

Statistics for Data Science-1

Week 7 Graded assignment

1. m boys and 2 girls are to be placed next to each other in the school ground for morning assembly. What is the probability that there are exactly 4 boys between the 2 girls?

a. $\frac{2m - 5}{{}^{m+2}P_2}$

b. $\frac{2m - 6}{{}^{m+2}P_2}$

c. $\frac{2m - 6}{{}^{m+3}P_2}$

d. $\frac{2m - 4}{{}^{m+2}P_2}$

Answer: b

Solution:

There are a total of $(m + 2)$ places to arrange the 2 girls.

Therefore, the number of ways in which 2 girls can be arranged $= {}^{m+2}P_2$

Positioning of the 2 girls such that there are exactly 4 boys between them can be done in the following ways:

Case 1: First girl at 1st place and second girl at 6th place and vice-versa, i.e. 2 ways.

Case 2: First girl at 2nd place and second girl at 7th place and vice-versa, i.e. 2 ways.

Similarly,

Case (m-3): First girl at $(m - 3)^{th}$ place and second girl at $(m + 2)^{th}$ place, and vice-versa, i.e. 2 ways.

Hence, Number of possible ways such that there are exactly 4 boys between the 2 girls $= 2 \times (m - 3) = 2m - 6$ ways.

Therefore, $P[\text{There are exactly 4 boys between the 2 girls}] = \frac{2m - 6}{{}^{m+2}P_2}$

Hence, option (b) is correct.

Example: $m = 8$

There are a total of 10 places to arrange the 2 girls.

Therefore, the number of ways in which 2 girls can be arranged $= {}^{10}P_2$

Positioning of the 2 girls such that there are exactly 4 boys between them can be done in the following ways:

Case 1: First girl at 1st place and second girl at 6th place and vice-versa, i.e. 2 ways.

Case 2: First girl at 2nd place and second girl at 7th place and vice-versa, i.e. 2 ways.

Case 3: First girl at 3rd place and second girl at 8th place and vice-versa, i.e. 2 ways.

Case 4: First girl at 4th place and second girl at 9th place and vice-versa, i.e. 2 ways.

Case 5: First girl at 5th place and second girl at 10th place and vice-versa, i.e. 2 ways.

Hence, Number of possible ways such that there are exactly 4 boys between the 2 girls
= $2 \times 5 = 10$ ways.

Therefore, $P[\text{There are exactly 4 boys between the 2 girls}] = \frac{10}{{}^{10}P_2} = \frac{10}{90} = \frac{1}{9}$

Hence, option (b) is correct.

2. In a Multiple Select Question, there are m options, of which one or more can be correct. Let us define an event E that the option 'A' is correct. What is the cardinality of E ?

Solution:

Case 1: Only option A is correct.

Number of possible elements = 1 i.e. $\{(A)\}$

Case 2: Two options are correct and A is one of them.

Number of possible elements = ${}^{m-1}C_1 = m - 1$

Case 3: Three options are correct and A is one of them.

Number of possible elements = ${}^{m-1}C_2$

Similarly,

Case m: All options are correct.

Number of possible elements 1

Hence, Cardinality of $E = 1 + {}^{m-1}C_1 + {}^{m-1}C_2 + \dots + 1 = 2^{m-1}$

Example: $m=4$

Case 1: Only option A is correct.

Number of possible elements = 1 i.e. $\{(A)\}$

Case 2: Two options are correct and A is one of them.

Number of possible elements = ${}^3C_1 = 3$ i.e. $\{(A,B), (A,C), (A,D)\}$

Case 3: Three options are correct and A is one of them.

Number of possible elements = ${}^3C_2 = 3$ i.e. $\{(A,B,C), (A,B,D), (A,C,D)\}$

Case 4: All options are correct.

