INDIAN INSTITUTE OF TECHNOLOGY MADRAS

सिद्धिर्भवति कर्मजा

IIT Madras

ONLINE DEGREE

Statistics for Data Science - 1
Professor. Usha Mohan
Department of Management Studies
Indian Institute of Technology, Madras
Lecture No. 7.2
Conditional Probability – Conditional Probability Formula

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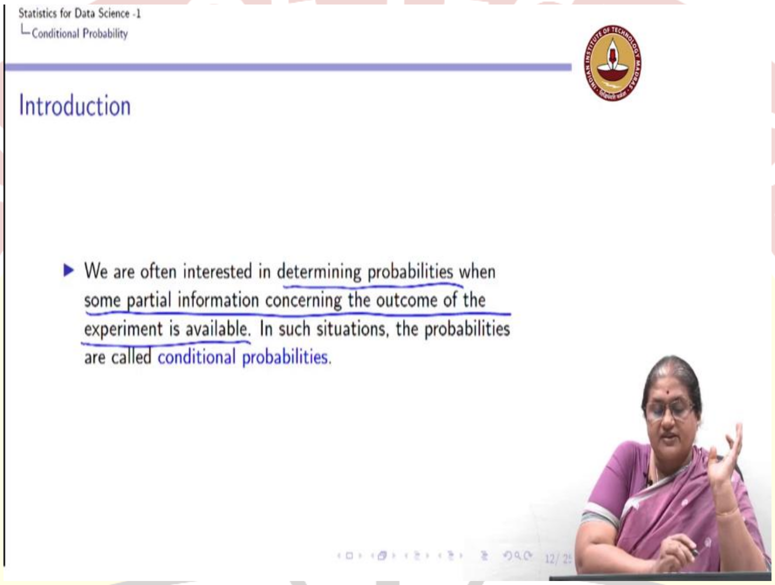

So, now let us actually introduce the notion of a conditional probability. Why do we have to learn about conditional probability? Because most of the times are very often we are interested in determining probabilities when some partial information concerning the experiment is available. So, let us understand it through an example.

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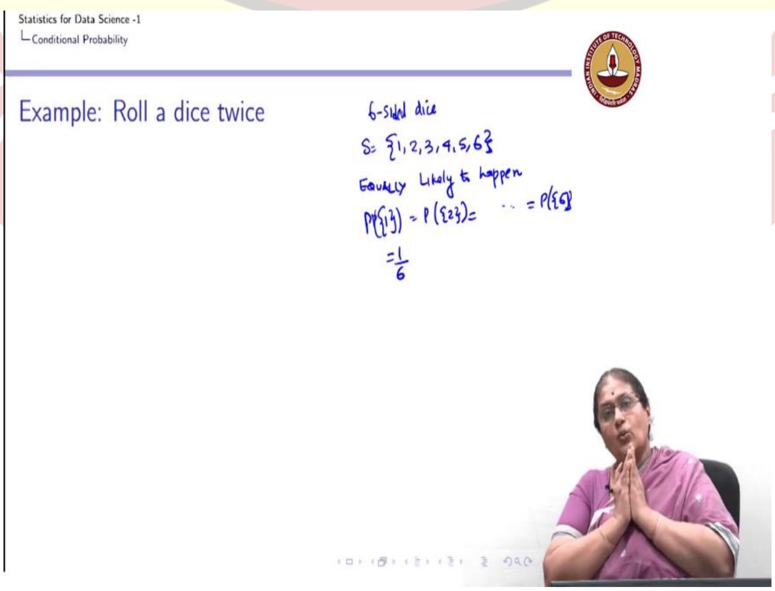

Let me have the experiment of rolling a dice twice I am assuming this dice is fair in the sense that we know that it is a 6 sided dice, so if I throw it once the sample space is I can have any one of the outcomes 1, 2, 3, 4, 5, 6, I again assume all the outcomes are equally likely to happen, we introduce what we mean by equally likelihood and this means that the probability of this 1 happening is the same as probability of 2 happening, which is the same as probability of each of the outcomes happening equally likely which is going to be $1/6$.

