

IIT Madras
ONLINE DEGREE

Mathematics for Data Science 1
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Lecture 4.1 B
Examples of Quadratic Functions

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Example

Let $f(x) = x^2 - 6x + 9$.


- Determine whether f has minimum or maximum value. If so, what is the value?
- State the domain and the range of f .

Observe that $a=1$, $b=-6$, and $c=9$.

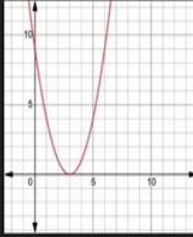
Since, $a > 0$, the function opens up and has the minimum value.


The minimum value is given by y-coordinate of the vertex. The x-coordinate of the vertex is $-b/(2a) = 3$. Therefore, the minimum value is $f(3) = 0$.

Domain = \mathbb{R} and Range = $\mathbb{R} \cap \{f(x) | f(x) \geq 0\}$.



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Let us say, this example we have been given a function $f(x) = x^2 - 6x + 9$ and we are asked to determine whether f has minimum or maximum values, if so what is the value and you need to state the domain and range of f . Let us first attempt the second question, what is a domain? Domain of f is entire real line we do not have to worry, what is the range of f ?

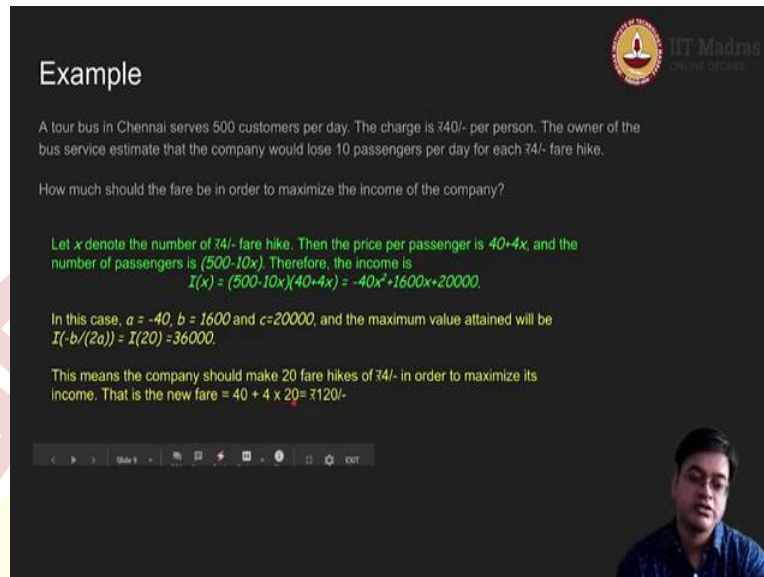
Let us take this function identify a, b, c so $a = 1, b = -6, c = 9$. Since $a > 0$, the function opens up, if the function opens up then it will have a minimum value. So, the answer to first question is whether f has minimum or maximum value, it has a minimum value, once it has a minimum value it cannot have maximum value, if so what is the value?

You need to figure out what is the vertex of this particular parabola. So, what is the formula for vertex of the parabola, $\frac{-b}{2a}$, $b = -6$, $a = 1$ so $\frac{-b}{2a}$ is $\frac{-(-6)}{2}$ which will give me minus 3. Sorry, this is wrong, it should give me +3, $\frac{-(-6)}{2}$, it should give me +3, which is written wrong here, but the graph is correct here where we are getting $x = 3$ is the vertex. So, if you substitute $f(3)$, what do you get? $f(3) = 3^2 - 6(3) + 9$ and therefore, the value of this is nothing but 0. So, this -3 is wrong it, should be +3.

And obviously, the range if it has a minimum value, the range is minimum upwards, this is the entire real line above this minimum. So, that is \mathbb{R} intersected with $f(x)$ such that $f(x) \geq$

0. So, we have understood how to find the minimum and maximum values of a function, if a is negative you can similarly find the maximum value.

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Example

A tour bus in Chennai serves 500 customers per day. The charge is ₹40/- per person. The owner of the bus service estimate that the company would lose 10 passengers per day for each ₹4/- fare hike.

How much should the fare be in order to maximize the income of the company?

