IIT Madras

ONLINE DEGREE

Mathematics for Data Science 1
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Lecture 4.4
Solution of quadratic equation using graph

In today's video, we are going to learn what are Quadratic Equations. And once we set up the Quadratic Equation, we are going to see, what are the solutions of the Quadratic Equations, that are called roots of the Quadratic Equation and how to solve these Quadratic Equations, using the technique that we have demonstrated, for quadratic functions. That is Graphing technique. So, let us start.

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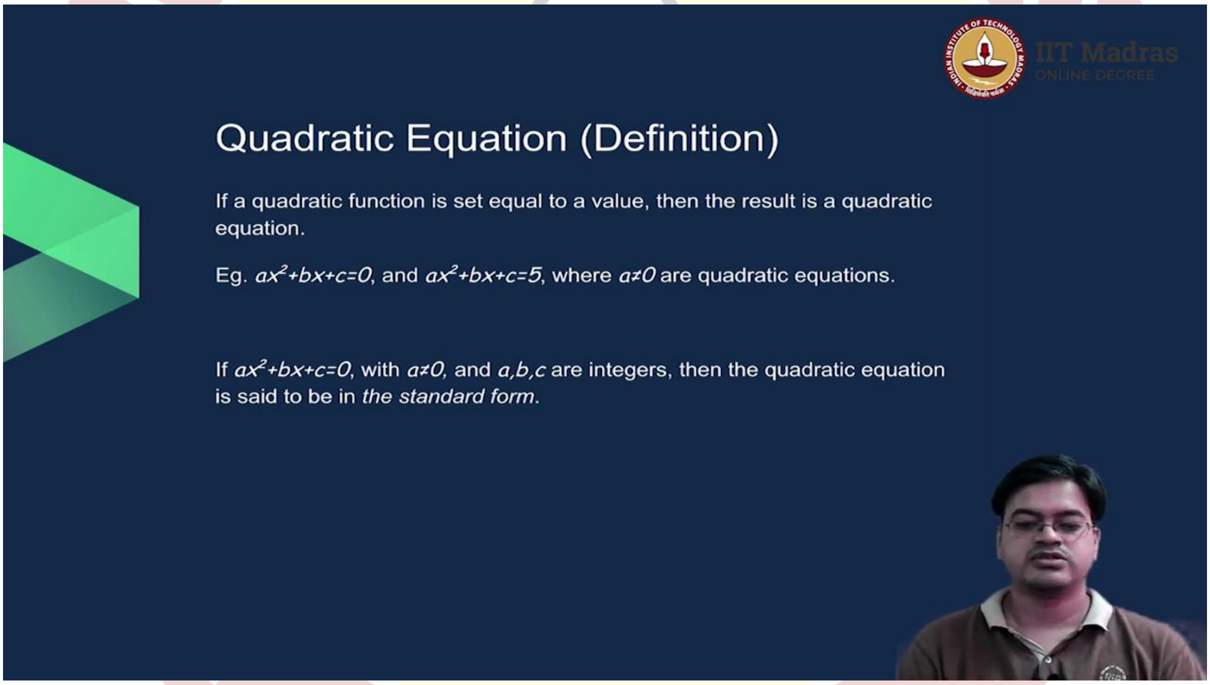

So, first of all, let us understand what is a quadratic equation and how it is related to quadratic function. So, here is a definition. If a quadratic function is set to be equal to a value, then the result is called quadratic equation. So, let us see one example. For example, $ax^2 + bx + c = 0$, is one quadratic equation, where $a \neq 0$.

In the similar manner, $ax^2 + bx + c = 5$, is another quadratic equation. Obviously a should not be equal to 0. Now, once we get the Quadratic Equation, if the coefficients, what are the coefficients. Coefficients are like a, b and c. These are called coefficients of the Quadratic Equation.

If the coefficients are from set of integers, which we have studied in week 1. So, if a, b, c, the coefficients are integers, and on the righthand side, it is equated to 0. That is, you have an equation, $ax^2 + bx + c = 0$, where a is not equal to 0 and a, b, c are integers. Then the Quadratic Equation is said to be in the standard form. So, on this slide, we have seen two definitions. One, what is Quadratic Equation. Quadratic Equation is nothing, but a quadratic function, where it is equated to some value.

