Week-2
Mathematics for Data Science - 2
Limits, Continuity, Differentiability, and the derivative
Graded Assignment

Note: Numbers may differ for some questions, but solution pattern will be the same.

1 Multiple Select Questions (MSQ)

1. Match the given functions in Column A with the equations of their tangents at the origin $(0, 0)$ in column B and the plotted graphs and the tangents in Column C, given in Table M2W2G1.

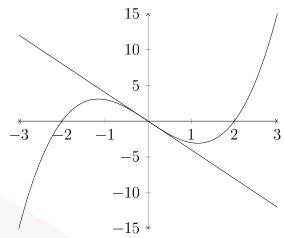
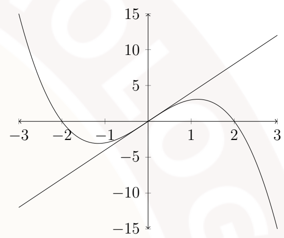
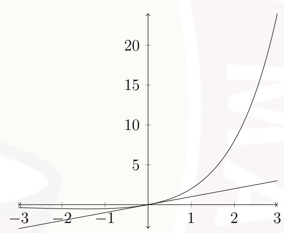
	Function (Column A)		It's tangent at (0,0) (Column B)		Graph (Column C)
i)	$f(x) = x2^x$	a)	$y = -4x$	1)	
ii)	$f(x) = x(x-2)(x+2)$	b)	$y = x$	2)	
iii)	$f(x) = -x(x-2)(x+2)$	c)	$y = 4x$	3)	

Table: M2W2G1

- ◯ **Option 1:** ii) → a) → 1.
- ◯ **Option 2:** i) → b) → 3.
- ◯ Option 3: iii) → b) → 1.
- ◯ **Option 4:** iii) → c) → 2.
- ◯ Option 5: i) → a) → 1.

Solution:
i) Given $f(x) = x2^x \implies f'(x) = 2^x + x2^x \ln 2$. So, $f(0) = 0$ and $f'(0) = 1$.
Hence the equation of the tangent at the origin is

$$y - 0 = 1.(x - 0) \implies y = x.$$

In Column C, figure 3 has the line $y = x$ and exponential graph.
Hence i) $\rightarrow$ b) $\rightarrow$3).
ii) Given $f(x) = x(x-2)(x+2) = x^3 - 4x \implies f'(x) = 3x^2 - 4$.
So, $f(0) = 0$ and $f'(0) = -4$
Hence the equation of the tangent at the origin is

$$y - 0 = -4(x - 0) \implies y = -4x.$$

In Column C, figure 1 has the line $y = -4x$.
Hence ii) $\rightarrow$ a) $\rightarrow$1).
iii) Given $f(x) = -x(x-2)(x+2) = -x^3 + 4x \implies f'(x) = -3x^2 + 4$.
So, $f(0) = 0$ and $f'(0) = 4$
Hence the equation of the tangent at the origin is

$$y - 0 = 4(x - 0) \implies y = 4x$$

In Column C, figure 2 has the line $y = 4x$.
Hence iii) $\rightarrow$ c) $\rightarrow$2).

2. Consider the following two functions $f(x)$ and $g(x)$.

$$f(x) = \begin{cases} \frac{x^3-9x}{x(x-3)} & \text{if } x \neq 0,3 \\ 3 & \text{if } x = 0 \\ 0 & \text{if } x = 3 \end{cases}$$

$$g(x) = \begin{cases} |x| & \text{if } x \leq 2 \\ \lfloor x \rfloor & \text{if } x > 2 \end{cases}$$

Choose the set of correct options.

- ◯ Option 1: $f(x)$ is discontinuous at both $x = 0$ and $x = 3$.
- ◯ Option 2: $f(x)$ is discontinuous only at $x = 0$.
- ◯ **Option 3:** $f(x)$ is discontinuous only at $x = 3$.
- ◯ Option 4: $g(x)$ is discontinuous at $x = 2$.
- ◯ **Option 5:** $g(x)$ is discontinuous at $x = 3$.