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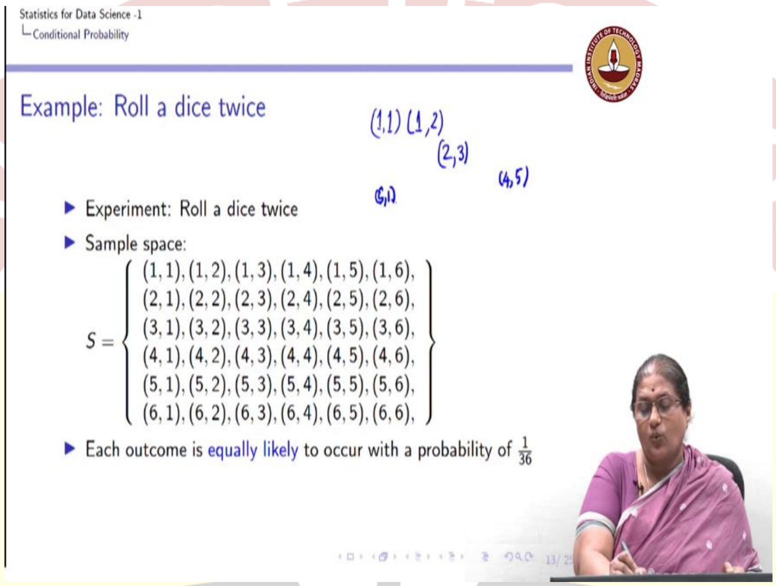

So, now suppose I toss a coin twice or so now suppose I am tossing or rolling this dice twice, so what are the outcomes so the again the experiment is to roll this fair dice twice, my sample space in this case is going to be, so if I my first toss is a 1, my second roll is also a 1, first is a 1, my second could be a 2, my second could be a my first toss could be a 1, my second could be a 3, my first toss could be a 4, my second could be a 5, my first toss would be a 5, my second could be a 1, so you can see that the set of possible outcomes is with every first toss.

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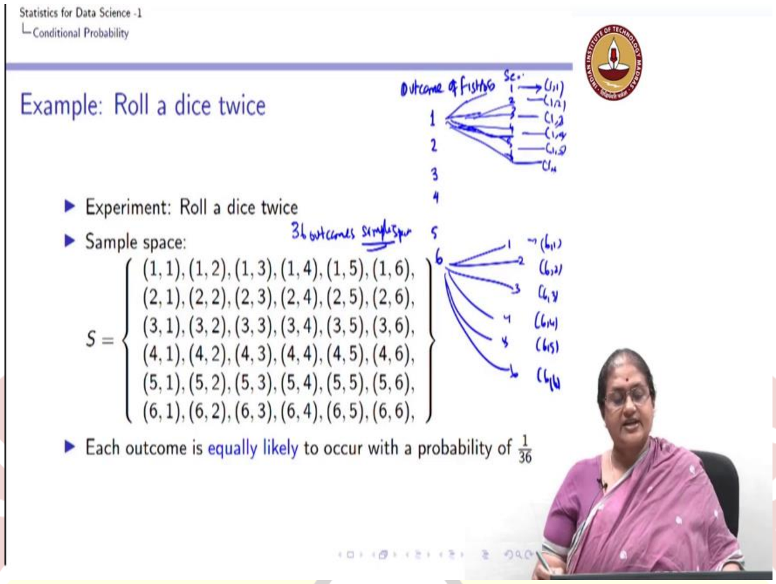

So, I have the first so I can depict this in this way the outcome of first toss the outcome of a first toss could be any one of this 1, 2, 3, 4, 5 and 6, now if my first toss is a 1 the second toss could again be so if I have second toss it could be a 1, 2, 3, 4, 5, 6, with 2 also I could have a 1 so this final outcome of the experiment is (1,1), this is (1,2), this is (1,3), this is (1,4),this is a (1,5), this is a (1,6), similarly with each one of these outcomes with 6 ,I could have a 1, 2, 3, 4, 5 and 6.

