IIT Madras

ONLINE DEGREE

Statistics for Data Science-I
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Lecture 5.2
Permutations and Combinations - Factorials

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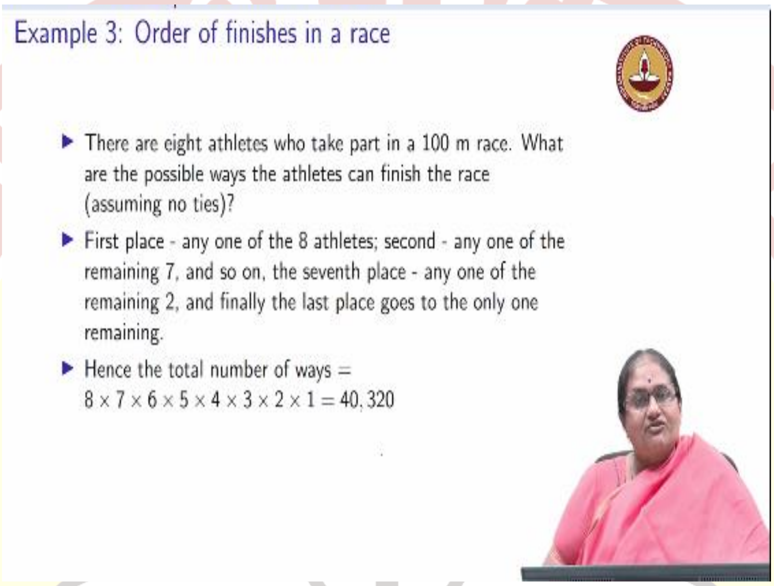

So now let us introduce a very important concept which is referred to as a factorial notation or a factorial. Now let us look at a race it is a 100 meter race and I have 8 people who are running in this race. So they are distinct people so I have person 1, person 2, person 3, person 4, person 5, person 6, person 7 and person 8. These are all distinct people they are running this race.

Now what are the possible ways these athletes can finish the race. So I have position 1, 2, 3, 4, 5, 6, 7, 8 these are the positions that is what I mean by this is the first position of finish, the person who come second and the person who comes 8 because they are only 8 people these are the only finishing position. So what is it I am interested in I am interested in knowing how many orders of finish that is possible when 8 people are participating in a particular race.

And not assuming any ties now I am assuming that there is a clear ranking or a clear order of finish. So now let us look at this position and I am also assuming that all of the 8 of them are equally capable so there could be anybody can come first on any given day and the difference is not going to be too much. Now how many choices do we have for the first person? So if I put person 1 here just a hypothetical situation.

Then person 1 cannot appear in any other position because he has already come in the first position. So given that person 1 is in the first position, person 1 is not available for any other position so for person 2 to position 2 I have only a choice of 7 people because person 1 has already occupied the first position. So, suppose person 2 comes to the second person for the third position.

I do not have both person 1 and person 2 I have only a choice from the other 6 remaining and suppose person 4 is in the third position for the fourth position I have only 5 choices it could be person 7 and so forth I have person 8, I have person 6, I will have person 3 and I have person 5. So this is one order. So if I am looking at it the number of choices to fill in the first position I have 8 choices.

If I fill up the position 1 with one of these people for the second thing I have only 7 choices. I have 6 choices here, I have 5 choices here, 4, 3, 2 and 1 these are the choices I have to fill in these positions from the available number of people. These are distinct people. So I can in other words if I turn the action first action is to choose from these 8 people number of people who would fill in the first position.

Second action is to come up with the number of people who will fill in the second position and so forth 8 action is number of people who will fill in the 8 position then I can apply my multiplication rule of counting to know that the total number of possible ways in which all these 8 athletes can actually complete the race is going to be $8 \times 7 \times 6 \times 5 \times 4 \times 3 \times 2 \times 1$ this is the large number.

So this is basically it helps me so instead of writing as $8 \times 7 \times 6 \times 5 \times 4 \times 3 \times 2 \times 1$. Can I have a simpler way to express this number. So what is the simpler way to express this number which is $8 \times 7 \times 6 \times 5 \times 4 \times 3 \times 2 \times 1$ this is where I define a notation which is referred to as a factorial notation.

