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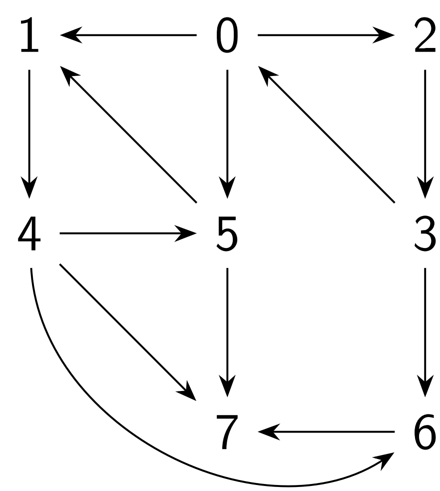

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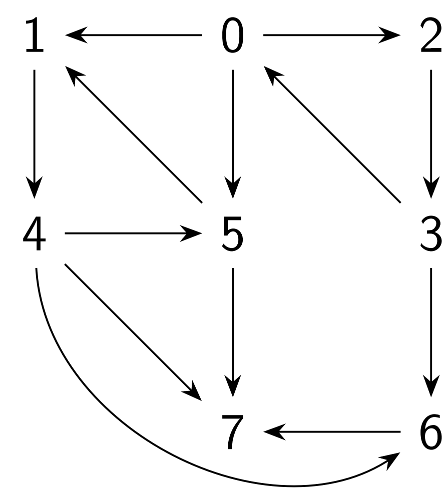

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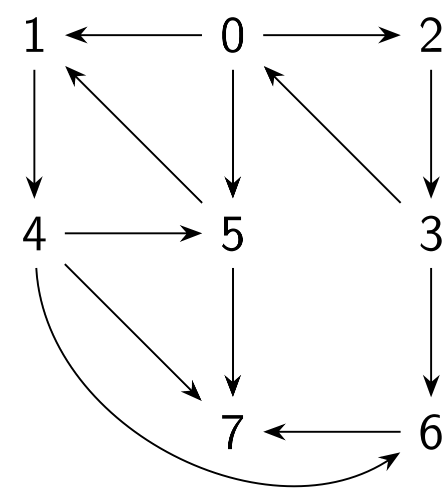

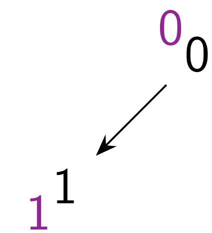

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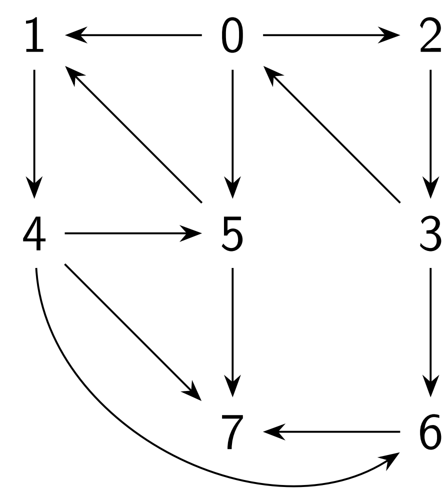

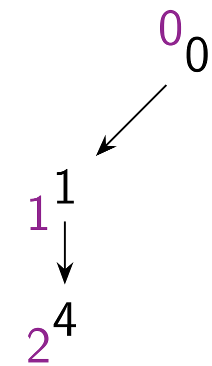

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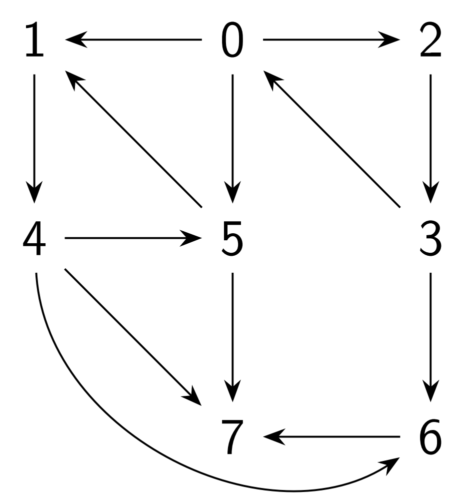

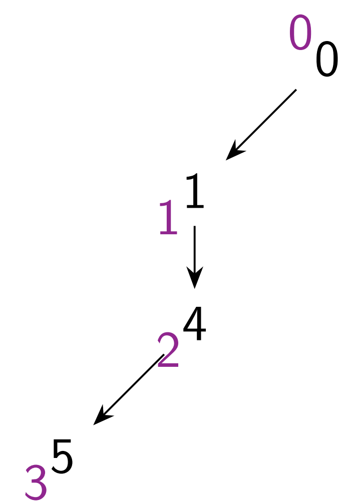

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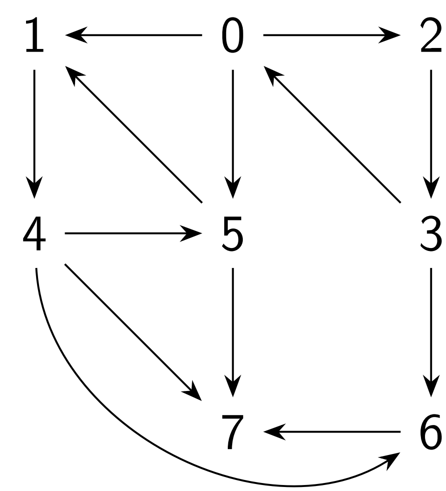