So, this outcome corresponds to (6 ,1), (6 2), (6, 3), (6, 4), (6,5) and (6, 6), so you can see with each one, I have 6 more outcomes and that gives me a total of 36 outcomes in my sample space, each outcome is equally likely to happen I have 36 outcomes, so each of these outcomes are equally likely to happen with the probability of 1/36.

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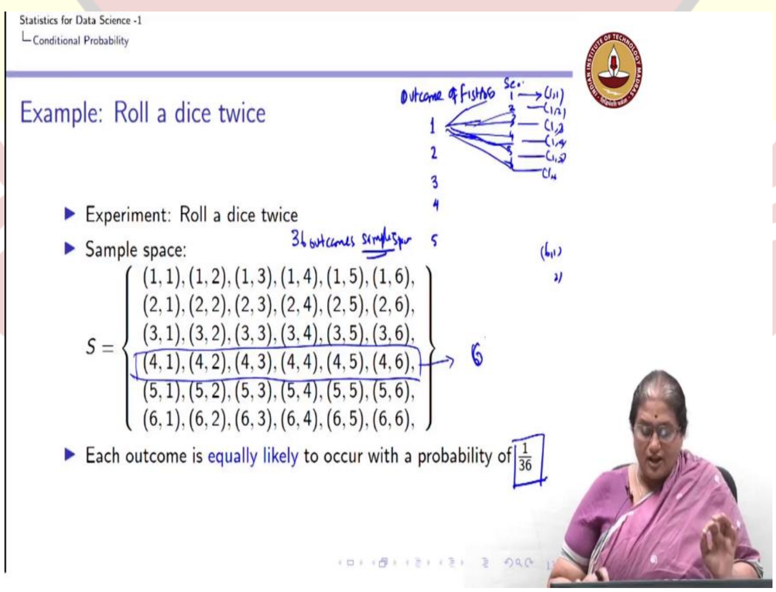

Now, suppose further I know that the first roll of the dice lands in a 4, so what are the chances? So, what are the outcomes? So, this you can see that these outcomes correspond to the first roll being a 4. How many outcomes do I have here? I have 6 outcomes and these 6 outcomes and these 36 outcomes actually correspond to the outcome that the first roll is a 4.

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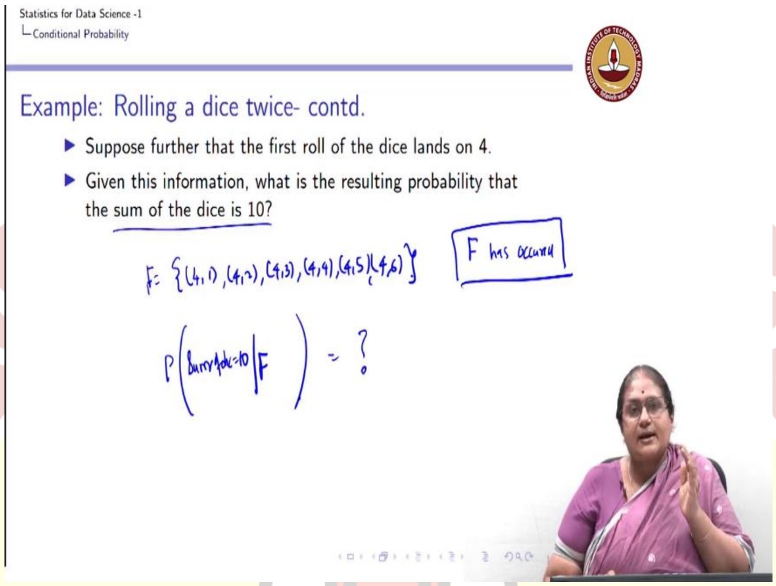

So, I want to know that given that the first outcome is a first role is a 4, what is the chance of the resulting probability that the sum or what is the chance that the sum of the dice is a 10? So, if I know that the first outcome is 4, let me define the event F to be $\{(4,1),(4,2),(4,3),(4,4),(4,5),(4,6)\}$, so what is given to us?

The information that is given to us is F has occurred, this is the partial information that has been given to us is F has occurred. So, given this information, so given this information I want to know what is the chance that some of dice is equal to 10? That is the question which we are asking. Given that the first roll is a 4, what is the chance that the sum is equal to a 10?

