IIT Madras

ONLINE DEGREE

Statistics for Data Science 1
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Lecture 6.2
Probability- Events and Basic Operations on Events

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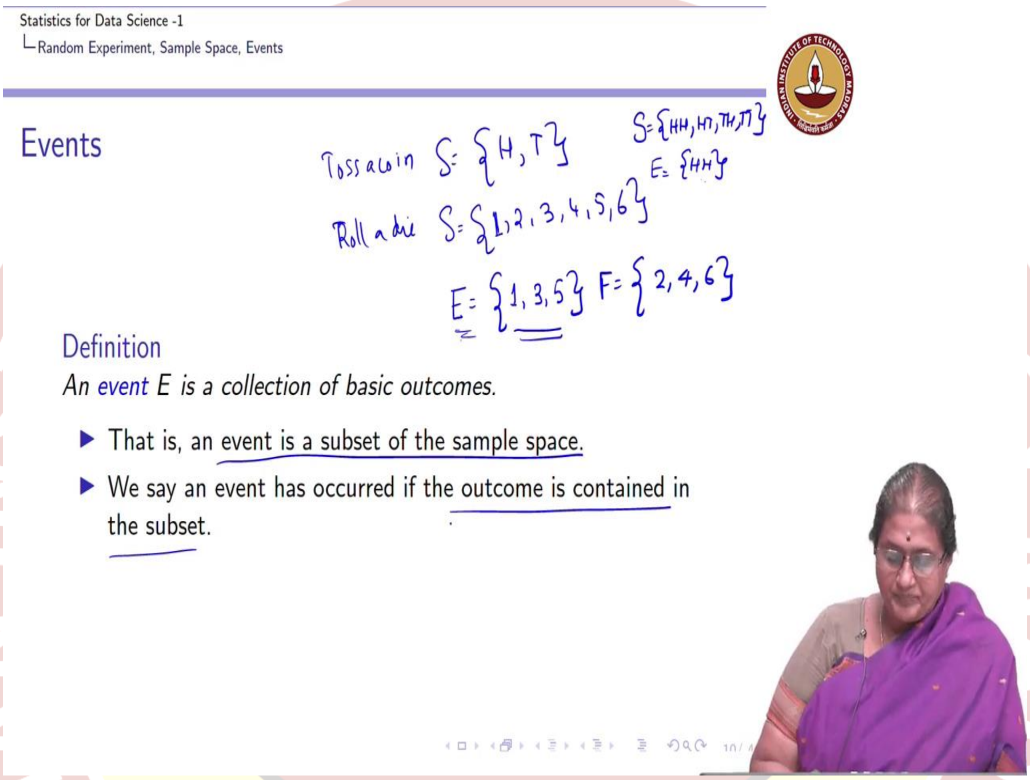

So, the next building block, we need to understand, is what is a event. So, event is a collection of basic outcomes. For example, when I toss a coin once, I know my basic outcome is a head or a tail, when I roll a die. So, this is a, when I toss a coin, when I roll a die; my collection is {1, 2, 3, 4, 5, 6}. A basic outcome is what I have, when I actually roll a die. That is what, is a basic outcome.

Now, I can define an event which is a collection of these basic events. So, for example, if I define an event {1, 3, 5}; I know the outcome of rolling a die is an odd number. So, it is again a collection of basic outcomes, but it is a subset of my total sample space. I could similarly define another event which is {2, 4, 6} which is again a subset of my total sample space. It is again a collection of the basic outcomes.

Now when I toss a coin twice, I know my sample space is $\{HH, HT, TH, TT\}$. Now, I can define an event which is $\{HH\}$, which I can see, it is again a. It is a set of basic outcome here it is a singleton. It is only 1 element, it is only 1 basic outcome, but nevertheless it is a event. So, we need to understand that, given a random experiment, we need to understand, how we define these events. We need to also understand that, these events are subsets of my sample space.

So, event is a subset of the sample space, and we articulate it as saying that, an event has occurred if the outcome is contained in the subset. So, I will say a event, that is a event of an odd number has occurred, if the outcome, what is a outcome, 1 is contained in my subset.

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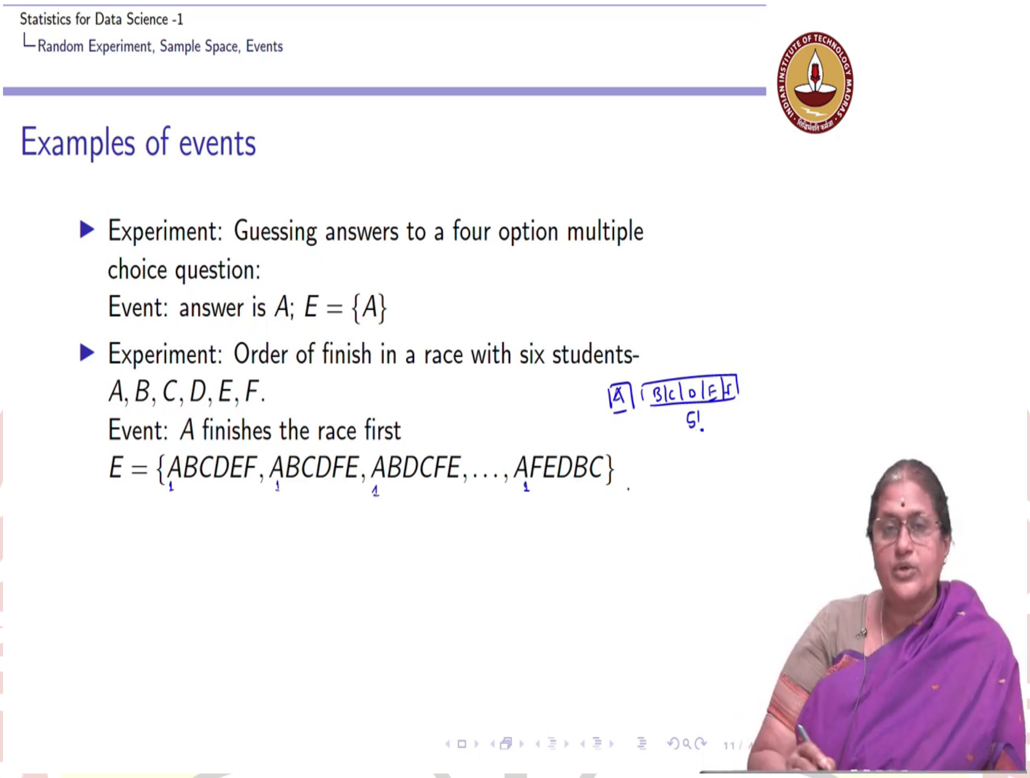

So now, let us go back to our examples. So, when I am guessing an answer to a 4 option multiple choice question. I can just define an event, that the answer is A. My event, I can represent it as a singleton set $\{A\}$. Again order of finish in a race with 6 students, suppose, I define my event to be A is the finisher, or the first person who has come first in the race.

So, you can see that, the event would be all possible permutations where A has appeared first. So, there are, we know that, if I fix A in the first position; the other B, C, D, E and F can be arranged in 5! ways. So, the event E, in itself will have about 5! outcomes. We know that, this is a subset of my original sample space.

