IIT Madras

ONLINE DEGREE

Mathematics for Data Science 1
Prof. Neelesh S Upadhye
Department of Mathematics
Indian Institute of Technology Madras – Chennai

Lecture-23
Distance of a point from the given line

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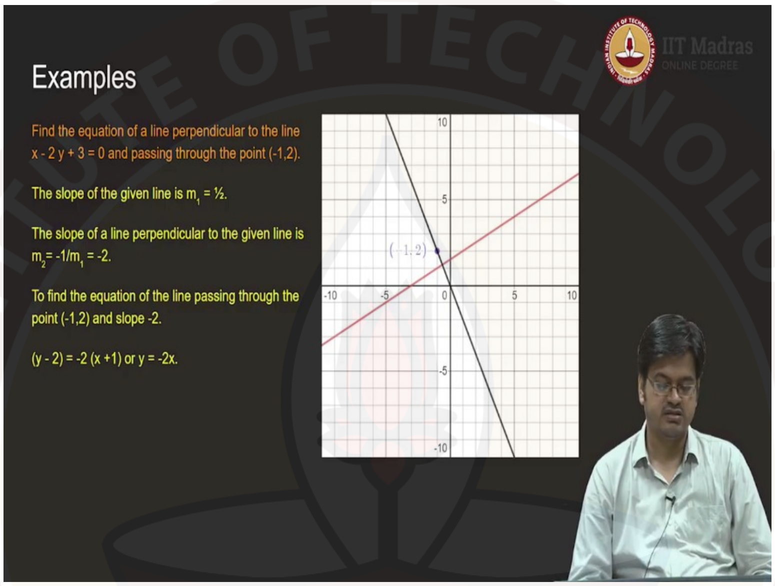

So, we have verified the answer now what you can see here is (-1, 2) is a point which is lying on the line which is perpendicular to the given line. An interesting question can be asked that what is the distance of this; point from the given line. Let us try to answer that question in the next slide.

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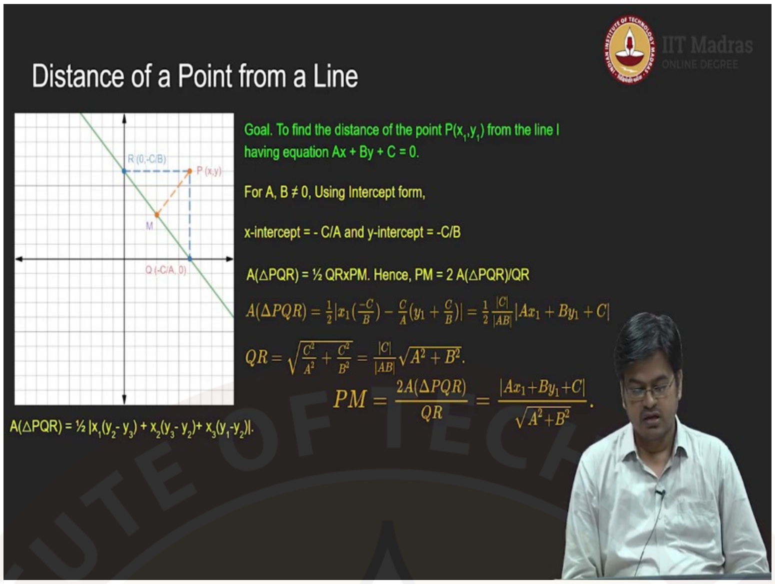

So, the question is given any point I have another line the point is not collinear to the line then what is the distance of that point from a line. So, let us take that as our goal. To be precise we are interested in finding the distance of the point P which has coordinates (x_1, y_1) of this point from the line l which has equation $Ax+By+C=0$ and this is a general form of equation.

Now how will we proceed? So, if I want to understand the location of P, I need to do some analysis because $Ax+By+C=0$ is a completely geometric object. So, I need to understand this line in terms of its geometric concepts. So, what are the geometric concepts associated with this line they are slope x intercept y intercept or points on this particular line. So, let us identify those things first.

