IIT Madras

ONLINE DEGREE

Mathematics for Data Science 1
Prof. Neelesh S Upadhye
Department of Mathematics
Indian Institute of Technology, Madras

Lecture – 34
Algebra of polynomials: Division

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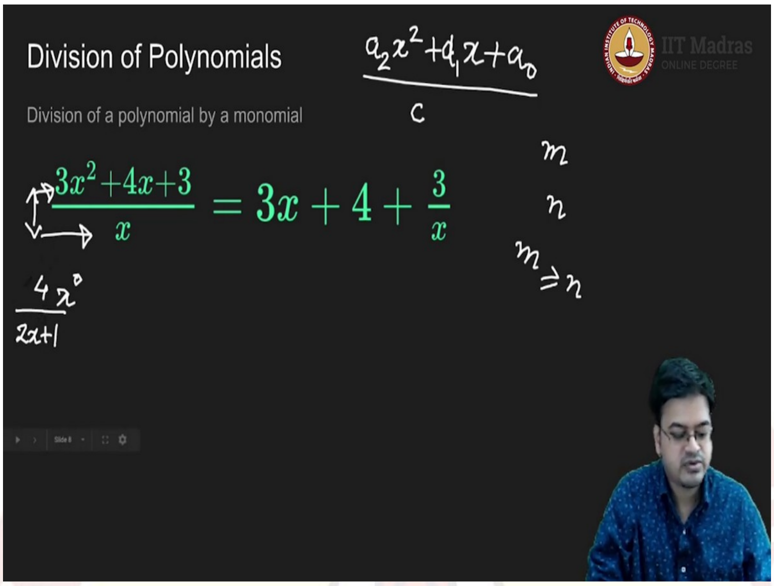

In this video, let us have look at Division of polynomials. What is a division of polynomial? We have already familiar with division of polynomials, but we have not done in a rigorous manner and we do not know all possible cases that can occur while considering division of polynomials, that is why it is important to look at division of polynomials.

We know some cases like for example, if I have been given a polynomial say $a_2x^2+a_1x+a_0$ and if I am told that if this polynomial is divided by a constant say c. Then, I know what is the resultant polynomial. It will be $\frac{a_2x^2}{c}+\frac{a_1x}{c}+\frac{a_0}{c}$ that will be the polynomial. So, this case, we are already familiar with.

Now, let us go to one more level of extension. Suppose, this polynomial is divided by a monomial; that means, we are considering a division of a polynomial by a monomial.

Monomial means, the polynomial that contains only one term, only one variable term. So, in that case, let us take this example. So, $\frac{3x^2+4x+3}{x}$, ok.

So, notice few factors here. In this case, when I am considering a division of two polynomials, the numerator and the denominator; the denominator should always have a degree smaller than the degree of the numerator. If it is not the case, let us say the numerator has degree m and the denominator has degree n, then what I am saying is the degree of the numerator m should always be greater than or equal to n. If it is not the case, then the division is not possible ok.

For example, let us consider one case, where I am considering a constant polynomial let us say 4 and I am dividing it by some polynomial which is $2x+1$. Here, I cannot divide this; I cannot divide by this polynomial because there is no corresponding x term. Here it is x^0.

So, I cannot divide this polynomial because the degree of the polynomial plays a crucial role. So, in this case, the division is not possible, I have to keep this function as it is. Let us keep this point in our mind and consider division of polynomials. So, now, I am dividing a polynomial with a monomial, how will you handle this?