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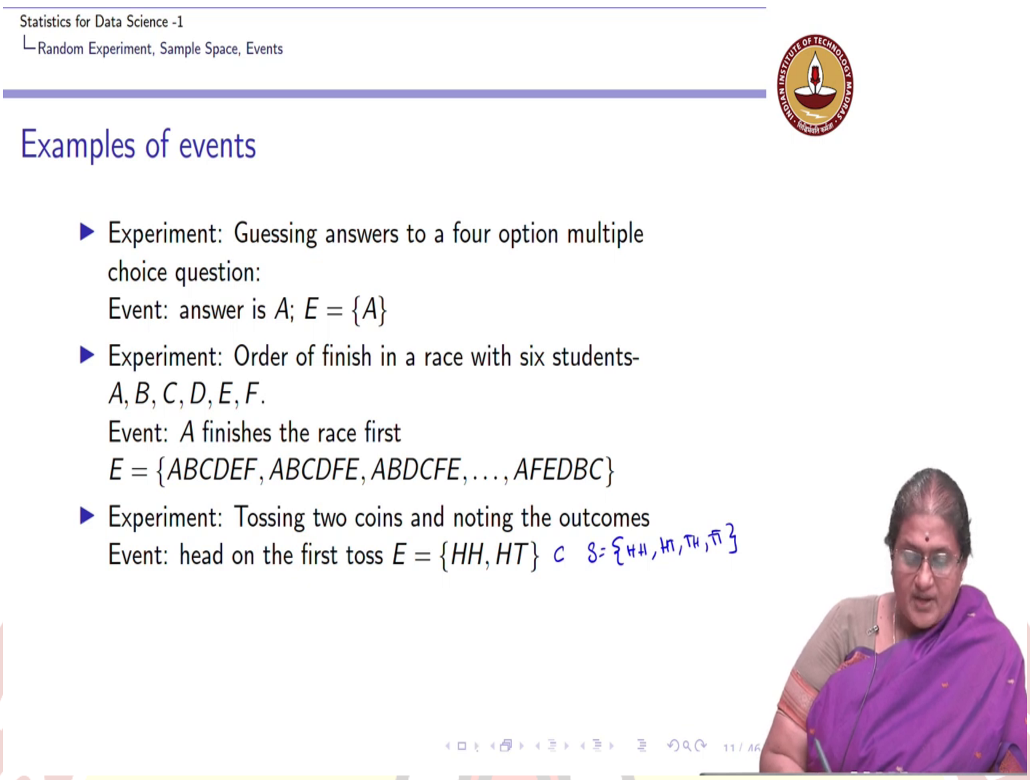

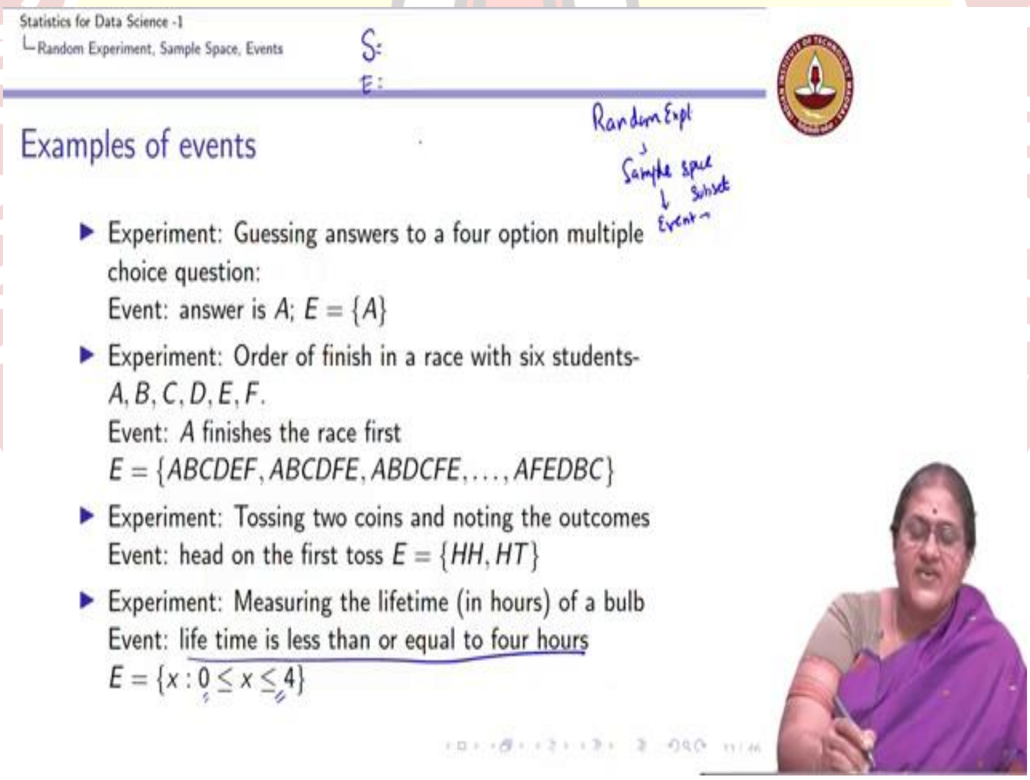

Now when I toss a coin twice, and note the outcomes; the event head on the first toss. So, I know that my sample space here is $\{HH, HT, TH, TT\}$. The event that, head appears in the first toss, again is a subset of my sample space. Measuring the life time, I can define an event, that my bulb has lasted for 4 hours, or has, is actually my bulb fails after 4 hours. In which case my event is, the life time is less than or equal to 4 hours, or my bulb has failed within 4 hours; my event can be represented by $\{x : 0 \leq x \leq 4\}$.

So, we can see that, coming from a random experiment and a sample space; sample space is the set of all possible basic outcomes. I define what is an event which is a subset of my sample space but this is also a set of basic outcomes. So, when we have a sample space and E is the set of basic outcomes we know that we can define all possible subsets of this sample space as events. Now, do all the possible subsets makes sense that is something which we need to see. But we know that, theoretically we can define all subsets as events. In a theoretical way, whether they make sense or not, we need to see, depending on the context of the experiment.

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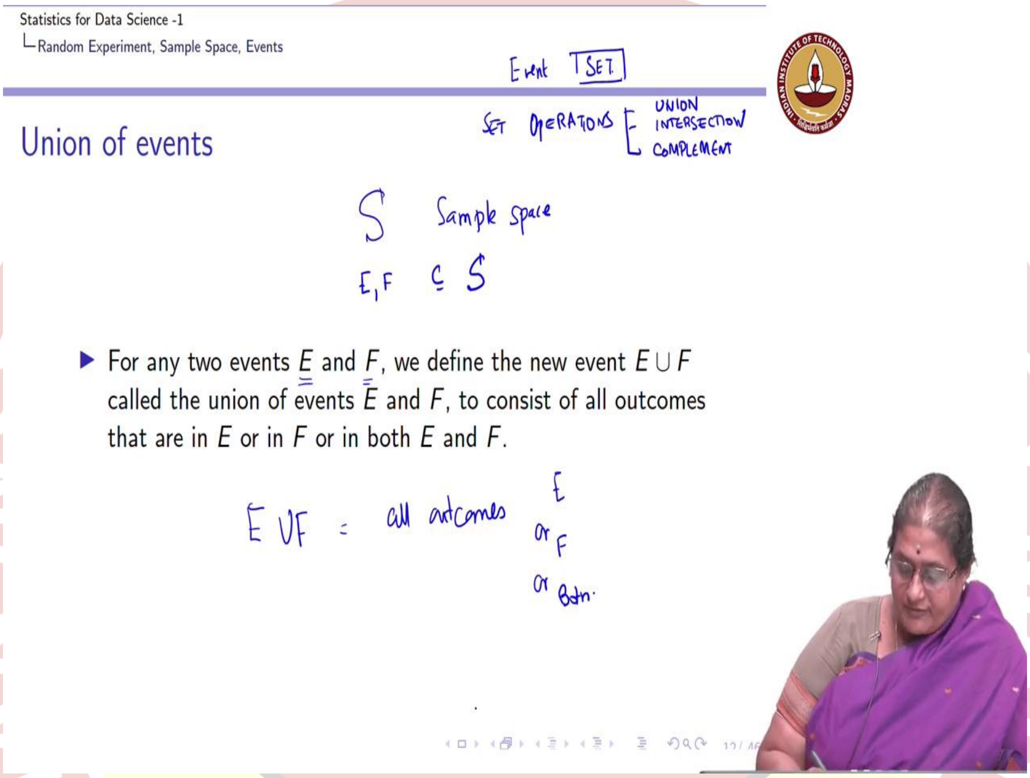

