INDIAN INSTITUTE OF TECHNOLOGY MADRAS

सिद्धिर्भवति कर्मजा

IIT Madras

ONLINE DEGREE

Mathematics for Data Science 1
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Lecture No. 54
Solving Exponential Equations

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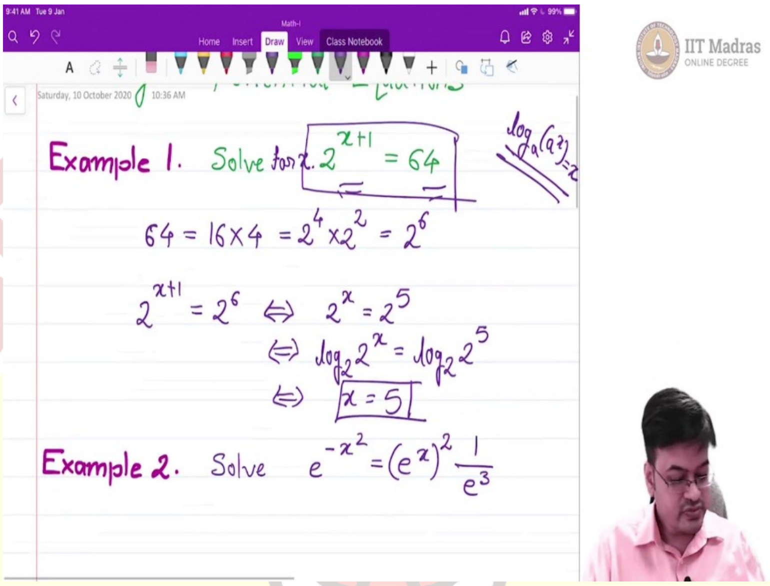

So, let us try to go back and solve our exponential equations. So, this is where the logarithms will come handy. In particular, we have seen one property of logarithm, for example, if I am talking about a^x., and if I talk about $\log_a a^x$. I get x, this property we have to emphasise and track everything in terms of this property and solve the exponential equations while solving the using logarithms.

So, let us try to get hands on these exponential equations using the logarithm. So, first you have to solve this equation naturally we have to solve for x. So, 2 raise, the equation is $2^{x+1} = 64$.. Now, as you know earlier that this is not more of logarithm, but more of inspection. So, 64 seems to be a nice number. So, you can write 64 as 16×4 is 64 but 16 is nothing but $2^4 \times 2^2$. So, what you got here is 2^6. So, my 64 can be written as 2^6.

Now, I have been asked to solve for this equation that is $2^{x+1} = 64$. So, essentially $2^{x+1} = 2^6$ if and only if, so you can take this 1 2 out $2^x = 2^5$, hit the function with a logarithm and use the property. So, use the property $\log_a x = x$ hit the function with logarithm and you know logarithm is one to one function.

So, nothing changes what should be my a, it should be 2, then I naturally I will get $\log_2 2^x = \log_2 2^5$. Using this property, you can actually write this as x =5 there is my answer, I have solved this expression. So, my answer to this question is x =5.

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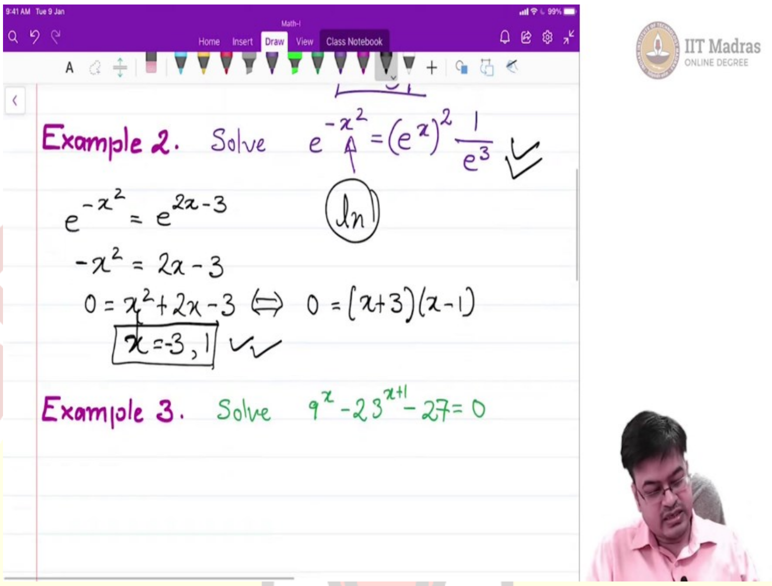

Let us look at the second example. This is also an exercise in computation where I will try to match these 2 but it becomes slightly complicated because as you can see, it involves the term containing x^2. So, again the question is to solve for x. So, let us solve for x. So, let me first simplify the right-hand side, $e^{(x^2)} = (e^x)^2$ using law of indices will become $2x$ and this 1 by e^3 will become-3.

