INDIAN INSTITUTE OF TECHNOLOGY MADRAS

सिद्धिर्भवति कर्मजा

IIT Madras

ONLINE DEGREE

Mathematics for Data Science 1
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Lecture – 37
Zeroes of Polynomial Functions

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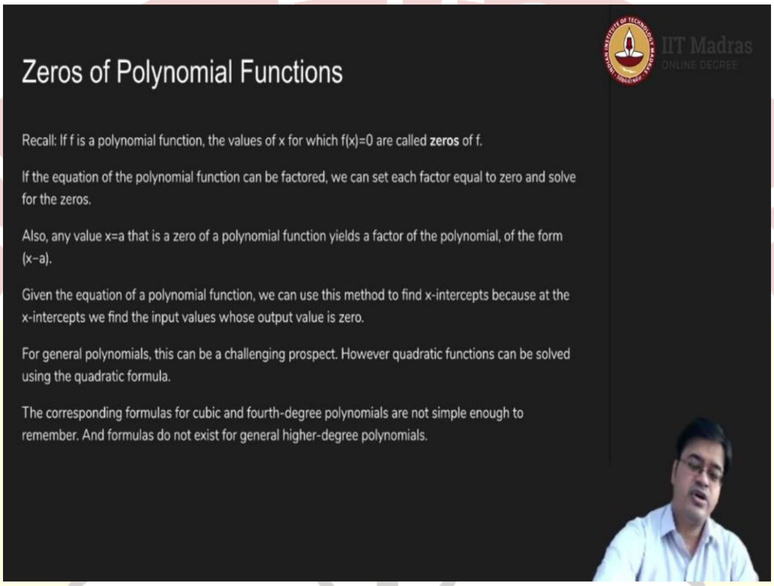

So, let us focus on Zeros of Polynomial Functions. So, for clarity, let us recall what is zero of a polynomial function. If f is a polynomial function, then the values of x for which $f(x) = 0$ is called zero of f. A value x of for which $f(x) = 0$ is called zero of f.

Now, when we studied quadratic functions, we had several methods of identifying the zeros of the quadratic functions. For example, we actually tried to graph the quadratic function because we knew some techniques, we actually plotted set of ordered pairs on a graph paper and join the curve smoothly, then we identified it is crucial to identify axis of symmetry and around axis of symmetry you can plot and wherever it intersects x axis, we will call that as a zero of a function. This is how we identified quadratic zeros of quadratic functions.

Another way that we used which will be helpful here is; factoring the quadratic function into factors given a quadratic function identify the factors and write the polynomial into intercept form. If you are able to do that, then you have again identified zeros of the polynomial because when you said that quadratic function to be equal to zero and if it is in a factored form, all the coefficients corresponding to that factor will be all the numbers corresponding to that factor will be zeros of the polynomial function.

So, now, we will focus on the factoring component of polynomial functions. So, if the equation of the polynomial function can be factored, then we can set that each factor to be equal to 0 and solve for zeros; this is an important step. But it as we have seen in quadratic functions, this is not always possible.

In such case, if you put some random values, if you throw in some random values in the function and you get something like $x = a$, you will get the value to be 0 that is also helpful.

Then, you can guess that is a factor and you can use the previous video to divide the polynomial by $(x - a)$ which will give you the remainder term and that remainder, you can actually figure out whether you can; you can consider factoring for that remainder or not all these things are possible or the other factor it is not remainder sorry it is the other factor. So, these are some possible ways.

Up to quadratic equations, we had some easy ways out easy way out like given the equation of a quadratic function, we can use this method to find x intercept because x for x intercepts, we get zeros that is what I explained earlier also. So, you can find x intercepts and you will easily get this. You can use the similar technique of finding x intercepts for a general polynomial function also, but it is very difficult to plot a polynomial function ok.

Given a graph of a polynomial function and you have identified based on our previous criteria, you have identified that this is a polynomial function, you can guess what are the zeros of the polynomial function that way this statement helps. But, if you go for higher order polynomials that is general polynomials, this can become messy, it can be really challenging.

Quadratic equations can be easily solved using quadratic formula we have a solution for quadratic equations. But, the cubic and four-degree polynomials have some formulae which you may study in your tutorials, but they are not easy enough to remember. And, for higher degree polynomials, you do not have any idea of how to approach finding zeros of the polynomial functions, you have to go by trial and error method and whatever knowledge you have about square, quadratic, linear and cubic polynomials.

