IIT Madras

ONLINE DEGREE

Computational Thinking
Prof. Madhavan Mukund
Prof. G. Venkatesh
Department of Computer Science
Chennai Mathematical Institute
Indian Institute of Technology Madras

Lecture – 3.4
Three price problem

So, let us try something new let us try to see if we can award prizes to the best students in the class. So, maybe we want to give not just one prize so let us give a few prizes. Let us maybe say the top three students in the class. So, what could be a good way to do that?

Top I mean I top three students, but they all could be girls all could be boys.

Yeah.

So, we should I think we should ensure that at least.

We could have two boys and one girl, two girls and one boy, but they should be representation I think that we should take care.

Ok, so that is one thing. So, they should be some.

They should be. So, it cannot be just the top whatever the top means it cannot be just the top three.

Ok.

So, we want to be fair also in some.

Want to be fair.

Two different categories.

So, let us say that we want the top three students, but there should be both boys and girls in the top three.

In the top three.

Ok.

Top three, but again top three means totals you take the total marks.

And, then find the best.

Three total marks if turns out that there are boys and girls in that then we are done, is that way?

But somebody might do very well in one subject and not well in another subject. So, we should also have some individual mark criteria and maybe, not just I mean of course, we want to go by the totals.

But, we should also maybe ask that they do well.

Individual subjects not just in total because there could be some wild fluctuations.

It may not it may not be required that if they do well in all subjects they toppers in all subjects.

Yeah.

They will be toppers all overall.

Yeah.

So, we do not need. So, can we just say that they are toppers in one subject? Toppers means top three let us say, in one subject.

Ok.

And also top three in.

Overall.

Overall.

Ok.

And, and there is a good there is a mixture of boys and girls in the set.

So, you have to give three prizes let us say.

Ok.

So, three prizes should satisfy the following three things – one that they all the three should be in the top three.

Overall.

Yeah.

I do not know that works or not may or may not work we will see as we go along. 2 – they should be a top three in at least one of the subjects.

Ok.

And, they should be a representation of boys and girls in the set.

Ok, so how do we start now? So, do we start by looking at the top three in each subject or do we look at the top three overall to start with and then?

We can first just find the top three overall.

Let us find the.

Then.

Top three overall and see at least if we have both boys and girls in that list, and then we can come back and see about the top three.

If it so, there could be many cases right, one is that the top three overall does not represent boys and girls.

So, there is let us say it is all girls, then what do we do? Then we may have to not just look at three we may have to look at four, five.

Yeah.

Till we find out boy right.

Yes.

Right.

We have to do that. The other possibility is that all top three are the top three have boys and girls in it is a good mix, but the top they are not the toppers in any subject.

Correct.

The toppers is in any subject the subjects are subject toppers are somebody else, right the other people who are subject toppers.

So.

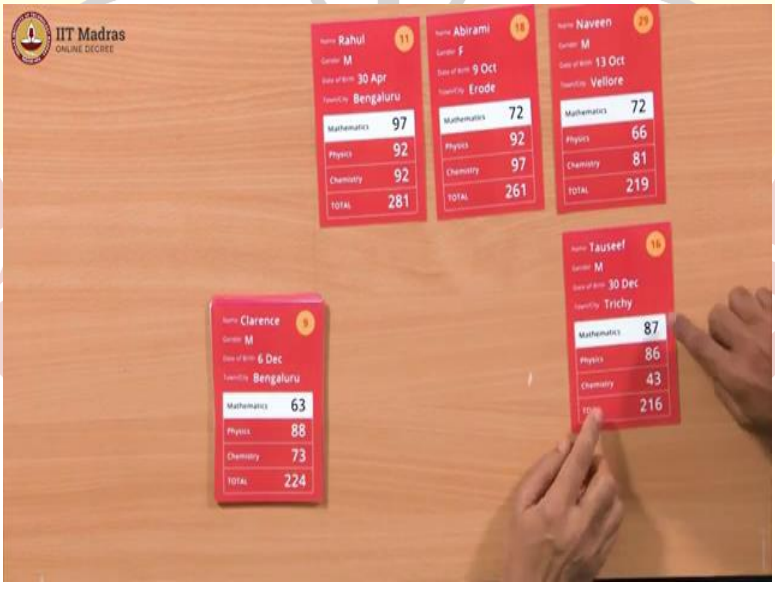
Then.

Then we will again go back and decide.

Then we may have to go back and redo the exercise against something we have to do.

Ok.

(Refer Slide Time: 03:40)



So.

So, let us first do this top three.

Let us do the top three and see what comes.

Yeah.

So, what we will do is we will go through and we will keep track of the top three marks that we have seen at any given time.

Yeah.

Right how do you do that?

So, I think.

We know.

We will find the max.

We have the max.

When we did the max we used to keep the largest one.

Compare each one with the largest one and replace it if it is larger than the largest one.

Yeah.

Now, here if it is larger than the largest one that becomes second, and then the second will become third. So, I guess the third will become fourth then we do not have to worry about it.

So, we keep three cards as set.

