IIT Madras

ONLINE DEGREE

Statistics for Data Science - 1
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Lecture No.10.2
Binomial Distribution- Independent and Identically Distributed Bernoulli Trials

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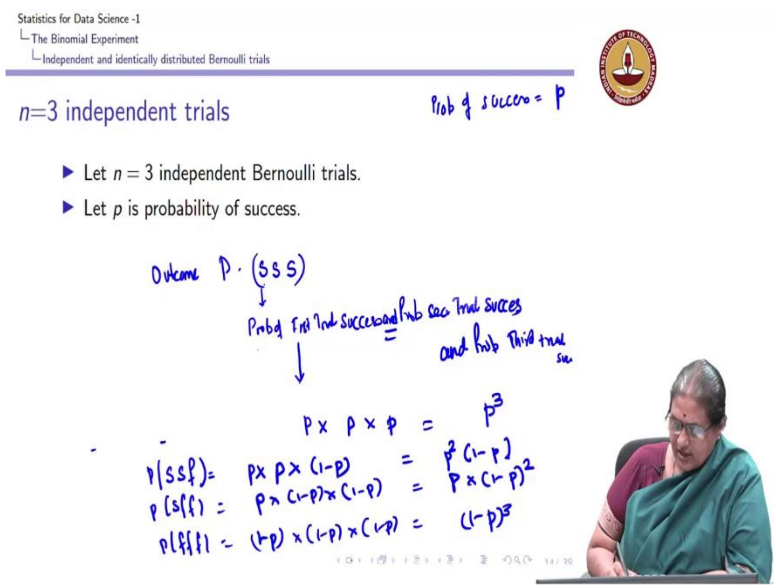

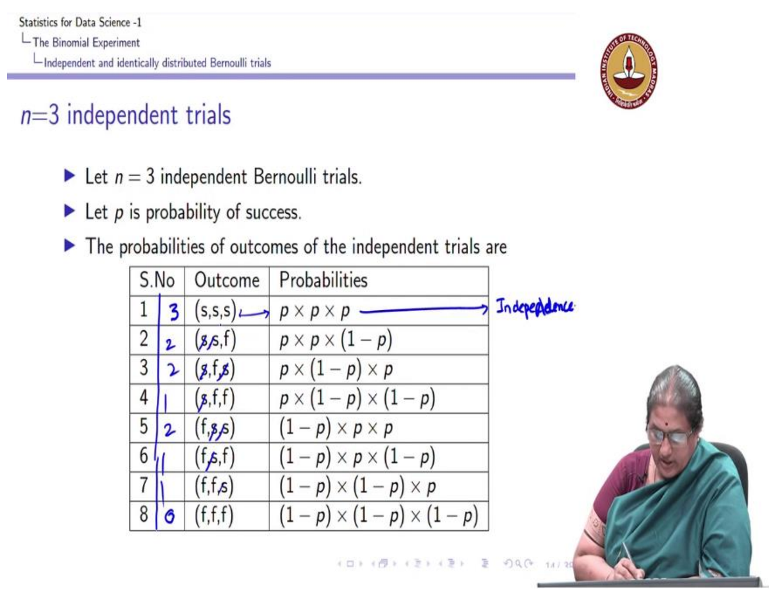

You can see that I can enumerate or I can find out the probability of each of these outcomes and to compute this probability, I use the fact that the independence of the trials, so I list out the probability of each of these outcomes. So, you can see that each of these outcomes I have listed out the probabilities.

But, what is a binomial random variable? The binomial random variable was again a model for counts what did I count I counted the total number of successes in the outcomes. Now, if I am going to map it I see in this outcome I have 3 successes. In this outcome I have 2, in this I have 2, in this I have 1, in this I have 2, in this I have 1, in this I have 1, in this I have no success. So, if what this counts is the number of successes in these outcomes.

