Inverse Functions

Monday, 14 September 2020 9:30 AM

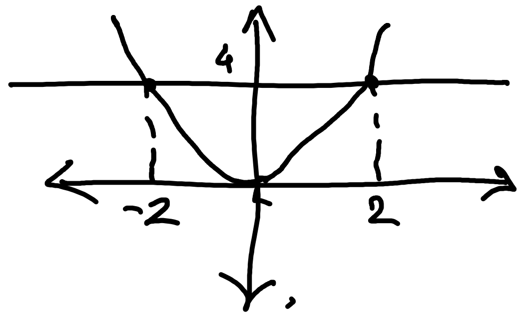
$f : \mathbb{R} \longrightarrow \mathbb{R}$

$f(x) = x^2$

☆ Not 'Reversible'

$f(-2) = f(2) = 4$

Reversible?



☆ **We now look at one-to-one functions**

$\mathbb{R} \to \mathbb{R}$

$\boxed{g(x) = 4x}$

$\boxed{h(x) = \frac{x}{4}}$ $\mathbb{R} \to \mathbb{R}$

$y = 4x$

$\frac{y}{4} = x$

$\boxed{g(x) = 4x}$

$y = \frac{x}{4}$

$4y = x$

$$I(x) = g \circ h(x) = g(\underbrace{h(x)}) = 4\,h(x)$$

$$= 4\,\frac{x}{4} = x$$

$$I(x) = h \circ g(x) = h(g(x)) = \frac{g(x)}{4} = \frac{4x}{4} = x$$

$$\boxed{g \circ h(x) = I(x) = h \circ g(x)}$$

Defⁿ. The Inverse of a function f, f^{-1} is a function such that

$$f^{-1}\circ f(x) = f^{-1}(f(x)) = x \quad \forall x \in Dom(f) = Range(f^{-1})$$

$$\& \; f(f^{-1}(x)) = x \quad \forall x \in Dom(f^{-1}) = Range(f)$$

$$f : \mathbb{R} \to [0,\infty) \qquad f^{-1} : [0,\infty) \to \mathbb{R}$$

Remark. f is one-to-one function

$\Rightarrow$ f^{-1} exists for f.

Warning: $f^{-1} \neq \frac{1}{f}$

$f^{-1}(x) \neq (f(x))^{-1}$

Example. $g(x) = x^3$ & $g^{-1}(x) = \sqrt[3]{x} = x^{1/3}$

$\mathbb{R} \to \mathbb{R}$ $\qquad$ $\mathbb{R} \to \mathbb{R}$

Verify

$$g^{-1}(g(x)) = g^{-1}(x^3) = (x^3)^{1/3} = x.$$

$$g(g^{-1}(x)) = g(x^{1/3}) = (x^{1/3})^3 = x.$$

Example 2. Verify f is the inverse of g

$$f(x) = \frac{x-5}{2x+3} \quad \& \quad g(x) = \frac{3x+5}{1-2x}$$

$$f(g(x)) = \frac{g(x)-5}{2g(x)+3} = \frac{\frac{3x+5}{1-2x} - 5}{2\,\frac{3x+5}{1-2x} + 3}$$

$$= \frac{3x+\cancel{5} - 5(\cancel{1}-2x)}{2(3\cancel{x}+5)+3(1-2\cancel{x})} = \frac{13x}{13} = x.$$

$$g(f(x)) = \frac{3f(x)+5}{1-2f(x)} = \frac{3\left(\frac{x-5}{2x+3}\right)+5}{1-2\left(\frac{x-5}{2x+3}\right)}$$

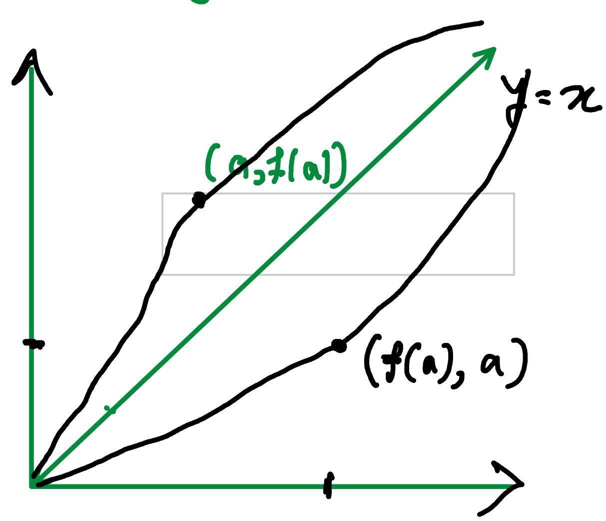
$$= \frac{3(x-\cancel{5})+5(2x+\cancel{3})}{2\cancel{x}+3-2(\cancel{x}-5)} = \frac{13x}{13} = x.$$

Graph f & f^{-1}

$$x = a = f^{-1}(f(a)) \xrightarrow{f} f(a) = y, \qquad f(a) \xrightarrow{f^{-1}} f^{-1}(f(a))$$

If $(a, f(a))$ is on the graph of f

then $(f(a), a)$ is on the graph of f^{-1}



Theorem. The graphs of f & f^{-1} are symmetric across $\underline{y=x}$ line