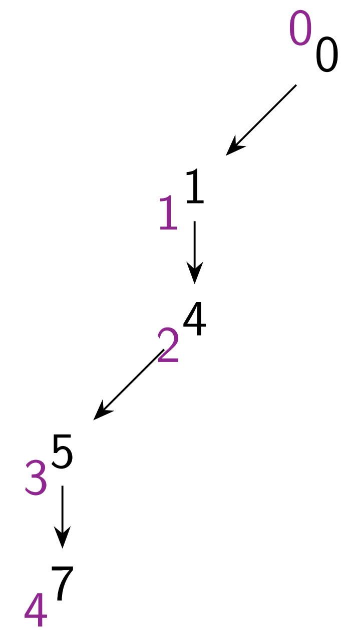

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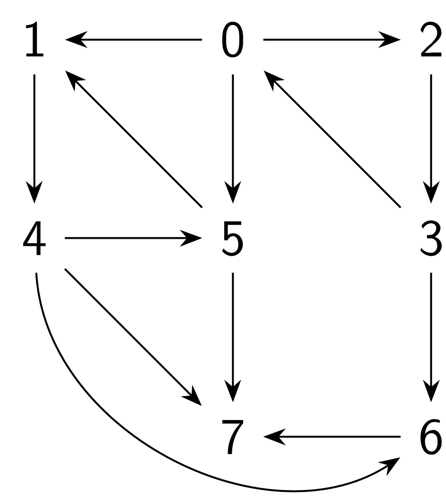

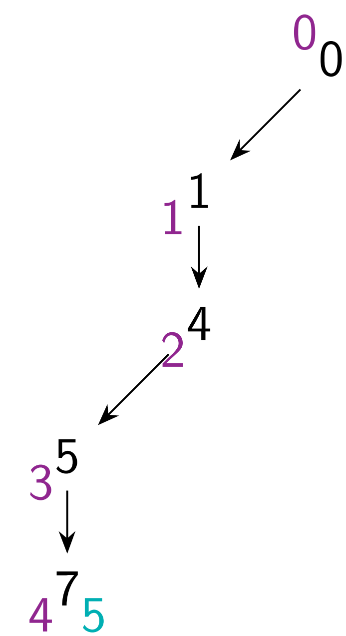

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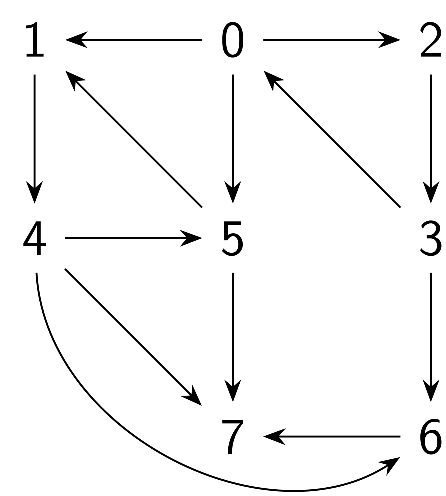

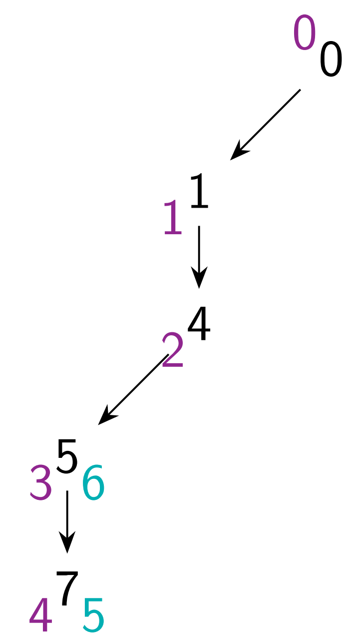

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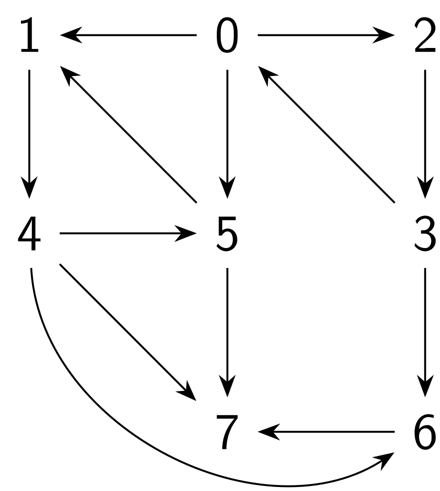

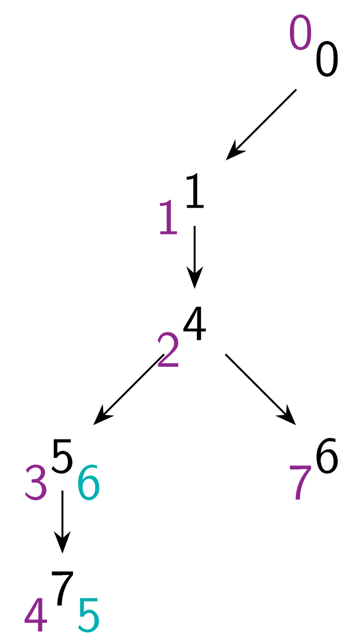

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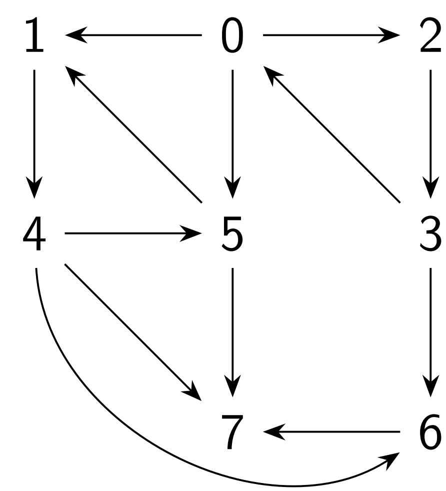

