IIT Madras

ONLINE DEGREE

Mathematics for Data Sciences 1
Professor. Madhavan Mukund
Department of Computer Science
Chennai Mathematical Institute
Professor. Neelesh S Upadhye
Indian Institute of Technology, Madras
Week 10 - Tutorial 01

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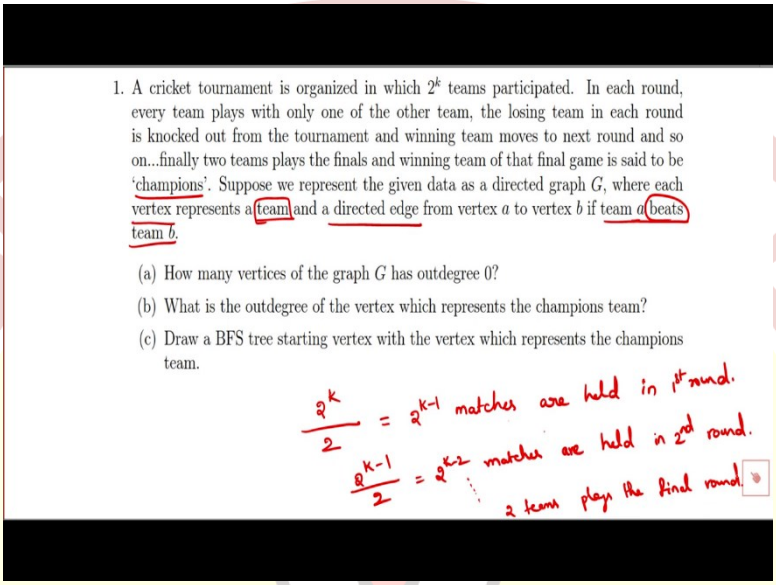

Hello, everyone. Today, let us solve few tutorial questions based on graph theory. A cricket tournament is organized in which 2^k teams participated. In each round, every team plays with only one of the other teams, the losing team in each round is knocked out from the tournament, and winning team moves to the next round, and so on. Finally, two teams plays the finals, and winning team of that final game is said to be champions. Suppose, we represent a given data as a directed graph G, where each vertex represents a team and a directed edge from vertex a to vertex b, if team a beats team b.

So, a knockout cricket tournament is organized and in each round, one team will play with one of the other team, like pair. If the team loses, it is out of the tournament. And if the team wins, it moves to the next round. And in the next round, in the same way, one team will play with the other team and so on. That means, in the initial stage, there are 2^k teams. In the first round, there will be $2^{(k-1)}$ matches are held, because there are 2^k teams. And each match is paired between two teams, $\frac{2^k}{2} = 2^{k-1}$matches are held in first round.

In the same way, in the second round, we have only 2^{k-1} teams, and each match is played between two teams. So, in the second round, 2^{k-2} matches are held in the second round. So, it

progresses and ultimately in the final round, there will be only two teams plays the finals, plays the final round. Now, how many vertices of the graph G has outdegree 0?

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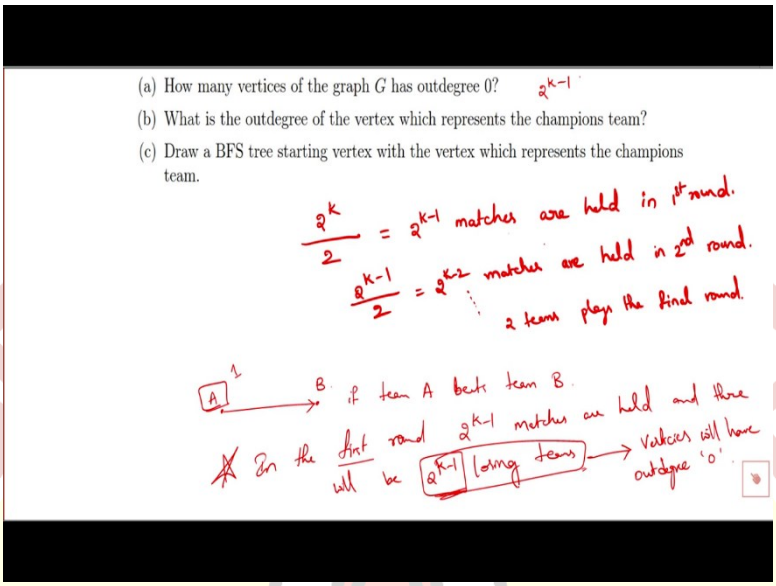

