



IIT Madras

ONLINE DEGREE

Computational Thinking
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Depth First Search (DFS) and Recursive Procedure Call (Part 3)

Professor Madhavan Mukund: So, when we drew this graph, we said you could connect stations and you could connect trains.

Professor G. Venkatesh: Yes, trains can be nodes.

Professor Madhavan Mukund: Yeah, trains could be nodes connected by connecting change over stations.

Professor G. Venkatesh: Connecting change over stations.

Professor Madhavan Mukund: Or the other way. So, we just saw that you could take a problem like if I start from a station, and if I have some limited time I want to reach on the same day, which all stations can I reach on the same day by possibly changing trains. We could ask the same question of trains, right? If I start on a train.

Professor G. Venkatesh: Which all trains can I get on to?

Professor Madhavan Mukund: How many trains can I explore if I allow myself 1 or 2 days?

Professor G. Venkatesh: How many trains can I explore? Let us give it 2 days.

Professor Madhavan Mukund: Let us say 2 days.

Professor G. Venkatesh: We start on Monday, you are saying.

Professor Madhavan Mukund: We start on whichever day. We start with the train.

Professor G. Venkatesh: We start with the train.

Professor Madhavan Mukund: With the train, whichever day It leaves on. And.

Professor G. Venkatesh: So, but, the train, so we will take this slide.

(Refer Slide Time: 01:02)



Professor Madhavan Mukund: So, this train.

Professor G. Venkatesh: Eventually, it has Monday and Friday.

Professor Madhavan Mukund: So, let us take Monday.

Professor G. Venkatesh: Monday.

Professor Madhavan Mukund: Monday.

Professor G. Venkatesh: So, Monday, we are starting with 12269 on a Monday. So, it is like explore again, but exploring the train.

Professor Madhavan Mukund: We want to now explore which train we can get to. So what.

Professor G. Venkatesh: We want to see all the trains we can reach?

Professor Madhavan Mukund: Yes.

Professor G. Venkatesh: Either whole of Monday, I can take?

Professor Madhavan Mukund: Or whole of Tuesday.

Professor G. Venkatesh: Or whole of Tuesday and I can get anywhere and change the train. Change it to another train, right?

Professor Madhavan Mukund: Yeah.

Professor G. Venkatesh: So, connecting trains I want to find.

Professor Madhavan Mukund: Connecting trains, correct.

Professor G. Venkatesh: So, you are exploring the connecting train graph. You have made that graph, you are exploring it using the same Depth First Search technique.

Professor Madhavan Mukund: Correct, yeah.

Professor G. Venkatesh: We are starting a node called 12269, and you are doing a depth first search.

Professor Madhavan Mukund: Correct.

Professor G. Venkatesh: And presumably, we will use Recursive Call, right?

Professor Madhavan Mukund: Exactly.

Professor G. Venkatesh: So, you have a thing called explore 12269. And 12269 explore, we will call 12269 explore something else, explore something else. It will go on.

Professor G. Venkatesh: But again, if you come back we have problem.

Professor Madhavan Mukund: Yeah, so let us say we want to come back.

Professor G. Venkatesh: You have to make a scene table. But this time in the scene table, you will put train, train number.

Professor Madhavan Mukund: Train numbers. And maybe, see we saw that these trains like come in pairs, right? So, there will be a corresponding train probably called 12270, which will go backwards.

Professor G. Venkatesh: 12270.

Professor Madhavan Mukund: So that is.

Professor G. Venkatesh: That is the same train.

Professor Madhavan Mukund: So, that is the same train more or less.

Professor G. Venkatesh: We will see the same change, no point, no fun.

Professor Madhavan Mukund: So, if we see a train, we will say we have also seen.

Professor G. Venkatesh: We move the, we have seen the, we will put its pair also in the same. Alright. So, let us start. So, how do we go about it?

Professor Madhavan Mukund: Yeah.

Professor G. Venkatesh: So, we start with Chennai.

Professor Madhavan Mukund: Yeah, so, we will first say that we have seen, we do not want to worry about these 2 trains anymore.

Professor G. Venkatesh: 12269. And we will write it with explore or we will?

Professor Madhavan Mukund: We will call it something.

Professor G. Venkatesh: Okay.

Professor Madhavan Mukund: I just write X. EX, I will write.

Professor G. Venkatesh: EX, we are doing 12269, right?

Professor Madhavan Mukund: 12269 on a Monday. On Monday, and...

Professor G. Venkatesh: So, we can go up to Tuesday. Now, 00:00 that is there.

Professor Madhavan Mukund: I will just write 00:00.

Professor G. Venkatesh: Time is required, right? It is required, because.

Professor Madhavan Mukund: It is required, because we have to see when the next train has to be connecting. So now, we reach...

Professor G. Venkatesh: So, we first have Chennai.

Professor Madhavan Mukund: So...

Professor G. Venkatesh: We are starting at Chennai.

Professor Madhavan Mukund: So now, we will, like first place you could possibly change is Vijayawada.

Professor G. Venkatesh: We should not tell our change in Chennai, there is a intersect.

Professor Madhavan Mukund: We cannot change in Chennai, because we have not got on this train at all. We have to travel at least one station.

Professor G. Venkatesh: That is what it means, you have to travel some place. So, only if you travel a little bit on the train, you are saying that you are on the train.

Professor Madhavan Mukund: Correct.

Professor G. Venkatesh: So, I have seen the train if I have gone from Chennai to Vijayawada, otherwise you have not seen the train, right?

Professor Madhavan Mukund: Correct-correct.

Professor G. Venkatesh: So, the first place, you can get off is Vijayawada. So, going by what we have done so far, we should get off at Vijayawada.

Professor Madhavan Mukund: We should get off at Vijayawada, and see what all trains we can get.

Professor G. Venkatesh: And what all we can get. But remember that we are at Vijayawada. Come back here later. Okay, so at Vijayawada, what happens?

Professor G. Venkatesh: So, let us take off Vijayawada. Here is Vijayawada. So, at Vijayawada. We have already done Vijayawada, anyway it does not matter.

Professor Madhavan Mukund: On a Monday but after 12:31.

Professor G. Venkatesh: On a Monday. That is how it is different. Last time it was not like that. So, Vijayawada after 12:31, this is where we arrive.

Professor Madhavan Mukund: So, this is only one train.

Professor G. Venkatesh: So, on Mondays, only one train.

Professor Madhavan Mukund: So, I can go.