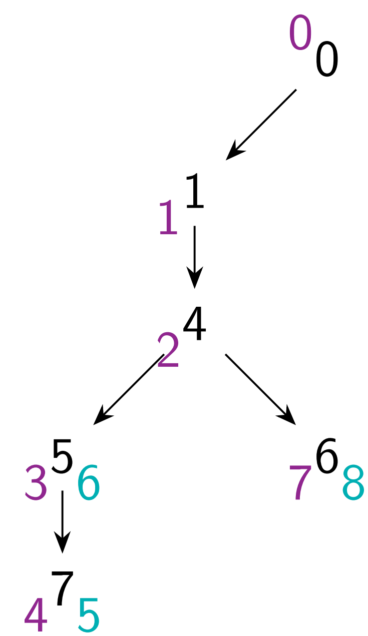

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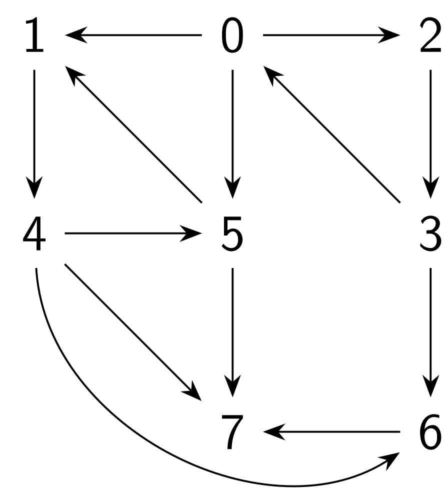

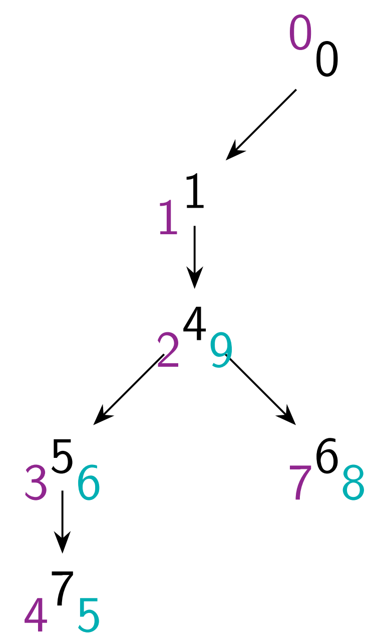

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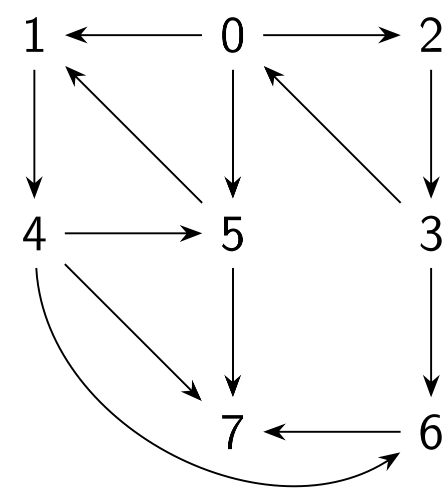

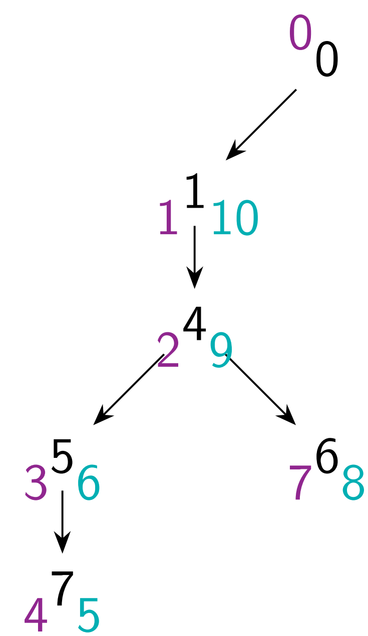

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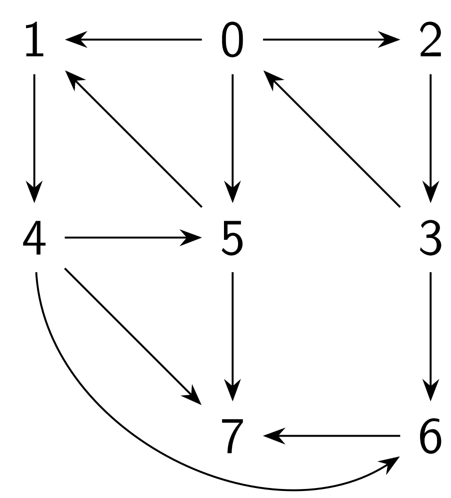

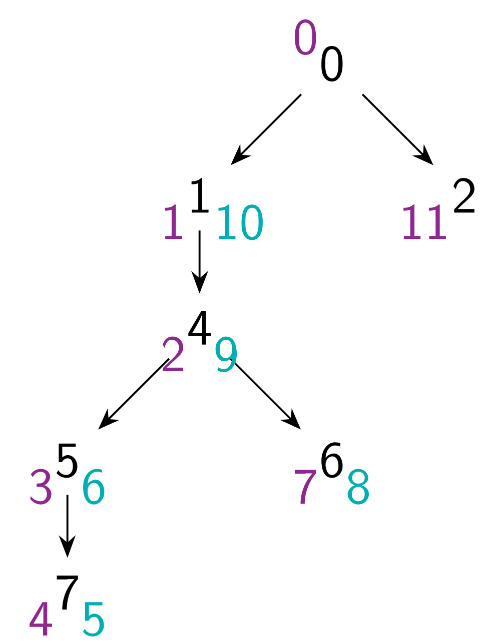

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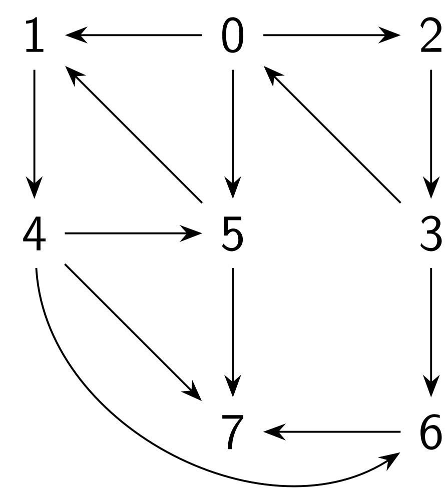