Number of possible elements 1 i.e. $\{(A,B,C,D)\}$

Hence, Cardinality of $E = 1 + 3 + 3 + 1 = 2^3 = 8$

3. A person predicts daily whether the price of stocks of wrist watch companies will go up or down. If his prediction on stock price of Titan is correct a times out of b , for Rolex it is correct p times out of q and for Fossil it is correct x times out of y , then what is the probability that at least two of his predictions are correct on a given day?

a. $\left[\frac{a}{b} \times \frac{p}{q} \times \left(1 - \frac{x}{y} \right) \right] + \left[\frac{a}{b} \times \left(1 - \frac{p}{q} \right) \times \frac{x}{y} \right] + \left[\left(1 - \frac{a}{b} \right) \times \frac{p}{q} \times \frac{x}{y} \right] + \left[\frac{a}{b} \times \frac{p}{q} \times \frac{x}{y} \right]$

- b. $\left[\frac{a}{b} \times \frac{p}{q} \times \left(1 - \frac{x}{y} \right) \right] + \left[\frac{a}{b} \times \left(1 - \frac{p}{q} \right) \times \frac{x}{y} \right] + \left[\left(1 - \frac{a}{b} \right) \times \frac{p}{q} \times \frac{x}{y} \right]$
- c. $\left[\frac{a}{b} \times \frac{p}{q} \times \frac{x}{y} \right] + \left[\frac{a}{b} \times \frac{p}{q} \times \frac{x}{y} \right] + \left[\frac{a}{b} \times \frac{p}{q} \times \frac{x}{y} \right] + \left[\frac{a}{b} \times \frac{p}{q} \times \frac{x}{y} \right]$
- d. $\left[\frac{a}{b} \times \frac{p}{q} \times \frac{x}{y} \right] + \left[\frac{a}{b} \times \frac{p}{q} \times \frac{x}{y} \right] + \left[\frac{a}{b} \times \frac{p}{q} \times \frac{x}{y} \right]$

Answer: a

Solution:

Let us define the following events:

A : Prediction for Titan is correct.

B : Prediction for Rolex is correct.

C : Prediction for Fossil is correct.

We are given that :

$$P(A) = \frac{a}{b}, P(B) = \frac{p}{q} \text{ and } P(C) = \frac{x}{y}$$

Case 1: Prediction for only Titan and Rolex is correct

$$P(A \cap B \cap C^c) = \frac{a}{b} \times \frac{p}{q} \times \left(1 - \frac{x}{y} \right)$$

Case 2: Prediction for only Titan and Fossil is correct

$$P(A \cap B^c \cap C) = \frac{a}{b} \times \left(1 - \frac{p}{q} \right) \times \frac{x}{y}$$

Case 3: Prediction for only Rolex and Fossil is correct

$$P(A^c \cap B \cap C) = \left(1 - \frac{a}{b} \right) \times \frac{p}{q} \times \frac{x}{y}$$

Case 4: All predictions are correct.

$$P(A \cap B \cap C) = \frac{a}{b} \times \frac{p}{q} \times \frac{x}{y}$$

Hence, $P(\text{At least two predictions are correct})$

$$= \left[\frac{a}{b} \times \frac{p}{q} \times \left(1 - \frac{x}{y} \right) \right] + \left[\frac{a}{b} \times \left(1 - \frac{p}{q} \right) \times \frac{x}{y} \right] + \left[\left(1 - \frac{a}{b} \right) \times \frac{p}{q} \times \frac{x}{y} \right] + \left[\frac{a}{b} \times \frac{p}{q} \times \frac{x}{y} \right]$$

Hence, option (a) is correct.

Example: $a = 4, b = 5, p = 6, x = 3, y = 4$

Let us define the following events:

A : Prediction for Titan is correct.

B : Prediction for Rolex is correct.

C : Prediction for Fossil is correct.

We are given that :

$$P(A) = \frac{4}{5}, P(B) = \frac{5}{6} \text{ and } P(C) = \frac{3}{4}$$

Case 1: Prediction for only Titan and Rolex is correct

$$P(A \cap B \cap C^c) = \frac{4}{5} \times \frac{5}{6} \times \frac{1}{4} = \frac{20}{120}$$

Case 2: Prediction for only Titan and Fossil is correct

$$P(A \cap B^c \cap C) = \frac{4}{5} \times \frac{1}{6} \times \frac{3}{4} = \frac{12}{120}$$

Case 3: Prediction for only Rolex and Fossil is correct

$$P(A^c \cap B \cap C) = \frac{1}{5} \times \frac{5}{6} \times \frac{3}{4} = \frac{15}{120}$$

Case 4: All predictions are correct.