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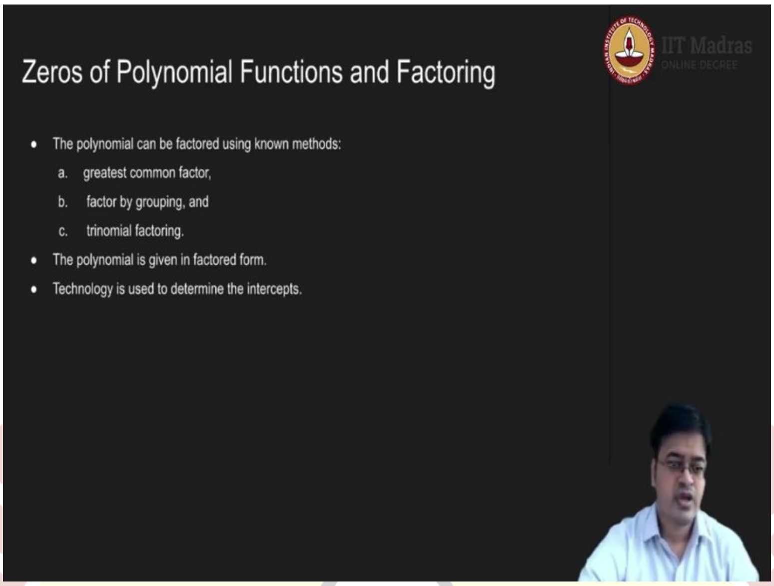

So, let us summarize what we have; what we have discussed just now. If I want to identify zeros of polynomial functions, the factoring technique is a crucial technique. So, what you can do is you can look at the polynomial and if you look at the polynomial, there is one easy way out that if you can identify the greatest common factor that is the greatest monomial that can be taken out common you can use that technique.

Once, if there is no such technique, if once that is available, the polynomial is more or less manageable, then you can use the technique of factor by grouping. So, you can create groups in that and see whether a anything is coming out common that is another technique. Another thing is you can instead of handling groups, you can decide to handle three terms at a time so, that is a trinomial factoring. This will be helpful when you have very high degree polynomial. So, these are the common methods for factoring the polynomials.

Once you can factor the polynomials each of them can be equated to 0 by writing a polynomial in a factored form. And then finally, if you are not very sure, then you can use some graphical tools which are available these days on computer or on the net one such tool is; Desmos which we are using in our presentations.

So, you can use those tools to determine the intercepts. In these tools basically, you will give of equation of a function and it will be graphed they will give the they will project the graph of a function right. So, this is our zeros of the polynomials and factoring play a crucial role.

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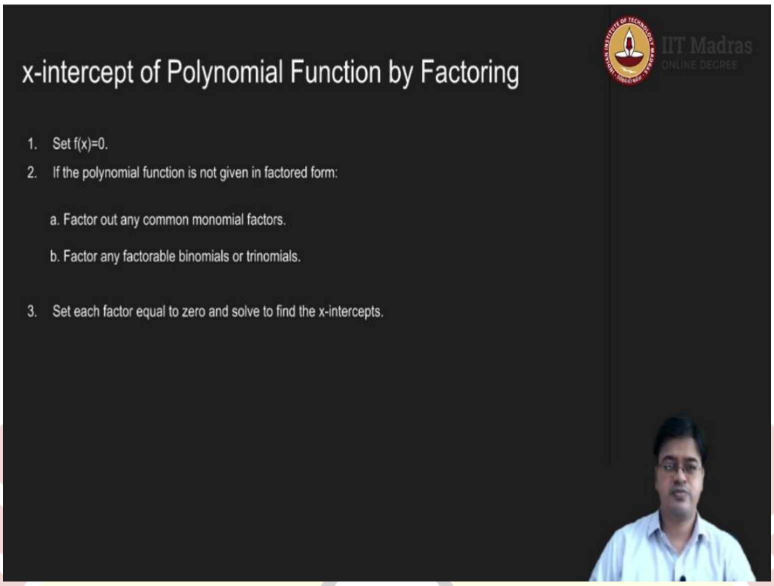

To understand this, let us see how to find x-intercept of a polynomial function by factoring. So, what we have discussed just now is we have set the equation that is $f(x) = 0$ in order to facilitate factoring $f(x) = 0$, then if the polynomial is given in factor form; factored form then equate each of them to be equal to 0 which we have seen for quadratic case also.

