INDIAN INSTITUTE OF TECHNOLOGY MADRAS

सिद्धिर्भवति कर्मजा

# IIT Madras

ONLINE DEGREE

**Mathematics for Data Science 1**
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**Week 08 – Tutorial 6**

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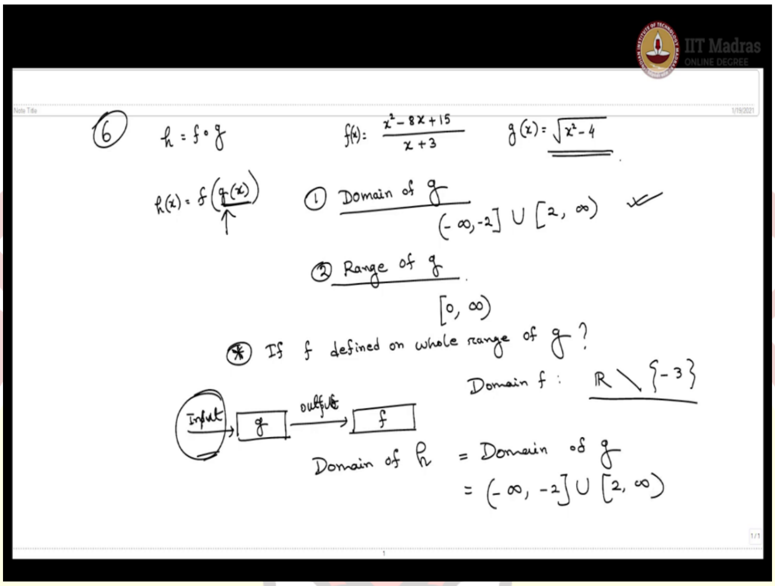


So, let us discuss the sixth question. So, in the sixth question we have to calculate the domain of the function h, where h is given as the composition function, the composite function fog, where $f(x) = \frac{x^2 - 8x + 15}{x+3}$. So, we can write it as $f(x)$ as it is a function of $x$ and $g(x) = \sqrt{x^2 - 4}$. So, we have to calculate what is the domain of the function $h$?

So, $h(x) = f(g(x))$. So, how will we compute the domain of $h(x)$? Observe that the input value of $h$ firstly come from the input value of $g$. So, let us begin with computing what is the domain of g x, I mean from where the g is taking its input. So, to begin with, calculate, so this is the first step, calculate the domain of $g$. So, you have seen earlier that domain of $g$ is nothing but $(-\infty, -2] \cup [+2, +\infty)$. So, this is the domain of $g$.

So, from this set $g$ is taking it input. Now, if we talk input from this set, then we will get some values as output from g and those output basically those are the, those are in the range of $g$ and those output must be inside the domain of f, otherwise the $f$ will not define. So, let us compute what is the range of $g$. So, you have seen that this is a square root function, so range of $g$ is nothing but 0 to ∞. So, it is nothing but the non-negative real numbers.

Now, let us ask the question, is f defined on whole range of $g$? This is the question. If $f$ is defined on the whole that means, if I am giving the input in the $f$ here in $f$ composite in the whole range of $g$, I mean whatever the outputs coming from $g$ is accepted as the input of f, then the domain of the function is nothing but the domain of $g$, because domain of $g$ means we are giving input, so let me write this is my $g$ and this is my $f$, so we are giving input, getting some output and if all the output elements, I mean if all this set, if everything is in the output set is taken as the input of f, then these inputs, these set is nothing but the domain of $h$.

So, here, let us observe what is the domain of f, I mean what inputs $f$ can take. So, domain of f is nothing but all the real numbers except $-3$, but observe that $-3$ is not in the range of g because range of g is nothing but non-negative real numbers. So, - 3 is not there. So, anything in the, which is in the range of $g$ can be taken as input in the $f$. So, whatever I am giving input in the g, we are getting some output and all the outputs are taken as the input of $f$.

So, the domain of $h$ here is nothing but domain of $g$. Remember, this is for this particular case. There may be some cases where this range of g, the whatever I am getting as the range of g cannot I mean all the elements may not be taken as an input of f. There can be some case, I will show an example there, but in this particular case, in this question number 6, we are seeing that domain of g is nothing but domain of h because the range, the range set is basically a subset of domain of f. So, let me write here. So, let me finish this at first. So, domain of $g$ is nothing but this thing (- ∞,- 2] ∪[ 2, ∞). So, this is the domain of $h$.