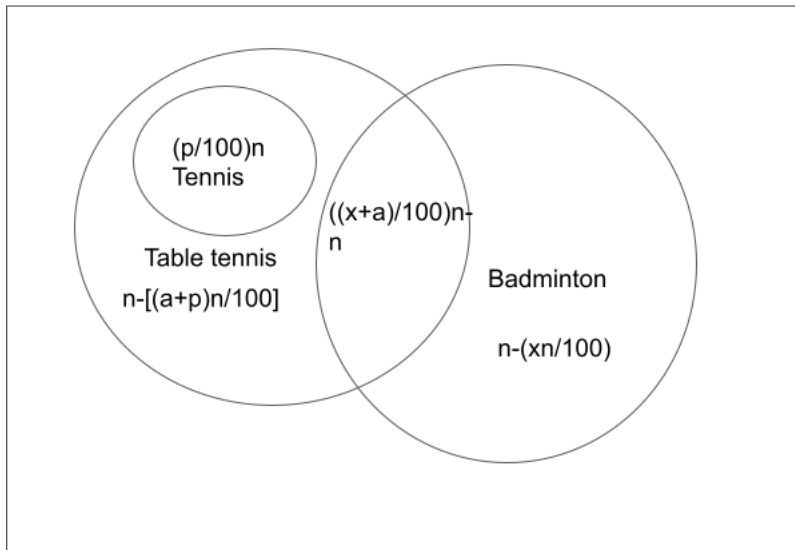
$$P(A \cap B \cap C) = \frac{4}{5} \times \frac{5}{6} \times \frac{3}{4} = \frac{60}{120}$$

$$\text{Hence, } P(\text{At least two predictions are correct}) = \frac{20}{120} + \frac{15}{120} + \frac{12}{120} + \frac{60}{120} = \frac{107}{120}$$

Hence, option (a) is correct.

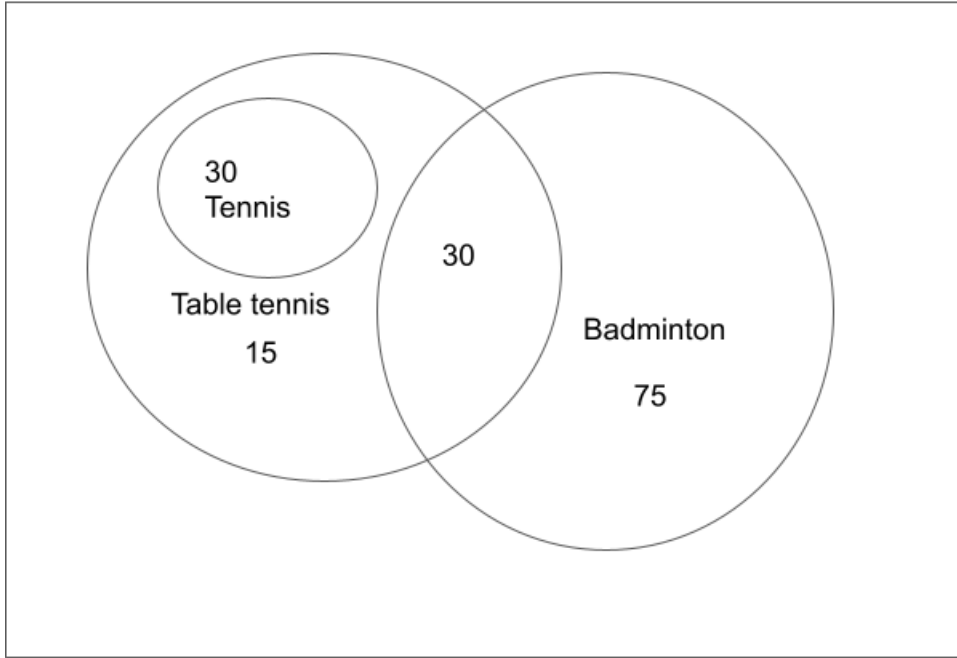
4. There are a total of n students who are part of badminton, table tennis and tennis team of the college. Of which $x\%$ of students play table tennis, $p\%$ play tennis and $a\%$ play badminton. It is also noticed that all students who play tennis also play table tennis, but not badminton. Now a student is selected at random, what is the probability that he/she is the part of table tennis team only? (Enter the answer correct to 1 decimal place.)

