IIT Madras

ONLINE DEGREE

Statistics for Data Science 1

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Lecture 6.6

Probability - Equally likely outcomes

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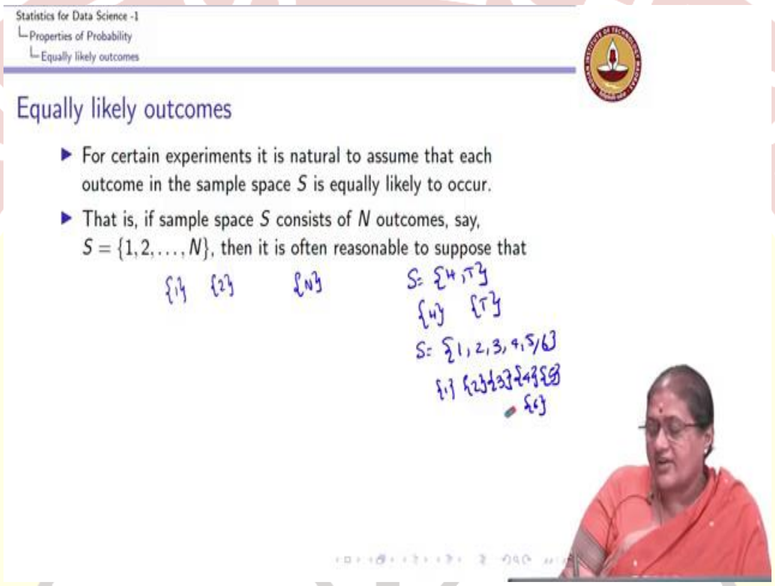

So, now we will focus on equally likely outcomes. Recall when we discussed about the classical interpretation of probability we said that we are assuming in an experiment the outcomes are equally likely. So how do we compute the probabilities of events when I have equally likely outcomes using the properties of probability.

So, suppose I have a random experiment and I am assuming that each outcome in the sample spaces is equally likely to occur then suppose my sample space has n outcomes, finitely many n outcomes and I am assuming that each one of these outcomes are equally likely to occur. What are the outcomes? I have 1 as an outcome, 2 as an outcome, n as an outcome, each one of them is an outcome.

For example, if I am tossing a coin I have head and tail; head is an outcome, tail is an outcome. When I am rolling a dice 1, 2, 3, 4, 5, 6; now 1, 2, 3, 4, 5 and 6 are the outcomes and we are assuming in each of these cases that each of the outcomes are equally likely to happen.

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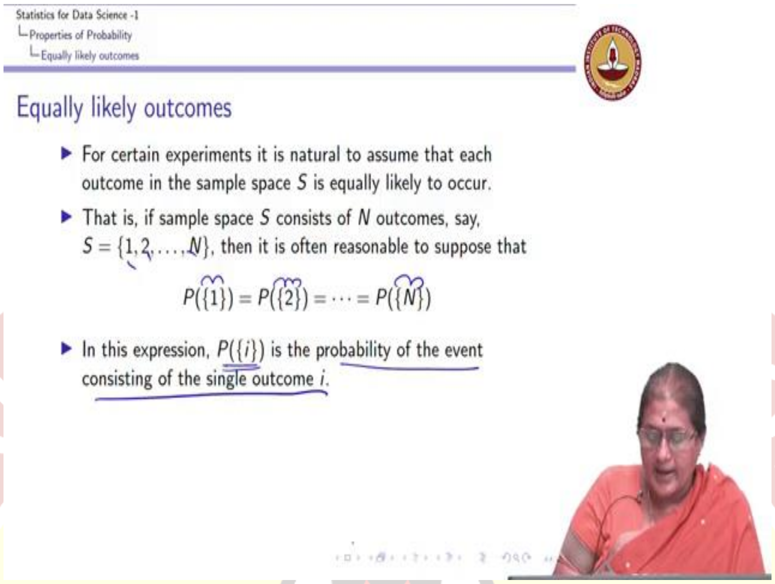