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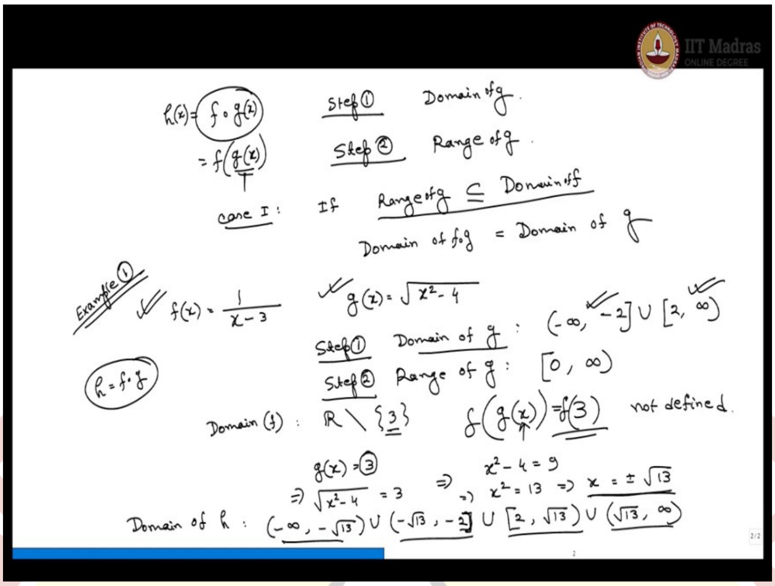


So, what we have seen here? We have seen that to find the domain of some composite function f composite, the step 1, we must calculate what is domain of g. If we calculate domain of g, then we have to calculate, so let me write this as $h(x) = fog\ (x)$. So, it is nothing but f of g x. So, these things are this g x, these are in range of g. So, step 2 must be calculating range of g.

Now, there some cases can arrive. So, if case 1, if range of $g$ is subset of domain of f which we have seen in the question number 6, which we have seen in the example we have just discuss, if range of g is subset of domain of $f$, then see every value which are in the output of $g$ can be taken as input in f, so no problem arises. So, domain of the composite function f composite is same as domain of $g$. So, this is domain of $g$, this is range of $g$. So, this is case 1.

Now, what other case can arrive? So, let us see another example before going to the case 2. So, I am taking my $f(x) = \frac{1}{x-3}$ and let g be the same, $\sqrt{x^2 - 4}$. So, what are the step, step 1 we have seen. Domain of $g$, domain of $g$ is nothing but we have already seen this ∪. Step 2, range of $g$. So, range of $g$ is nothing but 0 to ∞. Now, we come to the case 1, is this happen? Is range of g is a subset of range of domain of $f$?

So, let us calculate domain of $f$. Domain of $f$ is set of all real number except 3 because when we put 3, this will become undefined, otherwise it is a defined everywhere. So, domain of $f$ is real number, set of real number - this singleton set 3. So, you can see that this 3 is in range of g. So, we are getting some output from g which is not in the domain of f, so we cannot give that value in g for which this 3 is coming.

So, we cannot put that $x$ as input for which g(x) is 3 because in that case this f3, this is not defined. So, we have to find those $x$ in the domain of g for which g(x) is 3 and we will cut those elements out. So, what is g(x) is 3? g(x) is √over $x$ square - 4 equal to 3, so if we solve this, we will get $x^2 - 4 = 9$, so $x^2 = 13$ which implies $x = \pm\sqrt{13}$. So, for these two values, $+\sqrt{13}$ and $-\sqrt{13}$, we are getting the output as 3 which is cannot, which cannot be given as input in $f$.

So, we have to eliminate this from the domain. So, in this case we have already seen domain of g is this, so we have to eliminate this + √13 from this side and - √13 from this side. So, + √13 from this side and - √13 from this side. So, eventually, our domain of h is - ∞ to $-\sqrt{13}$ and again ∪ - √13 because we are eliminating $-\sqrt{13}$. So, $-\sqrt{13}$ to - 2. This should be closed interval as it is there, 2 to + √13. See, + √13 is get 2, you can check.

So, this ∪ + √13 to ∞. So, this is the ∪ of these 4 intervals, this is the domain of h in this case. So, where h is fog. whereas f is this one and g is this one. So, let us write me as example 1. So, here you have observed that here case 1 is not satisfying so we are going to some other case. So, let me write it.

