Statistics for Data Science -1

Lecture 5.3: Permutation formula-distinct objects

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Learning objectives

1. Understand basic principles of counting.
2. Concept of factorials.
3. Understand differences between counting with order (permutation) and counting without regard to order (combination).
4. Use permutations and combinations to answer real life applications.

Permutations

Permutation when objects are distinct

Permutation when objects are distinct- repetitions not allowed

Permutation when objects are distinct- repetitions allowed

Permutation

Definition

A permutation is an ordered arrangement of all or some of n objects.

Example

Example

Take A, B, C- Possible arrangements- taking all at a time

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First place	Second place	Third place
A	B	C
A	C	B
B	A	C
B	C	A
C	A	B
C	B	A

Example

Take A, B, C- Possible arrangements- taking two at a time

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First place	Second place
A	B
A	C
B	A
B	C
C	A
C	B

Example

Take A, B, C, D- Possible arrangements- taking all at a time

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Take A, B, C, D- Possible arrangements- taking all at a time

First place	Second place	Third place	Fourth place
A	B	C	D
A	B	D	C
A	C	B	D
A	C	D	B
A	D	B	C
A	D	C	B
B	A	C	D
B	A	D	C
B	C	A	D
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B	D	A	C
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C	A	B	D
C	A	D	B
C	B	A	D
C	B	D	A
C	D	A	B
C	D	B	A
D	A	B	C
D	A	C	B
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D	B	C	A
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First place	Second place
A	B
A	C
A	D
B	A
B	C
B	D
C	A
C	B
C	D
D	A
D	B
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Permutation formula

The number of possible permutations of r objects

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and is denoted by ${}^{n}P_{r}$

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  1. ${}^nP_0 = \frac{n!}{(n-0)!} = \frac{n!}{n!} = 1$ There is only one ordered arrangement of 0 objects.

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  3. ${}^nP_n = \frac{n!}{(n-n)!} = \frac{n!}{0!} = n!$. We can arrange n distinct objects in $n!$ ways- multiplication principle of counting.

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Example

Take A, B, C- Possible arrangements- taking all at a time

First place	Second place	Third place
A	B	C
A	C	B
B	A	C
B	C	A
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$$n = 3, r = 3, {}^nP_r = \frac{n!}{(n-r)!} = \frac{3!}{0!} = 6$$

Example

Take A, B, C- Possible arrangements- taking two at a time

First place	Second place
A	B
A	C
B	A
B	C
C	A
C	B

Example

Take A, B, C- Possible arrangements- taking two at a time

First place	Second place
A	B
A	C
B	A
B	C
C	A
C	B

$$n = 3, r = 2, {}^{n}P_{r} = \frac{n!}{(n-r)!} = \frac{3!}{1!} = 6$$

Example

Take A, B, C, D- Possible arrangements- taking all at a time

First place	Second place	Third place	Fourth place
A	B	C	D
A	B	D	C
A	C	B	D
A	C	D	B
A	D	B	C
A	D	C	B
B	A	C	D
B	A	D	C
B	C	A	D
B	C	D	A
B	D	A	C
B	D	C	A
C	A	B	D
C	A	D	B
C	B	A	D
C	B	D	A
C	D	A	B
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B	C	D	A
B	D	A	C
B	D	C	A
C	A	B	D
C	A	D	B
C	B	A	D
C	B	D	A
C	D	A	B
C	D	B	A
D	A	B	C
D	A	C	B
D	B	A	C
D	B	C	A
D	C	A	B
D	C	B	A

$$n = 4, r = 4, {}^{n}P_{r} = \frac{n!}{(n-r)!} = \frac{4!}{0!} = 24$$

Example

Take A, B, C, D- Possible arrangements- taking two at a time

First place	Second place
A	B
A	C
A	D
B	A
B	C
B	D
C	A
C	B
C	D
D	A
D	B
D	C

Example

Take A, B, C, D- Possible arrangements- taking two at a time

First place	Second place
A	B
A	C
A	D
B	A
B	C
B	D
C	A
C	B
C	D
D	A
D	B
D	C

$$n = 4, r = 2, {}^{n}P_{r} = \frac{n!}{(n-r)!} = \frac{4!}{2!} = 12$$

Example: application

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- $8 \times 7 = 56$

Example: application

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- How many of these will be even?

Example: application

- Find the number of 4-digit numbers that can be formed using the digits 1, 2, 3, 4, 5 if no digit is repeated.
- $5 \times 4 \times 3 \times 2 \times 1 = 120$
- How many of these will be even? 48

Example: application

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Example: application

- Six people go to the cinema. They sit in a row with ten seats. Find how many ways can this be done if
  - (i) they can sit anywhere: ${}^{10}P_6 = 1,51,200$
  - (ii) all the empty seats are next to each other: ${}^{7}P_6 = 5,040$

Example

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Take A, B, C- Possible arrangements- taking all at a time

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A	C	B
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B	A	A
B	A	B
B	A	C
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B	B	B
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B	C	A
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B	C	C
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The number of possible permutations of r objects from a collection of n distinct objects when repetition is allowed is given by the formula

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The number of possible permutations of r objects from a collection of n distinct objects when repetition is allowed is given by the formula

$$n \times n \times \ldots \times n$$

and is denoted by n^r

Example

Take A, B, C- Possible arrangements- taking all at a time.$n = 3$, $r = 3$, $n^r = 27$

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B	A	A
B	A	B
B	A	C
B	B	A
B	B	B
B	B	C
B	C	A
B	C	B
B	C	C
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Take A, B, C- Possible arrangements- taking two at a time

First place	Second place
A	A
A	B
A	C
B	A
B	B
B	C
C	A
C	B
C	C

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Take A, B, C- Possible arrangements- taking two at a time

First place	Second place
A	A
A	B
A	C
B	A
B	B
B	C
C	A
C	B
C	C

$n = 3, r = 2, n^r = 9$

Section summary

1. The number of possible permutations of r objects from a collection of n distinct objects is given by the formula

$$n \times (n-1) \times \ldots \times (n-r+1)$$

and is denoted by ${}^nP_r = \frac{n!}{(n-r)!}$

2. The number of possible permutations of r objects from a collection of n distinct objects when repetition is allowed is given by the formula

$$n \times n \times \ldots \times n$$

and is denoted by n^r