



IIT Madras
ONLINE DEGREE

Mathematics for Data Science 1
Professor Neelesh S Upadhye
Department of Mathematics
Indian Institute of Technology, Madras
Week 08 – Tutorial 7

(Refer Slide Time: 0:14)

7. Rohan (age 22) saw a birthday offer outside of a shop. The offer includes a discount of $D(a)\%$ on the payable amount if the customer has birthday on that particular day, where a is the age of the customer and $D(a) = (-a^2 + 50a - 600)$. The shop also has a Sunday offer which is flat discount of ₹1500, if the initial purchased amount is more than ₹12000. Suppose Rohan has a friend (age 25) who shares the same birthday with Rohan on a particular Sunday. Express the final payable amount as a function in terms of a and find the possible minimum amount needed to be paid if Rohan purchased some commodities of ₹15000 from the shop.

Assume the following :

- Any offer can be applied first.
- Rohan can use either his or his friend's birthday for the birthday offer.

Our seventh question says that Rohan who has age 22 and he saw birthday offer outside a shop. The offer includes a discount of $D(a)$ percent on the payable amount if the customer has birthday on that particular day, where a is the age of the customer and the $D(a)$ is a function. And $D(a)$ is given like this. The shop also has a Sunday offer which is flat discount of 1500 if the initial purchase amount is more than 12000.

Suppose Rohan has a friend whose age is 25 who shares the same birthday with Rohan on a particular Sunday. So, Sunday and both the friends had the birthday on that Sunday also. Express the final payable amount as a function in terms of a and find the possible minimum amount needed to be paid if Rohan purchase some commodities of 15000 from the shop. It is also given that any offer can be applied first and Rohan can use his birthday or his friend birthday for the offer.

(Refer Slide Time: 1:26)

• Rohan can use either his or his friend's birthday for the birthday offer.

22 **25** $D_1 \rightarrow$ Discount of $D(a)\%$

Discount $D_1 = m \left(\frac{D(a)}{100} \right)$

$P_1 = P - P \left(\frac{D(a)}{100} \right)$

$P_1 = P \left\{ \frac{100 - D(a)}{100} \right\}$

Sunday offer:
 $D_2 =$ Discount (1500)

$P_2 = \square - 1500$

$P_2 = 15000 - 1500$

$P_2 = P - 1500$

Case I \rightarrow

$P \xrightarrow{\text{Birthday}} P_1 \xrightarrow{\text{Second}} P_2$

$\alpha = 22$
 $\alpha = 25$

Case II \rightarrow

$P \xrightarrow{\text{Sunday}} P_2 \xrightarrow{\text{Birthday}} P_1$

$\alpha = 22$
 $\alpha = 25$

So, let us try to understand what the question is. So, question is about Rohan whose age is 22 and his friend whose age is 25. Rohan wants to use any of them the birthday. And the shop has offer, 2 offers. Offer 1 is the discount of $D(a)\%$. So, how will I apply offer 1 it means if the payable amount or purchase amount is m , then the $D(a)\%$ discount you will get and perceiving. So, this is how we write the discount when the percent discount is given. So, this is actually discount. So, offer 1 is a discount.

What is offer 2? Offer 2 is a Sunday offer which means if the purchase amount is less, more than 12000, so in that case the discount is 1500. So, offer 2 is the discount of 1500. So, this is the offer 2. The payable amount after applying first offer, first offer means individual first, only first offer if I apply that is, birthday offer, the payable amount will be the initially payable amount whatever the payable amount is our purchase amount-the discount and discount is what?

That p_m will be replaced by p because p is the payable amount so $\frac{D(a)}{100}$. And if you solve this you will get $p \left\{ \frac{100 - D(a)}{100} \right\}$. So, this is the offer 1. This is the payable amount if we apply the first offer. Now, what is the second offer? Second offer is same if the amount payable is this, then reduce 1500, this will be the payable amount p_2 . So, here the payable amount will be whatever the you purchase that is 15000 or I can say if I apply 15000 here, so -1500 .