So, we can keep the current first two first three and then.

Keep three cards I say.

And.

Like we did for max we get one card a set.

Yeah.

So, we will have.

So, we keep.

Like a max second max and third max.

Third max.

And then.

Keep three cards as a set.

Yeah.

And, we compare each time with all the three cards.

Because we might have to replace the second max and not the top.

We do not have to compare it with all the three cards we kept it in decreasing order.

Yeah.

We only have to compare with the last one.

Yes, but we still have to keep it second or third we have to know right, so.

We have to keep in order let us say we keep the top second.

Yeah.

Second and third.

If you either the card is.

If it is below the third we can discard.

Below the third we can discard it. So, we only compare with third card.

And.

But, if it is above the third then it replaces something.

Yeah.

Then you have to check against.

Yeah.

With all three.

So, let us go along.

Go through it.

Yeah.

Let us go through it let us go through it and see it.

So, the first one at the moment is 196 is the largest one ok. So, right now it is the highest mark.

Yeah.

Because you have not seeing anything.

Yeah.

Then we have 281 which is clearly higher. So, let us keep it maybe we can keep it from left to right.

Left to right.

So.

281 and 196 ok.

Now, we have 219 which is bigger than what we have there. So, now, we have the top three marks as 281, 219 and 196 alright. Now, we have 261. So, we certainly bigger this 196 can go aside and it is bigger than this. So, we now have to replace this in to second place.

Wow, this is interesting. So, you are shuffling the cards. You are moving the card. You are assuming you are making sure that this is always.

In top.

Arranged in such a way that this is the highest this is the next highest and this is the third highest.

Yeah.

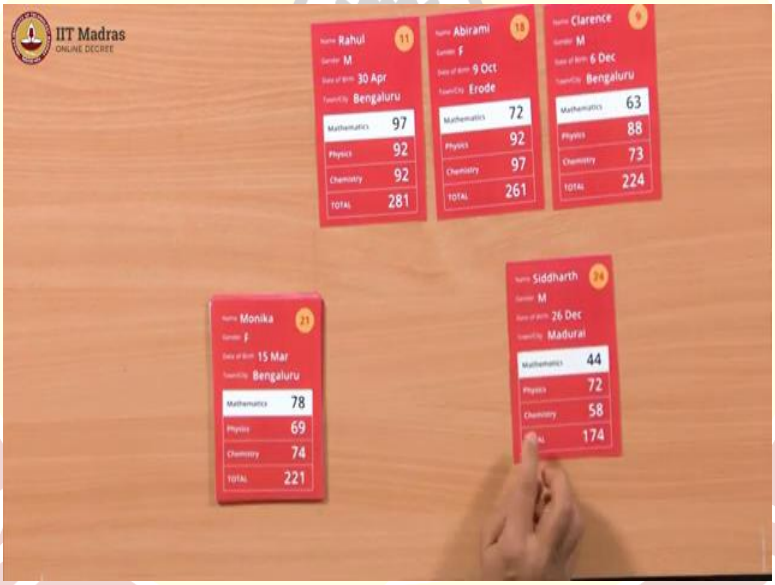
Ok.

Ok.

So, now, we have 216 which we start with the lowest one and since it is lower than the lowest one we do not.

We can discard it.

(Refer Slide Time: 05:49)



Now 224.

It is more than 219.

Yeah.

So, certainly that card must go.

Yeah.

But this next one is 261. So, it remains in third place.

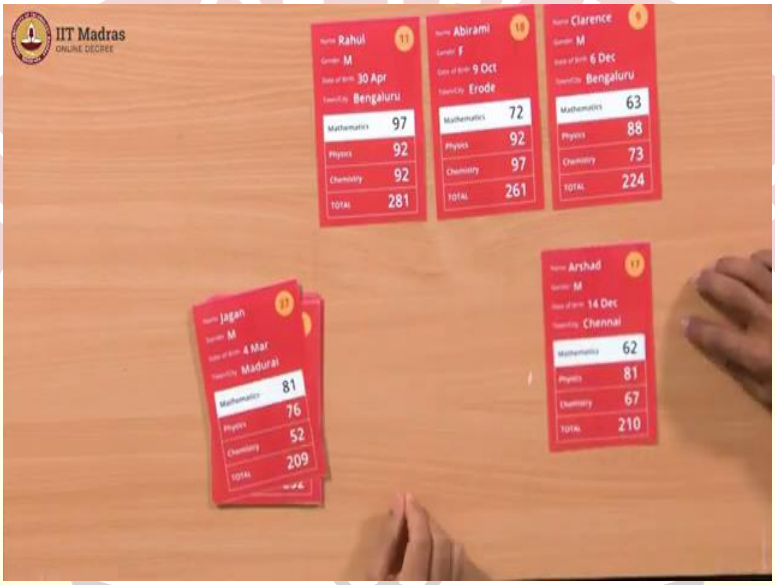
Now, alright ok.

We have 174 which is lower than third place. So, it goes.

Discarded.

Similarly, 221 can go.

