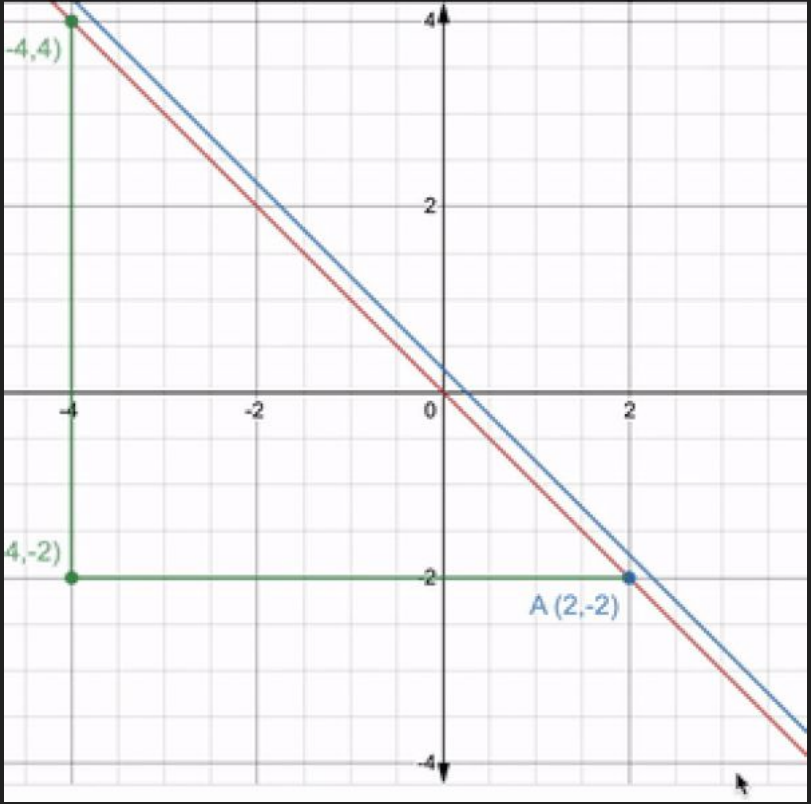
IIT Madras

ONLINE DEGREE

Can slope of a line uniquely determine a line?



Answer: No, it can not uniquely determine the line.

How is the slope useful?

To explore:

- Condition for parallel lines
- Condition for perpendicular lines

Characterization of Parallel Lines via slope

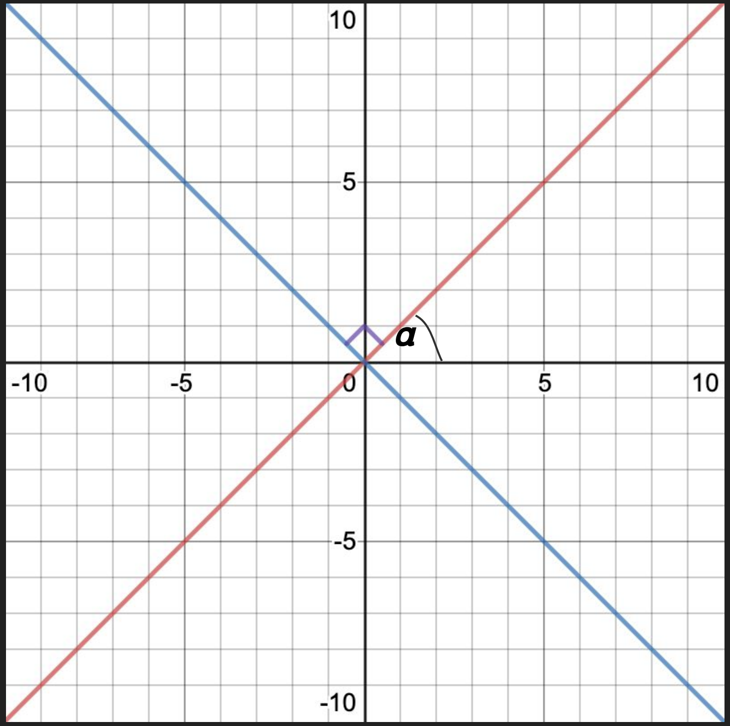
Let l_1 and l_2 be two non-vertical lines with slopes m_1 and m_2 with inclinations α and β respectively.