Solution:
(Options 1,2,3)
Given

$$f(x) = \begin{cases} \frac{x^3-9x}{x(x-3)} & \text{if } x \neq 0,3 \\ 3 & \text{if } x = 0 \\ 0 & \text{if } x = 3 \end{cases}$$

Now, $\lim\limits_{x\to 0} f(x) = \lim\limits_{x\to 0} \frac{x^3-9x}{x(x-3)} = \lim\limits_{x\to 0} \frac{x(x-3)(x+3)}{x(x-3)} = \lim\limits_{x\to 0} x+3 = 3 = f(0)$.
So $f(x)$ is continuous at $x = 0$.
Similarly, $\lim\limits_{x\to 3} f(x) = \lim\limits_{x\to 3} \frac{x^3-9x}{x(x-3)} = \lim\limits_{x\to 3} \frac{x(x-3)(x+3)}{x(x-3)} = \lim\limits_{x\to 3} x+3 = 6 \neq f(3)$.
So $f(x)$ is not continuous at $x = 3$.
Also observe that $f(x) = \frac{x^3-9x}{x(x-3)}$ if $x \neq 0, 3$, is continuous at all points except at $x = 3$.
Hence $f(x)$ is discontinuous only at $x = 3$.
(Option 5)

Given

$$g(x) = \begin{cases} |x| & \text{if } x \leq 2 \\ \lfloor x \rfloor & \text{if } x > 2 \end{cases}$$

Observe that, as $x > 2$, $g(x) = \lfloor x \rfloor$. And $\lim\limits_{x\to 3^+} g(x) = 3 \neq 2 = \lim\limits_{x\to 3^-} g(x)$, i.e, $\lim\limits_{x\to 3} g(x)$ does not exist.
Hence $g(x)$ is discontinuous at $x = 3$.
(Option 4)
Observe that $\lim\limits_{x\to 2^+} g(x) = \lim\limits_{x\to 2^+} \lfloor x \rfloor = 2$
and $\lim\limits_{x\to 2^-} g(x) = \lim\limits_{x\to 2^-} |x| = 2$.

Hence, $\lim_{x\to 2^+} g(x) = 2 = \lim_{x\to 2^-} g(x)$
i.e., $\lim_{x\to 2} g(x) = 2 = g(2)$.
So $g(x)$ is continuous at $x = 2$.

