Statistics for Data Science -1

Lecture 8.6: Discrete Random Variable: cumulative distribution function

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Learning objectives

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4. Cumulative distribution function, graphs, and examples.
5. Expectation and variance of a random variable.

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- If X is a discrete random variable whose possible values are $x_1, x_2, x_3, \ldots$, where $x_1 < x_2 < x_3 \ldots$, then the distribution function F of X is a step function.

Step function

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- Let X be a discrete random variable with the following probability mass function.

X	1	2	3	4
$P(X = x_i)$	$\frac{1}{4}$	$\frac{1}{2}$	$\frac{1}{8}$	$\frac{1}{8}$

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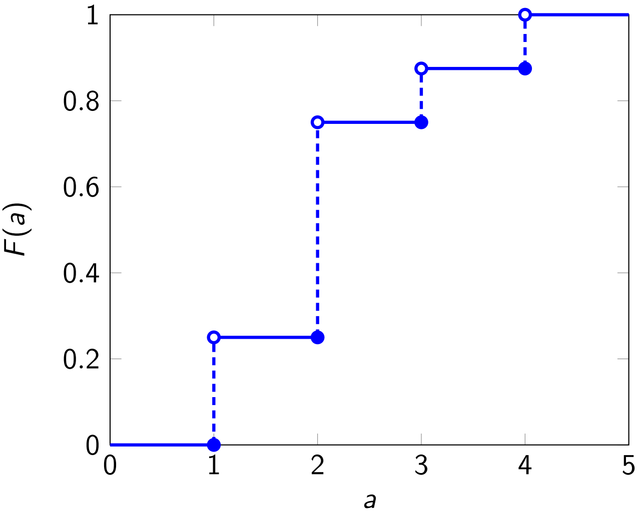
Note that the size of the step at any of the values 1, 2, 3, and 4 is equal to the probability that X assumes that particular value.


1
0.8
0.6
0.4
0.2
0
F(a)
0
1
2
3
4
5
a

Section summary

- Probability mass function- tabular form and graph.
- Cumulative distribution function- definition and graph.
- Key ideas:
    - Shape of distribution: skewed, symmetric, constant, etc.
    - Answer questions about distribution of random variable.