- If l_1 is parallel to l_2, then $\alpha = \beta$.
- It is clear that $\tan\alpha = \tan\beta$.
- Hence, $m_1 = m_2$.

- Assume $m_1 = m_2$. Then $\tan\alpha = \tan\beta$.
- Since, $0^o \leq \alpha, \beta \leq 180^o$, $\alpha = \beta$.
- Therefore, l_1 is parallel to l_2

Two non-vertical lines l_1 and l_2 are parallel if and only if their slopes are equal.

Characterization of Perpendicular Lines via Slope



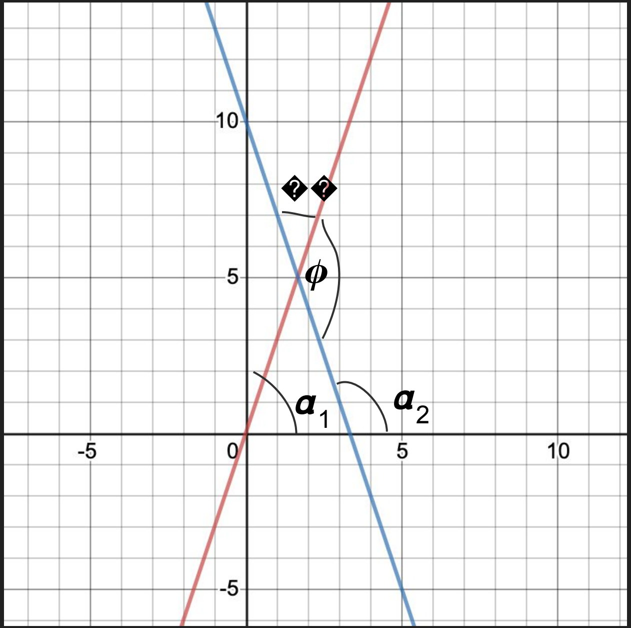
Let $\boldsymbol{l_1}$ and $\boldsymbol{l_2}$ be two non-vertical lines with slopes m_1 and m_2 with inclinations $\boldsymbol{\alpha}$ and $\boldsymbol{\beta}$ respectively.

- If $\boldsymbol{l_1}$ is perpendicular to $\boldsymbol{l_2}$, then 90 + $\boldsymbol{\alpha}$ = $\boldsymbol{\beta}$.
- Now, tan$\boldsymbol{\beta}$ =tan(90 + $\boldsymbol{\alpha}$) =-cot$\boldsymbol{\alpha}$ = -1/tan$\boldsymbol{\alpha}$.
- Hence, m_2= -1/m_1 or m_1m_2= -1.

- Assume m_1m_2= -1. Then tan$\boldsymbol{\alpha}$ tan$\boldsymbol{\beta}$.= -1.
- tan$\boldsymbol{\alpha}$ = -cot$\boldsymbol{\beta}$ = tan(90+$\boldsymbol{\beta}$) or tan (90-$\boldsymbol{\beta}$.).
- Hence, $\boldsymbol{\alpha}$ and $\boldsymbol{\beta}$ differ by 90^o which proves $\boldsymbol{l_1}$ is perpendicular to $\boldsymbol{l_2}$

Two non-vertical lines $\boldsymbol{l_1}$ and $\boldsymbol{l_2}$ are perpendicular if and only if m_1m_2=-1

Relation of Angles between the Two lines and their slopes



Let l_1 and l_2 be two non-vertical lines with slopes m_1 and m_2 with inclinations α_1 and α_2 respectively.

Suppose l_1 and l_2 intersect and let φ and θ be the adjacent angles formed by l_1 and l_2.

Now, $\theta = \alpha_2 - \alpha_1$, for $\alpha_1, \alpha_2 \neq 90^o$

Then,

$$\tan\theta = \tan(\alpha_2 - \alpha_1) = \frac{\tan\alpha_2 - \tan\alpha_1}{1 + \tan\alpha_1 \tan\alpha_2} = \frac{m_2 - m_1}{1 + m_1 m_2}, m_1 m_2 \neq -1.$$

$$\tan\phi = \tan(180 - \theta) = -\tan\theta = \frac{m_1 - m_2}{1 + m_1 m_2}$$