



**IIT Madras**  
ONLINE DEGREE

**Mathematical for Data Science 1**  
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**Lecture 49**  
**Composition Functions: Examples**

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②  $g(x)$  is in the domain of  $f$ .

$$(f \circ g)(x) = f(g(x))$$

$x \xrightarrow{g} g(x) \xrightarrow{f} f(g(x))$   
 $\text{Dom}(g) \quad \text{Dom}(f) \quad \text{Range}(f)$

$x \rightarrow [g] \rightarrow g(x) \rightarrow [f] \rightarrow f(g(x))$

So, we have understood the theory, roughly the theory behind the function, composition, composite functions or composition of two functions. So, it's time to get some practice.

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$x \rightarrow [g] \rightarrow g(x) \rightarrow [f] \rightarrow f(g(x))$

Example. Given  $f(x) = 3x - 4$ ,  $g(x) = x^2$ ,

find ①  $(g \circ f)(x)$  ②  $(f \circ g)(x)$ .

Sol<sup>n</sup>.

$$\begin{aligned}
 (g \circ f)(x) &= g(f(x)) \\
 &= (f(x))^2 \checkmark \\
 &= (3x - 4)^2
 \end{aligned}$$

$g(\square) = \square^2$

Sol<sup>n</sup>.

$$\begin{aligned}(g \circ f)(x) &= g(f(x)) \\ &= (f(x))^2 \checkmark \\ &= (3x-4)^2\end{aligned}$$

$$(g \circ f)(x) = g(f(x)) = g(3x-4) \quad \text{Replace } f(x)=3x-4$$

$$= (3x-4)^2$$

So, let me start with an example. And in that example, let us take you have been given two functions  $f(x)=3x-4$ , and  $g(x)$ , which is equal to let us say  $x^2$  these are the two functions that are given, then you are asked to find two things one is  $g \circ f(x)$ , and the other one is obviously  $f \circ g(x)$ , how to find this? Let us start, let us start with a solution.

So, what can be the solution let us take this function. So, let me write it properly, it is  $g \circ f(x)$ . So, as per our theory, we have to write this as  $g(f(x))$ . So,  $g(f(x))$ , you can treat this as, what is  $f(x)$  now?  $f(x) = 3x - 4$ , and  $g(x)$  is  $x$  square. So, naturally  $g(f(x))$ , so, you go to this function, you treat this  $g$  as  $g$ . So, let me write it here, you treat this  $g$  as a  $g$  of a box, and  $g(x)$  is nothing but this box squared. So, in particular, if I want to write something about this function, this box right now has an argument which is  $f(x)$ .

So, I will simply write this as  $f(x)$  squared, that is all. Now, the entire process is simplified. So, now, you do not have to worry about what  $g$  is, now it simply  $f(x)^2$  what is the  $f(x)$  fit that when you in and you will get  $(3x - 4)^2$ . Another way to handle this is you can simply write  $g \circ f(x)$  as  $g(f(x))$  fit in the value of  $f(x)$  that is  $g(3x - 4)$  and what is  $g(3x - 4)$  as per our question, it is  $x^2$ . So,  $g(3x - 4)$  will be  $(3x - 4)^2$ . So, anyway whichever way is convenient to you, you proceed and you will get this answer correct.

So, what I have done here is I have replaced  $f(x)$  in this particular case, I have replaced  $f(x) = 3x - 4$  in this particular case I have written  $f(x)$  and replaced what is  $g(x)$ . So, both ways you can go now.

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$$= (3x-4)^2$$

$$(f \circ g)(x) = f(g(x))$$

$$= 3g(x) - 4$$

$$= 3x^2 - 4$$

$$f(\Delta) = 3\Delta - 4$$



Sol<sup>n</sup>.

$$(g \circ f)(x) = g(f(x))$$

$$= (f(x))^2$$

$$= (3x-4)^2$$

$$g(\square) = \square^2$$

$$(g \circ f)(x) = g(f(x)) = g(3x-4)$$

Replace  $f(x) = 3x-4$

$$= (3x-4)^2$$



$x \rightarrow f \rightarrow g(x) \rightarrow f \rightarrow f(g(x))$

Example. Given  $f(x) = 3x-4$ ,  $g(x) = x^2$

Find (a)  $(g \circ f)(x)$  (b)  $(f \circ g)(x)$ .



Sol<sup>n</sup>.

$$(g \circ f)(x) = g(f(x))$$

$$= (f(x))^2$$

$$= (3x-4)^2$$

$$g(\square) = \square^2$$



Let us go to the second problem that is a  $f \circ g(x)$  and  $f \circ g(x)$  is again can be written as  $f \circ f(g(x))$ . Clear, there is no question, then there are two ways let us go it the first way, what is  $f(g(x))$ ? So, what is  $f(x)$  here?  $f(x) = 3x - 4$  here. So, I will write this as to be equal to  $3g(x) - 4$ .

So again, let me be very clear about this there should not be any confusion in this. So, what is  $f(\Delta)$ ?  $\Delta$  is an argument. So, this  $\Delta$  triangle will be  $3\Delta - 4$ . So, now this triangle is replaced with  $g(x)$ , that is all. Therefore, your answer is  $3x-3g(x)-4$ . But what is  $g(x)$ ? Again, go back to the question  $g(x)$  is  $x^2$  So, substituted here that means it will be  $3x^2 - 4$  and this is the final answer for you in terms of  $f \circ g(x)$ . So, we are seen how to write the compositions in both ways  $g \circ f$  and  $f \circ g$ .

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$f(g(x))$   
 $= 3g(x) - 4$   
 $= 3x^2 - 4$

Exercise  $f(x) = x + 1$   $g(x) = x^2 - 1$

find  $(g \circ f)(x)$   $(f \circ g)(x)$ .

So, here is a quick exercise for you pause the video, do the exercise and get back the get the answer. So,  $f(x) = x + 1$  and  $g(x) = x^2 - 1$ . Then simply find  $g \circ f(x)$  and  $f \circ g(x)$ . This is an exercise you stop and get the answer. It will be a good practice to revise the concepts.