Applications of BFS and DFS

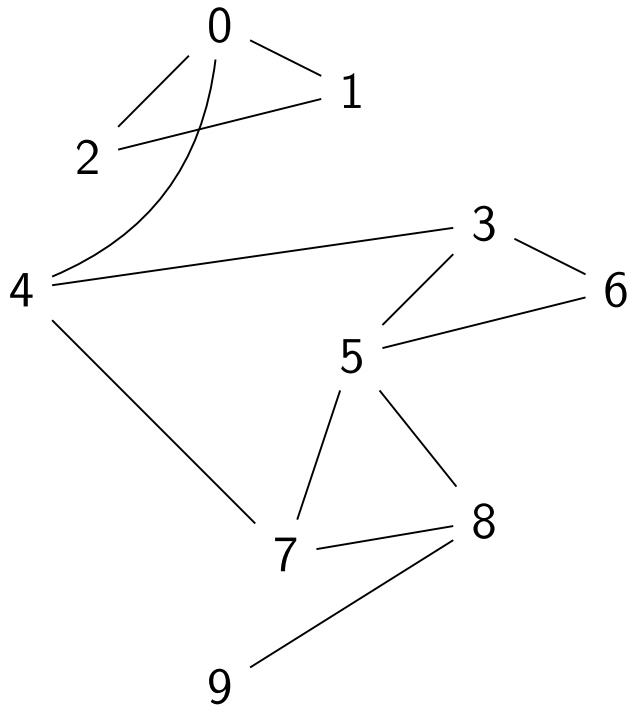
Madhavan Mukund

https://www.cmi.ac.in/~madhavan

Mathematics for Data Science 1
Week 10

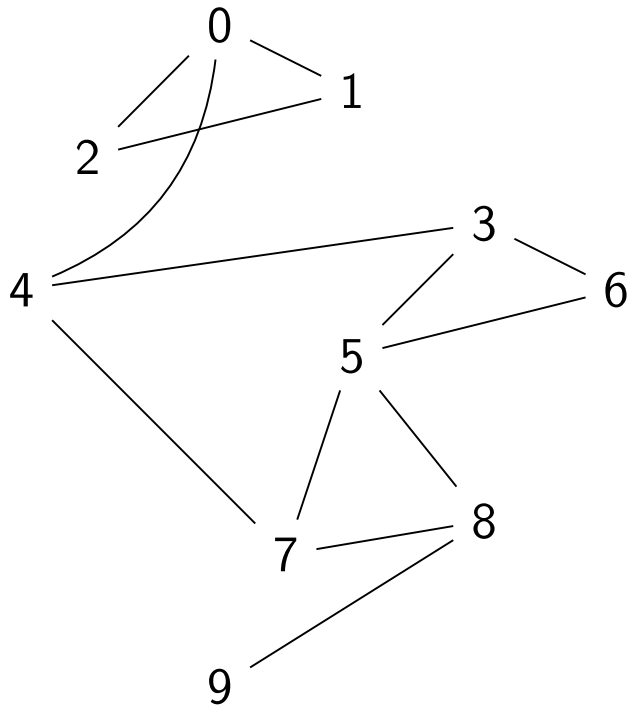
BFS and DFS

- BFS and DFS systematically compute reachability in graphs



BFS and DFS

- BFS and DFS systematically compute reachability in graphs
- BFS works level by level
  - Discovers shortest paths in terms of number of edges



BFS and DFS

- BFS and DFS systematically compute reachability in graphs
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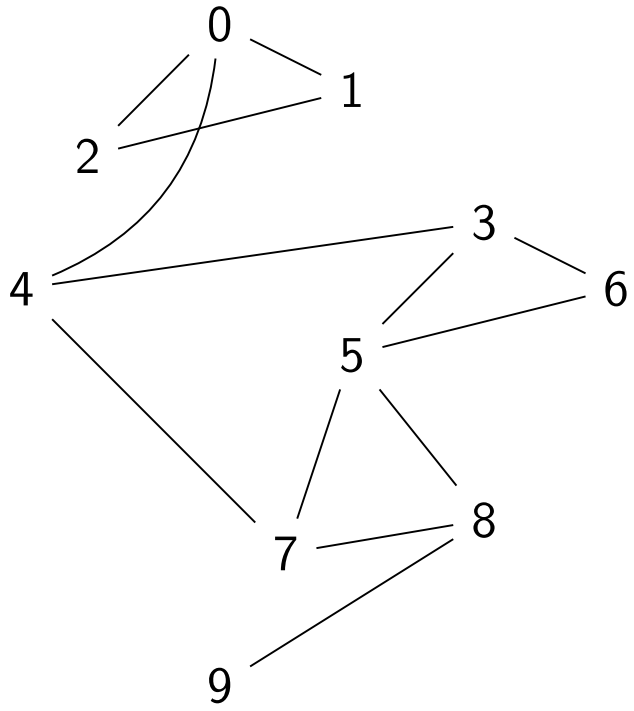
- DFS explores a vertex as soon as it is visited neighbours
  - Suspend a vertex while exploring its neighbours
  - DFS numbering describes the order in which vertices are explored



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  - Discovers shortest paths in terms of number of edges
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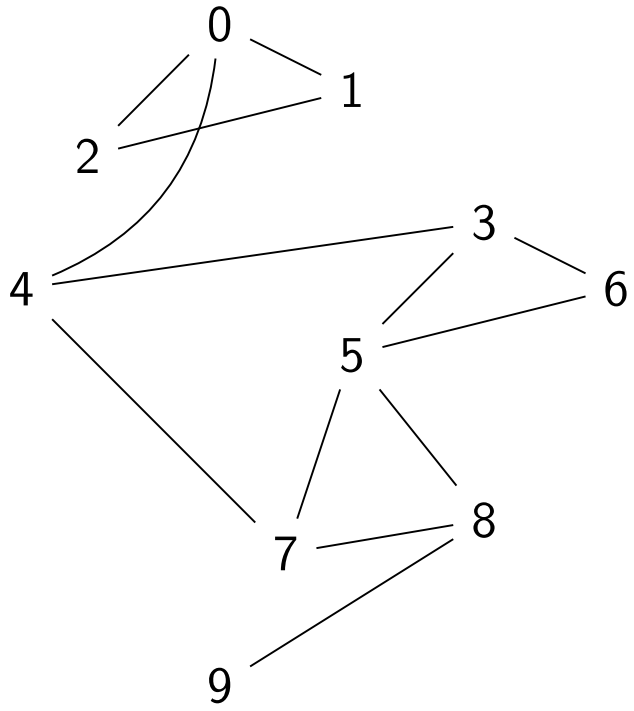
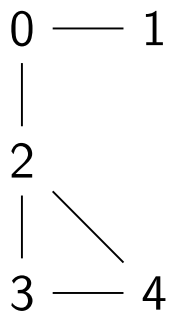
  - DFS numbering describes the order in which vertices are explored
- Beyond reachability, what can we find out about a graph using BFS/DFS?