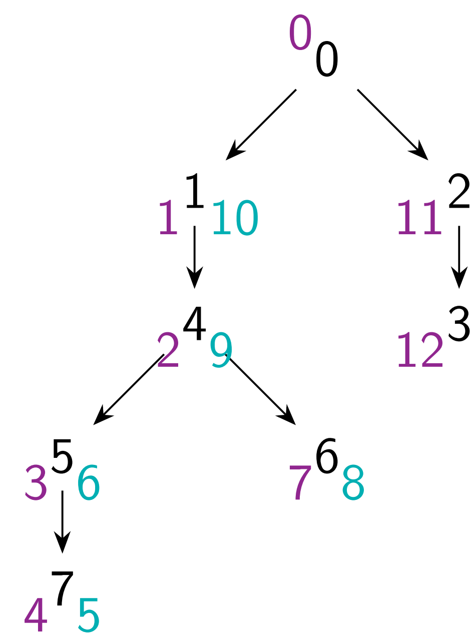

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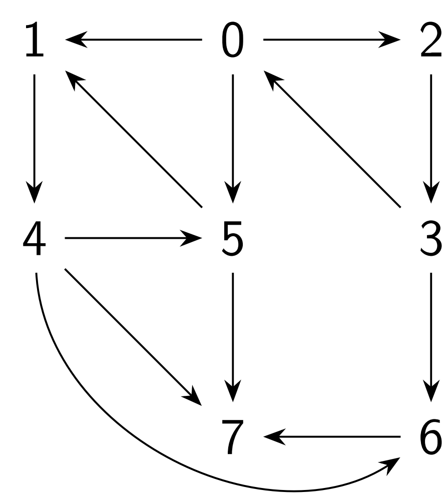

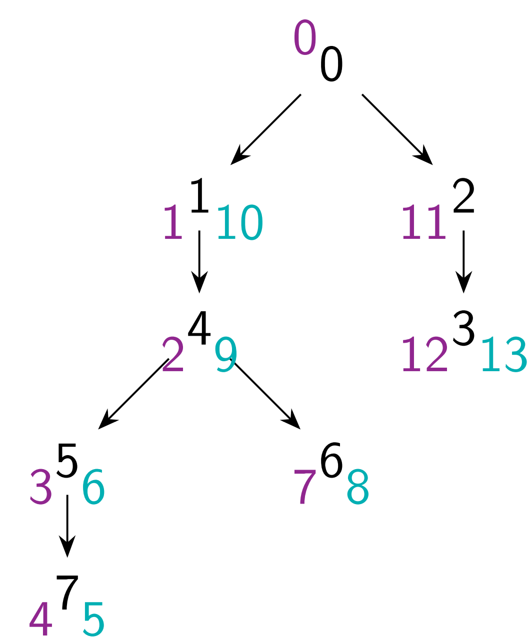

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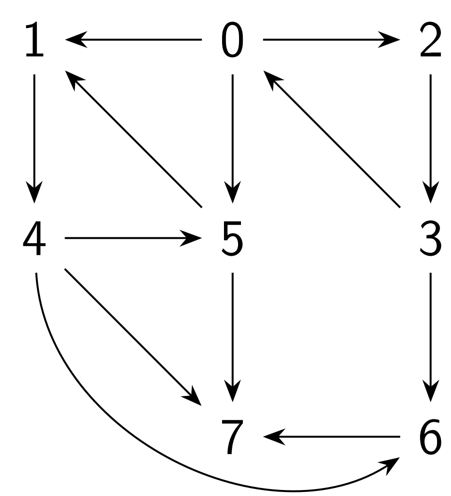

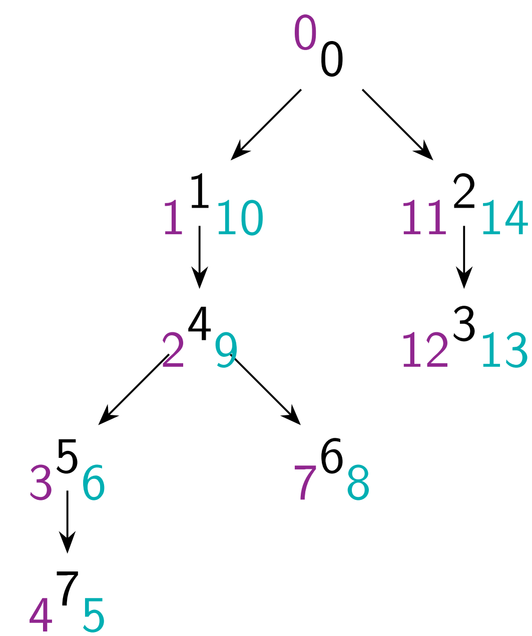

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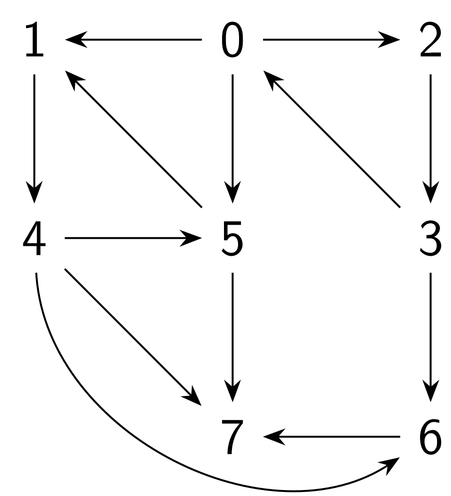

