IIT Madras

ONLINE DEGREE

Mathematics for Data Science 1
Prof. Neelesh S Upadhye
Department of Mathematics
Indian Institute of Technology Madras – Chennai

Lecture-20
General Equation of Line

So, far in our journey we have studied how to represent on line which is a geometric object in algebraic manner using various forms of equations. This is a time to recollect; what are the forms of equations that we have studied and understand some common properties commonalities in that equation of line and give a general equation of line which will be helpful for further analysis. So, let us see what are the different forms of line; equations of line that we have studied.

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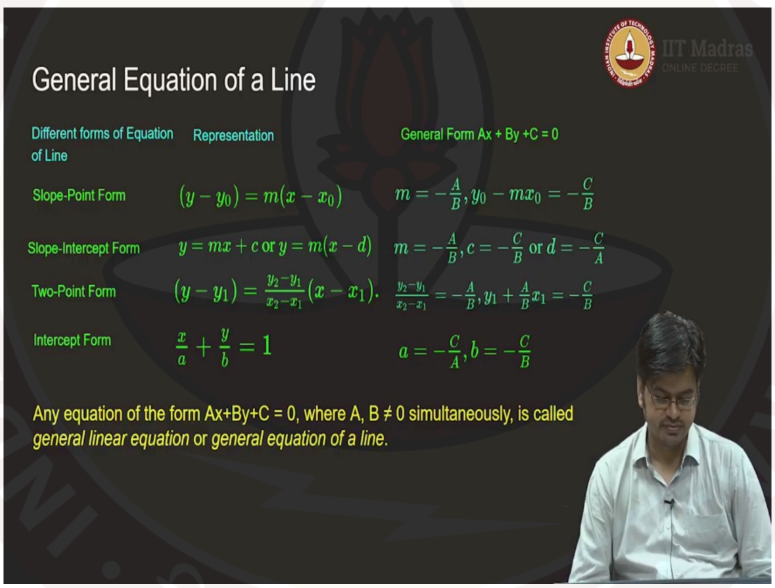

So, in particular we had two forms one is two-point form another when its slope point form. So, first I will list the slope point form, a specialized version of this is slope intercept form where instead of a point you have been given x intercept or a y-intercept. Then we have also studied two-point form given two points how to uniquely determine a line and a specialized version of that is nothing but intercept form.

So we can quickly review these forms like slope point form we have a point (x_0, y_0) which is given to us and a slope m that is given to us. So, we come up with an equation when we give the algebraic representation of this line with slope y with slope m and point (x_0, y_0), we will come up with a representation as $(y-y_0)=m(x-x_0)$. When you come to slope intercept form suppose the x intercept is given to me if I have been given an x intercept then the y coordinate of that point will be 0.

So let us say x intercept is d, in that case my equation from slope point form as slope intercept form is a specialized version of slope in point form. My equation will become $(y-0)=m(x-d)$ if the intercept is at d. So, $y=m(x-d)$, in a similar manner so the y intercept is given to me and that intercept is at c then my y_0 will be replaced by c and x_0 will be replaced by 0 therefore I will come up with an equation $y=mx+c$ that is what is listed here, given a y-intercept and given an x intercept the equation has a form $y=m(x-d)$.

Let us come to two point form we have also seen during the course that this two point form is closely related to slope point form. We also know that given any two points on a line we can determine the slope of a line so in this particular expression m will be replaced by the ratio of the change in y upon change in x. Therefore the two point form will be just replica of this instead of m you will have the difference between y-axis difference between the coordinates of y-axis and difference between the coordinates of x-axis that will be given in this form.

$$(y-y_1)=\frac{y_2-y_1}{x_2-x_1}(x-x_1)$$

Now remember here the points given are not (x_0, y_0) so the points given are (x_1, y_1) and (x_2, y_2) therefore x_0 is replaced by x_1 and y_0 is replaced by y_1 and this thing is nothing but a replacement of m that is how these two forms are also closely related. In an intercept form you will get two intercepts x intercept let us say x intercept is a and y intercept is b then how will these forms change?

If x intercept is a that means I have a point $(a,0)$ so my (x_1, y_1) will be nothing but $x_1=a, y_1=0$ and (y_2-y_1) will be b, x_2-x_1 will be minus a, so $\frac{b}{-a}$ is equal to $\frac{-b}{a}$ x, and if you simplify that you will come up with a very simple expression of the form $\frac{x}{a}+\frac{y}{b}=1$. So, here there is a no-brainer nothing to remember below x you write x-intercept below y you write y-intercept and equate it with 1.