Professor G. Venkatesh: This is 02246. So, I can see, I can see another train, which is 02246 also on Monday. This one is departing 22:31. 22:31? So, it makes sense, right?

Professor Madhavan Mukund: Yeah.

Professor G. Venkatesh: Because that is a time at which it is departing, okay?

Professor Madhavan Mukund: Yeah.

Professor G. Venkatesh: And in that case, this should be called something, it should be called time of departing, 06:40 you can put.

Professor Madhavan Mukund: Well actually, technically, okay.

Professor G. Venkatesh: Yeah, that is a better thing to do.

Professor Madhavan Mukund: 06:40.

Professor G. Venkatesh: Because then, because this is Monday 06:40 departure train, it could be Tuesday 06:40 departure or Monday another train could be there, the same number. We do not know.

Professor Madhavan Mukund: So, now anyway, we will say that we have seen this train.

Professor G. Venkatesh: We have seen now also 02246.

Professor Madhavan Mukund: 46 and 45, it's pair.

Professor G. Venkatesh: And it's pair, and it's pair.

Professor G. Venkatesh: So, I will take out 02246.

Professor Madhavan Mukund: And see where it goes.

Professor G. Venkatesh: And 45. No, 45, it is not 45.

Professor Madhavan Mukund: 02245.

Professor G. Venkatesh: Got it. So, 02245 and 46.

Professor Madhavan Mukund: So, 46 is the one we are following.

Professor G. Venkatesh: We are following 46, so 45, we do not need it.

Professor G. Venkatesh: We just removed it. We just marked it aside. So, 46 we are following. Where are we?

Professor Madhavan Mukund: Vijayawada.

Professor G. Venkatesh: Vijayawada. 22:31.

Professor Madhavan Mukund: So, we can.

Professor G. Venkatesh: We are allowed to continue, because we are allowing 1 more day.

Professor Madhavan Mukund: So, we can go to Vijayanagram.

Professor G. Venkatesh: So, I can take in this train, 02246, I can go to Vijayanagram.

Professor Madhavan Mukund: Get off at to Vijayanagram, next stop.

Professor G. Venkatesh: So, I should get off at Vijayanagram.

Professor Madhavan Mukund: You are supposed to get off at Vijayanagram. So, we have to look at what Vijayanagram has.

Professor G. Venkatesh: My goodness, this will require us through very carefully tracking. So...

Professor Madhavan Mukund: Yeah, but look at this, Vijayanagram has only this 1 train. So, Vizianagram is not a good, is not useful.

Professor G. Venkatesh: It is nothing, it is not useful. But I can, oh I am not marking cities.

Professor Madhavan Mukund: No, you are not marking.

Professor G. Venkatesh: I am only marking, so Vijayanagram is nothing.

Professor Madhavan Mukund: That is it.

Professor G. Venkatesh: So, then I continue to Bhubaneswar.

Professor Madhavan Mukund: Yes, so Bhubaneswar. So now, we are in Bhubaneswar.

Professor G. Venkatesh: So, I just got off on the platform, I saw that got back into the train again. That is what I did in Vizianagram. Now, in Bhubaneswar what happens?

Professor Madhavan Mukund: So, Bhubaneswar, now we have reached.

Professor G. Venkatesh: I reached Bhubaneswar at.

Professor Madhavan Mukund: 02246, I have reached at 09:50 in the morning.

Professor G. Venkatesh: 09:50 I arrive.

Professor Madhavan Mukund: So now, I can take on a Monday nothing.

Professor G. Venkatesh: Monday, you cannot do anything. Or Tuesday, I mean.

Professor Madhavan Mukund: Tuesday at 5 o'clock.

Professor G. Venkatesh: I have reached at Tuesday or what, have I reached at Tuesday?

Professor Madhavan Mukund: You should have reached on Tuesday, it is the Day 2. So, you reach Tuesday at here 10 o'clock. And now, Tuesday at 5 o'clock and at 22, 20:15, there are 2 different trains. But 5 o'clock is a pairing train, so that I do not want to take, the train going back.

Professor G. Venkatesh: That we would have marked it actually, 02246, you have marked, 02245.

Professor Madhavan Mukund: Yeah, so this train is not allowed.

Professor G. Venkatesh: So, this is marked. So since it is marked, seems certain, skip it. So now, we can take 02202 on.

Professor Madhavan Mukund: 02202.

Professor G. Venkatesh: So, 02202, it is required, there is lot of tracking. 02202, it is this train.

Professor Madhavan Mukund: This is on Tuesday, this is what time, 20:15. Now, I have to write off these 2 also, 02202 and 02201.

Professor G. Venkatesh: Its Bhubaneshwar-Kolkata, Kolkata-Bhubanewshwar. 1 and 2 are gone, okay?

Professor Madhavan Mukund: Okay.

Professor G. Venkatesh: Where are we now?

Professor Madhavan Mukund: We are in Bhubaneshwar on Tuesday at 20:15.

Professor G. Venkatesh: Then it is Wednesday.

Professor Madhavan Mukund: Wednesday. So now, so we cannot take any more trains. So that, we can get on this train, but we cannot go anywhere with it.

Professor G. Venkatesh: So, I have seen, which I have seen 02202, I am done. Done. Okay?

Professor Madhavan Mukund: Yeah.

Professor G. Venkatesh: Now, I am done with 02202, so what happens now?

Professor Madhavan Mukund: So now, we are done with this train, so actually, we are finished with Vijayawada, the 46 train, this last train and that is it.

Professor G. Venkatesh: Vijayawada.

Professor Madhavan Mukund: There are no more trains to try on Vijayawada. No, there is a Tuesday train 22204. We can wait on, we can wait in Vijayawada and take this Tuesday train.

Professor G. Venkatesh: Arrived at, we arrived on Monday.

Professor Madhavan Mukund: We arrived on Monday.

Professor G. Venkatesh: So, I can wait.

Professor Madhavan Mukund: And take the Tuesday.

Professor G. Venkatesh: Tuesday 01:05 train, which is here.

Professor Madhavan Mukund: Correct.

Professor G. Venkatesh: So, 22204.

Professor Madhavan Mukund: Yeah, and it's pair. So, where should we write this now? This is again from here only.

Professor G. Venkatesh: 12269 Vijayawada, this is where we are.

Professor Madhavan Mukund: So now, we have 22204 on Tuesday at 01:15.

Professor G. Venkatesh: 22204 on Tuesday at 01:15.

Professor Madhavan Mukund: And I have to write that number down.

Professor G. Venkatesh: So, you are writing 02204, 00220, 22204 and 22 let us see, is that the pair 22203?