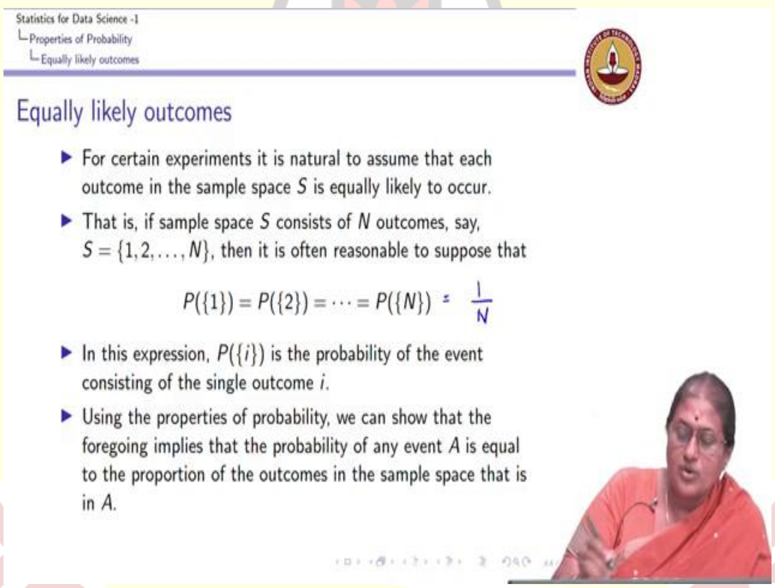

In other words what we are trying to say is a probability with which this outcome happens is the probability with which this outcomes happens with the probability with which this outcome happens.

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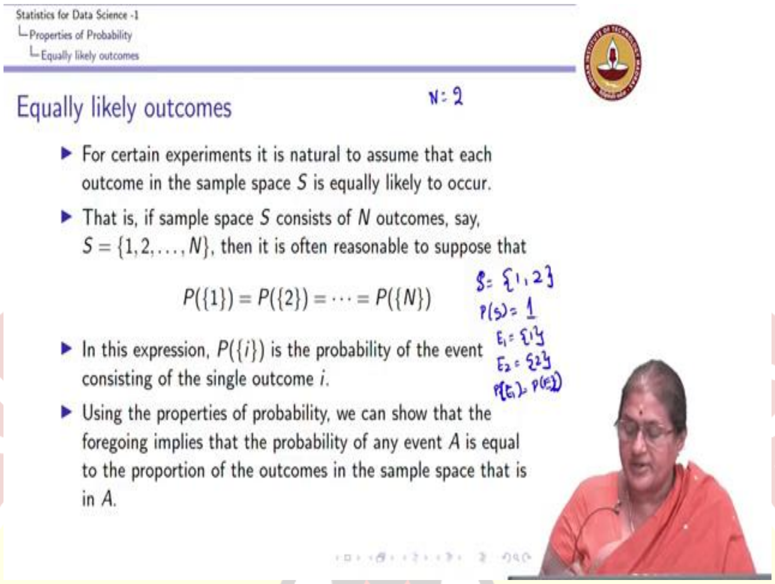

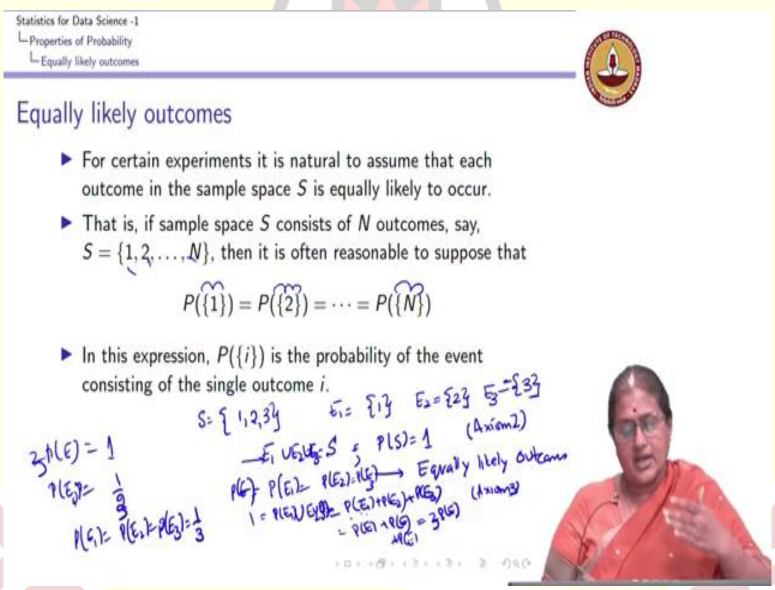

There are n outcomes. so let us find out what would be the probability of the event consisting of a single outcome.

So, let us look at a simple example where I have s equal to 1 and 2 just n equal to 2, I have two outcomes. Now I am assuming let me define E1 to be the outcome 1, E2 to be the outcome 2. Now, I know $E1 \cup E2$ is my sample space and I also know my probability of my sample space is equal to 1, this is from my axiom 2, this is what I have from my axiom 2.

Now I also know $P(E1) = P(E2)$ this is from my assumption of equally likely outcomes. Now from my axiom 3 I know $P(E1 \cup E2) = P(E1) + P(E2)$. Now this $P(E1 \cup E2) = 1$ and I know probability of E1 plus probability of E2 and E1 equal to E2 let me call it some probability of E so this equal to probability of E plus probability of E which is equal to 2 times probability of E.

