IIT Madras

ONLINE DEGREE

Mathematics for Data Science 1
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Lecture No. 55
Logarithmic Functions: Properties - 1

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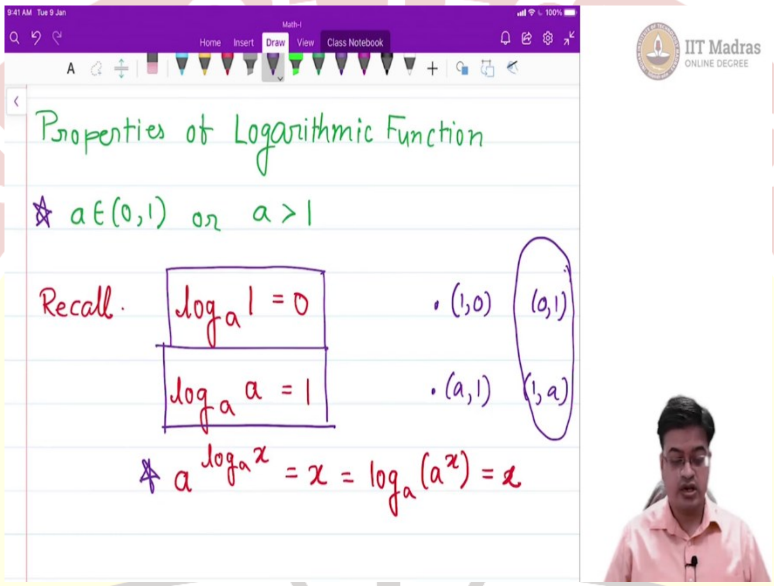

So, in this video we are going to look at further properties of logarithmic functions. So, when we look at the logarithmic functions in general we have a standard set of conditions that are imposed that is $a\epsilon(0,1)$ and a > 0. And we already know few properties of logarithmic function namely logarithmic function is actually an inverse of exponential function which is conveyed through this property that is if you take $a^{\log_a x}$, then you will get x back and if you use $a^x$and take log $a^{\log_a x}$ you will get x back.

So, essentially our logarithmic function is an inverse of exponential function. Another thing that you need to recall based on the graphical representation of logarithms is log to the base a of 1 is 0, that means we have already located that point 1, 0 is on the graph of the logarithmic function independent of whether a < 1 or a is bigger than 1. Another thing that you need to remember or recollect is log to the base a of a is equal to 1. What does this mean?

The point a1 is on the graph, which you have also seen, so when you are talking about logarithmic function, these two points are on the graph, when you are talking about exponential function because it is a reflection along y is equal to x axis 0, 1 and 1a are the points that you will look for exponential functions. So, this we have done enough. So, these are the basic logarithmic properties that we are already aware of.

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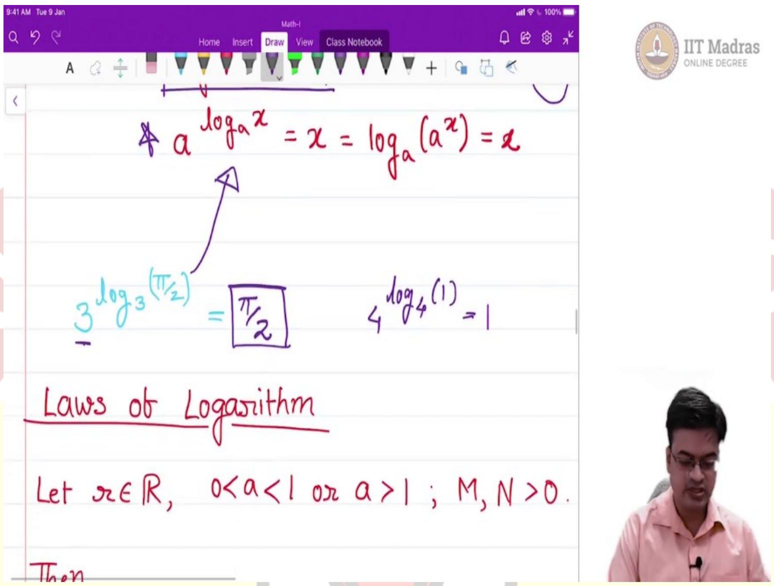

Now, let us go further and explore something which is called laws of logarithm. Prior to that now because we have mentioned log to the base a of a^x is x, you can also ask a question that what if I have been given a function like this, which is $3^{\log_3 \frac{\pi}{2}}$, so based on this you can use this particular formulation which is where a is 3 log to the base 3 of pi by 2 naturally this should be equal to pi by 2. So, the sometimes some complicated numbers may simplify in this way, it is simple demonstration of use of these properties.