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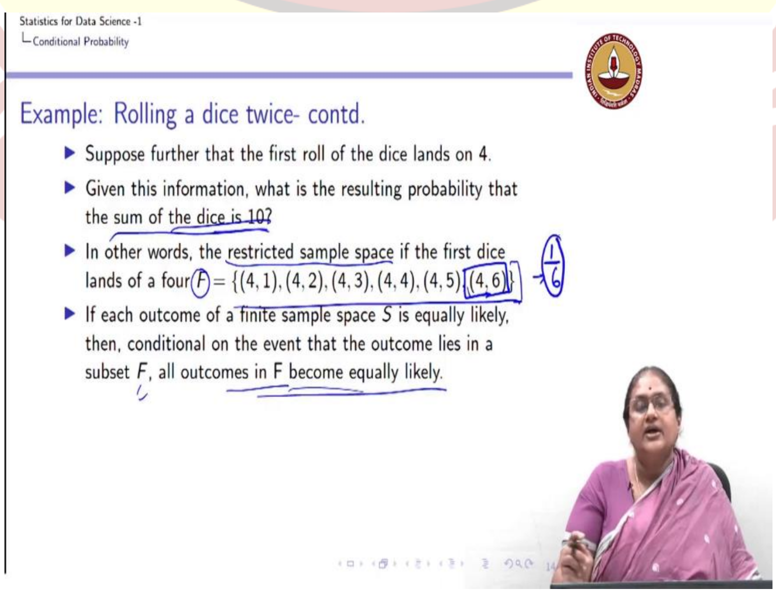

So, if an outcome of a finite sample space, what is the finite sample space we consider here I have 36 outcome, so if an outcome of a finally finite sample space is equally likely then conditional on the event that the lies in a subset F the outcomes in F are also equally likely. So, if I am conditioning at on this event the outcomes of this event or this subset are also equally likely.

So, conditioned on what I refer to as a restricted sample space, what is the restricted sample space? I have this full sample space, but I am not interested in all these outcomes here, I am only interested in those outcomes that are favourable to this event that the first roll is a 4 and I am looking at these events also being equally likely, now within this restricted sample space you can see that the outcome that satisfies that the sum of dice is 10 is this outcome. And what is the chance of that assuming all the outcomes here are equally likely in this space I have 6 outcomes of which one of the outcome satisfies this event giving me a probability of $1/6$.

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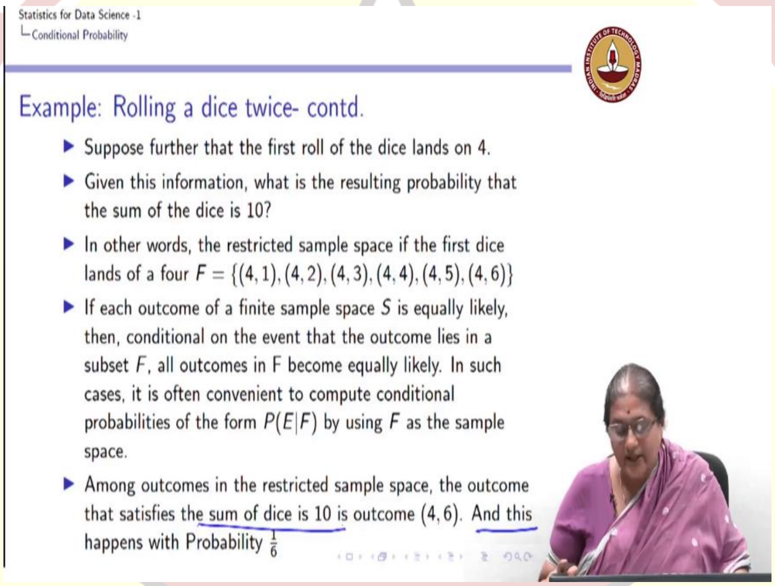

So, you can see that the $P(E|F)$ can be obtained by this logic as $1/6$. So, among the outcomes in the restricted sample space, since the outcomes in the restricted sample space are also equally likely, I have 6 outcomes in my restricted sample space, I can say that the probability of sum of dice is equal to 10 is as $1/6$.