3. Consider the graphs given below:

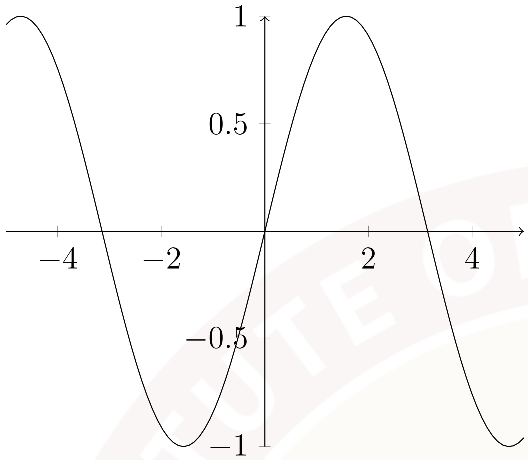


Figure: Curve 1

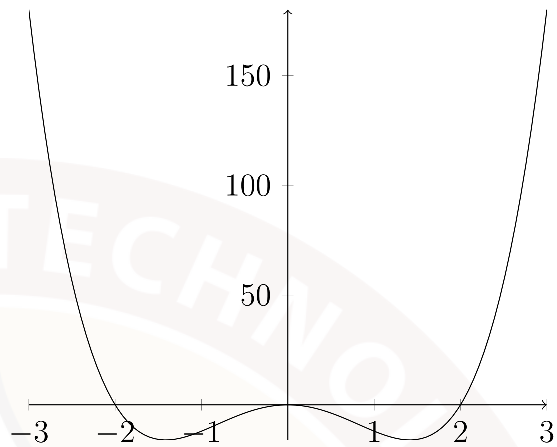


Figure: Curve 2

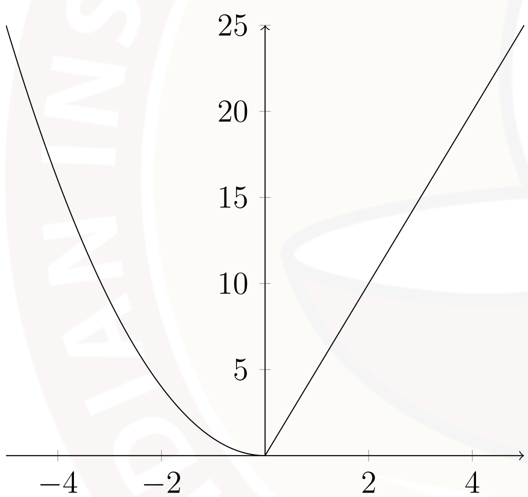


Figure: Curve 3

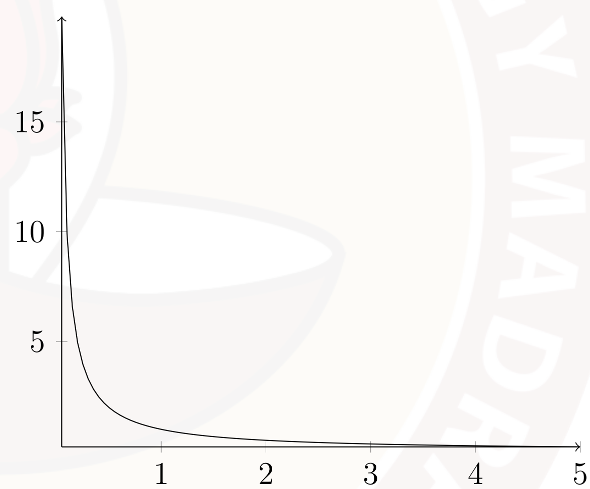


Figure: Curve 4

Choose the set of correct options.

◯ **Option 1:** Curve 1 is both continuous and differentiable at the origin.

◯ Option 2: Curve 2 is continuous but not differentiable at the origin.

◯ **Option 3:** Curve 2 has derivative 0 at $x = 0$.

◯ **Option 4:** Curve 3 is continuous but not differentiable at the origin.

◯ Option 5: Curve 4 is not differentiable anywhere.

◯ Option 6: Curve 4 has derivative 0 at $x = 0$.

Solution:
Option 1: Observe that if x approaches 0 from the left or from the right the value of the function represented by Curve 1 approaches 0. So, the limit of the function exists at $x = 0$ which is 0. Since $f(0) = 0$, the function represented by Curve 1 is continuous at $x = 0$.
We can draw a unique tangent to Curve 1 at the origin as shown in Figure M2W2GS (also observe that at $x = 0$, the graph has no sharp corner).
Hence function is differentiable at $x = 0$.

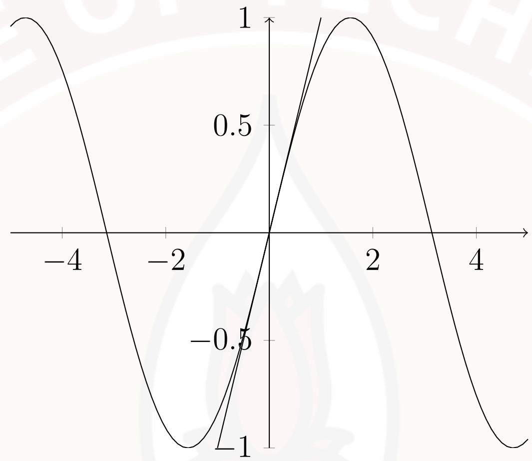


Figure M2W2GS

Options 2, 3: Observe that there is a unique tangent to the curve at the origin which is the $X-$axis itself and we know that slope of the $X-$axis is zero. Hence the function represented by Curve 2 is differentiable at $x = 0$ with derivative 0.
And we know that a differentiable function is continuous.
Hence function represented by Curve 2 is continuous at the origin.

