IIT Madras

ONLINE DEGREE

Computational Thinking
Professor. Madhavan Mukund
Department of Computer Science
Chennai Mathematical Institute
Professor. G.Venkatesh
Indian Institute of Technology, Madras
Undirected Graph and Cliques

Professor. Madhavan Mukund: So, we have been looking at this mentoring problem. So, the mentoring problem is trying to find when students are separated in marks,…

Professor. G.Venkatesh: Dissimilar.

Professor. Madhavan Mukund: Dissimilar, so that one student can help another student but you should not be too bad, they should be at least some separation but not too much separation.

Professor. G.Venkatesh: Right.

Professor. Madhavan Mukund: But the other problem is also possibly something useful which is when our two students similar so we can find out whether they are actually performing in a related way and see whether there are groups of students this group of student is doing well…

Professor. Madhavan Mukund: That student that group of students…

Professor. G.Venkatesh: They should not only be similar, they should be similar in all subjects.

Professor. Madhavan Mukund: Across all subjects, yes.

Professor. G.Venkatesh: All subjects. So similar 10 marks, can we say 10 marks?

Professor. Madhavan Mukund: Last time we said dissimilar is more than 10…

Professor. G.Venkatesh: 10.

Professor. Madhavan Mukund: So let us say that…

Professor. G.Venkatesh: 1 to 9.

Professor. Madhavan Mukund: This is 1 to 9, so upto 9 marks.

Professor. G.Venkatesh: So which means that…

Professor. Madhavan Mukund: So we are saying similar.

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Professor. G.Venkatesh: Let us say if I have Harish and Gautami, just to compare here.

Professor. Madhavan Mukund: Yeah.

Professor. G.Venkatesh: I would say Harish and Gautami are similar…

Professor. Madhavan Mukund: Yeah.

Professor. G.Venkatesh: If their maths marks are within 10, less than 10 marks.

Professor. Madhavan Mukund: 9 marks.

Professor. G.Venkatesh: 1 to 9 marks, Physics is within 10 and chemistry in this case is obviously 76 minus 62 is 14 so they are not similar.

Professor. Madhavan Mukund: So they are not similar.

Professor. G.Venkatesh: That means we have to compare everybody?

Professor. Madhavan Mukund: So, we have to see whether we can find this…

Professor. G.Venkatesh: But this is I mean, this was that was a thing there we are creating a relationship we just…

Professor. Madhavan Mukund: Yes.

Professor. G.Venkatesh: Directed.

Professor. Madhavan Mukund: Yes.

Professor. G.Venkatesh: A is mentoring B, here when we say similar, we are not directing or anything, right?

Professor. Madhavan Mukund: Correct, they are an equal in some sense, they are you could either say 1 is similar to 15 or 15 is similar to 1, it dos not make…

Professor. G.Venkatesh: So this if you are going to represent it with notes and all, so be a…

Professor. Madhavan Mukund: So we would for example, supposing we just take two numbers may or may not be, so supporting 22 and say 13, we say they are similar. Then we will connect them…

Professor. G.Venkatesh: Undirected edge.

Professor. Madhavan Mukund: But, so we could of course do this but it makes no sense because this thing 13 is similar to 22 and 22 is similar to 13 so we can just remove this…

Professor. G.Venkatesh: No arrows.

Professor. Madhavan Mukund: No arrows and just put a single…

Professor. G.Venkatesh: One single line.

Professor. Madhavan Mukund: Single line.

Professor. G.Venkatesh: Single line connecting to everybody.

Professor. Madhavan Mukund: So this is an undirected.

Professor. G.Venkatesh: So undirected edge. So should be find, all the pairs?

Professor. Madhavan Mukund: Let us try it, maybe we can find some.

Professor. G.Venkatesh: Let us see. So, how do we do? Every pair every card we have to do.

Professor. Madhavan Mukund: That we have to do.

Professor. G.Venkatesh: So, Gautami Harish we have compared.

Professor. Madhavan Mukund: Straight away Rahul is out because maths is too high.

