IIT Madras

ONLINE DEGREE

Mathematics for Data Science 1
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Lecture – 35
Division Algorithm

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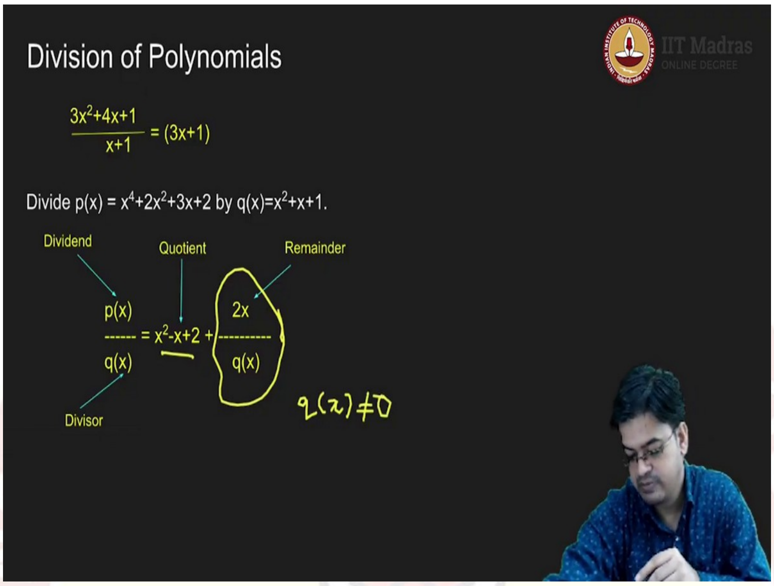

So, let us go to the next slide and emphasize the algorithm that we have derived just now. In order to understand the algorithm, you need some terminology. For example, this $p(x)$ is called the divident; the $q(x)$ is called the divisor. The term that you get over here, here is called the term that you get over here is called the quotient. And the $2x$ that you have got is called the remainder.

Remember you will declare something as a reminder only when the degree of the denominator is higher than the degree of the numerator, this is the strategy that we will follow.

So, now, you are very clear about the terminology, the numerator is the divident, the denominator is the divisor, the term the polynomial term that you get after dividing is called the quotient, and the rational and the remainder is something that where the degree of the numerator is smaller than the degree of the denominator.

This is also called a rational function. If you look at polynomial as a function, then division of two polynomials is a rational function, only condition that we are enforcing is $q(x)$ cannot be equal to 0, this is the condition is which is always in place. Let me eliminate this and let us go and study the algorithm.

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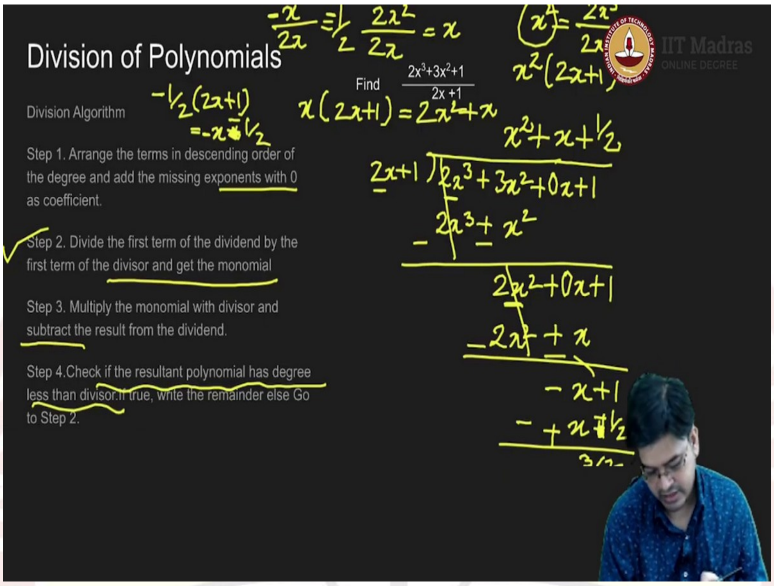

So, for division of polynomials, we will use the following division algorithm which we have derived just now, where in the first step what we will do is we will arrange the terms in the descending order of the degree, and add the missing exponent with 0 as a coefficient.

Then after adding the missing 0, 0 as a coefficient after adding the missing exponents, next what we will do is, we will take the first leading terms or the leading monomials, and we will divide the dividents monomial, the leading monomial of the divident and the leading monomial of the divisor together. And we will get some number which is which we will call as quotient, temporary quotient and that quotient we will multiply with our divident.

Once we multiply with our divident, what we will actually do is we will subtract that from the original expression for the polynomial that is our numerator. Whatever is remaining, we will treat that as the next divident. Once we treat that divident, then we will check if the degree of that new divident is higher than the degree of the denominator or divisor. If yes, then we will continue with the procedure; if no, we will terminate the procedure; this is how we will give the division algorithm.

Let us understand this division algorithm by using one example. So, here is an example. This is the numerator $2x^3+3x^2+1$ divided by $2x+1$, and I want to find the answer to this question. Let us figure out how to find the answer.