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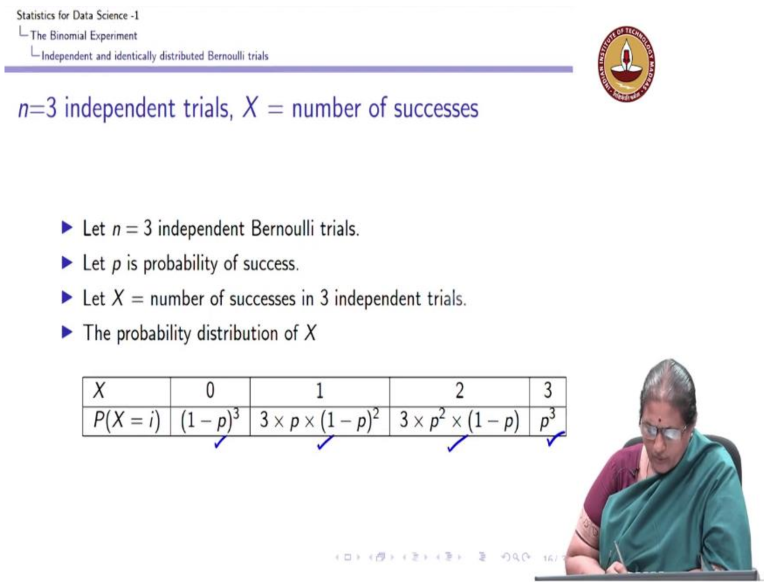

So, I can map this X, or the random variable to be the number of successes so I have a 3, 2 ,2, 1, 2, 1, 1, 0 the probability is remain the same. So, I now I can see that the chance of X, where X is the number of successes taking the value 3, 3 appears only 1 with just p^3. Now, the chance again they are independent they are identically distributed the chance of X taking the value 2 is equal to this plus this plus this. So, how many times does that occur? 1, 2, 3 so 1, 2, 3 so it would be $3 \times p^2 \times (1-p)$.

Similarly, I see that the chance of X taking the value 1 equal to $p \times (1-p)^2$ and that appears 3 times and the chance of X taking the value 0 or no success is $(1-p)^3$. So, I have now what is what I refer to as the probability distribution of the number of successes in $n = 3$ independent Bernoulli trials with a p which is a probability of success,$(X = i) = (1-p)^3, 3 \times p \times (1-p)^2, 3 \times p^2 \times (1-p), p^3$.

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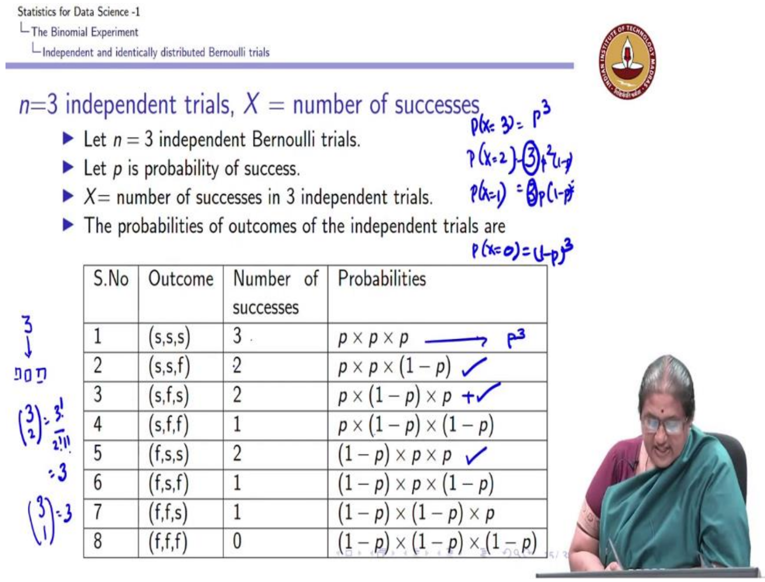

Now, what I want you to notice is, how did I get this 3, now when I look at the number of successes is equal to 2. Now, what I mean by number of successes equal to 2 is the way I can I have 3 trials, I need 2 of these trials to be successful. So, I am in other words I am choosing 2 positions out of the 3 positions, where I can have a success and I know I can do this in ${}^3C_2$ ways which is $\frac{3!}{2!.1!}$ which is 3 and that reflects in this 3. Similarly, choosing 1 success is also can be done in ${}^3C_1$ way, which is again the same as 3 and that reflects here.

