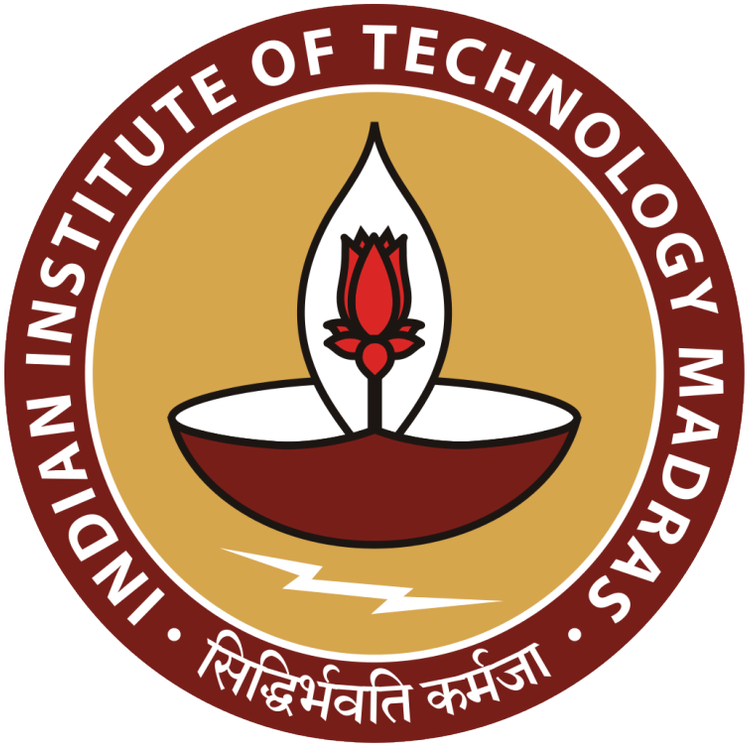

IIT Madras

ONLINE DEGREE

Real numbers

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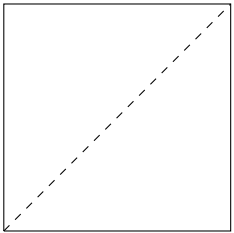
Mathematics for Data Science 1
Week 1

Beyond rationals

- Rational numbers are dense
  - Between any two rationals we can find another one
- Is every point on the number line a rational number?
- For an integer m, its square is $m^2 = m \cdot m$
- Square root of m, $\sqrt{m}$, is r such that $r \cdot r = m$
- Perfect squares — $1, 4, 9, 16, 25, \ldots, 256, \ldots$
- Square roots — $1, 2, 3, 4, 5, \ldots, 16, \ldots$
- What about integers that are not perfect squares?

Beyond rationals . . .

- $\sqrt{2}$ cannot be written as $\frac{p}{q}$
- Yet we can draw a line of length $\sqrt{2}$
  - Diagonal of a square whose sides have length 1
- $\sqrt{2}$ is irrational
- Real numbers: $\mathbb{R}$ — all rational and irrational numbers
- Like rationals, real numbers are dense
  - If $r < r'$, then $\frac{(r + r')}{2}$ lies between r and r'



Beyond reals

- Some well known irrational numbers
  - $\pi = 3.1415927\ldots$
  - $e = 2.7182818\ldots$
- Can we stop with real numbers?
  - What about $\sqrt{-1}$
  - For any real number r, r^2 must be positive — law of signs for multiplication
- $\sqrt{-1}$ is a complex number
- Fortunately we don't need to worry about them!

Summary

- Real numbers extend rational numbers
- Typical irrational numbers — square roots of integers that are not perfect squares
- Real numbers are dense, like rationals
- Every natural number is an integer
- Every integer is a rational number
- Every rational number is a real number
- Complex numbers extend real numbers, but we won't discuss them