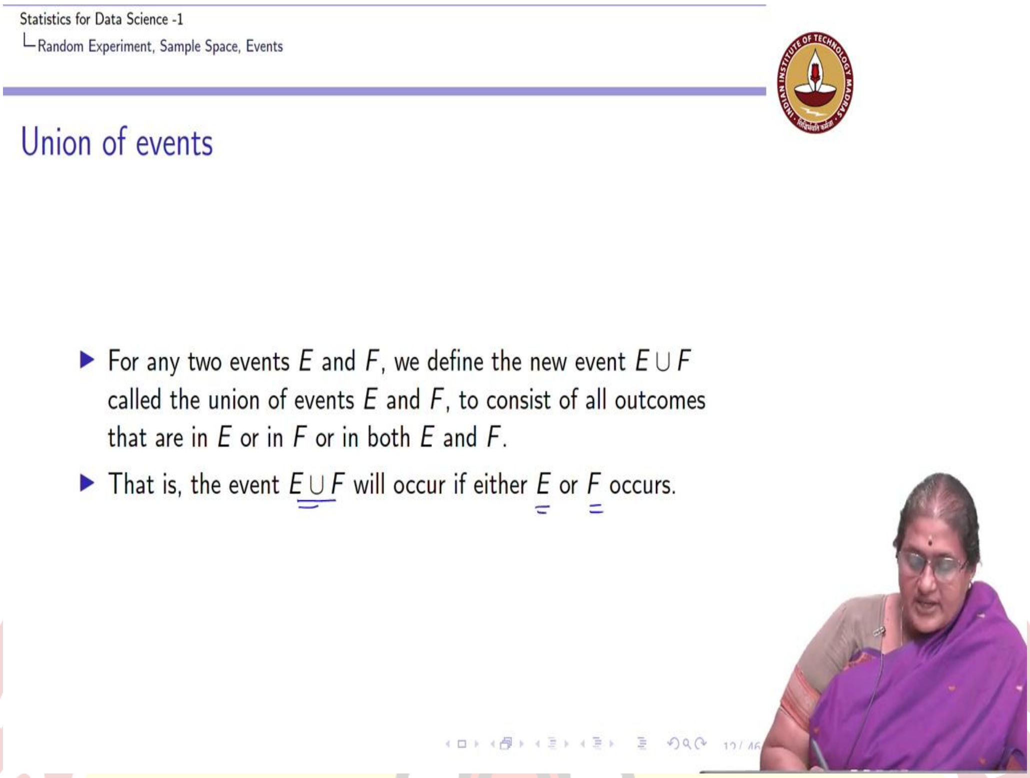

Now, because events are again subsets. So, I have an event. Event is also a set. Now I know, the minute I say that, E is an event; then I can talk about all possible set operations. So, the 3 key set operations we are going to see, are. So, I have what I can refer to as set operations on my events because I realize an event is a set. What are the possible set operations? The 3 set operations, basic set operations, we are going to talk over.

What do we mean by union of two events? What do we mean by intersection of two events? And what do we mean by complement of a event? So, these are the basic set operations, which we are interested in knowing. So, suppose I am given two events; E and F. I know that S is my sample space. Given S is my sample space I know both E and F are subsets of my sample space because they are events.

Now I can be define a new event $(E \cup F)$ to consist of all outcomes that are in $E \; or \; F$ or both; that is what I refer or I mean by $(E \cup F)$. So, the event $(E \cup F)$ will occur, if either $E \; or \; F$ occurs.

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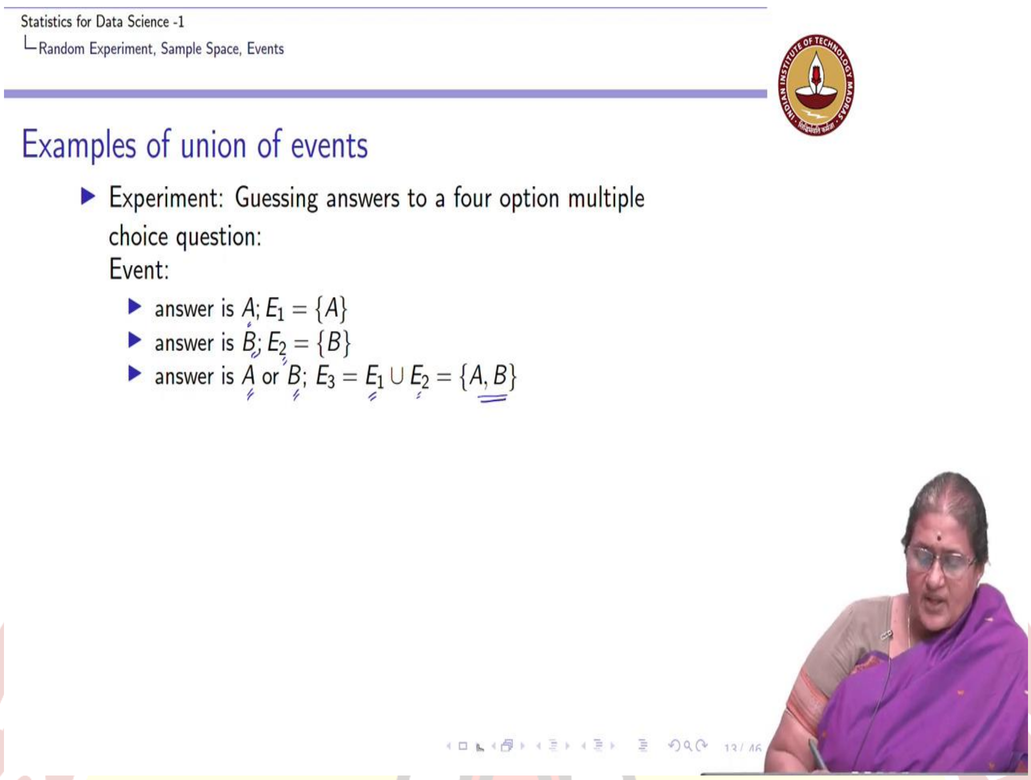

So now, let us look at a few examples. Suppose my event E is E_1, suppose my event E_1 is answer A, event E is answer is B, answer is either A or B can be defined by the event $(E_1 \cup E_2)$ which can be represented by $\{A, B\}$.

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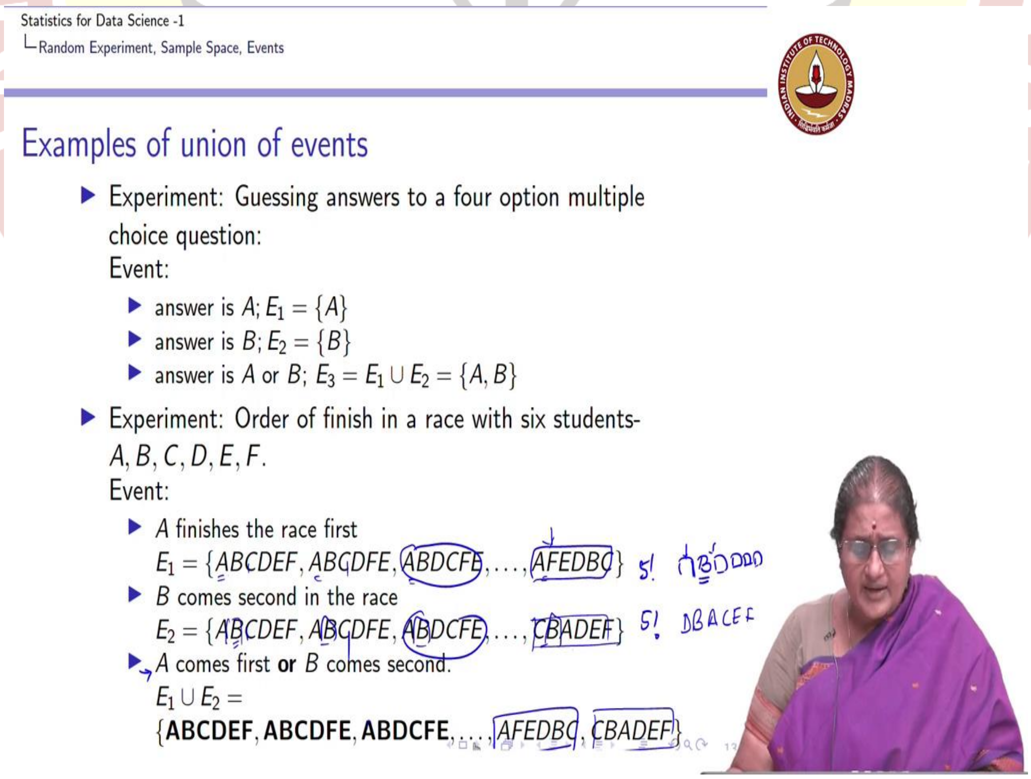

Let us go to the next example. Order of finish in a race, suppose my E_1 event is A finishes the race first. So, in this I have all the outcomes where A is finishing first. Event B is, B is coming second in the race. So, you can see that B is occupying the second position here. Again, I fix B in the second position. Total number of outcomes here is again 5! because I fixed B in the second position.