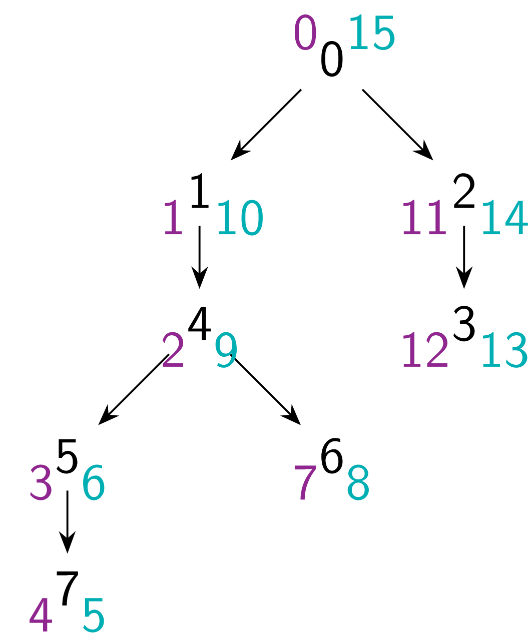

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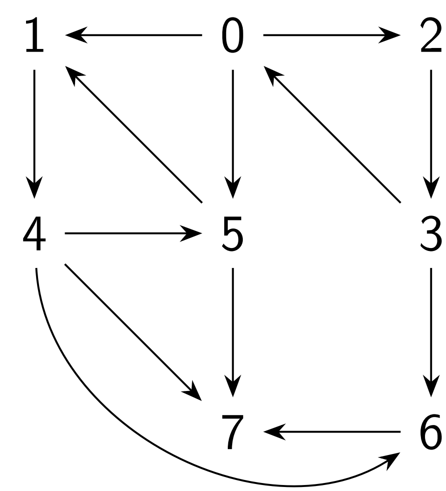

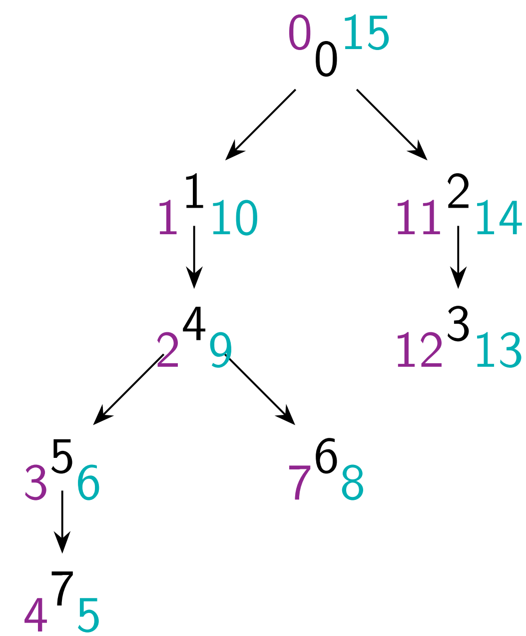

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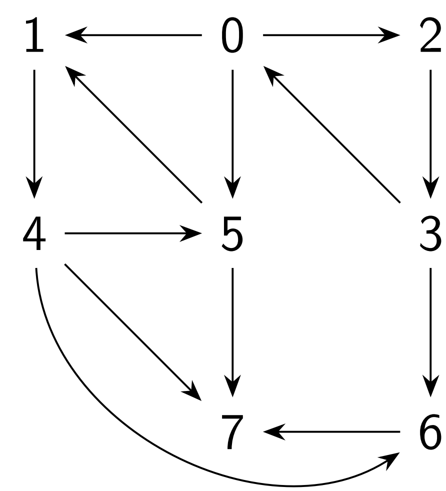
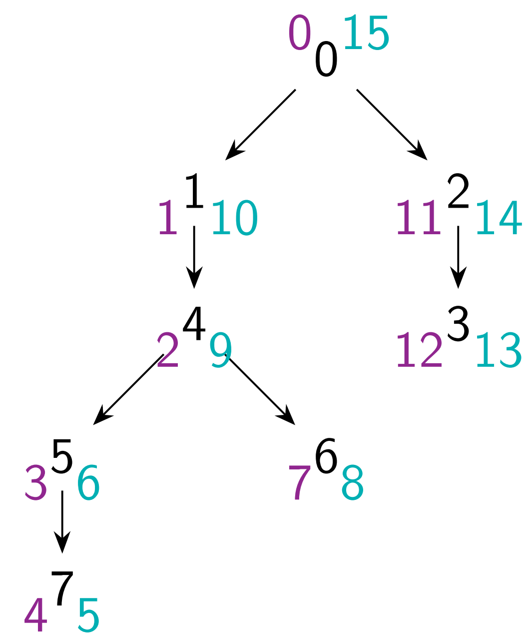
- Different types of non-tree edges





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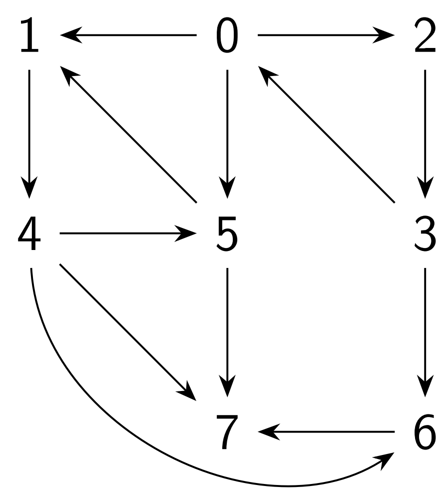

