IIT Madras

ONLINE DEGREE

Statistics for Data Science - 1

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Permutations and Combinations - Basic Principles of counting

Now, we start the module 2 of this course. So, you recall that in the module 1 we spent our time understanding about descriptive statistics, in particular we looked at how we summarize a categorical variable and a numerical variable and we also spend time to understand how we describe association between variables when both the variables are categorical, when both of them are numerical and one of them is categorical and numerical.

So, till this time we spent our energy and understanding to look at how we describe data that is given to us, we did not use the data to look into the future, there was everything was clear about the data, it was certain about the data, there was no element of uncertainty that was there in our data that we looked so far.

But however, we also know that we want to use the statistics as a tool to infer about the unknown. In other words, we are faced with uncertainty most of the times and whenever we are faced with uncertainty we want to see whether we can have a framework or a tool that would help us predict about the uncertain or understand the uncertainty with a certain degree of confidence.

Now, probability is a very powerful tool to help us understand this uncertainty. So, in this module we are going to understand about probability, we are going to introduce the notion of probability and we are going to look at how we can formalize certain intuitions which we already have in an uncertain world in the domain or in the framework of probability.

But before we go and understand probability what we are going to do is to try and understand about certain basic counting principles. All of us would have studied in high school about permutations, combinations and fundamental principles of counting, we just revisit that basic fundamental principle of counting along with a revision of concepts from permutation and combination this week. So, let us get started.

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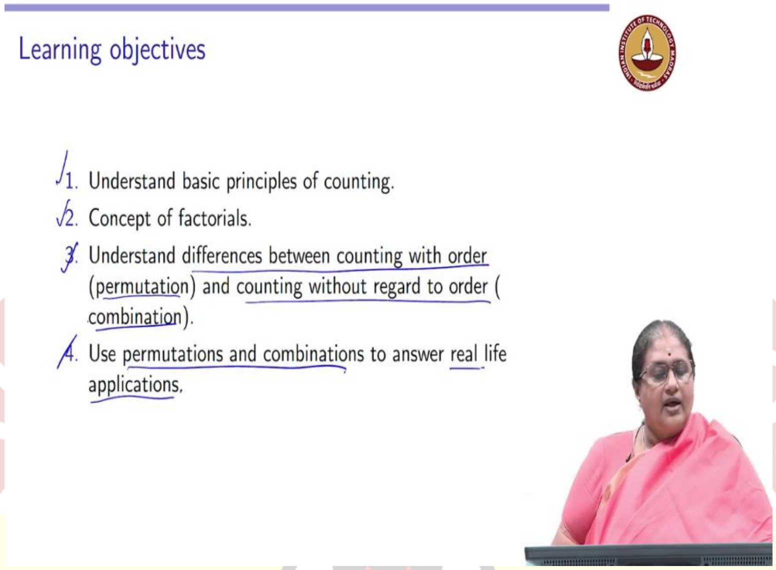

So, what are the learning objectives here? So, you can see that the learning objectives I am going to have here are the following. So, the basic objectives, the learning objectives of this week are we understand what are the basic principles of counting, then we introduce the concept of factorials and what are the simplified expressions that we can have using the concept of factorials.

Then this is the key thing will understand what is the counting with order and counting without order. In other words what is the difference between permutation and combination. And finally we will use the concepts of permutation and combinations to answer a few applications that arise in real life. So, this is the agenda for the week.

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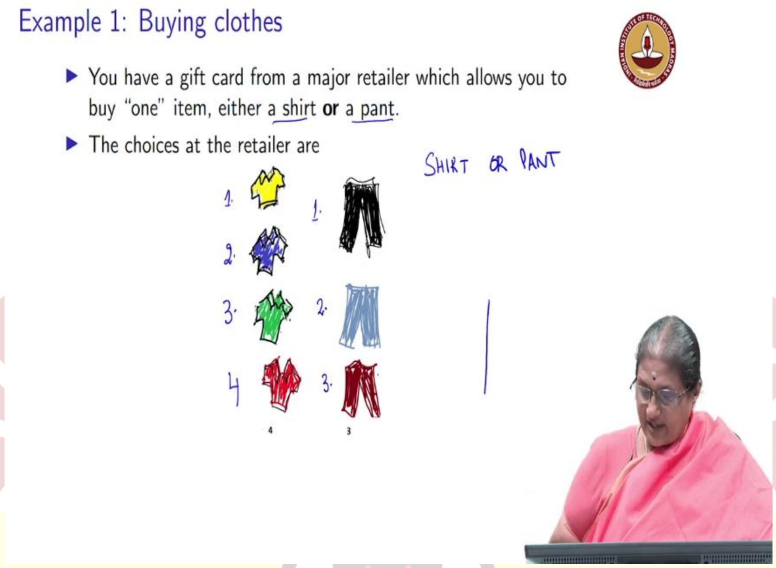