Another thing that you can also see is say let us say 4 raised to log to the base 4 of 1, now you already know log to the base 4 of 1 is 0 based on this property and therefore it is 4 raised to 0 which will naturally give you 1, so all these simple simple tricks you should solve, you should solve more and more problems and gain more confidence in while using the logarithms, because while solving the problems on logarithms and exponential functions applying a log function or applying an exponential function will play a crucial role while solving the problems.

So, let us focus on some simple laws of logarithm and in fact when I, why these are called laws? Because these are the principles for which the logarithms were invented by Napier. So, let us see what are the laws of logarithms.

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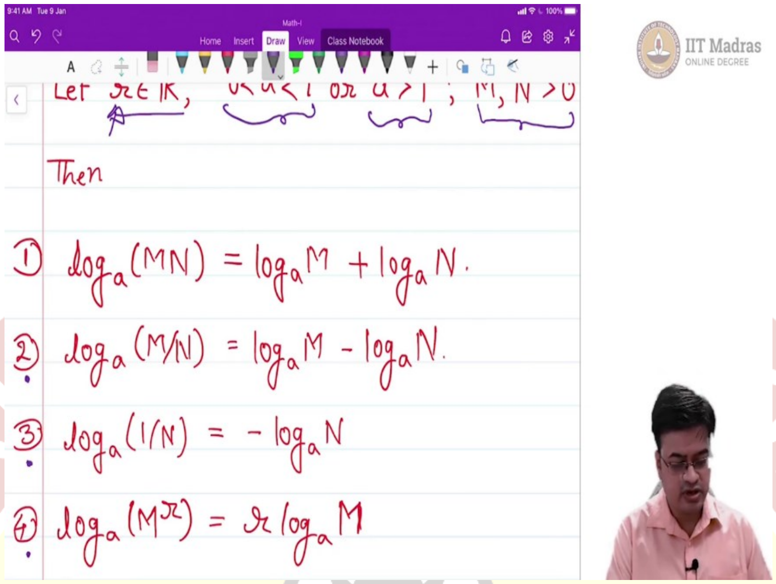

So, in order to define laws of logarithms first we need to restrict to the valid zone, so we will restrict to the value zone in such a way that my a is between 0 and 1 open interval or my a > 1, then because the logarithmic function is always defined on the positive side that means the argument that is applied to logarithmic function is positive, so my M and N are actually the arguments for the logarithmic function, so they are always positive.

And here is one more thing that is some are real number r is given to you. If all these conditions are satisfied then there are 3 laws, we will see each of them and we will try to prove each of them. So, first law is, logarithm of a product of two positive numbers is sum of the logarithms, verbally you can state this as logarithm to the base a of product of two numbers positive numbers is nothing but the sum of the logarithms of the individual numbers.

In a similar manner if you go ahead and do some simplifications you can also come up with a second law that is, logarithm of a quotient of two positive numbers is nothing but difference between the logarithms of those two numbers. In a similar manner this is not a new law but we will state it for the sake of clarity that logarithm of reciprocal of a number is nothing but negative of a logarithm of the original number.

And the fourth one which uses this number r, which we have defined here and remember this is any real number, then the logarithm of M raised to r or any number raised to r any positive number is to r is nothing but r times logarithm of that number. So, when many astronomist where doing some calculations and they wanted to do the product of the two distances which are very high in the power of 10^{32}or something like that. And in that case the multiplication of two numbers becomes a tedious task, so in order to handle these tasks they have actually invented these logarithms.

So, if you search on the Google why the logarithm were invented you will come to know about many references from astronomy where they are successfully using logarithms. And remember they were doing this in around eighteenth century, so there was an absence of computational power. So, these laws were helpful and therefore they are governed as laws of logarithms. Now, let us try to prove each of them, one by one, let us take the first law which is logarithm of the product of two numbers positive numbers is equal to logarithm of is equal to some of the logarithm of these two numbers.