So, if I apply p_2 directly, so I will get this much amount if p is a payable amount rather than 1500, not in general case, so this will be 1500. Do not get confuse with this p and this p . So, these are two different cases. Now, the question, let us try to understand the question. There is

a different scenarios. So, in case 1, this is the purchase amount which can I say p , then apply the first offer which is the birthday offer, I will get some p_1 , then apply the second offer which is Sunday offer, I will get some p_{12} .

Then, there will be two case which $a=22$ or $a = 25$ should be used. So, for minimum payable amount this will be the case for case 1 when we go with this first. What is the case 2? p , the purchase amount-Sunday offer which is second offer, but I will write Sunday offer which means this will be p_1 and this will p_2 because we are getting the Sunday offer first, so p_2 and then this will be p_{21} if I apply the birthday offer, which is the first offer, birthday offer.

Then again I will have to decide what should I take, $a = 22$ his birthday or his friend birthday equal to 25. These are the two cases I need to think about. So, should we work on first case 1?

(Refer Slide Time: 5:42)

Case 1! \rightarrow $[P] \xrightarrow{\text{Sunday}} [P_2] \xrightarrow{\text{Birthday}} [P_1]$ $\begin{cases} a=22 \\ a=25 \end{cases}$

Case 1 - $p = 15000 \rightarrow p_1 = p \left(\frac{100 - D(a)}{100} \right)$

$$p_{12} = p_1 - 1500$$

$$p_{12} = p \left\{ \frac{100 - D(a)}{100} \right\} - 1500$$

$$= 15000 \left\{ \frac{100 - D(a)}{100} \right\} - 1500$$

$$p_{12} = 150 \left\{ 100 - D(a) - 10 \right\}$$

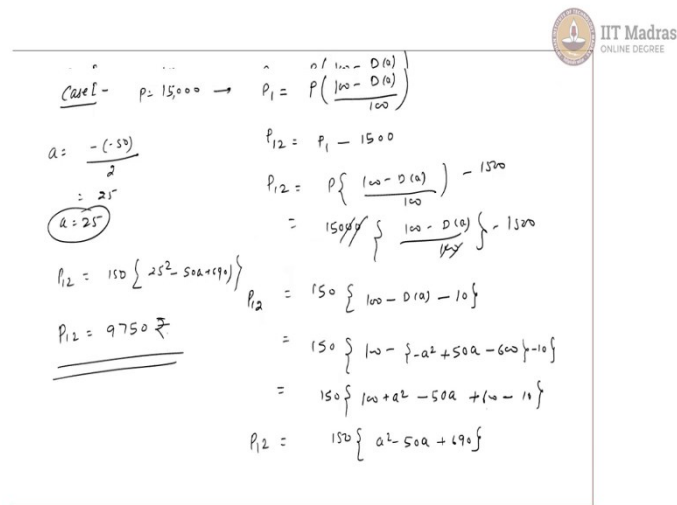
$$= 150 \left\{ 100 - (-a^2 + 50a - 600) - 10 \right\}$$

$$= 150 \left\{ 100 + a^2 - 50a + 600 - 10 \right\}$$

Case 1 is a p , so p is 15000. What is the birthday offer that discount of some percentage amount and that is p_1 actually. So, $p_1 = p \left(\frac{100 - D(a)}{100} \right)$, we calculated this 100. So, this is p_1 . What will be p_{12} ? p_{12} is equal to reduce from here to here to get p_{12} . So, $p_1 - 1500 = p_{12}$. So, $p_{12} = p \left[\frac{100 - D(a)}{100} \right] - 1500$. So, what is p ? $p = 15000 \left[\frac{100 - D(a)}{100} \right] - 1500$. Take 150 common, then you will get $100 - D(a) - 10$.

This is the final payable amount for the case when we apply first offer first and then second offer second. Now we need to choose what should be the a and what will be the minimum possible amount. For that purpose, I need to write a here and a $D(a)$ is $-a^2 + 50a - 600 - a$. So, $150 (100 - a^2 + 50a - 600 - 10)$. So, 150, it will be $100 + a^2 - 50a + 600 - 10$.