Professor Madhavan Mukund: Yeah.

Professor G. Venkatesh: I assumed that is the pair. This is Vishakhapatnam to Secunderabad. So, we are in Vijayawada.

Professor Madhavan Mukund: And we are taking 04 forward.

Professor G. Venkatesh: So, this one we do not locate. We left at 01:15.

Professor Madhavan Mukund: 01:15. So, we can reach.

Professor G. Venkatesh: So, we can reach Vishakhapatnam at 06:30.

Professor Madhavan Mukund: But 06:30 is still on Tuesday.

Professor Madhavan Mukund: So, we are still on the same day, so we can go to Vishakhapatnam and see whether we can do anything.

Professor G. Venkatesh: There is nothing further to do. I am putting this card back. It was to keep, okay?

Professor Madhavan Mukund: So, we go back to Vishakhapatnam.

Professor G. Venkatesh: We go back to Vishakhapatnam.

Professor Madhavan Mukund: And that is it, Vishakhapatnam has no other trains.

Professor G. Venkatesh: Vishakhapatnam has no other trains, only the reverse trains.

Professor Madhavan Mukund: Reverse train, we do not want to write that.

Professor G. Venkatesh: We do not want to look at that.

Professor Madhavan Mukund: So, that is it.

Professor G. Venkatesh: So, we are done with Vishakhapatnam.

Professor Madhavan Mukund: So, we are done with Vijayawada also.

Professor G. Venkatesh: We are done with Vijayawada. We went back and let on Tuesdays. Nothing else. So, we are done with Vijayawada.

Professor Madhavan Mukund: So, now we are done with this Vijayawada.

Professor G. Venkatesh: So, Vijayawada is done, so I put Vijayawada back.

Professor Madhavan Mukund: So, maybe, I will just write, although I do not know whether we want to keep track of it or not, but I will just write that this is Vijayawada.

Professor G. Venkatesh: Vijayawada, Just like we put train on that thing, we would write, previous thing we wrote the train. Now, we should write the station. Makes sense actually.

Professor Madhavan Mukund: And this one was what? Bhubaneshwar.

Professor G. Venkatesh: Bhubaneshwar, yeah.

Professor Madhavan Mukund: Yeah. Okay?

Professor G. Venkatesh: Yeah.

Professor Madhavan Mukund: So now, we get to Balharshah.

Professor G. Venkatesh: And we come to Balharshah.

Professor Madhavan Mukund: So, what Balharshah.

Professor G. Venkatesh: We are still on Monday. We have reached at 06:06.

Professor Madhavan Mukund: So, here is Balharshah.

Professor G. Venkatesh:: Balharshah. 06:06 Balharshah on Monday.

Professor Madhavan Mukund: Cannot do much on Monday, but you can go and wait for 02286 on Tuesday.

Professor G. Venkatesh: This is, Strictly speaking, aligned here, I can take this time.

Professor Madhavan Mukund: Yeah, but it this is not on a Monday.

Professor G. Venkatesh: But this train, we have seen already.

Professor Madhavan Mukund: No, we have not seen the train.

Professor G. Venkatesh: Oh, this train is not on Monday.

Professor Madhavan Mukund: But we can go and wait to that train.

Professor G. Venkatesh: We can wait and come on this train, 02286, have we seen it?

Professor Madhavan Mukund: No.

Professor G. Venkatesh: We have not seen.

Professor G. Venkatesh: All right, so let me find 02286. What is 02286?

Professor Madhavan Mukund: And the pair is 85.

Professor G. Venkatesh: And the pair is 85.

Professor Madhavan Mukund: So, this is on Tuesday at 08:15.

Professor G. Venkatesh: Secunderabad to New Delhi, this is Secunderabad-New Delhi. So, Balharshah, we are, we are, which direction we are in? We are, 85 or 86?

Professor Madhavan Mukund: We are doing 86.

Professor G. Venkatesh: 85.

Professor Madhavan Mukund: 86.

Professor G. Venkatesh: 86. Okay. So, I will just put 85 back. So Balharshah is going to go to, go back to Secunderabad, right? But it is okay, it is a different train.

Professor Madhavan Mukund: Yeah, that is right, it is a different train.

Professor G. Venkatesh: You do not have problem, it might seem.

Professor Madhavan Mukund: So now, we have to go to Secunderabad and see what we can do at 14:00 at Secunderabad.

Professor G. Venkatesh: Exactly. This is interesting, now we are allowed to go to the same city again.

Professor Madhavan Mukund: So...

Professor G. Venkatesh: We keep going back to same city, it is all right, different train.

Professor Madhavan Mukund: So right now, we have not seen Secunderabad, I think, but this is going backwards, so let us see what happens in Secunderabad.

Professor G. Venkatesh: So, 02286, this is not, we have not done? We have done.

Professor Madhavan Mukund: Yeah.

Professor G. Venkatesh: 02086, we went to Secunderabad. We are at Balharshah, Balharshah, Balharshah, we have this train, 02286. You have to see Balharshah, is it not?

Professor Madhavan Mukund: Yeah.

Professor G. Venkatesh: We are going to Secunderabad.

Professor Madhavan Mukund: So, Secunderabad, we have reached at 08:15 on a Tuesday.

Professor G. Venkatesh: definitely is not enough. We have reached 08:15 on a Tuesday, where is it.

Professor Madhavan Mukund: No-no, so now we reach 14:00 on a Tuesday. So, anything after 14:00 on a Tuesday is there nothing. No, here there is 21:31, we can get onto 12214.

Professor G. Venkatesh: Which one? The Balharshah we reached? On second day, yeah, Tuesday, correct. So, 08:15, we reached Balharshah, Tuesday there is one more.

Professor Madhavan Mukund: No-no, no-no, 2 o'clock we reached Secunderabad, that was the 02286.

Professor G. Venkatesh: 2 o'clock we reached.

Professor Madhavan Mukund: And now on same Tuesday, we can get on to 12214.

Professor G. Venkatesh: So, explore 12214 and it is pair, which is 12213.

Professor G. Venkatesh: Have we seen it again?

Professor Madhavan Mukund: No.

Professor G. Venkatesh: No? My goodness. So, 12214, we are on 12214, right?

Professor Madhavan Mukund: Yeah.

Professor G. Venkatesh: 12214, so I will put 12213 back.

Professor G. Venkatesh: 12214, where are we?

Professor Madhavan Mukund: Tuesday at some, we are at Secunderabad, so, 21:31.