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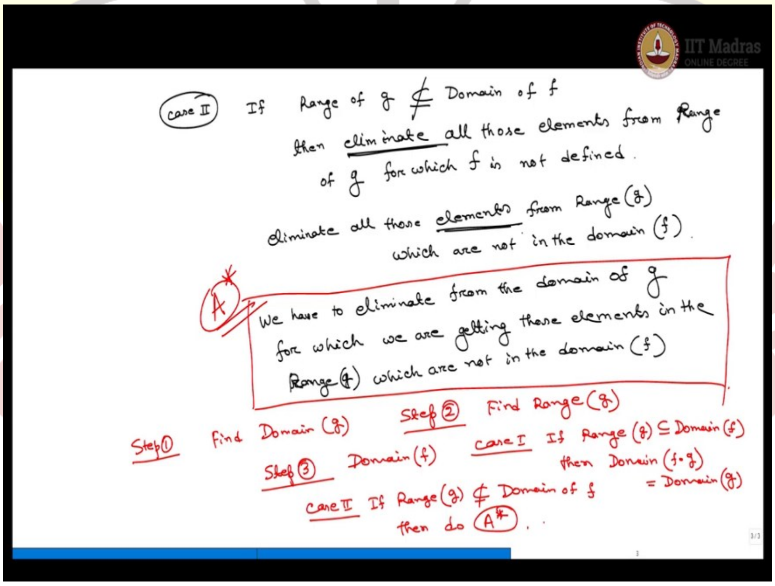


Case 2, if range of g is not a subset of domain of $f$, then eliminate all those element, all those elements from range of $g$ for which f is not defined. What does it mean? It means eliminate all those element from range of $g$ which is, which are not in the domain of $h$. What does it mean by elimination here? So, we have to eliminate from the domain of $g$, which element from the domain we have to eliminate? For which we are getting those element in the range of g which are not in the domain of $f$. So, this is my final conclusion.

So, let us recall the steps. Step 1, find domain if $g$. Step 2, find range of $g$. Now, after that two cases will arrive so, before going to the two cases, is better to calculate what is domain of $f$, what is domain of $f$. Now observe if, now case 1, if range of $g$ is subset of domain of $f$, then it is easy, then domain of $fog$ is same as domain of $g$. Case 2, if range of $g$ is not subset of domain of $f$, then what we have to do? We have to do this thing. So, let me write it as A*, then do A*. So, this is the stepwise procedure to calculate the domain of f compo g.

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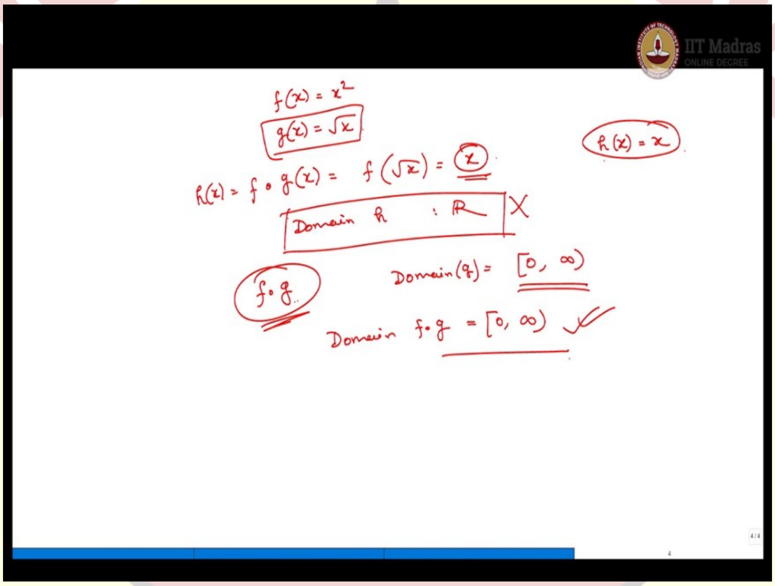


Now, you may ask was this the problem if we calculate the f composite beforehand and then solve this, find the domain? So, what I mean here let $f(x) = x^2$, $g(x) = \sqrt{x}$. So, what is $fog$ here? So, $f(g(x))$ is $f(g(x))$ of is $\sqrt{x}$ so, $(\sqrt{x})^2$ whole square that means, $x$. So, you see that if we calculate this f composite beforehand you can see, so this is our $h(x)$.

So, as a function $h(x)$, the domain of $h(x)$ if I write only this thing, so what is the domain of this thing? This is whole real number, domain of h, let me write this, domain of $h$ is whole real number. But when I am getting $h(x)$ as a composite function, as a composite function fog, then is $g$ defined on whole $\mathbb{R}$, it is not the case. So, domain of $g$, what is domain of $g$? Domain of $g$ is only non-negative real numbers because we cannot put negative real numbers under the square root, otherwise it will give us complex values.

So, domain of g for real valued function is $[0,\infty)$. Only the non-negative real numbers. So, this fog, the domain of $fog$ must be a subset of this and if you calculate as we have stated the steps previously, then you will get domain of $f o g$ is nothing but domain of $g$ which is 0 to $\infty$. But if you calculate the f compose beforehand, and then try to find what is the domain, then you will get domain of $h$ is $\mathbb{R}$ which is not the case. So, this is the correct one. So, you have to follow the procedure which I have stated in this page. So, this is the correct procedure to find domain of fog. Thank you.