Now, let us start going, so the first thing is we start with a basic example of counting. Now, what you can see on the screen is suppose I have a gift card from a major retailer which specifies the following, you can buy 1 item and that item can either be a shirt or a pant, so the specification is you can buy a shirt or a pant, we assume now that the shirts and the pants are priced the same, we are not assuming it, so you have a gift card, a gift card for certain denomination and what is specified in the card is you can buy a shirt or a pant.

Now, when you go to the shop, you notice the following, you see that you have a choice the first choice which is a yellow shirt or a blue shirt or a green shirt or a red shirt but when it comes to pant I have 3 choices I can either buy a black pant or a blue pant or a brown pant. Now, if I exercise my gift card or I use my gift card to buy a yellow shirt I have exhausted my option. Similarly, if I use my gift card to buy a blue pant I have exhausted my option.

So, the question here we are asking is how many different ways can I use my card? So the answer is I can either buy a yellow shirt or a blue shirt or a green shirt or a red shirt. So, I have basically the number of options available to me, so the question is how many different ways can I use my card?

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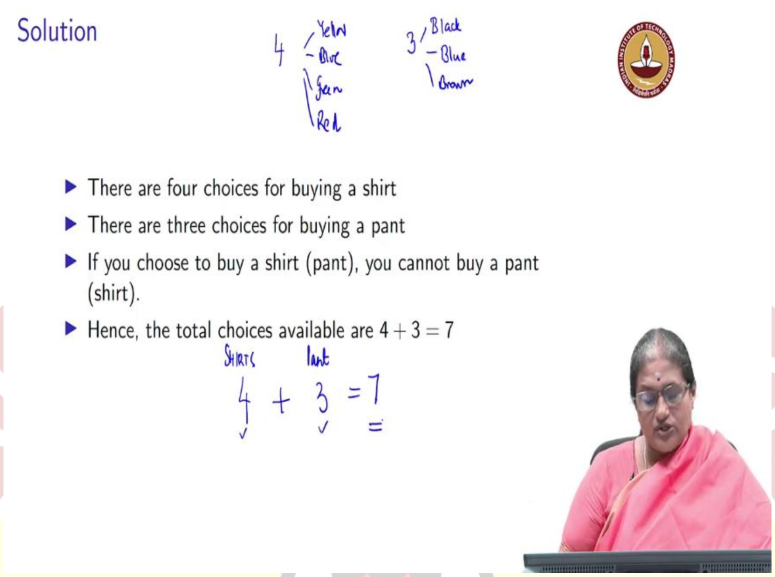

To answer this question I have how many choices to buy a shirt? I have 4 different choices to buy a shirt, I could have bought a yellow shirt or a blue shirt, those are the choices that are available to me or a green shirt or a red shirt these are the 4 different choices available to me, I have 3 different choices when it comes to a pant I can buy a black pant or a blue pant or a brown pant.

But what I need to remember is if I buy a shirt I cannot buy a pant, if I buy a pant I cannot buy a shirt, so the total number of choices that are available are the 4 choices which come from the shirts and the 3 choices that come from the pant and the total number of choices 4 plus 3 which is equal to 7.

So, here when I am talking about a dependency that is what do I mean by dependency, here the actioner is you either buy a shirt or you buy a pant but I cannot, the actions are dependent what do I mean by dependent, I cannot buy a pant if I have bought a shirt and I cannot buy a shirt if I bought a pant, so in a sense that actions are dependent on each other I have 4 choices for my first action which is buying a shirt, I have 3 choices for the second event which is buying a pant and the total number of choices available here are the addition of the choices available for each of the action which is 4 plus a 3 which is equal to 7.