If it is not given in factor factored form, first in that you will look for is you take out some common monomial that is available in all the terms if that is that is there and you have taken out or if that is not there still you can go to the second step that is whatever at the rest of the terms you can factor them into factorable binomials or trinomials, you look for try to look for combinations which we have done successfully for quadratic equations well doing the factoring. So, you can do a similar thing over here.

And then finally, set each factor equal to zero that will give you the x-intercept. This is the; this is the strategy that we will follow for finding x-intercept of polynomial function by the method of factoring.

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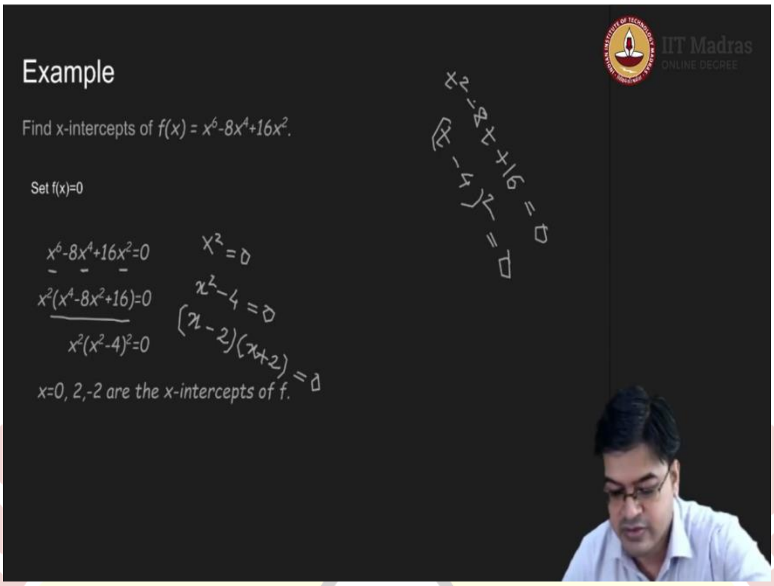

So, let us look at this example where we will follow the steps of the algorithm. So, the a question says; find x-intercepts of a function $x^6 - 8x^4 + 16x^2$. So, as per our algorithm or as per the steps given in the previous slide, I will set $f(x) = 0$ that is; $x^6 - 8x^4 + 16x^2 = 0$.

Now, you look at greatest common factor, a monomial that is common in all these terms that is x^2. So, what I will do is I will separate out this x^2, I have taken out this x^2 and now, you look at the other factor that is $x^4 - 8x^2 + 16$.

Now, this factor can be related to our quadratic equation of the form $t^2 - 8t + 16$. Can I factor this quadratic equation because there is no term corresponding to x^1 and there is no there are no odd terms essentially. So, I can use this and I can leverage the skill of quadratic equations to solve this equation and from quadratic equation point of view, I know this is $(t - 4)^2 = 0$. So, instead of t here, it is x^2. So, that will give me $x^2 \times (x^2 - 4)^2 = 0$.

Now everything is looks in the form of x^2. So, what are the values of x? What are the feasible values of x? Those will be the x-intercepts. So, you can put x^2 is so, this will give me $x^2 = 0$ or $x^2 - 4 = 0$. So, $x^2 - 4$ can further be factored into $(x - 2)(x + 2) = 0$. And with this understanding, I can write $x = 0, 2, -2$ are the intercepts of f x-intercepts of f ok.