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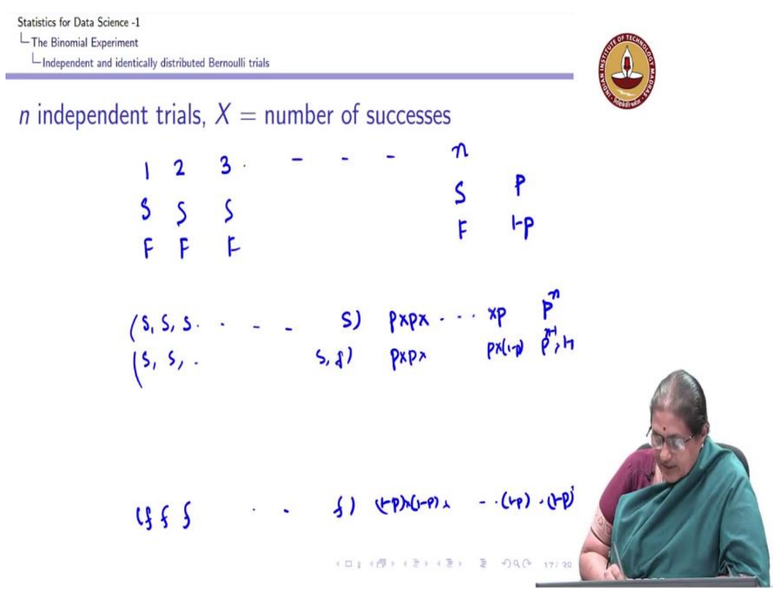

So, in other words if I am going to extend the notion of 3 independent trials to n independent trials, again I have trial 1, trial 2, trial 3 up to n independent trial each 1 of them results in a success or failure the probability of getting a success is p and failure is $(1-p)$. So, the possible outcome is I could have all the n successes, the probability with which it happens is $p \times p \times p$ ... which is p^n.

The other extreme is all failure, which can happen with probability $(1-p) \times (1-p) \times (1-p)$ ... which is $(1-p)^n$. I could have my first $(n-1)$ successes and the last failure this would happen with $p \times p \times p \ldots \times (1-p)$ which is $p^{n-1} \times (1-p)$.. In other words, I have to choose $(n-1)$ positions from n positions and that can be done in n ways or ${}^nC_{n-1}$ way.

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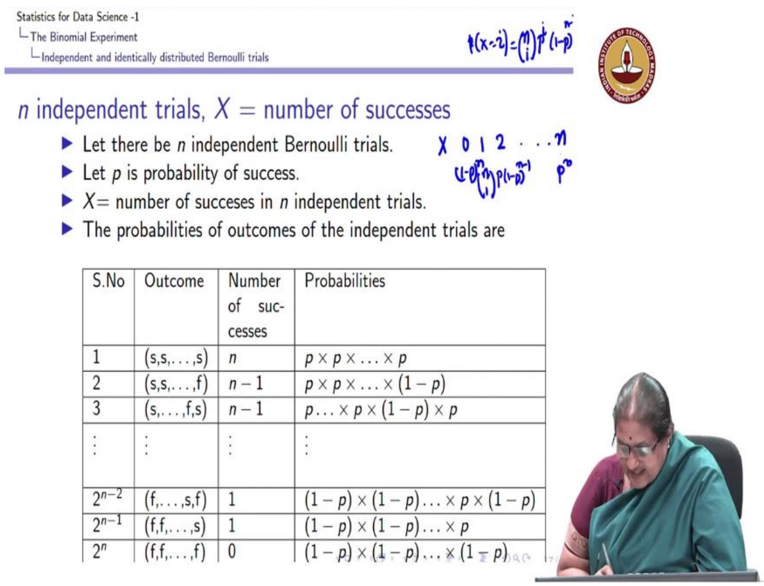

So, I can continue it in this form and I can show that the way I have obtain for each of these outcomes and I have n independent trials. So, I have my variable X, which now takes the value 0 1, 2 up to n. Now it takes the value 0 with probability $(1-p)^n$, it takes the value n or all of them or successes with probability p^n. It takes the value 1 it tells me that if I take the value 1 all I have $(n-1)$ failures and 1 success this success can occur in any 1 of these points I can choose it in 1 way, I have $p \times (1-p)^{n-1}$.

