



**IIT Madras**  
ONLINE DEGREE

## Mathematics for Data Science 1

### Week 06 - Tutorial 03

(Refer Slide Time: 00:16)

3. Which of the following polynomials (may also be monomial or constant) should be added to the polynomial  $P(x) = 2x^3 + 23x^2 + 40x$  to make it divisible by  $x+9$ ?

☒ A.  $2x^2 + 9x$

☒ B.  $-45$

☒ C.  $5x$

☒ D.  $x^2 - 126$

$$\begin{array}{r}
 2x^2 + 5x - 5 \\
 x+9 \overline{) 2x^3 + 23x^2 + 40x} \\
 \underline{2x^3 + 18x^2} \phantom{+ 40x} \\
 5x^2 + 40x \\
 \underline{5x^2 + 45x} \\
 -5x - 45 \\
 \underline{+ 45} \\
 45
 \end{array}$$

$5x + 45 = 5(x+9)$

$$P(x) = (x+9)(2x^2 + 5x - 5) + 45$$

$$p(x) + 2x^2 + 9x = (x+9)(\dots) + 2x^2 + 9x + 45$$



$$p(x) + 2x^2 + 9x = (x+9)(\dots) + 2x^2 + 9x + 45$$

$\therefore 2x^2 + 9x + 45$  divisible by  $x+9$

$$2(81) + 9(-9) + 45 = 162 - 81 + 45 > 0$$

$$x^2 - 126 + 45 = x^2 - 81 = (x+9)(x-9)$$

Now, we have this problem, which of the following polynomials should be added to the polynomial  $p(x)$  to make it divisible by  $x+9$ . So, we need to recognize that it is not necessary that there is only one polynomial that you add, because since it is only divisibility, we can add a number of polynomials to  $p(x)$  and make it divisible by  $x+9$ . So, we have to check for each of these cases.

So let us see, or what we can additionally do is, we can look at the remainder that we get by dividing  $p(x)$  with this and then see what to do with that remainder. So, if we did the division,

now, we have  $2x^3 + 23x^2 + 40x$  and we are dividing it with  $x + 9$ . So, start with  $2x^2$ , so we get  $2x^3 + 18x^2$ . So, this cancels off, this gives us  $5x^2 + 40x$ .

So, we do  $+5x$  additionally, then we get  $5x^2 + 45x$ , so negative and negative so we are left with  $-5x$  and then that gives us a  $-5$  additionally here, so we have  $-5x - 45$ , therefore these two go off and we are left with 45 as our remainder. So,  $p(x)$  is essentially  $(x + 9)$  into the quotient  $+45$ . So, if we subtracted 45 from  $p(x)$ , we will get divisibility by  $(x + 9)$ .

So, B is necessarily correct. Let us look at what happens if we added A, if we added A,  $p(x) + 2x^2 + 9x$  is some multiple of some product of  $(x + 9)$ , and some quadratic plus  $2x^2 + 9x + 45$ . So, unless  $2x^2 + 9x + 45$  is divisible by  $(x + 9)$ ,  $p(x)$  would not be divisible by  $(x + 9)$ .

So, what we should really be checking is  $2x^2 + 9x + 45$ . Is it divisible by  $(x + 9)$ ? And the direct way to check it is to substitute  $x = -9$ , so you will get  $2 \times 81 + 9 \times -9 + 45 = 162 - 81 + 45$ , which is greater than 0, it is not equal to 0. So, no, A does not give us divisibility by  $(x + 9)$ .

What happens if we added  $5x$ , we get  $5x + 45$ . So, we have this 45 remainder, so we are getting  $5x + 45$ , which is equal to  $5(x + 9)$ , which is directly divisible by  $(x + 9)$ . So, this is correct too, c is also correct. And what happens if we added  $x^2 - 126$ , then we would get  $x^2 - 126 + 45$  as the additional part upside from  $(x + 9)$  into that quadratic, so this is equal to  $x^2 - 81$ , which is equal to  $(x + 9)(x - 9)$ . So,  $(x + 9)$  is dividing this particular polynomial. So, we can add  $x^2 - 26x - 126$  also, and get divisibility by  $(x + 9)$ .