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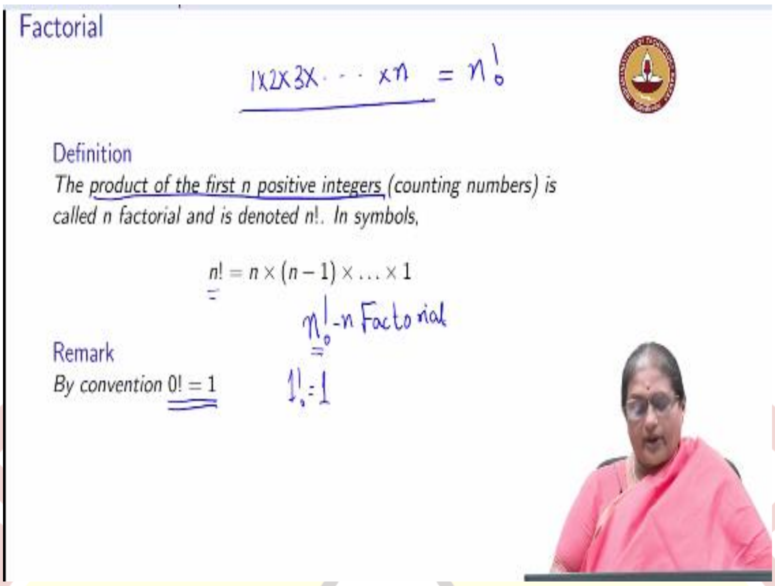

What is a factorial notation? Factorial notation is the product of the first n positive integers. What are the first n positive integer $1 \times 2 \times 3 \times \ldots \times n$. This product is what I referred to as n factorial $n!$. So this is the notation as n with a exclamation mark and I refer to it as a it is called n factorial and it is written as n factorial $n!$. by convention $0! = 1$, $1!$ is also equal to 1.

So now if I have this factorial notation my earlier problem becomes very simple I can write that the total number of ways I can express it as equal to 8 factorial which is far more elegant than writing it as $8 \times 7 \times 6 \times 5 \times 4 \times 3 \times 2 \times 1$ I can just express it as 8 factorial, but this factorial notation is extremely useful.

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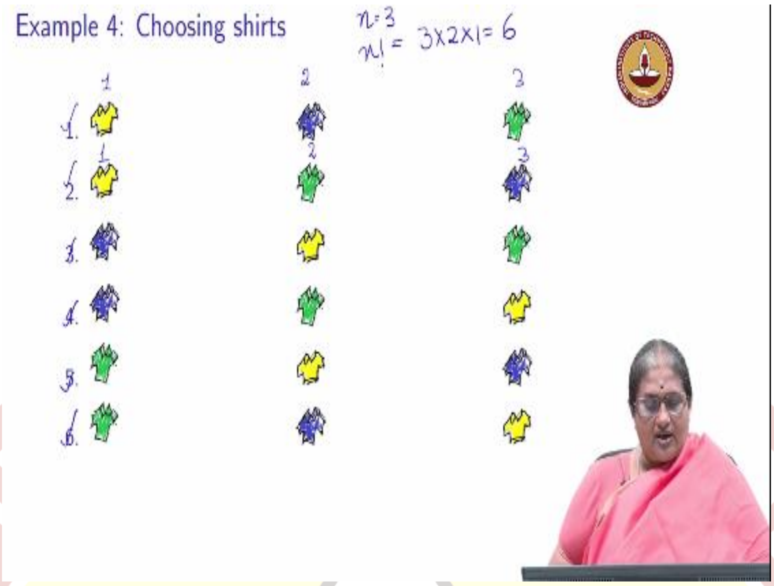

Again let us go back to looking at this example where I have three shirts. Now the first choice is again suppose in the same choosing this one I have only 3 people and this is again the order of finish of these 3 people or wearing a yellow t shirt, a blue t shirt and a green t shirt. The first order could be the yellow t shirt as 1 the blue t shirt is second, the green t shirt is third. The second is yellow t shirt is first, green is second, blue is third.