And what is a standard form of Quadratic Equation? That is $ax^2 + bx + c = 0$, where $a, b, c \in \mathbb{Z}$ and a is not equal to 0. Then the Quadratic Equation is said to be in a standard form.

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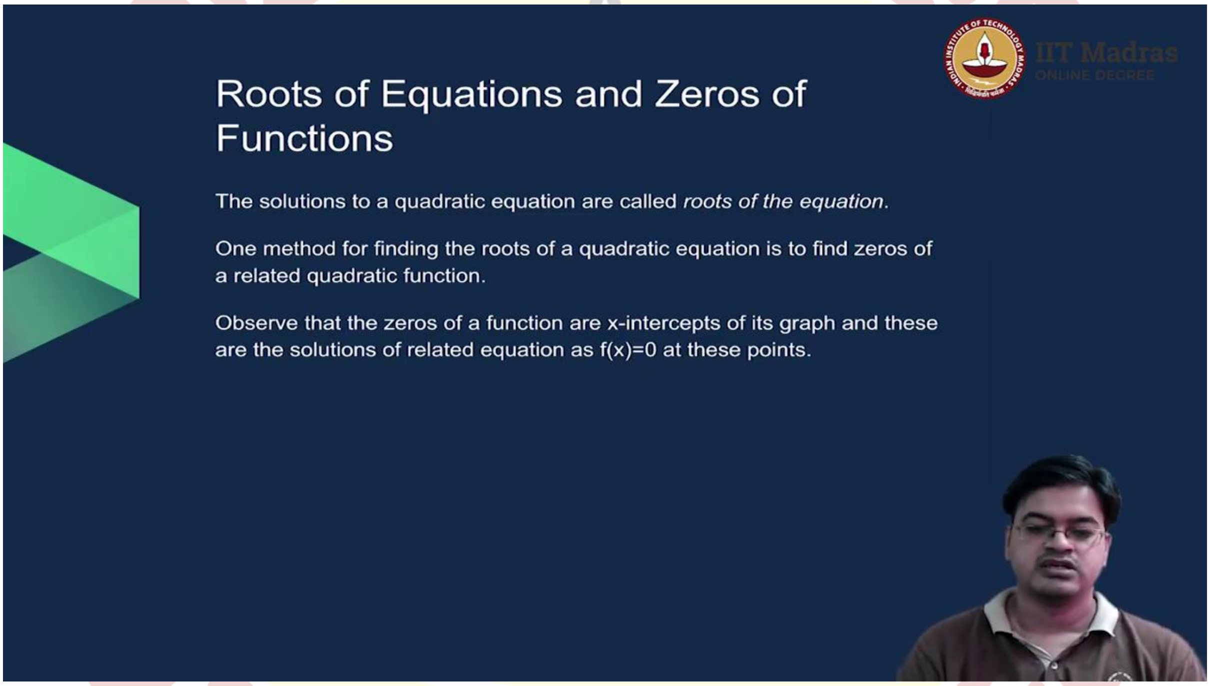

Now, once we have a Quadratic Equation in standard form, we can discuss about roots of the Quadratic Equation or zeroes of the functions. And we will see how the concept of roots of Quadratic Equation and zeroes of quadratic function are related, in this slide. So, the solutions of the Quadratic Equation are called roots of the equation.

What do I mean by that? If $ax^2 + bx + c = 0$, then what is the value of x, that gives me 0, is called the solution to the Quadratic Equation. And also, that value of x will also be known as root of the Quadratic Equation. So, this way we get the root of the Quadratic Equations.

So now, which way you can find the roots of the Quadratic Equations? One method, which is very easy. If you have a quadratic function associated with this Quadratic Equation, then you just plot the quadratic function and find its zeroes. What is a zero of a quadratic function? Zero of a quadratic function is nothing but its x intercept. So, in particular, if you observe that, zeroes of the functions are x intercepts of its graph and these are the solutions to the related equation, $f(x) = 0$, at these points?

So, if you are having a quadratic function, what you need to do is just plot it and see where it intersects x axis. If it intersects x axis, then you got the solution or the root of the Quadratic Equation. So, let us try to see this through some examples.

