

Week-8
 Mathematics for Data Science - 1
 Exponential and Logarithm
Tutorials

1. If $3 \log_p 9 - 2 \log_q 27 = 2 \log_{p^{-1}q} 81$, where p and q are two distinct positive real numbers, then show that $p = q^3$ or $q = p^3$.
2. Consider the equation $\log_5(x^2 + x + 5) + \log_4(x) = 3$. How many integers x satisfy the equation?
 - ☐ 0
 - ☐ 1
 - ☐ 2
 - ☐ 3
3. For isomerization reaction of compound F to compound G , Arrhenius constant is given by equation $A = A_0 e^{-\frac{E_a}{RT}}$ where A_0, E_a, R, T are pre-exponential factor ($A_0 = e^{35}$), activation energy (kJ), universal gas constant ($8.314 J mol^{-1} K^{-1}$) and temperature ($^{\circ}K$) respectively. Arrhenius equation gives the temperature dependence of the reaction rates. For different values of temperature T , Arrhenius constant (A) are given in the Table M1W9A-1. If we use the method to minimise the Sum Squared Error (SSE), what is the most likely activation energy from the given options?

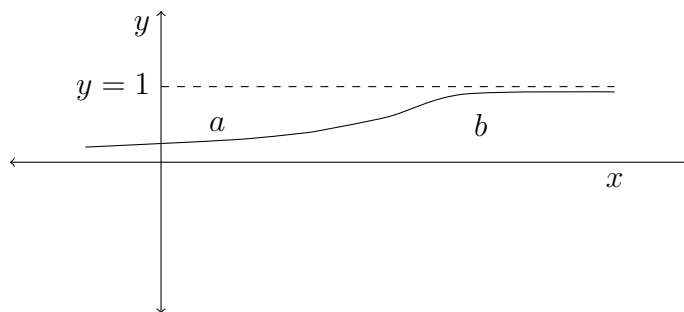
$A(T)$	0.00018	0.0027	0.030	0.26
T	750	796	850	896

Table 1: Table M1W9A-1

- ☐ $E_a = 272 kJ$
 - ☐ $E_a = -272 kJ$
 - ☐ $E_a = 438 kJ$
 - ☐ $E_a = -438 kJ$
4. The table shows the different points on a coordinate plane. All the points can be divided in two groups $G1$ and $G2$. It is sure that points a and b belong to groups $G1$ and $G2$ respectively. Anita found a way that if she can draw a curve which divides all the points in two groups as shown in figure. She some how found the equation of the curve to be $f(x) = \frac{1}{1+e^{-2x+9}}$.

Point	x	y	Group
a	1	0.5	$G1$
b	5	0.5	$G2$
c	4	0.2	
d	4	0.8	
e	3	0.8	
f	7	0.8	

- (a) Find what was the logic to separate the points.
(b) Assign $G1$ or $G2$ to each point based on that particular logic.



5. Suppose p and q are two non-zero natural numbers and $q = p + 1$. Consider two functions $f(x) = \log_p(q^x - 1)$ and $g(x) = \log_q(p^x + 1)$. Find the co-ordinates where f and g will intersect.
6. Find the roots of the equation $x^{\left(\frac{3}{4}(\log_3 x)^2 + \frac{5}{4} \log_3 x - 4\right)} = 3^3$.