Option 4: Observe that there is a sharp corner on Curve 3 at the origin. So function represented by Curve 3 is not differentiable at the origin.
But if x approaches 0 from the left or from the right the value of the function represented by Curve 3 approaches 0. So, the limit of the function exists at $x = 0$ which is 0. Since the value of the function $f(x)$ is 0 at $x = 0$, the function represented by Curve 3 is continuous at $x = 0$.
Option 6: If the derivative of the function represented by Curve 4 is 0 at the origin then at the origin the slope of of the tangent must be 0 i.e., the tangent must be parallel to the $X-$axis. For Curve 4, the tangent (if exists) at the origin can never be parallel to the $X-$axis. Hence this statement is not true.

Option 5: Observe that at $x = 1$, there does not exist any sharp corner and at that

point, there exists a unique tangent (which is not vertical).
Hence the function represented by Curve 4 is differentiable at $x = 1$.
Hence option 5 is not true.

4. Choose the set of correct options considering the function given below:

$$f(x) = \begin{cases} \frac{sin\ x}{x} & \text{if } x \neq 0, \\ 1 & \text{if } x = 0 \end{cases}$$

- ◯ Option 1: $f(x)$ is not continuous at $x = 0$.
- ◯ **Option 2:** $f(x)$ is continuous at $x = 0$.
- ◯ Option 3: $f(x)$ is not differentiable at $x = 0$.
- ◯ **Option 4:** $f(x)$ is differentiable at $x = 0$.
- ◯ **Option 5:** The derivative of $f(x)$ at $x = 0$ (if exists) is 0.
- ◯ Option 6: The derivative of $f(x)$ at $x = 0$ (if exists) is 1.

Solution:
We know that $\lim\limits_{x \to 0} \frac{\sin x}{x} = 1 = f(0)$. So $f(x)$ is continuous at $x = 0$.
Hence option 2 is true.
Now, $\lim\limits_{h \to 0} \frac{f(0+h)-f(0)}{h} = \lim\limits_{h \to 0} \frac{f(h)-1}{h} = \lim\limits_{h \to 0} \frac{\frac{\sin h}{h} - 1}{h} = \lim\limits_{h \to 0} \frac{\sin h - h}{h^2} = \lim\limits_{h \to 0} \frac{\cos h - 1}{2h} = \lim\limits_{h \to 0} \frac{-\sin h}{2} = 0$
(using L'Hopital's rule twice).
Hence the derivative of $f(x)$ at $x = 0$ is 0.
So options 4 and 5 are true.

5. Let f be a polynomial of degree 5, which is given by

$$f(x) = a_5x^5 + a_4x^4 + a_3x^3 + a_2x^2 + a_1x + a_0.$$

Let $f'(b)$ denote the derivative of f at $x = b$. Choose the set of correct options.

- ◯ **Option 1:** $a_1 = f'(0)$
- ◯ **Option 2:** $5a_5 + 3a_3 = \frac{1}{2}(f'(1) + f'(-1) - 2f'(0))$
- ◯ **Option 3:** $4a_4 + 2a_2 = \frac{1}{2}(f'(1) - f'(-1))$
- ◯ Option 4: None of the above.