Connectivity

- An undirected graph is connected if every vertex is reachable from every other vertex

Connected Graph



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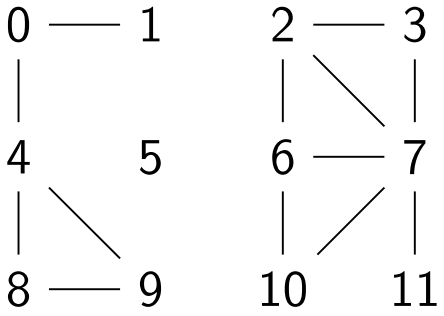

Connectivity

- An undirected graph is connected if every vertex is reachable from every other vertex
- In a disconnected graph, we can identify the connected components

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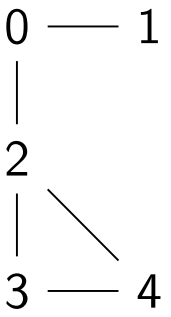

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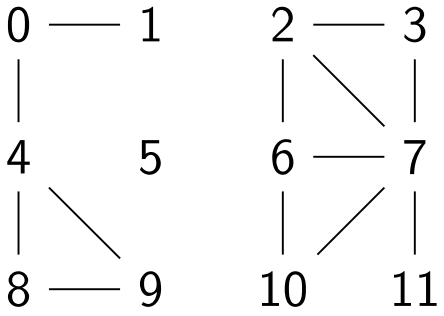

Connectivity

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- In a disconnected graph, we can identify the connected components
  - Maximal subsets of vertices that are connected

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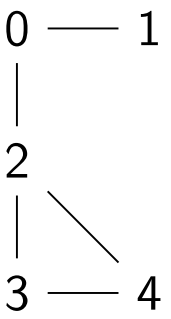

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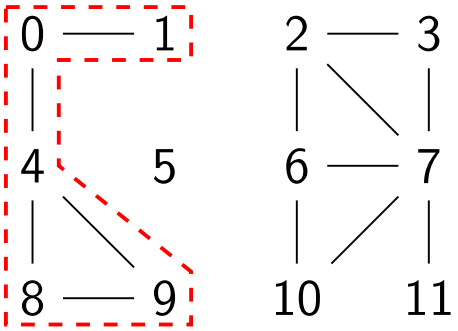

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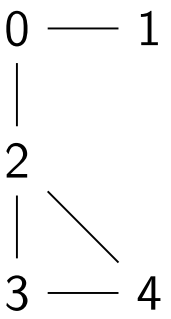

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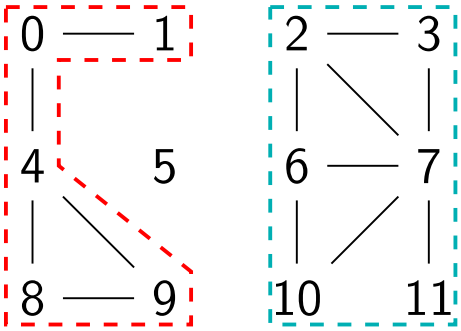

Connectivity

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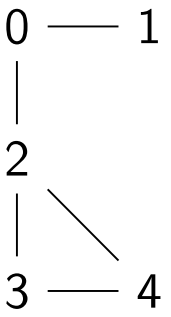
- In a disconnected graph, we can identify the connected components
  - Maximal subsets of vertices that are connected
  - Isolated vertices are trivial components

Connected Graph



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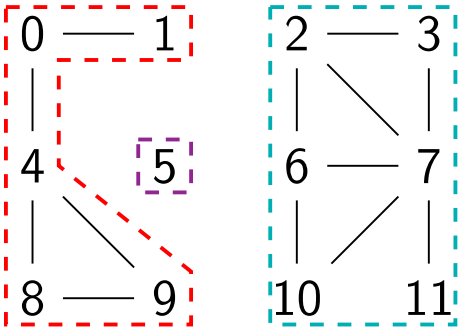
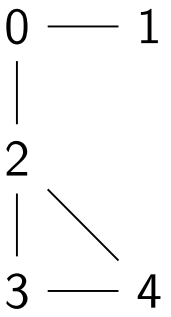


Identifying connected components

- Assign each vertex a component number

Connected Graph



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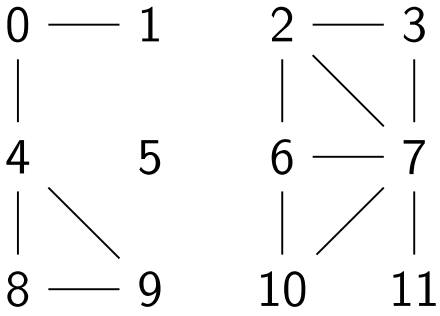
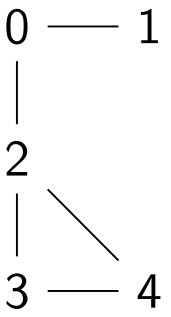


Identifying connected components

- Assign each vertex a component number
- Start BFS/DFS from vertex 0

Connected Graph



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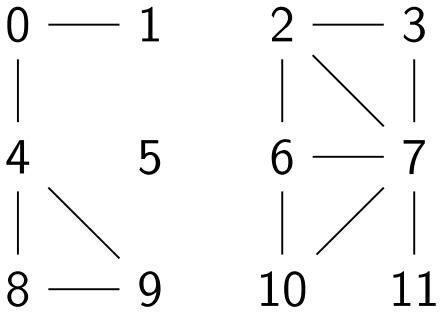

Identifying connected components

- Assign each vertex a component number
- Start BFS/DFS from vertex 0
  - Initialize component number to 0

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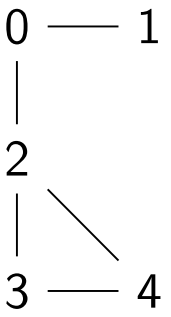

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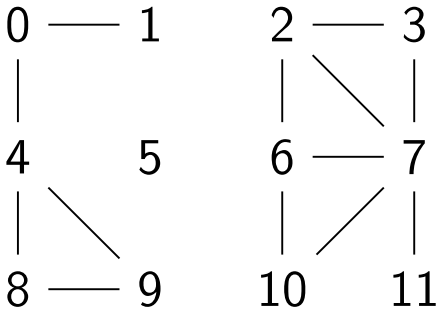
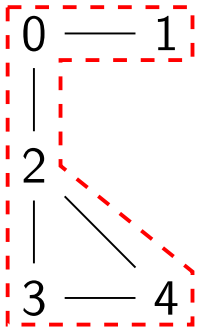


Identifying connected components

- Assign each vertex a component number
- Start BFS/DFS from vertex 0
  - Initialize component number to 0
  - All visited nodes form a connected component

Connected Graph



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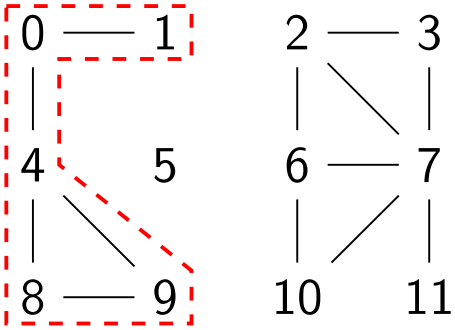

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- Start BFS/DFS from vertex 0
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  - Assign each visited node component number 0

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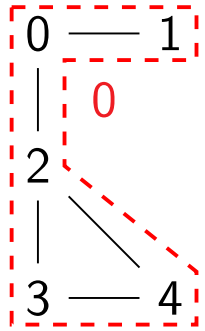

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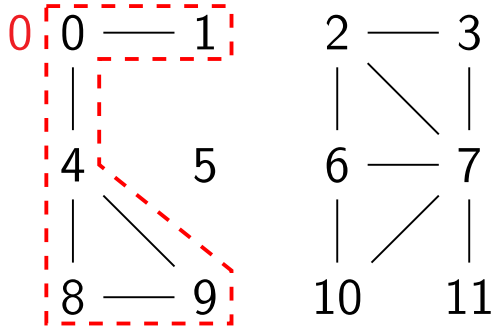
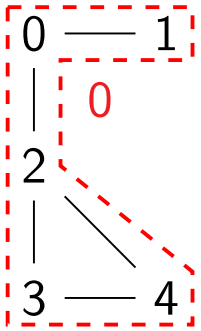


Identifying connected components

- Assign each vertex a component number
- Start BFS/DFS from vertex 0
  - Initialize component number to 0
  - All visited nodes form a connected component
  - Assign each visited node component number 0
- Pick smallest unvisited node j

Connected Graph



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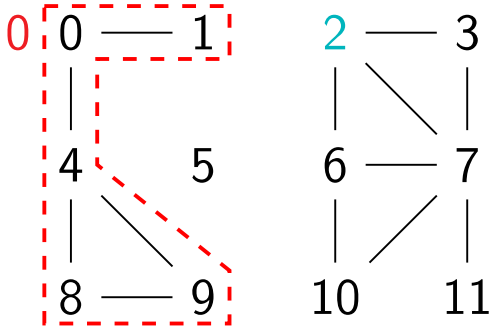

