Exponential function.

12 March 2020 10:58

$\boxed{\text{Exponent}}$

Recall a^{r} exponent 'base'

$a > 0, \; r \in \mathbb{Q}$

What if $r \in \mathbb{R} \setminus \mathbb{Q}$? Irrational?

Why $a > 0$?

$a = -1$

$a^{1/2} = (-1)^{1/2} = \boxed{i \in \mathbb{C}}$

$a^{r}, \; r \in \mathbb{R} \setminus \mathbb{Q}$

Q. Can we define a^{x} $(a > 0)$ for $x \in \mathbb{R} \setminus \mathbb{Q}$?

Eg. $2^{\sqrt{2}}$, 5^{π}

$\sqrt{2} = 1.41\ldots$

2^{1}

$2^{1.4}$

$2^{1.41}$

$\vdots$

a^{x} is defined for $\boxed{x \in \mathbb{R}}$

$\pi = 3.14159\,2635\ldots$ (Non-repeating)

$5^{\pi} = ?$

5^{3}

$5^{3.1}$

$5^{3.14}$

$5^{3.141\ldots}$

$\vdots$

$\downarrow$

5^{π}

seq^n

Existence of 5^{π} is assured.

Laws of Exponents.

For $s, t \in \mathbb{R}$ and $a, b > 0$,

(i) $a^s \cdot a^t = a^{s+t}$

(ii) $(a^s)^t = a^{st}$

(iii) $(ab)^s = a^s b^s$

Recall. $1^s = 1$, $a^{-s} = \frac{1}{a^s} = \left(\frac{1}{a}\right)^s$ and $a^0 = 1$, $a > 0$

0^0 is undefined

Definition.

An exponential function in standard form is given by $f(x) = a^x$, where $a > 0$, $\boxed{a \neq 1}$

Observations.

(i) Domain of f is $\mathbb{R}$

(ii) $a \neq 1$? $f(x) = 1^x = 1$ (constant)

Exercise.

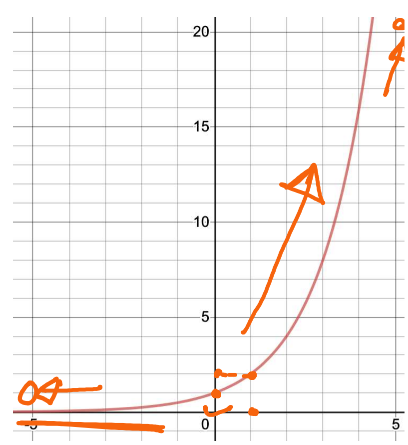
Graph the following functions (Graphing tool)

1. (a) 2^x (b) 3^x (c) 5^x together

2. (a) $(1/2)^x$ (b) $(1/3)^x$ (c) $(1/5)^x$. together

Identify properties of the graphs.

1(a) $f(x) = 2^x$



Domain of f = $\mathbb{R}$

$2 > 1$

$\boxed{\begin{array}{l} 2^x > 2^0 = 1 \\ 0 < 2^x < 1 \end{array}}$ $x > 0$

Range of f = $(0, \infty)$

y-intercept = $(0, 1)$

x-intercept = Nil

$\boxed{y = 0 \quad \text{Horizontal Asymptote.}}$

End-behavior $x \to \infty$ $2^x \to \infty$

$x \to -\infty$ $2^x \to 0$

Roots $\boxed{\text{No roots}}$

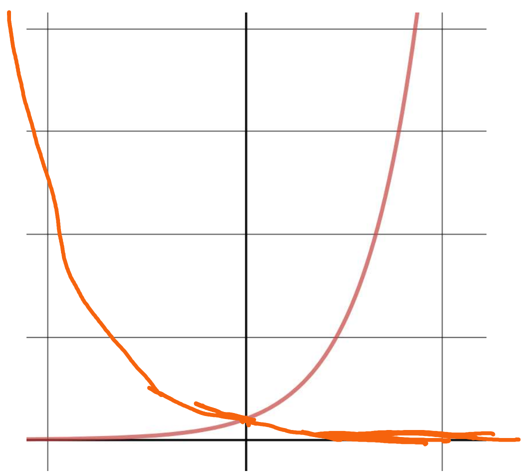
$x_1 \neq x_2$ $x_1 < x_2$

increase / decrease $\quad 2 \quad$ increasing $\boxed{2^{x_1} < 2^{x_2}}$

Fact.

Every $f(x) = a^x$, $a > 1$ has same properties as 2^x.

Graph of $f(x) = a^x$, $a > 1$.



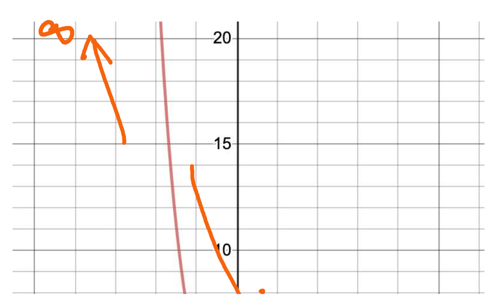
$0 < a < 1 \qquad g(x) = a^x$

2 c $\quad g(x) = \left(\frac{1}{5}\right)^x = 5^{-x}$

Compare with $\boxed{5^x}$

Domain $= \mathbb{R}$

Range $= (0, \infty)$



y-intercept = (0, 1)

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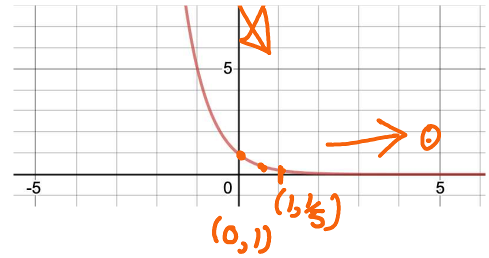


x-intercept = Nil

Roots No roots

End-behavior

$x \to \infty$ $\left(\frac{1}{5}\right)^x \to 0$

$x \to -\infty$ $\left(\frac{1}{5}\right)^x \to \infty$

Increase/decrease Decreasing f^n

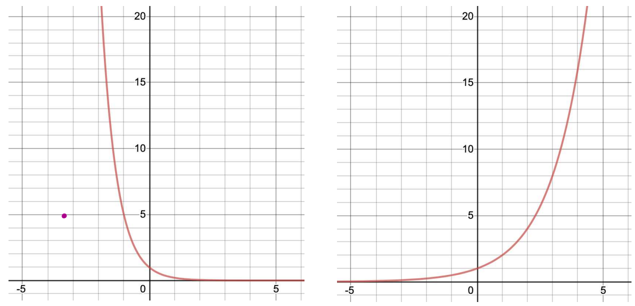
Fact.

Every $f(x) = a^x, 0<a<1$ has same properties as $\left(\frac{1}{5}\right)^x$.

Summary

$f(x) = a^x$	$0<a<1$	$a>1$
Domain	$\mathbb{R}$	$\mathbb{R}$
Range	$(0, \infty)$	$(0, \infty)$
x-intercept	Nil	Nil
y-intercept	$(0, 1)$	$(0, 1)$
Horizontal Asymptote	$y=0$	$y=0$
Increase/decrease	decreasing	increasing
End behavior		
$x \to \infty$	$f(x) \to 0$.	$f(x) \to \infty$.
$x \to -\infty$	$f(x) \to \infty$.	$f(x) \to 0$.

Graphs



Example. Graph $f(x) = 3^{-x} + 2$.