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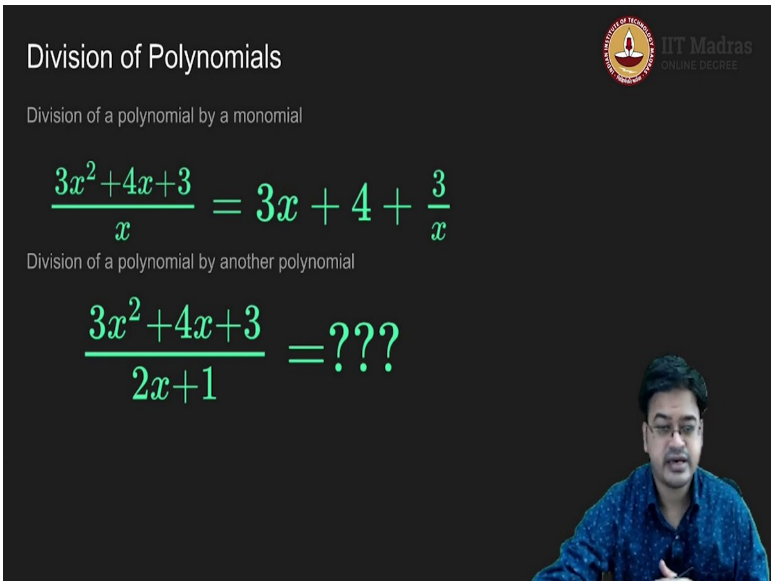

So, a monomial is simply x here. So, what I will do is I will split this with this addition sign, I will split each of them in separate terms. So, I will consider the term $\frac{3x^2}{x}$ that will give me a term, when I consider this term it will give me a term $3x$.

When I consider $\frac{4x}{x}$, I will get a term 4.

Now, as I mentioned earlier when I consider the term $\frac{3}{x}$, the degree of 3 is a constant because 3 is a constant polynomial the degree is 0. So, I cannot divide this polynomial. So, this will automatically influence this decision that it will remain as it is, that is $\frac{3}{x}$. So, these are some key things while dividing polynomial by a monomial.

Now, the key idea is I want to divide a polynomial with another polynomial. Let me erase this first. So, now, I want to divide a polynomial with another polynomial. So, how will I go about this?

That is I want to find something of this sort. Let us address this question in a video ok. So, apparently, I do not have any practical way to divide this right now; but from whatever theory I learnt about quadratic functions, can I derive something? That is what the question is. So, we will try to figure out some more methods in this video.

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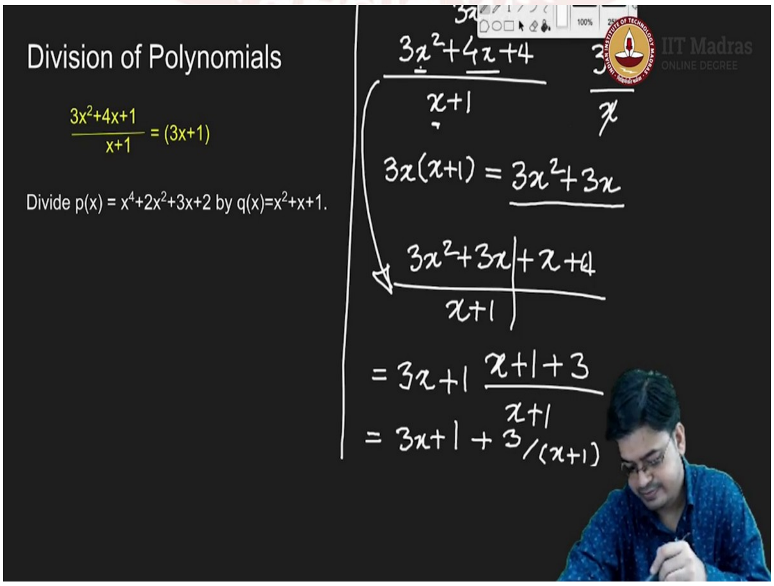

So, let us continue and take a question, this is the numerator that is given to me divided by another polynomial which is $x+1$ and I want to figure out what this will be equal to? Let me take this polynomial over here and try to figure out what this polynomial will be equal to. So, now, I have $3x^2+4x+1$ and it is divided by $x+1$. Now, if I want to divide the numerator by the denominator, what I should see is ok, the denominator has the highest degree which is x. The numerator has highest degree which is x^2 and now, how will I be able to get rid of the denominator for some at least for some terms?