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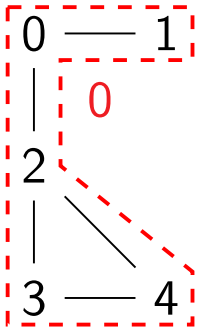
Connected Graph



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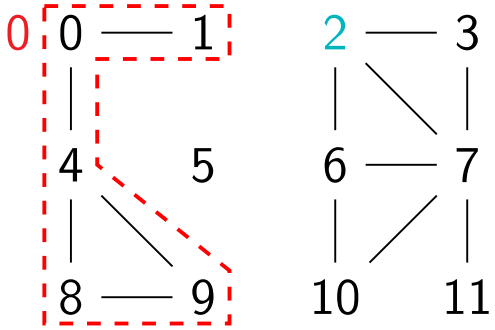

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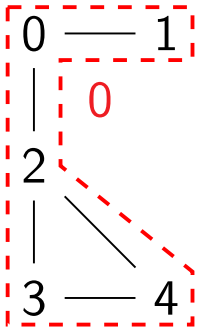
  - Increment component number to 1
  - Run BFS/DFS from node j

Connected Graph



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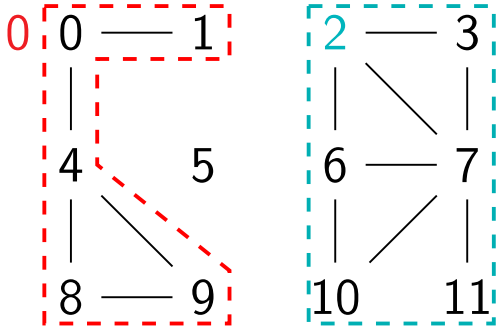

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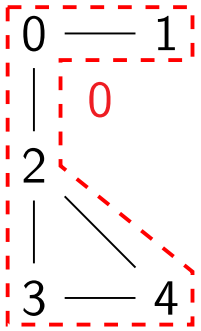
Connected Graph



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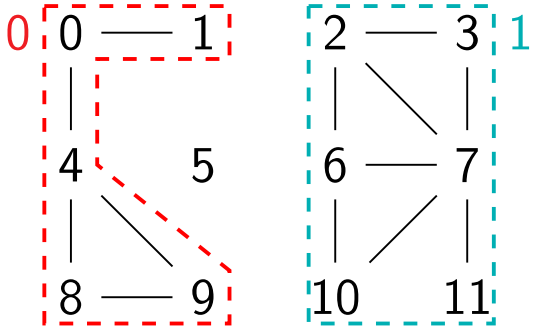

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  - Run BFS/DFS from node j
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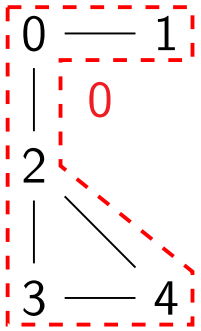
- Repeat until all nodes are visited

Connected Graph



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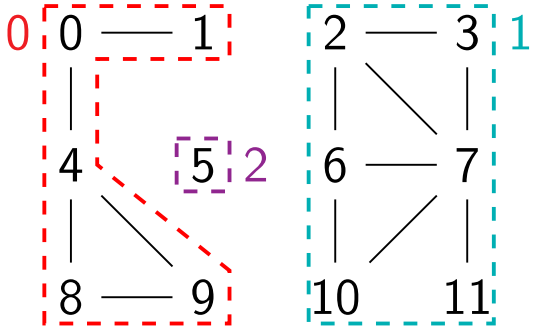
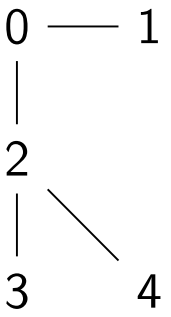
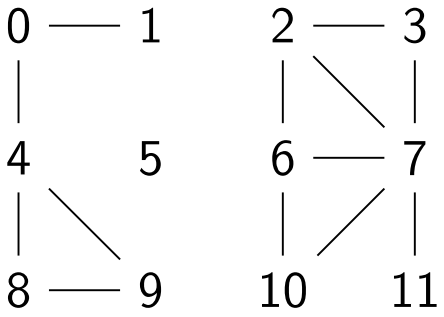


Detecting cycles

- A cycle is a path (technically, a walk) that starts and ends at the same vertex





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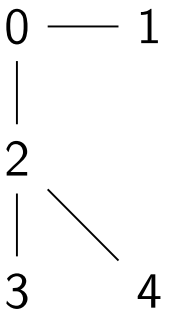
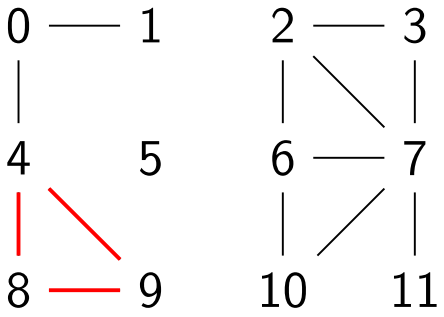
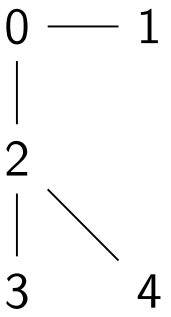
  - $4 - 8 - 9 - 4$ is a cycle

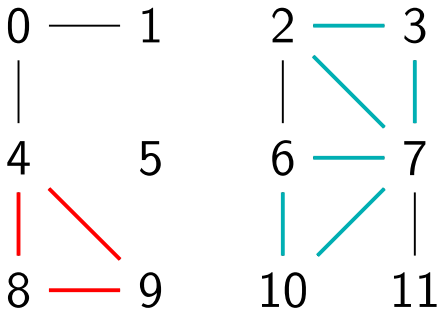




Detecting cycles

- A cycle is a path (technically, a walk) that starts and ends at the same vertex
  - $4 - 8 - 9 - 4$ is a cycle
  - Cycle may repeat a vertex: $2 - 3 - 7 - 10 - 6 - 7 - 2$





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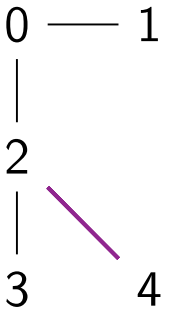

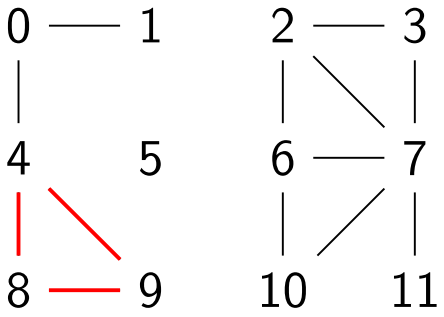

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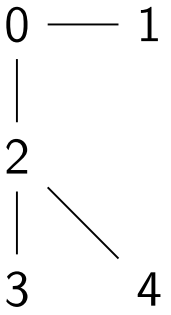
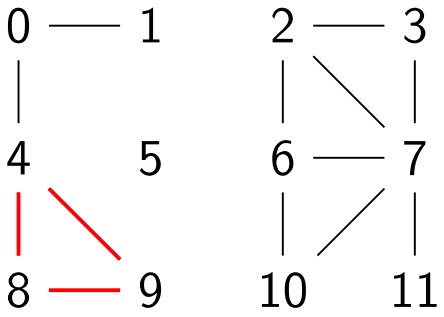
  - Simple cycle — only repeated vertices are start and end