Professor. G.Venkatesh: So, leave it out, 76 42 rule out, 76 56 ruled out, 76 62 ruled out, 89 ruled out, 74 this is possible, now we have to compare 58 with 64 possible, 19 51 ruled out, 76 52 ruled out, 58 64.

Professor. Madhavan Mukund: But this is ruled out.

Professor. G.Venkatesh: But this is rules out, 87 ruled out, 76 68 ruled out.

Professor. Madhavan Mukund: 68 is possible, let us see…

Professor. G.Venkatesh: 76 68 is possible.

Professor. Madhavan Mukund: 8, and this is…

Professor. G.Venkatesh: 58 64…

Professor. Madhavan Mukund: But this is 12.

Professor. G.Venkatesh: 90 78, we are not finding anybody, 76 93, 58, 68.

Professor. Madhavan Mukund: No.

Professor. G.Venkatesh: No, 90 91, this is actually there, but other are gone.

Professor. Madhavan Mukund: This maybe…

Professor. G.Venkatesh: 76 78 possible, 58 69…

Professor. Madhavan Mukund: Is gone, too much.

Professor. G.Venkatesh: Too much. 76 74, 58 71, gone.

Professor. Madhavan Mukund: Gone.

Professor. G.Venkatesh: 76 64 gone.

Professor. Madhavan Mukund: 64 is okay, no so 12.

Professor. G.Venkatesh: 76 65 is also gone, 76 87 is also gone, that 81 is possible.

Professor. Madhavan Mukund: Yeah.

Professor. G.Venkatesh: 58…

Professor. Madhavan Mukund: 76 is too much.

Professor. G.Venkatesh: Too much, gone. 76 56 is gone, 76 72 is possible, 66 is 8, 98 81 is…

Professor. Madhavan Mukund: So, we found somebody, so now

Professor. G.Venkatesh: Gautami and Navin.

Professor. Madhavan Mukund: So, 15 is similar to…

Professor. G.Venkatesh: 29.

Professor. Madhavan Mukund: 29, very good.

Professor. G.Venkatesh: Some doing, alright. Let us take this 76 62 is gone, 76 81 is possible, 58 82…

Professor. Madhavan Mukund: Gone.

Professor. G.Venkatesh: Gone, 76 74, 83, 87 gone, 76 63 gone, 76 97 gone, 76 84.

Professor. Madhavan Mukund: 84 is possible, right? 8 But 58 is 92.

Professor. G.Venkatesh: 76 71 is possible…

Professor. Madhavan Mukund: Your next one is too much.

Professor. G.Venkatesh: Next one is too much, 76 72 is possible…

Professor. Madhavan Mukund: But again…

Professor. G.Venkatesh: Again too much.

Professor. Madhavan Mukund: Physics is too much.

Professor. G.Venkatesh: 44, alright. So we found…

Professor. Madhavan Mukund: So, Gautami is only similar to one other student…

Professor. G.Venkatesh: Who is?

Professor. Madhavan Mukund: Mister Naveen I think number 29.

Professor. G.Venkatesh: Number 29, so let me take out Naveen. Now let us see if Naveen is similar to somebody, just like that, for the heck of it. Of course Naveen is similar to Gautami.

Professor. Madhavan Mukund: So, that we already know.

Professor. G.Venkatesh: We already checked. We checked this with this, this with that same thing. Now we have to check with Naveen is similar to somebody. We go through that?

Professor. Madhavan Mukund: Yeah.

Professor. G.Venkatesh: So, 72 44, ruled out, 72 72, 66 92…

Professor. Madhavan Mukund: 66 92 is out, so 92…

Professor. G.Venkatesh: You know I mean Gautami is not similar to any of these, so Naveen may also not be similar to any of them.

Professor. Madhavan Mukund: We will see but Gautami has a very high chemistry marks so it is not because it is 9 and then 9…

Professor. G.Venkatesh: So, it is not a…

Professor. Madhavan Mukund: It is not transitive.

Professor. G.Venkatesh: Tranitive, not a transitive.

Professor. Madhavan Mukund: Because somebody could be within 9 marks or Naveen but more than 9 marks away from Gautami.

Professor. G.Venkatesh: I see okay. Alright let us see.