So, in that quest, what I will see is I will simply take the first term over here and the first term over here and I will see like monomial, I will see what is $\frac{3x^2}{x}$. This I can do very easily because both are monomials. So, x vanishes with this square and I will I am left with $3x$. So, next thing that I will do is I will consider $3x(x+1)$. So, this actually gives me the answer $3x^2+3x$. Now, I will try to figure out this term in the expression that is given in the numerator.

So, if I want to figure out the expression that is given in the numerator, I can easily split this $4x$ as $3x+x$. If I can do so, that means, I can take this term and based on this logic, I can actually write this as $\frac{3x^2+3x+x+1}{x+1}$.

Now, I can intelligently split the term over here and I can divide this and I can take this as a separate term and divide this. So, now, you can readily see the answer will be here, $3x(x+1)$ that will get cancel off with $x+1$ and over here, it will be 1. So, the answer is $3x+1$. Therefore, such a division is possible, ok.

Let us verify whether the answer is $3x+1$; yes. So, I have demonstrated you how to divide a polynomial using simple method by the method of factorization that we have already used. Now, this is because $x+1$ was the factor of $3x^2+4x+1$.

What if $x+1$ is not a factor of $3x^2+4x+1$, what would have happened? Let us use this example to understand our findings. So, let me take a eraser and let me write that this instead of 1, let me put ok, let all other things remain constant, what makes it a factor; that $x+1$.

So, I will simply what I will simply do is I will simply change the term to 4 ok. Now, $x+1$ is no longer a factor, still I will continue with the same method, I will take this $\frac{3x}{x}$ 3x by x. So, I can consider this x^2+3x , only difference is this will be $x+4$. In that case, what happens is $3x^2+3x$. So, that will give me $3x$ into let me rewrite this as $\frac{1(x+1+3)}{x+1}$. So, that again gives me an edge that is this is nothing but $3x+\frac{1(x+1)+3}{x+1}$ getting cancelled.

So, this will remain as $\frac{3}{x+1}$. So, this is how even if it is not a factor, I can divide the polynomial. Now, as we have started by giving some for addition, multiplication, subtraction, we have given some algorithms. So, now, we need to identify such algorithm for division of polynomials. To do that, let us first solve this complicated problem in this simple manner and try to derive an algorithm and try to derive an algorithm for by solving this problem.

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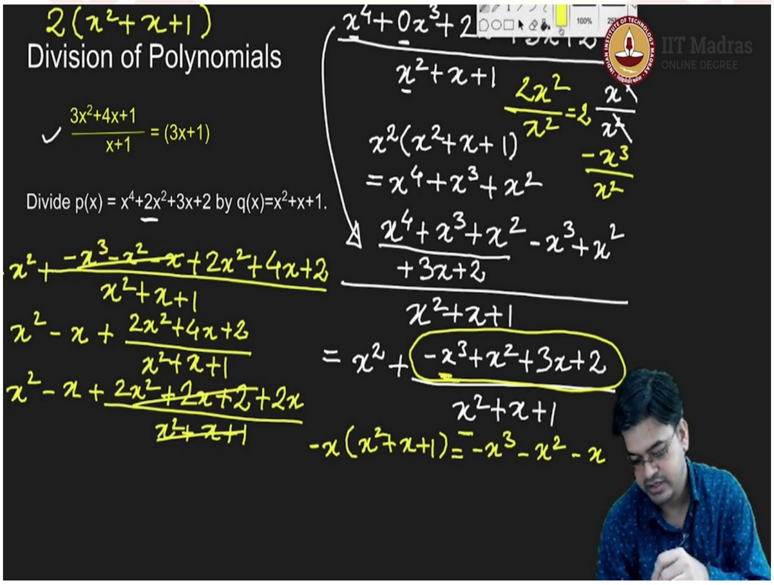

So, the problem is, I want to divide the terms divide a polynomial $p(x)=x^4+2x^2+3x+2$ by $q(x)=x^2+x+1$. So, the $p(x)$ is a polynomial of degree 4; $q(x)$ is a polynomial of degree 2. So, how will I go about this? So, again, I will apply a strain strategy that is I will start by writing x^4 plus remember here, it directly goes to $2x^2$. So, but I want the term containing x^3 also to be present $0x^3+2x^2+3x+2$ and this term is divided by x^2+x+1.

