

IIT Madras
ONLINE DEGREE

Mathematics for Data Science 1

Week 05 - Tutorial 05

(Refer Slide Time: 0:14)

5. Let M and N be the sets of all values of m and n respectively such that both equations $x^2 + mx + 4 = 0$ and $x^2 - nx + 1 = 0$ have always two real distinct roots each, then find the sets of M and N .

Let C be a set of integers and values of m and n to be chosen randomly from C , then define the set C such that both the equations have two real distinct roots each.

$$ax^2 + bx + c = 0 \quad b^2 - 4ac > 0$$

$$m^2 - 16 > 0 ; \quad n^2 - 4 > 0$$


$$m^2 > 16 ; \quad n^2 > 4$$

$$\Rightarrow \begin{array}{ll} m > 4 & n > 2 \\ m < -4 & n < -2 \end{array}$$

In this question we have 2 sets capital M and capital N which are sets of all values of small m and small n respectively such that these two equations have always two distinct real roots each, then find the sets M and N. Let us finish this part first. So, for a quadratic equation $ax^2 + bx + c = 0$ to have distinct real roots, the discriminant which is basically the value $b^2 - 4ac > 0$.

So, for this first equation that would be $m^2 - 16 > 0$ and simultaneously, we need for the second equation $n^2 - 4 > 0$. So, $m^2 > 16$ and $n^2 > 4$ and this would imply m is positive and greater than 4 or m is negative and lesser than -4. And here this would imply similarly n is positive and greater than 2 or n is negative and lesser than -2. So, these are all the possible values for which you will have two real distinct roots for these equations.

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$$\begin{aligned} m^2 > 16 & ; \quad n^2 > 4 \\ \Rightarrow \quad m > 4 & \quad n > 2 \\ m < -4 & \quad n < -2 \end{aligned}$$
$$\begin{aligned} M &= (-\infty, -4) \cup (4, \infty) \\ N &= (-\infty, -2) \cup (2, \infty) \end{aligned} \quad M \subset N$$
$$C = \{n \mid n \in \mathbb{Z} \mid n| > 4\}$$

So, your set M would be the union of two intervals, one is a $(-\infty, -4) \cup (4, \infty)$. And set N is similarly $(-\infty, -2) \cup (2, \infty)$. Now the next part of the question, C is a set of integers and values of m and n are to be chosen randomly from C , then define the set C such that both equations have two distinct real roots each.

So, this is necessarily one single set we are taking and m and n should be chosen from that set. So, we clearly cannot have m being -2 or 2 or even -3 or 3 . The set we are looking for is some sort of an intersection of capital M and capital N because both small m and small n should be drawn from this. And in this case that intersection will just be the set capital M because M is necessarily a subset of N .

However, C is also given out to be a set of integers, so it is not just the intersection of m and n , it is the set of integers which belong to the intersection and this case that intersection is only capital M where therefore, we have this set coming up as C .