1. Vinod has n registers and m cover papers of different colours. In how many ways can he cover all the registers with cover papers?
   Answer: $m \times (m - 1) \times (m - 2) \times ... \times (m - n + 1)$
   Solution:
   For the 1st register, he can choose any of the m covers. So, number of ways to cover the first register with cover paper $= m$ ways.
   For the 2nd register, he can choose any of the remaining $(m - 1)$ covers. So, number of ways to cover the second register with cover paper $= m - 1$ ways.
   Similarly, for n^{th} register, he can choose any of the remaining $(m - n + 1)$ covers. So, number of ways to cover the second register with cover paper $= m - n + 1$ ways.
   Hence, total number of ways in which he can cover all the registers with colour papers$=m \times (m - 1) \times (m - 2) \times ... \times (m - n + 1)$ ways.

   For example:
   Suppose, we substitute values of n and m as 6 and 10 respectively.
   For the 1st register, he can choose any of the 10 covers. So, number of ways to cover the first register with cover paper $= 10$ ways.
   For the 2nd register, he can choose any of the remaining $(10 - 1)$ covers. So, number of ways to cover the second register with cover paper $= 10 - 1 = 9$ ways.
   Similarly, for 6th register, he can choose any of the remaining $(10 - 6 + 1)$ covers. So, number of ways to cover the second register with cover paper $= 5$ ways.
   Hence, total number of ways in which he can cover all the registers with cover papers is
   $= 10 \times 9 \times 8 \times 7 \times 6 \times 5 = 151200$ ways.

2. n classmates could not agree on who would stand in the group photo along with the teacher for the yearbook. How many possible groups can be made such that there is at least one student with the teacher in the photo?
   Answer: $2^n - 1$
   Solution:
   Either a student be in that group photo or not. So, each student has 2 possibilities.
   The possibility that there may be 0 students along with the teacher in the group photo=1, as there is only 1 possible case of not selecting any of the n students.
   Hence, the total number of possible sets for a group photo where there is at least one student with the teacher$=2^n - 1$

For example:
Suppose, we substitute values of n as 7.
Either a student be in that group photo or not. So, each student has 2 possibilities.
The possibility that there may be 0 students along with the teacher in the group photo=1, as there is only 1 possible case of not selecting any of the 7 students.
Hence, the total number of possible sets for a group photo along with a teacher such that there is at least one student with a teacher in the photo $= 2^7 - 1 = 127$ ways.

Jay bought a new car in New York where a license plate can be created with alphabets A, B, C, D, E, W, X, Y, Z and numbers 0 to 9. He can either select a normal license plate or a VIP license plate. The VIP license plate begins with m alphabets followed by n numbers with repetition allowed. The normal license plate begins with a numbers followed by b alphabets without repetition. Based on this information, answer questions (3) and (4):

3. In how many ways can he select the VIP license plate?
   Answer: $9^m \times 10^n$
   Solution:
   Total choice of alphabets $= 9$ i.e. A,B,C,D,E,W,X,Y and Z
   Number of ways to select m alphabets with repetition $= 9^m$
   Number of ways to select n numbers with repetition $= 10^n$
   Hence, number of ways he can select VIP License Plate $= 9^m \times 10^n$

   For example:
   Suppose, we substitute values of m and n as 2 and 4 respectively.
   Number of ways to select 2 alphabets with repetition $= 9^2$
   Number of ways to select 4 numbers with repetition $= 10^4$
   Hence, number of ways he can select VIP License Plate $= 9^2 \times 10^4 = 81000$ ways.

4. In how many ways can he select the license plate(normal or VIP)?
   Answer: $[9^m \times 10^n] + [9(9-1)...(9-(b-1)) \times 10(10-1)...(10-(a-1))]$
   Solution:
   For the normal license plate:
   Number of ways to select b alphabets without repetition $= 9 \times (9-1) \times ... \times (9-(b-1))$
   Number of ways to select a numbers without repetition $= 10 \times (10-1) \times ... \times (10-(a-1))$
   Hence, number of ways he can select the normal license plate
   $= [9 \times (9-1) \times ... \times (9-(b-1))] \times [10 \times (10-1) \times ... \times (10-(a-1))]$
   Therefore, total number of ways he can select the license plate $=$ Number of ways he can select VIP License Plate + Number of ways he can select the normal license plate
   $= [9^m \times 10^n] + [9(9-1)...(9-(b-1)) \times 10(10-1)...(10-(a-1))]$

