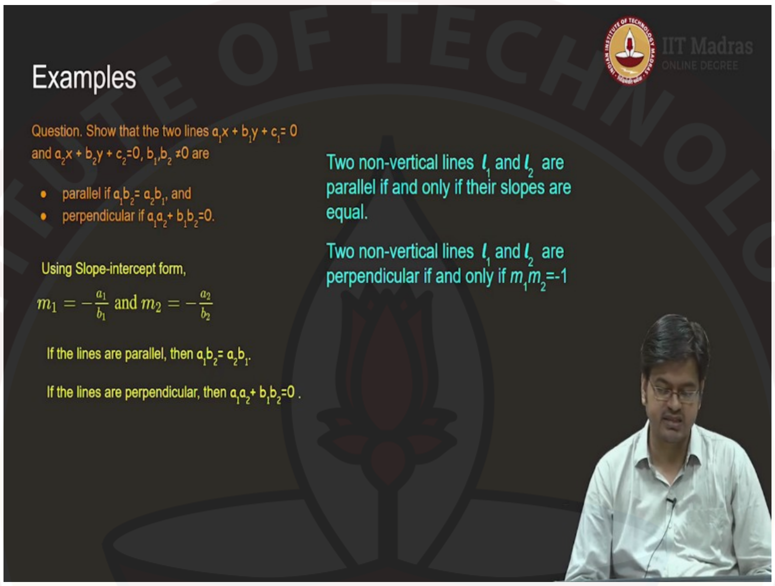
IIT Madras

ONLINE DEGREE

Mathematics for Data Science 1
Prof. Neelesh S Upadhye
Department of Mathematics
Indian Institute of Technology Madras – Chennai

Lecture-21
Equation of Parallel and Perpendicular Lines in General Form

(Refer Slide Time: 00:19)



Let us look at next example which is another application of a general form of equation of a line. The example is stated in the form of a question that is if I have been given two lines $a_1 x + b_1 y + c_1 = 0$, $a_2 x + b_2 y + c_2 = 0$, $b_1, b_2 \neq 0$. What does this mean? That means the lines are non-vertical. $b_1, b_2 \neq 0$ means the lines are non-vertical you can verify for yourself.

Now two such lines are parallel if $a_1 b_2 = a_2 b_1$and perpendicular if $a_1 a_2 + b_1 b_2 = 0$. This is an interesting application of general form of equation of line. And if you recollect, we have derived some characterization of line in terms of slope. So, let us try to see this problem so let me first identify if I want to characterize parallel and perpendicular lines what should I do?

What is a parallel line, how will I identify a parallel line when I will have their slopes to be equal and how will I identify a perpendicular line, when the product of the slopes of the two lines is -1? So, if you remember this then the job reduces to finding the slopes of the two lines. Can I find a slope of these lines? Let us first consider this line $a_1x+b_1y+c_1=0$. You should be immediately able to identify this with slope point form which is $y=mx+c$.

So if I want to adjust this equation in the form of $y=mx+c$ then what should I do? Because b_1 is nonzero I can divide throughout by b_1 and shift this coordinate of y to their right-hand side of the equation. So, I will get $y=\frac{-a_1}{b_1}-\frac{c_1}{b_1}$. So what is the slope $\frac{-a_1}{b_1}$. A similar trick you can apply here and therefore you will get $m_2=\frac{-a_2}{b_2}$. So using slope intercept form you have got $m_1=\frac{-a_1}{b_1}\wedge m_2=\frac{-a_2}{b_2}$.

Now let us recollect the famous fact because $b_1\wedge b_2$ are not equal to 0 we are not considering vertical lines. So, two non-vertical lines are parallel if and only if their slopes are equal. So, what you will do you will just put $m_1=m_2$because you have been given that the lines are parallel. So if you put $m_1=m_2$, minus sign will cancel each other $\frac{a_1}{b_1}=\frac{a_2}{b_2}$. Multiply both sides by b_1b_2, b_1,b_2 are nonzero.

So multiply both sides by b_1b_2, you will get $a_1b_2=a_2b_1$. Therefore, the lines are parallel then $a_1b_2=a_2b_1$. In a similar manner we also know something about perpendicular lines that the product of their slopes is -1, if the lines are perpendicular. So, just multiply m_1,m_2 and equated to -1. Minus sign will cancel each other so you will get $\frac{a_1}{b_1}\times\frac{a_2}{b_2}=-1$.

So take the denominator on the right hand side that is b_1b_2, so $a_1a_2=-b_1b_2$ which essentially means $a_1a_2+b_1b_2=0$. Therefore, we have proved the result. So, now what we have done right now is we have related our result about the characterization of perpendicular and parallel line via slope to a general form of equation and this is the new condition that we are coming up with if the lines are parallel and you have been given to two non -vertical lines and their general forms then you just need to check that $a_1 b_2 = a_2 b_1$ for the lines to be parallel and $a_1 a_2 + b_1 b_2 = 0$ for the lines to be perpendicular. This you can consider as another characterization of parallel and perpendicular lines using a general form of the equation of lines.