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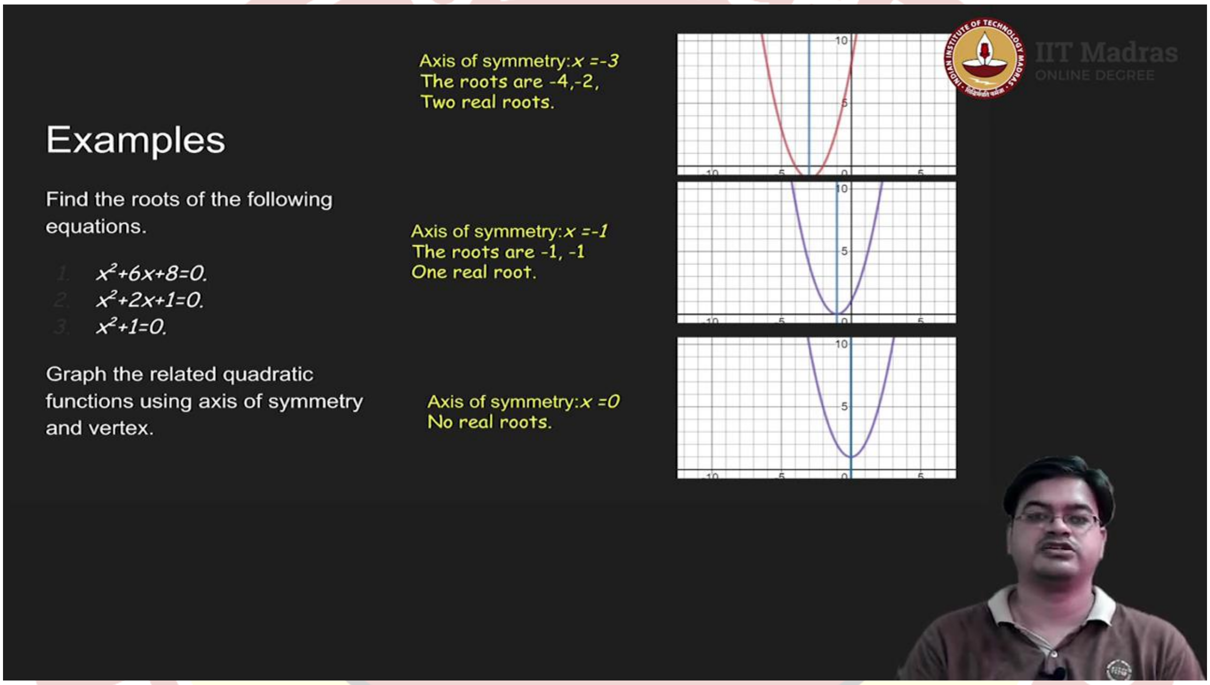

So, here are some examples. So, the question is to find the roots of the following equations. First equation is $x^2 + 6\,x + 8 = 0$. Second one is $x^2 + 2\,x + 1 = 0$. And third one is $x^2 + 1 = 0$. Now, we will take these equations one by one. So, essentially what we are proposing is, we want to chart these equations or we want to plot these expressions on a graph paper.

So, if you recollect from our last few videos, in order to plot a quadratic function, we need to understand the axis of symmetry of the quadratic function. So, let us take the first example, where you have $x^2 + 6x + 8$. Now, I want to understand, what is the axis of symmetry of this particular function.

Let us see. So, in this case, for our standard notation, our related quadratic function is $x^2 + 6x + 8$. So, $a = 1, b = 6$ *and* $c = 8$. So, y intercept is obviously 8. And axis of symmetry is $x = -\frac{b}{2a}$, which obviously means it is $-\frac{6}{2}$, which is -3. So, axis of symmetry is x = -3.

So, axis of symmetry is x = -3 and a, the value of a is positive. So, what are the things that we can conclude from our previous videos? That is, if a > 0, the curve opens up, the graph of the function opens up. It attends the minimum.

And the axis of symmetry in this particular example is x = -3. So, the simplest thing that we can do here is, put x is equal to minus 3 in this expression. And you will see that, the expression will take a negative value. That means the y value taken is negative. That means if the y value taken is negative, you can easily see, the curve opens up. That means it will intersect x axis in two points.