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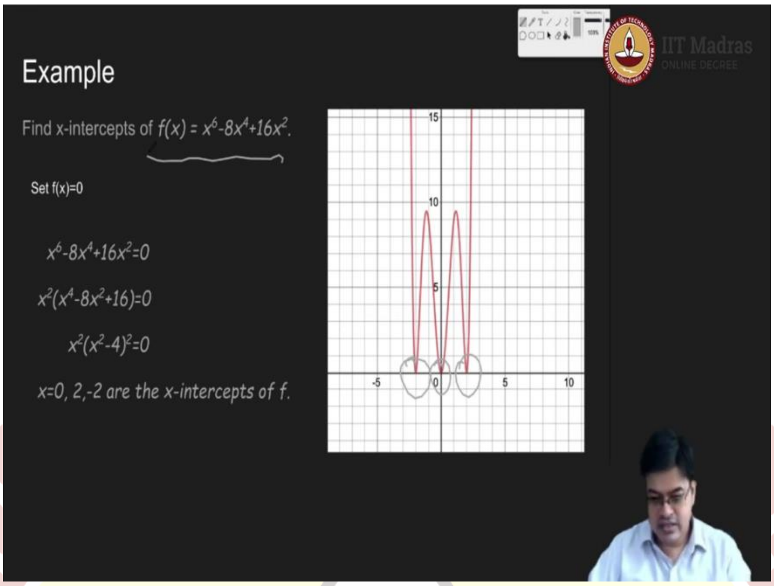

Now, as per the last step in the algorithm, you want to verify this result. How will you verify this result? Using the technology; so using Desmos, I have drawn this graph and you can verify that $x = -2$ which is here, $x = 0$ which is here and $x = 2$ which is here are all x-intercepts of a polynomial function given by these $f(x)$ ok. So, this is how we will identify x-intercepts.

Let us understand this strategy by looking at one more example.

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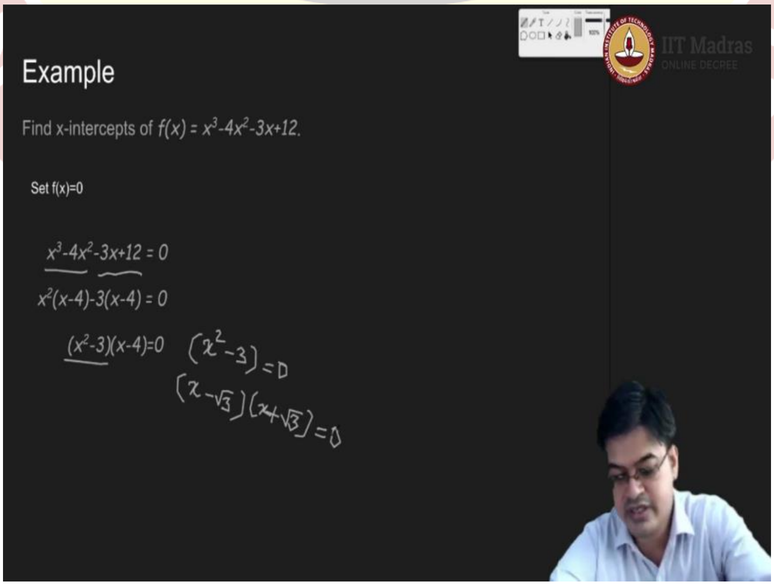

So, now here, we have been asked to find x-intercept of a polynomial function which is a cubic polynomial function $x^3 - 4x^2 - 3x + 12$ fine. So, as per our set up, this first step is set $f(x) = 0$. So, you have set $f(x) = 0$ that essentially gives me $x^3 - 4x^2 - 3x + 12 = 0$.

Then, the second a step if you have any common monomial, there is no common monomial because the last term is a constant term so, you cannot figure out a common monomial. Then is there any pattern? Can you look at two-two terms each binomials or trinomials because there are four terms, it is better to look at binomial terms.

So, if you look at the first two terms, you can see that you can throw out x^2 as a common thing, if you throw out x^2 as a common thing, then you will be stayed with $(x - 4)$ as a term as a one factor. And if you look at these two terms, then again if you take out 3 common - 3 common, then you will get $(x - 4)$. So, using the technique of binomial, binomials in this case, I am able to see this kind of factoring possible.

Good, that essentially means I can rewrite this expression as $(x^2 - 3)(x - 4) = 0$. Then, I want to solve this $(x^2 - 3)$ that is all is remaining which is a quadratic equation. So, you can easily solve using quadratic formula or a factoring, but here in this case, I know the factors so, that will be $(x - \sqrt{3})$ and $\left(x + \sqrt{3}\right) = 0$.