Professor G. Venkatesh: Secunderabad, 21:31.

Professor Madhavan Mukund: But now we will cross over to the next is.

Professor G. Venkatesh: 02:58, so nothing to do.

Professor Madhavan Mukund: Nothing to do. So, we will stop.

Professor G. Venkatesh: So, we, so this train is over, right?

Professor Madhavan Mukund: So now, we go back and check whether.

Professor G. Venkatesh: Now, we are done with Secunderabad.

Professor Madhavan Mukund: Secunderabad.

Professor G. Venkatesh: Are we done with Secunderabad?

Professor Madhavan Mukund: Yeah.

Professor G. Venkatesh: Okay.

Professor Madhavan Mukund: And I think we are done Balharshah also.

Professor G. Venkatesh: This train also we are done. Yes. This train we are done, because it is the last station, nothing to do. So, this train also is done. So, we put this train back and then.

Professor Madhavan Mukund: And then we go back to Balharshah, and we see whether there is anything more to be done. So, I think there is no more trains, because Balharshah, we have arrive on Tuesday. There is one more train here.

Professor G. Venkatesh: 12214, is not it?

Professor Madhavan Mukund: Yeah, we already seen that. So, we do not take that.

Professor Madhavan Mukund: So, Balharshah is done

Professor G. Venkatesh: Balharshah is also done.

Professor Madhavan Mukund: So, we can put it back. So now, we go back to our old train and what comes. And what comes out of Balharshah.

Professor G. Venkatesh: Hey, fun, we have seen a lot of trains.

Professor Madhavan Mukund: I mean, yeah.

Professor G. Venkatesh: See, because of the, I mean, of course, the reverse, we have put some reverse in this, but still its interesting.

Professor Madhavan Mukund: Now we go to Nagpur.

Professor G. Venkatesh: Now, we are at Nagpur, all right.

Professor Madhavan Mukund: So, we have reached Nagpur on the same day 21:06. So, we are on day 1 at Nagpur.

Professor G. Venkatesh: 21:06.

Professor Madhavan Mukund: Yeah. So, after that, on Monday, so we are here on Nagpur, so there is a 23:20 in Nagpur, so we should try that first.

Professor G. Venkatesh: 12262.

Professor Madhavan Mukund: So, let me draw a line from here maybe.

Professor G. Venkatesh: Explore 12262 and 23:20 on Monday, 12262 and it is pair is 61 Kolkata to Mumbai. 61. my...

Professor Madhavan Mukund: This will take us the new directions.

Professor G. Venkatesh: my, we have reached Bombay, there are so many trains or Kolkata, either of them.

Professor Madhavan Mukund: So, 12262, we are doing. So, we are going towards Mumbai, I think here. So, now we can.

Professor G. Venkatesh: Nagpur, 43:15, we are on Monday.

Professor Madhavan Mukund: Yeah. So, we can go to Bhusawal.

Professor G. Venkatesh: We can go to Bhusawal.

Professor Madhavan Mukund: So, we have to now look at what we can do in Bhusawal.

Professor G. Venkatesh: So, I put this back, because we are not interested in this time. So, 12262, we are in Bhusawal. There is 12262, one second. So, we are on Tuesday.

Professor Madhavan Mukund: So, we can wait till here and take 12261, which is a pair. So, we cannot do anything.

Professor G. Venkatesh: 12261 is not interesting, its already seen, right?

Professor Madhavan Mukund: Yeah.

Professor G. Venkatesh: So, finish, Bhusawal is done. Put it back. And we come back to 12262. Bhusawal is done.

Professor G. Venkatesh: Igatpuri. We can get off with Igatpuri at 07:40 on Tuesday, and see what to do. Igatpuri, Igatpuri on Tuesday.

Professor Madhavan Mukund: We got off at?

Professor G. Venkatesh: 07:40, we got off Tuesday.

Professor Madhavan Mukund: Okay.

Professor G. Venkatesh: And then there is another train here at 07:31 on Tuesday.

Professor Madhavan Mukund: Which is same pair again.

Professor G. Venkatesh: It is 12261, pair.

Professor Madhavan Mukund: But there is another train at 21.

Professor G. Venkatesh: 12289.

Professor Madhavan Mukund: Yeah, which is in.

Professor G. Venkatesh: You could get into the train, but will be on the same day, which is all right. At least we will get on the train, right?

Professor Madhavan Mukund: Yeah.

Professor G. Venkatesh: 12289.

Professor Madhavan Mukund: So...

Professor G. Venkatesh: And 12289 presumably as a pair which is 90? Must be, right?

Professor Madhavan Mukund: Yeah, 90. It could be Nagpur-Mumbai.

Professor G. Venkatesh: Nagpur-Mumbai, 12289. We are on 12289, right? Yeah.

Professor Madhavan Mukund: Tuesday at 21, 22:31.

Professor G. Venkatesh: It goes to the next day. So anyway, it is going back to Bhusawal, which we have seen, but does not matter actually.

Professor Madhavan Mukund: So, we will stop on.

Professor G. Venkatesh: So, we stopped.

Professor G. Venkatesh: So, we are done with Tuesday.

Professor Madhavan Mukund: With Igatpuri.

Professor G. Venkatesh: We are done with Igatpuri.

Professor Madhavan Mukund: So, now we come and check Mumbai.

Professor G. Venkatesh: Mumbai.

Professor Madhavan Mukund: So, Mumbai, we have reached what? 10:31 on Tuesday, yeah?

Professor G. Venkatesh: 10:31 on Tuesday.

Professor Madhavan Mukund: Now, there are a lots of trains that we have to look at.

Professor G. Venkatesh: Mumbai is huge.

Professor Madhavan Mukund: So, on tenth, so on Tuesday, we have to look what? All departures after 10:31 first.

Professor G. Venkatesh: So, departures after, this is all arrivals. Departures after 10:31 on Tuesday.

Professor Madhavan Mukund: 12.

Professor G. Venkatesh: So, we have 12263, have we seen it?

Professor Madhavan Mukund: No.

Professor Madhavan Mukund: We have seen 62 but not 63. This is 63 and 64. It is probably going to Delhi or something.

Professor G. Venkatesh: Here is 63. So, this is from Nagpur you came to Mumbai, right? So 02:40, right? 63?

Professor Madhavan Mukund: Yeah.

Professor G. Venkatesh: So, you can take 12263 to Vadodara. We are on Tuesday?

Professor Madhavan Mukund: Tuesday.

Professor G. Venkatesh: So, you are still on Tuesday, so you can go Vadodara at 06:34, you will land.