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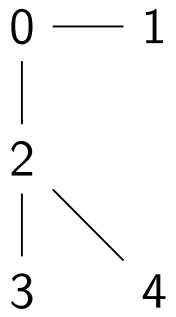
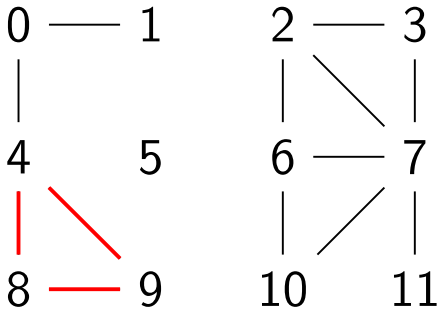
  - Cycle should not repeat edges: $i - j - i$ is not a cycle, e.g., $2 - 4 - 2$
  - Simple cycle — only repeated vertices are start and end
- A graph is acyclic if it has no cycles

Acyclic Graph



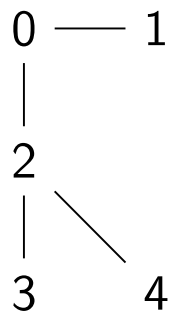
Graph with cycles



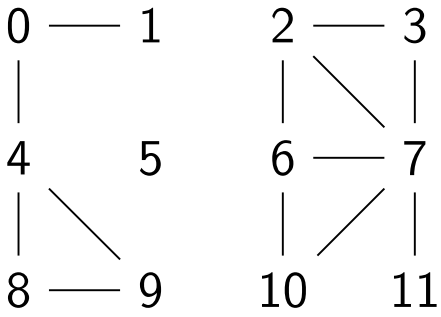
BFS tree

- A tree is a minimally connected graph

Acyclic Graph



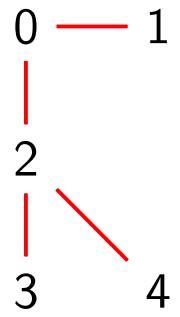
Graph with cycles



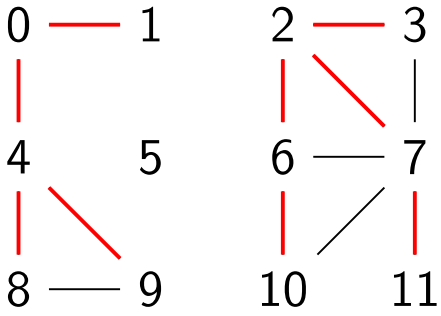
BFS tree

- A tree is a minimally connected graph
- Edges explored by BFS form a tree
  - Technically, one tree per component
  - Collection of trees is a forest

Acyclic Graph



Graph with cycles



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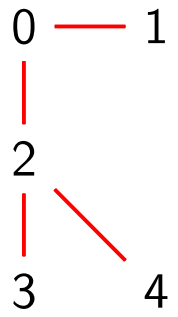
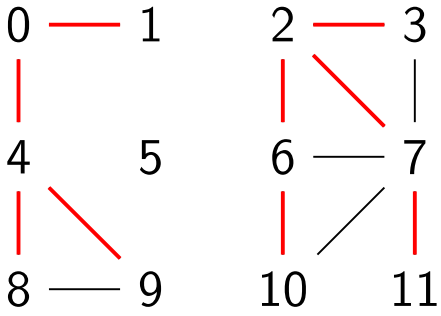
- Facts about trees

Acyclic Graph



Graph with cycles



BFS tree

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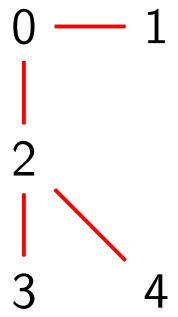
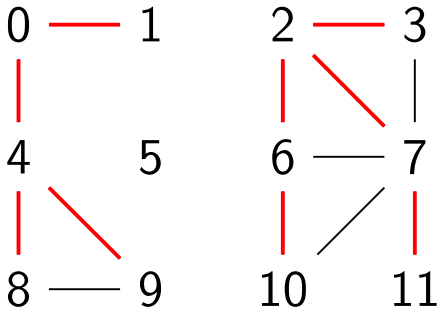
  - Technically, one tree per component
  - Collection of trees is a forest
- Facts about trees
  - A tree on n vertices has $n - 1$ edges

Acyclic Graph



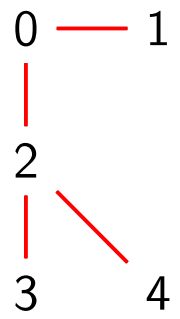
Graph with cycles



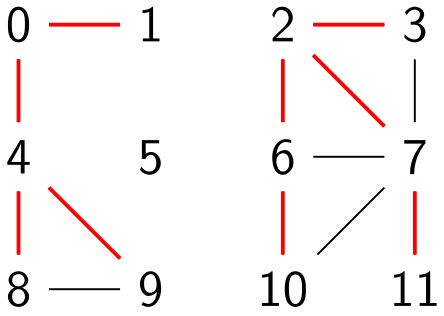
BFS tree

- A tree is a minimally connected graph
- Edges explored by BFS form a tree
  - Technically, one tree per component
  - Collection of trees is a forest
- Facts about trees
  - A tree on n vertices has $n - 1$ edges
  - A tree is acyclic

Acyclic Graph



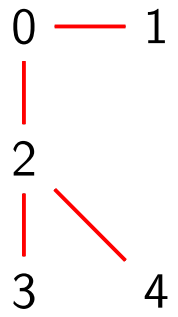
Graph with cycles



BFS tree

- A tree is a minimally connected graph
- Edges explored by BFS form a tree
  - Technically, one tree per component
  - Collection of trees is a forest
- Facts about trees
  - A tree on n vertices has $n - 1$ edges
  - A tree is acyclic
- Any non-tree edge creates a cycle
  - Detect cycles by searching for non-tree edges

Acyclic Graph



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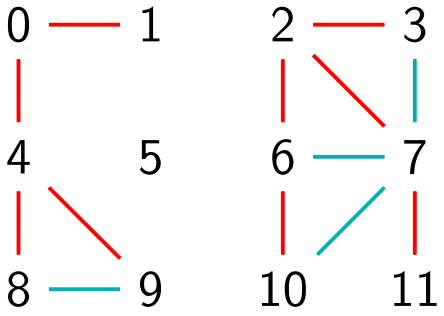
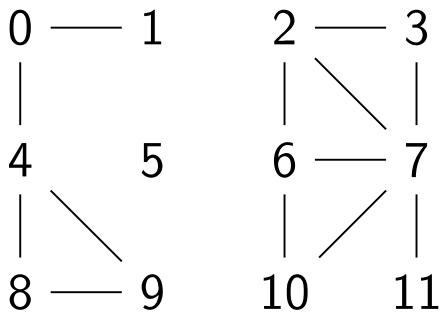


DFS tree

- Maintain a DFS counter, initially 0



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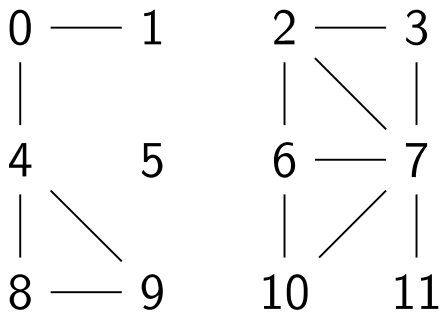

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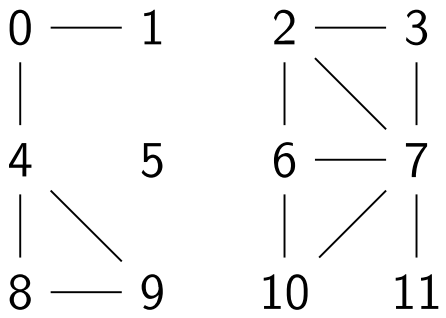

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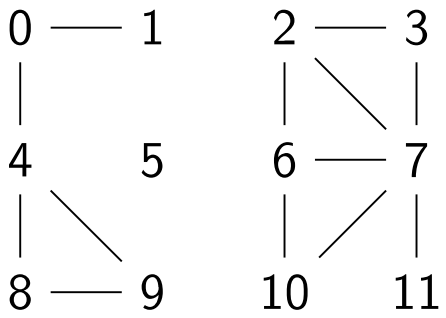

pre

0 0

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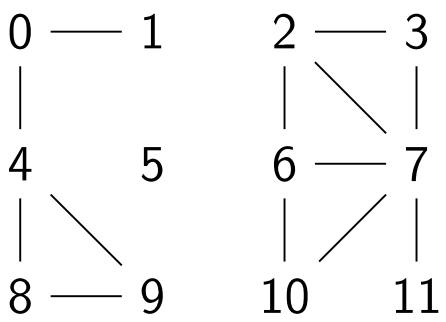