Solution:
Given $f(x) = a_5x^5 + a_4x^4 + a_3x^3 + a_2x^2 + a_1x + a_0 \implies f'(x) = 5a_5x^4 + 4a_4x^3 + 3a_3x^2 + 2a_2x + a_1$
So $f'(0) = a_1$, $f'(1) = 5a_5 + 4a_4 + 3a_3 + 2a_2 + a_1$, and $f'(-1) = 5a_5 - 4a_4 + 3a_3 - 2a_2 + a_1$
Hence $5a_5 + 3a_3 = \frac{1}{2}(f'(1) + f'(-1) - 2f'(0))$ and $4a_4 + 2a_2 = \frac{1}{2}(f'(1) - f'(-1))$

2 Numerical Answer Type (NAT)

6. Let f be a differentiable function at $x = 3$. The tangent line to the graph of the function f at the point $(3, 0)$, passes through the point $(5, 4)$. What will be the value of $f'(3)$?
[Answer: 2]
Solution: Slope of the line passing through two points (x_1, y_1) and (x_2, y_2) is $\frac{y_2 - y_1}{x_2 - x_1}$.
So slope of the tangent at $x = 3$ is $\frac{4-0}{5-3} = 2$.
Since derivative of a function at a point equals the slope of the tangent at that point.
Hence $f'(3) = 2$

7. Let f and g be two functions which are differentiable at each $x \in \mathbb{R}$. Suppose that, $f(x) = g(x^2 + 5x)$, and $f'(0) = 10$. Find the value of $g'(0)$. [Answer: 2]

Solution:

Given $f(x) = g(x^2 + 5x) \implies f'(x) = g'(x^2 + 5x)(2x + 5)$

So $f'(0) = 5g'(0) \implies g'(0) = \frac{10}{5} = 2$

3 Comprehension Type Questions:

The population of a bacteria culture of type A in laboratory conditions is known to be a function of time of the form

$$p : \mathbb{R} \to \mathbb{R}$$

$$p(t) = \begin{cases} \frac{t^3-27}{t-3} & \text{if } 0 \leq t < 3, \\ 27 & t = 3 \\ \frac{1}{e^{81}(t-3)}(e^{27t} - e^{81}) & \text{if } t > 3 \end{cases}$$

where $p(t)$ represents the population (in lakhs) and t represents the time (in minutes).
The population of a bacteria culture of type B in laboratory conditions is known to be a function of time of the form

$$q : \mathbb{R} \to \mathbb{R}$$

$$q(t) = \begin{cases} (5t-9)^{\frac{1}{t-2}} & \text{if } 0 \leq t < 2, \\ e^4 & t = 2 \\ \frac{e^{t+2}-e^4}{t-2} & \text{if } t > 2 \end{cases}$$

where $q(t)$ represents the population (in lakhs) and t represents the time (in minutes).
Using the above information, answer the following questions .

8. Consider the following statements (a function is said to be continuous if it is continuous at all the points in the domain of the function). (MCQ)

   - **Statement P:** Both the functions $p(t)$ and $q(t)$ are continuous.
   - **Statement Q:** $p(t)$ is continuous, but $q(t)$ is not.
   - **Statement R:** $q(t)$ is continuous, but $p(t)$ is not.
   - **Statement S:** Neither $p(t)$ nor $q(t)$ is continuous.

   Find the number of the correct statements. [Ans: 1]
   Solution:
   Given

$$p(t) = \begin{cases} \frac{t^3-27}{t-3} & \text{if } 0 \leq t < 3, \\ 27 & t = 3 \\ \frac{1}{e^{81}(t-3)}(e^{27t} - e^{81}) & \text{if } t > 3 \end{cases}$$

   and

$$q(t) = \begin{cases} (5t-9)^{\frac{1}{t-2}} & \text{if } 0 \leq t < 2, \\ e^4 & t = 2 \\ \frac{e^{t+2}-e^4}{t-2} & \text{if } t > 2 \end{cases}$$

   It is enough to check the continuty of $p(t)$ at $t = 3$ and of $q(t)$ at $t = 2$.

