INDIAN INSTITUTE OF TECHNOLOGY MADRAS

सिद्धिर्भवति कर्मजा

IIT Madras

ONLINE DEGREE

Mathematics for Data Science 2
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Week 2 Tutorial

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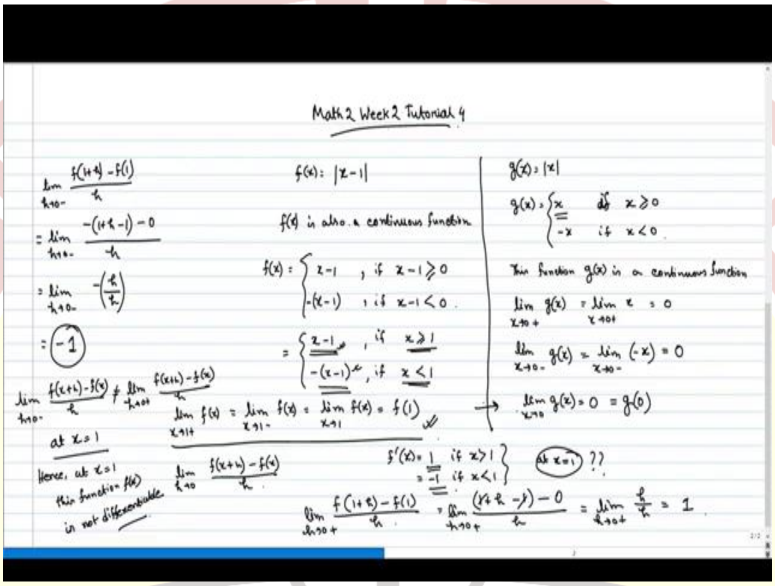

Hello everyone. Welcome to the fourth tutorial video of Math 2 week 2. So, let us consider the function $f(x) = |x-1|$. So, you know that the function $g(x) = |x|$ that we can write as $g(x) = x$ if $x \geq 0$ and $g(x) = -x$ if $x < 0$ and you have already seen that this function $g(x)$ is a continuous function.

Why it is continuous because $\lim_{x\to 0} g(x)$, if we come from the positive side $g(x)$ that will give us this x, so this is $\lim_{x\to 0+} g(x) = 0$ and again if we come from the left side, from the negative side of x axis that will again give us $\lim_{x\to 0-} g(-x) = 0$ again. So, both the limit are same, $\lim_{x\to 0} g(x) = 0 = g(0)$. So, this function is continuous.

Similarly, in this case the $f(x)$ what we have considered here, which is a $|x-1|$, this is also a continuous function. So, $f(x)$ is also a continuous function. Why? Because again this we can write it as $f(x) = (x-1)$ if $x-1 \geq 0$ and $f(x) = -(x-1)$ if $x-1 < 0$ is strictly less than 0. That means this again we can write it as $(x-1)$ if $x \geq 1$ and $-(x-1)$ if $x < 1$.

So, if we calculate the $\lim_{x\to 1+} f(x) = \lim_{x\to 1-} f(x) = \lim_{x\to 1} f(x) = f(1)$. The similar kind of checking for $g(x)$ what we have done here the same thing will work here.

So, $f(x)$ is continuous on 1 and for the other places it is anyway continuous because this is a linear function $x-1$, and this is again a linear function $-(x-1)$, whenever x is not 1. So, only for 1 we have to check it here, so that we have already checked. So, $f(x)$ is a continuous function.

Now, what about the differentiability, so we will check through the definition of derivative, so $\lim_{h\to 0} \frac{f(x+h)-f(x)}{h}$, if this limit exists, then the function is differentiable. So, again for the, all other places it is obviously differentiable because $x-1$, we can see that this is a linear function, so $\acute{f}(x) = 1$ if $x > 1$ and $\acute{f}(x) = -1$ if $x < 1$

So, only problem can arise at $x = 1$, so we will have to check what is it differentiable at $x = 1$ or not, so let us do this. We have to check this one, at $x = 1$ we have to check whether this limit exist or not. So, this at first we check when we approach 1 from the right side, so from the right side $f(1+h)$ means x is obviously greater than 1, so this first thing will come.