Professor G. Venkatesh: So, this one I will leave, because we are not interested in this train. 06:34, so you can actually exit at Vadodara and see what over there.

Professor Madhavan Mukund: Wow, this is taking us a long way off our route.

Professor G. Venkatesh: Gone all the way, place, we came to Vadodara. So, Vadodara, 06:34.

Professor Madhavan Mukund: We have the same train continuing at 06:45, which you do not want. And there is no other train on Tuesday.

Professor G. Venkatesh: So, Vadodara is done.

Professor Madhavan Mukund: Vadodara is done.

Professor G. Venkatesh: So, after that, we have Ratlam, which also on the same day.

Professor G. Venkatesh: So, strictly speaking, you could get down in Ratlam at 10:11 and try your luck. Write down that you will find anything. 10:11 is the last train.

Professor Madhavan Mukund: Yeah, that is the same train.

Professor G. Venkatesh: Nothing to do.

Professor Madhavan Mukund: Nothing to do.

Professor G. Venkatesh: So, Ratlam there is nothing. And then after that, you are over, the day is over. So, 12263 is finished.

Professor Madhavan Mukund: So, 12263 is finished. So now, we go back to Mumbai and see what else is there on.

Professor G. Venkatesh: So, what else is there? This one is over in Mumbai, 02:40. We are coming down now ledges. Next is 12261.

Professor Madhavan Mukund: 12261, we have already.

Professor G. Venkatesh: Seen, so nothing to do.

Professor Madhavan Mukund: No.

Professor G. Venkatesh: This is not there on Tuesday, 08:15.

Professor Madhavan Mukund: 122289, 89 also we have seen.

Professor G. Venkatesh: You have seen? All right.

Professor Madhavan Mukund: That is new there. 12223, that is new.

Professor G. Venkatesh: All right. Wow, lot of trains. We have seen plenty of trains.

Professor Madhavan Mukund: Well a lot of.

Professor G. Venkatesh: Did you think in 2 days, we can see so many trains? Get on to so many trains if you really think about it. We have not taught together, right?

Professor Madhavan Mukund: Yeah-yeah.

Professor G. Venkatesh: Can possibly, because either this or that. Does not mean both, right?

Professor Madhavan Mukund: Yes.

Professor G. Venkatesh: Either this or that, because we are coming back in the recursion. So 12223, what time was it?

Professor Madhavan Mukund: 22, that gets, that 20:50. So, that is it, we cannot do much on that.

Professor G. Venkatesh: It is gone to the next day, which is gone, right?

Professor Madhavan Mukund: Yeah. So, I will, we got on to it, so it is.

Professor G. Venkatesh: We got onto, so. And it is pair, which is 4, Mumbai.

Professor G. Venkatesh: What time was it by the way? Do you remember?

Professor G. Venkatesh: 12223 was 08:50.

Professor Madhavan Mukund: So, that is a Mumbai station.

Professor G. Venkatesh: Got Mumbai station. Careful. So, where are we now? 12223, 08:50 on Tuesday, right?

Professor Madhavan Mukund: Yeah.

Professor G. Venkatesh: So, after that no more, right? We would have gone.

Professor Madhavan Mukund: Yeah, that is it.

Professor G. Venkatesh: There is one more, 11:25 departure.

Professor Madhavan Mukund: 12267.

Professor G. Venkatesh: We just barely make it.

Professor Madhavan Mukund: Okay, we will get onto it at least.

Professor G. Venkatesh: 12267 and there is nothing again too clearly, but anyway, I will just open the card just for fun. And it is pair. New Delhi to Jammu Tawi, no 12267 is Rajkot. So, 87 and 68. And it is clearly going on to Wednesday, so nothing much to do here. But anyway, 12267 12268 also you can mark as seen, all right?

Professor Madhavan Mukund: Yeah.

Professor G. Venkatesh: So, we are done with Mumbai.

Professor Madhavan Mukund: Yeah.

Professor G. Venkatesh: What a relief. Then we come back to, where were we?

Professor Madhavan Mukund: So, we were looking at trains from Nagpur, right? So, we explored on 12262, we had a train with us, 12262, I think you put it back.

Professor G. Venkatesh: Anyway, that was the last train, it was the last, it was the last stop. Mumbai is the last stop.

Professor Madhavan Mukund: So, 12262 is done.

Professor G. Venkatesh: It is done.

Professor Madhavan Mukund: So now, we have to see any more trains on this.

Professor G. Venkatesh: So, 12262 Mumbai is last stop, which is, where is 12262?

Professor Madhavan Mukund: 61 actually we started with, no, let us see. 12262. This is the last one.

Professor G. Venkatesh: So, we are done with that.

Professor Madhavan Mukund: We are done now. So, Nagpur is done.

Professor G. Venkatesh: Nagpur is done. This one?

Professor Madhavan Mukund: So now, we go to Bhopal.

Professor G. Venkatesh: So, we are back to this train, 12269 again. Now, this recursion is fun. So, the calling, you called 12269 that basically called 12269 something something something.

Professor Madhavan Mukund: Yeah, we went into a long thing here, finished all that came back, now we have processed this edge.

Professor G. Venkatesh: We have come up to this edge.

Professor Madhavan Mukund: Yeah. So now, we have to look at what trains we can reach via Bhopal.

Professor G. Venkatesh: The edge via Bhopal. All right. This is only 3 4, we will do it.

Professor Madhavan Mukund: I think Bhopal may not have many. So, Bhopal now we reach at on this train net 02:00 on Tuesday. So, there is 12214, which we have already seen.

Professor G. Venkatesh: We seen it?

Professor Madhavan Mukund: Yes.

Professor G. Venkatesh: Okay.

Professor Madhavan Mukund: Then 12213, we already seen. And then anyway, that is not there, so the only train which we could have taken was 12214 but that we have already explored here.

Professor G. Venkatesh: So, there is not any route Bhopal. We are done with Bhopal?

Professor Madhavan Mukund: Then Jhansi.

Professor G. Venkatesh: Then we come to Jhansi. We are still on Tuesday, right?

Professor Madhavan Mukund: Yeah, we have on Tuesday.

Professor G. Venkatesh: Tuesday, now.

Professor Madhavan Mukund: So, Tuesday we have reached at 05:35.

Professor G. Venkatesh: 05:35.

Professor Madhavan Mukund: 12270.

Professor G. Venkatesh: No-no.

Professor Madhavan Mukund: We have not seen. Yeah, so 69 and 70. No-no, we have seen it, sorry. This is the pair of this train itself.

