IIT Madras

ONLINE DEGREE

Statistics for Data Science – 1
Professor. Usha Mohan
Department of Management Studies
Indian Institute of Technology, Madras
Lecture No. 7.6
Conditional Probability - Independent events: properties

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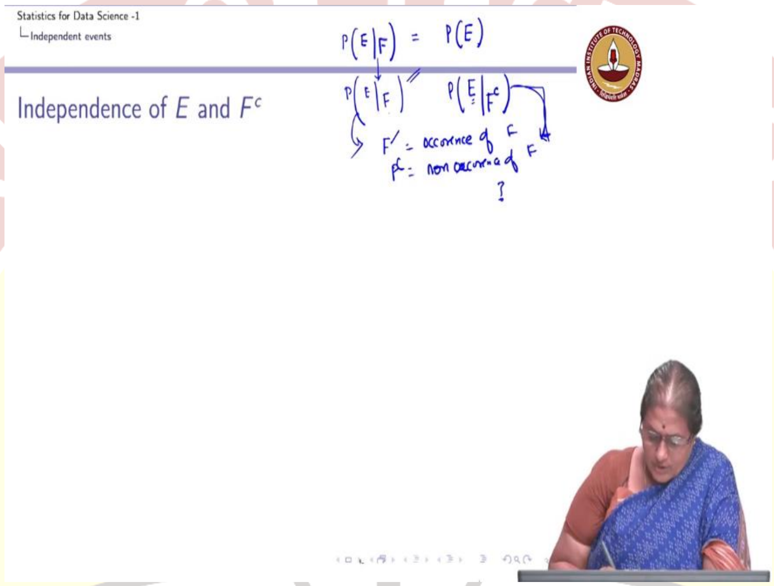

So, the question now we want to ask is, now independence means that the conditional probability of a event E happening conditioned on the occurrence of F is equal to the unconditional probability of an event happening. Recall this says that E conditioned on the occurrence of F, so the natural question to ask is what would happen with if E is conditioned on occurrence of F is does not affect the unconditional probability of E, then what can you say about the probability of this event happening on the non occurrence of this event F.

Recall F is occurrence of event F, so F^c is non occurrence of event F, F is the occurrence of event F andF^c is the non occurrence of event F, so the question we are asking is if a event is conditioned on the occurrence of the event F and this is the event E conditioned on the nonoccurrence of event F. So, the question that is being asked is if E and F are independent can I say that E and F^c are also independent? That is the question, in other words if the occurrence of E is independent of the occurrence of F can I say that the occurrence of E is independent of the non occurrence of F also? So, that is the question that is being asked.

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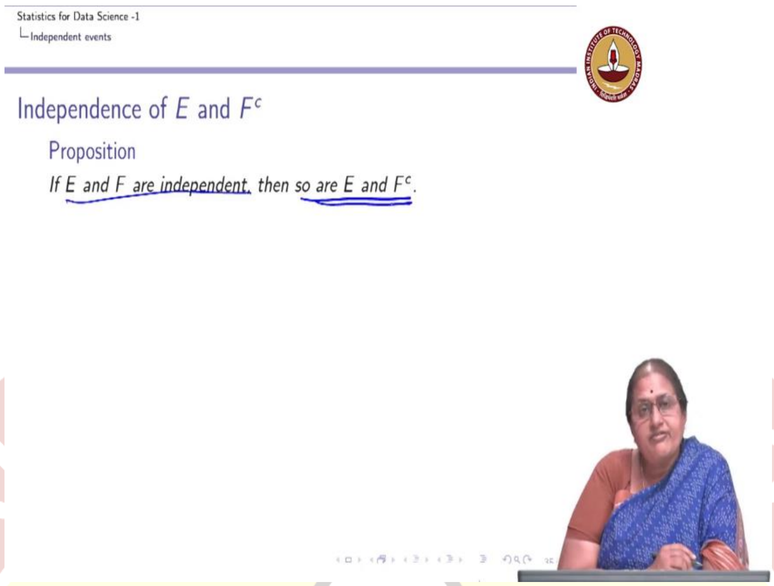