So if we assume that A and B both are not equal to 0 then I can rewrite this equation in the form of intercept form that is $\frac{x}{a}+\frac{y}{b}=1$ so in that case my a is actually $\frac{-C}{A}$ that is x intercept and my small b will be $\frac{-C}{B}$ which is y intercept. So, I have identified this line as I have identified the 2 points and these 2 points uniquely determine the line. So, I know how the line is located.

Let us try to visualize this line in terms of the graph of a function. So, as you can see I have mentioned that x intercept is $\frac{-C}{A}$ so it is mentioned as a point Q which is $\left(\frac{-C}{A},0\right)$, y intercept is $\frac{-C}{B}$ which is identified here so $\left(0,-\frac{C}{A}\right)$, the point P is located here it may be located anywhere but right now the point P is located here it has coordinates $\left(x_1,y_1\right)$. So, now I want to identify a distance of this point from this line, the line joining the points Q and R.

So, what is the distance? It should be the shortest distance from the line, so the shortest distance in this case if you move along this line the shortest distance in this case is a point where the point is actually perpendicular to the line. So, what I want to say is the shortest distance is the one which is the perpendicular distance. So, the entire question reduces to how to find this perpendicular distance PM.

So, let us try to see what are the geometric objects associated with this. So, you can see from the dotted lines the geometric object that I can associate with this particular distance is a triangle PQR. Now if I want to find the distance PM, I can take help of this triangle PQR so that I will be able to find the distance PM. So, how will I do that? First you see if I want to compute the area of triangle PQR what do I need to know?

I need to know the base and the height and the area of a triangle is half base into height. So, half base into height means $\frac{1}{2}\times QR\times PM$. I do not know what is PM. But we have already seen in this course how to find area of a triangle where its coordinates are given. So, even though I do not know what is PM I know how to compute the area of a triangle. The next question is do I know how to compute the length QR?

Yes of course because this is x intercept this is y intercept and these are the lines which are the distances on x and y axis and all of them form a right-angled triangle. So, by Pythagorean theorem I will be able to find the length of QR. So, I can reformulate the question as

$PM = 2 \times \frac{A(\Delta PQR)}{QR}$. So, now I know how to compute the length PM if I know how to compute area of triangle and how to compute the length of line segment QR both of which I know.

So, let us go ahead and try to compute area of triangle PQR. So, here is our formula for area of triangle PQR which has coordinates $(x_1, y_1), (x_2, y_2), \wedge (x_3, y_3)$. So, let us start with (x_1, y_1), the (x_1, y_1) is the first coordinate remember you will always take this in anti-clockwise direction. So, I will start with this coordinate then I will go to R and then I will go to Q. So, this is (x_1, y_1) this is (x_2, y_2) and this is (x_3, y_3) according to the notation that is given in the formula.

So you will see $x_1(y_2 - y_3)$ so x_1 is first coordinate it will remain x_1 because P has coordinate (x_1, y_1), y_2 is $\frac{-C}{B}$, y_3 is zero. So, you will get $x_1\left(\frac{-C}{B} - 0\right)$ then the next term that is x_2, x_2 here is 0 so this entire thing vanishes then you go to x_3, what is x_3? x_3 is $\frac{-C}{A}$, into y_1, which is y_1 as it is $-\left(\frac{-C}{B}\right)$ so $\left(y_1 + \frac{C}{B}\right)$, this is how I got the formula.

So, if you look at this formula closely you can actually take C common from all within the mod sign so you can take $\left|\frac{C}{B}\right|$, denominator has terms containing B and AB. So, if you want to take those terms out you multiply throughout by AB or you find the LCM is AB and you take AB out so you will get $\frac{1}{2} \times \frac{|C|}{|B|} \times |A x_1 + B y_1 + C|$, remember this is the term corresponding to general form of the equation.