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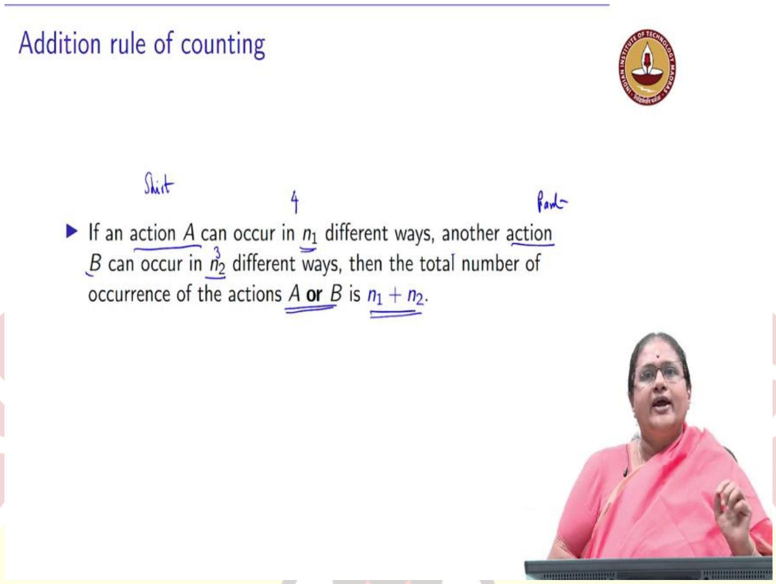

So, you can see that this is what I can formally state as the addition count rule of counting which I can state as if an action A, the action A in my example could be buying a shirt can occur in n1 different way, so n1 in my example was 4, another action B, action B could be buying a pant, this can occur in n2 different ways which is 3, then the total number of occurrence of actions A or B is given by n1 plus n2 which is given by n1 plus n2. So, now let us, this is what we refer to the addition rule of counting. Now, let us revisit the same example.

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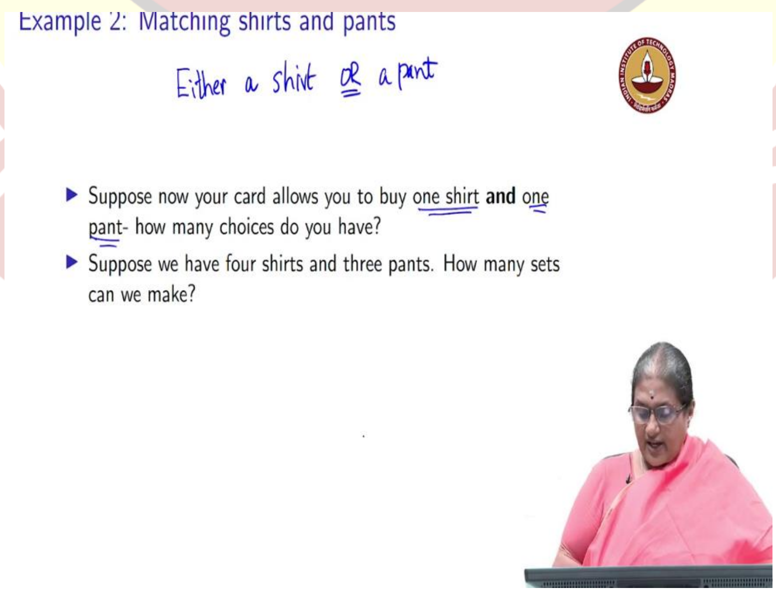

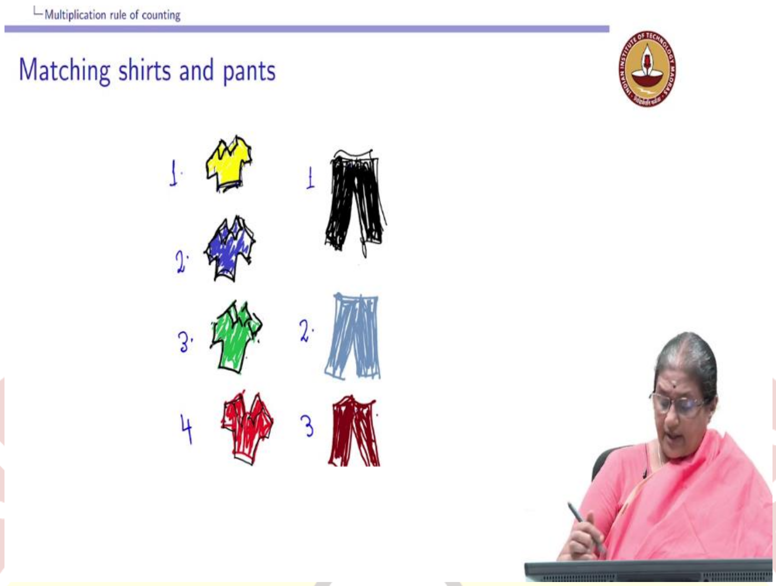

Now, suppose my card allows the following, initially my card said you choose either a shirt or a pant. So, it was very clear, the card said you choose either a shirt or a pant. But now, suppose the card allows you to buy 1 shirt and 1 pant, it is not allowing you to buy 2 shirts or 2 pants, it is allowing you to buy 1 shirt and 1 pant.