For example:
Suppose, we substitute values of m, n, a and b as 2, 4, 2 and 3 respectively.
For the normal license plate
Number of ways to select 3 alphabets without repetition $= 9 \times (9-1) \times (9-2)$
Number of ways to select 2 numbers without repetition $= 10 \times (10-1)$
Hence, number of ways he can select the normal license plate is
$= [9 \times 8 \times 7] \times [10 \times 9] = 45360$ ways.
Therefore, total number of ways he can select the license plate $=$ Number of ways he can select VIP License Plate + Number of ways he can select the normal license plate
$= [9^2 \times 10^4] + [9 \times 8 \times 7 \times 10 \times 9] = 810000 + 45360 = 855360$ ways.

5. Ram has n trophies that he wishes to place in his main cabinet, which has space only for two trophies. If the number of trophies is increased by 3, then the number of possible ways to arrange the trophies in the main cabinet becomes 5 times the number of ways to arrange n trophies. How many trophies does Ram have?
Answer: 3
Solution:
Number of ways Ram can place his n trophies in the main cabinet $= n(n-1)$
Number of ways Ram can place his $(n+3)$ trophies in the main cabinet $= (n+3)(n+2)$
It is given that:

$$5n(n-1) = (n+3)(n+2)$$
$$5n^2 - 5n = n^2 + 6n - n + 6$$
$$4n^2 - 10n - 6 = 0$$
$$(4n+2)(n-3) = 0$$

Therefore, $n = 3, -\dfrac{1}{2}$
Since, n can not be negative.
Hence, $n = 3$

6. There are N students in a class. The class teacher announced that the first n students who completes a given project within two days will be awarded. What are the possible number of ways the students will be awarded?

(a). $\dfrac{N!}{(N-n)!}$

(b). $\dfrac{N!}{(N-n)!n!}$

(c). $N!$

(d). $(N-n)!$

Solution:
Since first n students are to be awarded, i.e., there are n awards which are to be distributed among the N students in class. Therefore, for the distribution of the first award there are N ways and, for the second award, there are $(N-1)$ ways because one award is already distributed. Similarly, 3rd, 4th, . . . , n^{th} awards can be distributed in $(N-2)$, $(N-3)$, ... $(N-(n-1))$ ways.
Since here, events are occurring simultaneously and order doesn't matter (because, we are awarding the first n students and there will be no difference in awarding 3rd student as first and then 5th student as second or 5th student as first and then 3rd student as second etc.)

Thus, the total number of possible ways the student will be awarded,

$$= \frac{N \times (N-1) \times (N-2)... \times (N-(n-1))}{n!} = \frac{N!}{(N-n)! \times n!}$$

For example;
Suppose, we substitute values of N and n as 40 and 5 respectively.
Since first 5 students are to be awarded, i.e., there are 5 awards which are to be distributed among the 40 students in class. Therefore, for the distribution of the first award there are 40 ways and, for the second award, there are 39 ways because one award is already distributed. Similarly, 3rd, 4th and 5th awards can be distributed in 38, 37 and 36 ways.
Since here, events are occurring simultaneously and order doesn't matter (because, we are awarding the first 5 students and there will be no difference in awarding 3rd student as first and then 5th student as second or 5th student as first and then 3rd student as second etc.)

Thus, the total number of possible ways the student will be awarded,

$$= \frac{40 \times 39 \times 38 \times 37 \times 36}{5!} = \frac{40!}{35! \times 5!}$$

7. N students watched a patriotic movie. An analyst wishes to ask each student whether they liked the movie or not. Each student can either answer the question or refuse to respond. In how many ways, can the analyst get responses from the students?
   Answer: 3^N
   Solution:
   Since each student can choose to answer like or dislike or prefer not to answer. If he gives response then he has two options like or dislike the movie and another option is, he does not give any response. Thus, analyst can get feedback from each student in 3 ways.

From N students, analyst can get feedback in 3^N ways.

For example
Suppose, we substitute value of N as 6.
Analyst can get feedback from each student in 3 ways.
From 6 students, analyst can get feedback in $3^6 = 729$ ways.

8. If the value of sum of first n non-zero natural numbers is equal to $\dfrac{x(n+1)!}{2z}$, then find the value of $\dfrac{1}{x}$?.