So right limit, $\lim_{t\to 3^+} p(t) = \lim_{t\to 3^+} \frac{1}{e^{81}(t-3)}(e^{27t} - e^{81}) = \lim_{t\to 3^+} \frac{27e^{27t}}{e^{81}} = 27$ (Using L'Hopital's rule).
Left limit, $\lim_{t\to 3^-} p(t) = \lim_{t\to 3^-} \frac{t^3-27}{t-3} = \lim_{t\to 3^-} 3t^2 = 27$
Hence, $\lim_{t\to 3^-} p(t) = \lim_{t\to 3^+} p(t) = 27 = p(3)$.
So $p(t)$ is continuous at $x = 3$.

Now right limit, $\lim_{t\to 2^+} q(t) = \lim_{t\to 2^+} \frac{e^{t+2}-e^4}{t-2} = \lim_{t\to 2^+} e^{t+2} = e^4$ (using L'Hopital's rule).
Left limit, $\lim_{t\to 2^-} q(t) = \lim_{t\to 2^-} (5t-9)^{\frac{1}{t-2}}$, to get the left limit,
let $y = (5t-9)^{\frac{1}{t-2}}$.
Taking log with base e on both sides and $t > \frac{9}{5}$,
we get, $\ln y = \frac{\ln(5t-9)}{t-2} \implies \lim_{t\to 2^-} \ln y = \lim_{t\to 2^-} \frac{\ln(5t-9)}{t-2} = \lim_{t\to 2^-} \frac{5}{5t-9} = 5$ (using L'Hopital's rule)
Hence, $\lim_{t\to 2^-} \ln y = 5 \implies \lim_{t\to 2^-} y = e^5$.
So $\lim_{t\to 2^-} (5t-9)^{\frac{1}{t-2}} = e^5$.
Since $\lim_{t\to 2^+} q(t) \neq \lim_{t\to 2^-} q(t)$ i.e., $\lim_{t\to 2} q(t)$ does not exists, $q(t)$ is not continuous at $t = 2$.

9. If $L_p(t) = At + B$ denotes the best linear approximation of the function $p(t)$ at the point $t = 1$, then find the value of $2A + B$. [Ans: 18]

Solution:

$p(t) = \frac{t^3-27}{t-3}$ if $0 \leq t < 3 \implies p(1) = 13$

$p'(t) = \frac{(t-3)(3t^2)-(t^3-27)}{(t-3)^2} \implies p'(1) = 5.$

Therefore the best linear approximation $L_p(t)$ of the function $p(t)$ at the point $t = 1$ is $L_p(t) = p(1) + p'(1)(t-1) = 13 + 5(t-1) = 5t + 8.$

Observe that $A = 5, B = 8,$

So $2A + B = 18.$

10. If $L_p(t) = e^4(At + B) + Ce^5$ denotes the best linear approximation of the function $q(t)$ at the point $t = 3$, then find the value of $A + B + C$. [Ans: -2]

Solution:

$q(t) = \frac{e^{t+2}-e^4}{t-2}$ if $t > 2 \implies q(3) = e^5 - e^4$

$q'(t) = \frac{(t-2)e^{t+2}-(e^{t+2}-e^4)}{(t-2)^2} \implies q(3) = e^4$

Therefore the best linear approximation $L_q(t)$ of the function $q(t)$ at the point $t = 3$ is

$L_q(t) = q(3) + q'(3)(t-3) = e^5 - e^4 + e^4(t-3) = e^4t + e^5 - 4e^4 = e^4(t-4) + e^5$.

Observe that $A = 1, B = -4, C = 1$,

So $A + B + C = -2$.

11. Consider a function $f : \mathbb{R} \to \mathbb{R}$ defined as

$$f(x) = \begin{cases} \frac{\sin 14x + A\sin x}{19x^3} & \text{if } x \neq 0, \\ B & \text{if } x = 0. \end{cases}$$

If $f(x)$ is continuous at $x = 0$, then find the value of $114B - A$. [Ans: -2716]

Solution:

Given that the function is continuous that at $x = 0 \implies \lim_{x\to 0} f(x) = f(0) = B$.