So, I have $P(E) = 1$ which gives me $P(E) = \frac{1}{2}$. So $P(E1) = P(E2) = \frac{1}{2}$.. I can extend this logic to 1 2 3 I have E1 equal to E2 equal to E3 equal to 3, so $(E1 \cup E2 \cup E3) = S$, so I have now this is equal to probability of E I extend the logic I have a E3 here, so this would be probability of E3, so I have probability of E plus probability of E plus probability of E which is 3 times probability of E.

I get $P(E) = \frac{1}{3}$, so I get $P(E1) = P(E2) = P(E3) = \frac{1}{3}$. So we can extend the same logic to a sample space having n outcomes and we can show that the probability of each of these outcomes is equal to 1 by N, the probability of each of these outcomes if I have n outcomes we can show is equal to 1 by n.

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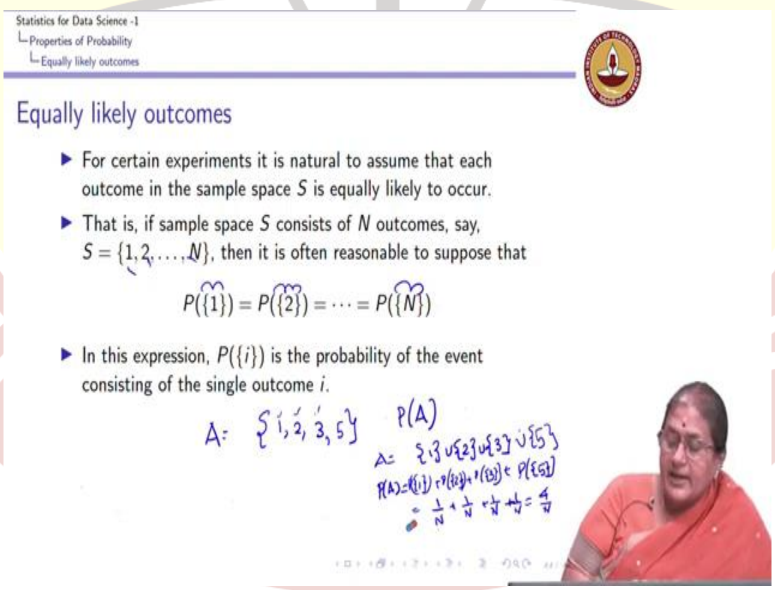

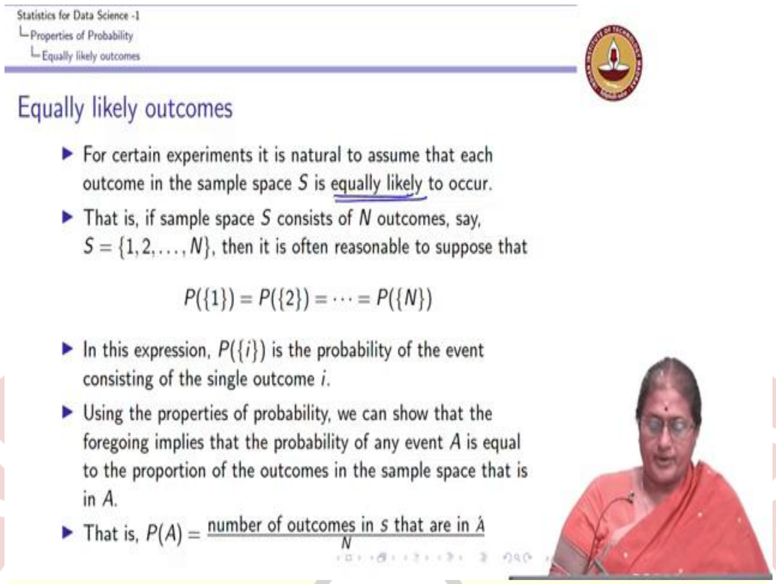

So, now the question is suppose I have an event A which is a subset of this, for example if I have an event A which is 1, 2 and 3. Now what is the probability of this A? I know outcome 1, outcome 2 and outcome 3 are equally likely to happen. Now I can express this A as 1 union 2 union 3 I can express A as 1 union 2 union 3 and by applying my earlier logic of mutually disjoint events I can get probability of A is probability of 1 plus probability of 2 plus probability of 3, plus probability of 3 and we have just worked out these probabilities to 1 by n plus 1 by n plus 1 by n which is equal to 3 by n.