Answer: $\dfrac{(n-1)!}{z}$

Solution:
As we know, the sum of first n non-zero natural numbers is $= \dfrac{n(n+1)}{2}$.

Given, $\dfrac{n(n+1)}{2} = \dfrac{x(n+1)!}{2z}$

$n(n+1) = \dfrac{x(n+1)n(n-1)!}{z}$

$1 = \dfrac{x(n-1)!}{z}$

$\dfrac{1}{x} = \dfrac{(n-1)!}{z}$

For example:
Suppose, we substitute values of n and z as 7 and 3 respectively.

The sum of the first 7 non-zero natural numbers is $= \dfrac{7(7+1)}{2}$.

Given, $\dfrac{7(7+1)}{2} = \dfrac{x(7+1)!}{2 \times 3}$

$7(7+1) = \dfrac{x(7+1)7(7-1)!}{3}$

$1 = \dfrac{x(7-1)!}{3}$

$\dfrac{1}{x} = \dfrac{(7-1)!}{3} = \dfrac{6!}{3} = \dfrac{720}{3} = 240$

9. Adam wrote down a n-digit university roll number on a piece of paper. On his way home from office, it rained heavily and the paper got wet. Later, he saw that the first m digits of the roll number had disappeared. In how many ways can Adam complete this university roll number if repetition of digits is allowed?

a. $m!$

b. 10^m

c. $10^{m-1} \times 9$

d. 9^m

Solution:
Adam has to complete the first m digits of a n-digit university roll number with repetition of digits allowed.
Now, the first digit can be filled with any number as it is a categorical variable and does not have any numeric meaning. For example: Roll number '000257' is also a 5-digit roll number.
Hence, number of ways to fill first digit $=10$ ways.
For the second digit, he can fill it with any number 0-9. Hence, number of ways to fill second digit $= 10$ ways.
Similarly, for the $3^{rd}, 4^{th}, ..., n^{th}$ digit also, he can fill it with any number 0-9.
Hence, the number of ways to fill any of these digits $= 10$
Therefore, the number of ways Adam can complete the n-digit university roll number $= 10^m$.
Hence, option (b) is correct.

For example:
Suppose, we substitute values of n and m as 10 and 3 respectively.
Adam has to complete the first three digits of a 10-digit university roll number with repetition of digits allowed.
Now, the first digit can be filled with any number as it is a categorical variable and does not have any numeric meaning. For example: Roll number '000257' is also a 5-digit roll number.
Hence, number of ways to fill first digit $=10$ ways.
For the second digit, he can fill it with any number from 0-9. Hence, number of ways to fill second digit $= 10$ ways.
Similarly, for the digit also, he can fill it with any digit 0-9. Hence, number of ways to fill third digit $= 10$ ways.
Therefore, the number of ways Adam can complete the 10-digit university number $= 10 \times 10 \times 10 = 10^3$.
Hence, option (b) is correct.

10. Let $x = \dfrac{5!}{4 \times 3!}$. Which of the following expressions is/are equal to x?

a. $5 \times 0!$

b. $5 \times \dfrac{1}{0!}$

c. $5 \times \dfrac{1}{0}$

d. $5 \times \dfrac{6}{3! + (3 \times 2) + (3 \times 2 \times 1)}$

e. $5 \times \dfrac{18}{3! + (3 \times 2) + (3 \times 2 \times 1)}$

Answer: a, b, e

Solution:

Let us first solve for x:

$$x = \frac{5!}{4 \times 3!} = \frac{5 \times 4 \times 3!}{4 \times 3!} = 5$$

Now,

a. $5 \times 0! = 5 \times 1 = 5 = x$

Hence, option a is correct.

b. $5 \times \dfrac{1}{0!} = 5 \times \dfrac{1}{1} = 5 = x$

Hence, option b is correct.

c. $5 \times \dfrac{1}{0} \neq x$

$\dfrac{1}{0}$ is not defined.

Hence, option c is incorrect.

d. $5 \times \dfrac{6}{3! + (3 \times 2) + (3 \times 2 \times 1)} = 5 \times \dfrac{6}{6 + 6 + 6} = 5 \times \dfrac{1}{3} \neq 5 \neq x$

Hence, option d is incorrect.

e. $5 \times \dfrac{18}{3! + (3 \times 2) + (3 \times 2 \times 1)} = 5 \times \dfrac{18}{6 + 6 + 6} = 5 = x$

Hence, option e is correct.