The other things are, I have 5 places which have, can be occupied by the 5 available alphabets, and that can be done in 5! way. So, the event A union, or $(E_1 \cup E_2)$ can be described as either A comes first or B comes second and that is. So, you can see that, this includes ABCDEF which is common to these two; ABCDFE is common to these two, this event ABDCFE is also common to both of them. But this event is not in B, but I am including it here because it is occurring in A.

So, this event AFEDBC does not have B in the second position, but I am including it in $(E_1 \cup E_2)$ because it is occurring in A. And this outcome CBADEF is not in A because A is not the first finisher here. In this event CBAD, in this outcome CBADEF, A is not the finisher. So, this outcome is not a part of A, but we are including it. It is not, this outcome is not a part of your E_1 but we are including it in $(E_1 \cup E_2)$ because it is an outcome in E_2.

So, the set of outcomes where either A comes first, or B comes second constitutes this $(E_1 \cup E_2)$ event. And I can describe it as either A comes first or B comes second. So, it may, it is a meaningful event to have. So, would this include the event where B comes first? The answer is would this include the event where B comes first? B comes second, so, B comes first is not included here. But would it include C coming first? Answer is yes because we have seen here.

Would it include D coming first? Yes, if the outcome is a DBACEF, yes because that is DBACEF is a outcome in my E_2 set. So, you can see that, it includes all the possible outcomes where either A is first or B is second.

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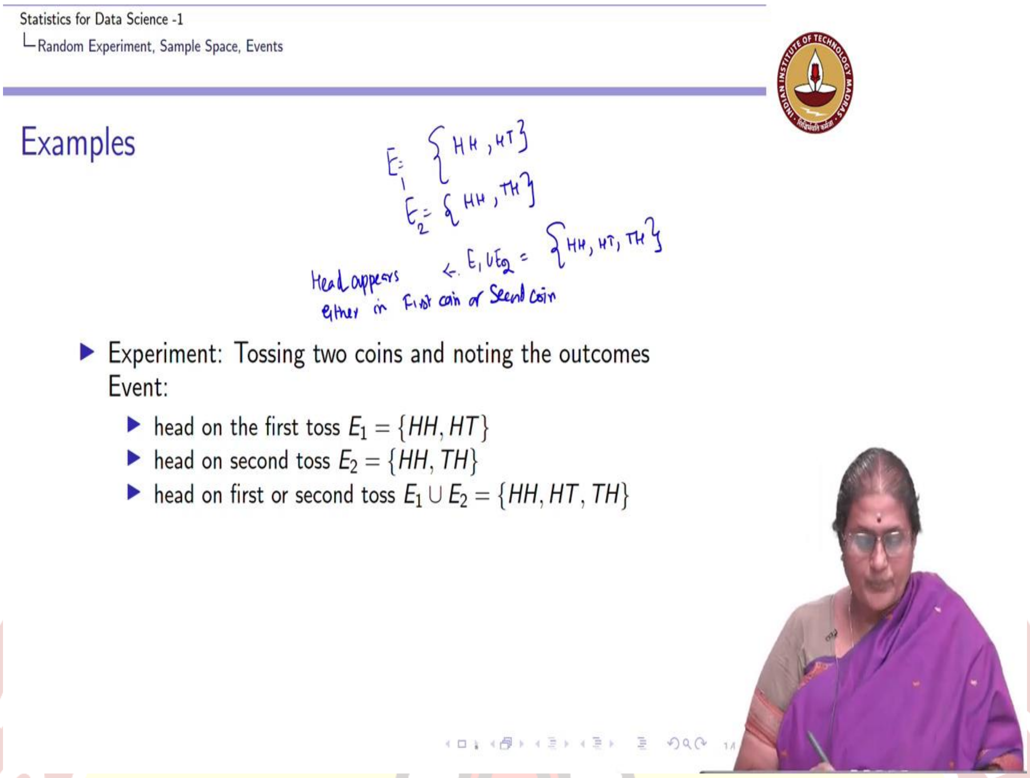

Now let us look at an example of tossing 2 coins, and noting the outcome. Let me define my first event as head on the first toss. I know if it is a head on the first toss, E_1 is {HH, HT}. Head on the second toss is {HH, TH}. This is the head on the second toss. So, if I am defining an event where I say that head occurs either in the first toss or in the second toss. So, my outcomes $(E_1 \cup E_2)$ is {HH, HT, TH}. So, this tells that head appears either in my first toss, or head appears either in first coin or second coin. That is how we can express this event $(E_1 \cup E_2)$.

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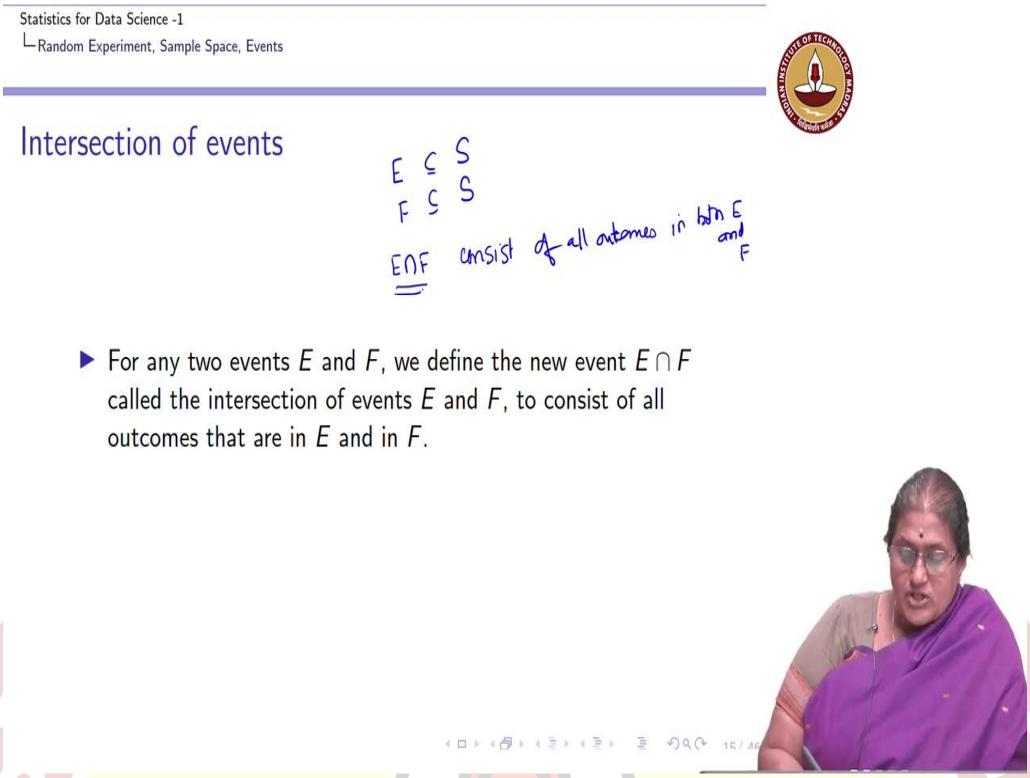

