



IIT Madras
ONLINE DEGREE

Representation of a Line

- How to represent a line uniquely?
- Given a point, how to decide whether the point lies on a line?

In other words, for a given line l , we should have a definite expression that describes the line in terms of coordinate plane.

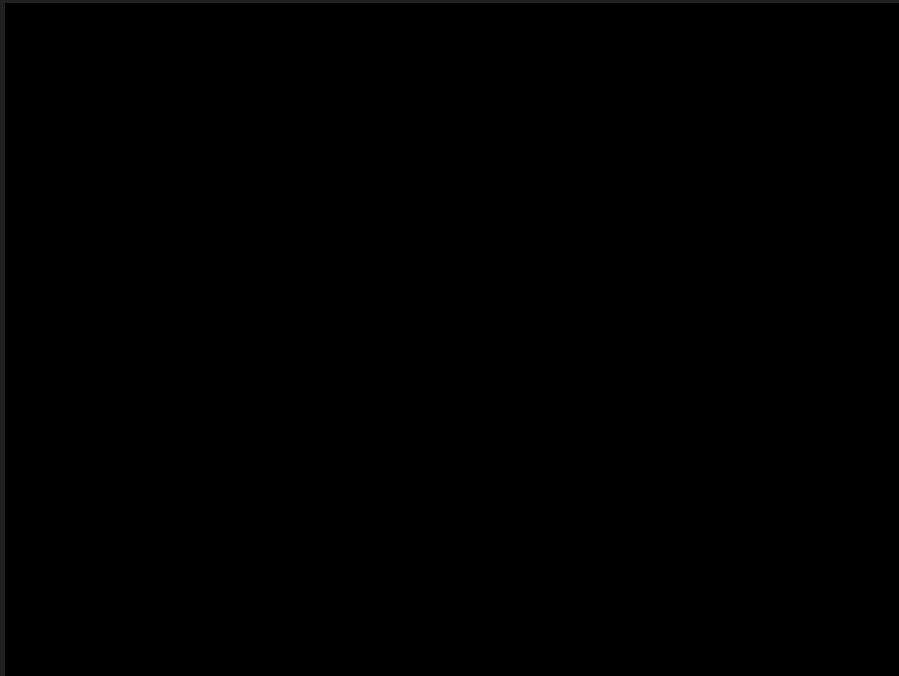
If the coordinates of a given point P , satisfy the expression for the line l , then the point P lies on the line l .

Horizontal and Vertical Lines

Horizontal Lines: A line is a horizontal line only if it is parallel to X-axis

- To locate such a line, we need to specify the value it takes on Y-axis.
- That is, the expression for such a line is of the form $y = a$.
- Then all points that lie on this line are of the form (x, a) .

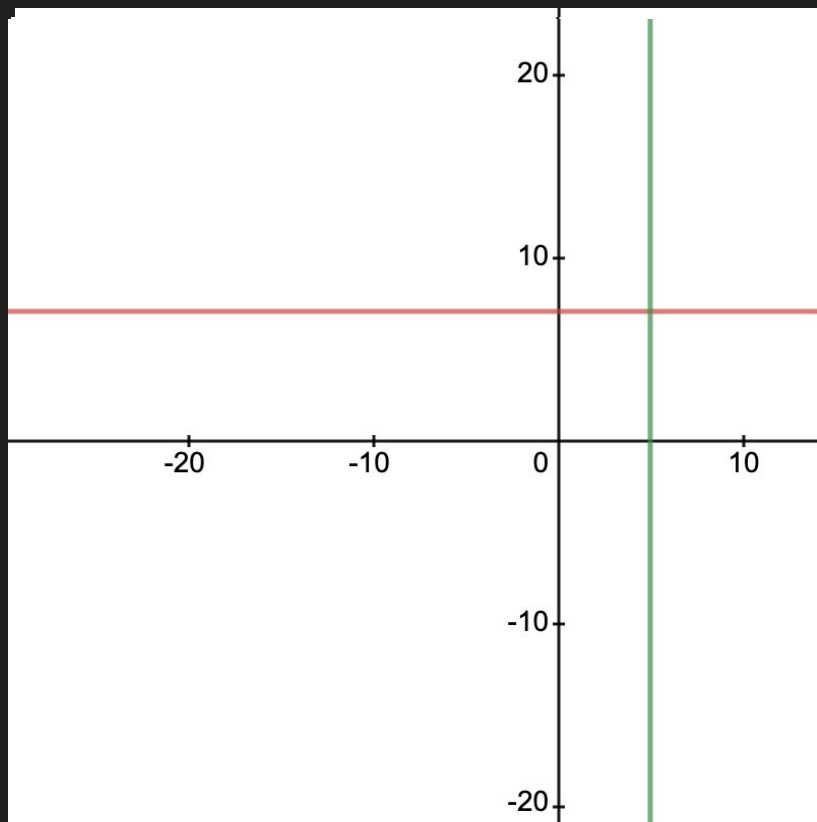
Horizontal and Vertical Lines



Vertical Lines: A line is a vertical line only if it is parallel to Y-axis

- To locate such a line, we need to specify the value it takes on X-axis.
- That is, the expression for such a line is of the form $x = b$.
- Then, all points that lie on this line are of the form (b, y) .

Example



Question: Find the equation of the lines parallel to the axes and passing through $(5,7)$.

The horizontal line is $y = 7$.

The vertical line is $x = 5$.

Equation of a Line: Point-Slope Form

For a non-vertical line l , with slope m and a fixed point $P(x_0, y_0)$ on the line, can we find the equation (algebraic representation) of the line?