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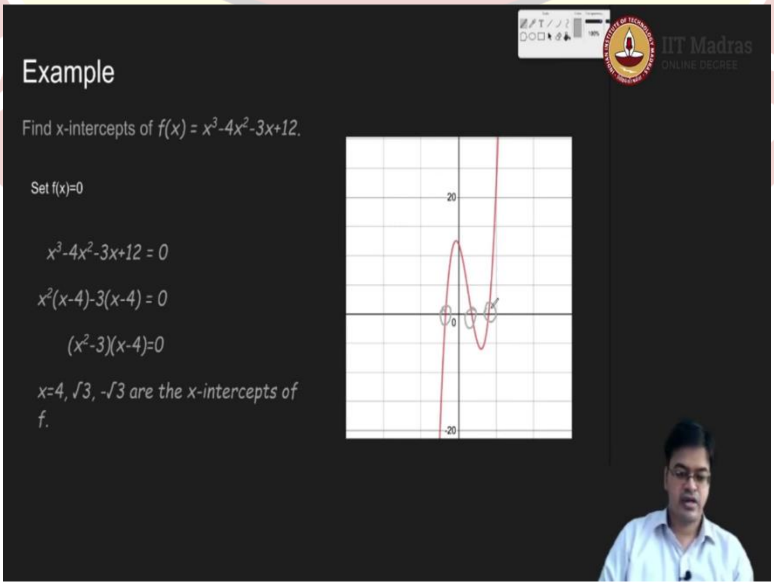

And therefore; therefore, the solution of this quadratic equation is well known that is $x = 4, +\sqrt{3}, -\sqrt{3}$ are the x-intercepts of the function. The final step is I want to verify using some technology or a graphing tool. This is the graph of a function.

So, in this case, you can easily verify there are three roots: first root this one which is a occurs, it occurs at , $-\sqrt{3}$, this one this is , $\sqrt{3}$, this one is 4. So, these are the four these are the three roots of a cubic polynomial. Roots or x-intercepts or a zeros of a cubic polynomial ok.

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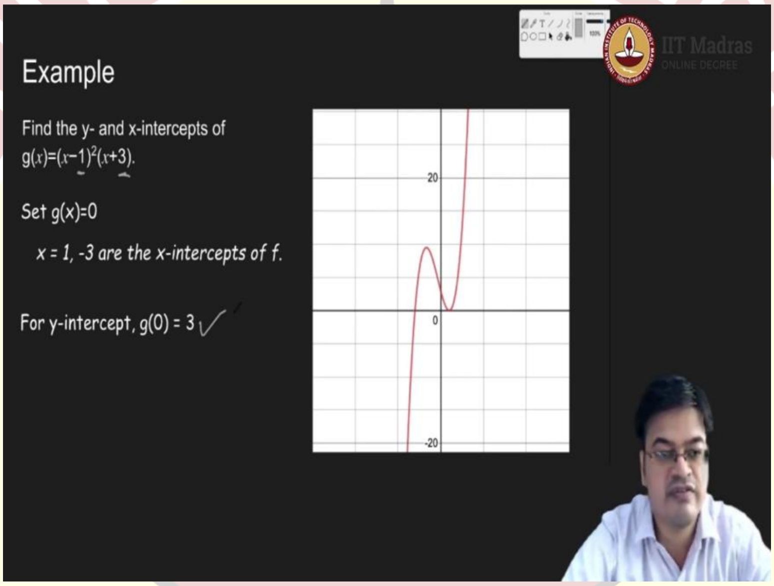

So, let us go ahead and see; let me remove this blocks. Another example where I want, I am interested in finding x-intercepts as well as y-intercepts of a polynomial function which is given in a factored form.

So, the polynomial is given in factored form. So, visually you will be able to guess the roots. So, as a standard set up, we will set $g(x) = 0$. Once you said $g(x) = 0$, it is very clear that $x = 1$ and $x = -3$ are the x-intercepts of f.

What about y-intercept? What is the y-intercept at all? So, y-intercept is where x is given to be 0. So, simply substitute $x = 0$ in the expression of $g(x)$, you will get $g(0)$, which is $0 \times (-1)$; the whole square that is 1+0+3 that is 3 so, 1×3 is 3 so, your $g(0) = 3$.

So, this is how you will figure out x-intercepts and y-intercepts of the function and this is the graph of that function. Using technology, I have verified.