Professor G. Venkatesh: Okay.

Professor Madhavan Mukund: So, It is the same distance.

Professor G. Venkatesh: Distance one train may buy.

Professor Madhavan Mukund: Yeah, same train.

Professor G. Venkatesh: So, Jhansi is done?

Professor Madhavan Mukund: Jhansi is done. Then, Gwalior.

Professor G. Venkatesh: Gwalior.

Professor Madhavan Mukund: Gwalior is nothing, same train only going back.

Professor G. Venkatesh: Okay, leave it.

Professor Madhavan Mukund: So, you can see it is only the same train going back.

Professor G. Venkatesh: This is going the next day? no?

Professor Madhavan Mukund: No.

Professor G. Venkatesh: The New Delhi is there.

Professor Madhavan Mukund: New Delhi is there. So New Delhi and then we are.

Professor G. Venkatesh: New Delhi on Tuesday.

Professor Madhavan Mukund: Yeah, so we can take more trains from New Delhi on Tuesday. So, there will be a whole new collection of things coming up. So, is New Delhi at 10 o'clock on a Tuesday?

Professor G. Venkatesh: 10:40 on Tuesday, right arrived? So, we have to look departures past 10:40. So, 12264, have we seen?

Professor Madhavan Mukund: 12264.

Professor G. Venkatesh: Yeah, we have seen it.

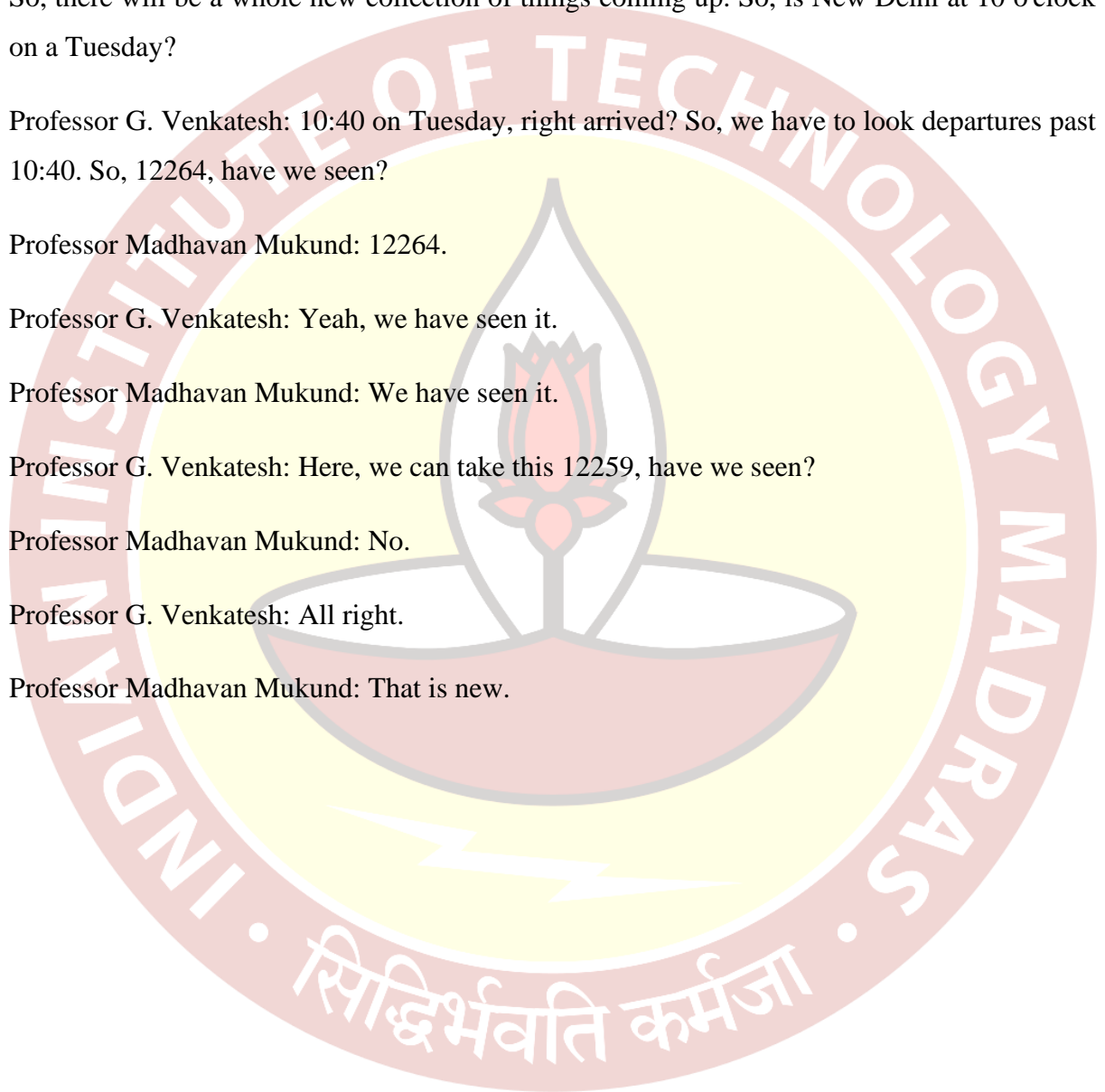
Professor Madhavan Mukund: We have seen it.

Professor G. Venkatesh: Here, we can take this 12259, have we seen?

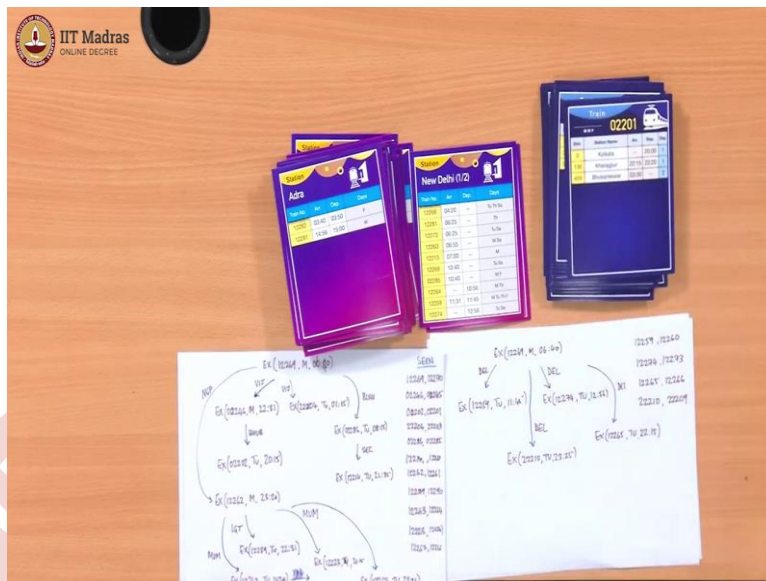
Professor Madhavan Mukund: No.