Let x denote the number of ₹4/- fare hike. Then the price per passenger is $40+4x$ and the number of passengers is $(500-10x)$. Therefore, the income is

$$I(x) = (500-10x)(40+4x) = -40x^2 + 1600x + 20000.$$

In this case, $a = -40$, $b = 1600$ and $c = 20000$, and the maximum value attained will be

$$I(-b/(2a)) = I(20) = 36000.$$

This means the company should make 20 fare hikes of ₹4/- in order to maximize its income. That is the new fare = $40 + 4 \times 20 = ₹120/-$.

So, let us try to make this example more realistic. So, let us take 1 realistic example. Where a tour bus in Chennai serves 500 customers per day, they charge rupees 40 per person. Now, they want to revamp their strategies, so the owner of the bus service estimate that the company would lose 10 passengers per day for each Rs 4 hike in the fare.

So, if they hike the fare by 4 rupees, then they will lose 10 customers per day, this is the estimate. Now, the company wants to maximize the profit, so how much should be the fair in order to maximize the income of the company is the question. So, let us try to answer this question using our knowledge of quadratic equation.

So, let us say, 1 unit of hike is 4 rupees so let x denote the number of Rs 4 fare hikes. So, what will this impact? This will impact the number of passengers because we are losing 10 passengers per fare hike. So, what will be the corresponding fair price for the passenger? It will be $40 + 4x$, 40 rupees is the fees that we are charging per person, the company charging per person and if I hike the fare it will be four times x , this will be charged per person.

Now, the number of passengers with this hike if you increase x units, that means, you will lose 10 passengers every x units increase. That means $500 - 10x$ is the passengers that still remain. So, in this case, the income of the company will be the number of passengers into the fare, they have charged so that is $(500 - 10x)(40 + 4x)$. If you open this, open the bracket

and multiply them, then you will get the expression to be $-40x^2 + 1600x + 20000$. This is the income.

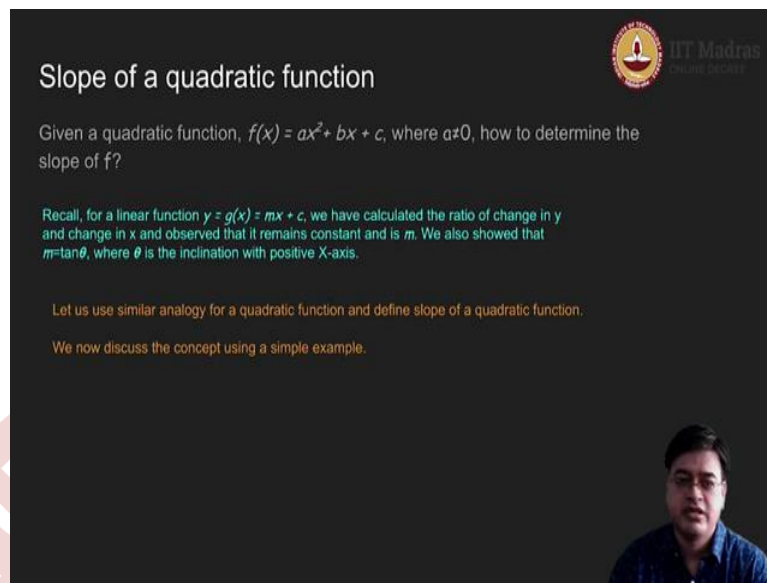
Now, the company wants to maximize the profit, first of all after getting this quadratic equation, can you tell me is the maximum possible? The answer is yes, and why the answer is yes, because it lies in the coefficients a, b and c . So, what is a here? $a = -40$, $b = 1600$ and $c = 20,000$. Because $a = -40$, $a < 0$ so, the curve will open downwards that means the maximum is possible.

And what will be the maximum value attained then that is what we have to figure out. So now, the next question is okay. So, where this maximum will be attained? The maximum is possible, maximum will be attained on the vertex, y coordinate of the vertex will give me the maximum. So, I will simply figure out what is the x coordinate of the vertex, x coordinate of the vertex is point of intersection of the axis of symmetry, what is the axis of symmetry $x = \frac{-b}{2a}$, what is b ? 1600, c is 20,000 and a is -40 .