So, if you see the graph G, there are two vertices and it is represented by two teams A and B, there will be an outward edge going from A to B. If team A beats team B. So, the outdegree of this vertex A becomes 1. We are supposed to find the vertices, which is outdegree 0. That means the team which represents those vertices has not won any one of the matches in the tournament. That means, it says that the teams that launched in the first round itself has not won any one of the matches in the tournament.

And the vertices which represent this first round lost teams are the vertices having outdegree 0. So, in the first round, 2^{k-1} match are held and there will be 2^{k-1}losing teams. The vertices which represent these teams will have outdegree 0, vertices which represent these 2^{k-1}losing teams in the first round will have outdegree 0. So, there will be 2^{k-1} vertices on the graph G that has outdegree 0. Coming to the part B, what is the outdegree of the vertex which represents the champions team?

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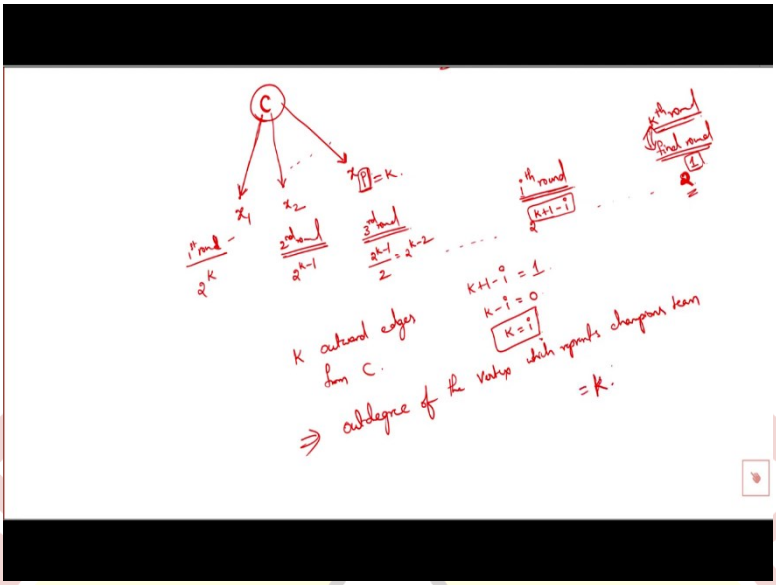

So, if we observe the champions team. Champions team has won all the matches that it played in the tournament. So, in the first round, let C be the champions team vertex. In the first round, it might have won with some other team x_1. And from C to x_1, there will be an outward edge. And in the second round, it might have won with some team x_2, and there will be an outward edge from C to x_2. In the same way, it goes in the final round, it might have played with some other xp, there will be an outward edge from C to x_p.

So, we have to find this P or it is same as saying, total number of rounds played in this tournament. So, we will go through the number of teams left, in order to find the number of rounds happen. In the first round, 2^kteams played. In the second round, as 2^{k-1} teams eliminated, we were left with 2^{k-1}teams, and these 2^{k-1} teams played in the second round.

After the second round, in the third round, we will be left with $\frac{2^{k-1}}{2} = 2^{k-2}$. And in the same way, if we observe in the i'th round, 2^{k+1-i}. So, if we progress in the final round, there are only two teams left. So that means, the power of 2 is 1 in the final round. And if we see here, my cape is 1 minus i becomes 1, then that is the final round.

So, if I equated $k + 1 - i$ should become 1. And my $k - i = 0$, and $k = i$. So, if my i becomes k, that means if it is the k'th round, we will be left with two teams. And these two teams plays the final round. That means, k'th round is nothing but the final round, we got our P, which is nothing but equal to k. That means there are k rounds happened in this tournament and in each round, this champions team has won against one of the other team, and the vertex which

represents this champions team has an outgoing edge from this champions team to the team, which lost with this champions team.