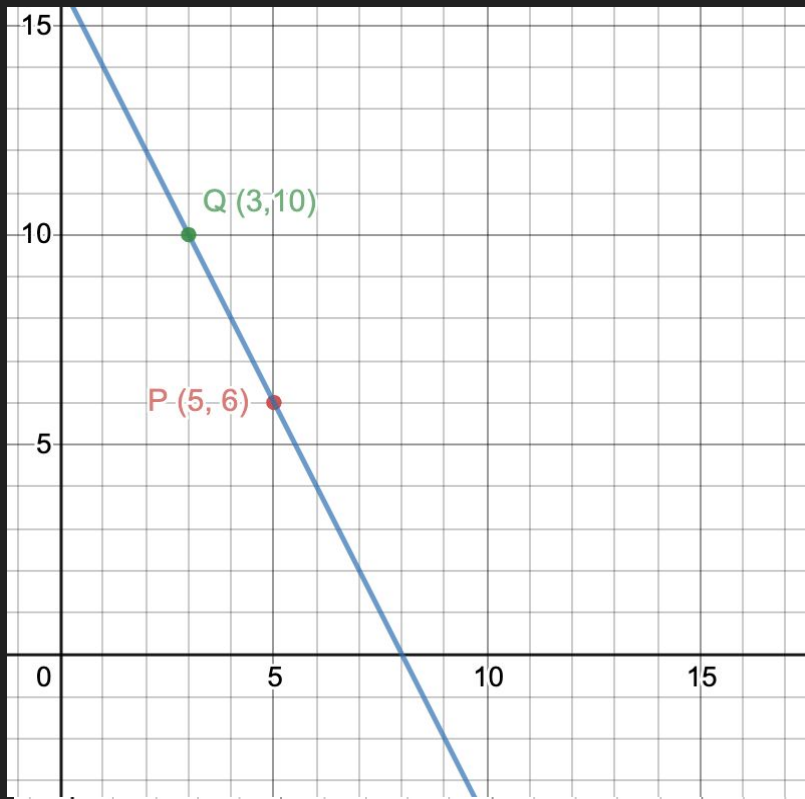
- Let $Q(x, y)$ be an arbitrary point on line l . Then, the slope of the line is given by

$$m = \frac{y - y_0}{x - x_0}$$

$$(y - y_0) = m(x - x_0) \quad (\text{Point-Slope form})$$

Any point $P(x, y)$ is on line l , if and only if the coordinates of P satisfy the above equation.

Example



Q. Find the equation of a line through the point $P(5,6)$ with slope -2 .

Let $Q(x,y)$ be an arbitrary point on this line. Then, using Point-Slope form, we get

$$-2 = \frac{y-6}{x-5}$$

$$(y - 6) = 2(5 - x) \text{ or } y = 16 - 2x.$$

Equation of a Line: Two-Point Form

Let the line l pass through the points $P(x_1, y_1)$ and $Q(x_2, y_2)$.

Assume that $R(x, y)$ is an arbitrary point on the line l .

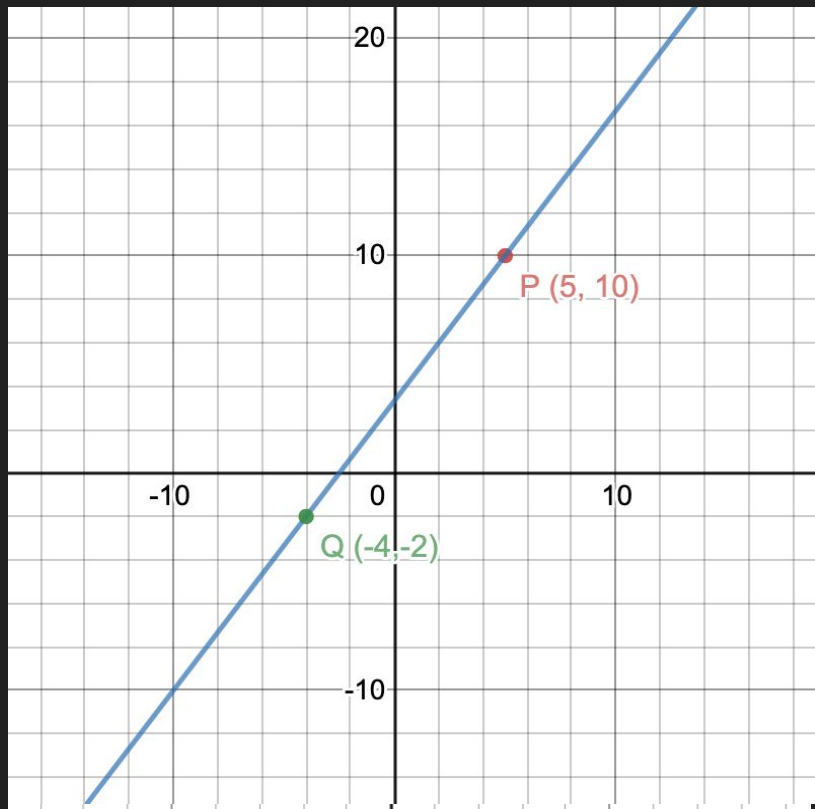
Then, the points P , Q , and R are collinear.

Hence, Slope of PR = Slope of PQ . Therefore,
$$\frac{y - y_1}{x - x_1} = \frac{y_2 - y_1}{x_2 - x_1}$$

$$(y - y_1) = \frac{y_2 - y_1}{x_2 - x_1} (x - x_1). \quad \text{(Two-Point form)}$$

Any point $R(x, y)$ is on line l , if and only if, the coordinates of R satisfy the above equation.

Example



Q. Find the equation of a line passing through (5,10) and (-4, -2).

Let (x,y) be an arbitrary point on this line. Then by two-point form, we get

$$(y - 10) = \frac{-2-10}{-4-5}(x - 5)$$

$$3y = 4x + 10.$$