IIT Madras

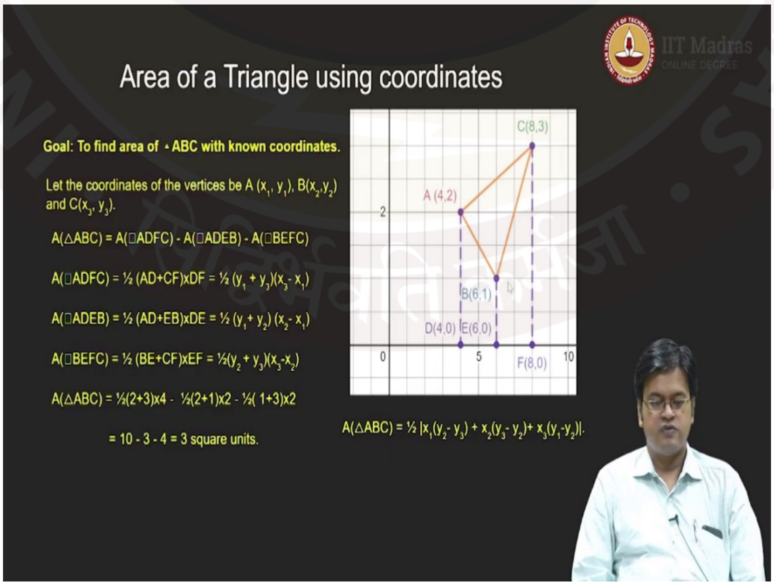
ONLINE DEGREE

Mathematics for Data Science 1
Prof. Neelesh S Upadhye
Department of Mathematics
Indian Institute of Technology, Madras

Lecture – 16
Slope of a Line

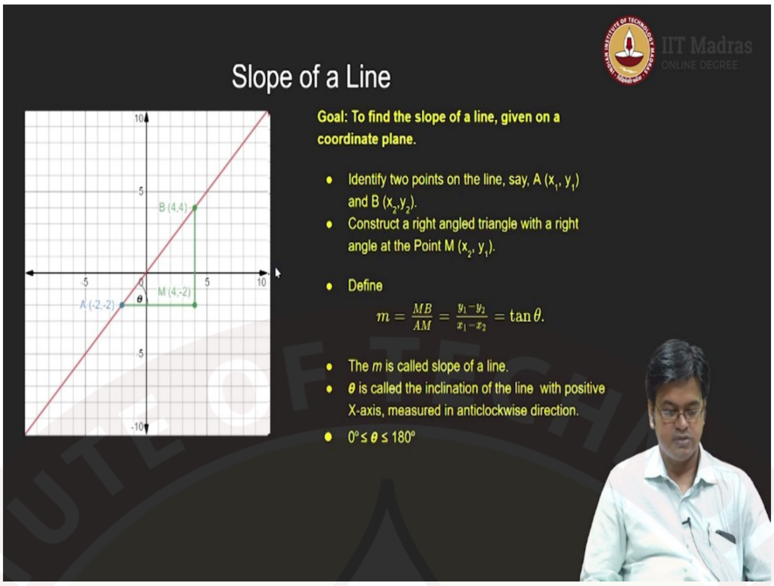
(Refer Slide Time: 00:05)

(Refer Slide Time: 00:14)



So, after looking at area of the triangle using coordinates, let us now focus our attention to again a two-point system and one-dimensional objects that is a line.

(Refer Slide Time: 00:24)



We have already seen in our basic classes that two points uniquely determine line. Now, if I want to characterize a line, and if I give you two points, I should be able to find a line passing through these two points. How is the geometric object algebraically related to the coordinate geometry? That is what we want to explore now, to explore that I need a concept of a slope of a line.

So, what essentially is the slope of a line? In a vague manner, what we understand by slope of a line? If you look at this coordinate plane which is displayed here. If I am moving some units in x directions; the question can be asked with respect to this change in x direction what is the corresponding change in y direction.

So, if I want to answer that question then I need to consider a ratio of change in y direction to change in x direction; some people call it as rise by run ratio, run is in the horizontal direction, rise is in the vertical direction. So, you can consider slope of a line as a rise by run ratio. So, let us try to make this work concept clearer by showing some examples.

So, now, here is a line with two points given onto it. Again, our standard conventional method we will construct a right-angled triangle using these two points. Now, the question that I posed is what is a rise by a run can be answered over here. For example, you look at this right-angled triangle, what is happening? This is the movement of a line in moving from one point to another point in y direction, this vertical length is the direction, is the movement of a line in moving from one point to other point in y direction and this horizontal line is a movement in x direction while moving from point A to B on a line.

So, essentially what I need to capture is the change in y direction that is from point $(4,-2)$ to point $(4,4)$, that is -6 and moving in x direction from $(-2,-2)$ to $(4,-2)$ that means, -6 here also. So, the slope of a line can be equal to 1. This we can make it more precise by giving some formal definitions.

So, if I want to find the slope of a line given the coordinate plane, I can always identify these two points as (x_1, y_1) and (x_2, y_2). I will construct a right-angled triangle which intersects the point at(x_2, y_1). And once I constructed, as I mentioned you know what is the change in x direction and what is the corresponding change in y direction, therefore, you can actually compute the ratio of this. But while computing the ratio, you can also think remember some concept from trigonometry.

For example, when I constructed this right-angle triangle there is some angle formed over here this arc denotes that angle. Let us call that angle as theta. Now, what I am saying is change in y upon change in x, but can you relate some quantity related to this trigonometric ratio that is tan of θ, right. So, what I can say is my m or the slope of a line is MB by AM which is $\frac{y_1 - y_2}{x_1 - x_2}$and which is also equal to$\tan\theta$.