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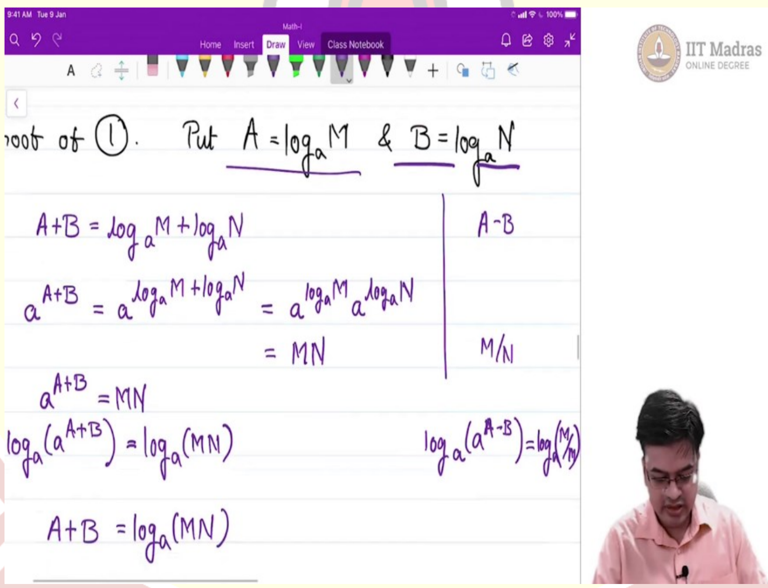

So, in order to prove this let us put $\mathrm{A} = \log_a M$ and B is equal to log to the base $\mathrm{B}{=}\log_a N$. And now what you do is you actually consider you actually consider A+B, so my A+B is nothing but log to the base a of M$\log_a M{+}\log_a N$, so what I will do now is I will simply go back and consider some properties of logarithms that I have already considered.

So, I will consider these kind of properties and let us see how this property can help me in proving this particular identity. So, I have used that particular property and let us say I have raised this as a^{A+B}. In this case, what I am using? I am using actually an exponential function is inverse of logarithmic function, so left hand side I have raised to the power a, so naturally right hand side will also be raised to the power a, so I will get this+$\log_a N$, no confusion here.

Now, you can actually see this particular thing when I am looking at this particular thing what I am getting over here is a raised to $a^{\log_a N}$. So, now what is this actually, if you look at our definition of a raised to log to the base a of M, you will get this to be equal to M and you will get this a raised to $a^{\log_a N} = N$, so you got MN.

So, now what you got here is MN and $a^{A+B}$is MN, so what I have got here is $a^{A+B} = MN$. Now, if you look at what you want to prove, if you want to prove the left hand side is log to the base a of MN, so how will I get that? Again use the similar property which is given here and by using this property take logarithm on both sides. So, you take log to the base a of a raised to A+B which is equal to log to the base a of MN.

Now, my claim is we have proved this result, how? Because $\log_a a^B$ is nothing but A+B and I am saying A+B=$\log_a MN$, now what is a? Just substitute what we have put A as, log to the base a of M and B as $\log_a N$. Therefore, I can rewrite this as log to the base a of M+log to the base a of N is equal to log to the base a of MN. Clear.

So, my first result is proved first law is proof, now you can easily guess what modification do I need to make for proving the second law. So, if you look at the second law that is log to the base a of M upon N is log to the base a of M-log to the base a of N. So, if you want to prove this what modifications you need to do? You simply use this A and B, there is no change in this only thing that you will have is A-B over here.

If you have A-B over here all the things correspondingly all the things will change and instead of MN what you will get is $\frac{M}{N}$, rest of the things are just same you can practice it as an exercise and therefore you will get a similar result which says $\log_a M$log to the base a of a raise to M a raised to A-B is equal to $\log_a \frac{M}{N}$. And you are done, then you will again apply the you again use the inverse function property of logarithms and cancel with this off and you will get the second result.

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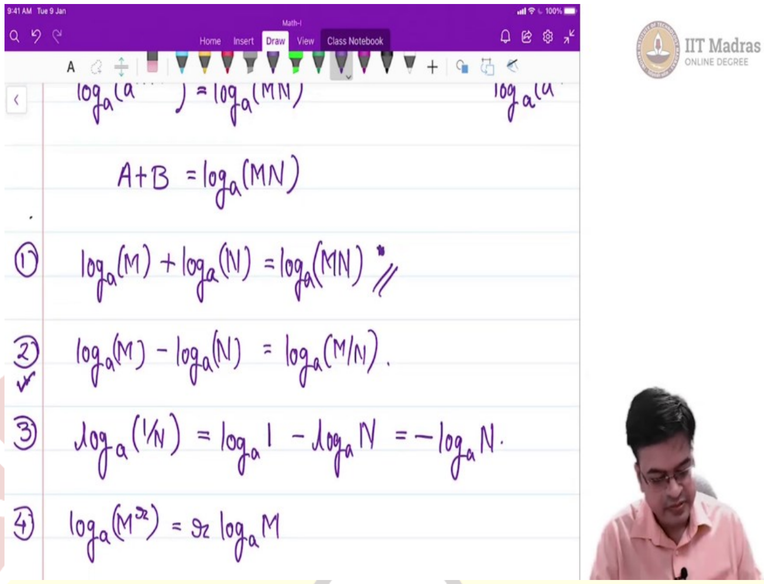