Now, how many choices do we have in this case? Again let us go back and we can see that I have 4 shirts, so this is my shirt 1 a yellow shirt, a blue shirt, a green shirt and a red shirt, I have a black pant, I have a blue pant and I have a brown pant. I can combine a yellow shirt with a black pant, a yellow shirt with a blue pant, a yellow shirt with a brown pant, this is 1, so how many ways can I buy it?

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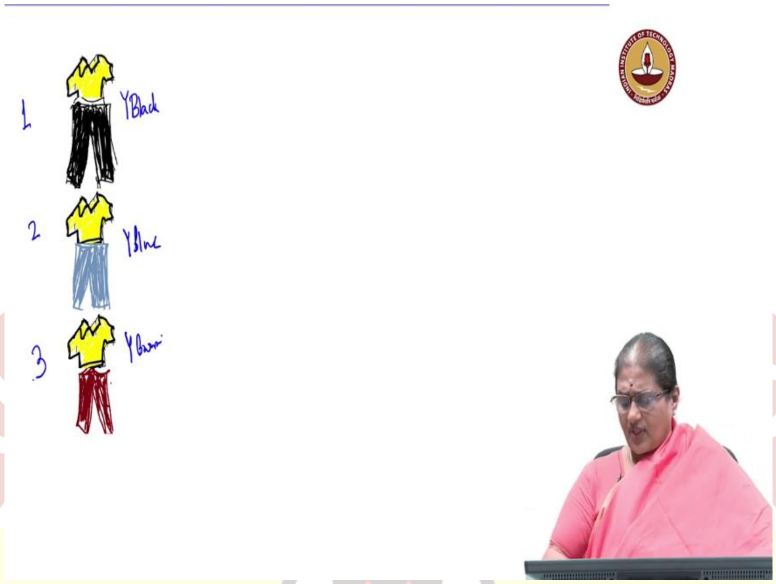

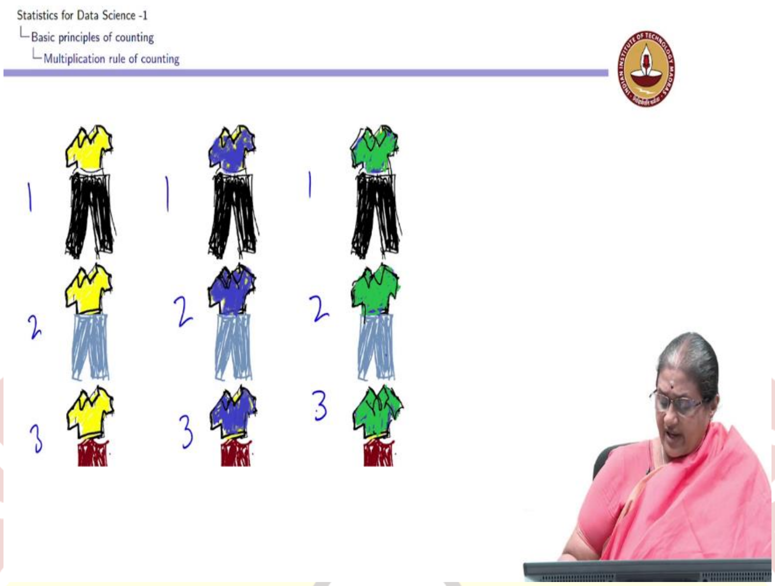

I can, when I am looking at the yellow shirt then I can see that a yellow black, a yellow blue and a yellow brown. So, that will give me 3 options, 1, 2 and 3. Similarly, what I can do is I can have a blue, so you can see a blue with black, a blue with blue and a blue with brown. So, I have 3 options from here 1, 2, 3. Blue again I have another 3 options I go to the next 1, a green with a black, green with a blue and a green with a brown, so I had 3 options here, I had 3 options with blue, I have 3 options with green and I finally have 3 options with orange also.

So, this is my 3, I have 3 options here, I have 3 options here, I have 3 options here, and I can see that what is the total number of things I have 3 here plus a 3 plus a 3 plus a 3 so I have a total of

12 options that are available for me to pick a shirt and a pant together and buy a pair of shirt and pants, these are 12 options that are available. Now, what do we mean by this?