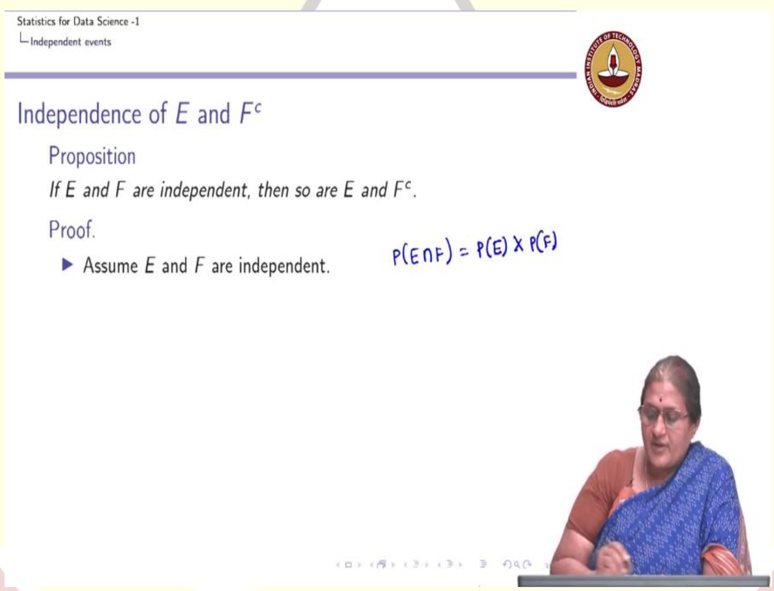

And the proposition is if E and F are independent then so are E and F^c. So, this requires a very simple proof. So, we can see that E and F are, let us assume E and F are independent, so if E and F are independent we have seen that $P(E \cap F) = P(E)P(F)$, so that is what we can see.

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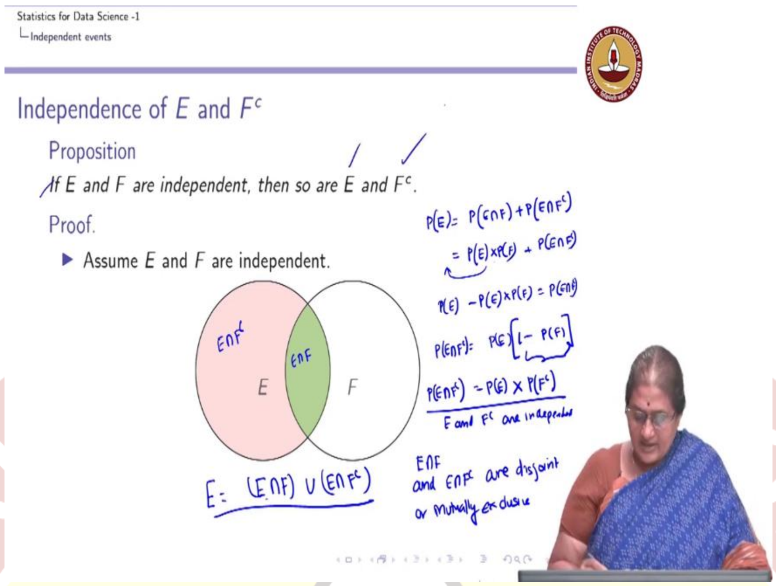

So, now you can look at this Venn diagram this green area is my$(E \cap F)$, I can write the pink area as $(E \cap F^c)$, so that I can write my E as $[(E \cap F) \cup (E \cap F^c)]$. Notice that $(E \cap F)$ and $(E \cap F^c)$ are disjoint or mutually exclusive.

Now, once I write E as $[(E \cap F) \cup (E \cap F^c)]$ I can apply my addition law to the disjoint sets to get $P(E) = [P(E \cap F) + P(E \cap F^c)]$. Now, I apply $[(E \cap F)]$, I apply my multiplication rule for independent events, $P(E \cap F) = P(E)P(F)$, I want to, I retain probability of $P(E \cap F^c) =$ the same way.

Now, I take this term to my left hand side, I have $P(E) - P(E)P(F) = P(E \cap F^c)$. Hence, I can write $P(E \cap F^c)$ is $P(E)[1 - P(F)]$. We know $1 - P(F)$ is $P(F^c)$ and hence I get $P(E \cap F^c) = P(E)P(F^c)$ and this tells me that E and F^c are independent. So, we can see that if E and F are independent so are E and F^c.