So, in the earlier quest, what I did is I have used the standard numerator denominator. Now, there is a popular method for division of the polynomial which is called long division, which works in a similar manner and the same division algorithm works, but you will have a better handle over the terms.

So, in this long division, what you will do is you will put a parenthesis over here, and you will put $2x+1$ outside the parenthesis, and you will put this term that is $2x^3+3x^2$. Now, remember the first step plus $0x+1$ ok. So, this is how we will write. Now, according to our standard terminology, what we will do is we will take the leading terms $2x$ and $2x^{3.}$ So, somewhere in the rough you do that. What is $2x^3$ divided by $2x$? This will give you $x^{2.}$.

So, you write x^2 over here, multiply x^2 with $2x+1$. Once you multiply x^2 with $2x+1$, write that term over here, $2x^3+x^2$. Now, according to our algorithm divide the first term of the dividend by the first term of the divisor and get the monomial that monomial is x^2 over here. Next step, multiply the monomial with the divisor and subtract the result from the divident. So, this is the result from the divident, result from multiplication, and you are subtracting it from the divident.

So, this will cancel off. So, this will give me 0 and $3x^2-x^2$ will give me $2x^2+0x+1$. This is the result ok. So, now, this result, I will check whether the degree of this result this polynomial that I have obtained is greater or smaller than this ok, that is what we will do. Check if the resultant polynomial has a degree less than the divisor that is not true.

So, we will go to step 2. What is the step 2? Which is this, divide the first term, first term of this divident with this that is you will divide $\frac{2x^2}{2x}$. So, what you will get here is

x . So, you will simply add x over here. And then you will multiply that x with $2x+1$. Once you do that, you will get $2x^2+x$. So, you write here $2x^2+x$.

Then what is the next step? You subtract it from the result, so minus, minus $2x^2$ vanishes, this gives me $-x+1$, ok. So, $-x+1$, again I will go to the same step because this degree is same, it is not less than the degree of the denominator.

So, I will again follow the same procedure; $\frac{x}{2x}$ which will give me $\frac{1}{2}$. So, naturally I will add $\frac{1}{2}$ over here. And once I add $\frac{1}{2}$ over here, when I multiply $\frac{1}{2}$ with $2x+1$, what I will get here is $x+\frac{1}{2}$. So, I will write that $x+\frac{1}{2}$.

But remember over here the thing was $-x$. So, I should what I should have done is I should have multiplied -1 to the x that means, $\frac{-x}{2}$. So, the answer is $\frac{-1}{2}$, and you will multiply $\frac{-1}{2}$ over here, so $-x$ this will not be plus this will be $\frac{-1}{2}$, so $-x-\frac{1}{2}$ which will be given a negative sign. So, this will be $x+\frac{1}{2}$. So, I will get the answer to be equal to $\frac{3}{2}$ ok. So, the answer is $\frac{3}{2}$.

So, what is what will be the resultant answer? This should not be plus, 1 minute, let me make it very clear. This cannot be plus; this should be minus, because I have to multiply with $\frac{-1}{2}$. And here it is the remainder is $\frac{3}{2}$. So, what I got here is $x^2+x-\frac{1}{2}$ and the as a quotient, and the remainder is $\frac{3}{2}$.

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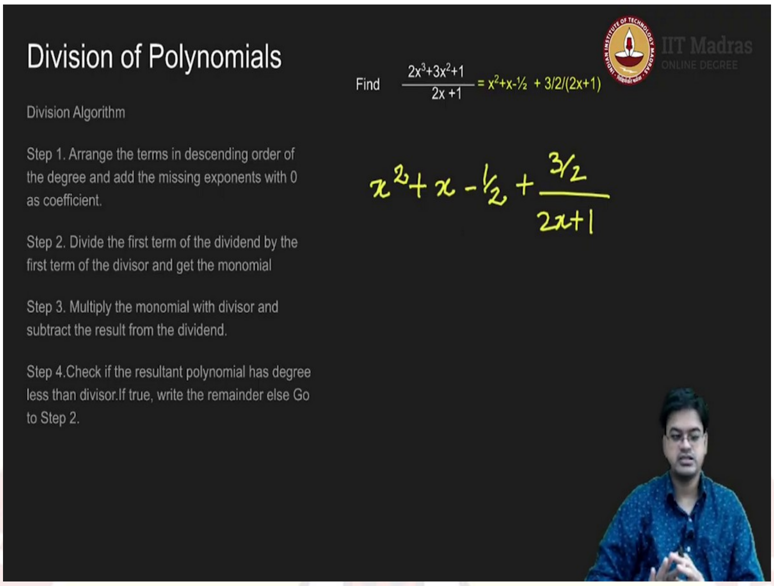

So, let me rewrite it again that is I got $x^2+x-\frac{1}{2}+\frac{\frac{3}{2}}{2x+1}$, this is what I got. Let me verify this result. And we have demonstrated the algorithm, yes, $x^2+x-\frac{1}{2}+\frac{\frac{3}{2}}{2x+1}$, this is how we will consider division of polynomials in general.