Professor. Madhavan Mukund: So, I think we might find somebody.

Professor. G.Venkatesh: Let us see, this is gone, 72 71, 66 92, ruled out, 72 84 ruled out, 72 97 ruled out, 72 63 is possible.

Professor. Madhavan Mukund: But 88 is too much.

Professor. G.Venkatesh: This is ruled out, 72 87 is ruled out, 74 is possible.

Professor. Madhavan Mukund: But 83 is too high.

Professor. G.Venkatesh: 83 is too high…

Professor. Madhavan Mukund: 81…

Professor. G.Venkatesh: 81 is 9 marks, 82 is too high, this is just about, 72 56 is ruled out, 81 is possible, 76 is…

Professor. Madhavan Mukund: 10.

Professor. G.Venkatesh: 10 marks. 87, 65, 73.

Professor. Madhavan Mukund: These are okay.

Professor. G.Venkatesh: 89.

Professor. Madhavan Mukund: So, now we have Gopi…

Professor. G.Venkatesh: Naveen is linked to Gopi.

Professor. Madhavan Mukund: So, I will draw it in different color because we found it differently, Naveen is linked to Gopi.

Professor. G.Venkatesh: Naveen is similar to Gopi, Gautami is similar to Naveen, we will just finish Naveen…

Professor. Madhavan Mukund: We will finish Naveen but you can see now Gautami because this is 4 and this is another 7, so this was 11…

Professor. G.Venkatesh: Totally added up 11...

Professor. Madhavan Mukund: So therefore, is actually not transitive.

Professor. G.Venkatesh: So, this did not form, I mean if Gopi were also connected to Gautami for some reason, then it would have been a nice group of all similar…

Professor. Madhavan Mukund: Similar to each other.

Professor. G.Venkatesh: But now we cannot really say that they are similar.

Professor. Madhavan Mukund: Yeah.

Professor. G.Venkatesh: All 3 are similar to each other you can say these two are similar these two are similar but not all 3 are not similar.

Professor. Madhavan Mukund: Correct.

Professor. G.Venkatesh: Kavya 72 64 is possible, 66 72 is possible, 82.

Professor. Madhavan Mukund: It is too much.

Professor. G.Venkatesh: 72 74, 66 71.

Professor. Madhavan Mukund: 26.

Professor. G.Venkatesh: Jk Naveen seems to be well located, better located. Then Naveen with Monica 6 is possible, 69 74.

Professor. Madhavan Mukund: So, 21.

Professor. G.Venkatesh: Another one. Kalyan is gone. I am comparing Naveen right gone. 68 is okay, 64 is okay, 78 is okay.

Professor. Madhavan Mukund: We are finding quite a few.

Professor. G.Venkatesh: Ritika.

Professor. Madhavan Mukund: 87 is too much.

Professor. G.Venkatesh: 87 is too much gone. Mathematics 72 gone, then 74 is okay, 64 is okay 51…

Professor. Madhavan Mukund: Chemistry is too much.

Professor. G.Venkatesh: 89 is too much right? 62 is out of the range, just out of the range. 57 is out of the range, 42 is definitely out of the range, 97 is out of the range, 62 is also out of the range.

Professor. Madhavan Mukund: So, these are have now are 6 people who are all connected, 5 people who are all collected to Naveen.

Professor. G.Venkatesh: Naveen.

Professor. Madhavan Mukund: So, now we should try and see whether any of them are connected to each other.

Professor. G.Venkatesh: We leave these cards.

Professor. Madhavan Mukund: So let us look are 13, let us look at 13. So, 13 we have already compared with these two, so let us see 13 and these top 3.

Professor. G.Venkatesh: Top 3, so 65 78 ruled out, 65 74…

Professor. Madhavan Mukund: Yes possible.

Professor. G.Venkatesh: Possible, 73 71, so Gopi and Jk.

Professor. Madhavan Mukund: So, that is 13 26. So, here we have now what we are looking for…

Professor. G.Venkatesh: We have a group of 3.

Professor. Madhavan Mukund: Group of 3 who are all…

Professor. G.Venkatesh: Naveen, Jk and Gopi are all 3…

Professor. Madhavan Mukund: Similar to each other.

