Statistics for Data Science -1

Lecture 7.2: Conditional Probability: Definition

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Learning objectives

1. Understand notion of conditional probability, i.e find the probability of an event given another event has occurred.

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2. Distinguish between independent and dependent events.
3. Solve applications of probability.

Conditional Probability

Multiplication rule

Independent events

Bayes' rule

Introduction

- We are often interested in determining probabilities when some partial information concerning the outcome of the experiment is available. In such situations, the probabilities are called conditional probabilities.

Example: Roll a dice twice

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- Experiment: Roll a dice twice

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- Sample space:

$$S = \left\{ \begin{array}{l} (1,1),(1,2),(1,3),(1,4),(1,5),(1,6), \\ (2,1),(2,2),(2,3),(2,4),(2,5),(2,6), \\ (3,1),(3,2),(3,3),(3,4),(3,5),(3,6), \\ (4,1),(4,2),(4,3),(4,4),(4,5),(4,6), \\ (5,1),(5,2),(5,3),(5,4),(5,5),(5,6), \\ (6,1),(6,2),(6,3),(6,4),(6,5),(6,6), \end{array} \right\}$$

- Each outcome is equally likely to occur with a probability of $\frac{1}{36}$

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- Among outcomes in the restricted sample space, the outcome that satisfies the sum of dice is 10 is outcome $(4,6)$. And this happens with Probability $\frac{1}{6}$

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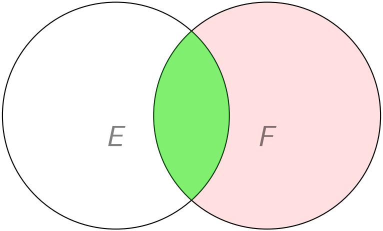
$$P(E|F) = \frac{P(E \cap F)}{P(F)}; \; P(F) > 0$$

Conditional probability: Venn diagram illustration



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- $P(E|F) = \frac{P(E \cap F)}{P(F)}$
- $\frac{P(E \cap F)}{P(F)} = \frac{P(\{(4,6)\})}{P(\{(4,1),(4,2),(4,3),(4,4),(4,5),(4,6)\})} = \frac{1/36}{6/36} = \frac{1}{6}$

Section summary

1. Introduced notion of conditional probability
2. Formula: $P(E|F) = \frac{P(E \cap F)}{P(F)}$