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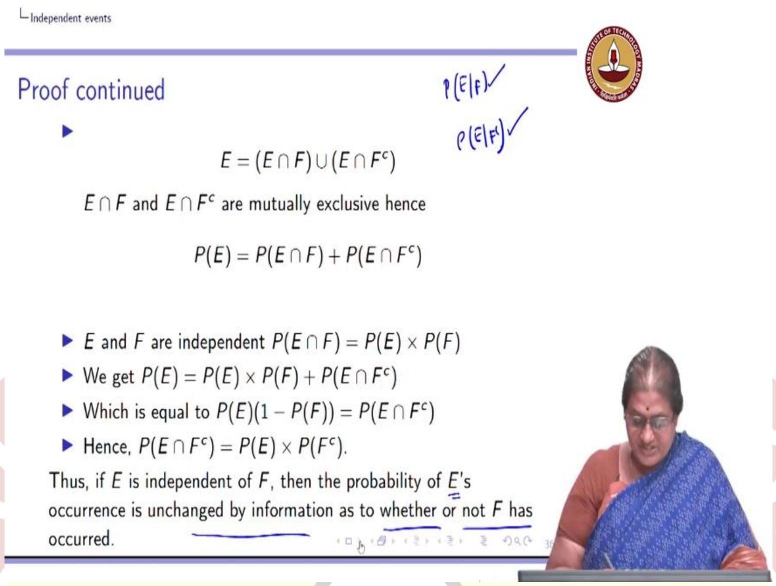

So, this is the proof which we have, we have just discussed, we have $P(E \cap F^c) = P(E)P(F^c)$. So, what this entails is the probability of E's occurrence is unchanged by the information as to whether F has occurred or F has not occurred. So, $P(E|F)$ or $P(E|F^c)$ whether F has occurred or F has not occurred if E and F are independent the probability of E's occurrence is unchanged, that is what this means.

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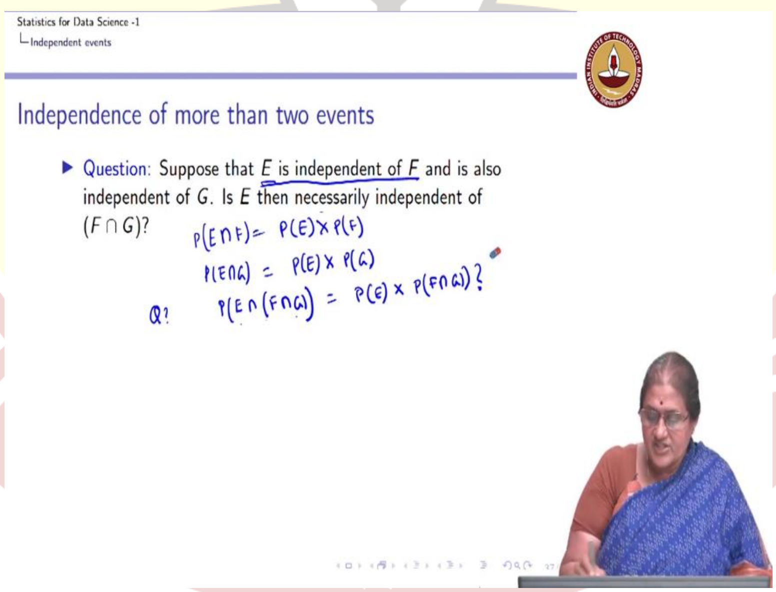

So, now we said that independence of 3 or more events is slightly more complicated than just discussing independence of 2 events. When we talked of independence of 2 events we said that , if I have 2 events probability of E and F, if probability of E is, if the probability of the intersection is equal to the product of probabilities then I can say E and F are independent and the converse is also true that if E and F are independent then the product; probability of the intersection as the product of the probabilities, this is what we have for 2 events.

Now, suppose I have, I want to extend this notion of multiplication or intersection of events to more than 2 events. So, let us, the question that is being asked is suppose E is independent of F, given that E is independent of F, so I know the $P(E \cap F) = P(E)P(F)$ this is what is given to us, E is also independent of G, so I have $P(E \cap G) = P(E)P(G)$, then the question we are asking is, is E necessarily independent of $F \cap G$.

So, I am asking if $P(E \cap (F \cap G)) = P(E)P(F \cap G)$, this is the question we are asking. So, let us look at it. So, this is an intersection E intersection F intersection G, this is the question we are asking. And we answer this question through a example, we are not going into the detailed mathematical implication here but we would like to establish the multiplication rule for more than 3 events through an example.