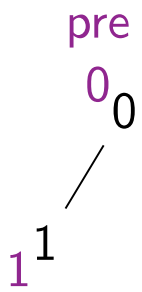

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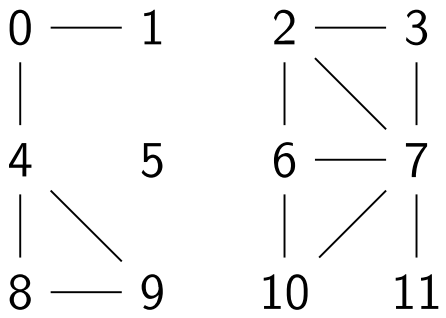

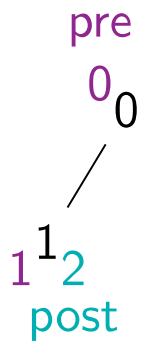

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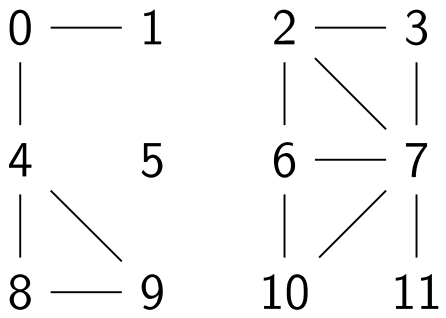

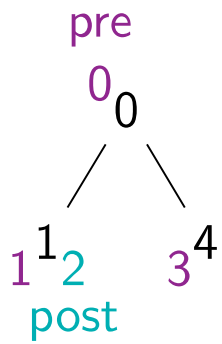

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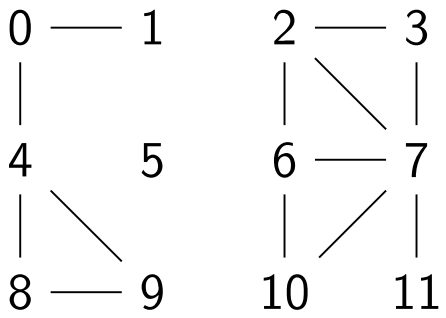

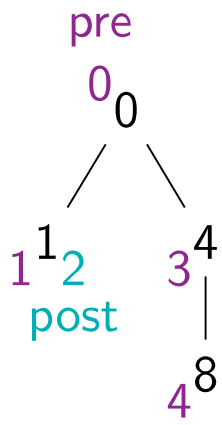

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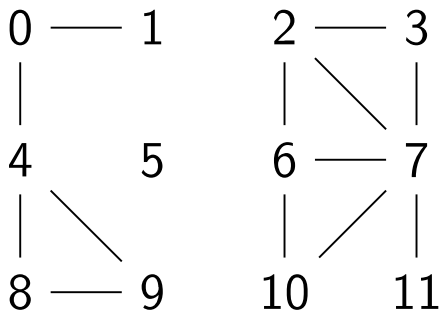

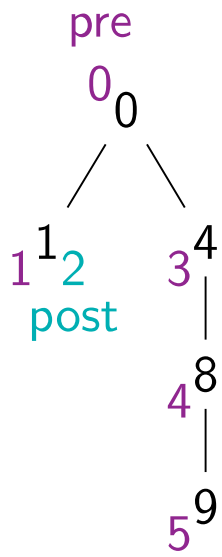

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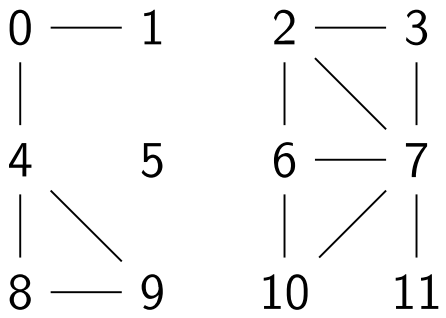

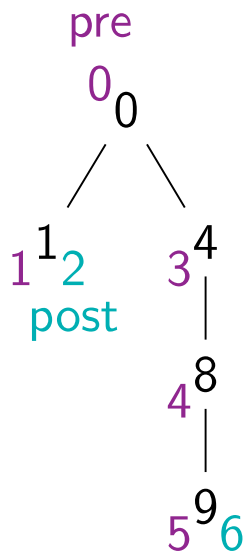

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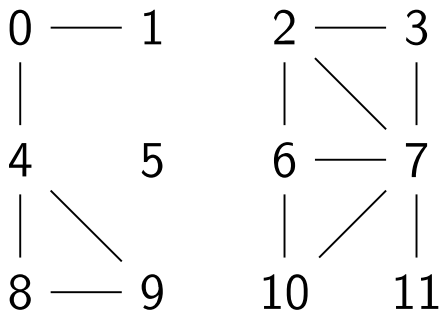

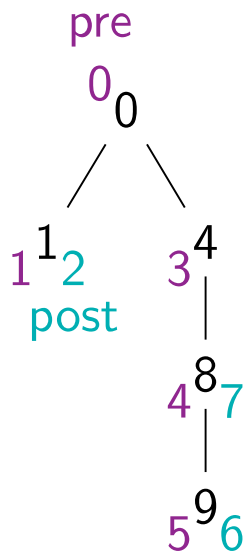

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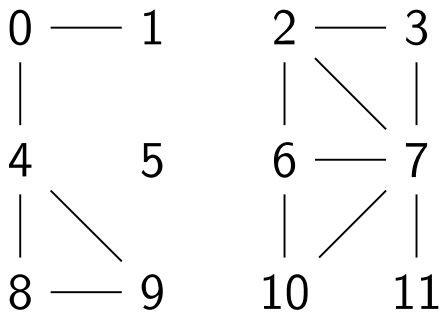

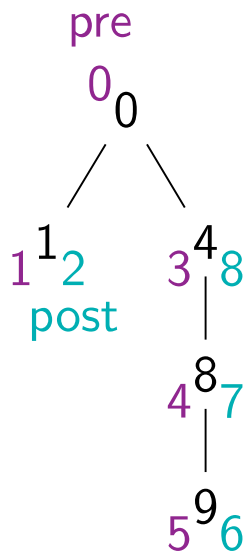

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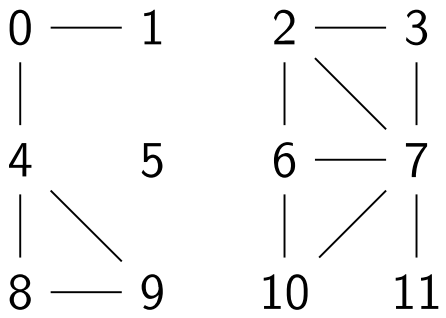

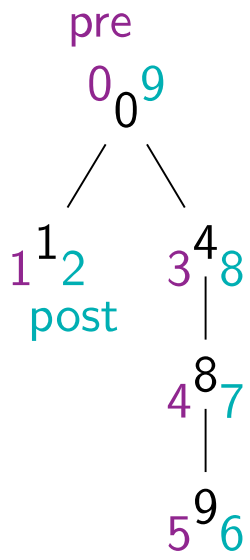

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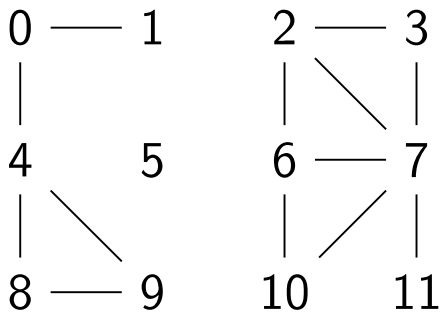