Now if you look at all these forms there is one common feature, let us take the slope point form given a point (x_0, y_0) this $x_0 \wedge y_0$is fixed. The slope of a line is fixed. So, now what we are identifying is we are identifying in a condition in the form of (x, y) what these coordinates should satisfy. So, the variables are x and y.

If you look at all these forms the same feature is visible, the variables are x and y and I have an expression of the form some constant times y some constant times x and added with another constant. Let us take this feature for example $y-y_0=m(x-x_0)$now I want to differentiate between variables and constant. So, I can simply write this as $y-mx=y_0-m x_0$. $y_0-m x_0$ will be the constant associated with this particular equation and y and one variable y is associated with real coefficient 1 and variable x is associated with real coefficient -m.

So in particular I can have a general form of the equation and similar story is true for all this. For example, if you come here, with variable x, $\frac{1}{a}$is a real coefficient that is associated, with variable y, $\frac{1}{b}$that is a real coefficient that is associated and the constant is c. So, I can discuss same things about all these features but one thing is common that I can have a general form of equation which will be of the form $Ax+By+C=0$.

Now let us identify this particular general form with our various expressions like slope point form, the way I discussed the slope point form we already know. In this case we have assumed that b is equal to 1 but I can as well multiply by a constant term throughout the equation and we will have the same equation. So, assuming this holds true let us discuss about this particular expression. So, in this case you can easily see if I relate this equation with this equation that is you rewrite this as $y-mx=y_0-m x_0$.

In that case you can have this expression which will give the value of m when you compare with respect to this expression as $\frac{-A}{B}$ and value of $y_0 - m x_0$, now remember this is a constant term because all these are constants. So, $y_0 - m x_0 = \frac{-C}{B}$. If you are able to understand this then you can easily understand the slope intercept form. Because in the slope-intercept form, $y = mx + c$you have y-intercept which is c therefore your y_0 will be replaced by c and x_0 will be replaced by 0 so if you look at this expression m will still remain $\frac{-A}{B}$, when I am identifying this equation m will still remain by minus $\frac{-A}{B}$, y_0 is identified with $C - \left(\frac{-A}{B}\right) x_0$is 0 so this becomes irrelevant so y_0 is c so $c = \frac{-C}{B}$. In a similar manner you can do for x-intercept and you will get these expressions.

So m as I mentioned $c = \frac{-C}{B}$ and for getting d you just put $x_0 = d$and $y_0 = 0$, you will get this expression. So, same exercise can be done for two point form and intercept form remember this m will be replaced by a ratio of these two differences. So, m is replaced by a ratio of these two differences there is no (x_0, y_0) there will be (x_1, y_1) therefore you will have an expression of this form.

But remember this $\frac{-C}{B}$is common everywhere the slope is $\frac{-A}{B}$ everywhere so essentially, we have got one simple general equation. Similar things you can do for $\frac{x}{a} + \frac{y}{b} = 1$ that is intercept form and you will get $a = \frac{-C}{A}, b = \frac{-C}{B}$. So, what we have seen here is an exact matching one-to-one correspondence of a general equation with respect to this equation.

Now why should I consider general equation? Remember when we figured out this representation our assumption was these are non vertical lines. For vertical lines our slope do not exist but in this case if you; and those lines are where the slope do not exist those lines are vertical lines. They are of the form x is equal to some constant. If you look at this equation which is a general form of this equation you just put B to be equal to 0 you will get $Ax+c=0$ that means x is equal to some constant x is equal to $\frac{-C}{A}$, you will get that is what our intercept form also reveals.

So all these lines are actually vertical lines, so this general equation is capable of handling vertical lines also, horizontal lines are anyway handled here because if you put m is equal to 0 the horizontal line is handled. While we were deriving these forms we were always assuming non-vertical lines. So, non-vertical lines are covered as well as vertical lines are covered therefore this equation is a general form of equation of a line.

Also, in your earlier classes you might have studied this as a polynomial in without this equal to 0 Ax + By + C is a polynomial in two variables and it is a linear polynomial in two variables. Therefore you will hear a term called linear equation in two variables. So, in particular if this has to represent general form of a equation of line then A and B cannot be simultaneously equal to 0.

If A and B are simultaneously equal to 0 then I am actually equating constant with a zero which is invalid therefore the assumption will always be A and B cannot be simultaneously equal to 0. Though individually they can be equal to 0 for example you can put A is equal to 0 then you will get y is equal to some constant which is a line parallel to x axis. You can put B is equal to 0 then you will get a line x is equal to constant, x is equal to constant is parallel to y axis.

