

Statistics for Data Science -1

Lecture 6.1: Probability- Sample space

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7. Distinguish between independent and dependent events.
8. Solve applications of probability.

Random Experiment, Sample Space, Events

Introduction

- ▶ There is a 50% chance that India will win the toss.
- ▶ My guess is answer "a" is the right choice.
- ▶ Party ABC will probably win the next election.
- ▶ There is a 30% chance of rain tomorrow.
- ▶ We routinely see or hear claims as the ones mentioned above. What do they mean?
- ▶ Indeed, as a general rule, to be able to draw valid inferences about a population from a sample, one needs to know how likely it is that certain events will occur under various circumstances.
- ▶ The determination of the likelihood, or chance, that an event will occur is the subject matter of **probability**.

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Remark

However, although the outcome of the experiment will not be known in advance, let us suppose that the set of all possible outcomes is known.

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- ▶ Experiment: To throw a dart on a unit square and note the point where it lands.
Outcome: Any point in the square (assuming the dart lands within the square).

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- ▶ Basic Outcomes: the possible outcomes that can occur must be:
 1. mutually exclusive: only one basic outcome can occur
 2. exhaustive: one basic outcome must occur

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- ▶ Experiment: To throw a dart on a unit square and note the point where it lands.
Sample space: $S = \{(x, y) : 0 \leq x \leq 1, 0 \leq y \leq 1\}$

Section summary

- ▶ Random experiment
- ▶ Sample space: set of all basic outcomes of a random experiment.