IIT Madras

ONLINE DEGREE

Statistics for Data Science - 1
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Week - 5 Tutorial - 3

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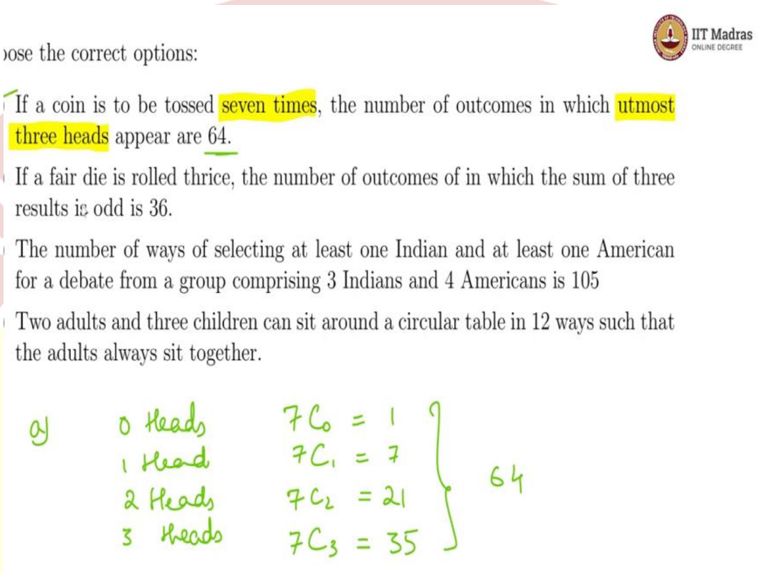

In this question, we are supposed to choose the correct options from these 4. So let us look at the first option. In the first option, a coin is to be tossed 7 times, the number of outcomes in which utmost 3 heads appear. So we need a maximum of 3 heads. So that gives us 4 cases which is 0 heads, 1 head, 2 heads, and 3 heads.

So 0 cases of the 7, we choose no toss at all. So that will become ${}^7C_0$ which is equal to 1. And for 1 head, we choose 1 toss of the 7 which is 7. And for 2 heads, we choose 2 tosses of the 7 which is 21. And finally, 3 heads is 3 choices from 7 which is 35. So all of these put together gives us 64 outcomes. So this is 64 and this is correct.

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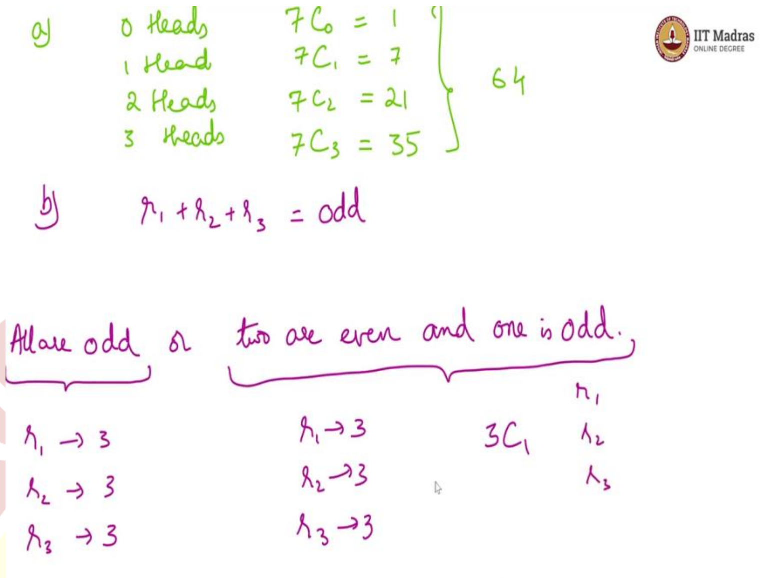

Now, for the second part, if a fair die is rolled thrice, the number of outcomes in which the sum of the three results is odd. So the sum of the results should be odd and that we are looking for the number of outcomes.