So, I have not doing it properly, but you can easily derive it from it. So, let us write the second result, this is the first result, first law of logarithm, second law of logarithm actually follows by replacing+sign with a-sign. So, $\log_a M - \log_a M = \log_a \frac{M}{N}$.

Now, you look at the third result which actually talks about log to the base a of 1 upon N. now, in this case you simply apply the second rule and you simply apply the second law logarithm that we have just proved and where the M is equal to 1. So, naturally I will get log to the base a of 1-log to the base a of N. Now, what is log to the base a of 1? We have already seen that 1, 0 is on the graph of a log, so based on this particular property we can easily conclude that this particular thing is going to be 0, so my final answer should be-log to the base a of N, that is all.

So, on let us look at what is the fourth result, $\log_a M^r = r \log_a M$. How will you prove this? Again you will apply the same modus operandi, that is you will first isolate some term and then you look at the term.

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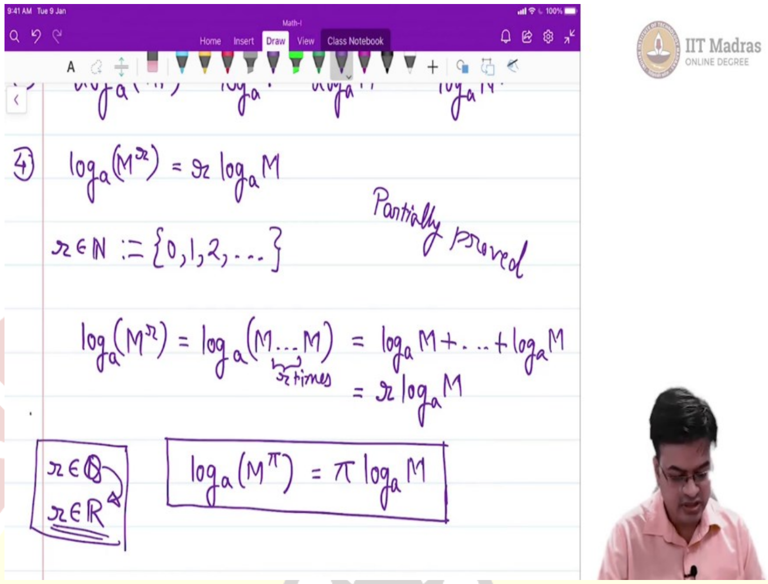

So, let us first look at this r belonging to set of natural numbers. If r belong to set of natural numbers, how will you proceed? The answer is very easy, what is set of natural numbers? In our case in our course we have defined set of natural numbers to be equal to 0, 1, 2 and so on. So, in this case our if r belongs to this particular set then you can easily see that I can write this log of M raised to r as log of log to the base a of M into M into M this is done r times, this is done r times and then I can apply the law of or the multiplication rule of the logarithm which is this.

And I can simply get this as $\log_a M^r = \log_a(M \ldots\ldots M)[r\ times] = r\log_a M$. And therefore you will get the answer to be equal to this is equal to log to the base a of M r times. Now, if r is set of rational numbers, this is a situation becomes tricky still can be managed, but this will not prove, we will assume for the sake of convenience and naturally the proof or the answer lies when you study calculus when r belongs to set of real numbers, you can actually construct a sequence of rationals which will converge to real number.

So, what I have done here is I have partially proved this is partially proved the law of logarithm when r belongs to set of natural numbers. When you study the math 2, or math for data science 2, in that you will come to know how to handle these particular objects. But for us what is important is if you give me $\log_a M^\pi = \pi\log_a M$.

This is this particular property of log will be very handy while solving the problems, you imagine an irrational number which was in the index of some positive number is taken into the multiplication with respect to log, so this simplifies the calculations significantly and as I already mentioned when we discuss exponential function how the number e has arrived natural exponential function how the number e has arisen, the same logic applies when we will prove the result for a set of real numbers. So, let us not get into those details right now, but for our purposes this law is very crucial and we will use this law left and right.