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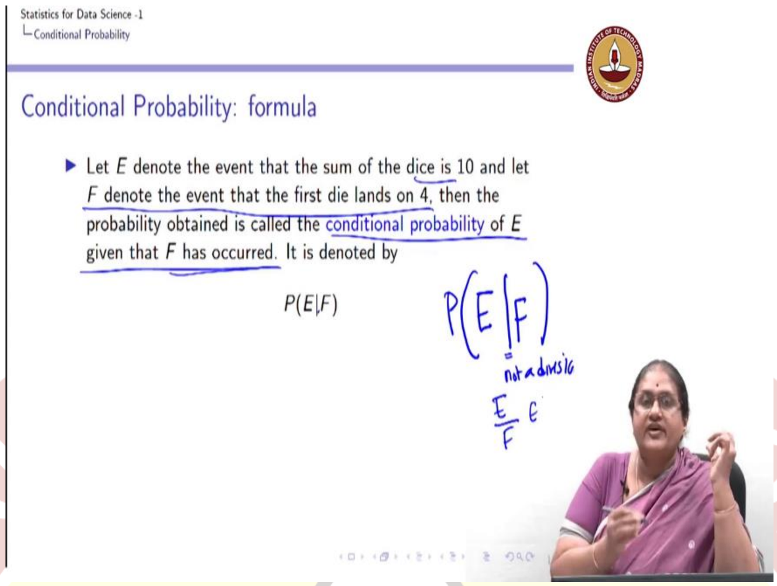

So, now let us come to a formal formula for the conditional probability. Now, what do we have here? If E denotes the event that the sum of dice is 10, so this is the event on which I am conditioning or this is the conditioning event and let F denote the event that the first die lands on a 4, so I am interested in knowing what is the conditional probability of E given that F has occurred. I denote that by $P(E|F)$. I repeat I write that or denote it as probability of E conditioned on F, please remember that this is not a division symbol, it is not E divided by F or it should not be told E by F, always practice yourself to articulate it as probability of E conditioned on F or probability E given F.

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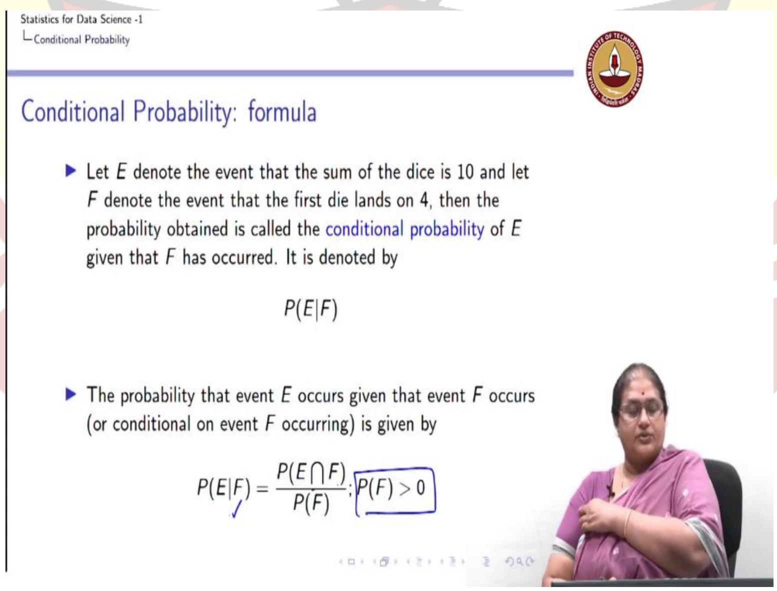

So, what is this $P(E|F)$? What is the formula for this probability? The probability of an event E occurring given that an F occurring is given by $\frac{P(E \cap F)}{P(F)}$and we know that this is defined if I have

$P(F) > 0$ or in other words I always condition on a non null event. So, the conditional probability formula is $P(E|F) = \frac{P(E \cap F)}{P(F)}, P(F) > 0$

