IIT Madras

ONLINE DEGREE

Mathematics for Data Science 1
Week 07 - Tutorial 01

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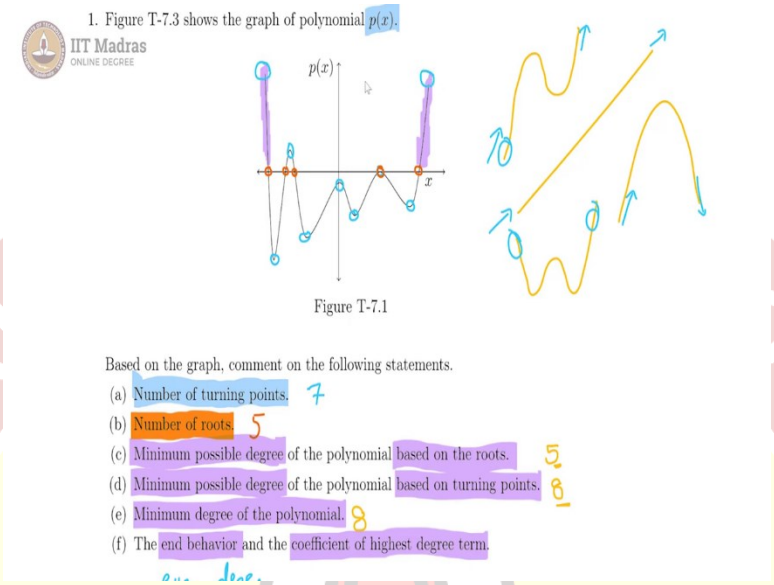

Hello mathematics students. In this tutorial we are going to look at questions based on graphs of polynomials. So, in this question there is this polynomial $p(x)$ whose graph is given here and we are supposed to comment on the following statements, the number of turning points, so that is easy so there is a turn here, 1, 2, 3, 4, 5, 6 and 7. So, there are 7 turning points. And then we are asked the number of roots, so roots would be where the polynomial touches or cuts the x axis so that is 1, 2, 3 and 4 and 5, so there are 5 distinct roots.

Now, what is the minimum possible degree? Minimum possible degree of this polynomial based on the number of roots. So, the minimum possible degree would be the same as the number of roots so if there are n roots to a polynomial then it should have a degree of at least n, so 5 is the minimum possible degree of this polynomial based on the roots. But now they are asking what is the minimum possible degree based on the turning points.

So, here we see this thing a straight line has no turning points, a quadratic equation has 1 turning point and a cubic would have 2 turning points at most likewise a quartic that is a fourth degree polynomial would have 3 turning points at most. So, if you have n turning points, then the minimum possible degree of the polynomial would be $n + 1$. So, here that is 8.

Now, what would be the minimum degree of the polynomial given all the information we have? Then you know that it has to be at least 8 the five which is on the basis of the roots is a lesser number than it and we know already that it has to be at least 8, so the minimum degree of the polynomial should be the greater of these two which is 8 because 6 and 7 and 5 are not allowed on the basis of turning points.

And then we are being asked what is the end behavior and the coefficient of the highest degree term. So, the end behavior shows that the polynomial is coming from ∞ and going to ∞ which means the degree of the polynomial is definitely even. So, we can say that it is an even degree polynomial.

So, as you can see we have just drawn these basic raw curves for the linear and quadratic and cubic and quartic polynomial. So, linear which is an odd degree polynomial it comes from -∞ and it goes to +∞ whereas quadratic a parabola here it is coming from -∞ and it is going to -∞ .

So, when it is even degree you see that the ends of the curves are in the same directions. Similarly, for quartic here this is coming from ∞ and going to ∞, whereas for a cubic this is coming from -∞ and going to +∞. So, here this is coming from ∞ and going to infinity.

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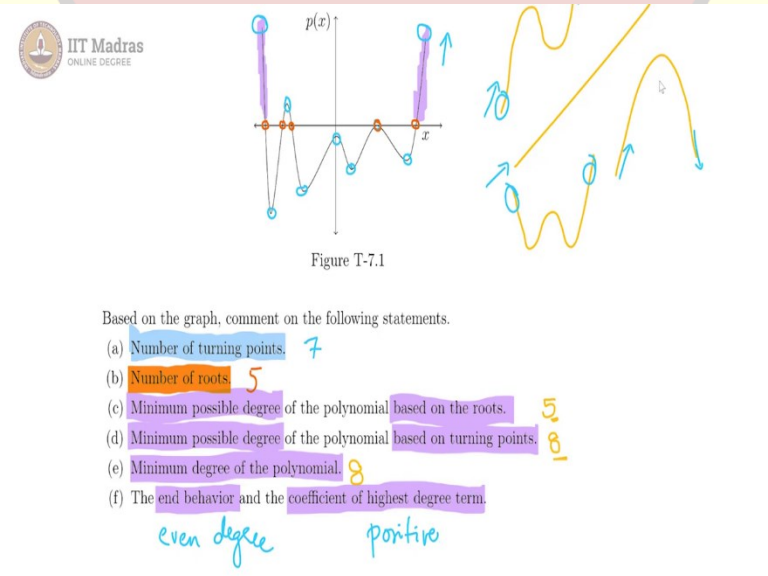

Therefore, this is a even degree polynomial and the coefficient of the highest degree term. So, the coefficient of the highest degree term determines whether the behavior of the polynomial as x

increases whether it is going to +∞ or -∞ , if the coefficient of the highest degree term is positive, it goes to +∞. So, if this is going to +∞ so this has to be positive coefficient for the highest degree term.