Suppose I have a 1 2 3 5 I will have a union 5 here, I will have a plus probability 5 here I will have a plus probability 5 here and I will have a 1 plus 1 by n here which will give me 4 by n. So if I have equally likely events I can find out the probability of any event by applying the loss of probability and what we can see is I can use the probabilities of probability to show that the probability of any event A is equal to the number of outcomes from S that are in A to the total number of outcomes which I give by n.

This actually is what we refer to in the classical interpretation of probability and hence if you recall I said the assumption of equally likely events or equally likely outcomes is extremely important.

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Example: Rolling a dice

- Experiment: Roll a fair dice
- Sample space: $S = \{1, 2, 3, 4, 5, 6\}$ $N=6$
- Let E_i denote the event of outcome i. Since the dice is fair, $P(E_i) = \frac{1}{6}$.

$P(\{1\}) = P(\{2\}) = \ldots = P(\{6\}) = \frac{1}{6}$

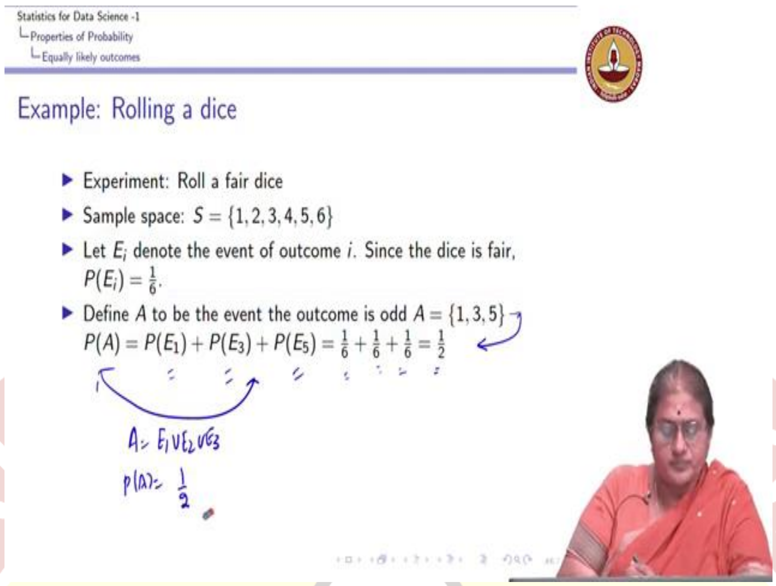
$P(E_1) = P(E_2) = \ldots = P(E_6) = \frac{1}{6}$

Statistics for Data Science -1
Properties of Probability
Equally likely outcomes

Example: Rolling a dice

- Experiment: Roll a fair dice
- Sample space: $S = \{1, 2, 3, 4, 5, 6\}$
- Let E_i denote the event of outcome i. Since the dice is fair, $P(E_i) = \frac{1}{6}$.
- Define A to be the event the outcome is odd $A = \{1, 3, 5\}$

$$P(A) = \frac{3}{6} = \frac{1}{2}$$



Now, let us look at a few examples where my outcomes are equally likely in a day to day life. So the first is I roll a fair dice, my sample space is 1 2 3 4 5, the minute I say fair I assume it is unbiased, it is not loaded or it is not weighted dice.

The chance I get any one of these outcomes is equal to same, so any one of these outcomes are equally likely to happen. So I have an experiment wherein the outcomes are equally likely to happen I can show that from our earlier, so my n equal to 6 in this case, so I can show that from my earlier discussion probability of 1 occurring equal to probability of 2 occurring which is equal to the probability of 6 occurring and all of them are same and equal to 1 by 6.

So, if Ei is the probability of an outcome i probability of E1 equal to probability of E2 which is equal to the probability of E6 which is equal to 1 by 6. So that is what we can see. Now suppose I define the event that the outcome is odd, so I know that the outcomes are 1, 3 and 5. Now how do I find out the probability of this event A? From my expression I find out how many so if you go back by my definition if I have equally numb likely outcomes probability of A is number of outcomes from S that are in A to the total number of outcomes.

I just apply that logic, so the number of outcomes from S, the number of outcomes in S that are in A are 1, 3 and 5 which is equal to 3. And the total number of outcomes is 6 giving me a probability of A is 1 by 2 which is also very evident that when I toss a die the chance that I get an odd number is 50%, it makes a lot of sense we have just got it from the properties which we have just defined which is probability of A, I can get it from the definition and I can verify that this is the same as probability of E1 plus probability of E3 plus probability of E5 which is 1 by 6, 1 by 6, 1 by 6 which is 1 by 2. This is from my axiom I have expressed A to be $E1 \cup E2 \cup E3$ and I work out probability of A is $\frac{1}{2}$.