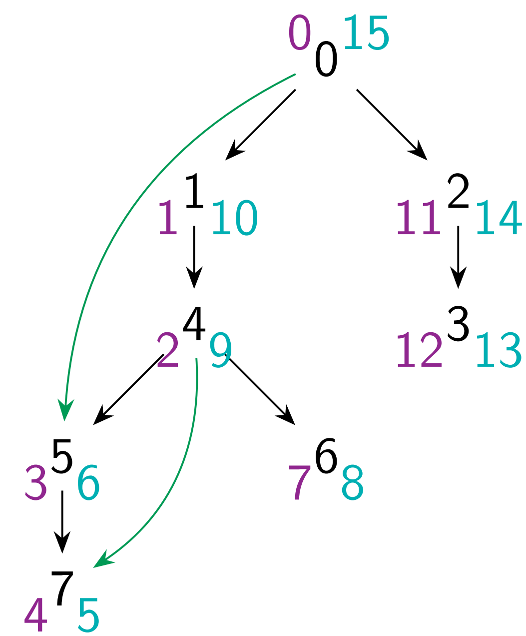

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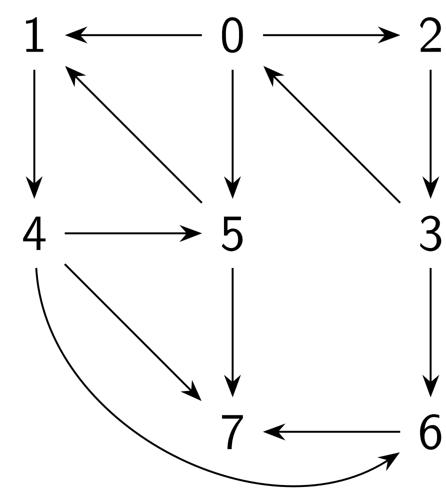

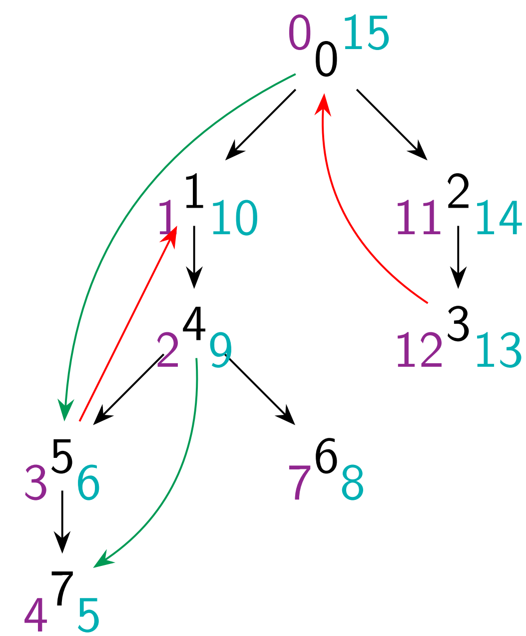

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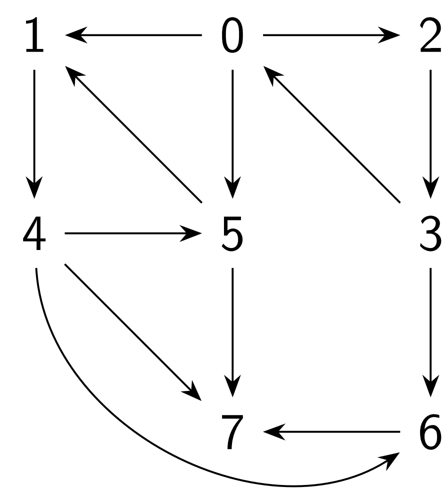

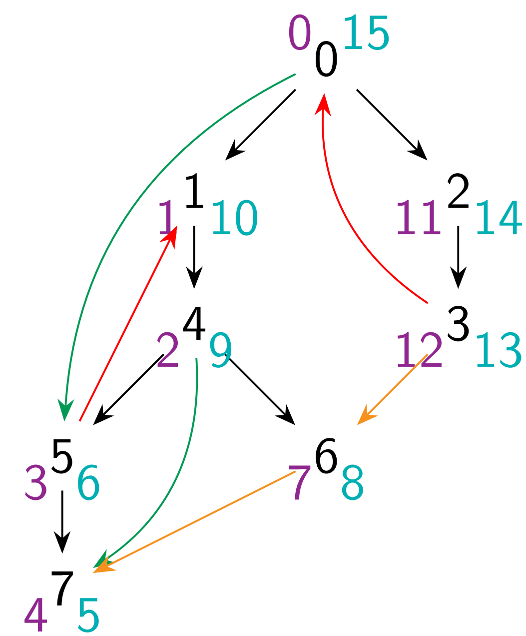

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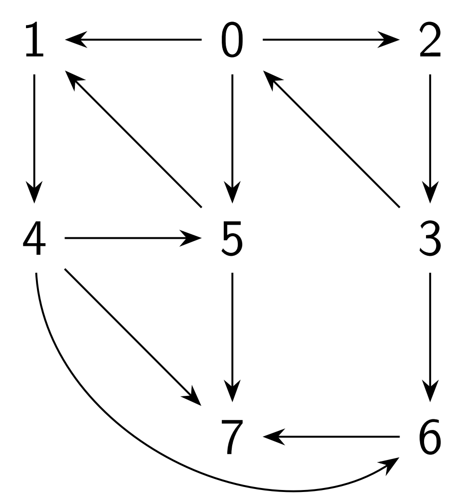
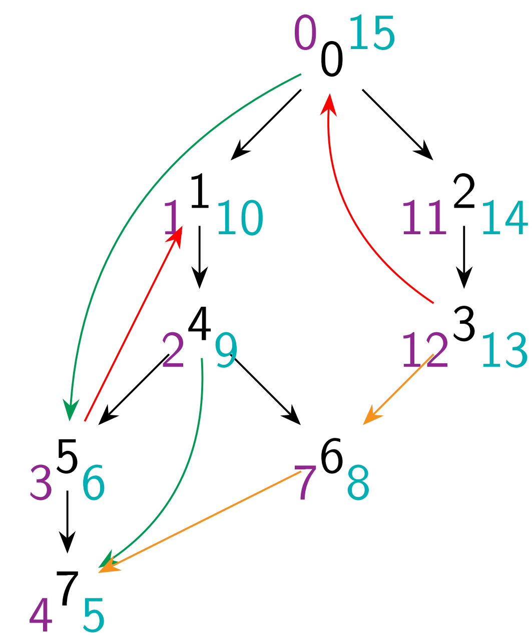
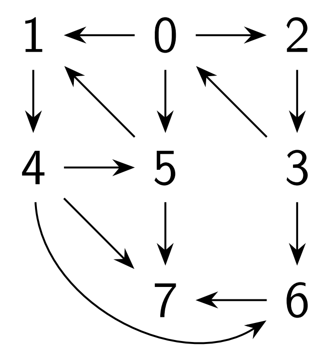