Again, you use the natural log take ln on both sides. If you take ln on both sides using the same property that I mentioned earlier, I will get $x^2 = 2x - 3$. That simply means, I have a quadratic equation which says x^2+2x-3 =0. Use our knowledge of quadratic functions or quadratic equations. And in this case, the quadratic equation I think can be solved with by a very easy solution that is x+3x+3×x-1.

This is simply by factoring you will just look at the lectures on quadratic functions and see how to solve the equation by factoring. Actually, I will split this× $x^2 + 3x - x - 3$ that will give me-1 as common and the first factor is x+3. So, this is how I will solve done. So, now, what I have as an answer is $x = -3$ *and* $x = 1$.

Now, remember, we are solving some equations, where the domains and co domains may not be defined properly, then we may land up in infeasible solution. So, always check whether these two functions will work here or not, let us substitute this $x = -3$ over here and you can verify that it perfectly works and x =1 also perfectly works. Therefore, these both are actually the solutions to the problem.

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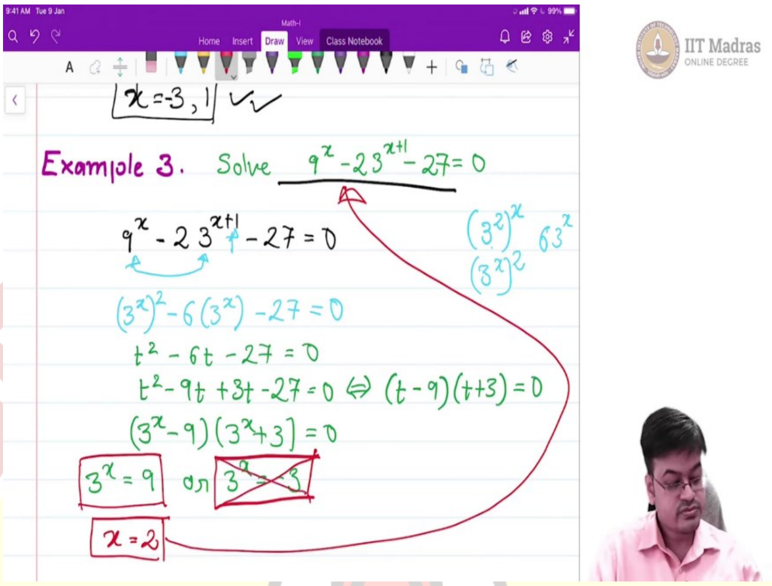

Let us complicate the matters a bit more by putting in this kind of equation. So, let me write this equation and now, obviously, the question is to solve for x $9^{x-2} \times 3^{x+1} - 27 = 0$. So, here if you look at the equations closely before actually handling them, you can see that there is some common feature between this exponent and this exponent. What is that I have something like $3^2 = 9^x$, correct and $3^{(x+1)}$, I can take the+1 out, so that these 2 will become 6. So, 6 times 6×3^x.

Now, this thing you can rewrite as$3^{x\ 2}$. the whole square using this trick, it is very easy now to see that this particular thing is $3^{x^2} - 6 \times 3^x \quad - 27 = 0$. Now, there are 2 ways this equation is actually very similar to a quadratic equation or you can reword it as it is a quadratic inform equation. Now, using this equation, you can actually solve for 3^x not for x.

So, let us go ahead and put 3^x. as t and is $t^2 - 6t - 27 = 0$ again resort to a method which is like factoring. So, 27 can be factored×9 $\times$ 3, 9 3s are 27 with the-sign, so, it will be $t^2 - 9t + 3t - 27$ which is going to be equal to 0 this will give me this is if and only if t-9×t+3 equal to 0 correct. Now, the question comes t+9 =0. So, I have solved it for t what is t? t is 3^x. So, I have to resubstitute that and if I substitute that, then I get $3^{x-9} \times 3^{x+3} = 0$..