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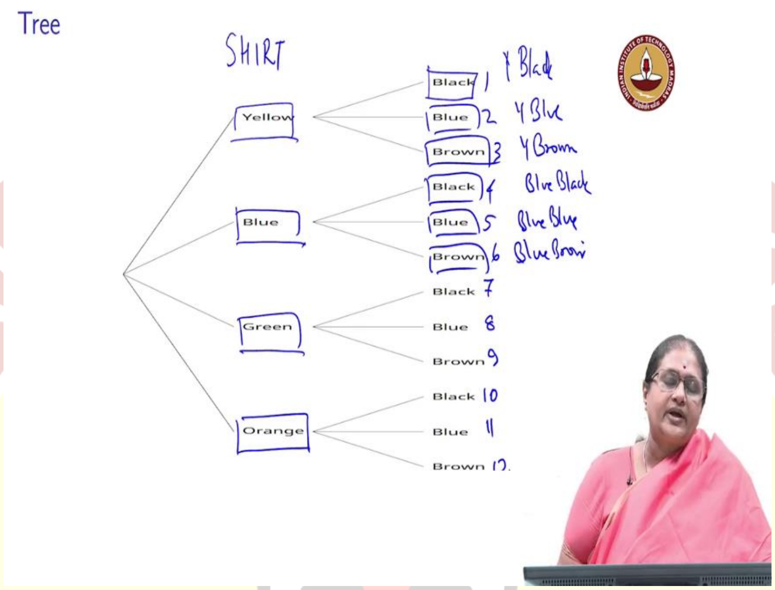

So, now let us look at this, I can represent it by what I call is a tree. So, if you look at the shirt options which is given at this level I have a yellow shirt, I have a blue shirt, I have a green shirt and I have a orange shirt. With every yellow shirt I could either have a black pant, so this is yellow with black, a blue, so I have a yellow with blue, a brown, yellow with brown. Similarly, I have a blue with black, blue with blue and blue with brown.

And you can see that the total number of ways I can do it is 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12. So, this sets the ground to understand what is the, what we refer to as the multiplicative rule of counting. What is the multiplicative rule of counting tell me? Before I go to multiplicative rule of counting, let us go to another example.

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So, suppose I am having now I have 4 shirts, I have 3 pants and now I am allowing with the gift card to choose either black pair of shoes or brown pair of shoes, so I have 2 pairs of shoes I can choose. Now, I know that I can have 12 if I am pairing or matching shirts and pants, now with every pair of your pant and shirt you can either go with a black shoe or a brown shoe.

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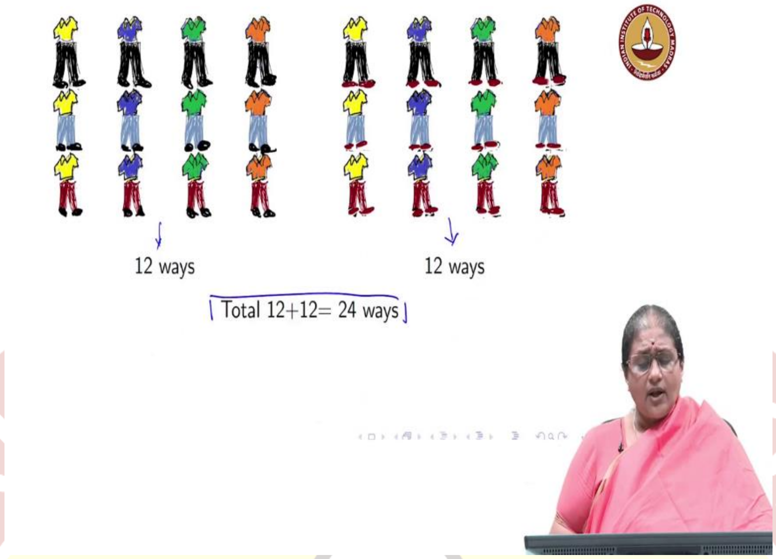

So, what I get is with each I had 12 pairs of my yellow and black, pant and shirt. And now you can see that I have for each of those 12 pairs I am having a black shoe, so this gives me 12 ways, the same 12 pairs with a brown shoe gives me 12 ways, so I have total 24 ways of matching the shirts with the pant and the shoes.