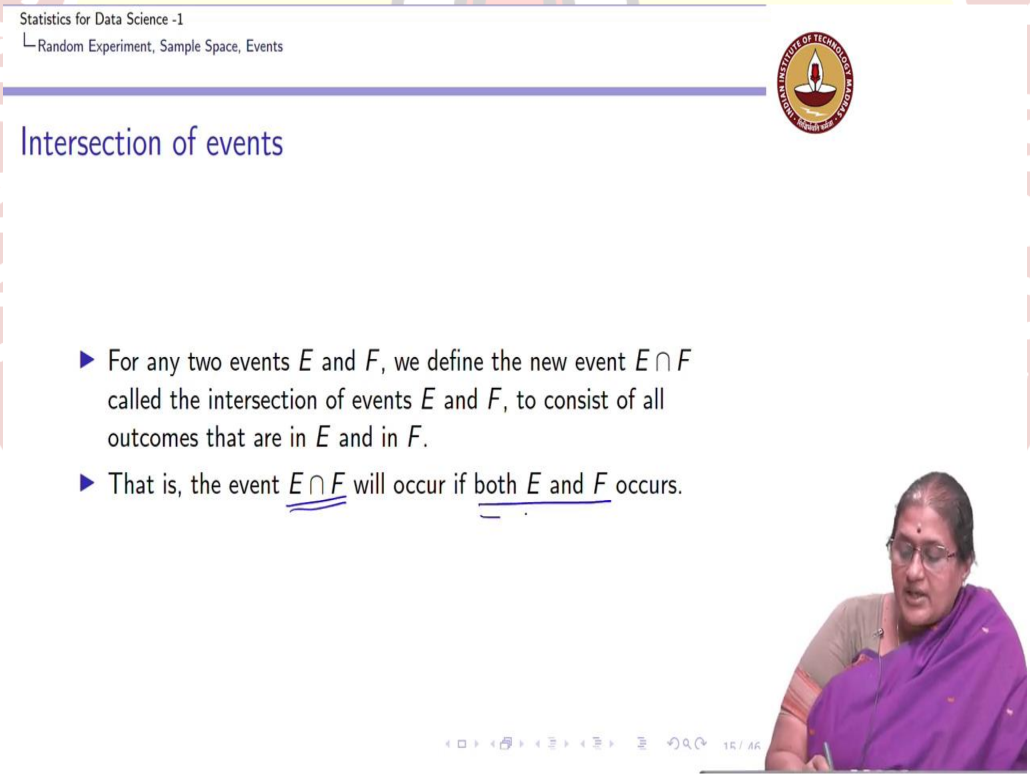

Now, the next set operation is what we refer to as a intersection of events. So, now given 2 events E and F. Again, I know these are subsets of my sample space. I can define a new event, and I represent it by $(E \cap F)$. Now, this event $(E \cap F)$ consists of all outcomes, that are in outcomes that are in both E and F. Earlier it was either E or F. Here it is both E and F. So, that is, so E, the event $(E \cap F)$will occur if both E and F occur. So that is how we define an event $(E \cap F)$.

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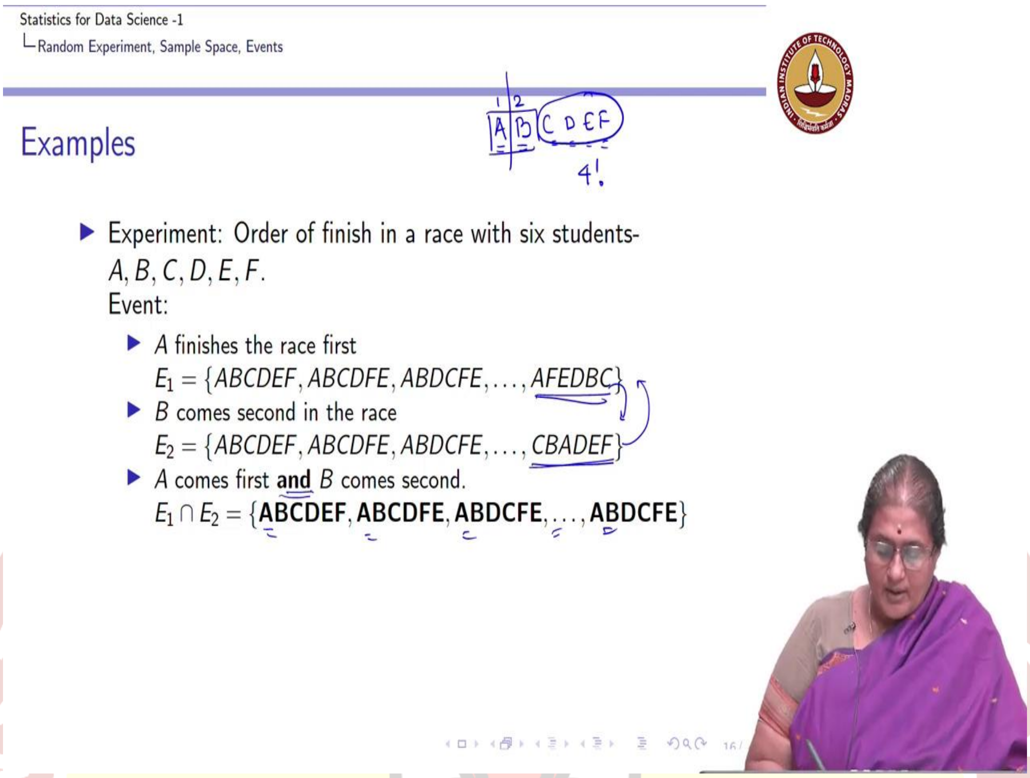

Let us look at examples. Again, when I have an order of finish in a race with 6 students. Let me go back, and define the events in a similar way. I have A finishes the race, which is ABCDEF. B comes second in the race is ABDCEF. Now if I am looking at events where A comes first, and B comes second.

Then I am looking at all possible arrangements where A is in my first place, B is in my second place, and the rest of the elements are arrangements of C, D, E, and F which can happen in 4! ways, because I am fixing A and B to be in my first and second space. And I am looking at all other possible arrangements of C, D, E, F among themselves. So, this is an event where A comes first, and B comes second.

Notice that this element CBADEF will not be an outcome in the set. Similarly, AFEDBC which was an event will not be an element of the $(E_1 \cap E_2)$, because this is not an element of E_2, and this is not an element of E_1.

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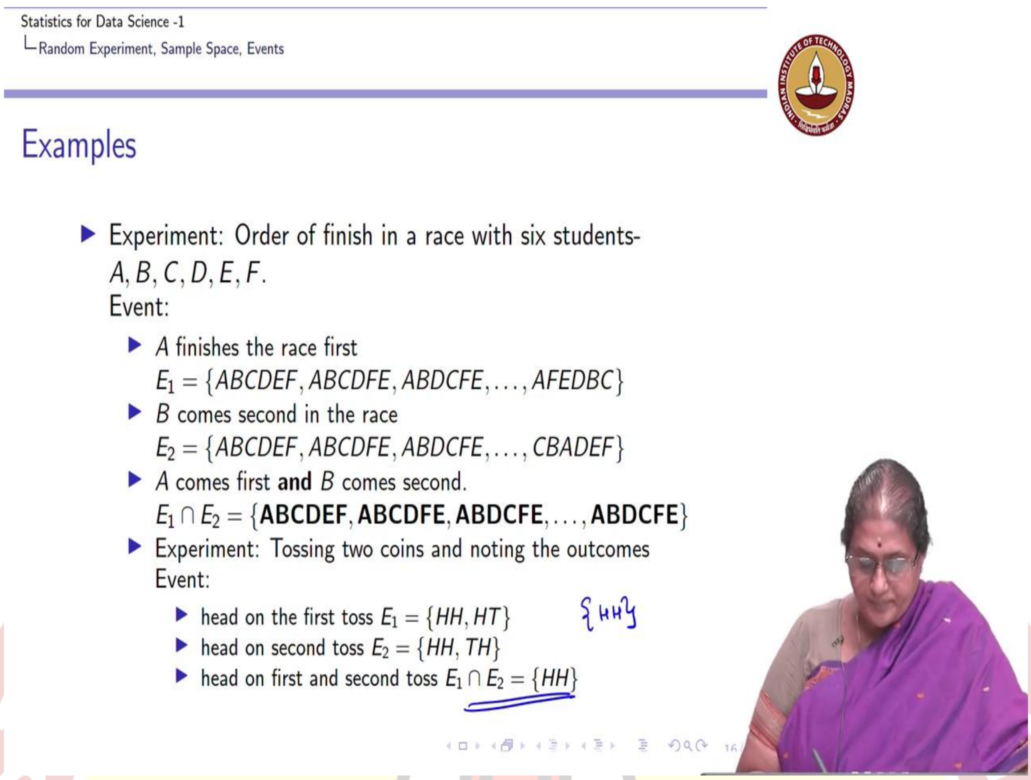

Now, let us toss two coins and note the outcomes. Again, let head on the first toss, head on the second toss, head on both the tosses would just be the event heads {HH}, which you can see is $(E_1 \cap E_2)$, would represent the event that head appears in both the tosses.