(Refer Slide Time: 06:05)



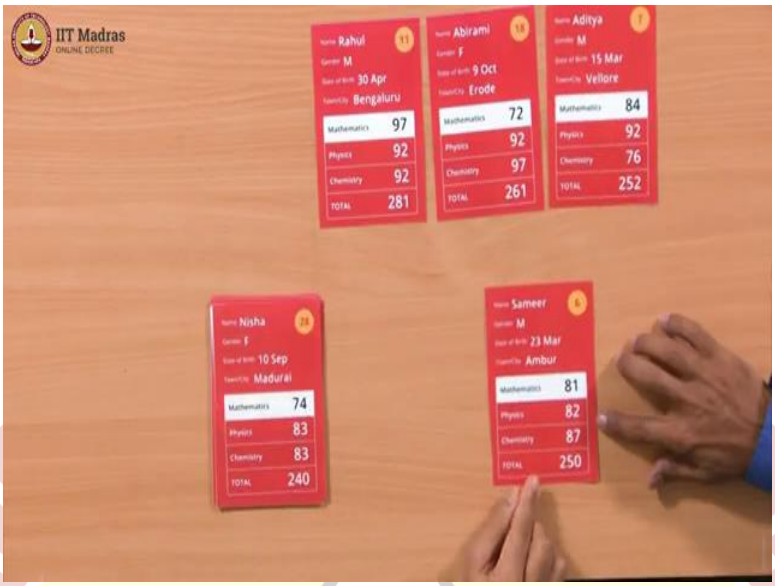
Ok.

210 is also lower.

Ok.

209 is lower.

(Refer Slide Time: 06:12)



Now, we have 252 which is.

More than higher than that.

So, this goes.

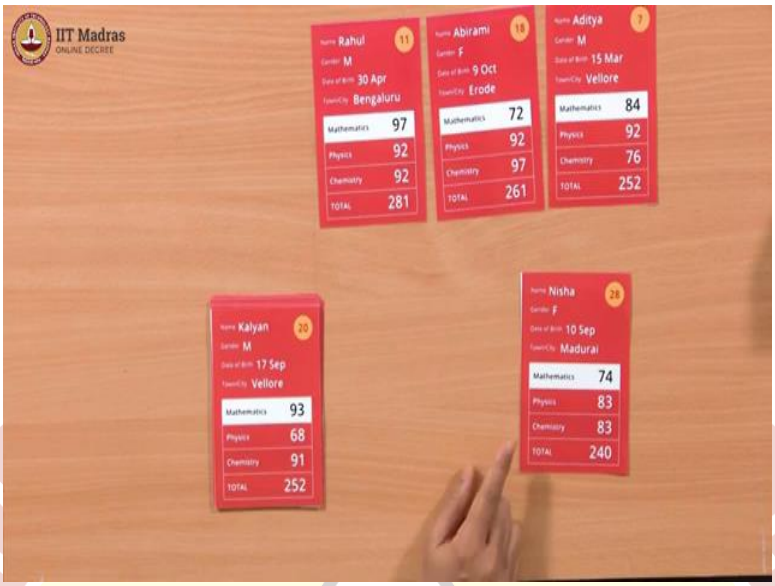
But, it is less than 261. So, it stays in third place.

Alright.

250 which is lower than that.

Lower than this.

(Refer Slide Time: 06:23)



240 which is lower than that. This is gone.

Ok.

Now, we have an interesting situation this is 252 which is the same as.

We saw this earlier also similar type of thing. So, what do we do? We keep both.

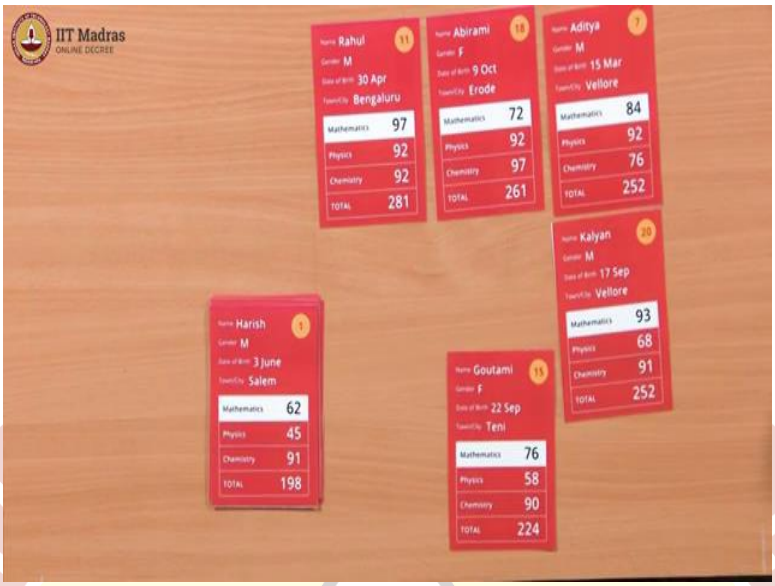
I guess we cannot rule out one based on this.

(Refer Time: 06:28).