So you can see that the total number of ways is 1, 2, 3, 4, 5, 6. The number of object distinct object n is equal to 3 and you can see that n factorial the way I defined it was $3 \times 2 \times 1$ which is giving me a 6 here total number of choices which is $n!$ which is equal to 6.

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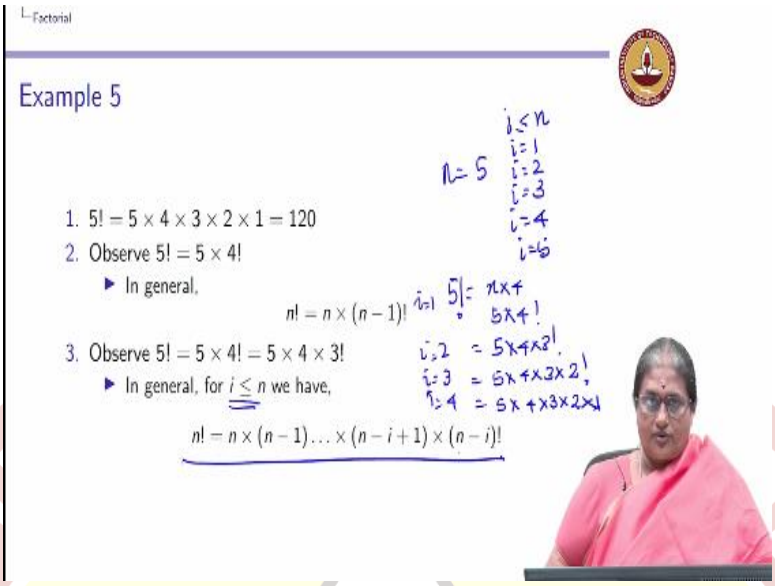

So now let us go back and look at the factorial notation 5!. What is 5!? 5! is $5 \times 4 \times 3 \times 2 \times 1$ now if you look at this portion if you look at this portion it is $4 \times 3 \times 2 \times 1$. I know this is nothing, but 4!. Hence I have 5 factorial which can be written as $5 \times 4!$. In other words what we say is similarly if I look at 6! Is $6 \times 5 \times 4 \times 3 \times 2 \times 1$. Now this portion is nothing but 5!.

So I can write 6! as $6 \times 5!$. In other words for any integer $n! = n \times (n-1) \times (n-2) \ldots . \times 1$ this is $n \times (n-1)!$. So the first expression is $n! = n \times (n-1)!$. So I have $n! = n \times (n-1)!$. Now again look at the following I 5! is $5 \times 4 \times 3 \times 2 \times 1$. So I can write this also as $5 \times 4 \times 3 \times 2 \times 1$ where this is nothing but 3 factorial.

So I can write this as $5 \times 4 \times 3!$ Similarly if I have a 6! is $6 \times 5 \times 4 \times 3 \times 2 \times 1$ I see that this is equal to 4!. So I can express 6! as $6 \times 5 \times 4!$. So you can see that if I have 5! I can write it as $5 \times 4 \times 3!$. 6! or $6 \times 5 \times 4!$. So in general for any $i \leq n$ so my n if it is 5 what are the i's that are less than or equal to n.

I can have $i = 1$. I can have $i = 2$, $i = 3$, $i = 4$ or $i = 5$ I have $n!$ which is 5! which is n which is equal to 5×4 so my $i = 1$ $n - i$ is 4! it is $5 \times 4!$, $i = 2$ it is $5 \times 4 \times 3!$, $i = 3$ $n - i$ I have $5 \times 4 \times 3 \times 2!$, $i = 4$ is $5 \times 4 \times 3 \times 2 \times 1!$. So this is another expression which we can use.

And this will help us simplify a lot of expressions when we are encountering counting when we do our permutations and combinations and probability problems.