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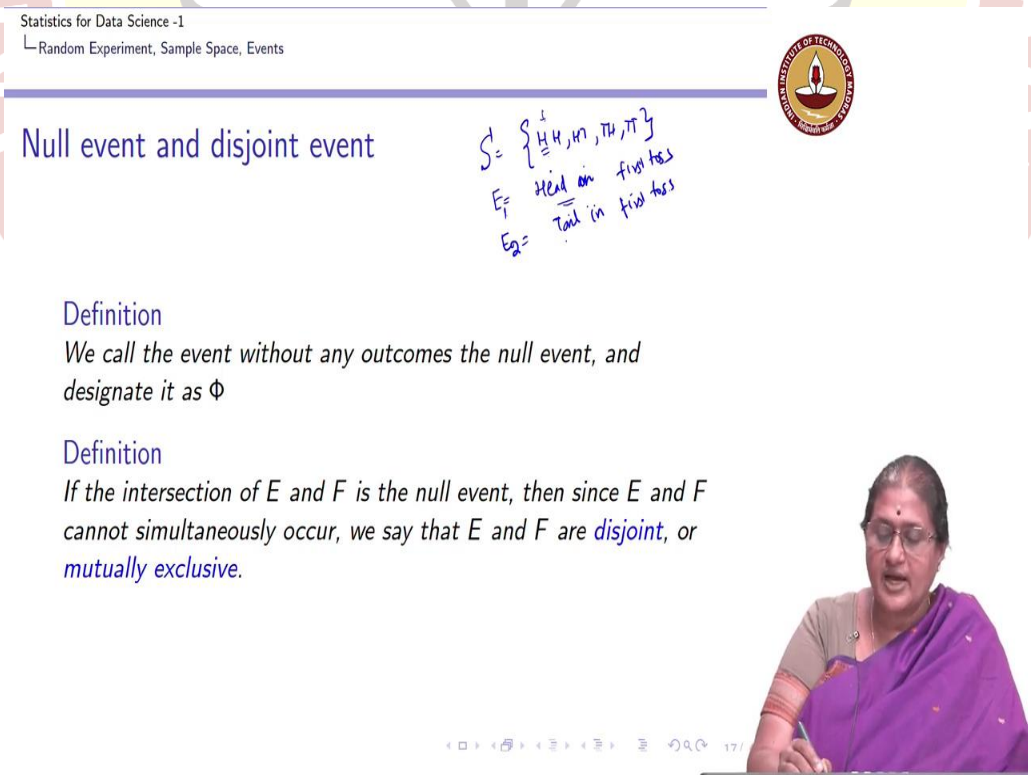

So, now suppose I go back here, and I define here that let S be my sample space again. And let me define an event E which is head in my first toss and E_2 to be head in second toss, or I can define my E_2 to be tail in my first toss. Now, I know that, this event $E1$ an E_2 cannot happen at the same time because if I have a head in my first toss, I cannot have a tail. If I have a head in my first toss, I know that I cannot have a tail also in my first toss. So, there are events where which cannot occur together.

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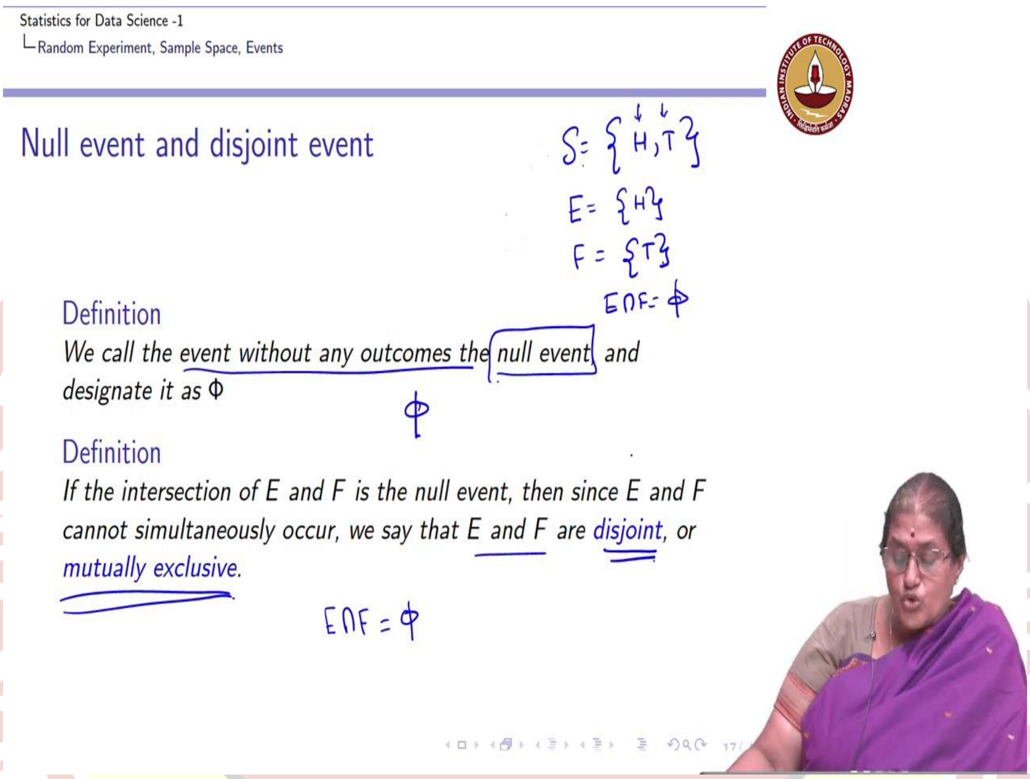

Similarly, when I have an answer to a multiple-choice question, and my sample space is one of these answers, or one of these guesses. I cannot have an event which would say A and B are correct answers, where only exactly one of the answers are correct to my multiple-choice questions with 4 options. So, an event without any outcome is called a null event, and you can determine or designate it by this alphabet or by this symbol $\emptyset$.

So, if the intersection of $(E \cap F)$is a null event, then we say E and F are disjoint events, or mutually exclusive events. For example, I have just toss a coin once, and I have head or tail. I can define my event E to be the outcome of a head, I can define my event F to be outcome of a tail, I can see that $(E \cap F)$is my disjoint set.

In other words, I can say the outcome head and tail are mutually exclusive that if I get a head, I cannot get a tail, if I get a tail, I do not get a head. So, these are mutually exclusive events.

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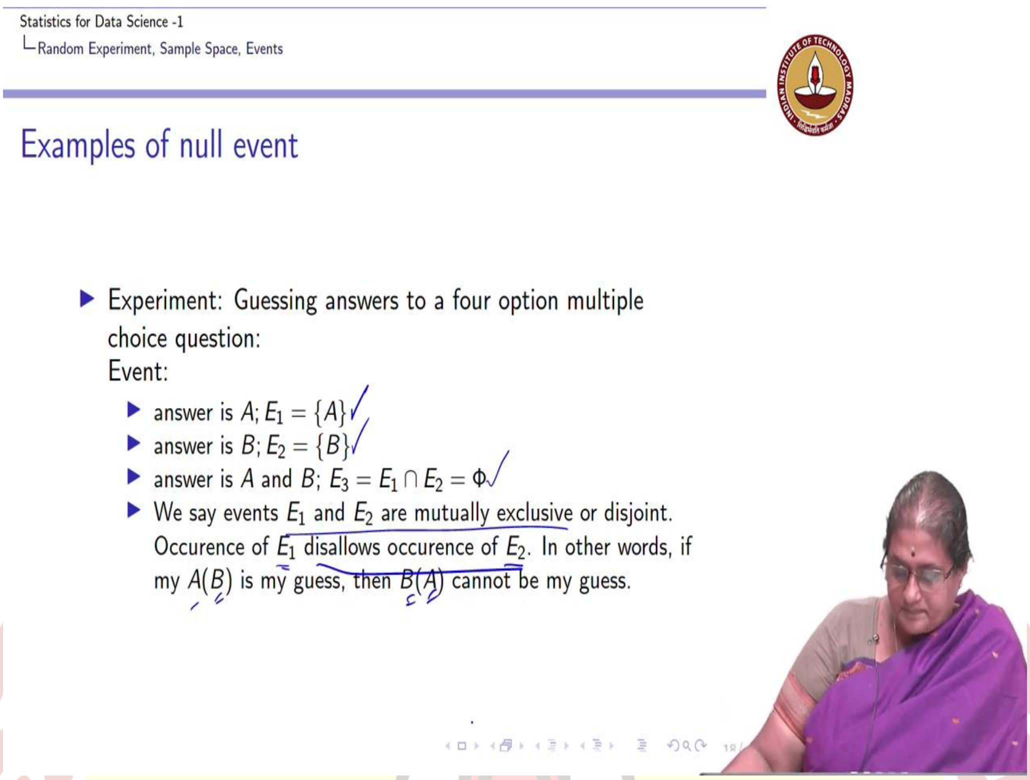

So, suppose I am guessing an answer to a four option multiple choice question. If E_1 is $\{A\}$, E_2 is $\{B\}$, E_3 which is $(E_1 \cap E_2)$, is a null set, $(E_1 \cap E_2)$, are mutually exclusive. If A that is occurrence of an event disallows the occurrence of E_2. So, if my guess is A or B. If my guess is A then B cannot be my guess. If my guess is B then A cannot be my guess. So, these are mutually exclusive events. Again, a concept that is very-very important for us to understand about probability of events.