So, let us keep them both for now. So, we have two places, two tie for third place.

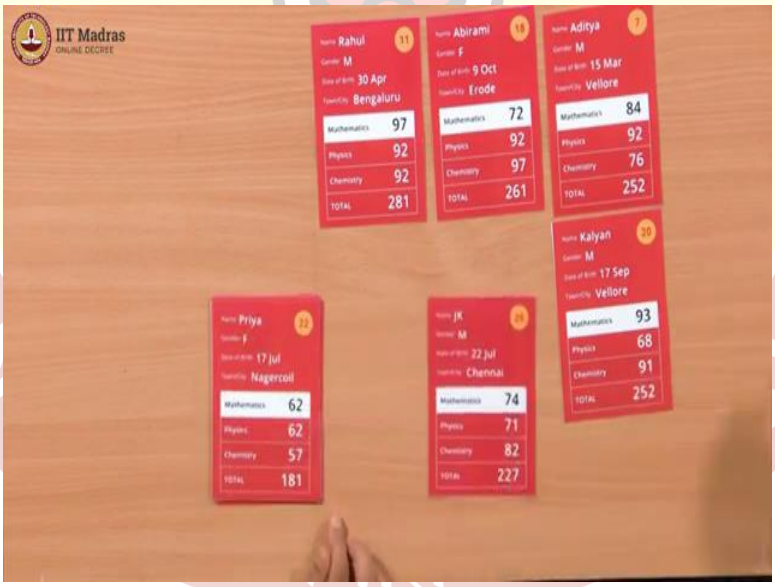
Ok.

(Refer Slide Time: 06:44)



224 is smaller. So, it should go. 198 is smaller. So, it should go.

(Refer Slide Time: 06:50)



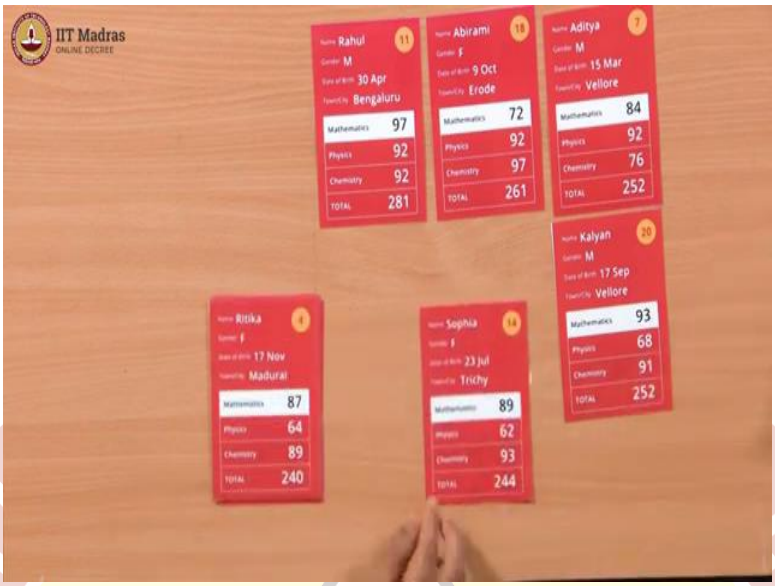
227 is smaller. So, it should go.

Ok.

181 is smaller. So, it should go 244 is smaller.

Ok.

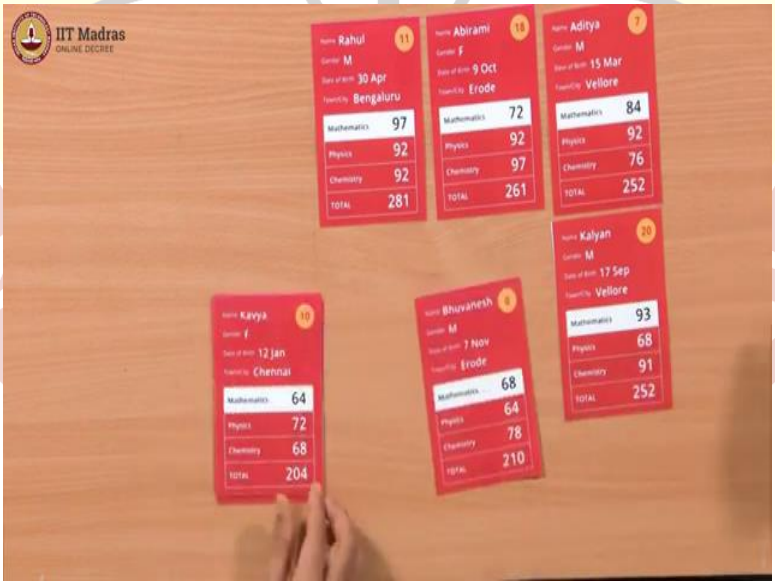
(Refer Slide Time: 06:57)



240 is smaller.

Ok.