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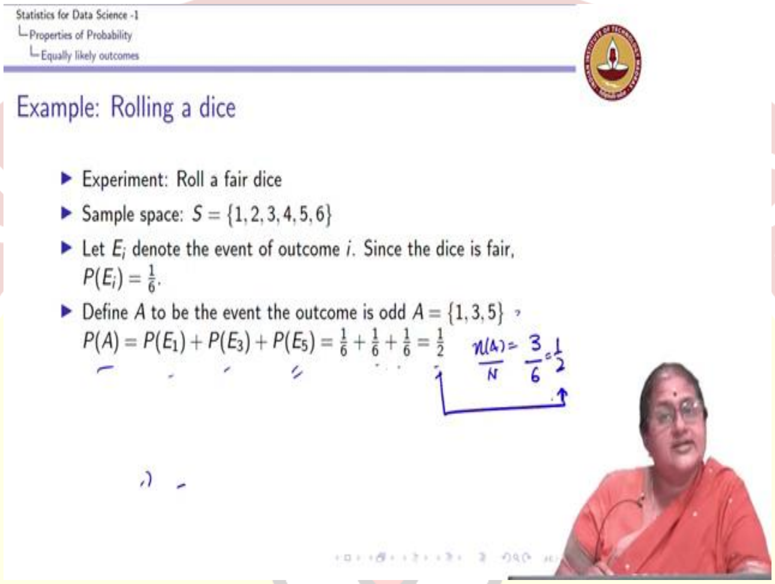

And this we have verified is the same as number of elements of A which is 3 to total number of elements which is 6 which is 1 by 2 and we can see that these two are the same.

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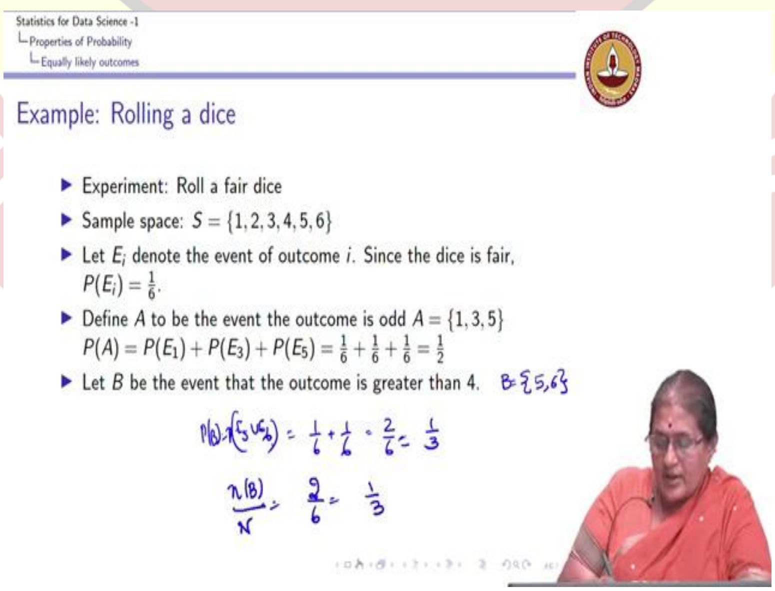

So, this, let us look at another event which says that the outcome is greater than 4. So I know the elements in B are greater than 4, so this is 5 and 6. Again I can express this as E5 union E6 probability of B equal to $P(E5 \cup E6)$where E5 and E6 are again the outcome is 5 and 6 so I get a 1 by 6 plus 1 by 6 which is a probability so I get this is 2 by 6 which is 1 by 3 I can also find out that as number of elements in B to total number which is 2 by 6 which is again 1 by 3.

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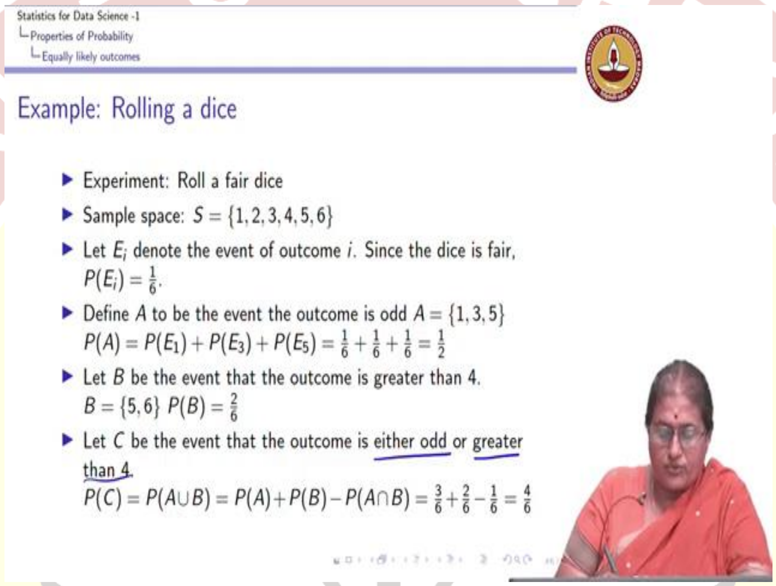