Now, you have to be a little bit careful. So, this simply means using the factor logic $3^x = 9\ or - 3$.. Now, this should give you an alarm in your head that this particular thing is not possible for any real x. So, this option is infeasible. So, I cannot solve for this, what about this?

Do I know something about this? Again, simple thing is you hit with a log you will get the answer or in this case, it is more obvious that x =2 should be the answer.

So, now once you got this you substitute this x =2 in the original equation and verify that it satisfies the equation that is that will be a good cross check. For example, $9^2$is 81, 3^2 is $3^3 =$ 27. This is 27, 27×2 is 54, 54+27 which should give you 81? Yes. So, it is a verification and we have solved this problem successfully, but remember here the occurrence of infeasible solution.

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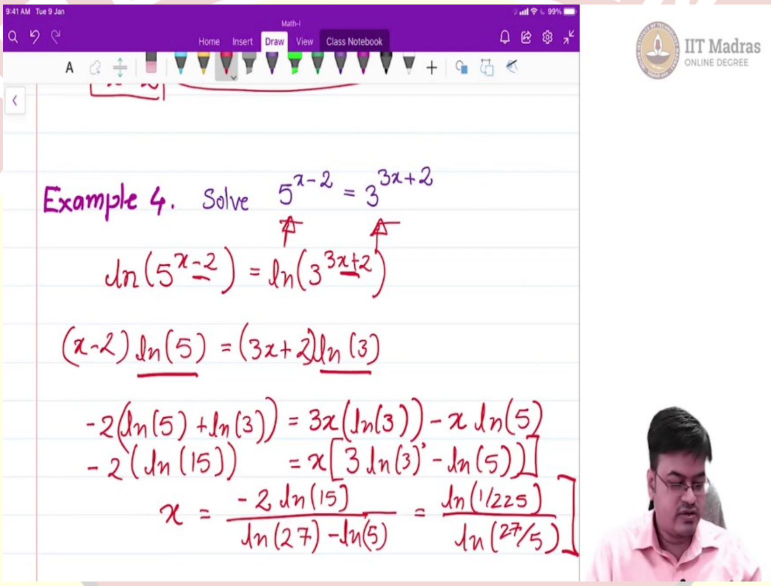

Let us take it a step further and see whether I can do something about an equation of this form, which is 5^{x-2}, 3^{x+2} unlike previous problems, the bases are not the same. So, what should we do about it is the question. So, in this case also we can actually rely on logarithm and we can blindly hit with a logarithm, but in this case, let us hit both sides of the equation with natural logarithm or you can take common logarithm it does not matter 5^{x-2} =ln 3^{3x+2}.

Now, if you have noticed that when you hit with the log of the same base the other thing gets vanished, but if you hit with a log of some other base that number will remain unperturbed, but this x-2 and 3x+2 will come in front that is x-2 ln 5 =3x+2 ln 3. Now, this ln of 5 and ln of 3 are nothing but merely some numbers which are getting multiplied with x and 2. So, they are just constants.

So, now my equation is actually a linear equation let us try to simplify this. So, here there are there is 3x here there is x so, we what we will do is we will take both parties on one side that is the parties corresponding to x on one side and parties corresponding to constant on one side.

So,-2 times ln of 5 will remain as it is+2 when it comes here becomes-2 and ln of 3. So,+ln of 3 which will be equal to 3x was already there multiplied with ln of 3 and from here comes x which is-x times ln of 5.

So, now what I got here is 3x-x times ln of 5, I want x to come out common So, I can process it further which will give me x as a common factor 3 times ln of 3-ln of 5 and here it is-2 times now, ln of 5+ln of 3 you will learn ahead it will be ln of 15. And if you look at this particular expression this equation will not, this expression in the square bracket will not be equal to 0. So, I can take this expression here another thing that you may notice is 3 here, can be raised to the power of 3 over there and you can further simplify.

So, you will get in particular $x = -2\ln\frac{15}{3\ln 3} = \ln 27 - \ln 5$ which can further be simplified to, if you want you will have more precise structure later 15^2which will be 225, $\frac{1}{2}$ and because of this-sign it will be $\frac{\ln\frac{1}{225}}{\ln\frac{27}{5}}$.

