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ONLINE DEGREE

Mathematics for Data Science 1
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Lecture - 19
Representation of a Line-2

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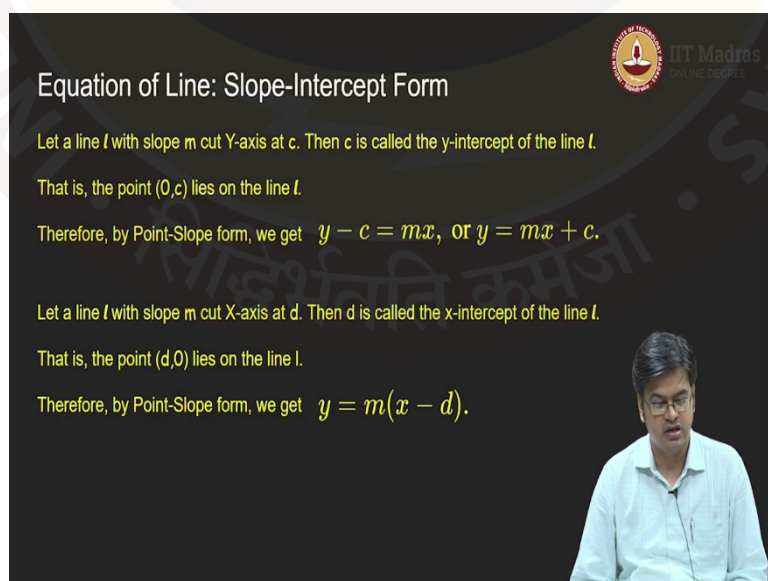
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Equation of Line: Slope-Intercept Form

Let a line l with slope m cut Y-axis at c . Then c is called the y-intercept of the line l .

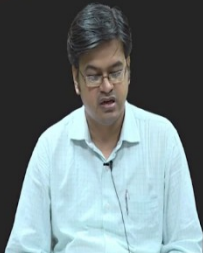


That is, the point $(0, c)$ lies on the line l .

Therefore, by Point-Slope form, we get $y - c = mx$, or $y = mx + c$.

Let a line l with slope m cut X-axis at d . Then d is called the x-intercept of the line l .

That is, the point $(d, 0)$ lies on the line l .

Therefore, by Point-Slope form, we get $y = m(x - d)$.



Now let us go ahead and try to figure out some spatial variations where the calculations become extremely easy for. These two forms are primary two-point form and slope-point

form. So, when you consider slope point form you can also consider a special case that is slope-intercept form. So, this is the methodology that we will use for considering slope-intercept form, before that let me define what is an intercept.

So, let l be the line with slope m that cuts Y - axis at point c . Then this c is called y intercept of the line l . So, what is the meaning that it cuts Y - axis at c ? The y coordinate of that point is c and the x coordinate is 0 ; that means, any point that it cuts through Y - axis of line l will be of the form $(0, c)$ and that $(0, c)$ will lie on line l .

Now we have our slope point form instead of having any point (x, y) you have a specific point which is $(0, c)$. So, I apply the slope point form or point slope form in this expression. What you will get instead of $y - y_0$ you have $y - c$ which is equal to $m(x - x_0)$, m is the slope of the line m times $x - x_0$. What is x_0 ? Zero.

So, so we will get $y - c = mx$ and therefore, I will get a form $y = mx + c$, this is a standard form that we generally deal with when we are dealing with straight lines. So, you have got a slope-intercept form which is of the form $y = mx + c$.

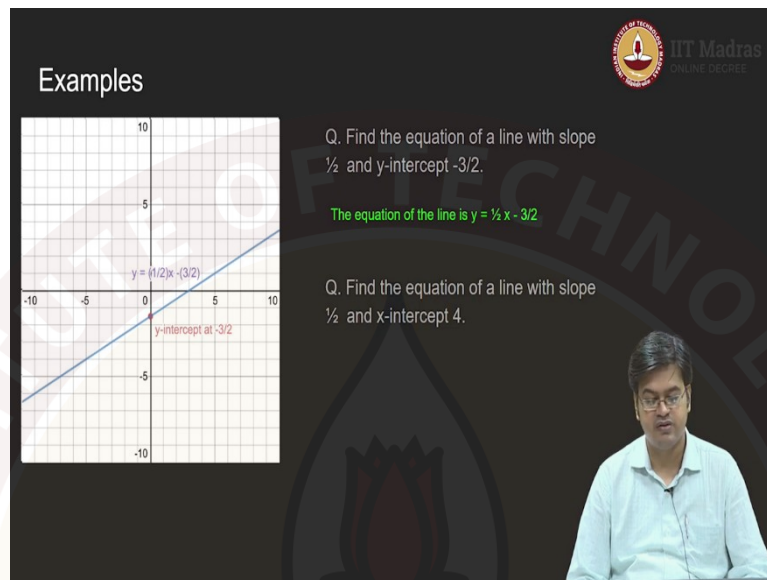
The interesting fact is the calculations are very simple whenever you are given the slope-intercept form. For example, now if you know the y intercept is at c and the slope is m you do not have to do any calculations, but straight away write this expression that is y is equal to take the slope m , take the intercept c ; $y = mx + c$ will be your answer.

Therefore, the calculations simplify significantly when you are considering a slope-intercept form. If the intercept is not available then you may have to go to that point slope form and figure out what it is. Now there can be if the line cuts Y - axis the line can as well-cut X - axis. So, there can be another variation of this formulation that is if a line l with slope m cuts X - axis at point d . Then d will be called as x intercept of the line l .