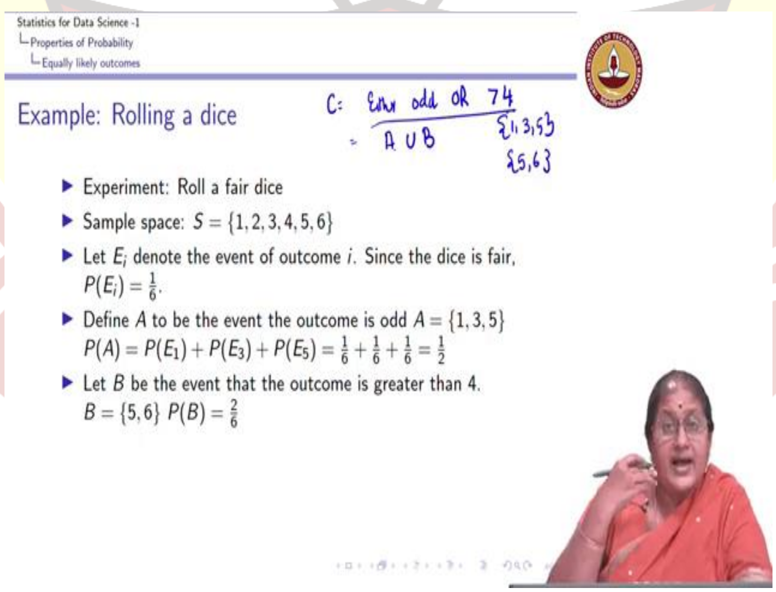

So, now let us look at the third event which says that it is either odd or greater than 4. So now we have defined an event C which is either a odd number or it is greater than 4. So I know C can be expressed as A union B. A was the outcome 1, 3, 5. B was the outcome 5 and 6. So you can see that A union B are not disjoints, so either odd and greater than 4, A union B are not mutually exclusive.

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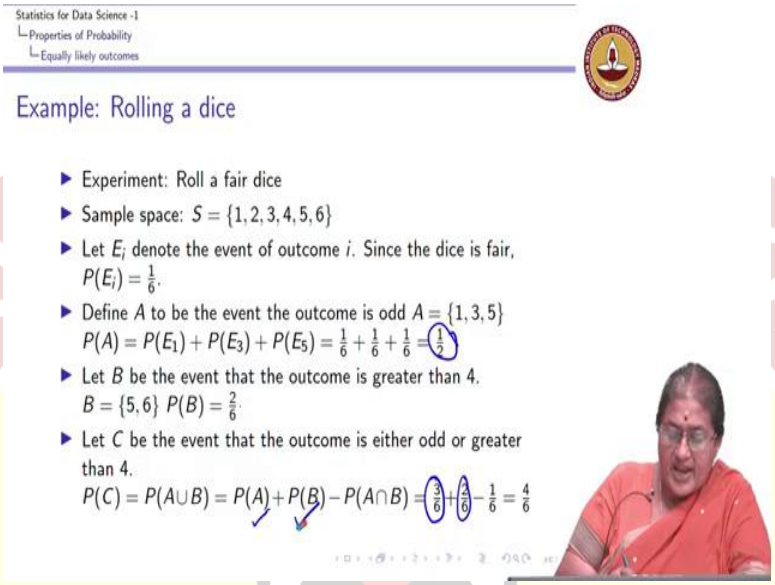