Now, we want to guess those two points. Without plotting, right now based on our visual interpretation of this curve, can we guess the two points? Okay. So, -3, the value is negative. That means, for -3 it is negative. Then let us check it for x is equal to -2. If you substitute x is equal to - 2, you will get $4 - 12 + 8 = 0$. So, one root I have got, which is -2. If -2 is one root, -3 is one, -3 is axis of symmetry. That means, at a distance one apart from this, there will be another root. That means - 4 will be the second root.

Wow. So, we were able to understand, that -4 and -2 will be the roots of this equation, without even drawing, just on the basis of what we have understood. So, what we have understood here is, -2 and - 4 will take the value 0 and for x is equal to -3, you will get one negative value. And based on that, you have prepared a table. And therefore, you can plot this graph easily. Right?

So, we will graph the related quadratic function, using axis of symmetry and vertex. We have already discussed this. So now, axis of symmetry x = - 3, the roots are - 4 and - 2. And therefore, the Quadratic Equation given here, $x^2 + 6x + 8$ has two real solutions, two real roots. How will the graph look like? It is very easy. We have already imagined the graph. Yes, so this is the graph, where - 4 is a point here and -2 is a point here. -4, - 2 are the roots. And here, it achieves the minimum, which is - 3.

So, you can easily plot this graph. Let us go to the second equation. Now, in this second equation, again we will consider the associated quadratic function. What is the associated quadratic function? $x^2 + 2x + 1$. What will be the axis of symmetry for this? $-\frac{b}{2a}$, that will be - 1. Because b is 2 and a is 1. So $-\frac{b}{2a} = -1$.

So, x = - 1, is the axis of symmetry for this particular quadratic function. Let us substitute the value of x = - 1, in this quadratic function. So, you will get $(-1)^2$, which is 1, 2×-1, which is – 2, + 1. So, you will get 0. Oh! so, -1 itself is a zero. Correct? But that is a point of the vertex, where it achieves the minimum. So, there using axis of symmetry, you can conclude that there cannot be any other point, other than - 1, where it will take the value 0. Because that's the point, where the vertex arises.

That means the axis of symmetry for the second equation is x is equal to -1. a is greater than 0. So, the curve opens up. And therefore, it achieves the minimum. And therefore, the roots are -1 and -1. What is the value at -1? It is 0. So, that is that itself is a root. And therefore, it has only one real root, which is repeated twice. So, in particular, the graph of a function will look like this.

Now, the next problem is very interesting.$x^2 + 1$, where if you compare this with a standard form of the equation, $ax^2 + bx + 1$, then you will get b to be equal to 0. That means this curve or this, the graph of this function will be symmetric about x = 0, that is y axis. And since a is greater than 0, the curve will open upwards. So, the curve is opened up.

Now, a > 0, it will achieve the minimum value. Where it will achieve the minimum value? At the vertex. So, what is the vertex of this particular function? Because x = 0, so, where it, you substitute x is equal to 0 here. So, that value is 1,$x^2 + 1 = 1$. So, (0, 1), so 1 is the minimum value of this function. Can this function be equal to 0 then? It cannot be. So, this will give us the answer, that axis of symmetry x is 0. There are no real roots for this particular function, because it never intersects x axis.

And the function will look like this. So, this in short summarizes, what are possible solutions in any scenario, $ax^2 + bx + c = 0$ is given to you. In particular, if $ax^2 + bx + c$, if you are able to find the vertex and the vertex takes the negative value and a is greater than 0, the curve opens up. So, it will have two roots which are real numbers. If the curve opens up, but the value at the vertex is 0, then it has only one root.

And if the curve opens up and it is above the X-axis, that is it takes a positive value on the vertex, y coordinate of the vertex is positive. Then it will never intersect x axis. In the similar manner, you it is for you to study, that when a is less than 0, what will happen. So, I can give you the rough interpretation. If a is less than 0 and it achieves the maximum on the vertex.

And if that maximum is positive, then it will have two real roots. If a is less than 0 and at the vertex, the value is 0, then it will have a single real root. And if a is less than 0 and it is below

X- axis, then it will have no real roots. So, these are the scenarios, that we can cover using this, this graphing technique.