Similarly, $P(X=i)$, I have i successive p^i, $(n-i)$ failures $(n-i)$ and I can choose this i successes as ${}^nC_i$ way.

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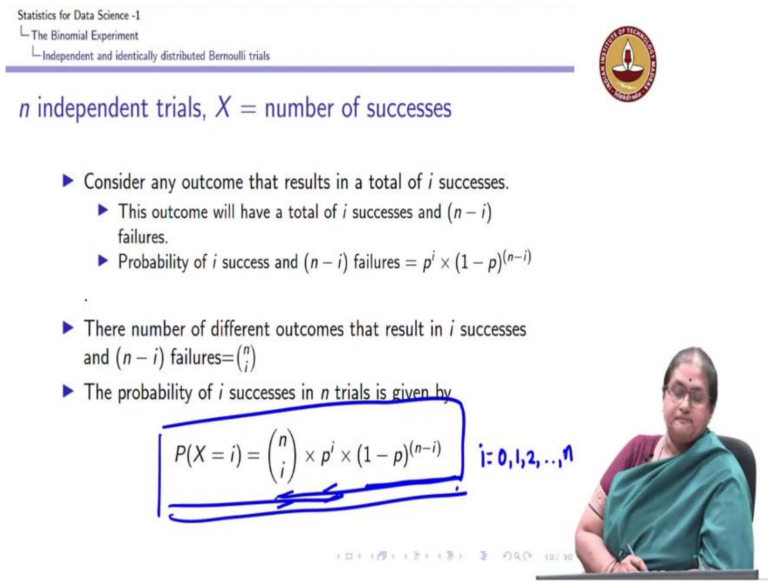

So, this gives me the general formula for the probability of i successes and $(n-i)$ failure to be $p^i \times (1-p)^{n-i}$ and this appears in ${}^nC_i$ outcomes. Hence, the probability of i successes in n trials is ${}^nC_i \times p^i \times (1-p)^{n-i}$. This is the probability mass function of a binomial random variable, i can take any value 0, 1, 2 up to n.

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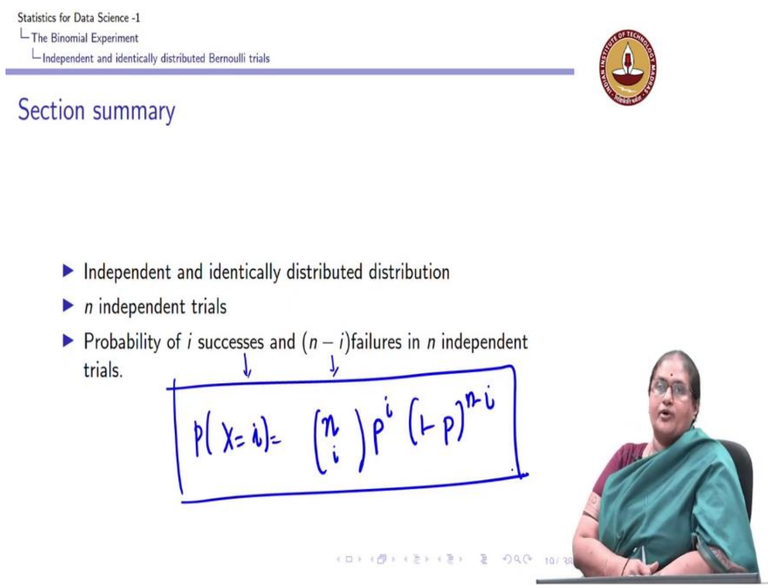

So, this is how we can see that independent and identically distributed Bernoulli random variables, how the binomial distribution arises naturally out of these n independent trials, where probability of i successes and $(n-i)$ failures is given by the expression $P(X=i) = {}^nC_i \times p^i \times (1-p)^{n-i}$. this is the key derivation of the binomial probability mass function.