Solution:



Therefore, probability that he/she is the part of table tennis team only = $\left(1 - \frac{a+p}{100}\right)$

Example: $n = 150, x = 50, p = 20, a = 70$



Therefore, probability that he/she is the part of table tennis team only = $\frac{15}{150} = \frac{1}{10}$
Hence, option a is correct.

5. The chance that a student will clear the quiz 1 paper is a and the chance that he will clear both quiz 1 and quiz 2 papers is b . The chance that he will clear at least one quiz paper is c . What is the chance that he will clear quiz 2 paper? (Enter the answer correct to 2 decimal accuracy)

Solution:

Let us define the following events:

A: Student will clear the quiz 1 paper ; B: Student will clear the quiz 2 paper.

We are given that:

$P(A) = a, P(A \cap B) = b, P(A \cup B) = c$ and we want, $P(B)$

Now, $P(A \cup B) = P(A) + P(B) - P(A \cap B) \implies c = a + P(B) - b$

$P(B) = d = c - a + b$

For example: $a = 0.4$, $b = 0.3$, $c = 0.5$

Let us define the following events:

A: Student will clear the quiz 1 paper ; B: Student will clear the quiz 2 paper.

We are given that:

$P(A) = 0.4$, $P(A \cap B) = 0.3$, $P(A \cup B) = 0.5$ and we want, $P(B)$

Now, $P(A \cup B) = P(A) + P(B) - P(A \cap B) \implies 0.5 = 0.4 + P(B) - 0.3$

$P(B) = 0.5 - 0.4 + 0.3 = 0.4$

Therefore, the chance that the student will clear the quiz 2 paper is 0.4

6. If $P(A) = x$ and $P(B) = y$ and probability of the complement of $(A \cup B)$ is z , then calculate $P(A \cup B)$? (Enter the answer correct to 2 decimal point accuracy)

Solution:

$$P(A \cup B) = 1 - P(A \cup B)^c = 1 - z$$

For example: $x = 0.2$, $y = 0.5$, and $z = 0.4$

$$P(A \cup B) = 1 - P(A \cup B)^c = 1 - 0.4 = 0.6$$

7. a cards are drawn at random from a pack of 52 cards. Find the probability that b are black and c are red. (Enter the answer correct to two decimal places)

Solution:

Since there are 26 black cards (of spades and clubs) and 26 red cards (of diamonds and hearts) in a pack of cards, the required probability = $\frac{{}^{26}C_b \times {}^{26}C_c}{{}^{52}C_a}$

Example: $a = 4$, $b = 2$ and $c = 2$

Since there are 26 black cards (of spades and clubs) and 26 red cards (of diamonds and hearts) in a pack of cards, the required probability = $\frac{{}^{26}C_2 \times {}^{26}C_2}{{}^{52}C_4} = 0.39$

Pramod goes to a shop to buy some clothes. Shopkeeper shows him x shirts, y pants and z t-shirts. If he selects three clothes at random, then based on the information, answer the questions 8, 9 and 10.

8. Find the probability that the randomly chosen clothes are of different type. (Enter the answer correct to three decimal places)

Solution:

The total number of cases are ${}^{x+y+z}C_3$.

Since the number of favourable cases of getting one cloth of each type is

$$= {}^xC_1 \times {}^yC_1 \times {}^zC_1$$

Therefore, the required probability = $\frac{{}^xC_1 \times {}^yC_1 \times {}^zC_1}{{}^{x+y+z}C_3}$

For example: $x = 5$, $y = 4$, $z = 10$

The total number of cases are ${}^{19}C_3$.

Since the number of favourable cases of getting one cloth of each type is

$$= {}^5C_1 \times {}^4C_1 \times {}^{10}C_1$$

$$\text{Therefore, the required probability} = \frac{{}^5C_1 \times {}^4C_1 \times {}^{10}C_1}{{}^{19}C_3} = \frac{200}{969} = 0.2064$$

9. Find the probability that the randomly chosen clothes does not contain pant. (Enter the answer correct to two decimal places)

Solution:

The total number of cases are ${}^{x+y+z}C_3$.

If randomly chosen clothes does not contain pant, then all the three clothes must be from shirts and t-shirts, i.e., from $x + z$ clothes. Hence, the number of favourable cases for this event is ${}^{x+z}C_3$.

$$\text{Therefore, the required probability} = \frac{{}^{x+z}C_3}{{}^{x+y+z}C_3}$$

For example: $x = 5, y = 4, z = 10$

The total number of cases are ${}^{19}C_3$.

