IIT Madras
ONLINE DEGREE

(Refer Slide Time: 00:14)

ndots

nC_2

if they are

not collinear

$$= {}^{n}C_2 + 1$$

$$= 190 - 3 + 1$$

$$- 6 + 1$$

$$= 190 - 9 + 2$$

$$= 183$$

In this question, they are saying that there are 4 black dogs and 16 brown dogs and they are running on a ground such that no three dogs are in a straight line. So basically, no three of them are in a straight line. Any two of them will be on a straight line because a straight line is defined by two points. So I need two dogs will be on a straight line but they are saying no three are collinear together.

Now, suddenly, two of the brown dogs started following one black dog. So we have two brown dogs following one black dog. So these three have now become collinear. And then further, the remaining three black dogs started following one of the remaining brown dog.

So the other three black dogs are now following one brown dog. So these four are also on a straight line. And then they are asking how many straight lines can be found passing through the dogs.

So now, suppose there are n dots which are all non-collinear; suppose there are n dots. So in order to make a straight line, you just have to pick two of these n dots. And how many ways can you pick is the number of straight lines you get, that is basically $^{n}C_2$. And this is if they are not collinear.

Now, of course, any two points will be collinear. The funda is no three should be collinear. So if no three of them are collinear, you choose any two you will get a unique straight line. If the points are not collinear, you get $^{n}C_2$ straight lines.

However, in our problem, there are 20 dots. And so, in this case, the each dot is a dog but there are some collinear. So what we do is we will calculate the number of straight lines for all 20 when they are not collinear, which is $^{20}C_2$ minus the number of lines which should have come from this, which is $^{3}C_2$ but we include the one line that is all coming from this particular three dogs being in a straight line. So we will add 1.

And then similarly, we will subtract the lines that were supposed to come from this which is $^{4}C_2$ but we will add the one individual line that they make anyway, which is 1. So we now have $^{20}C_2$ - $^{3}C_2$ + 1 - $^{4}C_2$ + 1 = 190 – 3 + 1 – 6 + 1 = 190 – 9 + 2 =183. So there are a 183 unique lines you can draw through these dogs.

Here, the questions is how many straight lines can be formed passing through the dogs. If you are only considering one dog per line, then you can get infinite. So what it actually is, is supposed to be how many straight lines can be found passing through at least two of the dogs; that is the actual question and that number is 183.