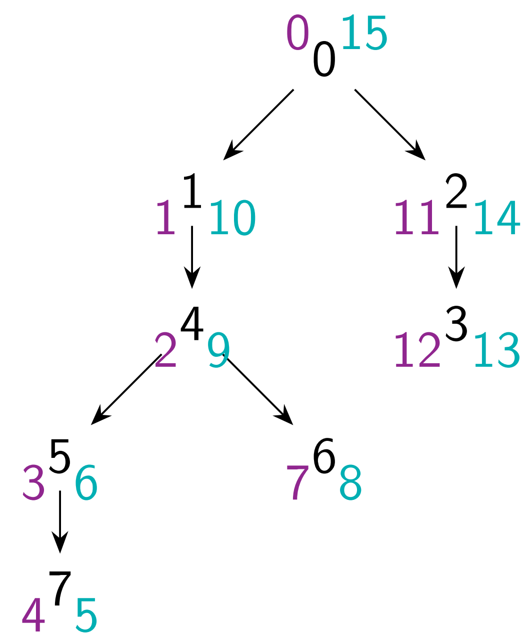


- Only back edges correspond to cycles

Classifying non-tree edges in directed graphs

- Use pre/post numbers





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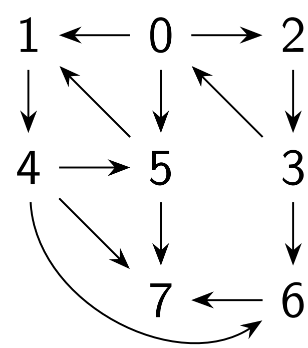

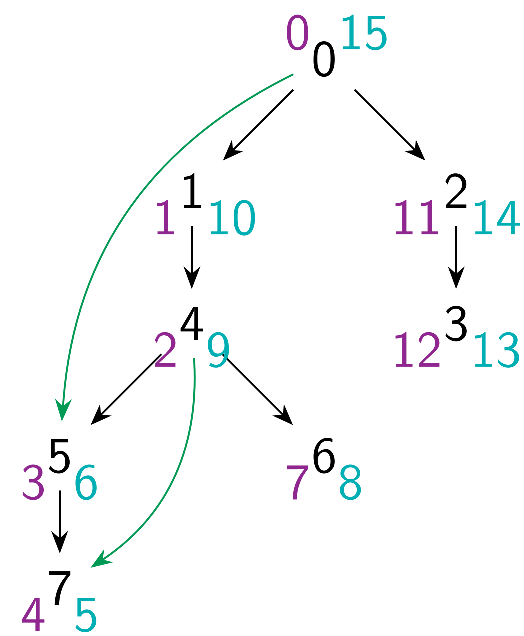

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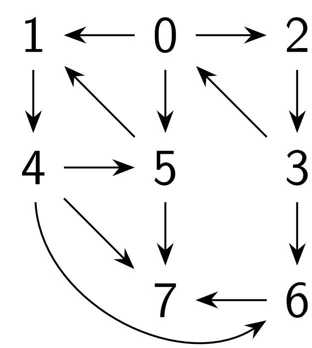

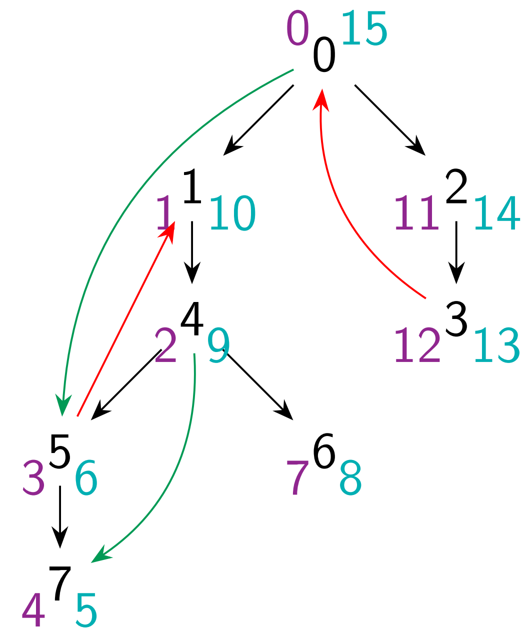

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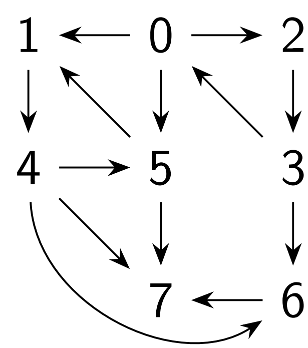

