Statistics for Data Science -1

Lecture 10.4: Modeling situations as Binomial distribution

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Learning objectives

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3. Expectation and variance of the binomial distribution.

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1. Derive the formula for the probability mass function for Binomial distribution.
2. Understand the effect of parameters n and p on the shape of the Binomial distribution.
3. Expectation and variance of the binomial distribution.
4. To understand situations that can be modeled as a Binomial distribution.

Application: Pack of three goods

- Consider a company that sells goods in packs of three.
- The production process of the goods is not very good and results in 10% of goods being defective.
- The company believes that customers will not complain if one out of three in a pack is of bad quality, however, will complain if more than two out of three are of bad quality.
- The company wants to keep number of complaints low, say at 3%.
- How do we help the company analyse the situation?

Application: Pack of three goods

- Random experiment: Choosing an item and noting its quality. $S = \{Good, Bad\}$
  - Success: good
  - Failure: Bad
- Given probability of a defective item is 0.1. Hence, Probability of good=p= 0.9.
- We want to know number of good items in a pack of three. Hence $n = 3$
- Let X = number of good in pack of three. X is a Binomial random variable with $n = 3, p = 0.9$.

Application: Pack of three goods-pmf

- The distribution of X is given by

X	0	1	2	3
$P(X = x_i)$	$\binom{3}{0}\frac{9}{10}^{0}\frac{1}{10}^{3}$	$\binom{3}{1}\frac{9}{10}^{1}\frac{1}{10}^{2}$	$\binom{3}{2}\frac{9}{10}^{2}\frac{1}{10}^{1}$	$\binom{3}{3}\frac{9}{10}^{3}\frac{1}{10}^{0}$

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$P(X = x_i)$	0.001	0.027	0.243	0.729

Application: Pack of three goods-Probability of complaint

- Customers will complain if they find more than one defective in the pack of three.
  - $P(X \leq 1)$
- The distribution of X is given by

X	0	1	2	3
$P(X = x_i)$	0.001	0.027	0.243	0.729

- $P(X \leq 1) = 0.001 + 0.027 = 0.028$
- 2.8% is less than 3% which was the goal set by the company-goal achieved.
- However, if the company set 2.5% as their threshold then 2.8% would have been more than 2.5% and company would not have achieved its goal.

Effect of n- size of packs

n							
3	X	0	1	2	3		
	$P(X = i)$	0.001	0.027	0.243	0.729		
4	X	0	1	2	3	4	
	$P(X = i)$	1E-04	0.0036	0.0486	0.2916	0.6561	
5	X	0	1	2	3	4	5
	$P(X = i)$	1E-05	0.00045	0.0081	0.0729	0.32805	0.59049

Rolling a dice

Roll four fair dice. Define success as getting a six. Find the probability that

a 6 appears at least once.

b 6 appears exactly once.

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- Let X = the number of sixes in four rolls of the dice. Then $X \sim B(4, 1/6)$.
- The pmf is given by

X	0	1	2	3	4
$P(X = i)$	0.4823	0.3858	0.1157	0.0154	0.0008

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b Two or more are defective=$P(X \geq 2) = 0.0225$

Example: Satellite functioning

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$X \geq 4, P(X \geq 4) = 0.0156$

Section summary

- Application of binomial model to real life examples.