(Refer Slide Time: 07:00)

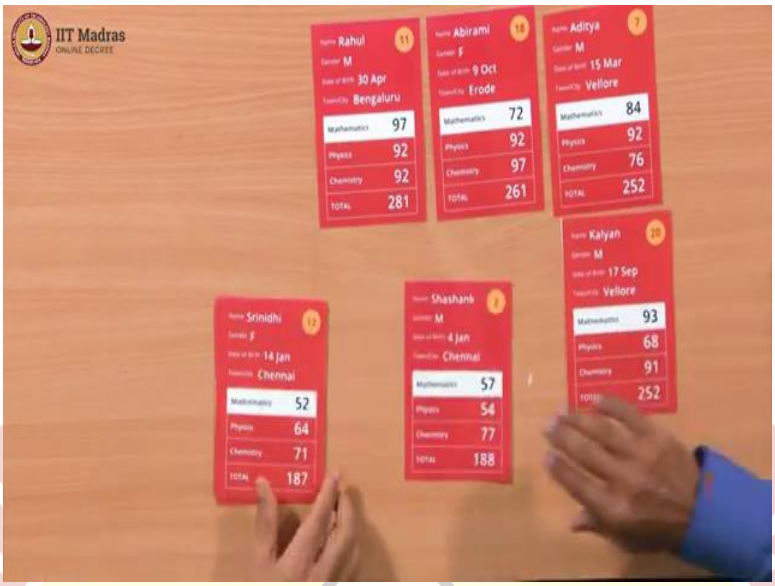


210 is smaller.

204 is smaller.

I think we have got it.

(Refer Slide Time: 07:07)

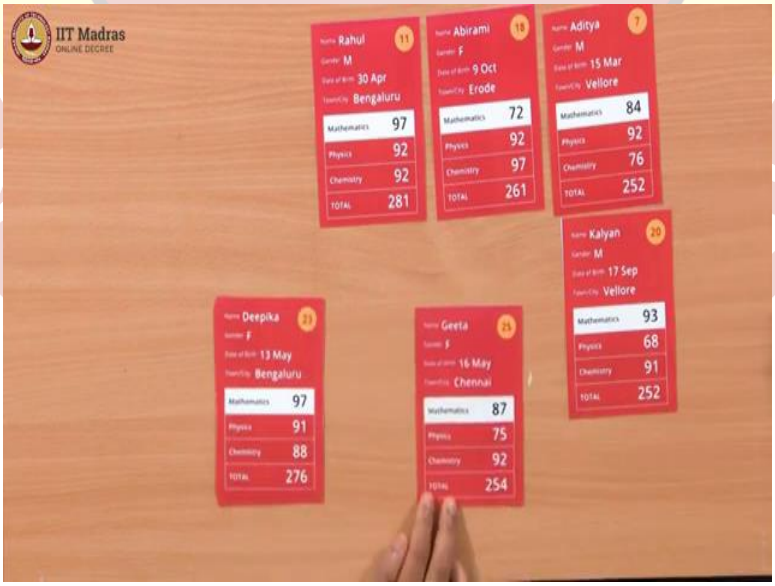


188 is smaller.

Ok.

187 is smaller.

(Refer Slide Time: 07:12)



Now, we have 254 which is.

Bigger than.

So, these two.

Both of them are gone.

Go, but we should remember that that is a situation that we would have had to worry about.

So, we will come back and discuss what to do about that.

Now, we have 276 which is.

So, first.

Which is bigger?

Bigger than.

Bigger than this.

This is gone.

And this.

This also.

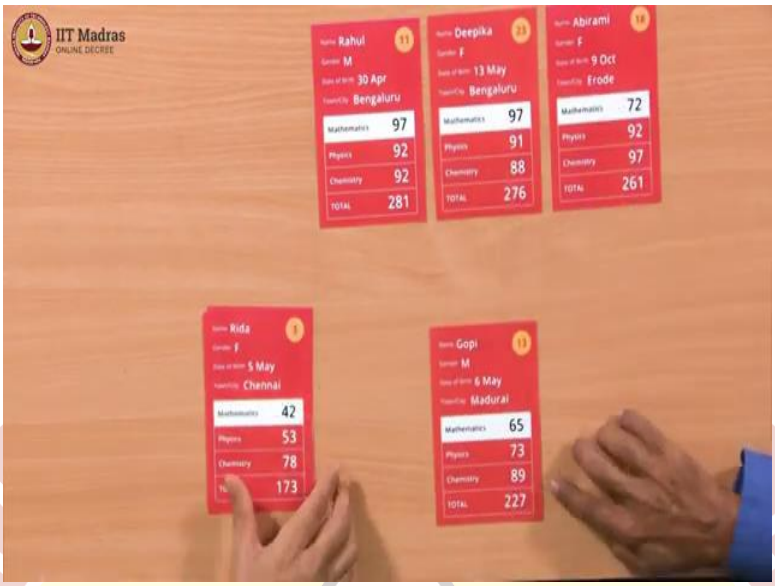
So, now we have to rearrange that.

This.

And make that third place.

Ok.