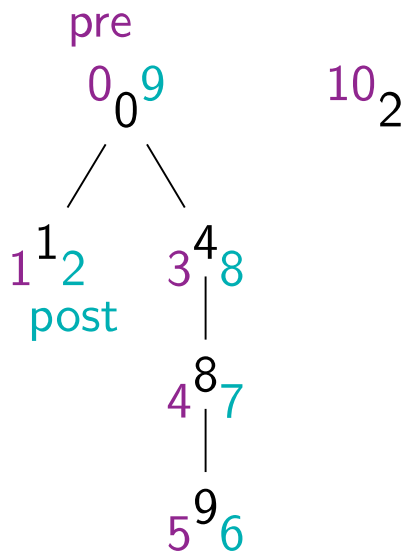

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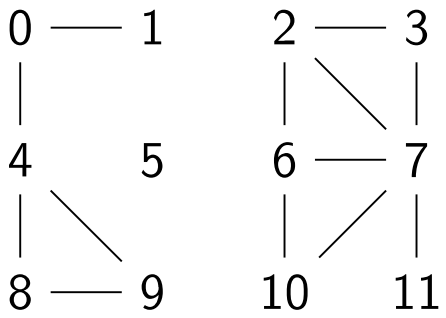

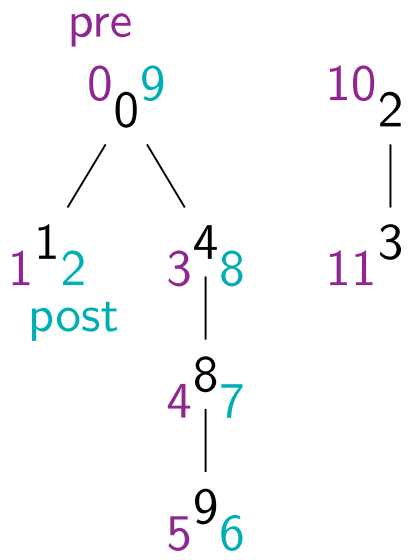

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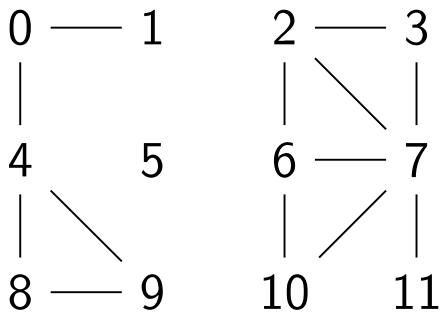

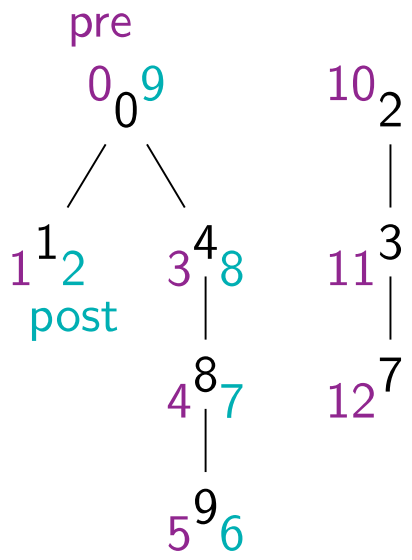

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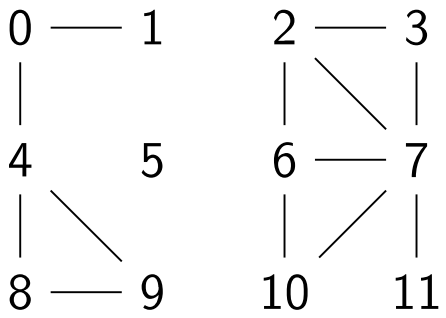

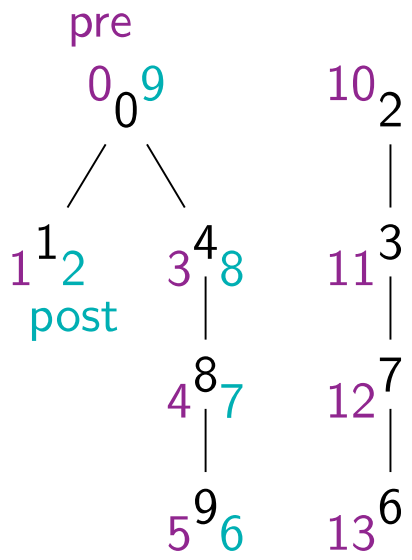

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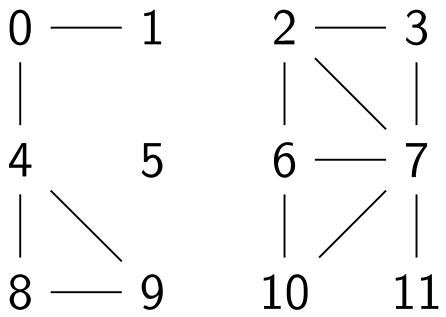

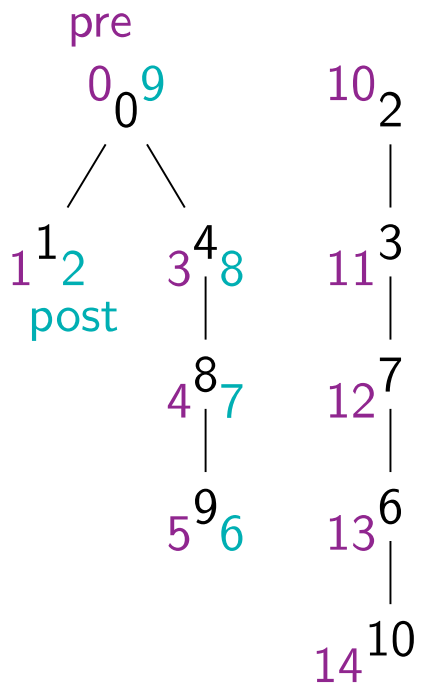

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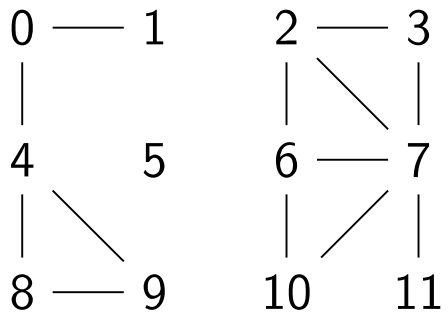

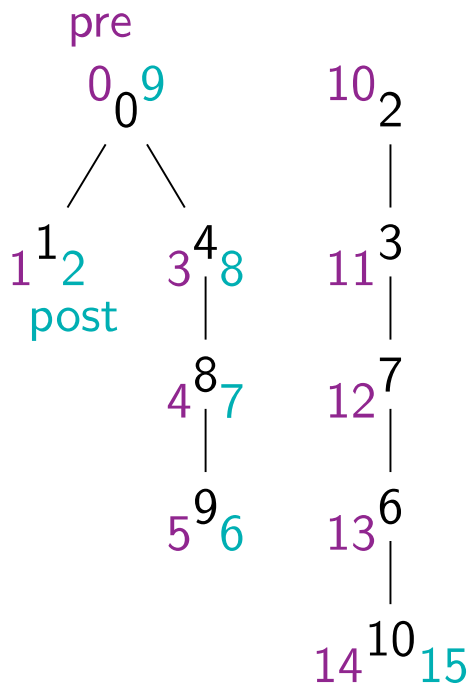

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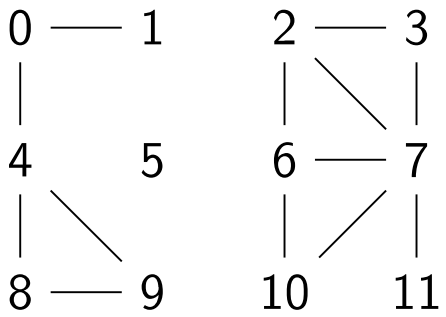