Professor G. Venkatesh: All right.

Professor Madhavan Mukund: That is new.



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Professor G. Venkatesh: Good. The more, the merrier. 12259.

Professor Madhavan Mukund: And 60, so 59 and 60.

Professor G. Venkatesh: So, this is Bikaner to Kolkata. So, I am not interested in 60, I am looking at 59, right?

Professor Madhavan Mukund: Yeah.

Professor G. Venkatesh: So, 59 New Delhi 11:45 on a Tuesday. Next stop is Loharu. So, we get off at Loharu.

Professor Madhavan Mukund: So, what was the train? 12259?

Professor G. Venkatesh: 12259.

Professor Madhavan Mukund: Which left on a Tuesday at 11:00.

Professor G. Venkatesh: No-no New Delhi, 11:45.

Professor Madhavan Mukund: 11:45. So now, we have to go to Loharu and see what happens in Loharu?

Professor G. Venkatesh: So, I get off at Loharu and try my luck.

Professor Madhavan Mukund: Why not?

Professor G. Venkatesh: I can go Loharu, Sadulpur, Churu, Ratangarh.

Professor Madhavan Mukund: Yeah, but Loharu, there is nothing. Loharu is the same, one second, let me just.

Professor G. Venkatesh: Same train going back, right?

Professor Madhavan Mukund: Yeah.

Professor G. Venkatesh: I can see Loharu. So, we leave Loharu and go to Sadalpur.

Professor Madhavan Mukund: I suspect this is all on 1 line. I think there is only 1 train coming.

Professor G. Venkatesh: There is only 1 train coming back, I think, right?

Professor Madhavan Mukund: Yeah.

Professor G. Venkatesh: That should be, we should be able to quickly check that.

Professor Madhavan Mukund: Satanah, Sadalpur, yeah 59:60.

Professor G. Venkatesh: Same thing, Churu, same I think.

Professor Madhavan Mukund: Churu also is very likely it is the same.

Professor G. Venkatesh: This Bikaner line is all 1 track.

Professor Madhavan Mukund: Yeah.

Professor G. Venkatesh: Going to Bikaner and coming back from Bikaner, right

Professor Madhavan Mukund: Churu only 1 train.

Professor G. Venkatesh: Ratangarh.

Professor Madhavan Mukund: Ratangarh, also, I suspect is very much the same. Prayagraj, Rajkot, Pune, Ratangarh, yes, same.

Professor G. Venkatesh: Same track.

Professor G. Venkatesh: Sri Dungangarah, Sri Dungangarah.

Professor Madhavan Mukund: Same.

Professor G. Venkatesh: All trains, Bikaner.

Professor Madhavan Mukund: Finally, Bikaner. So, I think this is.

Professor G. Venkatesh: Bikaner is I think only 1 train, right?

Professor Madhavan Mukund: It is going to go and turn around.

Professor G. Venkatesh: Same train would come back.

Professor G. Venkatesh: All right.

Professor Madhavan Mukund: So, this is.

Professor G. Venkatesh: So, we are done with 12259.

Professor G. Venkatesh: So, 12259 is over means, we come back to New Delhi.

Professor Madhavan Mukund: New Delhi, and see what else happens after Tuesday.

Professor G. Venkatesh: After 12259, 09:45, there is 12274.

Professor Madhavan Mukund: 12274, have we seen? Let me see.

Professor Madhavan Mukund: 12274, no.

Professor G. Venkatesh: No?

Professor Madhavan Mukund: 73 74, no.

Professor G. Venkatesh: This is Kolkata. 12273, 12274, we are on 74, so I am leaving 73, okay?

Professor G. Venkatesh: And we are at 12:56 at Delhi on Tuesday?

Professor Madhavan Mukund: 12:56.

Professor G. Venkatesh: So, we have to be on Tuesday only, right? So, Kanpur, I can go.

Professor Madhavan Mukund: Okay.

Professor Madhavan Mukund: Let us see what we can do in Kanpur. So, we can actually leave from Chennai and reach Kanpur on the same day, second day.

Professor G. Venkatesh: So, I reached Kanpur at 05:48, which is this train. And that is on Tuesday and there is nothing for that Kanpur. So, I am done with Kanpur. So, I can continue on this train to Varanasi.

Professor Madhavan Mukund: Varanasi. And we can, may, just may be able to make a train, but because Varanasi, we are reaching pretty late.

Professor G. Venkatesh: Yeah, we are reaching at 10:11, and its Tuesday, so same train.

Professor Madhavan Mukund: So, same train.

Professor G. Venkatesh: So, nothing to do. So, Varanasi is nothing to do.

Professor Madhavan Mukund: Yeah.

Professor G. Venkatesh: And we go to Patna next day.

Professor Madhavan Mukund: Next Day.

Professor G. Venkatesh: So, finished.

Professor G. Venkatesh: So, 12274 is over. 12274 is over.

Professor Madhavan Mukund: So now, we do.

Professor G. Venkatesh: Now, we explore the next train in Delhi, right?

Professor Madhavan Mukund: On a Tuesday, yeah.

Professor G. Venkatesh: On a Tuesday. So, Tuesday.

Professor Madhavan Mukund: 12270, which looks familiar. Surely, we have seen that before, yeah.

Professor G. Venkatesh: We have seen it?

Professor Madhavan Mukund: It is a pair of our same train, this guy's pair.

Professor G. Venkatesh: It's the pair, yeah. What about 12260, is not it?

Professor Madhavan Mukund: This is also, we have seen. That is the train going to Bikaner, I think.

Professor G. Venkatesh: And then last, this is probably the last one, 12265, have we seen?

Professor Madhavan Mukund: That looks new. Yeah, that is new.

Professor G. Venkatesh: We have these trains still in the night, but 12265, 2 more of them, 12265.

Professor G. Venkatesh: We can just get in, we cannot do anything, because.

Professor Madhavan Mukund: Yeah, because the next station is bound to be after.

Professor G. Venkatesh: Yeah, exactly.

This is going to Ludhiana, 10:15 reaches at 02:45, so next.

Professor G. Venkatesh: So anyway, we can only see the train, nothing else, all right?

