INDIAN INSTITUTE OF TECHNOLOGY MADRAS

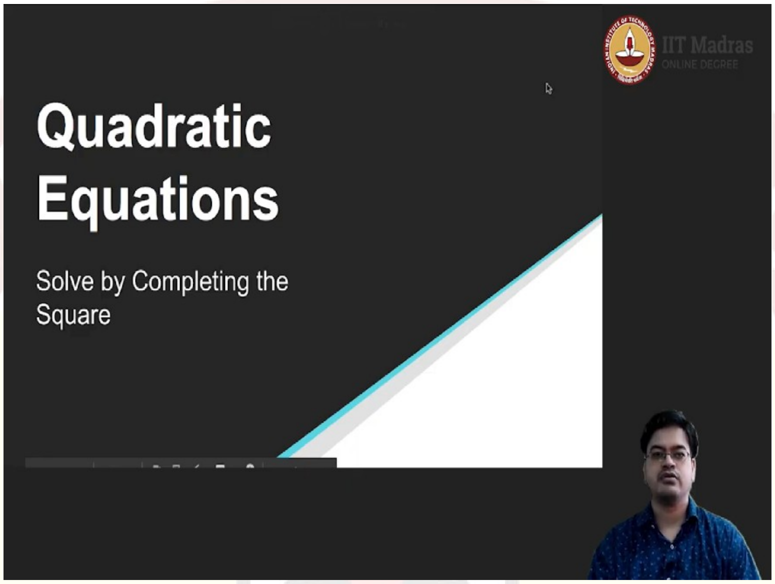
सिद्धिर्भवति कर्मजा

IIT Madras

ONLINE DEGREE

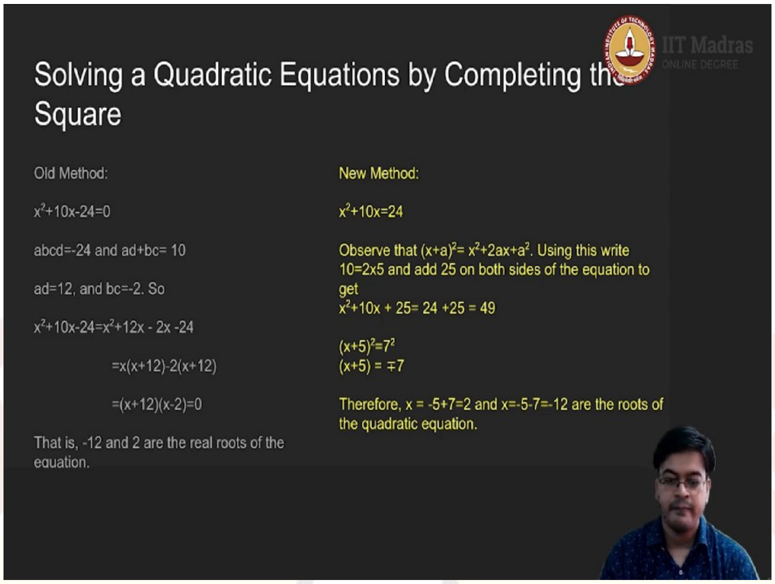
Mathematics for Data Science 1
Professor Neelesh S Upadhye
Department of Mathematics
Solution of quadratic equation using Square method
Indian Institute of Technology, Madras
Lecture 28

(Refer Slide Time: 00:14)



In this video we are going to learn one more interesting method called Solving quadratic equations or for finding the roots of quadratic equation. The method named completing the Square method also it has a very good connection with a very well-known or very popularly known as Quadratic formula. So, we will explore that connection towards the end of this video.

(Refer Slide Time: 00:44)



So, let us start I will demonstrate this method through some examples. So, let us first understand or revise what we done in the earlier stage or in the earlier video. We have used a method of Factoring that I have called as old method. So, let us take this example $x^2+10x-24=0$. If you use the method of factoring you need to identify what is this term 24 and 1. So, -24×1 so - 24 is the product of the leading coefficient and the constant term and $ad+bc=10$.

So, I have this setup which is $abcd=24$ and $ad+bc=10$. So, I will essentially use a prime factorization theorem and get the prime factors of -24 so that if you rearrange the prime factors in such a way that the sum should be equal to 10. One such rearrangement is 12 and -2. So, ad is 12 and bc is -2. So, I got this and then based on our factorization technique I will substitute this 12 and 2 for this coefficient if x and I will get this expression which is $x^2+12x-2x-24$.

Then I will use the greatest common factor technique that is I will take out x common, 2 common from the last 2 terms and I will get this expression. So, finally, I got $(x+12)(x-2)=0$ and therefore, I will get the roots of the equation are -12 and 2. Now, somebody came up and thought, that why should I bother what is this last term? It is just a constant right, so I will replace this constant with something and I will work on it.

So, from that particular though comes the new method, which is the method of completing the square. So, what that person did is just rewrite this expression into this form that is $x^2+10x=24$. Now, the next question that person asked is if I look at this x^2+10x, do I know something that will make this particular expression as a complete square. So, what do I mean by complete square let us understand?

So, complete square means $(x+a)^2$ if I want to write $(x+a)^2$ then what I need to do here is to add some number and subtract some number or to add this same number on both sides right. So, in this case if you look at this expression that is $(x+a)^2$ which is $x^2+2ax+a^2$. So, this a is the number that I am looking for. Now, in this case if I consider this expression and if I want to add a number which will typically be a^2 what that a^2 should be, Is the first question.

So, to answer that let us equate this 10 to this 2a, so $10=2a$ therefore $a=5$ is the answer. So, what will be a^2? a^2 will be 5^2 which is 25. So, now I got a number a^2 to add and subtract from both sides. So, what I will do is I will add the number 25 on both sides so once I add the number 25 on both sides for this expression. I get $x^2+10x+25=24+25$, it turns out here in this case that the number is 49 which is also a perfect square.

But it need not be the case all the time. So, now what I know here is this number this particular expression is nothing but $(x+5)^2$ from this formula. Formula that is given here and then what is other side is 7^2. So, I can rewrite this expression as $(x+5)^2=7^2$ wonderful.

So, I got something in terms of squares, now had it been only one square then in the situation was easy I would have equated to $x+5=\pm 7$. But there will be two situations because both the terms on the left-hand side and on the right-hand side are squares. Now, you just write $x+5=\pm 7$ and $-(x+5)=\pm 7$ that will give us four cases.

But if you look at these two expressions, they will eventually reduce to the same two expressions that is $x+5=\pm 7$. So, it is sufficing to consider only two equations $x+5=\pm 7$. Once I have considered this then I know the solution right, so it is just a matter of substituting the values and doing some little bit of algebra. So, you subtract 5 from both sides so $x = -5 + 7$ which will give me 2, and $x = -5 - 7$ which will give me -12.

These are the roots of the quadratic equation, these exactly match with the roots that we have got -12 and 2, and here 2 and -12. So, the solution set is same therefore now it is a personal choice which method to prefer but what is a choice that is available if you have some difficulty in factoring this. Let us say this is not 24 but some absurd number and you have some difficulty in factoring this finding prime factorization.

What you are doing here is you are not using this particular property when you are doing this example. When you are solving this example through this method you are not using this particular property so you can get rid of this property and you do not have to worry about. One note of caution is you cannot use this method when the number given here becomes negative in this side because square root of a negative number is not defined.

So, after adding this a and the number still remains negative you cannot use this method because according to this method there is no real solution whereas we do not know. So, this method had some limitations but it is quite powerful in solving the problems okay. We will come to how to overcome the limitations in a certain in the next few slides and we will see its beautiful connection with the quadratic formula.