So, remember our first step in the last example was first you take the first term over here and take the first term over here. Then, take a consider a division of these monomials. This will give me x^2. So, what I will do now is, I will consider $x^2(x^2+x+1)$. This will give me actually $x^4+x^3+x^2$.

Now, as per our earlier strategy while solving this problem, we have adopted a strategy that I will add these terms over here. So, if I add these terms over here, then I will subtract appropriate terms over here. So, in this case, x^4 is already there, x^3 was not there and here, x^3 is there.

So, I need to subtract that x^3 from this expression and then, I need to subtract from $2x^2$, I will split this into two. So, let us rewrite this expression, that is the numerator of this expression, x^4 is already there. In order to cancel the denominator, I need x^3 over here.

So, I need to add x^3+x^2 ok; but this x^3 was not present here, it was $0x^3$. So, naturally the next step will be to eliminate x^3 from here. So, that it will retain a legacy of this term. So, if I have eliminated x^3, then it is $2x^2$ of which $1x^2$, I have taken out, so this will be another x^2+3x+2 as it is.

So, let me write that term as it is $3x+2$ and now, if I divide this term by x^2+x+1, then what I will get here is take these first three terms and keep the remaining term as it is ok. So, if I do that, then what will happen is this term x^2 will come out as common plus now, what happens?

This term vanishes, this term vanishes, this term because x^2, I can take out common; from these three terms, I can take out x^2 common that is what I have written and it cancels with the denominator. So, whatever is remaining are the remaining term that is $-x^3+x^2+3x+2$ and this thing is divided by x^2+x+1.

Now, is our division over? No, because the numerator over here has a higher degree than the denominator. Therefore, our division is not over. So, again, I will follow a similar step, I will simply change the color so that I will have a better view ok. So, let us change the color and have a better view of this.

So, let me write it here from this; from this step, I can go here and say ok. So, this is in fact, equal to x^2 plus now you look at this term $-x^3$ and x^2. So, you divide $\frac{-x^3}{x^2}$ which will give you $-x$. So, in this case, you will multiply $-x(x^2+x+1)$.

So, if you multiply $-x(x^2+x+1)$, what you will get over here is $-x^3-x^2-x$, this is what you will get. So, you write this term as it is, that is $-x^3-x^2-x$.

Now, from this term, you adjust the terms. So, $-x^3$ is already there, so I do not have to compensate for this term. But there is a plus x^2, there is a plus x^2 and here there is a $-x^2$. So, that will give me plus $2x^2$ because I am compensating for this extra $-x^2$ added in this term, then there is a $-x$ and over here it is plus $3x$.

So, I have to add one x for this $-x$. So, that will give me plus $4x$ plus and there is no competition for a constant term upon x^2+x+1. Now, you can take out x common and this will cancel off, this term will cancel off with this term by taking x common. So, it is x^2-x is in common plus what you are left with here is $2x^2+4x+2$ upon x^2+x+1 ok. Again, you will apply a similar procedure that is you will actually divide $\frac{2x^2}{x^2}$. So, you will get 2. So, essentially what you; so, when you do that, when you divide $\frac{2x^2}{x^2}$, you will get 2.

So, when you will multiply this number by 2, let me write it here that is $2(x^2+x+1)$ ok. So, in this case, what you will get is $(2x^2+2x+2)$ of which $2x^2$ is already there. So, I will continue over here itself x^2-x+2x^2 is already there, $2x^2$. So, let it be $2x^2+2x$, over here there is plus $4x$. So, I can split $2x$ over here plus $2x$

plus 2. So, that will again come plus 2 as it is here. So, what is remaining now is $2x$ upon x^2+x+1.