So, in each round, we have one outward edge. So, in k rounds, we will be having k outward edges from champions team. So, this implies, outdegree of the vertex which represents champions team is equal to k.

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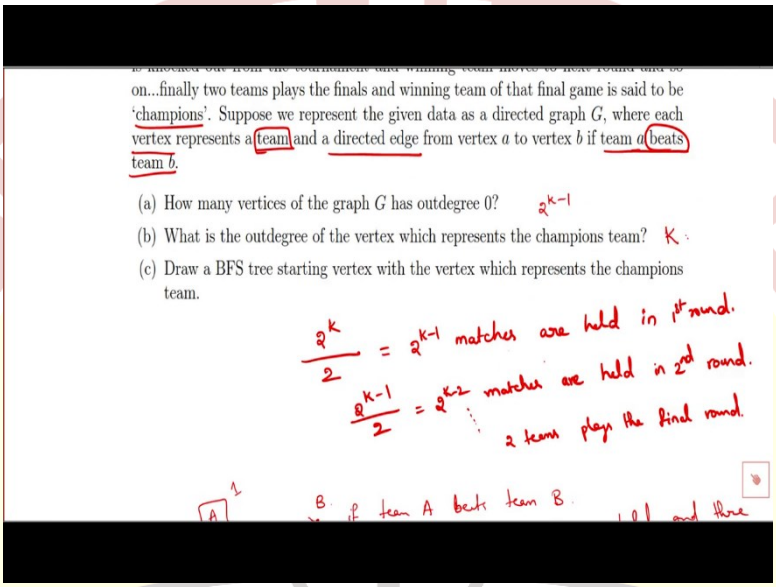

So, answer to our part B is k. And part C, draw a BFS tree starting vertex with the vertex which represents the champions team. So, we have to draw a BFS tree starting with the vertex, which represents the champions team.

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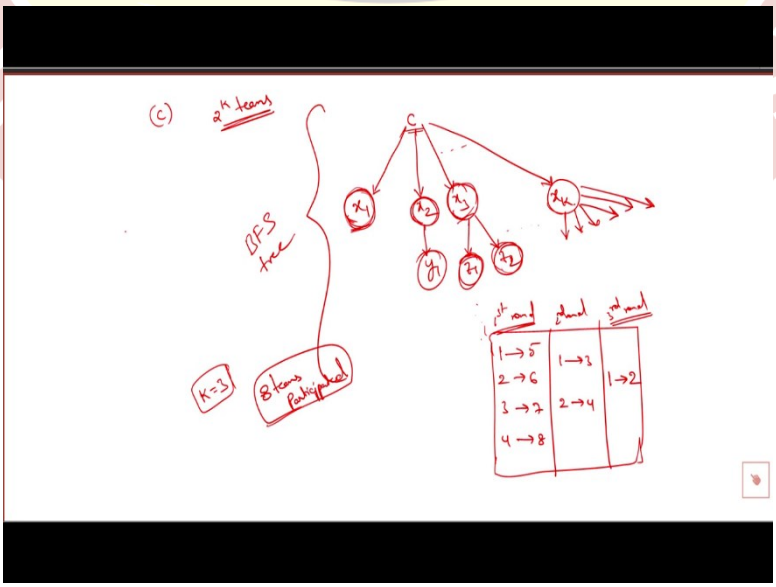

So, now, there are 2^k teams, we know that. And we start with the champions team vertex, this champions team vertex has won against some x_1 in the first round, some x_2 in the second round, some x_3 in third round, and in the same way some x_k in the final round. And this x_1 if you observe, as this is played against champions team in the first round, the x_1 is eliminated in the first round itself. So, there will not be any other vertex, that are adjacent to x_1.