So the sum of the three results should be odd which means, let us call these results $r_1 + r_2 + r_3$, and this is odd. And this can only happen if all three of them are odd or two are even and one is odd. These are the only two cases which is, all are odd or two are even and one is odd.

So why are these only two cases? Suppose we consider the other cases where all or even. If all are even, you are going to get the sum as even, and if there is only one even and two odd, the sum of two odds will be even; so even plus even will give you even. So these are the only cases we have and now, let us look at them.

All are odd. So in the first result, we have 3 options; in the second result, we have 3 options. And in the third result, we have 3 options. Because you have 1 3 5, 1 3 5, 1 3 5 in all of these. Whereas, in the next case, again you will have, first result will have 3 options; second result will have 3 options, and third result will have 3 options.

So these options are 3 3 3 because there are also 3 even numbers. So let us assume that r_1 is odd, then r_1 has the option of being 1 3 5, but r_2 and r_3 will then have the options of being 2 or 4 or 6. And now, there is a further concern of which one is the odd result. So that can be chosen in $^3C_1$ ways which is basically r_1 is odd or r_2 is odd or r_3 is odd.

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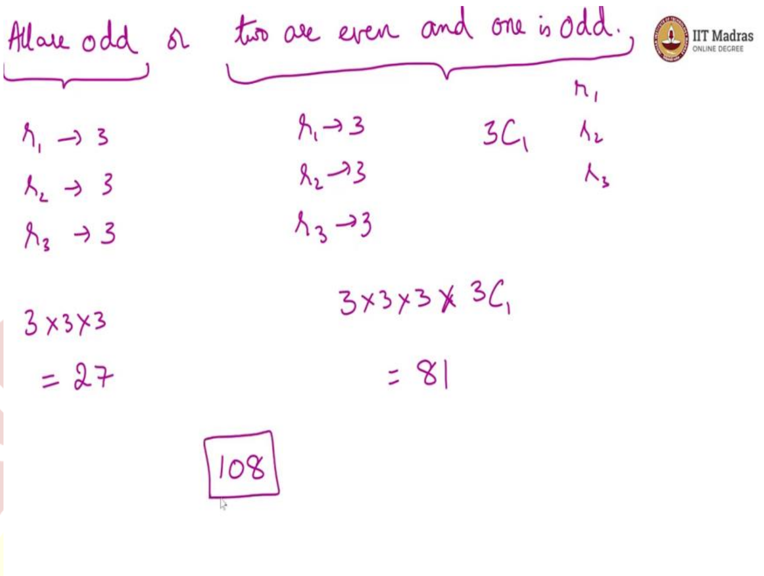

Now, if we count the total number, will have $3 \times 3 \times 3 = 27$ for all are odd. Whereas, in the other case, will have $3 \times 3 \times 3 \times {}^3C_1$=81. The sum is then $27 + 81 = 108$.

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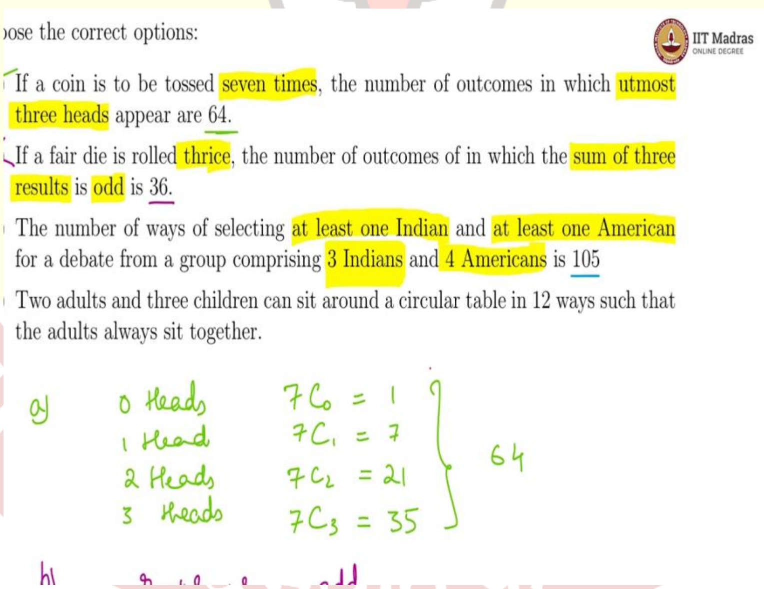

But here, they are saying it is 36 which is not true. So this is not true. Going further. The number of ways of selecting at least one Indian and one, at least one American for a debate from a group comprising of 3 Indians and 4 Americans is 105. So we have at least one Indian and at least one American.