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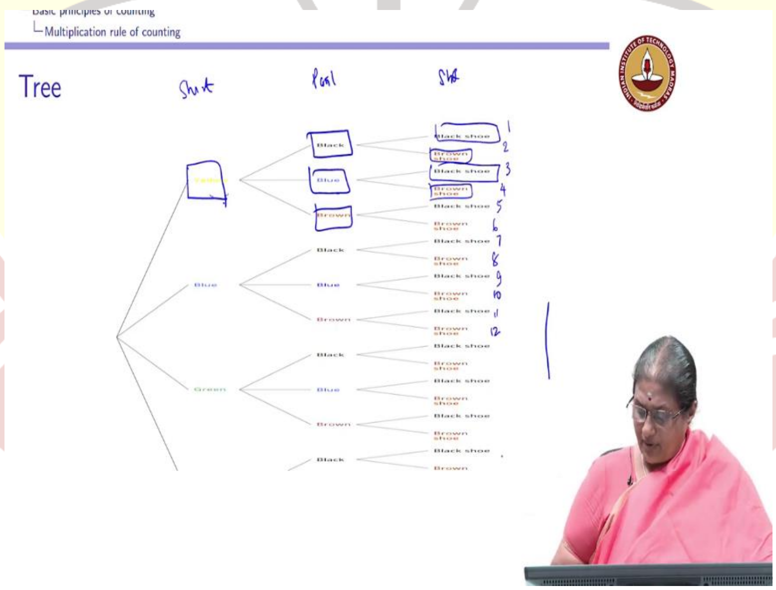

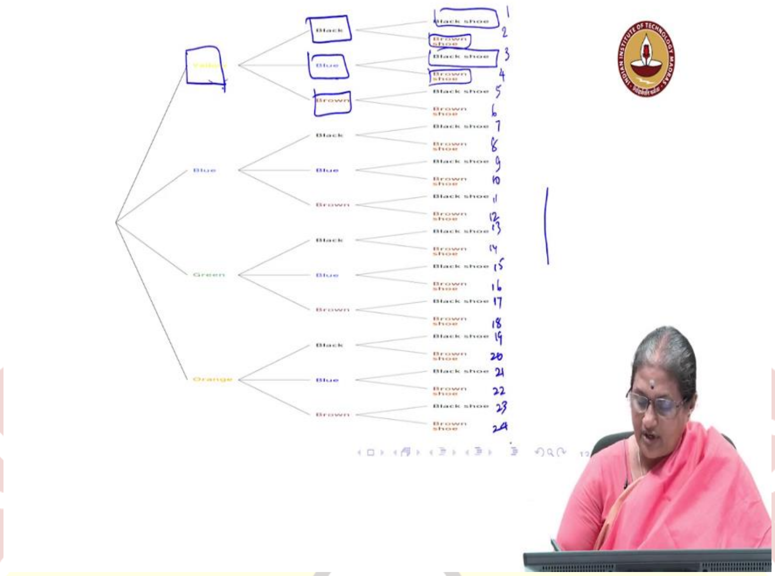

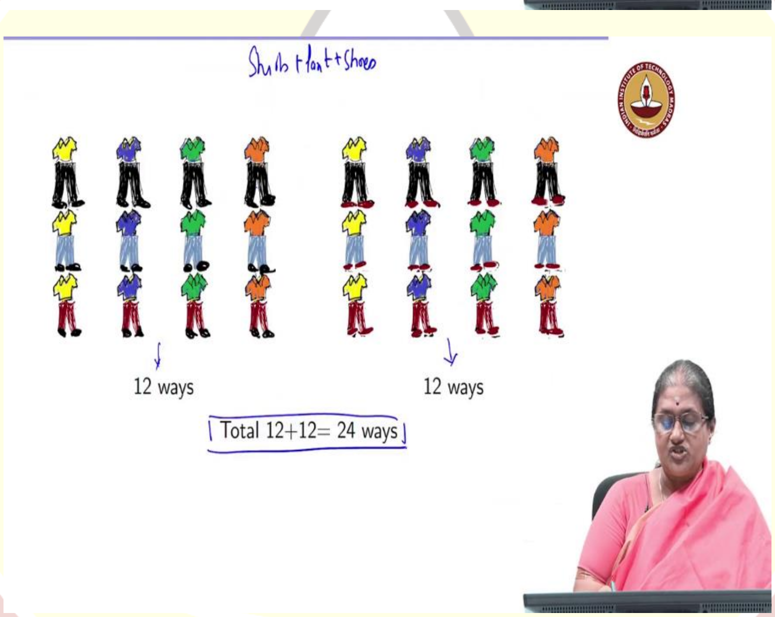

So, now if you go back and look at a decision or a tree, if you go back and look at a tree diagram, this is the huge tree. So, I have a yellow shirt, with a yellow shirt I could either have a black pant, a blue pant or a brown pant, with a yellow shirt and a black pant I could either have a black shoe or a brown shoe, with a yellow shirt a blue pant, so this is my pant, this is my shirt, this is my shoe. With the yellow shirt and a blue pant I could again have a black shoe or a brown shoe.