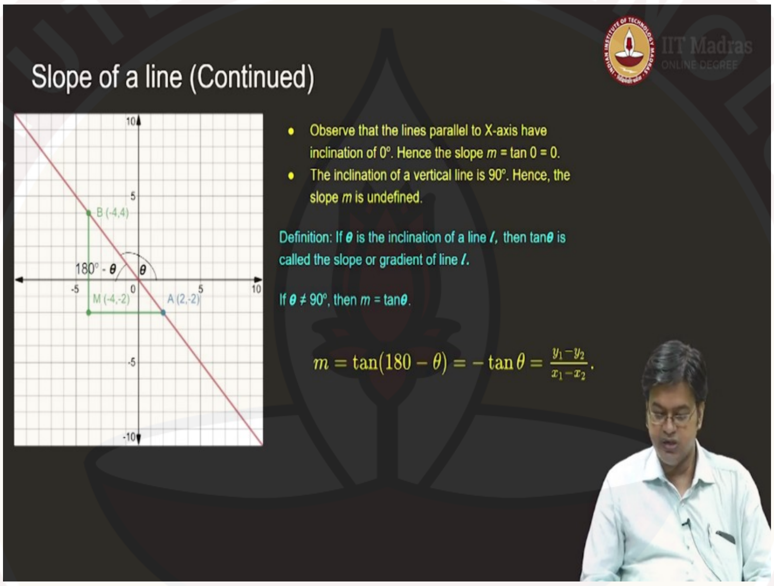
So, I have defined one thing that is m which is the ratio of these two, but which in turn turned out to be equal to$\tan\theta$. So, if it is $\tan\theta$, see here it does not matter whether I take $y_1 - y_2 \vee y_2 - y_1$whatever I am doing I should do synonymously. For example, if I have taken $y_2 - y_1$then I should take $x_2 - x_1$or if I have taken $y_1 - y_2$ then I should take $x_1 - x_2$.

So, it does not matter which order you are swapping because finally you are taking the ratio so whatever you are doing you do it asynchronously, so that there will not be any confusion. So,$m = \tan\theta$. Now, I have introduced two terminologies here m and θ. So, let us define them properly. This m is called slope of a line, which is the topic of this discussion. And then this θ is called inclination of a line with respect to positive X - axis measured in an anti-clockwise direction.

Now, somebody may say I have drawn this angle over here, but if you look at this particular line, this line is parallel to X - axis. And this line is intersecting X - axis here, that means even if I consider this angle, this angle also will be $\theta$from the basics of geometry, correct.

So, now the question can be asked how far the θ can go? So, to answer that question let us try to see if I am considering a θ then θ can be equal to 0, θ equal to 90 degrees tan is not defined. As you can see tan of 90 is not defined, but it can go up to 180 degrees. So, the variation of θ allowed is 0 to 180 degrees.

(Refer Slide Time: 07:42)



So, now let us have a look at the salient features of the slope of a line. In particular, let us see if the line is parallel to X - axis the angle of inclination is 0 degrees; therefore, the slope of a line should be 0. Now, if the angle is 90 degrees; that means, 90 degrees with respect to X - axis; that means, eventually I am on Y - axis or in fact, I am on Y - axis in such case tan 90 is undefined, right. Therefore, slope is undefined.

As you can see if I have an angle which is 90 degrees that is Y - axis; that means, x is equal to constant is the equation of the line. And you cannot have any movement in y direction or you can have infinite movement in y direction without any change in x direction. That itself creates a problem therefore, the slope is undefined for theta is equal to 90 degrees or the inclination is equal to 90 degrees.

So, with respect to inclination there is another definition of slope. If theta is the inclination of a line l then tan theta is called slope or gradient of the line. This is the second definition of our slope of a line which matches exactly with the original definition, but there will be some glitch, there may be some confusion, ambiguity.

So, let us try to resolve that ambiguity because this theta is the angle made with respect to positive X - axis. And theta not equal to 90 degrees I can define $m=\tan\theta$. That is perfectly fine and it is well-defined over there whenever it is not equal to 90 degrees. What is the ambiguity? The ambiguity can be shown in the figure. For example, now what is θover here? θ over here is actually this particular angle.

Now, if I you look at this particular angle which is θ you can see that this is an obtuse angle. Now, how to evaluate a tan of this angle? We already know some methods, but will that contradict with our definition of slope. That is the question. So, if I use the rise by run formula or the change in y to up on change in x formula, how will I figure out the slope? So, the answer is I will simply drop a perpendicular or I will construct a right-angle triangle with right angle at point M which is $(-4,-2)$.

In that case, I will be interested in this angle that is angle at A in our older definition or this angle is essentially equal to $180-\theta$. So, let us go further. This angle is equal to this angle. What is the measurement of this angle? It is$180-\theta$. That means, if I want to find a slope according to our definition that is $\frac{\delta y}{\delta x}$or change in y by change in x, then I need to consider the angle of this particular structure that is $\tan(180-\theta)$. So, $m=\tan(180-\theta)$.

Now, what is $\tan(180-\theta)$? If you use simple trigonometric formula you will get $\tan(180-\theta)$ is nothing, but $-\tan\theta$. But what is $-\tan\theta$? You can easily see what is $-\tan\theta$which will be $\frac{y_1-y_2}{x_1-x_2}$. So, in short, our formula for slope is consistent no matter which definition we use, therefore a slope of a line is uniquely determined given a line.