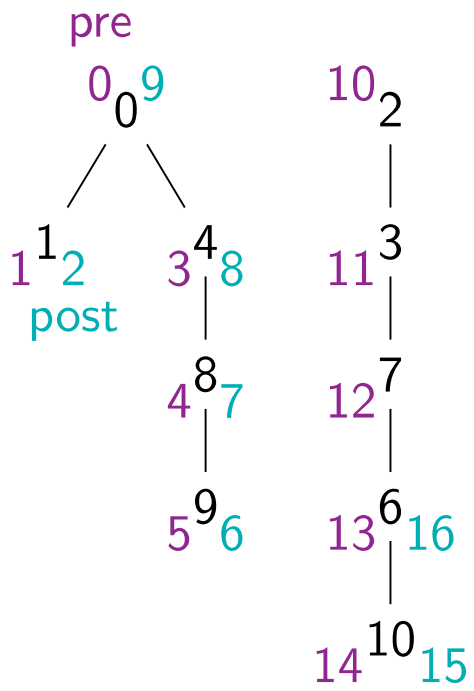

DFS tree

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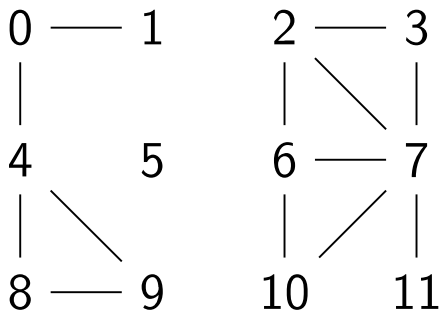

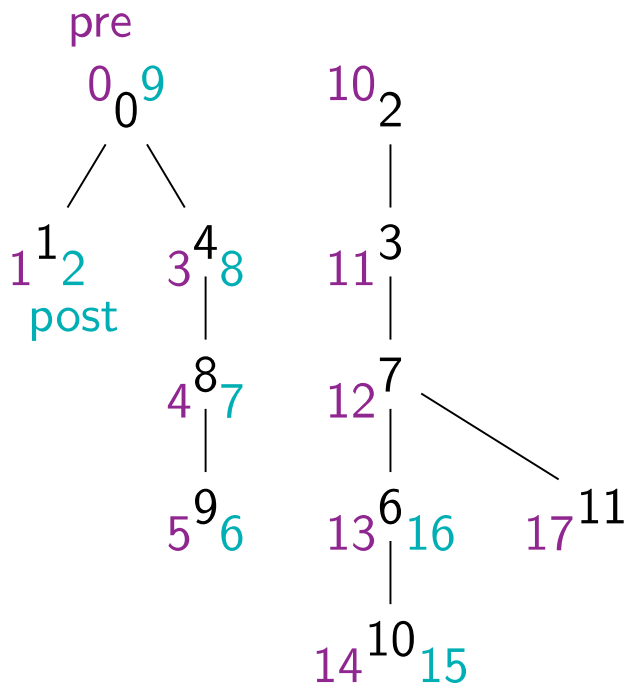

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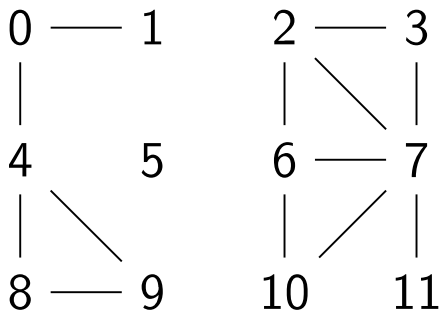

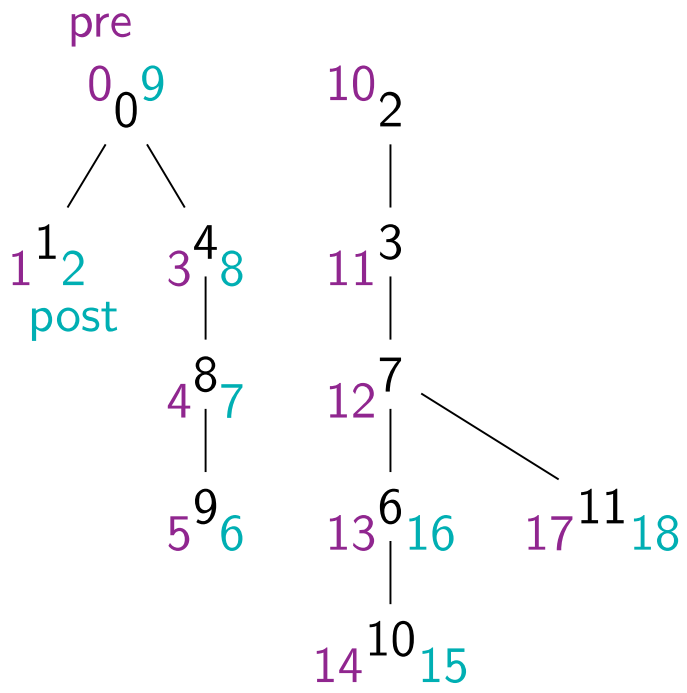

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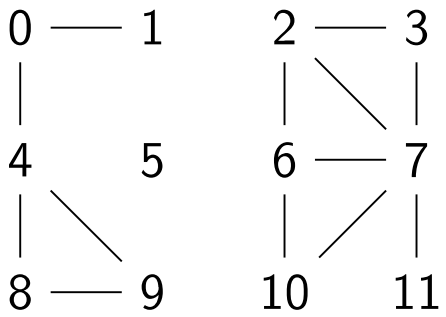

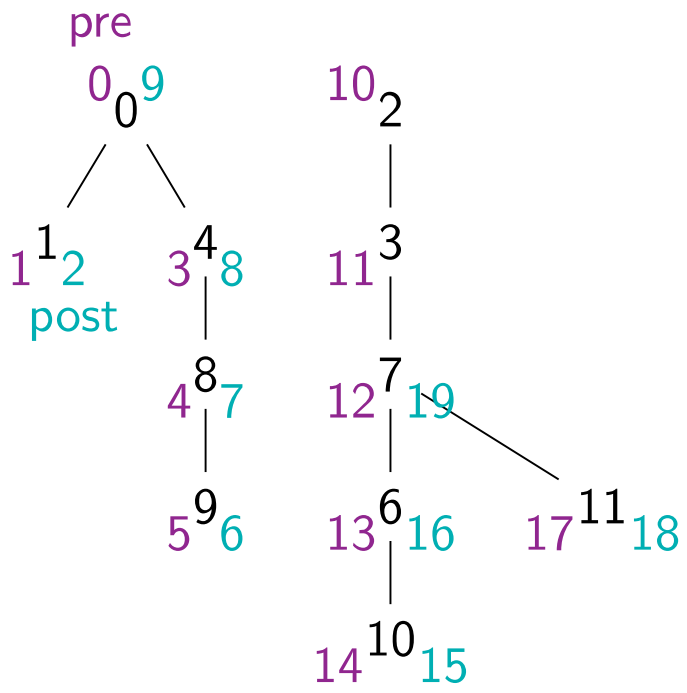

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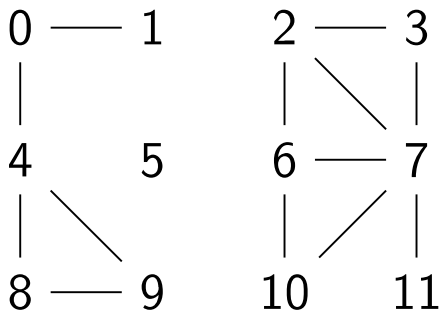