Now, the problem does not say how many people need to be selected. So presumably, you can choose any number.

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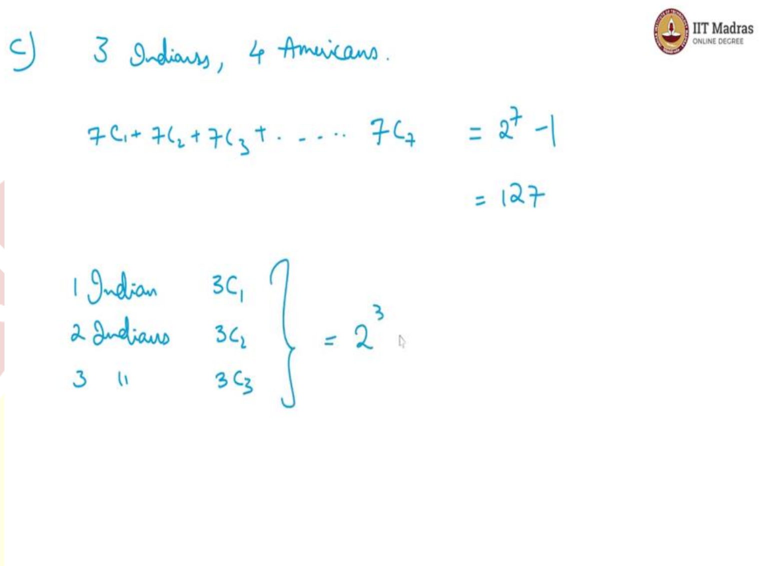

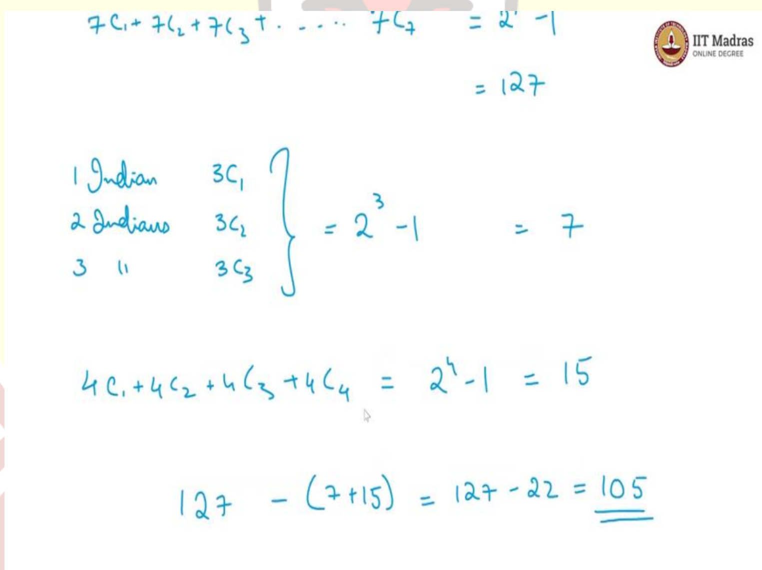

So if you can choose any number, there are 3 Indians and 4 Americans. So there are 7 in all. So if you can choose any number, what you are getting is $^7C_1$ plus $^7C_2$ plus $^7C_3$, so on till $^7C_7$. This is if there are no restrictions. And this is equal to 2^7-1, that is, 127.

