

# **IIT Madras**

## **ONLINE DEGREE**

**Statistics for Data Science 1**  
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**Week 7 Tutorial 5**

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The probability that a new car battery functions for over 10,000 miles is 0.8, the probability it functions for over 20,000 miles is 0.4, and the probability it functions for over 30,000 miles is 0.1. If a new car battery is still working after 10,000 miles, find the conditional probability that



i) Its total life will exceed 20,000 miles.

20000

$$P(B|A) = \frac{P(B \cap A)}{P(A)}$$

ii) Its additional life will exceed 20,000 miles

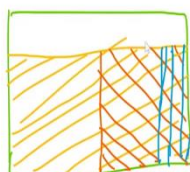
30000

$$P(C|A) = \frac{P(C \cap A)}{P(A)}$$

A → > 10000

B → > 20000

C → > 30000



A //  
 B //  
 C //

The probability that a new car battery functions for over 10,000 miles is pointed. And the probability that it functions for over 20,000 miles is 0.4. And lastly, the probability that it functions for over 30,000 miles is 0.1. And if a new car battery is still working after 10,000 miles, find the conditional probability that its total life will exceed 20,000 miles and additional life will exceed 20,000 miles that means, case 1 is for the 20,000 scenario, and case 2 is for the 30,000 scenario.

So, if we consider these as 3 events, where A is exceeding 10,000, B is exceeding 20,000 and C is exceeding 30,000 what is being asked of us here is  $P(B|A)$  and this is  $P(C|A)$ . And we know this is equal to  $\frac{P(B \cap A)}{P(A)}$  and likewise, this would be equal to  $\frac{P(C \cap A)}{P(A)}$ . And for this, if we draw Venn diagrams, we will observe the following.

So, let this be the entire universe. And if there is 80% probability of this happening, so yeah, let us say this is A, this is A. And B is a case which is entirely within this and it is only 40% of the total universe. So, B would be something like this, because B has to happen only when A happens. So, B is completely within A and lastly, the 30,000 miles C happens within B. So, within A is B and within B is C. So, C is also within A and that would be something like this.

So, we have a situation where A is these lines and B is these lines and C is these lines. However, C will also have these lines. And these lines, because C is within embedded within

B and within A it and likewise B is going to have these lines because B is embedded within A.

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50000

$A \rightarrow > 10000$   
 $B \rightarrow > 20000$   
 $C \rightarrow > 30000$

$$= \frac{P(C \cap A)}{P(A)}$$

$P(A \cup B) = P(A) + P(B) - P(A \cap B)$   
 $\Rightarrow P(A \cap B) = P(A) + P(B) - P(A \cup B)$   
 $P(A \cap C) = P(A) + P(C) - P(A \cup C)$

$P(A) = 0.8$   
 $P(B) = 0.4 = P(A \cap B)$   
 $P(C) = 0.1 = P(A \cap C)$

30,000 miles is 0.1. If a new car battery is still working after 10,000 miles, find the conditional probability that  
 i) Its total life will exceed 20,000 miles.  
 $P(B/A) = \frac{P(B \cap A)}{P(A)} = \frac{0.4}{0.8} = 0.5$   
 ii) Its additional life will exceed 20,000 miles  
 $P(C/A) = \frac{P(C \cap A)}{P(A)} = \frac{0.1}{0.8} = 0.125$

$A \rightarrow > 10000$   
 $B \rightarrow > 20000$   
 $C \rightarrow > 30000$

$P(A \cup B) = P(A) + P(B) - P(A \cap B)$   
 $\Rightarrow P(A \cap B) = P(A) + P(B) - P(A \cup B)$   
 $P(A \cap C) = P(A) + P(C) - P(A \cup C)$

$P(A) = 0.8$   
 $P(B) = 0.4 = P(A \cap B)$

Anyway, now looking at this case, you know,  $P(A)$  is 0.8 and  $P(B)=0.4$  and  $P(C)$  is equal to point how much was that 0.1 this is what is given to us. Now, for finding the intersection probabilities, what we should use is this probability of union is equal to  $P(A) + P(B) - P(A \cap B)$ . And that would give us  $P(A \cap B)$  is equal to  $P(A) + P(B) - P(A \cup B)$ .

Likewise,  $P(A \cap C)$  also will be  $P(A) + P(C) - P(A \cup C)$ . So, once we have this, what we have left to observe is this  $P(A \cup B)$ , the probability of the union is going to be just  $P(A)$  because B is fully embedded within A so the union is just A. So, therefore,  $P(A)$  and  $P(A \cup B)$  cancelled

here and likewise  $P(A \cup C)$  and  $P(A)$  are the same thing because  $C$  is fully embedded within  $A$ . As you can see,  $B$  is completely within  $A$  and  $C$  is completely within  $B$  and  $A$ .

So, therefore  $P(B)=0.4$  is also  $P(A \cap B)$  and  $P(C)$  which is  $0.1$  is also  $P(A \cap C) = 0.4/0.8$  which is  $0.5$ . And this would give us  $0.1/0.8$  which is  $0.125$ . So, these are the probabilities we are looking for.