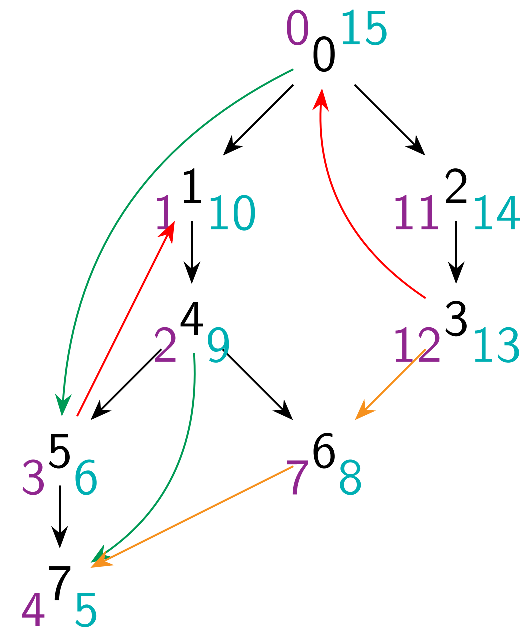

Connectivity in directed graphs

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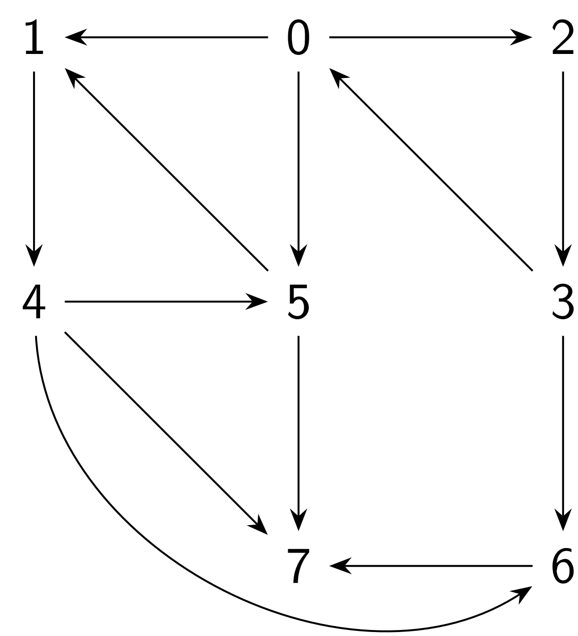

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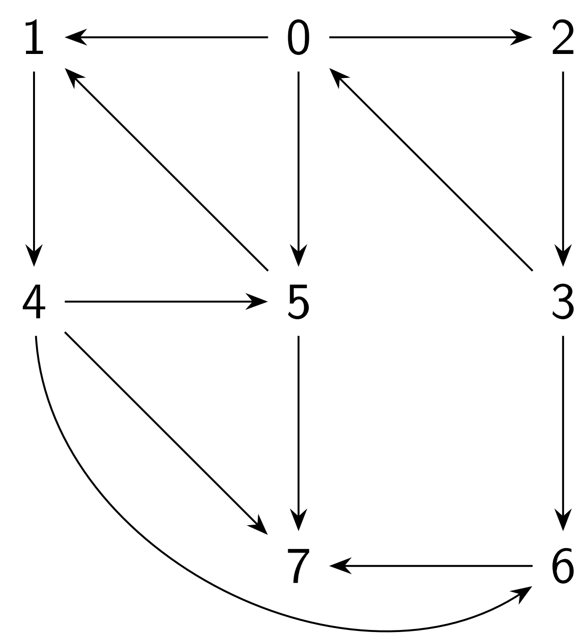

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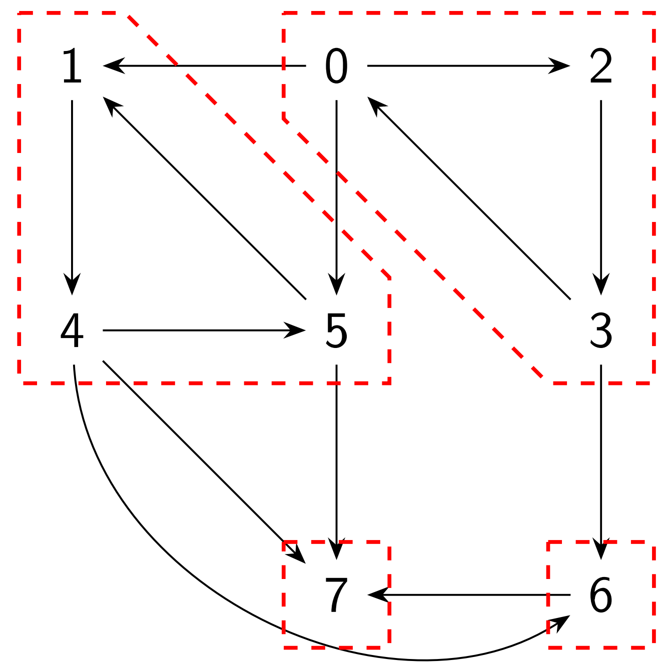

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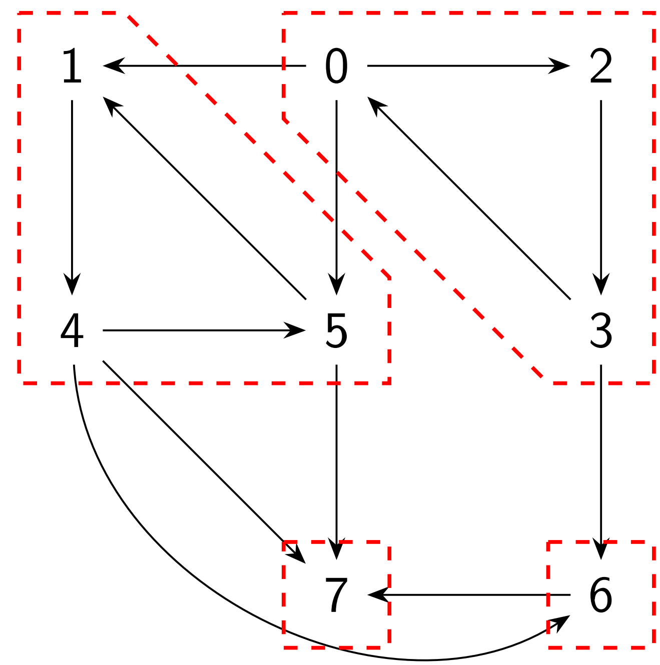

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  - BFS and DFS identify an underlying tree, non-tree edges generate cycles
- In a directed graph, non-tree edges can be forward / back / cross
  - Only back edges generate cycles
  - Classify non-tree edges using DFS numbering
- Directed graphs decompose into strongly connected components
  - DFS numbering can be used to compute SCC decomposition
- DFS numbering can also be used to identify other features such as articulation points (cut vertices) and bridges (cut edges)

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- DFS numbering can also be used to identify other features such as articulation points (cut vertices) and bridges (cut edges)
- Directed acyclic graphs are useful for representing dependencies
  - Given course prerequisites, find a valid sequence to complete a programme