$\lim_{x\to 0} f(x) = \lim_{x\to 0} \frac{\sin 14x + A\sin x}{19x^3} = \lim_{x\to 0} \frac{14\cos 14x + A\cos x}{57x^2}$ (using L'Hopital's rule)

Observe that $\lim_{x\to 0} \frac{14\cos 14x + A\cos x}{57x^2}$ exist, if $(14\cos 14x + A\cos x) \to 0$ and $(57x^2) \to 0$ as $x \to 0$

Now, $14\cos 14x + A\cos x \to 0$ as $x \to 0 \implies 14 + A = 0 \implies A = -14$

So $\lim_{x\to 0} \frac{14\cos 14x + A\cos x}{57x^2} = \lim_{x\to 0} \frac{14\cos 14x - 14\cos x}{57x^2} = \lim_{x\to 0} \frac{-196\sin 14x + 14\sin x}{114x}$ (using L'Hopital's rule)

Now, $\lim_{x\to 0} \frac{-196\sin 14x + 14\sin x}{114x} = \lim_{x\to 0} \frac{-2744\cos 14x + 14\cos x}{114} = \frac{-2744+14}{114} = \frac{-2730}{114}$ (using L'Hopital's rule)

So $B = \frac{-2730}{114}$

Hence $114B - A = -2716$.

12. The distance (in meters) traveled by a car after t minutes is given by the function $d(t) = g(4t^3 + 2t^2 + 5t + 2)$, where g is a differentiable function with domain $\mathbb{R}$. Find the instantaneous speed of the car after 5 min, where $g'(577) = 2$. [Ans: 650]

Solution:

The instantaneous speed of the car after t min $= d'(t) = g'(4t^3+2t^2+5t+2)(12t^2+4t+5)$.
(use derivative property of composition of two functions)
So the instantaneous speed of the car after 5 min $= g'(577) \times 325 = 2 \times 325 = 650$

13. Consider the following two functions

$$p : \mathbb{R} \to \mathbb{R}$$

$$p(t) = \begin{cases} \frac{2e^{(t-2)}-2}{t-2} & \text{if } 0 \leq t < 2, \\ 2 & t = 2 \\ 2(t^2-4)^{\frac{1}{\ln(t-2)}} & \text{if } t > 2 \end{cases}$$

and

$$q : \mathbb{R} \to \mathbb{R}$$

$$q(t) = |t(t-7)(t-8)|$$

and the following statements (a function is said to be continuous (respectively differentiable) if it is continuous (respectively differentiable) at all the points in the domain of the function).

- **Statement P:** Both the functions $p(t)$ and $q(t)$ are continuous.
- **Statement Q:** Both the functions $p(t)$ and $q(t)$ are not differentiable.
- **Statement R:** $p(t)$ is continuous, $q(t)$ is differentiable.
- **Statement S:** $q(t)$ is continuous, $p(t)$ is not differentiable.
- **Statement T:** Neither $p(t)$ nor $q(t)$ is continuous.

Find the number of correct statements. [Ans : 2]

Solution:

Right limit of $p(t)$ at 2, $\lim_{t\to 2^+} p(t) = \lim_{t\to 2^+} 2(t^2-4)^{\frac{1}{\ln(t-2)}} = 2\lim_{t\to 2^+}(t^2-4)^{\frac{1}{\ln(t-2)}}$

Let $y = (t^2-4)^{\frac{1}{\ln(t-2)}}$
taking ln both sides,
$\ln y = \ln(t^2-4)\frac{1}{\ln(t-2)}$
Now, $\lim_{t\to 2^+} \ln y = \lim_{t\to 2^+} \frac{\ln(t^2-4)}{\ln(t-2)} = \lim_{t\to 2^+} \frac{2t(t-2)}{(t+2)(t-2)} = 1$ (using L'Hopital's rule)
So as $t \to 2^+$, $y \to e^1$
hence $\lim_{t\to 2^+} p(t) = \lim_{t\to 2^+} 2(t^2-4)^{\frac{1}{\ln(t-2)}} = 2e^1 = 2e \neq 2 = p(2)$
So function $p(t)$ is not continuous and so $p(t)$ is not differentiable.