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Complement of an event

Toss a coin $S = \{H, T\}$

$E = \{H\}$ $S \setminus E = \{T\}$

Definition

The complement of E, denoted by E^c, consists of all outcomes in the sample space S that are not in E.

Statistics for Data Science -1
└ Random Experiment, Sample Space, Events

Complement of an event

Toss a coin $S = \{H, T\}$

$E = \{H\}$ $E^c = \{T\}$

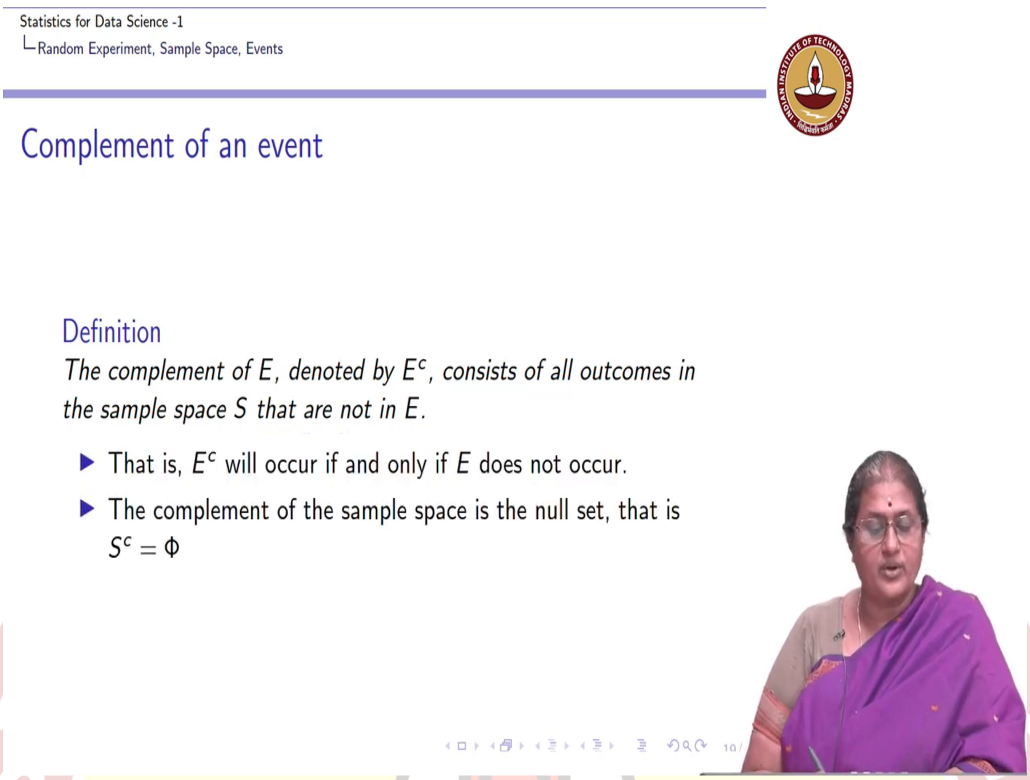
Definition

The complement of E, denoted by E^c, consists of all outcomes in the sample space S that are not in E.

Toss a coin twice $S = \{HH, HT, TH, TT\}$

$E = \{HH, HT, TH\}$ → At least one coin is a head

$E^c = \{TT\}$ → Both the coin are Tail



The next set operation which we are going to talk about is, what we refer to as a compliment of a set. So, I have my sample space. Again, I have $\{H, T\}$. I toss a coin once. I know this is my sample space. If I define E to be the outcome of a head, I know the complement of E is $(S \setminus E)$ set, which is $\{T\}$. I can refer to this as E^c, complement of a set.

Similarly, what is the E^c? It consists of all those outcomes that are not in E. So again, I toss a coin twice, and it define, I know S is $\{HH, HT, TH, TT\}$. If I define an event that at least one of the coin is a head, then I know these events are $\{HH, HT, TH\}$ are the outcomes in this event E. So, the E^c is $\{TT\}$, that is this element which is not in E, and I can see that this corresponds to both the coins are tail.

Now, this is at least one coin is a head. So, the complement of E is both the coins are tail. So, you can see that this is a very important operation again, and how we have represented this operation also. So now let us go back, and look at our example.

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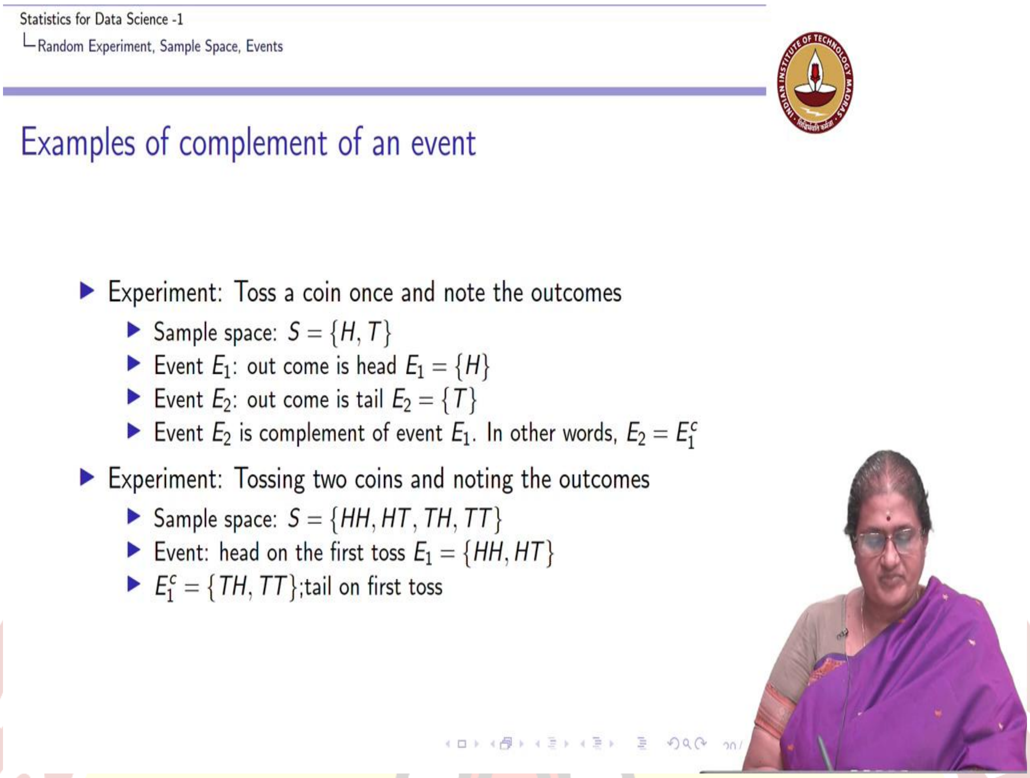