(Refer Slide Time: 07:36)

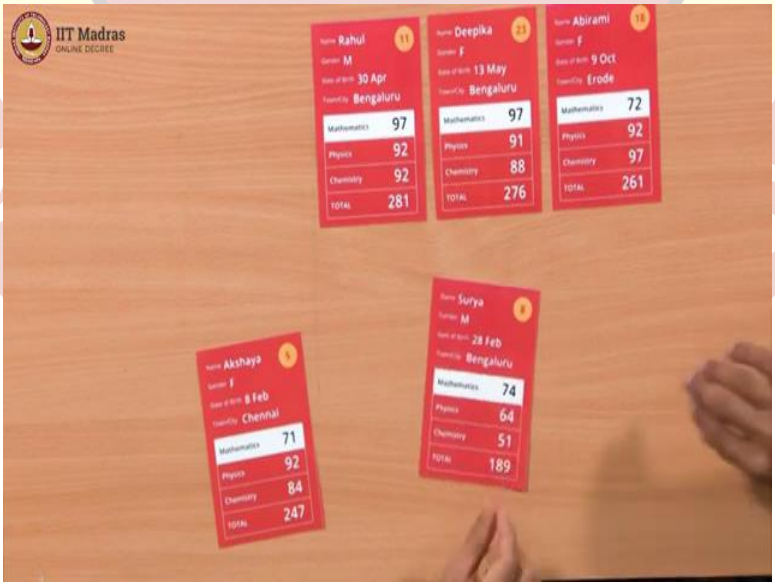


Now, we have 227 which is smaller.

173 which is smaller.

Ok.

(Refer Slide Time: 07:41)



189.

And. finally, 247.

Ok, So we have got the top three.

Now, we got the top three and we have.

Rahul, male.

Rahul – boy, Deepika and Abirami – two girls. So, this was.

So, we have got our mix.

We got our mix. So, now, what remains is to check.

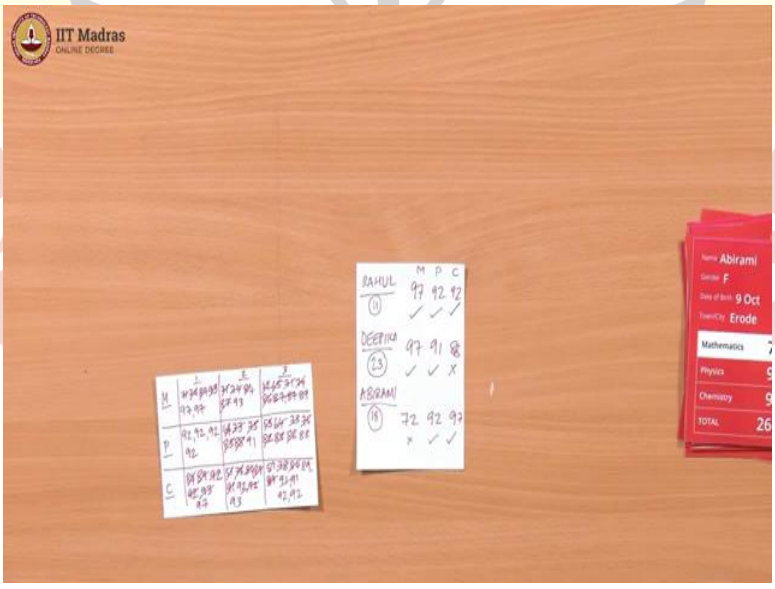
Check whether.

Whether each of them has got within the top three in at least one subject.

At least one subject.

So, that means, that maybe we should make note of these peoples marks maybe for now. So, that we do not have to come back and do this again.

(Refer Slide Time: 08:15)



So, we have Rahul right. So, Rahul has in order of maths physics and chemistry has 97, 92, 92 right and second is Deepika. She has 97, 91, 88 and finally, we have Abirami who has.

72.

72.

92.

92.

And, 97.

97. Now, you know what we need only take the lowest of this if we are going to do it actually right.

Yeah.

And, check whether that is for the highest.

For the lowest.

Lowest right.

But we want to only we want at least one to be there.

At least want to be.

So, for instance if we expect that 97.

Ok.

Will be in the top three in maths.

Ok.

Then we do not have to worry about physics and chemistry.

So let us keep all three we have to keep all three.

Yeah.

Let us keep.

Because distribution of marks in each subject may also be different.

Yeah.

So, we have to see.

We have to see.

Yeah.

Ok, so what do we do now? So, we got. So, now, we have to do this same thing we have to do subject wise. So, we have to find the top three in maths, top three.

So.

In physics, top three in chemistry.

We only need to check that 97 is the max, what do we need to check. So, we have to check that the 97.

Is not the top three in maths.

Is in the top three maths.

Otherwise we have to check 92 is in top three in physics and so on. So, I think the best is that for each subject we calculate the top three and we can eliminate those who have passed the tests.

So, we should do one pass just finding the.

Yeah.

Top three for all subjects.

Let us just do this.

Just like what we do here.

Exactly. So, it is very similar to what we did for the total we have to do the same thing for each subject.

So.

So, let us do it for maths first and then see.

