INDIAN INSTITUTE OF TECHNOLOGY MADRAS • सिद्धिर्भवति कर्मजा •

IIT Madras

ONLINE DEGREE

Graphical Behavior of Polynomials at x-Intercepts

If a polynomial contains a factor of the form $(x-a)^m$, the behavior near the x-intercept a is determined by the exponent m. We say that $x=a$ is a zero of **multiplicity m**.

The graph of a polynomial function will touch but not cross the x-axis at zeros with even multiplicities. The graph will cross the x-axis at zeros with odd multiplicities.

The sum of the multiplicities is no greater than the degree of the polynomial function.

Graphical Behavior of Polynomials at x-Intercepts

Given the graph of a polynomial of degree n, how can one identify zeros and their multiplicities?

1. If the graph touches the x-axis and bounces off of the axis, it is a zero with even multiplicity.
2. If the graph crosses the x-axis, it is a zero with odd multiplicity.
3. If the graph crosses the x-axis and appears almost linear at the intercept, it is a single zero.
4. The sum of all the multiplicities is no greater than n.

Example

Use the graph of the function of degree 6 to identify the zeros of the function and their possible multiplicities.

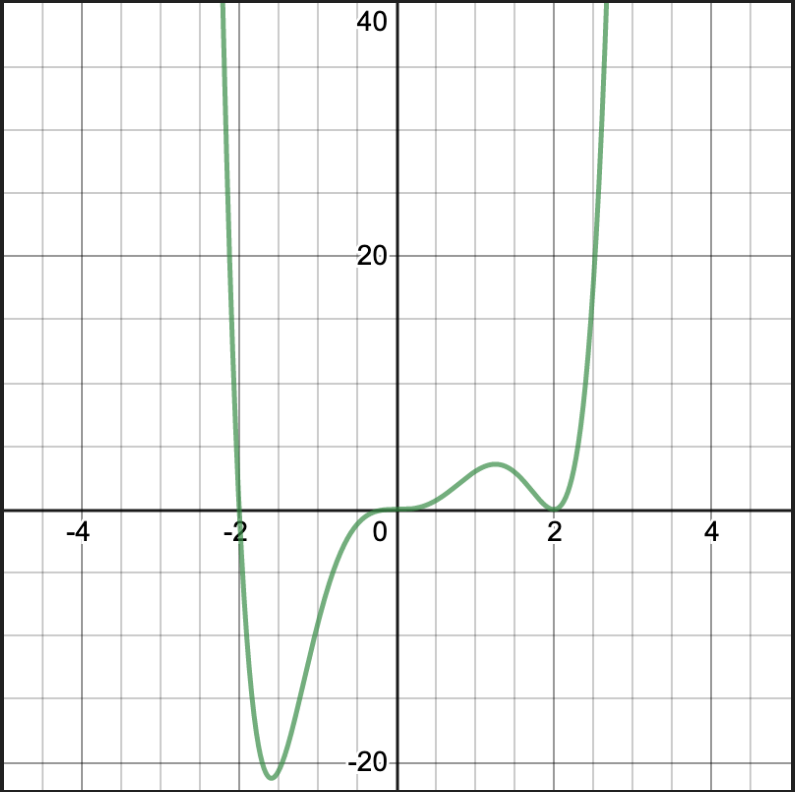
x = -2, 0, 2

x=-2, linear, 1

x=0, odd degree, 3 or 5

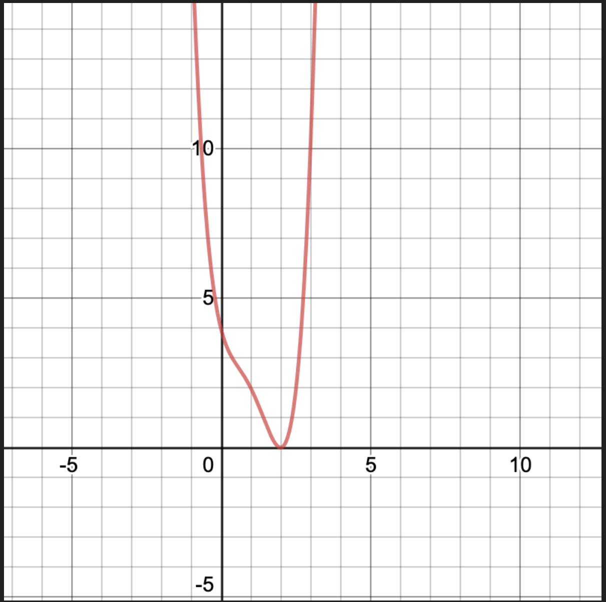
x=2, even degree, 2 or 4

x=0 with multiplicity 3 and x=2 with multiplicity 2 and x=-2 with multiplicity 1.



Example

Use the graph of the function of degree 4 to identify the zeros of the function and their possible multiplicities.



x = 2

x=2, even degree, 2 or 4

Hence, the function f(x) must have a factor $(x-2)^2$.