So, you can see if I number the total base of having this I have 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, I can keep going and you can see that the total number of ways is going to be so I have a 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, this is precisely what we had in our earlier slide that total twelve plus twelve is equal to 24 ways. So, what are we doing here?

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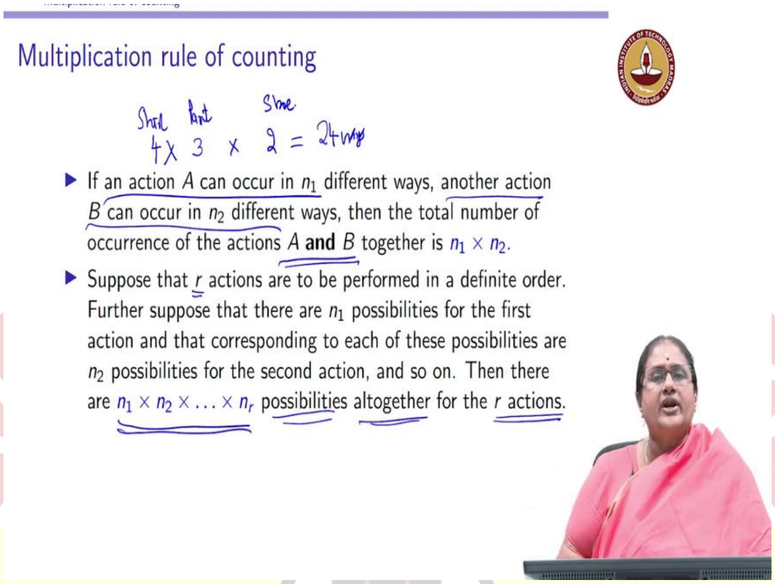

So, you can see that I can now I can state what is popularly known as the multiplication rule of counting. What does the multiplication rule of counting says? It says that if an action A can occur in n_1 different ways what is an action A, it was choosing a shirt. How many ways I could choose a shirt? I could have chosen a shirt in 4 different ways.

Another action B can occur in n_2 different ways, choosing a pant and that could have occurred in 3 different ways, then the total number of A and B, A and B earlier we looked at A or B, now we are looking at A and B is 4×3 is 12 ways. Suppose, I have r action, so in a third example we had 3 actions, what was it in addition to the shirt and pant?

I had to choose a shoe, so the shirt was occurring in n1 which is 4 ways, pant in 3 ways, shoe can be chosen in 2 ways, the total number of ways I can have, the all the 3 actions occurring that is choosing a shirt and a pant and a shoe is $4 \times 3 \times 2$ which is 24 ways. I can extend this to any sequence or order of r actions and $n_1 \times n_2 \times \ldots . \times n_r$ will give me the total number of possibilities all together for all these r actions to occur together. And this is what we refer to as the multiplication rule of counting.

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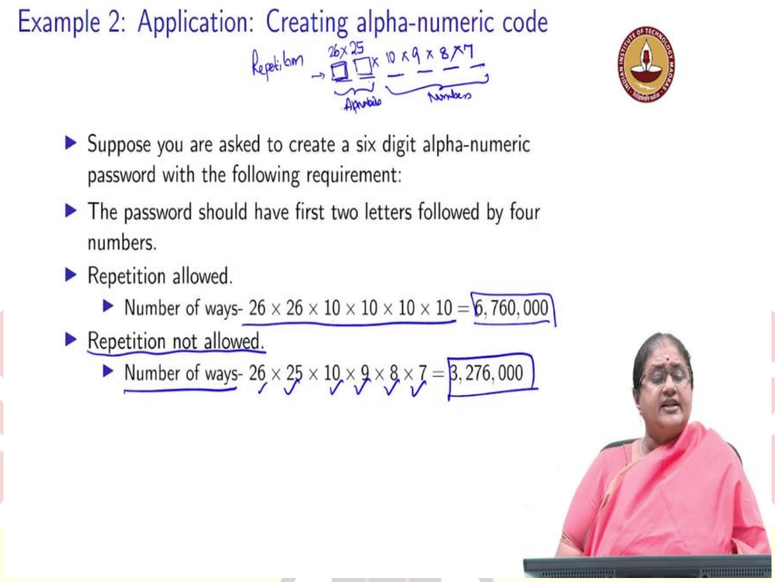