Now we have seen how to compute area of triangle PQR. Next, we will see how to compute the length QR. But length QR is actually very easy because I have a point Q which has only x-coordinate and I have a point R which has only y-coordinate. So, it will be as if computing the distance of length QR is $\sqrt{\frac{C^2}{A^2}+\frac{C^2}{B^2}}$ these are the 2 sides of the triangle and this QR is the hypotenuse of that right-angle triangle.

So, again you can simplify this to amend to this form so you can take out C common so you will get $|C|$, you take A and B common you will get $|AB|$ and then you will get $\sqrt{A^2+B^2}$ which is which is in the numerator and now if you look at this form PM which is the length of the line segment PM is $2\times\frac{A(\Delta PQR)}{QR}$, so just now it is just a matter of feeding the values this half will get cancelled with this 2 and area of triangle PQR is this and QR is this therefore this constants also will vanish because they are same.

And you will get the formula to be equal to $\frac{|Ax_1+By_1+C|}{\sqrt{A^2+B^2}}$ this is how you will calculate a perpendicular distance of a point from a line. Now this idea can be helpful in finding one more thing that is a distance between two parallel lines.

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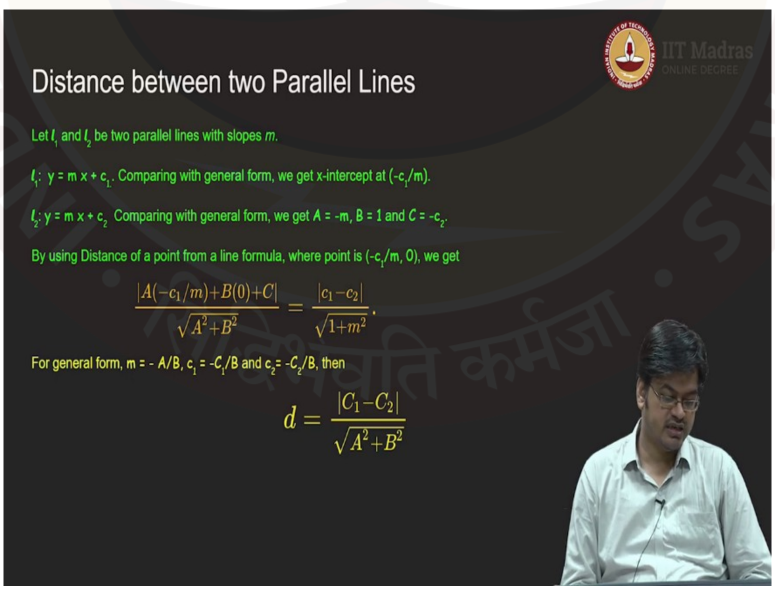

So, the question can be asked is I have two parallel lines what is the distance between two parallel lines. So, let us take the set up because the lines are parallel l1 and l2 they have common slope or the same slope, so their slope is m. Then you can use the slope point form which is

$y = mx + c_1$. Now I want to use the previous concept that I have introduced distance of a point from a line. So, I will first identify this with x intercept. So, what will be the x intercept in this case?

If you identify this line it is very easy to see go back to the general form and figure out that x-intercept is $\frac{-c_1}{m}$, because $B=1$ here $B=-m$ and $C=-c_1$, so the intercept is this $\frac{-c_1}{m}$. Let us take another line that is l2 it has same slope identify it with our standard form $A=-m, B=1, \wedge C=-c_2$. So, now given x-intercept what are the coordinates of this x-intercept $(\frac{-c_1}{m}, 0)\cdot$

So now the problem reduces to finding the distance of this point from this line ok. So, by using the distance of a point from a line formula where the point is $(\frac{-c_1}{m}, 0)$, you just need to substitute this point (x_1, y_1) into this formula for the distance of a line which is given as $\frac{|A x_1 + B y_1 + C|}{\sqrt{A^2+B^2}}$.