Professor. G.Venkatesh: Because each pair wise they are similar.

Professor. Madhavan Mukund: What about that one?

Professor. G.Venkatesh: Gopi 65 68, 73 64, 89 70.

Professor. Madhavan Mukund: That is not…

Professor. G.Venkatesh: How about Jk and Bhuvanesh, 74 68.

Professor. Madhavan Mukund: Yes.

Professor. G.Venkatesh: 71 64.

Professor. Madhavan Mukund: Yes.

Professor. G.Venkatesh: 82 78.

Professor. Madhavan Mukund: Yes, so that is…

Professor. G.Venkatesh: Jk and Bhuvanesh.

Professor. Madhavan Mukund: 26 and 0, so that is another different triangle that we have...

Professor. G.Venkatesh: And you try Monica, 21…

Professor. Madhavan Mukund: Okay.

Professor. G.Venkatesh: So, Monica we have tried with…

Professor. Madhavan Mukund: Gautami and Naveen already we know, with Gopi we said no for some reason.

Professor. G.Venkatesh: 78 and 65.

Professor. Madhavan Mukund: Jk.

Professor. G.Venkatesh: 78 74, 69 71, 74 82, yes.

Professor. Madhavan Mukund: Yes, so 21 26 is connected, very good.

Professor. G.Venkatesh: And Monica with Bhuvenesh 78 68, ruled out.

Professor. Madhavan Mukund: No.

Professor. G.Venkatesh: So, we have 29 26 0.

Professor. Madhavan Mukund: Yes.

Professor. G.Venkatesh: A similar, 29 26 13 are similar, 29 26 21 are similar.

Professor. Madhavan Mukund: So, we have 3 different groups here.

Professor. G.Venkatesh: But 29 21 26 13 0 we cannot put all of them into one group. 15 seems not connected actually strictly speaking.

Professor. Madhavan Mukund: So, 15 is kind out sticking out from this group.

Professor. G.Venkatesh: Ignored. Atleast these people are kind of more or less connected because more edges in there?

Professor. Madhavan Mukund: So now…

Professor. G.Venkatesh: So, if everybody is connected everybody else in a set is it a name in the graph for that.

Professor. Madhavan Mukund: So it is like a what we say about people, people who stick to each other and they kind of exclude other people so called a clique.

Professor. G.Venkatesh: Clique.

Professor. Madhavan Mukund: A clique is a group of…

Professor. G.Venkatesh: So, clique is a group of students…

Professor. Madhavan Mukund: Who kind of congregate together and they exclude other people from their conversation. So, here we have these cliques of size 3.

Professor. G.Venkatesh: We did not find one of size 4 yet.

Professor. Madhavan Mukund: So, if we go looking we might find a bigger group all of whom who are similar to each other.

Professor. G.Venkatesh: We might find 1 or 4.

Professor. Madhavan Mukund: Might find them and then that we can isolate and say that this group of students performs similar to...

Professor. G.Venkatesh: Similar.

Professor. Madhavan Mukund: So, here we have these three each triangle and then this red triangle which is the outer triangle.

Professor. G.Venkatesh: By enlarge and if there are many many edges in between, even though it does not form a clique, if you have a set of students and the number of edges is very large between them, kind of feel that they are very similar.

Professor. Madhavan Mukund: They may be dissimilar only in one subject, for example we have seen some of them where two of the subjects they are similar and then in one there is differences. So, we could have even made our criterion a little weaker…

Professor. G.Venkatesh: Weaker.

Professor. Madhavan Mukund: We could have said in two subjects within 9 marks, then we would have got…

Professor. G.Venkatesh: Many more edges.

Professor. Madhavan Mukund: More edges and then we would have found more cliques or we could have only look for similarity in 2 subjects not in 3, so only maths and physics are they doing similar and so on, so that could have been one way of…

Professor. G.Venkatesh: So, difference between this what we did here which is clique, clique finding cliques creating this relationship and the one we did earlier we were mentoring, they seem to be the exact opposite of each other. When some when two students are connected by an edge here, they are similar they will not be connected by an edge there because…

Professor. Madhavan Mukund: Because they are…

Professor. G.Venkatesh: Here less than 10

Professor. Madhavan Mukund: Where there you want to be strictly more than…

Professor. G.Venkatesh: More than 10.