If randomly chosen clothes does not contain pant, then all the three clothes must be from shirts and t-shirts, i.e., from $5 + 10 = 15$ clothes. Hence, the number of favourable cases for this event is ${}^{15}C_3$.

$$\text{Therefore, the required probability} = \frac{{}^{15}C_3}{{}^{19}C_3} = \frac{2730}{5814} = 0.4695$$

10. Find the probability that at least one of the clothes is a shirt. (Enter the answer correct to two decimal places)

Solution:

The total number of cases are ${}^{x+y+z}C_3$.

$P(\text{at least one of the clothes is shirt}) = 1 - P(\text{none of the three clothes is shirt})$

In order that none of the 3 clothes is shirt, all the 3 clothes must be from pants and t-shirts, i.e., from $y + z$ clothes and the number of favourable cases for this event is ${}^{y+z}C_3$.

$$P(\text{none of the three clothes is shirt}) = \frac{{}^{y+z}C_3}{{}^{x+y+z}C_3}$$

$$\text{Hence, } P(\text{at least one of the clothes is shirt}) = 1 - \frac{{}^{y+z}C_3}{{}^{x+y+z}C_3}$$

For example: $x = 5, y = 4, z = 10$

The total number of cases are ${}^{19}C_3$.

$P(\text{at least one of the clothes is shirt}) = 1 - P(\text{none of the three clothes is shirt})$

In order that none of the 3 clothes is shirt, all the 3 clothes must be from pants and t-shirts, i.e., from $4 + 10 = 14$ clothes and the number of favourable cases for this event is ${}^{14}C_3$.

$$P(\text{none of the three clothes is shirt}) = \frac{{}^{14}C_3}{{}^{19}C_3} = \frac{2184}{5814}$$

$$\text{Hence, } P(\text{at least one of the clothes is shirt}) = 1 - \frac{2184}{5814} = 0.624$$

11. An urn contains 3 balls numbered 1, 2 and 3. The co-efficients of the equation

$px^2 + qx + c = 0$ is determined by drawing the numbered balls with replacement. What is the probability that the equation will have imaginary roots?

Nature of roots:

Consider a quadratic equation: $ax^2 + bx + c = 0$

Compute $D = b^2 - 4ac < 0$

$$\text{Roots} = \begin{cases} D < 0 & \text{imaginary roots} \\ D \geq 0 & \text{real roots} \end{cases}$$

- a. $\frac{4}{27}$
- b. $\frac{23}{27}$
- c. $\frac{16}{27}$
- d. None of the above

Answer: b

Solution:

Since each coefficient in equation $px^2 + qx + c = 0$ is determined by drawing a numbered ball from the urn, each of the coefficients p, q and c can take values from 1 to 3.

Therefore, total number of possible outcomes $= 3 \times 3 \times 3 = 27$

$$P[\text{Imaginary roots}] = 1 - P[\text{Real roots}]$$

For a Quadratic equation to have real roots, the equation $q^2 - 4pc \geq 0$ should be satisfied.

The number of favourable cases for real roots are:

pc	p	c	$4pc$	q (such that $q^2 - 4pc \geq 0$)	No. of cases
1	1	1	4	2, 3	2
2	1	2	8	3	1
	2	1	8	3	1

Hence, Total number of favourable cases for real roots $= 2 + 1 + 1 = 4$

$$\text{Therefore, } P[\text{Real roots}] = \frac{4}{27}$$

$$P[\text{Imaginary roots}] = 1 - \frac{4}{27} = \frac{23}{27}$$

Hence, option (b) is correct.

12. If A and B are mutually exclusive or disjoint events, then which of the following is/are always true:

- a. $P(A) = P(B)$

- b. $P(A) < P(B)$
- c. $P(A) \leq P(B^c)$
- d. $P(A) \geq P(B^c)$

Answer: c

Solution:

Since, A and B are mutually exclusive or disjoint events.

Therefore, $A \cap B = \phi$

$$A = (A \cap B) \cup (A \cap B^c) = \phi \cup (A \cap B^c) = A \cap B^c$$

Therefore, $A \subseteq B^c \Rightarrow P(A) \leq P(B^c)$