So now we will bring up a definition that any equation of the form Ax + B y + C = 0 where A and B are not equal to 0 simultaneously individually they can be 0 or they can be nonzero as well is called general linear equation because we are handling a linear polynomial which is equated to 0 so it is an equation, general linear equation or general equation of a line. So, what we are summarizing here is a polynomial in two variables or and general linear equation in two variables gives you line.

So this is the identification of a geometric object called straight line with an algebraic representation of general linear equation. So, this will give us both the strength in our analysis because now you do not have to discuss about the line. But you can as well discuss about its algebraic representation or you can start with an algebraic representation of a line and then discuss about the geometric properties of the line. How let us see in the next slide.

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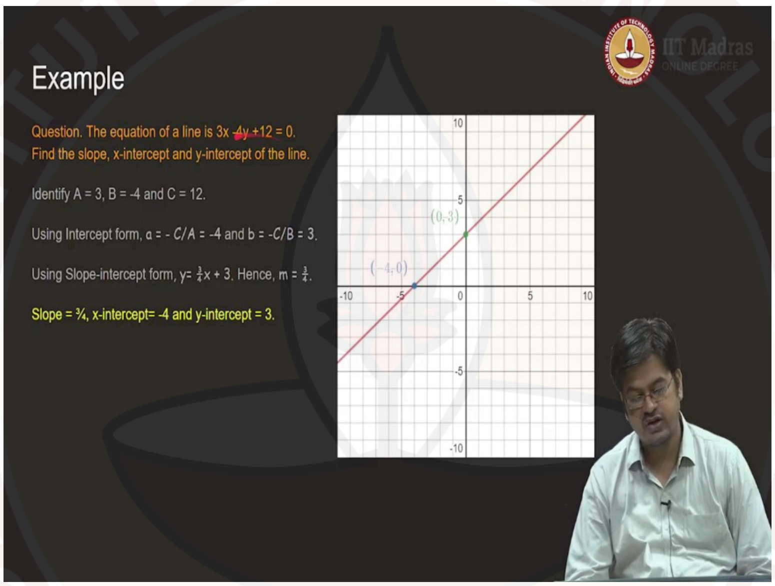

So, here is an example, the example gives you a question that the equation of a line is $3x-4y+12=0$. Now I do not know how this line behaves now I want to see how this linear equation represents a line. So, when I talk about a line what is the natural question we will talk about what are the two points that uniquely determine this line or you can ask what is the slope of a line and give me one point on a line because we have slope-intercept form or we have two point form any of them should be usable.

So in order to discuss about the geometric aspects we can ask a question that find the slope or x intercept or y intercept of a line. So, how will you find this the job is pretty simple let us go back and revisit the previous slide which will make the job very simple. Suppose I want to determine the x-intercept and y-intercept then I have this intercept form right which says that a is the x-intercept and b is the y-intercept.

Now you I have been given an equation in this form which is $Ax+By+C=0$ so I can image lately consider this equation and consider the values of a and b which is $-C/A$ and B is equal to $\frac{-C}{B}$. So, let us go and do the same thing on the on the our; now our problem so we have identified $Ax+By+C=0$. So, what is A, A is 3, B is minus4, C is positive 12. So, what should be my x intercept A as you have seen in the previous slide is $-C/A$.

So what is C? $\frac{12}{A}$ which is 3 so my a is 4, and a minus sign associated with it so $a=-4$. In a similar manner you can talk about y intercept which is $\frac{12}{-4}=-3$ but a minus sign because it is $\frac{-C}{B}$ so it will be 3. So, now we can readily answer the question what is on a x-intercept and y-intercept.

Now the question comes what is the slope of a line. So, for slope of a line you can use the slope intercept form $y=mx+c$. So, identify this equation in the form of $y=mx+c$ so if you look at this equation, I should push this 4y to the right hand side that gives me $y=\frac{3}{4}x+\frac{12}{4}$. So, my m should be $\frac{3}{4}$this is the answer. So, slope intercept form you have y is equal to 3 by 4 x plus 3 so the slope is naturally $\frac{3}{4}$, this easy is our calculation.

Now we have identified an algebraic object as a geometric object. Now let us see what we can do further and we can actually verify this graphically you know although it may be correct it is always better to verify it graphically. So, slope is $\frac{3}{4}$ x intercepts should be - 4 and y intercept should be 3 if you want to satisfy the equation of this line this should happen right.

So this is how we have drawn so the x-intercept is -4, y intercept is 3 and the line passes through this. Now you pick for verification purposes you can pick any point on this line and you can put the values of the coordinates into the equation of a line and verify that it will give you the value 0 that will be the identification that your answer is correct.