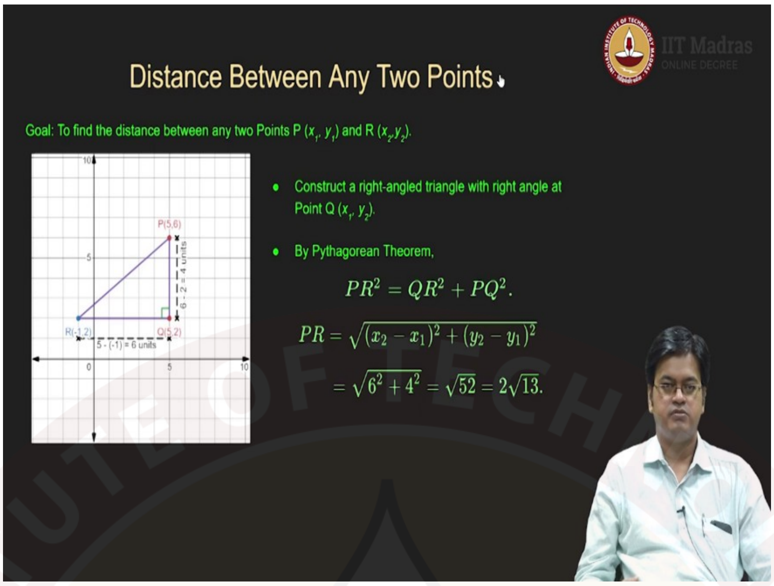

Now, let us take up the next concept. Now, we have handled two points; now let us take three points.

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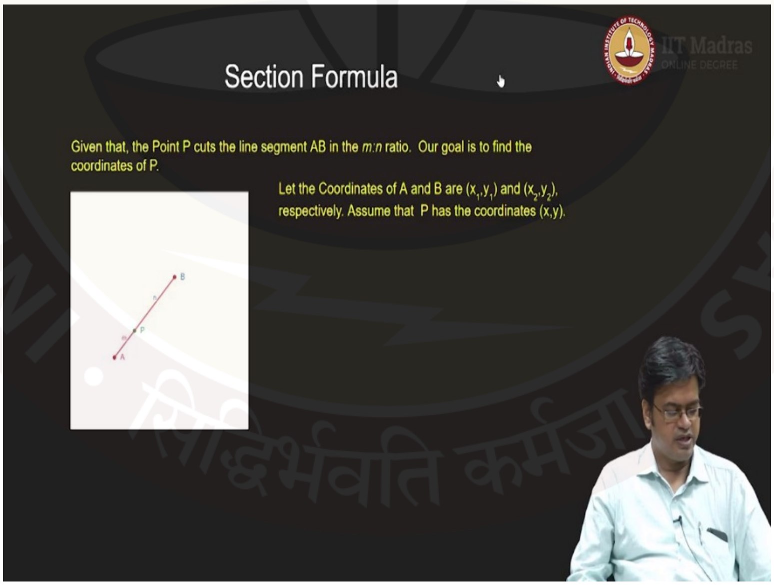

And, let us say those three points lie on a line and given that the point P cuts the line segment AB in the ratio m : n. Our goal is to know the coordinates of point P; this will give us the Section Formula. So, this is the graphical representation of the points. So, there are two, there is a line segment AB and point P cuts this line segment in the ratio m : n.

How will you find the coordinates of point P? This is the question; let us bring in our coordinate system. So, let the coordinates of A and B are ¿, y_1¿ and ¿, y_2¿, the coordinates of A are ¿, y_1¿ coordinates of B are¿, y_2¿,. I do not know what P is, let us assume it has some coordinates which are x and y ok. So, let us bring in them in the coordinate system which is this.

