IIT Madras

ONLINE DEGREE

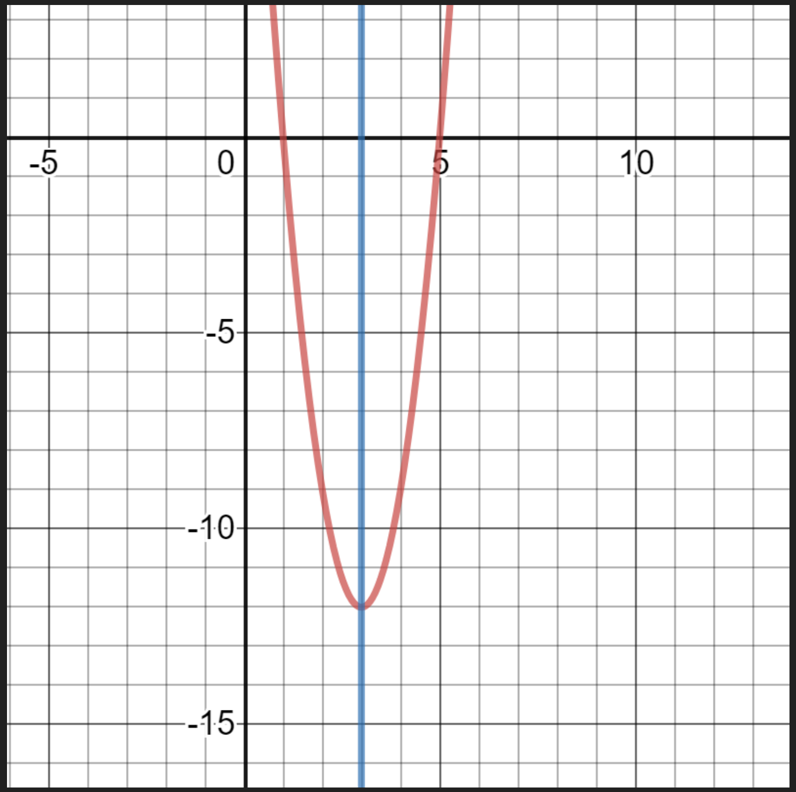
Quadratic Equations

Solve by Factoring

Quadratic Function: Intercept form

Let *y= f(x) = a(x-p)(x-q)*, where *p* and *q* represent x-intercepts for the function. Then the form *y = a(x-p)(x-q)* is called the *intercept form*.

Example: Graph *y=3(x-1)(x-5)*



Question: How will you convert the intercept form into the standard form?

Intercept form to the Standard form

Changing intercept form to standard form requires us to use FOIL method which can be described as follows:

The product of two binomials is the sum of the products of the first(F) terms, the outer(O), the inner(I) and the last(L) terms.

$$(ax+b)(cx+d) = \underbrace{ax.\,cx} + \underbrace{ax.\,d} + \underbrace{b.\,cx} + \underbrace{b.\,d}$$

F O

I L

Quick Observations:

The product of coefficient of x^2 and the coefficient of the constant is *abcd*.
The product of the two terms in the coefficient of x is also *abcd*.

Example

Question. Write a quadratic equation with roots, $\frac{2}{3}$ and -4, in the standard form.

Recall: By standard form, we mean $ax^2+bx+c=0$, where a,b,c are integers.

By intercept form, we know $(x-\frac{2}{3})(x+4)=0$.

By FOIL method, $(x-\frac{2}{3})(x+4)= x^2+(-\frac{2}{3} +4)x -\frac{2}{3}.4 = x^2+(10/3)x - 8/3=0$

For standard form, multiply both sides by 3, to get

$$3x^2+10x-8=0.$$

Standard form to Intercept form

Example: Convert the function $f(x) = 5x^2 - 13x + 6$ to intercept form.

Let us apply FOIL Method.

$5x^2 - 13x + 6 = (ax+b)(cx+d) = ac\,x^2 + (ad+bc)x + bd$.

Therefore, $ac = 5$, $ad+bc = -13$ and $bd = 6$. That is, $abcd = 30$ and $ad+bc = -13$.

$30 = 2 \times 3 \times 5 = 10 \times 3 = (-10)(-3)$. That is, $ad = -10$ and $bc = -3$.

$5x^2 - 13x + 6 = 5x^2 - 10x - 3x + 6 = 5x(x-2) - 3(x-2) = (5x-3)(x-2) = 5(x - \frac{3}{5})(x-2)$.

Examples

Solve: $x^2=8x$

That is, $0 = x^2-8x$

$= x(x-8)$

This means 0,8 are the roots of the given quadratic equation.

Solve: $x^2-4x+4=0$.

Using FOIL method, and comparing the coefficients, we get $abcd=4$ and $ad+bc=-4$. Therefore, $ad=-2$ and $bc=-2$.

So,

$$
\begin{aligned}
x^2-4x+4 &= x^2-2x-2x+4 \\
&= x(x-2)-2(x-2) \\
&= (x-2)^2=0
\end{aligned}
$$

Hence, 2 is the repeated real root of the given equation.

Solve: $x^2-25=0$

Note $abcd = -25$ and $ad+bc = 0$.

That is, $ad=5$ and $bc=-5$

So,

$$
\begin{aligned}
x^2-25 &= x^2-5x+5x-25 \\
&= x(x-5)+5(x-5) \\
&= (x+5)(x-5)=0
\end{aligned}
$$

Hence, -5, 5 are the roots of the given quadratic equation.