Professor. Madhavan Mukund: And in fact because we have said all 3 subjects they cannot be mentored in…

Professor. G.Venkatesh: So can we say that this is a compliment of that graph or something like that is different, opposite I mean just like in sets? You say, A and compliment of A something like that?

Professor. Madhavan Mukund: We could but the only difference is that here we are doing different type we have this undirected edges and there we have directed edges.

Professor. G.Venkatesh: Directed edges.

Professor. Madhavan Mukund: But it is true that if there is an undirected edge between these two students, we know that in every subject they are within 9 marks, 9 or below. So, in no subject can it be 10 or more, so there definitely cannot be a mentoring edge.

Professor. G.Venkatesh: Mentoring an edge.

Professor. Madhavan Mukund: So, wherever this is there there is no mentoring edge but it could be possible that neither is there.

Professor. G.Venkatesh: Right right.

Professor. Madhavan Mukund: It could be that you cannot mentor in any subject but at the same time you are not…

Professor. G.Venkatesh: Also there in that one we gave a thing that it has to be within 20 or…

Professor. Madhavan Mukund: That is why there is an upper bond so somebody is 30 apart of instance would not be mentored would not be similar.

Professor. G.Venkatesh: So, they could be.

Professor. Madhavan Mukund: So, you could have neither but…

Professor. G.Venkatesh: So, we saw directed graph we saw an undirected graph and there we were trying to find cycles, here also we are looking, not cycles but looking for pairwise everything.

Professor. Madhavan Mukund: So, at a triangle it looks like a cycle but actually we are looking for these groups of people who are all connected to each other, so that is this clique. So, we should probably write that word, so we are looking for this…

Professor. G.Venkatesh: Cliques, there we are looking for directed cycles, here we are looking for cliques.

Professor. Madhavan Mukund: So clique on so for example if we had 4 students who form a clique, then it will look like you have these 4 students and all of them are connected to each other, so we have these 4 edges but we also have these 4 edges…

Professor. G.Venkatesh: So this would be...

Professor. Madhavan Mukund: So, this would be a clique.

Professor. G.Venkatesh: I see.

Professor. Madhavan Mukund: And if you add a fifth student to join this clique, then this fifth student would have to be connected to these two but also have to be connected to these two, so we will have something like this. So, everybody is connected to everybody.

Professor. G.Venkatesh: 1, 2, 3 everything will have 4 edges.

Professor. Madhavan Mukund: Everything will have 4 edges because they are 5 of them and every student is similar to all the other 4. So, this is how so if we have 6 students, then every student will have 5 neighbours in this graph, so that what a clique would look like. So, for 3 it is just…

Professor. G.Venkatesh: So, in this set…

Professor. Madhavan Mukund: For 3 a clique is just a triangle…

Professor. G.Venkatesh: Nice to see, I mean I do not know we cannot do it mannualy tooks difficult to do, nice to run a procedure on this…

Professor. Madhavan Mukund: And see whether actually there are…

Professor. G.Venkatesh: Clique of size 4 or 5 or something…

Professor. Madhavan Mukund: What is the biggest clique we can find?

Professor. G.Venkatesh: What is a biggest clique we can find in this?

Professor. Madhavan Mukund: And then we can say that and it will also be interesting to know what that clique looks like in terms of marks, marks roughly in the middle or are they in the lower, if there is a clique of students who are doing badly, that means that the whole class performance somehow needs to improve and there is something…

Professor. G.Venkatesh: It is more likely that you would find cliques in the middle class.

Professor. Madhavan Mukund: Normally yes, normally I would expect that.

Professor. G.Venkatesh: Because more number of students are there, towards the edges you find extreme performance.

Professor. Madhavan Mukund: And the students in the middle tend to be roughly of similar…

Professor. G.Venkatesh: Similar…

Professor. Madhavan Mukund: Capability.

Professor. G.Venkatesh: Capability.