Do we keep this card separately, outside?

No, we have we have marked. We have not.

Ok.

Maybe I should write down their numbers so that we know that 11.

Sequence 11.

Abirami is 18.

Deepika is 23 ok.

So, let us go through this.

Yeah.

Let us go through this I am just randomly it moving around go through this cards and you may keep track of this maximum.

Correct.

Maths maximum three maths maximum three in physics. So, maths 1 2 3; physics 1 2 3.

Ok.

So, we will do this instead of keeping them and do it all in one shot right.

Right.

So, try to do it all in one shot so.

So, 1 2 3 you write like that.

(Refer Time: 10:46).

So, we have three categories.

Yeah.

So this is 1 2 and 3.

And, we will keep changing the number of the card right.

Yeah.

As we go along.

Yeah.

Alright. So, we are starting with Akshaya.

So, we have 71.

71.

92.

92.

And 84.

84.

So, currently these are the first in all three categories.

All three categories right.

Now, here this is higher in maths.

74.

So, we have to.

Replace 71 by 74.

So, we put 74 and 71.

So, 71 goes to the next one.

Yeah.

Yeah.

64 is now the second highest.

64.

And chemistry 51.

51.

So, second highest ok.

Now, Rida maths 42.

So, that is.

Third highest so far.

Physics 53.

That is also third highest.

Chemistry 78.

So, now this replaces the second highest which now moves to the third position. So, after seeing three cards these are our top three.

Ok, now we have Gopi 65, maths.

So, 65 replaces the third it is not second.

65; physics 73.

So, 73 is bigger than.

Smaller than 64.

So, we put 64 goes to second place and now 73 is in second place.

Alright. So, we can similar to what we did with the cards now you are doing.

We said we do not have and we are doing it all at one time we do not it.

All doing it in one shot.

Yeah.

Chemistry is 89.

So, now, this is actually biggest one.

This is biggest one.

So, now 89 comes here 84 moves here and 78 moves here ok.

Ok, Geeta 87.

So, 87 is the highest we have seen so far. So, again we move all of them one position down and put 87 here ok.

75.

So, 75 now comes in second place. So, we have 73 in third place and 75.

92.

92 is the highest. So, the 78 now goes. So, this is now 84, 89 and 92.

Aditya 84.

So, 84 now comes in second place.

Ok.

Compared to this.

Ok.

So.

This 74 goes.

Now, 74.

And, this is 84.

Physics 92.

92 so.

Already there.

There is one more 92. So, we will just.

Keep.

Should be recorded that we have two or we just want to know the top marks. So, we do not need to keep it twice it is ok.

No, but till top three.

Ok.

We need.

Yes (Refer Time: 13:10).

So, there are two actually already 92, 92 and.

Ok.

So, ok.

75 actually 75 may be knocked out.

Right now let us keep it.

Let us keep it.

Let us keep it.

And 76.

So, 76 is out.

Maths 93.

So, now, this becomes highest. So, this goes. So, now, we have 84 87 and 93 ok

Physics 68.

68 is lower.

91.

91 is replaces 89 now. So, 84 is gone and now we have 91.

Over here ok.

Srinidhi 52 in maths.

No.

64 in physics.

No.

71 in chemistry.

No.

Ok.

Shashank 57 in maths.

No.

54 in physics, 77 in chemistry.

No, ok.

Kavya 64 in maths.

No.

72 in physics.

No 73 is the third highest.

68 in chemistry.

No, alright.

Rahul was the topper I think.

97 in maths.

Now this one becomes 97. So, let us start from the right 84 goes becomes 87, 87 becomes 93 and the 93 becomes 97.

You are shifting right.

Yeah.

You are shifting it.

Just shifting it.

Physics 92. Again a 92.

One more 92.

Let us let us keep it chemistry also 92.

Here also we have one more 92 ok

Alright. Bhuvanesh 68 in maths.

No.

Physics 64.

No.

Chemistry 78.

No, alright.

Ritika maths 87.

Well, that is a duplicate let us keep it.

Physics 64.

No.

Chemistry 89.

That is also a duplicate.

Sophia maths 89.

So, now, both these 87s go and get replaced by 89 in third place ok.

This also 89.

They were 87. So, now, this is 89.

89. This is 93.

So, it is in third place.

Third place.

Yeah.

Third place is 89.

You are right actually.

Physics 62.

No, that is gone.

Chemistry 93 so.

Now, (Refer Time: 15:13).

So, now this is on top list. So, now.

Both 92s are.

We have 92.

So, we will.

Now, you move the 92s here.

Move the 91 here move the two 92s here.

And, now we have 93 here.

93 here you move the two 92s ok.

Yeah.

Two of them you have to move.

Yeah.

Priya 62.

No.

Physics 62.

No.

Chemistry 57.

No, ok.

JK maths 74.

No.

Physics 71.

No.

Chemistry 82.

Yes, physics is also below that.

Harish maths 62.

No.

Physics 45.

No.

Chemistry 91.

One more 91 we have a second one.

Ok.

Goutami 76 maths.