(Refer Slide Time: 8:13)



Case 1 - $p = 15000 \rightarrow p_1 = \frac{100(100 - D(a))}{100}$

$a = \frac{-(-50)}{2}$

$a = 25$

$p_{12} = 150 \{ 25^2 - 50a + 690 \}$

$p_{12} = 9750$

$p_{12} = \frac{100(100 - D(a))}{100} - 1500$

$p_{12} = 150 \{ 100 - D(a) \} - 1500$

$p_{12} = 150 \{ 100 - (a^2 - 50a + 600) \} - 1500$

$p_{12} = 150 \{ 100 + a^2 - 50a - 600 \} - 1500$

$p_{12} = 150 \{ a^2 - 50a + 400 \}$

So, p_{12} is actually $150(600 - 700 - 10)$ so $a^2 - 50a$, $600 + 700 - 10$ will be 690 , $+ 690$. Now you can understand this p_{12} is a quadratic function and possible minimum amount we need to find the vertex. As we did in week 4, so I will directly find the vertex. So, a will be for minimum possible amount, for paying minimum possible amount, of which will be $-(-50)/2a$ means 2 . It will be 25 .

So, $a = 25$ means Rohan should use his friend's birthday for the purchasing so that he can get, he should, he will have to pay minimum amount. Now what will be p_{12} when $a = 25$ so put $a = 25$ and you get $150, (25)^2 - 50a + 690$. So, after calculation, the payable amount for p_{12} will be 9750 . What if we apply the second case? First Sunday offer, then the birthday offer.

(Refer Slide Time: 9:49)

IIT Madras
ONLINE DEGREE

Case 2

$$\begin{aligned}
 p_2 &= p - 1500 \\
 &= 15000 - 1500 \\
 &= 13500 \\
 p_{21} &= p_2 \left(\frac{100 - D(a)}{100} \right) \\
 &= 13500 \left(\frac{100 - D(a)}{100} \right) \\
 &= 135 \left(100 - 50a + 600 \right) \\
 &= 135 \left(a^2 - 50a + 700 \right) \\
 &= 135 \left(25^2 - 50 \times 25 + 700 \right)
 \end{aligned}$$

IIT Madras
ONLINE DEGREE

$$\begin{aligned}
 &= 135 \left(100 - 50a + 600 \right) \\
 &= 135 \left(a^2 - 50a + 700 \right) \\
 &= 135 \left(25^2 - 50 \times 25 + 700 \right) \\
 p_{21} &= 10,125 \\
 p_{12} &= 9750
 \end{aligned}$$

Answer:

$a = 25$
9750 when Birthday \rightarrow Sunday

So, I will do it, in first as we have discussed first case, so for case 2, the p_1, p_2 will be $p-1500$. So, p is 15000-1500. It will be 13500. So, p_2 is 13500. Now, p_{21} will be applied on the p_2 and the discount will take $\frac{100-D(a)}{100}$ and that will be $\frac{13500-D(a)}{100}$. So, it is 135, 100-if I replace $D(a)$ with the function so it will be $a^2 - 50a + 600$. So, I will get $135a^2 - 50a + 600$.

Now, again we will follow the same procedure that minimal amount will be our $a = \frac{-b}{2}$ and again $a = 25$ we are getting. So, you should use thus use his friend's birthday for the second case also. What will be the minimum payable amount in this case? $135a^2$ means $(25)^2 - 50 \times 25 + 700$. So, after solving this we will get $p_{21}=10125$.

So, obviously p_{12} which was around 9000, we solved here 9750 and p_{21} is 10125. So, clearly this is the least amount to be paid, so the answer for this question will be what? $a = 25$ which means Rohan should use his birthday, his friend's birthday and the payable amount will be 9750 when he uses the birthday offer first and then the Sunday offer. So, this is the answer.