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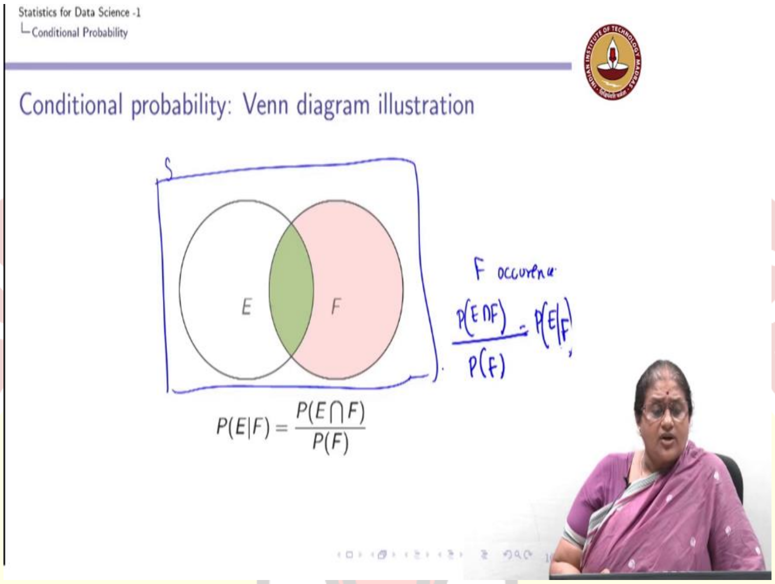

So, now let us understand it through a Venn diagram, we also saw that Venn diagrams illustrate the concept of probability well, so suppose I have a sample space and I have two events E and F which are subsets of my sample space, now I am interested in knowing so this is my E event and this is my F event, so the green portion is my $E \cap F$. Now, the way I can view this is suppose I want to know what is the chance of E happening given F has happened, so I can look at this.

So, this green portion is the portion of your entire pink portion entire pink portion is F occurrence and this E portion this actually this shaded green portion is the chance of E occurring because given F has occurred, pink portion is F occurring, the green portion is the chance of E occurring given F has occurred and we can see that the green portion is nothing but $E \cap F$, so $\frac{P(E \cap F)}{P(F)}$ this green portion to the entire pink portion is my $P(E|F)$, that is one way of visualizing the conditional probability formula.

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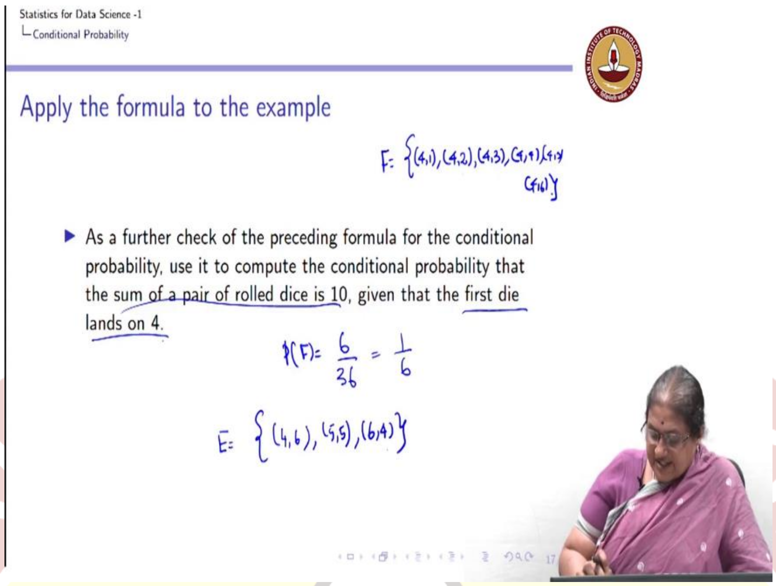

