IIT Madras

ONLINE DEGREE

Mathematics for Data Science 1
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Week 9 Tutorial

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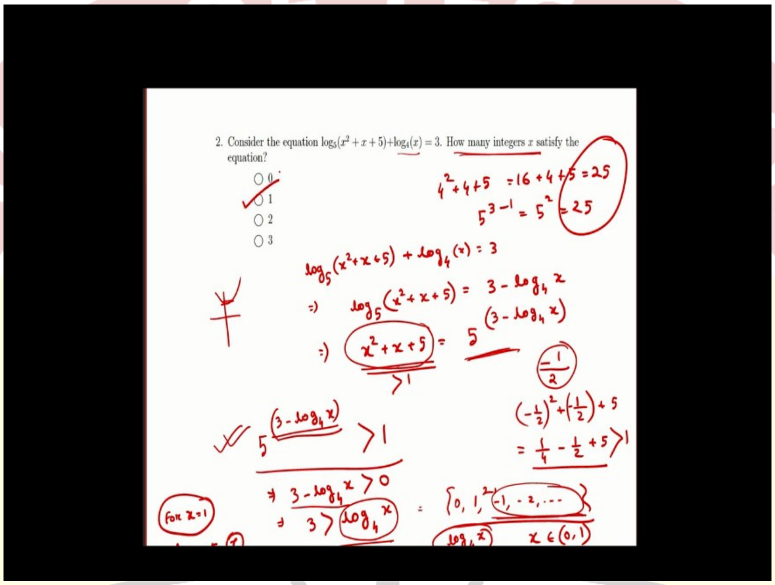

So, now let us see what the second question is. In the second question we have an equation log this is base 5x square + x + 5 and this is $\log_4 x$ = 3 and we have to find how many integers x satisfy this equation. So, let us write this equation, so this will give us we take this log base 4 to the other side and we can write it as this quadratic binomial = $5^3 - log\ 4x$. Now, this is a quadratic polynomial and the coefficient of x square is 1 which is positive, so it will open towards the positive direction of y axis, so it will attend some minimum at the vertex.

So, let us calculate what the minimum value is. So, the x coordinate of the vertex will be - v by a, which will give us - 1 by, - $\frac{-v}{2a}$, so this will give us $\frac{-1}{2}$. So, the x coordinate of the vertex is $\frac{-1}{2}$. Now, if we put that value to get the minimum, so we will get $\frac{-1}{2}^2 + \frac{-1}{2} + 5$ so it will give us $\frac{-1}{4} + \frac{-1}{2} + 5$, so whatever the value it is, it is strictly greater than 1.

So, this part this left hand side is strictly greater than 1, so our right hand side also have to be strictly greater than 1, so let us write the right hand side it is strictly greater than 1, when it will be strictly greater than 1? When this power $(3 - log\ 4x)$>0, when it will be strictly greater than 0?

When $\log_4 x < 3$. Now, observe that this $\log_4 x$ this has to be integer, otherwise the $3 - \log_4 x$ will not be an integer then this will not be an integer and then this part will not be an integer.

But this cannot happen because x is an integer we are considering only the integers, so this left hand side must be an integer, so this right hand side also must be an integer, so this is an integer which means the power should be an integer, so this $\log_4 x$ has to be integer only. So, and this is an integer which is strictly < 3, so our option is 0, 1 and all the negative integer. Now, when this $\log_4 x$ will be negative integer this is only possible if x is in between 0 and 1, so we are taking open one.

So, I have mistaken here, so there will be one more case, so we have to consider 2 also, 2 is less than 3, so in the positive part there are 3 integer 0, 1 and 2 and the whole set of negative integers is also there. But these negative cannot be attained because then x has to be in between 0 and 1 but we have seen that x is an integer, so this negative integer case is also not possible.

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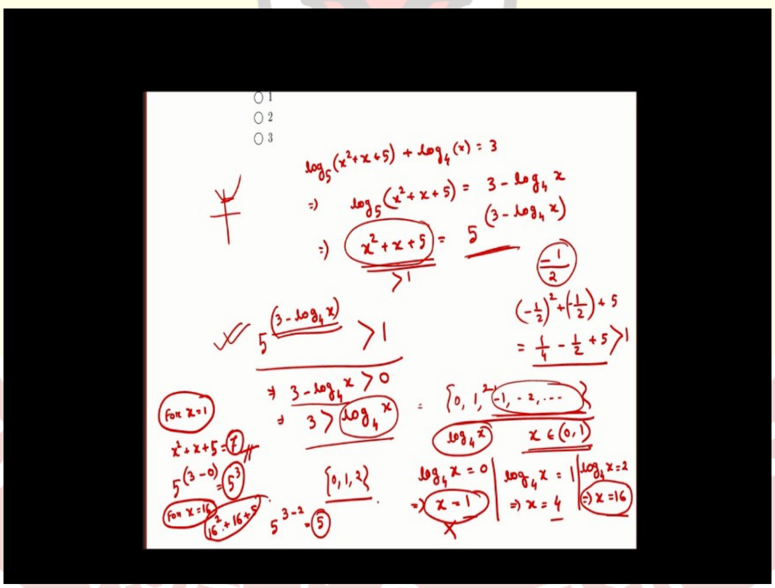

So, our only possible values are 0, 1 and 2, so there are three possible cases. Now, let us observe what this case is telling us. So, when log 4x is 0, it will give us x = 1, so this is the first case. And then when log x base 4 is 1 when x = 4 and when it is 2, it will be 2 when x = 16, so there are three possibilities for x, 1, 4 and 16. Now, let us substitute see what happens here. So, for x = 1, $x^2 + x$ + 5 it will be 7, which is the left hand side, but what the right-hand side is giving? This is 3 - 0, so this is 5^3, so clearly these two are not equal. So, x = 1 cannot be the case, so we will cancel this.

Now, see what happens for $x = 16$. So, at first we are checking for the extreme value, so we are taking $x = 16$, so for $x = 16$ observe that so the left hand side for $x = 16$ the left hand side will be 16 square + 16 + 5, so this will give us some value and the right hand side will give us $5^3 - 2$ which is so if we put 16 here it is 4^2, so it will be us 2, so 3 - 2 is 1, so it is 5. So, the right hand side is 5 and the left-hand side is much greater than 5, so it is not the case.

And the last option is $x = 4$. So, if we put $x = 4$ here, let us see, if we put $x = 4$ here we will get for the left hand side we will get $4^2 + 4 + 5$ which is 16 + 4 + 5 which is 25 and for the right hand side we will get $3 - \log_4 x$, so it is 5^2 again it is 25. So, this satisfies the equation. So, there are only one possibility that is $x = 4$. So, how many integer satisfies this? So, there is only one integer, which is satisfying.