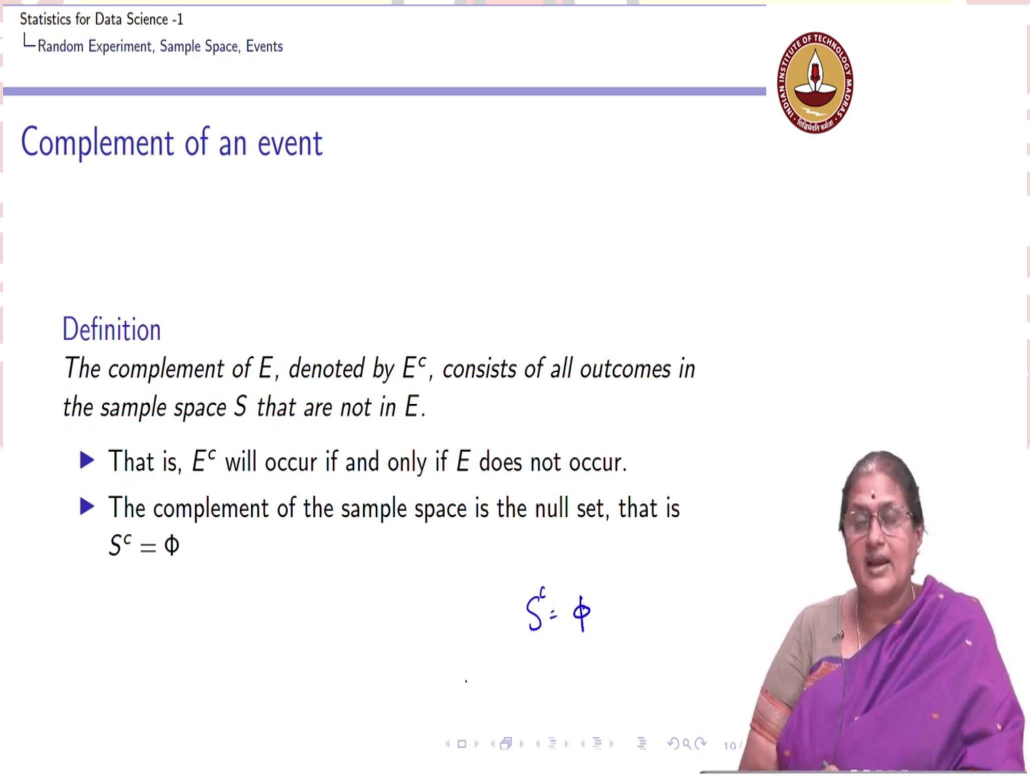

So, I toss a coin once, and note the outcomes. I know that $E_2^c$is E_1^c. Toss a coin twice, event is head on the first toss, event 2 is tail on the first toss and E_1^c is $\{TH,\ TT\}$. One thing we need to observe is the complement of the sample space is the null set. This is again, or the null event.

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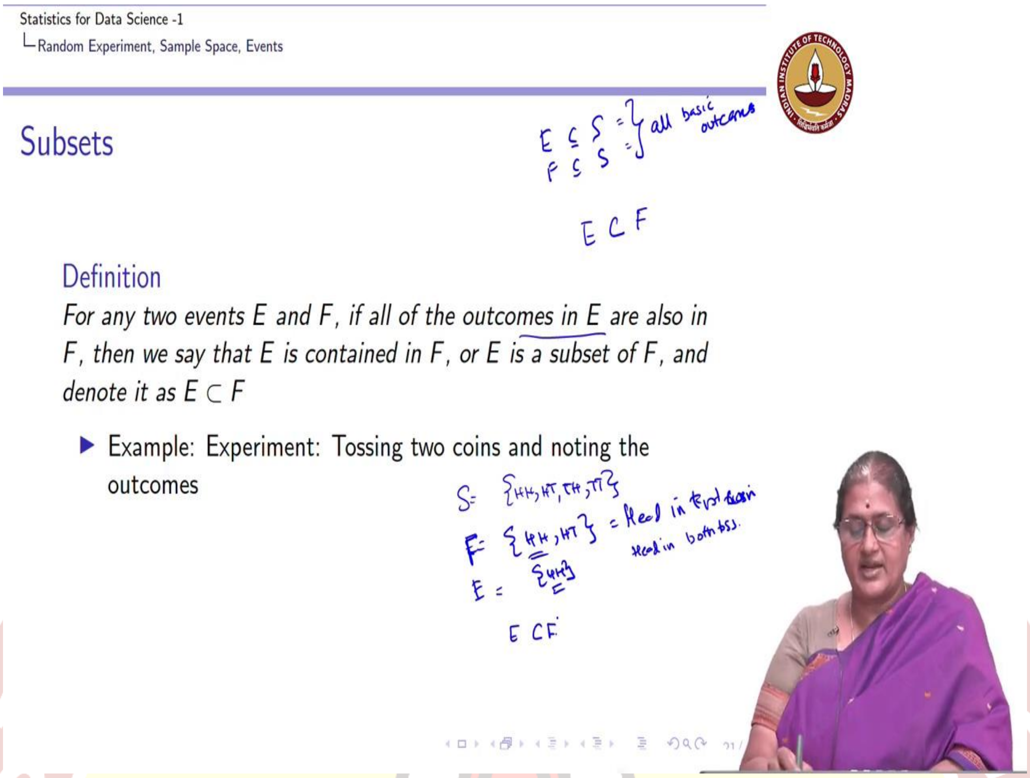

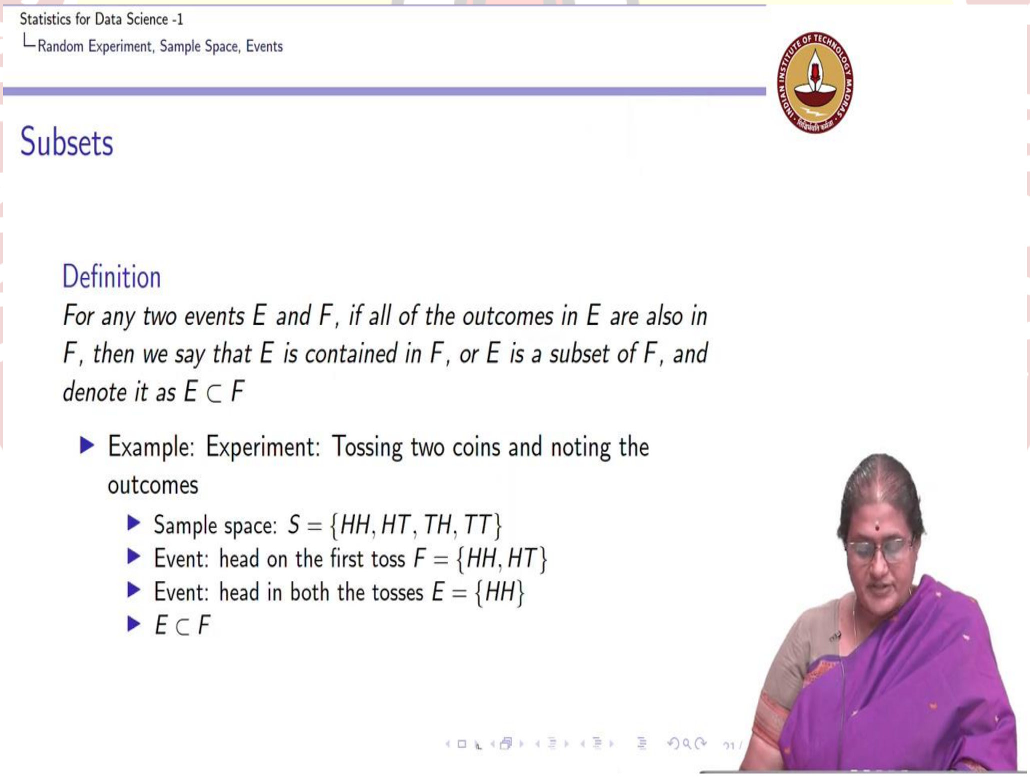

So, now suppose we are given 2 events. Again, remember events are subsets of my sample space, where sample space in itself is a set of all basic outcomes; all possible basic outcomes of my random experiment. So, if all the outcomes in E are also in my event F, then I say E is contained in F, and I denote it by $E \subset F$.

Suppose, I again go back, I toss a coin twice. I know $\{HH, HT, TH, TT\}$ are my outcomes of the sample space. Let me define an event which is head in my first toss. So, the event of head in my first toss is $\{HH,\ HT\}$. This is my event of my head in my first toss. Let me define or event F is my head in the first toss. Let me define E to be the event, head in the both coins, or head in both tosses. I am tossing 2 coins, so head in both toss, head in first toss of, first coin, head in both toss of the coin. Then I know E is $\{HH\}$. So, the outcomes in E, what is the outcome, I have only one outcome $\{HH\}$. It is also in F.

Then I know that, I can say, $E \subset F$. In terms of event, I know that head occurring in both the coins is a subset event of the event, where I am saying head is occurring in my first coin. So, head occurring in the first coin can happen in the outcome $\{HH, HT\}$, and head in both toss is a subset of this event. Now, these notions would become very important when we want to actually derive probabilities of the events.