No.

58 physics.

No.

90 chemistry.

No.

Marginal ok.

Nisha 74 maths.

No.

Physics 83.

83 is.

83.

So, now 75 moves down here and this comes into second place ok.

Chemistry 83.

83 no.

Sameer.

81.

81.

Yeah.

Math physics 82.

Yes. So, 82 now is in third place because they was already in 83.

Chemistry 87.

No ok.

Clarence maths 63.

No.

Physics 88.

Yes.

Chemistry.

88.

88 here.

88, yes. So, 82 gets replaced by 83 and 83 gets replaced by.

88.

Alright. Very good. Chemistry 73.

No.

Jagan maths 81.

No.

Physics 76.

No.

Chemistry 52.

No.

Arshad maths 62.

No.

Physics 81.

No.

Chemistry 67.

No.

Monica, maths 78.

No

Physics, 69.

No.

Chemistry 70.

No.

Siddharth 44, 72, 58.

No,

Naveen 72, 66, 81.

81.

No.

Tauseef 87.

86.

86 yes 86 is alright.

Ok.

Third place.

43.

No.

Vetrivel 56, 78, 62, no.

56.

78, 62.

No, ok.

Deepika.

Yeah.

Deepika is one of the topper right.

Yeah.

97 ok. So, we have 97 again in maths.

91.

So 91 now comes in second place in physics. So, 86 becomes 88 and 88 becomes 91.

88 ok.

88 is gone.

Ok.

Abirami another topper right Abirami.

Yeah.

72.

No.

92.

Yes another last 92.

Here right.

Chemistry 97 oh (Refer Time: 17:29).

Ok. So, now, we have to knock off.

92.

Two 91s we have two 92s instead then we have a 93 and we have a.

97.

97. So, now, we can go back to our.

Now.

Top three. So, Rahul's 97 is the top mark in maths.

Ok.

So, he is ok.

So we can tick that off.

Now we can take.

We can tick that off. I will just take it off underneath.

Similarly, Deepika's she has. So, Rahul actually does not have the other two because he does he has a actually we can just mark it like that. So, he has a top mark in chemistry also.

Ok.

Third mark.

Physics also.

Physics is 88.

92.

Sorry physics also he has top in all three ok. Deepika has 97 here she is second in this and she is not got a top three mark in chemistry.

So, it is worth ticking ok. Abirami?

And, Abirami has not got in maths for sure because chemistry, yes and physics yes she is got a top mark and chemistry she is the absolute topper.

So, this is good (Refer Time: 18:34).

So, we are actually.

(Refer Time: 18:35).

Yeah.

But, now we have these questions which we would have had had they happened like we found when the individual subject totals we found like four people already have the top mark in physics at 92 and if we had to give three prizes then what would we do? Would we give four prizes or would we have to use some other?

No, you could use a tiebreaker. So, for example, if all of them have equal mark in physics.

Then you can check who is got the better total.

Correct.

Which is what we have done actually this.

Yeah.

We started with the total first.

Yeah.

But had we gone with physics marks or.

Yeah.

Subject mark first.

Yeah.

We are got this card and then we are done a tiebreaker using.

Correct.

So, correctly yeah.

So, we managed to get all the criteria that we wanted for this particular data.

Set.

Set of data, but it could happen that some of the things that we noticed while we were going through could have happened.

So, one thing that could happen for example, is that somehow one of these people could have not got the highest in all three.

Correct.

Then what would we do then?

So, that is a very good question then do we bring in a fourth person who has the next highest total which is at least.

Or we do not give that award at all.

Yeah.

That is also. So we can decide that.

So, we say we will give three awards, but again you know there is a problem there. If you say I am going to give only two awards that both of them happen to be girls again that is a problem.

Correct. So, I think all of these situations need to be thought about. So, I think that is the most important lesson I guess that when you make these kinds of choices you have to also decide what is reasonable according to what you are trying to achieve.

So, again here we have what we are seeing right. We are seeing basically that we have a abstract problem we started with right. We are saying can we give three prizes to students? Very.

Top three top three students.

Top three students it is very common.

Yeah.

When you have when you want to give prizes it is a common problem right.

Yeah.

And, you see basically that it is I mean the minute you start looking at the data you find that it is not such an easy.

Correct.

Thing to do actually because you have to come up with the right criterion, it has to be fair.

Correct.

And, the criterion has to be such that it selects people unambiguously right I mean it is not like should not be that somebody comes and contests you are not able to defend why you are giving the prize to these three people and not to somebody else.

Yeah.

Right? So, you should be able to defend it and in this case we are able to defend it because we have the criteria is very clear.

We are saying that they should be a topper one of the top three in overall as well as.

The total.

One of the top three at least one subject.

And we are ensuring basically some amount of fairness by making sure that at least a boy and one boy and one girl is represented in the set right.

Yeah.

Good. So, I think we were lucky that we got away with it because if we had not done this then we would have come into a question.

Correct.

Where somebody.

Correct.

Where somebody would have asked.

That is right.

That question.

Yeah.

Ok.