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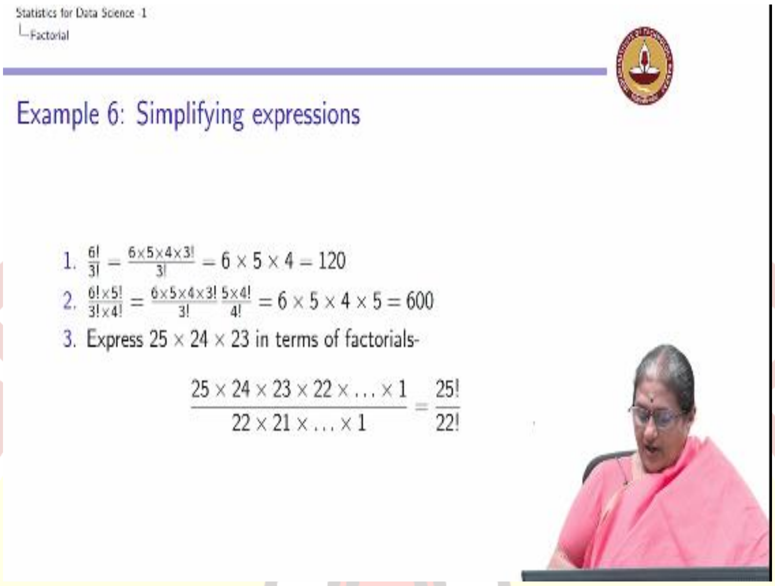

Now let us look at further simplification of factorial notation. Now what is the factorial notation we are looking at what is 6!. 6! I know is $6 \times 5 \times 4 \times 3 \times 2 \times 1$. From my first definition 3! I just retained that way. I recognize that this quantity is nothing, but 3! so I can cancel out this with this and I get $\frac{6!}{3!}$ is nothing but 6! is $6 \times 5 \times 4$ which is 120.

Now let us look at the next problem $6! \times 5!$ so I have a 6 into 5 into 4 we have already seen $\frac{6!}{3!}$ is $6 \times 5 \times 4$. Now $\frac{5!}{4!}$is going to be 5 factorial divided by 4 factorial into 4 factorial. So I can see that I can simplify this by cancelling out the 4! with 4! and what I have is 600 as the answer. Now sometimes you might want to express a product in terms of factorial.

So suppose the example I have taken here is to express $25 \times 24 \times 23$ in terms of factorials let me write it I am having $25 \times 24 \times 23$. I am going to multiply the numerator so I am going to write it as a fraction into 21. So forth with 1 I only want this portion so I need to divide the denominator with this portion which I have put with double line and what is that portion that portion is nothing, but $22 \times 21 \times 20 \times \ldots . \times 2 \times 1$.

Now what is this denominator? Denominator is nothing but 22!. My numerator is nothing but25!. So I can express this $25 \times 24 \times 23$ as $\frac{25!}{22!}$. So I can express this $25 \times 24 \times 23$ as $\frac{25!}{22!}$. So this notation of expressing the product in terms of factorial would come in news when we learn more about permutations and combinations.

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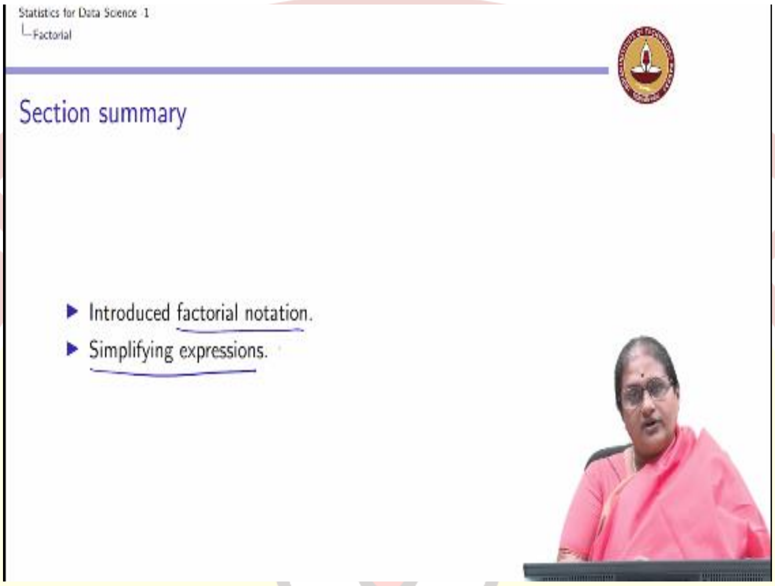

So, in summary what we have learned today is we introduced the factorial notation and how do we simplify expressions using the factorial notation, that is what we have learned.