So, now let us apply the multiplication rule of counting to create an alpha numeric code. Many at times whenever we login into some website has asked us to generate a password, suppose, I am logging in into a website which is asking me to generate a password and what it is telling me is you generate a 6 digit password, the first 2 digits should be alphabets and the next 4 digits should be numbers.

So, the first action is to fill in the first space. How many ways can this be done? I can choose any 1 of the 26 alphabets to be here. Again this second space is again an alphabet, I can choose any 1 of the 26 alphabets again here. So, in the first case if I allow repetition of my alphabets, the first can be chosen in 26 way, the second can be chosen in another 26 way, now numbers if I include 0 this has a 10 I can do this choice for this blank in 10 ways, this again and 10 ways, this again in 10 ways.

So, I have actually 6 actions, the first action is to choose an alphabet for the first blank, the second action is to choose an alphabet for the second blank, the third action is to choose a number for the third blank and so forth the 6th action is to choose a number for the 6th blank, so the total number of ways I can create or total number of codes I can create is $26 \times 26 \times 10 \times 10 \times 10$.

So, that is what I have here which is $26 \times 26 \times 10 \times 10 \times 10$ which is this number which is 6,760,000 ways to do it. Now, suppose I do not allow repetition. What do I mean? You can see that when I look at the alphabets here if I have chosen an alphabet to fill in the first blank I have only 25 alphabets available to fill in the second blank, so I have 25 here, so these 2 blanks or the first 2 blanks together can be filled up in 26×25 or 25×26 ways.

Similarly, the third blank which has to be a number for the first blank I have 10 choices, now once I have the 10 choices and I am not repeating that number, I only have a 9 choices for the second blank, I am again not repeating it, I have 8 choices that are left over for the third blank, and for the last blank I have only 7 choices that are left over.

Now, applying the same, so I have here r actions 26 choices for the first action, 25 choices for the second action, 10 for the third, 9 for the fourth, 8 for the fifth and 7 for the sixth action which gives me the total number of ways is $26 \times 25 \times 10 \times 9 \times 8 \times 7$ which is 3276, sorry which is this number 3,27,6000. So, you can see that how we can apply a basic principle of counting to something which we generate almost on a daily basis and every time you want to generate an alpha numeric code you know that you can choose from this number.

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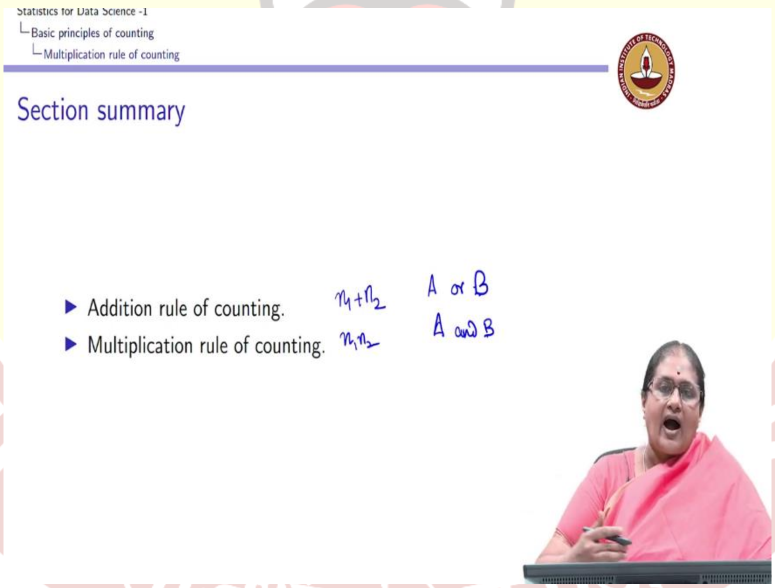

So, in summary what we have learnt so far is, first we have introduced what is the fundamental principle of counting, we say that the fundamental principle of counting is again we looked at the addition rule of counting here it is $n_1 + n_2$, here basically it is action A or action B, $n_1 \times n_2$where it is action A and action B and I can extend this logic to r actions.