So, now if you look at this term, what you will get is you can take out 2 common and this will cancel off with this denominator and therefore, the final expression, I am running short of space. So, let me erase some terms over here. Let me erase some terms over here so that I will get some space.

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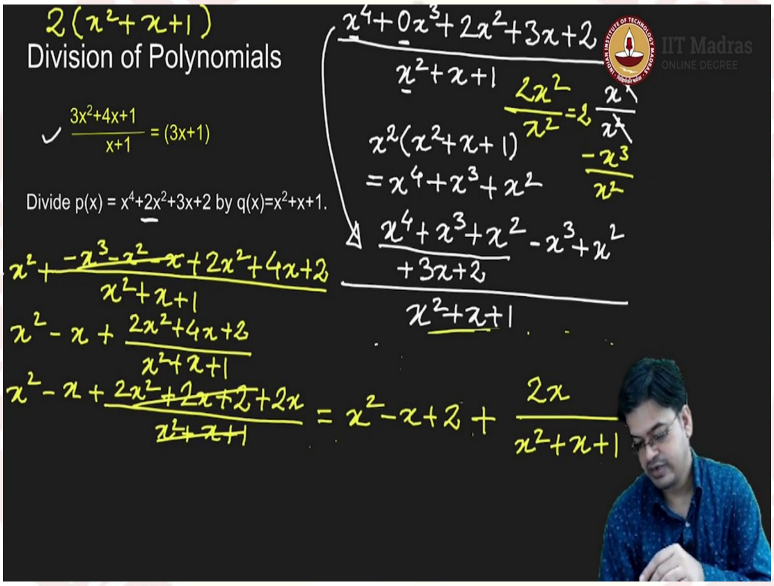

So, you can rewrite this as to be equal to $x^2-x+2+\frac{2x}{x^2+x+1}$. This will be the final answer to this division ok. So, this is how we can actually do a division of two polynomials ok. Let us remove this and see whether to verify whether we have got the final answer to be correct or not. So, I have removed it, you must have noted the answer.

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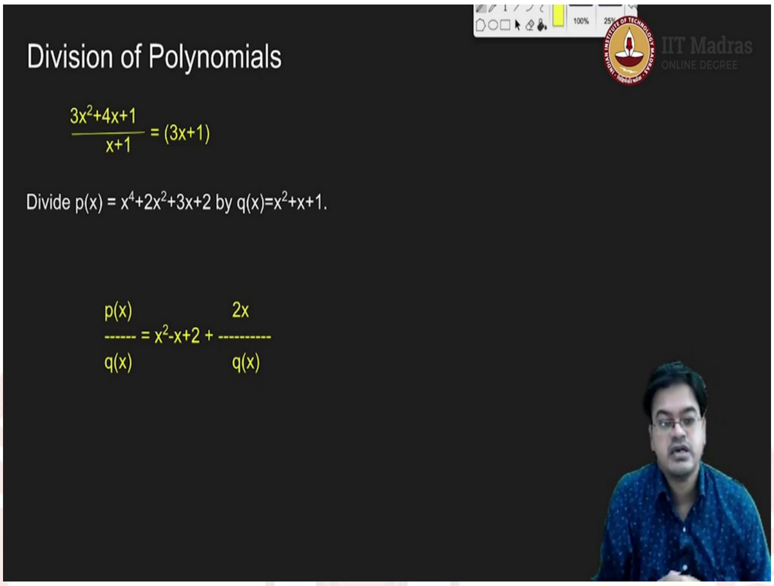

And the final answer that we have got here is $x^2 - x + 2 + \frac{2x}{q(x)}; q(x) = x^2 + x + 1$. Yes, so I have got the correct answer. So, here while doing this, we have derived one algorithm which we will emphasize in the next slide.