So, $\lim_{h\to 0+} \frac{(1+h-1)-0}{h} = \lim_{h\to 0+} \frac{h}{h} = 1$. Now, what about limit h tending to 0 minus? So, let us do it here, $\lim_{h\to 0-} \frac{f(1+h)-f(1)}{h} = \lim_{h\to 0-} \frac{-(1+h-1)-0}{h} = -1$.

So, you can see that $\lim_{h\to 0-} \frac{f(x+h)-f(x)}{h} \neq \lim_{h\to 0+} \frac{f(x+h)-f(x)}{h}$ at $x = 1$. Hence, at $x = 1$ this function $f(x)$ is not differentiable.

So, that we have checked here and for other cases, when x is strictly greater than 1 the function is differentiable and the derivative of the function is 1 and when x is strictly less than 1 the derivative of function is -1 and at $x = 1$ this function is not differentiable. Now, let us try to visualize this using graphs using GeoGebra.

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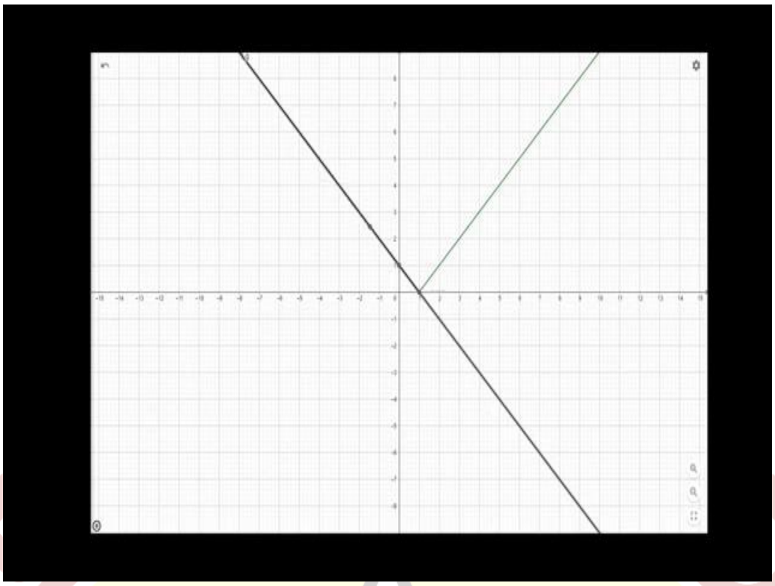