So, $x = \frac{-1600}{2(-40)}$ which will give me 20, so that is what 20 is yes. And of maximize y coordinate the maximum fare that we will get is 36,000. Right now, how much we are earning, how much the company is earning, it is 500 customers they are serving, where everybody is paying 40 rupees so they are simply earning only 20,000 rupees that is when you do not increase any fare $x = 0$, you get 20,000. So, the main question is how much the fare should be?

Now, what we are suggesting here by solving this problem is there should be a 20 units of hike of rupees 4 each that means, what we are suggesting is there should be 80 rupees hike in the fare. So, the new fare for the company should be 40 plus four times x that x is 20. So, $40 + 4 \times 20 = 120$ and that is what is the recommended hike by the company. So, now every person should be charged 120 rupees as opposed to 40 rupees and then the company will be profitable and you may have to serve less customers. This is how we are using real life, we are using quadratic equations to solve real life situations.

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Slope of a quadratic function

Given a quadratic function, $f(x) = ax^2 + bx + c$, where $a \neq 0$, how to determine the slope of f ?

Recall, for a linear function $y = g(x) = mx + c$, we have calculated the ratio of change in y and change in x and observed that it remains constant and is m . We also showed that $m = \tan \theta$, where θ is the inclination with positive X -axis.

Let us use similar analogy for a quadratic function and define slope of a quadratic function.

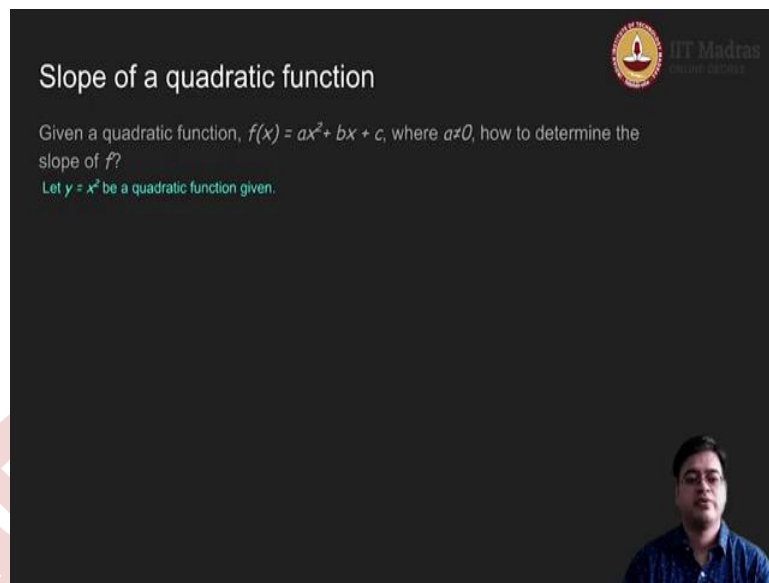
We now discuss the concept using a simple example.

Now, let us go back to our linear functions, where we studied the slopes of the lines. What was the slope of a line? Slope of a line was change in y by change in x . Let us see what the concept of slope has to do with a quadratic function. Let us try to analyse that. So, my goal in this set of slides is given a quadratic function $f(x) = ax^2 + bx + c$ where $a \neq 0$, how to determine the slope of a function f .

So, in order to generalize this notion of slope of a function, we will first recall what we do know about linear function. So, if you look at a linear function which is y which is equal to $g(x) = mx + c$, we know that this m represents the slope and m can be calculated by considering a ratio of change in x upon change in y .

We have spent a lot of time in understanding the slope and when I consider a linear function, I also know that the slope remains constant okay. We also know that the slope is nothing but \tan of some inclination and that inclination is with positive x axis. I want to relate all these concepts and try to figure out what is the slope of this quadratic function. Let us go ahead, we will use a similar analogy for a quadratic function and define the slope of a quadratic function. First let us take one example to discuss this concept of slope.

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Slope of a quadratic function

Given a quadratic function, $f(x) = ax^2 + bx + c$, where $a \neq 0$, how to determine the slope of f ?

Let $y = x^2$ be a quadratic function given.

Let us take our standard prototype example. We are trying to answer this question, $y = x^2$.