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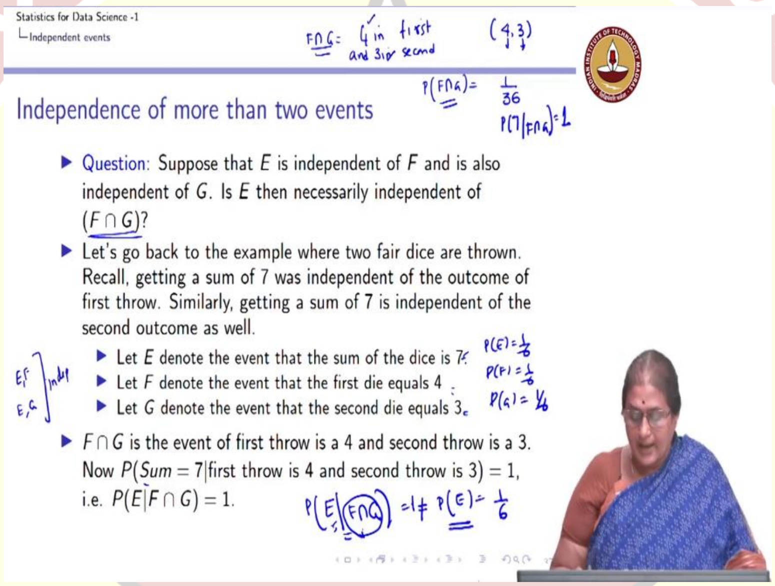

So, now let us recall this example. I rolled 2 fair dice, you also recall that we defined an event E, now let me define the event E is sum of 7 in the independent throw, so I am rolling a die twice and we saw that the sum of 7 probability of E was $1/6$ this is something which we saw, I had 6 out of 36 which is equal to $1/6$.

In other terms we saw that getting a sum of 7 is actually independent of whether your first throw was a 1 or a 2 or a 3 or a 4 or a 5 or a 6, the sum of 7 is independent of what your first throw was, I can use a similar logic to say that a sum of 7 is independent of what my second throw is also. So, if I define my events in the following way E is the event that the sum of dice is equal to 7 we know $P(E)$ is $1/6$, let me define F to be the event that the first dice equals a 4 again $P(E)$ is $1/6$, $P(F)$ is again 1 first, first dice is equal to 4 or the first outcome is a 4, again this is a $1/6$.

This G is the event that the second dice equals a 3, so $P(E)$is $1/6$, $P(F)$ is $1/6$, $P(G)$ is $1/6$, I know that E and F are independent because the sum is equal to 7 is independent of what was your first choice, the sum yet is equal to 7 is similarly independent of what is your second choice, so E and F are independent. Similarly E and G are also independent. Now, let us look at what is the event $(F \cap G)$.

So, now if I have defined all these events, the event$(F \cap G)$ is the event of getting a 4 in your first throw and a 3 in the second throw. So, the event $(F \cap G)$ G corresponds to the outcome 4 and 3 and I know that the $P(F \cap G)$is again $1/36$. This is the probability of F intersection G.

Now, the question that is being asked is what is the chance of E happening given the event $(F \cap G)$ has happened, I know that, if I know that this event $(F \cap G)$ has happened that is the first throw is a 4 and second throw is a 7 is 3, I know that the chance of me getting a 7 is given $(F \cap G)$ has happened is 1, given 4 and 3 has happened the sum equal to 7 is equal to 1. In other words, $P\big(E\big|(F \cap G)\big) = 1$. But I also know $P(E)$ is 1/ 6. So, we can see that the unconditional probability of E conditioned on this event F intersection G is not equal to the unconditional probability of E happening.

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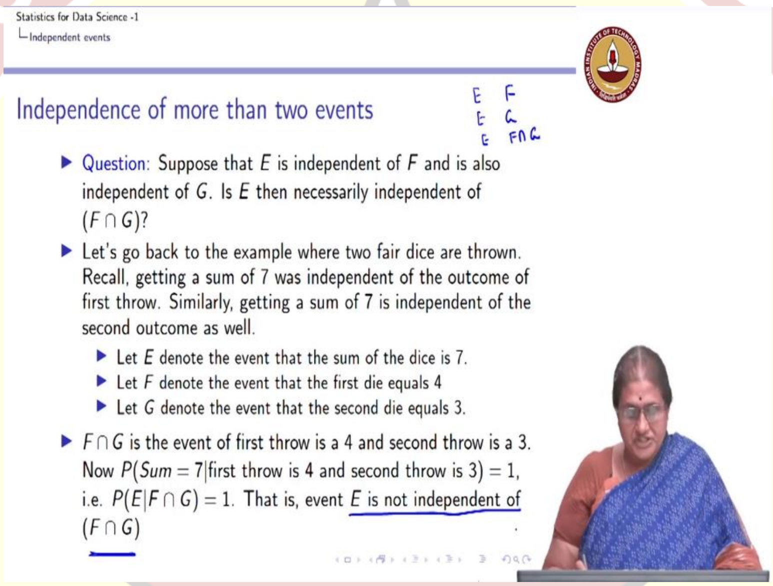

