The Natural Exponential Function.

Sunday, 9 August 2020 11:41 AM

From the theory of limits, it is known that

$\left(1+\frac{1}{n}\right)^n \to e$ as $n \to \infty$

Existence of 'e' is studied in calculus.

e is irrational number.

$e \approx 2.71828\ldots$

Q Why is 'e' so important?

Interest Rate Calculation

↓ Cont^s Compounding

n	$\left(1+\frac{1}{n}\right)^n$
1	2
10	2.5937
100	2.7048
1000	2.7169
10,000	2.7181
1,00,000	2.7182

$\to e$

Euler's number

Rel $\left(1+\frac{x}{n}\right)^{nt}$

e^{xt}

1% $\left(1+\frac{1}{100}\right)^n$

$\left(1+\frac{0.01}{4}\right)^4$

$\left(1+\frac{0.01}{12}\right)^{12}$

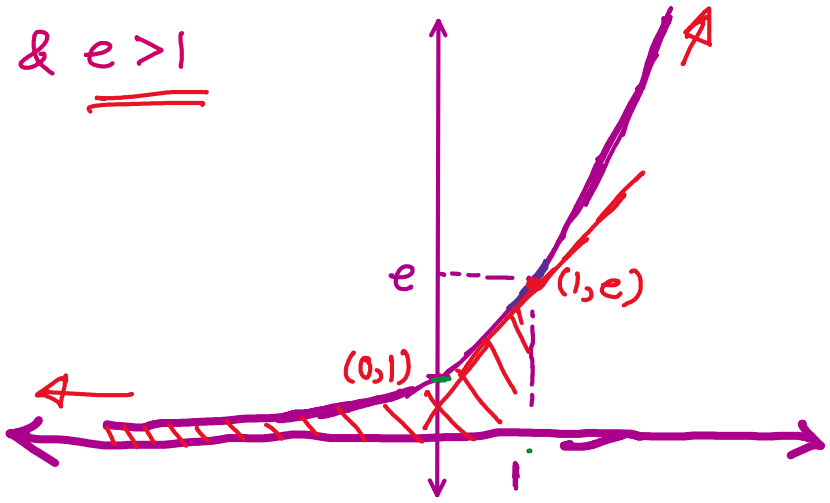
$\left(1+\frac{0.01}{n}\right)^n \quad n \to \infty$

$1 \times e^{0.01t}$

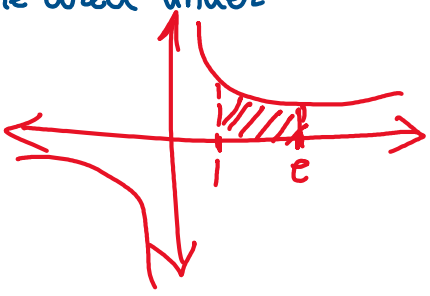
Definition.

The natural exponential function is defined as $f(x) = e^x$.

Properties. Domain of $f = \mathbb{R}$, Range $= (0, \infty)$ & $e > 1$



- e is the slope of the tangent line to $f(x) = e^x$ at $(1, e)$.
- The area under the $f(x) = e^x$ from $(-\infty, 1)$ is e.
- For $f(x) = 1/x$, $x \in (1, e)$, the area under the curve is 1.



Example.

Let R be the percent of people who respond

to affiliate links under YouTube descriptions &

purchase the product in t minutes is given by

$$R(t) = 50 - 100e^{-0.2t}.$$

(a) What is the percentage of people responding after 10 minutes?

(b) What is the highest percent expected? 50%

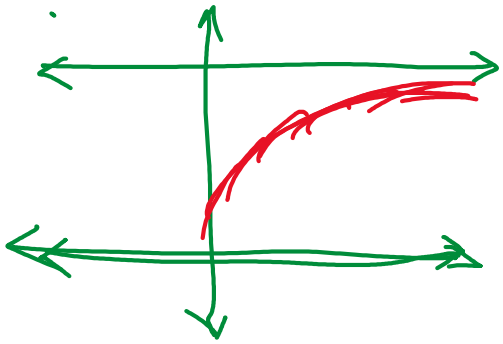
(c) How long before $R(t)$ exceeds 30%?

(a) $R(10) = 50 - 100e^{-0.2 \times 10}$

$= 50 - 100e^{-2} = 36.46$

(b) $R(t) = 50 - 100e^{-0.2t}$

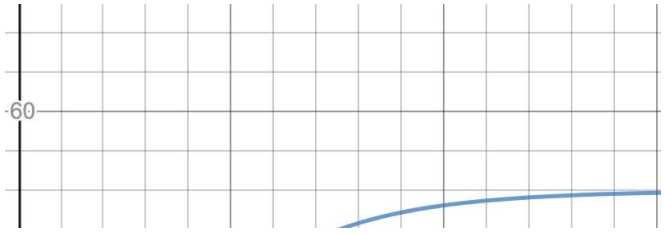
50%



(c) $R(t) = 50 - 100e^{-0.2t}$

- $R(t) = 30$

$30 = 50 - 100e^{-0.2t}$



$20 = 100e^{-0.2t}$

$\frac{1}{5} = e^{-0.2t}$ • STOP

→ $t \approx 8$ minutes