Professor Madhavan Mukund: Yeah. And what time was that by the way?

Professor G. Venkatesh: That was 22:15.

Professor Madhavan Mukund: Okay. And the last one is?

Professor G. Venkatesh: The last, 12214.

Professor Madhavan Mukund: No, that is on Monday. So.

Professor G. Venkatesh: So,

Professor Madhavan Mukund: 22210.

Professor G. Venkatesh: 22210

Professor Madhavan Mukund: And 22209.

Professor G. Venkatesh: This is going to Mumbai. So, this leaves 23:25.

Professor Madhavan Mukund: And again everything else is the next day.

Professor G. Venkatesh: So, we are done with New Delhi. Can you believe that? We are done with New Delhi.

Professor G. Venkatesh: So now, I think we have.

Professor Madhavan Mukund: And this train is over. Yes, finally.

Professor G. Venkatesh: Finally, over. So, I am putting back.

Professor G. Venkatesh: And you can put New Delhi back. And how many stations have we got? How many trains do you get actually?

Professor Madhavan Mukund: So, I potentially do, so we have, well we can just look down this list, because these are in pairs 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15 trains we can go.

Professor G. Venkatesh: 15 trains, we can connect.

Professor Madhavan Mukund: Including the 14 plus the 1 we started with.

Professor G. Venkatesh: if you give 2 days, Monday and Tuesday, 15 possible.

Professor Madhavan Mukund: Trains, we can get to.

Professor G. Venkatesh: Get to. Multiple connections.

Professor Madhavan Mukund: Yeah.

Professor G. Venkatesh: We may have to hop between many-many stations.

Professor Madhavan Mukund: And we can actually do like 3 hops.

Professor G. Venkatesh: So, Nagpur, Mumbai.

Professor Madhavan Mukund: We can get on this train, 12, start on this train, then get on to this train on Nagpur. And then for instance, get onto this train at Mumbai.

Professor G. Venkatesh: I see.

Professor Madhavan Mukund: So, you can do 2 hops or you can get on to this train. So, this is the best, the longest.

Professor G. Venkatesh: The longest.

Professor Madhavan Mukund: Or we can do Vijayawada-Bhubaneswar. So, we can do 2 trains there.

Professor G. Venkatesh: So again here, see, what we have done, we have taken, we took 1 train. We start exploring that train, which means that we started looking at all the places we can get off.

Professor Madhavan Mukund: Yes.

Professor G. Venkatesh: And you get off there and try another train.

Professor Madhavan Mukund: Yeah, the earliest one

Professor G. Venkatesh: Earliest one. After that you come back and try another one. Keep trying, I mean, you finish that completely exhausted there, come back to the train again and see the next stop.

Professor Madhavan Mukund: Yeah.

Professor G. Venkatesh: And keep doing that and when you finish with the train, come back.

Professor Madhavan Mukund: Correct, yeah. So, we did Vijayawada first. So, the first stop on this train is Vijayawada. So, we explored all the Vijayawada possibilities. Then we came back and looked at the second stop Balharshah, looked all the Balharshah. Then we came in third stop Nagpur, and then we saw a few stops, which did not help like Jhansi and all the Gwalior and all that.

Professor G. Venkatesh: Bikaner.

Professor Madhavan Mukund: And then we went to Delhi, and then in Delhi we got .

Professor G. Venkatesh: Very nice. So, 15 stations, we have, 15 trains, we were able to go to. So again here, this is example of depth first search starting with trains and going from train to train.

Professor Madhavan Mukund: Yeah, train to train.

Professor G. Venkatesh: And we are again using recursion to explore.

Professor Madhavan Mukund: So, the same way, we get to a new station at a fixed time.

Professor G. Venkatesh: Same way. So, you call explore. Explore with a train number this time, rather than the station number, station name.

Professor Madhavan Mukund: And the time.

Professor G. Venkatesh: And again, you are keeping track of the trains you have seen, so you do not come back again to it, so otherwise it will looped, right?

Professor Madhavan Mukund: Yeah.

Professor G. Venkatesh: And then.

Professor Madhavan Mukund: Yeah, and we added this constraint that we do not want to go back on the same.

Professor G. Venkatesh: Same train, also we added. So, an additional interesting constraint. And then you keep calling, explore keeps calling calling calling and eventually it makes a tree. It is making a tree, right?

Professor Madhavan Mukund: Yeah, it is a tree. It is just that, it is something.

Professor G. Venkatesh: So, if you had that connecting train graph, this tree actually is just kind of sitting inside.

Professor Madhavan Mukund: Sitting inside. Some of the edges of that graph of formal tree.

Professor G. Venkatesh: Interesting.

Professor Madhavan Mukund: Pretty interesting.

Professor G. Venkatesh: Skeleton of the tree, of the graph, right?

Professor Madhavan Mukund: Yeah.

Professor G. Venkatesh: Some kind of skeleton of that graph is being created, right?

Professor Madhavan Mukund: Yeah.

Professor G. Venkatesh: Is there a name for that? I mean, this kind of skeleton that you create, tree skeleton for a graph?

Professor Madhavan Mukund: Well, depends on is it going to cover the whole graph?

Professor G. Venkatesh: Let us say, its, if you do it long enough, I guess it will cover.

Professor Madhavan Mukund: Yeah. So, it is like it spans the graph, right?

Professor G. Venkatesh: Spans it, it covers.

Professor Madhavan Mukund: So, it covers like put your hand across.

Professor G. Venkatesh: I see.

Professor Madhavan Mukund: So, it was called a Spanning Tree.

Professor G. Venkatesh: Spanning Tree. So, you have made a spanning tree for the graph in a sense. I mean, if it (re), provided it reaches everything. This case we put a limit, because 2 days, we said it came out, right?

Professor Madhavan Mukund: Yes.

Professor G. Venkatesh: But if you did not put a limit, we could kind of span it.

Professor Madhavan Mukund: If we could reach all the stations then you would actually get a tree.

Professor G. Venkatesh: Spanning tree, you will get.

Professor Madhavan Mukund: Which is 1 way of reaching you all of them.

Professor G. Venkatesh: We could have stopped anywhere. I mean, if it was a graph, which is fully connected, I could, strictly speaking start anywhere.

Professor Madhavan Mukund: Yeah.

Professor G. Venkatesh: And I would have created spanning tree.

Professor Madhavan Mukund: So, the starting point can be anything. And from that starting point, you generate this tree.

Professor G. Venkatesh: I see. Very nice.