Of these, we should remove the cases where there are no Indians or no Americans. So the cases where there are no Americans let us take, for example. Then you can pick 1 Indian or 2 Indians or 3 Indians. So that will be $^3C_1$ plus $^3C_2$ plus $^3C_3$ which again is equal to 2^3-1.

And similarly, if we choose 1 American or 2 Americans or 3 Americans or 4 Americans without any Indians, you will get ${}^4C_1$ plus ${}^4C_2$ plus ${}^4C_3$ plus ${}^4C_4$ which is equal to 2^4-1. This is 7 and this is 15. So of the total possibilities, we are subtracting 7 plus 15 which gives us 127 minus 22 which is equal to 105.

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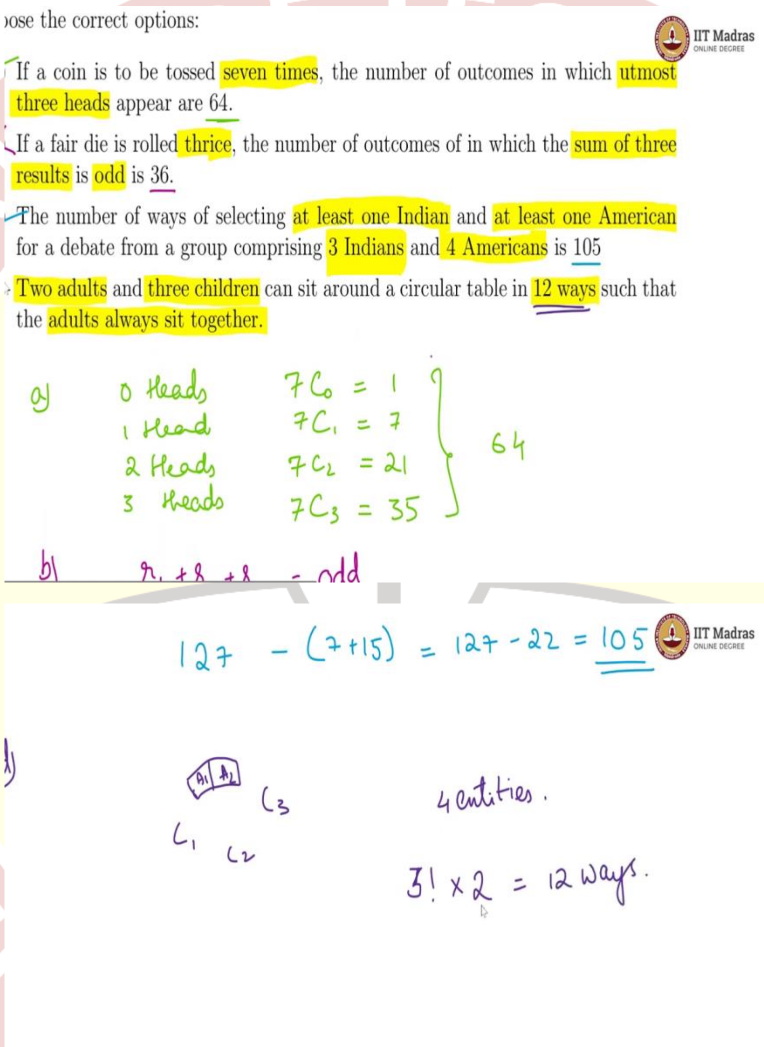

So this is a number of ways to pick a debate team in which there is at least one Indian or one American and 105 is correct. So C is also correct. Now, let us look at the last option which is, 2 adults and 3 children can sit around a circular table in twelve ways such that the adults are always sitting together.

So then, we have a circular permutation here, there are 2 adults. Now, these 2 adults are treated as one entity, and then they have C_1, C_2, and C_3 who can sit around them, the three children. So technically, these are 4 entities totally and in a circular permutation for n entities, you will get n minus 1 factorial, that is, 3 factorial ways in this case.

But the A_1 and A_2, the adult 1 and adult 2 can be interchanged in each of these permutations so you get additionally 2. That gives us 6 into 2, 12 ways. So there is these 12 ways, which means D option is also correct.