So, my point $x_1 \, is - \frac{c_1}{m}$ substituted here, y_1 is 0 substituted here you will get the formula to be equal to $\frac{|C_1 - C_2|}{\sqrt{A^2+B^2}}$ so in this case $\sqrt{A^2+B^2}$, B was 1, A is -m so it is $\sqrt{1+m^2}$.

Now you can actually identify this formula in the general equation form also. So, in the general form instead of B = 1 we have slope which is equal to -A/B and $c_1 = \frac{-C_1}{B}$ and $c_2 = \frac{-C_2}{B}$. So, this I am matching with both equations in general form these are slope point forms but now if you match these equations with a general form you will get this description of the line where you have $Ax + By + C1$ as one line is equal to 0 as one equation of line.

$Ax + By + C_2 = 0$ as equation of the second line. So, in that case this is the form and therefore now you just substitute these values into this expression. So, this m will be replaced by - A/B so you will get $\sqrt{A^2+B^2}$ here and some $|AB|$ will come out common and therefore finally that will cancel off with this B and you will get the expression of the form $\frac{|C_1 - C_2|}{\sqrt{A^2+B^2}}, C_1 \wedge C_2$ belong to general form of equation.

So, this gives us a clear-cut understanding of the interconnection between the slope point form and general form of equation and we have figured out what is a distance between two parallel lines using distance of a point from a line formula.

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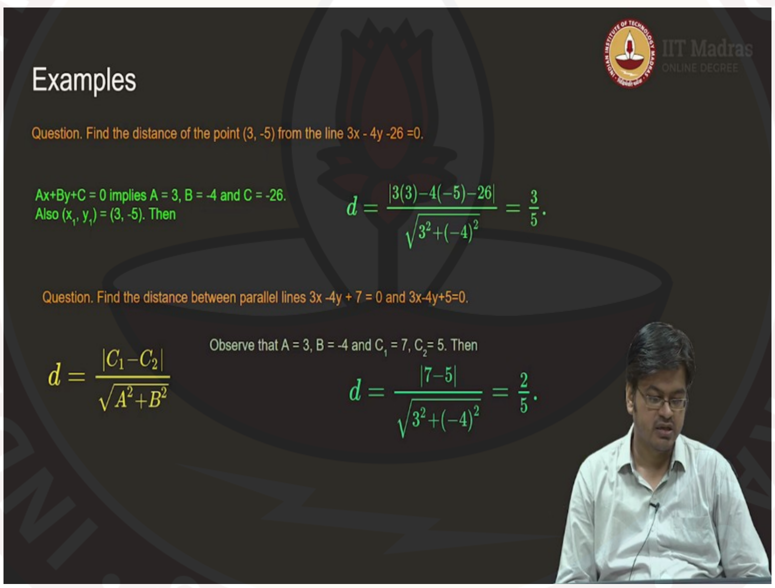

So, now we will solve some examples to concretize the concepts so here are the examples in line. So, you have been asked to find a distance of a point $(3, -5)$ from the line $3x - 4y - 26 = 0$. So, in this case you just need to apply the formula, what is a formula, $\frac{|Ax_1 + By_1 + C|}{\sqrt{A^2+B^2}}$. So, what is A, B, and C here is the key question? (x_1, y_1) is known to be $(3, -5)$. So, A is 3, B is -4, C is -26 then you just need to apply that formula which will give the denominator square root of 25 it will give me 5 the numerator will be 3.

In a similar manner you can ask a question what is the distance between two parallel lines $3x-4y+7=0$, $3x-4y+5=0$. So, what is a formula that we have derived it is $\frac{|C_1-C_2|}{\sqrt{A^2+B^2}}$ it is very straightforward. So, what is C_1 here? C_1 the first line the constant term is 7 the second line the constant term is 5. So, it will be modulus of $\frac{7-5}{\sqrt{3^2+(-4)^2}}$. So, you will get the answer to be 2 / 5.