So, this is again a number and this resolves the problem for x. Whether these numbers are feasible? Yes, they are very much feasible and therefore there is no visibility violation over here. This we will learn in a bit later how to do all these calculations, but as of now, this is the answer.

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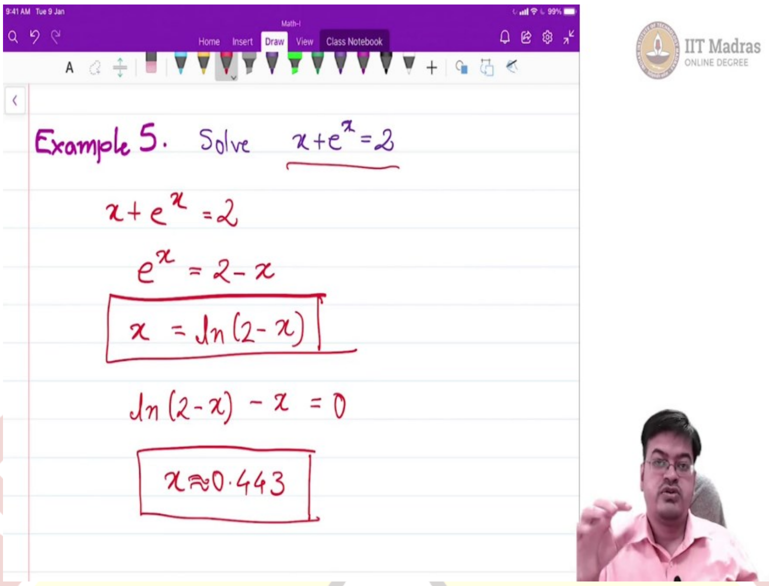

Now, let us look at this particular example, where you cannot solve on your own, but graphical solution may yield a better answer. So, for that let us look at x+$e^x = 2$. Now, if I try to hit this with a log, but before hitting this with log because I am encountering a + sign, let me put rewrite this equation as e raised to x equal to 2-x.

Now, hit this expression with a log, so you will get x =ln of 2-x and you are struck you cannot go anywhere. Now, if I want to solve this equation again the only thing that right now we are aware of is I will hit it with exponent and I will again get back the same equation. So, it is of no use. So, let us focus on this equation and let us let us try to graph this equation. So, in particular I have a function which is $ln\ 2 - x - x = 0$. Now, let us try to use some graphical tool like Desmos to graph these equations.

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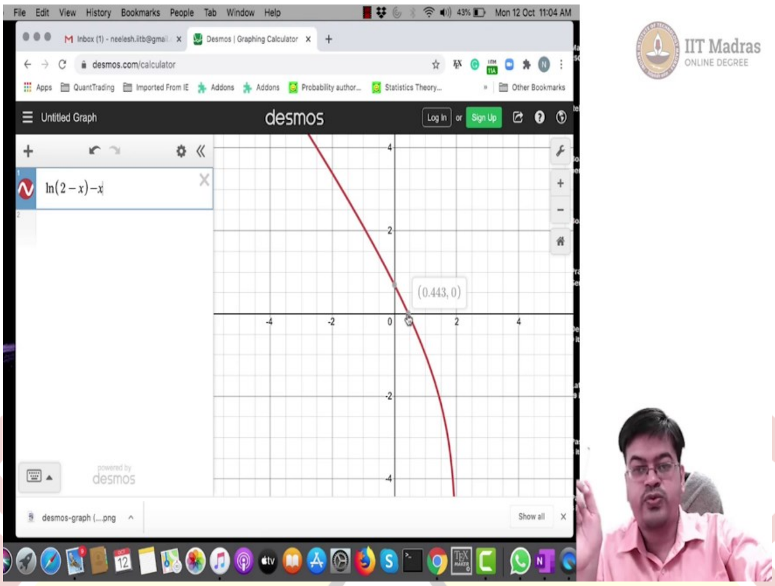

So, let us go ahead and use Desmos for graphing the equation and naturally the choice of log is only up to B. So, I will use ln or I will use whatever so, ln of 2-x you can see the graph-x. Now, the point where it cuts 0 is the solution to the problem and you can actually check using Desmos that the point is actually 0.443.

So, let us go back and draw and tell everybody that x =0.443 is the approximate solution to this problem, but in such problem this is the best that you can do. So, today we have seen how to solve exponential problems using graph or using algebraic methods. Thank you.