If d is called as x intercept of the line l then how will this point lie on the line l or what are the coordinates of the line that intersects X - axis and line l ? So, what is the point of intersection? That will be $(d, 0)$ and this $(d, 0)$ lies on line l . So, I will again use the point slope form of the line.

So, if I want to use point slope form $y - 0 = m(x - 0)$ will be the answer. So, that will be the form $y = mx - md$. So, let us try to use this and solve some problems for finding the equations of the line using slope-intercept form.

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So, typically some example like this. So, I want to find the equation of a line with slope is $\frac{1}{2}$ and y intercept is $-\frac{3}{2}$. Remember here things are very easy because you just need to know

$mx + c$. So, what is m? m is $\frac{1}{2}$ and c is $-\frac{3}{2}$. So, upfront I can tell you orally this, the equation of the line will be $y = \frac{1}{2}x - \frac{3}{2}$. Let us verify the result using the graphics and all other things.

So, here is the y intercept of this particular line. So, here the y intercept is at point $-\frac{3}{2}$. Now

slope is half correct. So, the equation of line you can easily see is $y = \frac{1}{2}x - \frac{3}{2}$. So, let us try to

figure out what is the x intercept of this line. So, $y = \frac{1}{2}x - \frac{3}{2}$. So, the x intercept of this line is

3. So, the question could have been asked that find the equation of a line with slope half and x intercept equal to 3 that also can be a question and the answer will be same.

So, let us see what is the next question that is find the equation of a line with slope half, but x intercept is 4 it is not 3. So, it is definitely not a same line because x intercept is 4, but the slope is half. So, can you relate it to some of the concepts? The slope is half; that means, the slopes are equal, we have seen that if the slopes are equal then lines must be parallel to each other.

So, therefore, I can easily see that the line must be parallel to this line with some different intercept which is at 4 for this the intercept is 3 so, intercept is 4. So, what can be the y intercept can also be an interesting question. We will answer it later. Right now, let us see how we can answer the question that is asked here. Find the equation of line with slope half and x intercept 4.

So, according to our formulation $y = mx - md$. So, where d is the intercept that is 4 so and this is half. So, $y = \frac{1}{2}x - 4$ is the equation of this line.

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Examples

Q. Find the equation of a line with slope $\frac{1}{2}$ and y-intercept $-\frac{3}{2}$.

The equation of the line is $y = \frac{1}{2}x - \frac{3}{2}$

Q. Find the equation of a line with slope $\frac{1}{2}$ and x-intercept 4.

The equation of the line is $y = \frac{1}{2}(x - 4)$ or $2y - x + 4 = 0$.

You can simplify this which will give you $2y - x + 4 = 0$. So, this will be the expression for the line. This is the slope-intercept form of the line, now we can go to two-point form that is suppose I have been given x intercept and y intercept how will I identify the line.

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Equation of a Line: Intercept Form

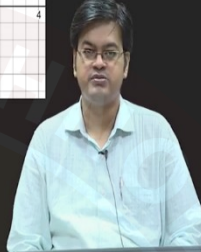
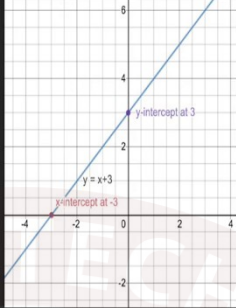
Suppose a line makes x-intercept at a and y-intercept at b . Then the two points on the line are $(a,0)$ and $(0,b)$.

Using two-point form,

$$(y - 0) = \frac{b-0}{0-a}(x - a) \text{ or } \frac{x}{a} + \frac{y}{b} = 1$$

Example

Q. Find the equation of a line having x-intercept at -3 and y-intercept at 3.

$$\frac{x}{-3} + \frac{y}{3} = 1 \text{ or } y = x + 3.$$


So, let us now go to the form of intercept that is intercept form, how to find equation of line when you have been given two intercepts x and y. So, let us formulate the hypothesis, suppose a line makes x intercept at a , y intercept at b , then naturally the coordinates of these two points are $(a, 0) \wedge (0, b)$. So, we will use two-point form to derive the equation of line.

So, I will take this point as the first point therefore, the y coordinate is 0. So,

$$(y - 0) = \frac{b - 0}{0 - a}(x - a).$$

Now, if you divide this expression throughout by b then you will get $\frac{y}{b} = \frac{-x}{a} + 1$. Because this has a minus sign shift it to the left hand side and you will get this expression which is

$\frac{y}{b} + \frac{x}{a} = 1$, now you see how beautiful is this expression; x intercept is a so, below x you put a y intercept is b . So, below y you put b .

Therefore, there is nothing to memorize, it is just a simple trick that

$\frac{x}{x\text{-intercept}} + \frac{y}{y\text{-intercept}} = 1$ that is how you will get the intercept form. So, it is very easy to solve the problems if you remember this trick.

Now, let us take one example where we need to find this. So, find the equation of line having x intercept at -3 and y intercept at 3. So, you do not have to do any complicated calculations, you can simply say $\frac{x}{-3} + \frac{y}{3} = 1$, multiply throughout by 3 you will get the expression $y = x + 3$.

So, let us verify whether this satisfies because it is always better to verify using graph. So, x intercept is -3 y intercept is 3, the line that passes through these two points is $y = x + 3$. This is what the intercept form is, it is very simple and you can practice more and more problems.

That is all for today.

