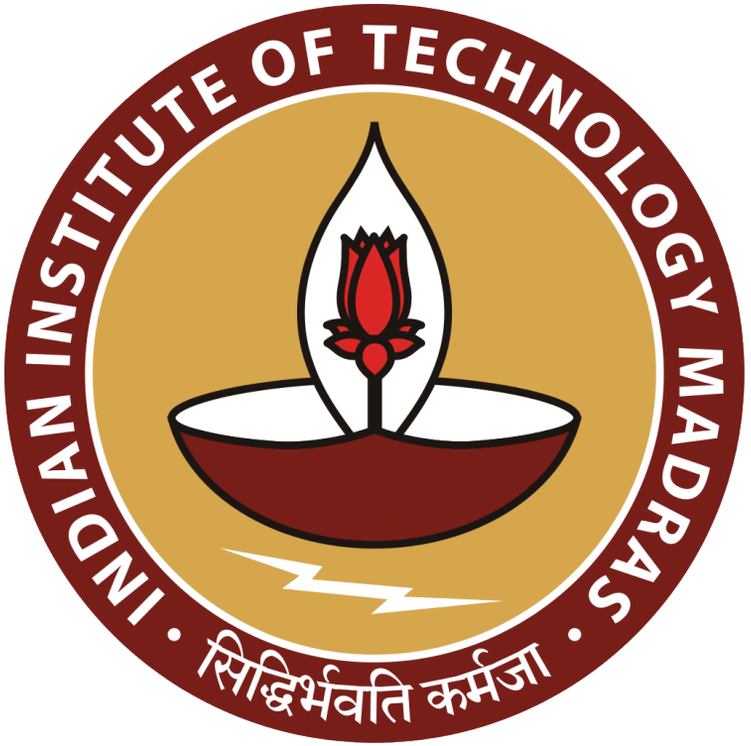

IIT Madras

ONLINE DEGREE

Rational numbers

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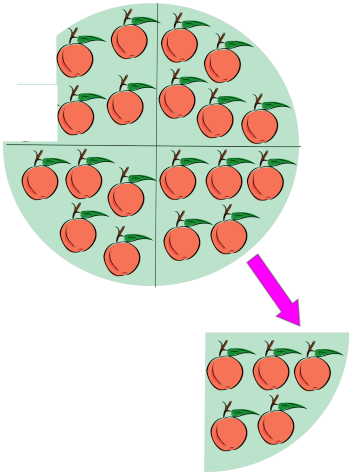
Mathematics for Data Science 1
Week 1

Rational numbers

- Cannot represent $19 \div 5$ as an integer
- Fractions : $3\ \frac{4}{5}$
- Rational number: $\frac{p}{q}$, p and q are integers
  - Numerator p, denominator q
  - Use $\mathbb{Q}$ to denote rational numbers
- The same number can be written in many ways
  - $\frac{3}{5} = \frac{6}{10} = \frac{30}{50} = \cdots$
- Useful to add, subtract, compare rationals
  - $\frac{3}{5} + \frac{3}{4} = \frac{12}{20} + \frac{15}{20} = \frac{27}{20}$
  - $\frac{3}{5} < \frac{3}{4}$ because $\frac{12}{20} < \frac{15}{20}$



Reduced form

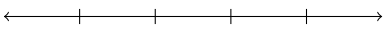
- Representation is not unique
  - $\frac{3}{5} = \frac{6}{10} = \frac{30}{50} = \cdots$
- Reduced form : $\frac{p}{q}$,
  where p,q have no common factors
  - Reduced form of $\frac{18}{60}$ is $\frac{3}{10}$
- Greatest Common Divisor: $gcd(18, 60) = 6$
  - Recall prime factorization
  - $18 = 2 \cdot 3 \cdot 3$, $60 = 2 \cdot 2 \cdot 3 \cdot 5$
  - Common prime factors are $2 \cdot 3$
  - Can find $gcd(m, n)$ more efficiently

Density

- For each integer, we have a next integer and a previous integer
  - For m, next is $m+1$, previous is $m-1$

- Next: No integer between m and $m+1$
  Previous: No integer between $m-1$ and m
- Not possible for rationals
  - Between any two rationals we can find another one
  - Suppose $\frac{m}{n} < \frac{p}{q}$

    Their average $\left(\frac{m}{n} + \frac{p}{q}\right)/2$ lies between them



- Rationals are dense, integers are discrete

Summary

- $\mathbb{Q}$: rational numbers
- $\frac{p}{q}$, where p, q are integers
- Representation is not unique $\frac{p}{q} = \frac{n \cdot p}{n \cdot q}$
- Reduced form, $gcd(p, q) = 1$
- Rationals are dense — cannot talk of next or previous