

# Statistics for Data Science -1

## Lecture 7.4: Conditional Probability: Independent events

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## Learning objectives

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2. Distinguish between independent and dependent events.
3. Solve applications of probability.

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- ▶ In the cases where  $P(E|F)$  is equal to  $P(E)$ , we say that  $E$  is independent of  $F$ .
  - ▶ In other words, event  $E$  is independent of event  $F$  if knowing whether  $F$  occurs does not affect the probability of  $E$ .

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Two events that are not independent are said to be *dependent*.



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- ▶ In other words, two events are independent if and only if the probability that both occur equals the product of their individual probabilities.
- ▶ The definition of independence for three or more events is more complicated than that for two events. We will discuss this later.

## Section summary

- ▶ Independent events
- ▶ Multiplication rule for **two** independent events.