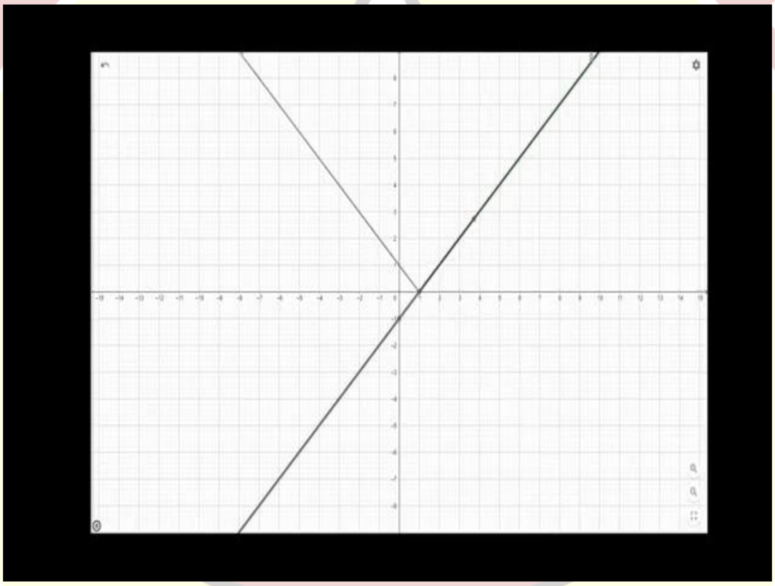

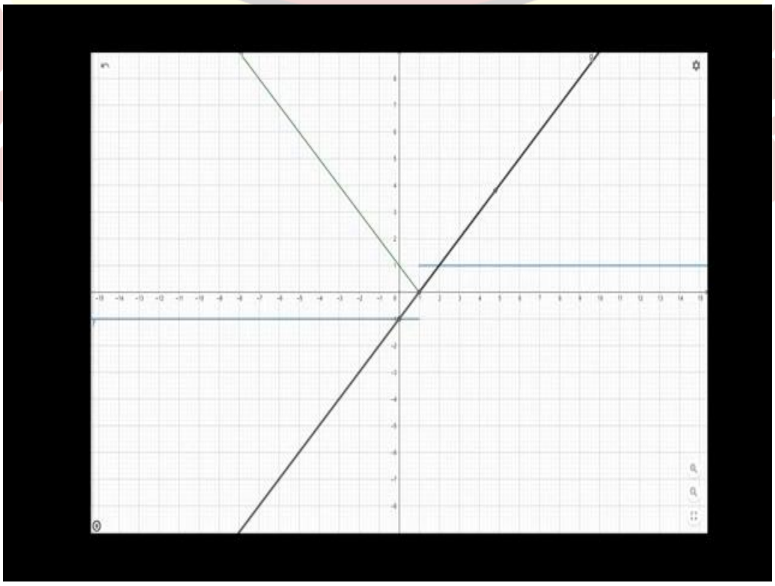

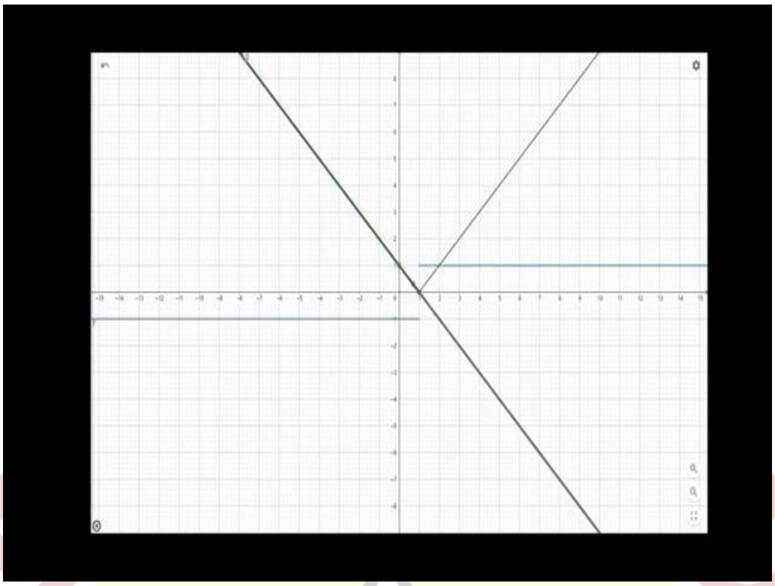

So, let us see how $|x-1|$ look like. So, it is like $|x|$, but it is shifted towards a positive x axis by 1 unit, so this is $|x-1|$ and now if we try to see the tangent of this function, how the slope of the tangent changes, so we can see that when we are in the positive side of 1, the tangent remains same, that when we are coming to the left hand side of one the slope of the tangent changes otherwise the slope of the tangent remain constant as you can see in this animation.

So, till 1 the slope is -1 and from right hand side of 1, I mean, 1 towards the positive side the slope is 1 as we have seen in our calculation. So, the derivative of the function will look like this. So, at 1 the function is not differentiable except that when x is greater than 1, strictly greater than 1, the derivative of the function is a constant function 1 and when x is strictly less than 1, the derivative of the function is a constant function -1. Thank you.