Hence, we can say that event E is not independent of event $(F \cap G)$. So, we have that, even though I have E is independent of F and E is independent of G, we have a condition where E is not independent of $(F \cap G)$.

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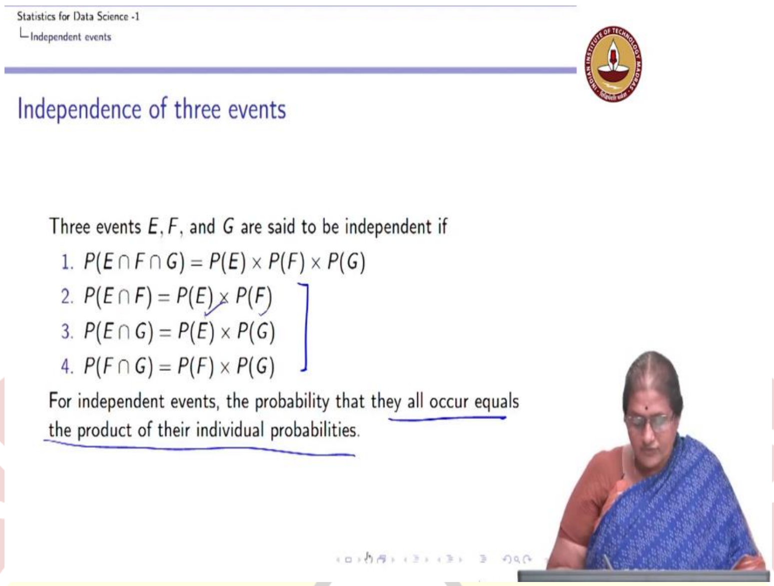

So, this helps us actually come up with the rule for 3 events to be independent, we are only stating the rule here, explaining and proving it is beyond the scope of this course but I prove, I state the independence of 3 events rule the following way. 3 events are said to be independent if probability of the intersection equal to the product of the probabilities, not only that I need to check the pairwise probabilities $P(E \cap F)$ is $P(E)P(F)$, $P(E \cap G)$ is $P(E)P(G)$, $P(F \cap G)$ is $P(F)P(G)$. If these 4 happen then I say the events E, F and G are independent events.

So, for independent events the probability that they all occur equals the probability of their individual probabilities and these probabilities which are looking at pairwise intersections that probability of pairwise intersection should be the product of the individual probabilities.

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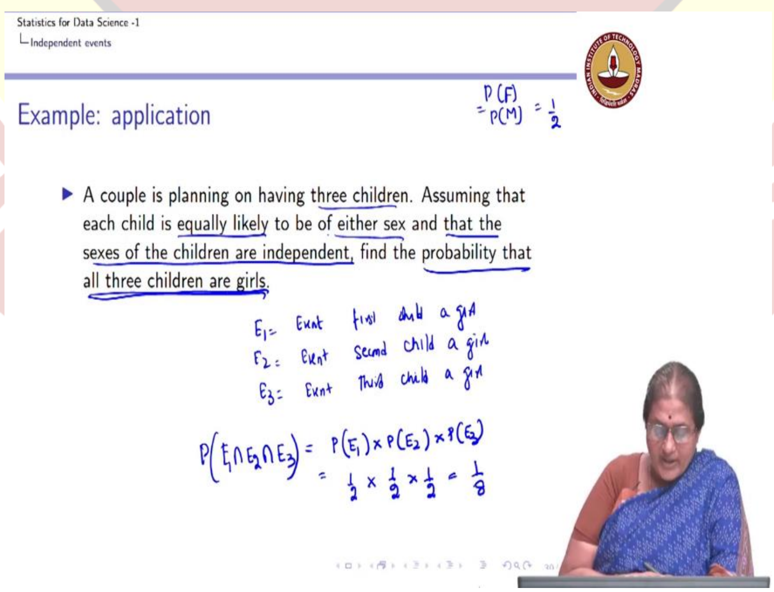

So, let us look at an application of the multiplication rule for more than 3 events. So, a couple is planning on having 3 children. Assuming that each child is likely to be of either sex, I am assuming female and male and the probability of female is equal to the probability of male equal to half that is what we mean by equally likely to be of either sex and that the sexes of the children are independent, then find the probability that all the 3 children are girls.