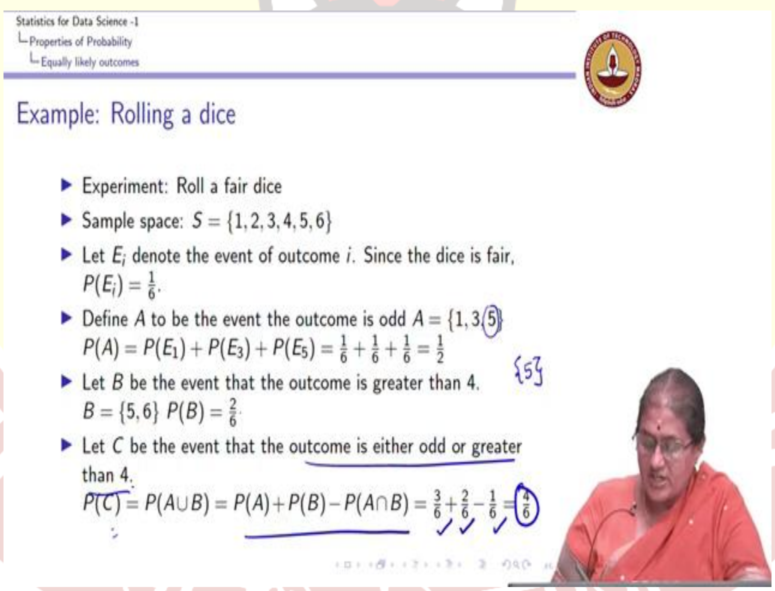

I apply the additive rule of probability to get probability of C is probability of A union B which is same as probability of A which I have obtained here which is 1 by 2 or 3 by 6. Probability of B which is 2 by 6, what is $P(A \cap B)$? So what do I mean by A intersection B? I mean that the event is both it is just this 5 which is greater than 4 and odd which is just the event 5 and that probability is 1 by 6 because $P(E5) = \frac{1}{6}$ and I see that the probability of C which is either odd or greater than 4 by application of the additive rule is $\frac{3}{6}+\frac{2}{6}-\frac{1}{6}$ which is equal to 4 by 6 which is equal to further equal to 2 by 3.

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Now, let us look at another example which we have been looking at right from the beginning which is about taking a card from a deck of 52 cards. So again when you draw a card from a standard deck of 52 cards, the question is what is the probability that the card is either a red or a queen? Now, first when I am having a card or a deck of 52 cards and I am drawing a card from these 52 cards the chance that I draw any one of these 52 cards is the same.

So, if I define my sample space as drawing a card so I know my sample space is going to be these 52 outcomes and each one of these outcome is equally likely to happen. So the chance that I am going to draw an A club is the same as the chance I am going to draw 9 hearts which is the same as the chance I am going to get a 10 clubs which is the same as I could get a queen diamond and so forth.

So, each one of these outcomes are equally likely to happen. With what probability; I have 52 cards, so each one of them can happen with the probability of 1 by 52. So now the question is what is the chance that the card is either a red or a queen? So I need to define the events.

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So, the first event I define is the event that the card drawn is a red card. So I can see that I have 26 of these cards which are red cards. So the number of cards which are red cards are 26, the total number of cards are 52 which gives me the probability of drawing a red card is 26 by 52 which is half. Another way to look at it is I have exactly 26 of cards of black and 26 are red and so hence the chance I draw a red card is going to be 50% which is given by $\frac{1}{2}$.

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Now, what is the chance of me drawing a queen? So I can see that I have 4 queens and drawing each queen so the chance of drawing a queen is I either get this card or this card or this card or a this card and I can again apply my laws of probability to know that the probability of drawing a queen is nothing but $\frac{4}{52}$ again applying the equally likely outcomes and number of outcomes that satisfy this event that it is a queen which is 4 I get which is $\frac{1}{13}$.

But that is not what we want, we want either a red or a queen. So I can express this as $R \cup Q$, now are they disjoint events? In other words, can I have a card which is both red and a queen so let us look at this, the answer is yes. I have a card, I have in fact two cards which are both red and queen hence red and queen are not mutually exclusive or not disjoint.

So, how do we find out what is probability of red union Q? The way I find out the probability of red union Q is I find out probability of red plus probability of Q minus probability of red intersection Q I have which is $\frac{26}{52} + \frac{4}{52} - \frac{2}{52}$ which gives me $\frac{28}{52}$ which I can further see that this is $\frac{7}{13}$. So hence I have the probability of red or a queen to be $\frac{7}{13}$. So what have we done so far?

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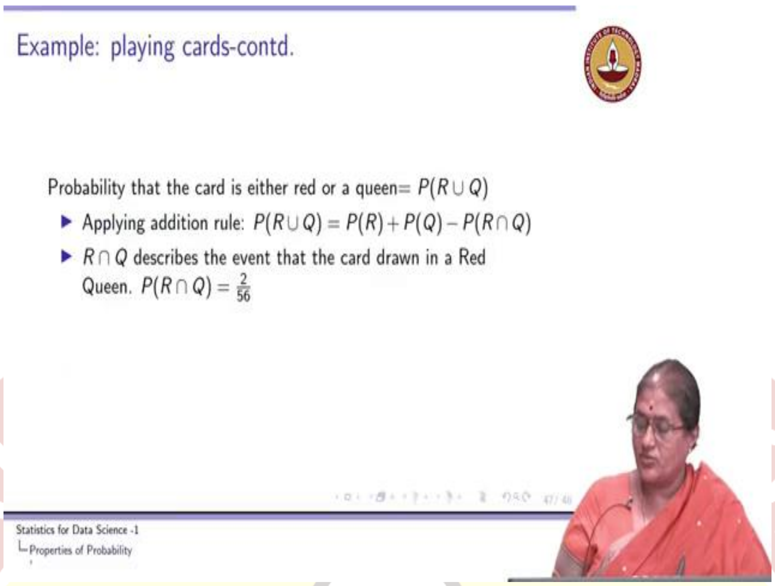