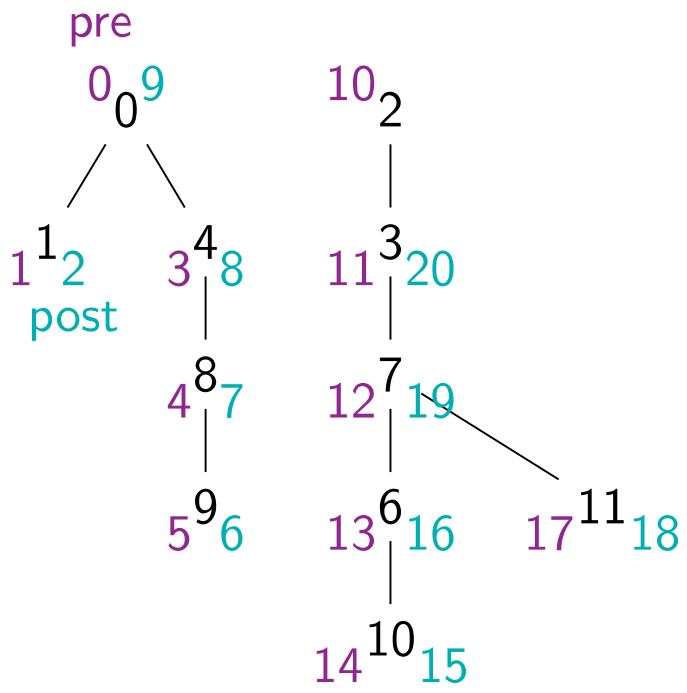

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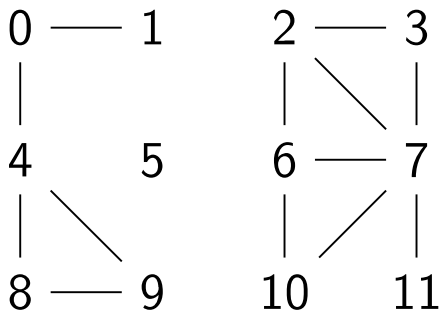

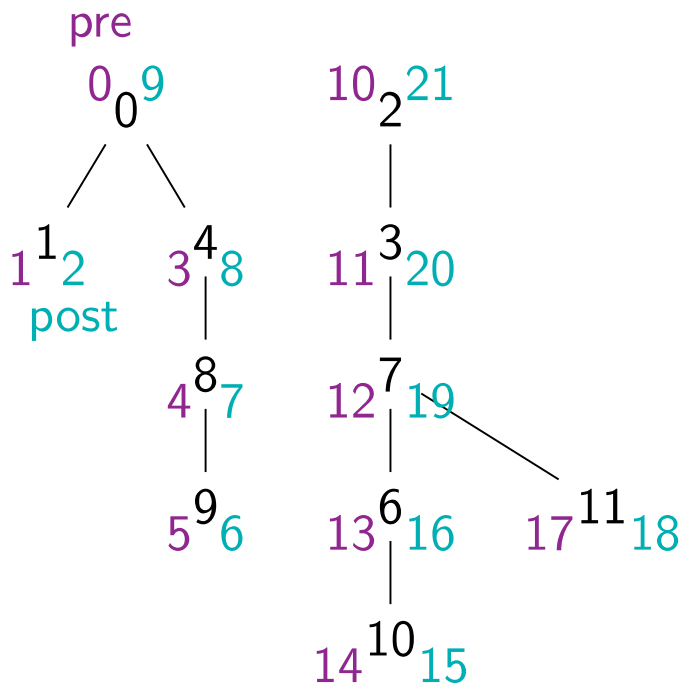

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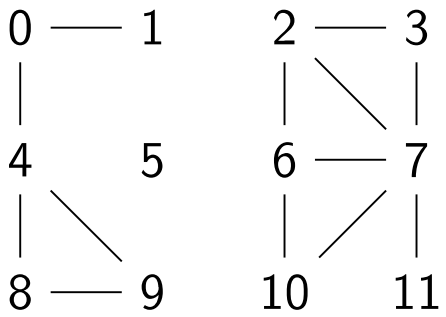

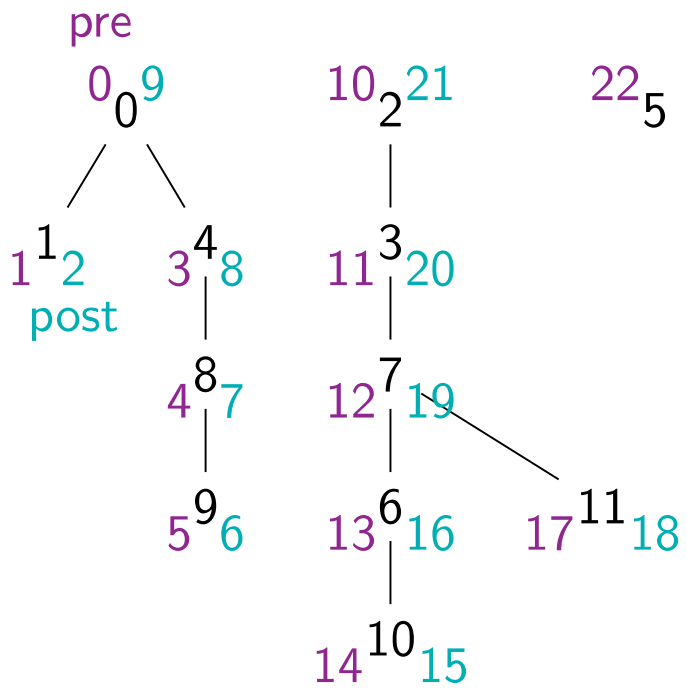

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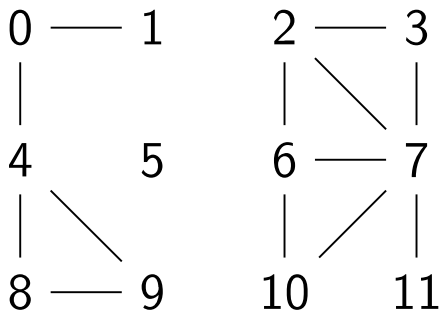

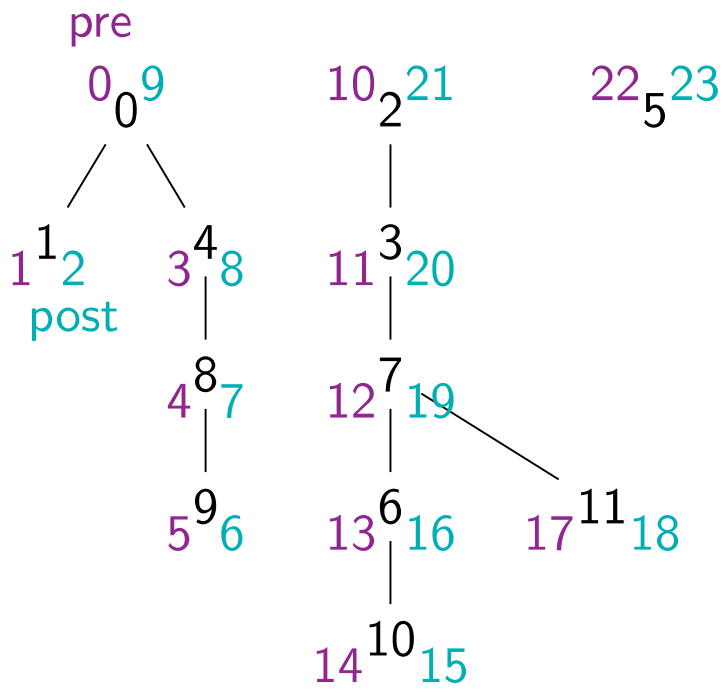

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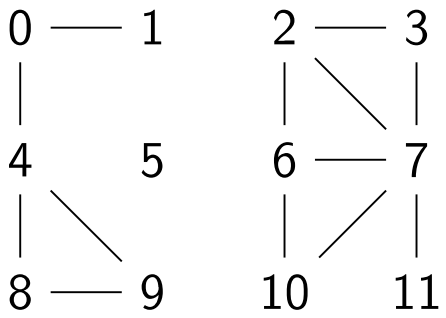

- As before, non-tree edges generate cycles