Now, let us define the events, the events are let us define E_1 to be the event that the first child is a girl, E_2 is the event second child is a girl, similarly, E_3 is the event third child is a girl. Now, the probability, now the event that all 3 children are girls are is the intersection, the event that all 3 children are girls are the intersection of these 3 events and what we require to find is the probability of this intersection.

So, applying our multiplication rule I know that probability if they are independent I know this is equal to $P(E_1)P(E_2)P(E_3)$. What is $P(E_1)$? I know $P(E_1)$=1/2, $P(E_2)$=1/2, $P(E_3)$=1/2 giving me a probability of 1/8.

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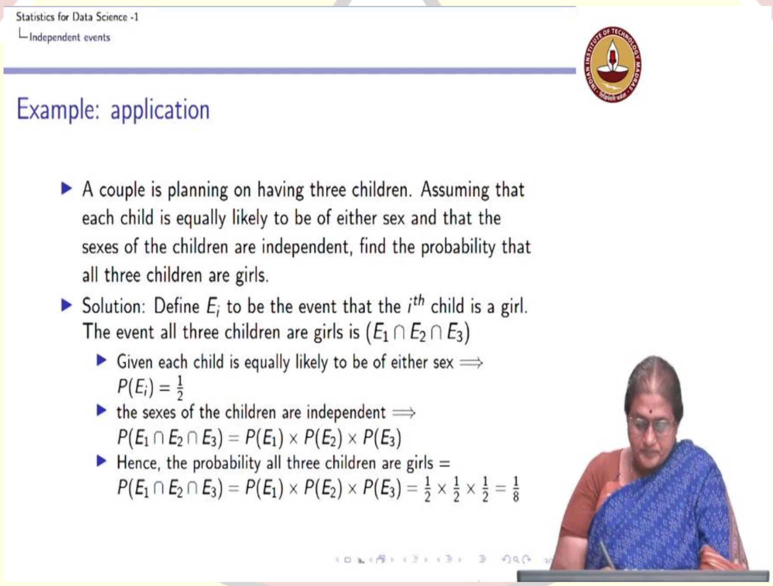

So, I have that the event the probability of all girls are independent is 1/2 into 1/2 into 1/2 which is equal to 1/8.

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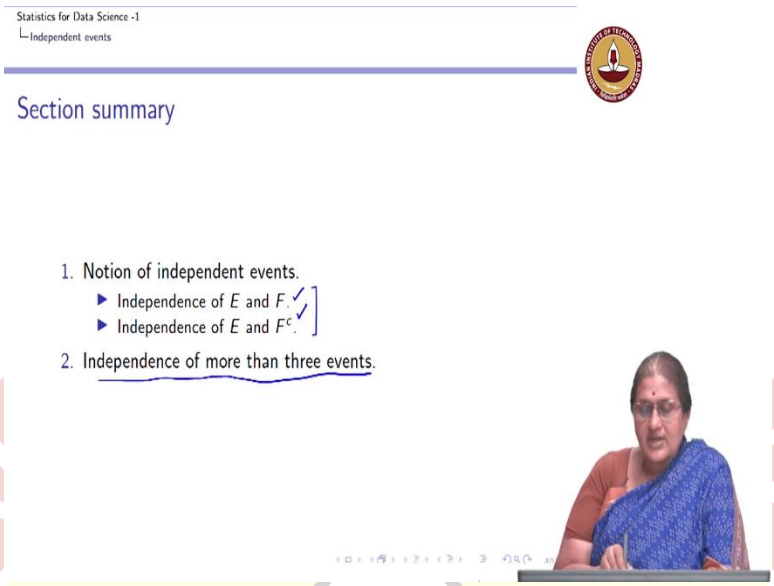

So, what we have seen so far is we introduce the important notion of independent events, we looked at the independence of E and F and E and F^c namely the occurrence of event E, given occurrence of event F and namely the independence of event E conditioned on occurrence of F, independence of event E conditioned on non occurrence of event F and we also extended the notion of independence to more than 3 events.