Now, consider the function $q(t)$,

$$q(t) = |t(t-7)(t-8)| = \begin{cases} -t(t-7)(t-8) & \text{if } t < 0, \\ t(t-7)(t-8) & \text{if } 0 \leq t < 7, \\ -t(t-7)(t-8) & \text{if } 7 \leq t < 8, \\ t(t-7)(t-8) & \text{if } t \geq 8, \end{cases}$$

So discontinuity can be possible at $x = 0, 7, 8$ but
observe that $\lim\limits_{t\to 0^-} q(t) = \lim\limits_{t\to 0^+} q(t) = q(0)$,
$\lim\limits_{t\to 7^-} q(t) = \lim\limits_{t\to 7^+} q(t) = q(7)$
and $\lim\limits_{t\to 8^-} q(t) = \lim\limits_{t\to 8^+} q(t) = q(8)$.
Hence $q(t)$ is continuous.

For differentiabilty of $q(t)$,
observe that left derivative,
$\lim\limits_{h\to 0^-} \frac{q(0+h)-q(0)}{h} = \lim\limits_{h\to 0^+} \frac{q(-h)-0}{-h} = \lim\limits_{h\to 0^+} \frac{-(-h)(-h-7)(-h-8)-0}{-h} = -56$
and right derivative
$\lim\limits_{h\to 0^+} \frac{q(0+h)-q(0)}{h} = \lim\limits_{h\to 0^+} \frac{q(h)-0}{h} = \lim\limits_{h\to 0^+} \frac{h(h-7)(h-8)-0}{h} = 56$.
So, Left derivative $\neq$ Right derivative.
Hence $q(t)$ is not differentiable.

14. Consider the following function

$$p : \mathbb{R} \to \mathbb{R}$$

$$p(t) = \begin{cases} \frac{2e^{(t-2)}-2}{t-2} & \text{if } 0 \leq t < 2, \\ 2 & t = 2 \\ 2(t^2-4)^{\frac{1}{\ln(t-2)}} & \text{if } t > 2 \end{cases}$$

If linear function $L_p(t) = At + B$ denotes the best linear approximation of the function $p(t)$ at the point $t = 1$, find the value of $\frac{-2}{e^{-1}-1}(A+B)$. [Ans: 4]

Solution:

Observe that $p(t) = \frac{2e^{(t-2)}-2}{t-2}$ if $0 \leq t < 2$.

Linear approximation of the $p(t)$ at $t = 1$ is $L_p(t) = p'(1)(t-1)+p(1) = p'(1)t-p'(1)+p(1)$

So here $A = p'(1), B = -p'(1) + p(1)$.

Therefore $A + B = p(1)$

Hence $\frac{-2}{e^{-1}-1}(A+B) = \frac{-2}{e^{-1}-1}p(1) = 4$

15. Consider the following function

$$q : \mathbb{R} \to \mathbb{R}$$

$$q(t) = |t(t-7)(t-8)|.$$

If m is slope of the tangent of the function $q(t)$ at point $t = \frac{3}{2}$, find the value $m - \frac{27}{4}$.
[Ans: 11]

Solution:

From question 13, observe that $q(t) = t(t-7)(t-8) = t^3 - 15t^2 + 56t$ if $0 \leq t < 7$.
So $q'(t) = 3t^2 - 30t + 56 \implies q'(\frac{3}{2}) = \frac{27}{4} - 45 + 56 = \frac{27}{4} - 11$.
Now, slope of the tangent of the function $q(t)$ at point $t = \frac{3}{2}$ is $q'(\frac{3}{2})$.
Hence $m = q'(\frac{3}{2})$.
So $m - \frac{27}{4} = 11$