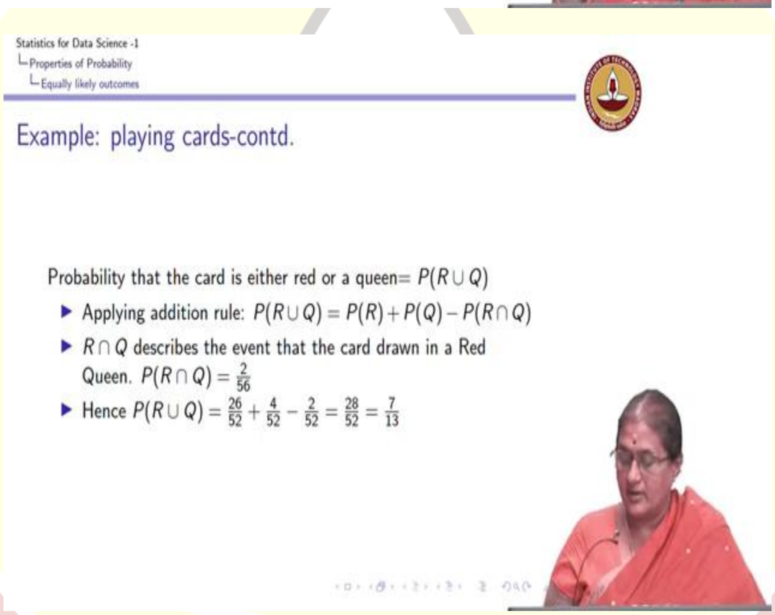

So, I apply the addition rule and after applying the addition rule I get that $P(R \cup Q) = \frac{7}{13}$.

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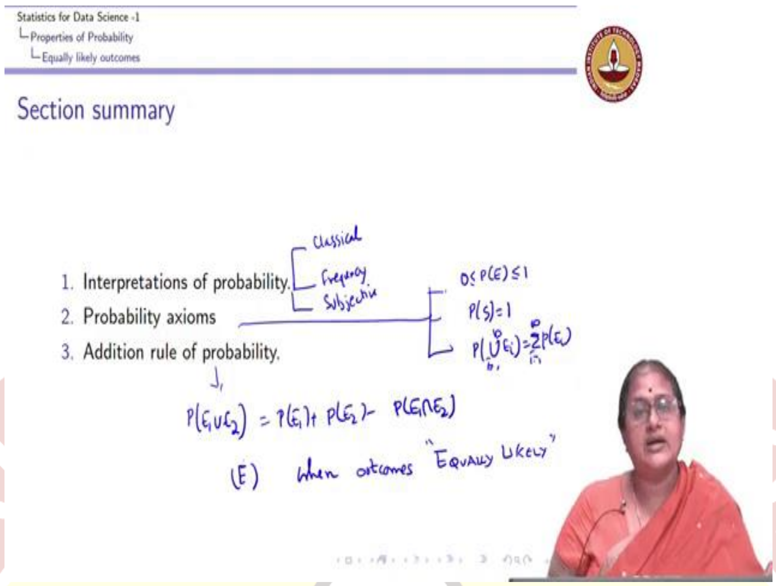

So, quick summary of what we have done in this session; we first introduce a 3 interpretations of probability, we started with a classical, so here we said the three popular interpretations are classical. The relative frequency or the empirical and the subjective interpretations of probability then we introduce the probability axioms by probability axioms we have three main axioms; one is given an event from a sample space probability of the event would be a number between 0 and 1.

Probability of the sample space equal to 1 and probability of the union of a sequence of events is equal to the sum of the probabilities of the events these are the three axioms. Then we extended this axiom and or derived the probability of a union of any two events not necessarily disjoint to be probability of E1 plus probability of E2 minus probability of E1 intersection E2 and looked at applications of this addition rule of probability.

We finally ended by how to compute the probability of an event when the outcomes of an experiment are equally likely when the outcomes of an experiment are equally likely to happen. So the next thing which we are going to address is the notion of conditional probability.