So, if you come to x_2, this played against the champions team in the second round. That means, this x_2 might have played against some other teams y_1 in the first round, and this x_2 won against y_1. And when we come to x_3, x_3 played against this champions team in the third round, that means this x_3 has won in the first round and the second round. So, there will be 2 vertices that are adjacent to x_3, let us say z1 and z_2, z_1 played against x_3 in the first round and z_2 played against x_3 in the second round.

And in the both matches x_3 won. But unfortunately, it lost to the champions team in the third round and it got eliminated. This is how we proceed further, if you see this x_k, this x_k has won all the matches except the final round. So, there will be k minus 1 vertices that are adjacent to this x_k. This is how a BFS tree looks like. So, let us take an example to understand it more clearly.

If my k is 3 that means, if B teams participated, let us try to draw a BFS tree with this B teams. And this is how the table shows the picture. In the first round, 1 won against 5. 2 won against 6, 3 won against 7 and 4 won against 8. And in the second round, 1 won against 3, 2 won against 4. And third round and it is the final round, 1 won against 2. So, if we have this data, we try to draw the BFS tree.

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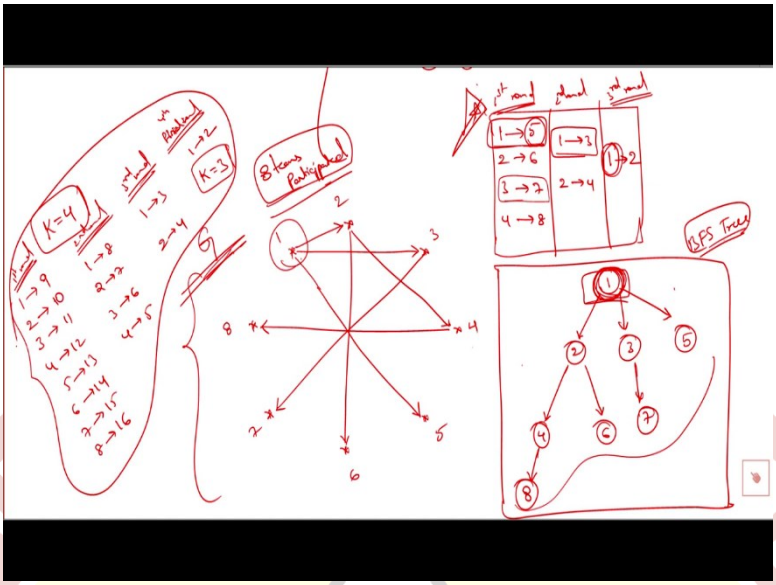

So, if you observe this table. From this table, we can clearly see that, team 1 is the champions team. Let us try to draw the graph for this given table. So, we have 8 vertices with us, namely 1,2,3,4,5,6,7,8, these are the vertices which represents these teams. So, 1 won against 5, so there will be an outward edge going from 1 to 5 and 1 won against 3, in the second round. So, 1 to 3 and 1 to 2, 2 to 6. So, if we draw all the edges, this is how the graph G looks like. So, from this graph, let us try to draw the BFS tree, taking 1 as the starting vertex.

So, from 1, we have 3 directly outgoing edges to 2, 3, and 6. From 2, we have two directed edges, outgoing edges to 4, and to 6. And from 4, we have only one directed edge to 8. And, if we come back, from 3 there is a directed edge to 7 and this is the BFS tree. If the pictures are like this, and 8 teams participated in that tournament, so our graph will be like this. So, if you observe this, from 1, which is the champions' team, we can reach to every other vertex in the graph. And this is the only vertex in this graph G that is reachable to every other vertex.

So, try this, and also try this by taking k is equal to 4 and having pictures like this 1 beats 9, 2 beats 10, 3 beats 11, 4 beats 12, 5 beats 13, 6 beats 14, 7 beats 15, 8 beats 16, in the first round. And in the second round, 1 beats 8, 2 beats 7, 3 beats 6, 4 beats 5. And in the third round, 1 beats 3, 2 beats 4. And in the final round, final round, or fourth round, 1 beats 2. So, try to draw a graph that represents these pictures and draw a BFS tree. So, work it out. Thank you.