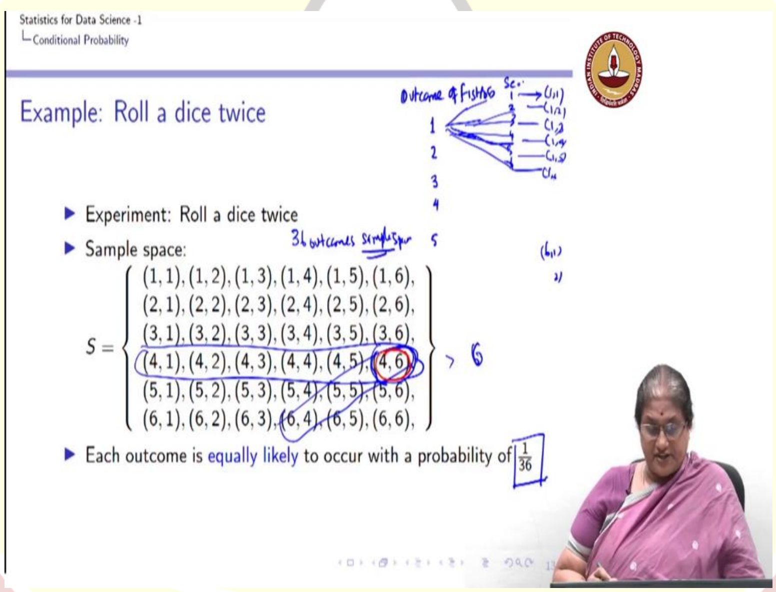

So, now let us apply the formula to the example that we have stated. The example was again here we check the preceding formula, so what is it we are interested in knowing? I have this event F which is the event that the first dice lands on a 4 and I know the outcomes of this experiment are 4 of this event is $\{(4,1),(4,2),(4,3),(4,4),(4,5),(4,6)\}$. Now, the probability of this F is 6 outcomes all of them are equally likely again recall this is going to be 6 /36 which is my 1 / 6.

Now, let E be the event that the sum is 10 and I know the events or the outcomes that satisfy this are $\{(4,6),(5,5),(6,4)\}$, these are the 3 events that satisfy that the sum is equal to 10, I can see that those events here are though outcomes here are this outcome, so I can write down that these 3 outcomes are the outcomes that satisfy my event E and this are my outcomes that satisfy my event that the first roll is a and we can see that the intersection of these 2 events is this outcome which is $\{(4,6)\}$.

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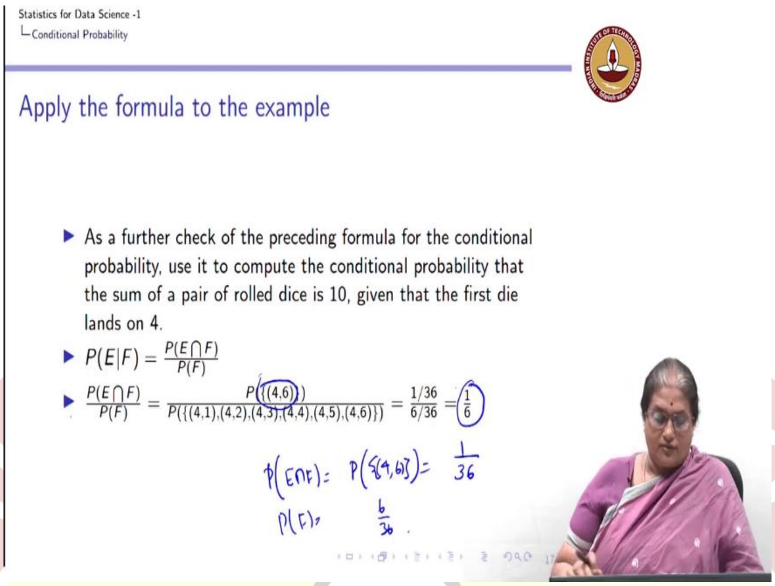

So, if we define E to be the event which has the so you can see that $E \cap F$ is my outcome which is $\{(4, 6)\}$ and the $P(E \cap F)$ is the same as the probability of this outcome (4, 6) happening, which is $1/36$ again I am assuming all my outcomes are equally likely, probability of F we have already computed it to be $6/36$, so by applying the conditional probability formula this is going to be $\frac{1/36}{6/36}$which is $1/6$, which is what we obtained when we restricted the sample space and assume that the outcomes of the restricted sample space are also equally likely and computed the probability.