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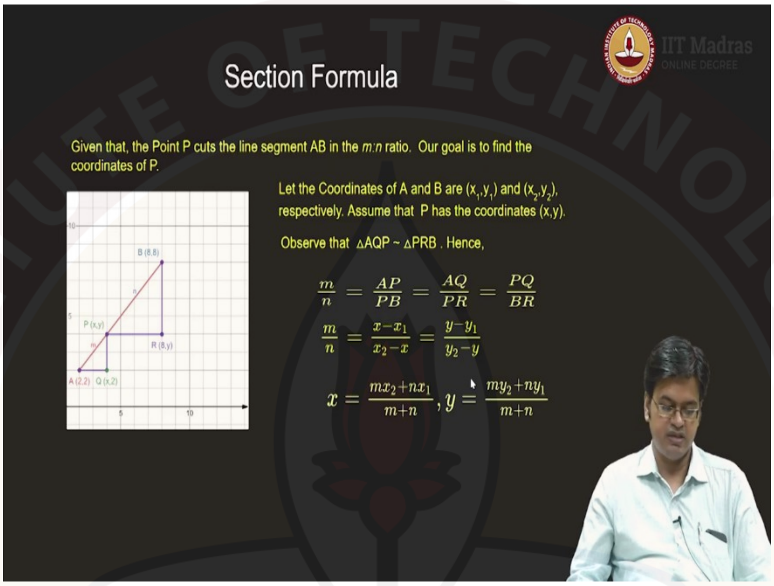

Let us try to understand this particular coordinate system by putting up some triangles around. So, what I have done is I have actually constructed two triangles using the same logic that I used in the distance formula. If the coordinates of point P are (x,y), then I will construct a right angled triangle in this direction; where the x coordinate will be x and y coordinate will be the coordinate of y coordinate of A that is y_1.

Similarly, I will do the same thing with respect to point B; I will drop a perpendicular which will meet at this particular point. So, basically I will drop a perpendicular which will meet the X axis and again I will draw a perpendicular here. But, let us for sake of simplicity we have constructed a right angle triangle, where the y coordinate of this point will be y and the x coordinate of this point will be the x coordinate of point B which will be x_2, (x_2, y).

With this understanding we can proceed further and see that the triangles, these two triangles are similar to each other. How? First of all let us see this line is parallel to X axis and this line is parallel to X axis as well. Therefore, these two are parallel lines and this is a transversal that is passing through these two parallel lines. Therefore, these two angles the angle A and angle P will be same or equal.

Next these two are right angles, then we know the sum of the angles in a triangle is 180 degrees, therefore this angle, angle B must be equal to angle P. Therefore, triangle AQP must be similar to triangle PRB by angle test that essentially means I have their sides in some ratio, correct. So, for simplicity I have plotted these points with some coordinate references.

So, this is A is (2,2), B is (8,8); then whatever I mentioned the coordinates of Q are (x,2) and coordinates of R are (8,y). So, now these two things will be in some ratio that is $\frac{AP}{PB}$, these are the hypotenuse of these two right angle triangles is equal to $\frac{AQ}{PR}$and this thing is $\frac{QP}{RB}$ right or you can see $\frac{AP}{PB}$ is equal to $\frac{AQ}{PR}$which is equal to $\frac{PQ}{BR}$.

Now, I already know $\frac{AP}{PB}$ have a ratio m : n. So, their ratio is m by n that is already known to us, that is given to me. Now, can I calculate the length of AQ and PR? The answer is yes, because AQ is parallel to x axis. It is just subtracting the highest x coordinate from the low.

So, it will be x -2 in the figure and in our theory it is x - x_1. Similarly, you can look at PR; it will be 8 - x or in our theory it will be x_2- x. For y axis or the lines that are parallel to y axis PQ and BR you can see you will go to the highest value that is y - 2 or y - $y_1$and the other one BR will have $y_2 - y$right.

So, together I will have a representation of this form: $\frac{m}{n}=\frac{x-x_1}{x_2-x}=\frac{y-y_1}{y_2-y}$. Now, take one equality at a time; that means, $\frac{m}{n}$is equal to let us say these the consider these x coordinates. So, we will just cross multiply them, rearrange them you will get what x is equal to.

In a similar manner just take $\frac{m}{n}$ is equal to this, these y coordinates ratio and then cross multiply and rearrange them. You will get the following values which are given by $x=\frac{m x_2+n x_1}{m+n}$. And, similarly $y=\frac{m y_2+n y_1}{m+n}$. This gives me the section formula, when a point divides the line in the ratio m : n.

Another interesting question is suppose I know the coordinates of x and y; can I find in what ratio the line divides? Obviously, yes because you know the coordinates of the line, you just need to use this formula for finding the ratio ok; that will be more clear when you solve more problems ok.