IIT Madras

ONLINE DEGREE

Statistics for Data Science 1
Professor. Usha Mohan
Department of Management Studies
Indian Institute of Technology, Madras
Week 7 Tutorial 4

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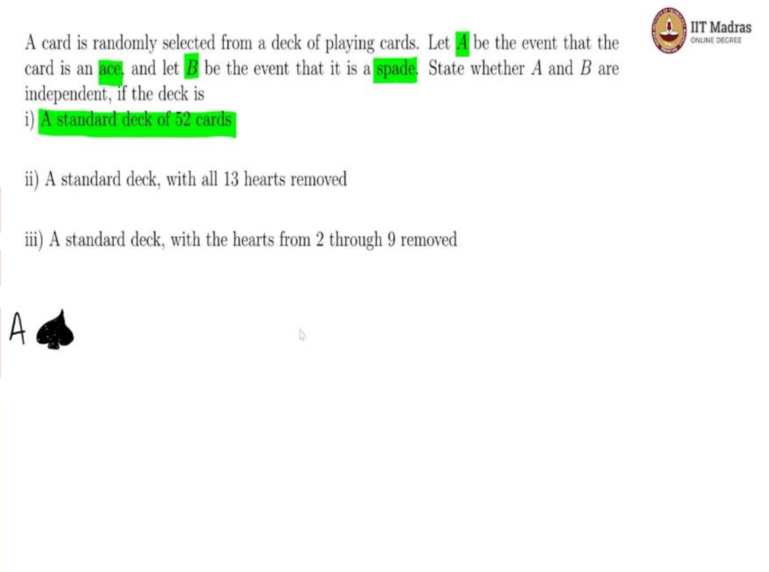

A card is randomly selected from a deck of playing cards, let A be the event that this card is an ace. And let B be the event that this card is a spade. So, if the card is a spade, then B is true. And if the card is an ace, then A is true. So, both are true when if it is the ace of spades. I do not know if you are familiar with the nomenclature of cards. So, spade is this particular symbol, which looks something like this filled with black.

So, the ace of spades would be both A and B being true, any other card of spades B would be true, but A would not be true. And any other suit, an ace A would be true, but not B. Now, state whether A and B are independent if the deck is a standard deck of 52 cards. Now, the condition for independence is that the probability of one event should not change whether or not the other event has taken place, which means P(A) given that B happened should be the same thing as P(A). And likewise, P(B) given that A happened should be the same thing as P(B).

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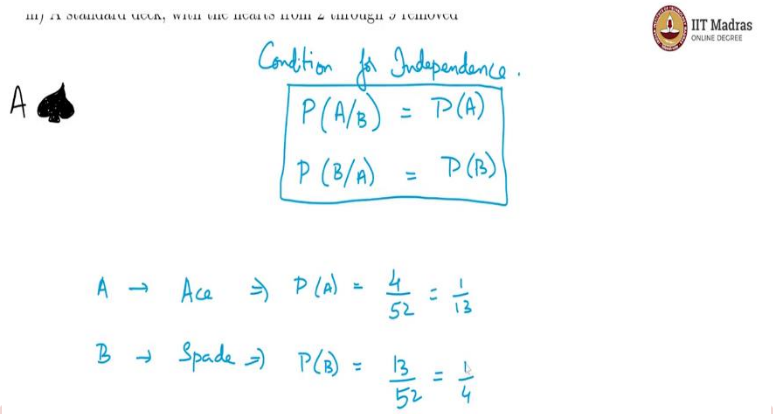

Now, our events are such that A is an ace the card being an ace, whereas B is the card being a spade. So, this would give us this is the condition for independence. This would indicate that P(A) that is the number of the probability of the card being an ace would be 4 by 52 in a standard deck, there are 52 cards in a standard deck and of them for races. So, that is 1/13. And P(B) is 13 by 52. There are 13 spades out of 52 cards, so you have 1 by 4 as a probability for a spade coming up.

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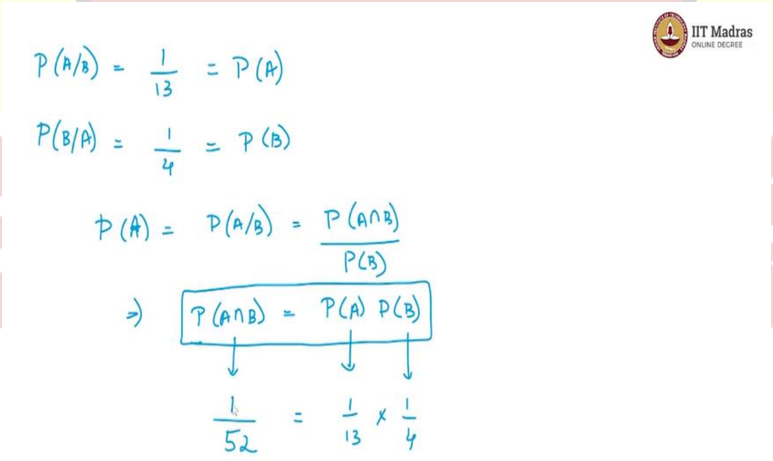

Now what is P(A) given B so in this case there is we know that the card is already a spade, so there are 13 spades, so your denominator is 13. And how many aces are there among the spades there is only 1 this is 1/13, which is equal to P(A). Likewise, if you did P(B) given A if you know your card is already an ace, there are 4 aces, how many spades would be there among these aces only 1 and that is equal to P(B).

So, that condition for independence is satisfied and we can further simplify this by saying that since P(A) is equal to P(A) given B, which is incidentally equal to P(A) intersection B divided by P(B), this condition can be simply written as P(A) intersection B of both of them happening together is simply the product of these two probabilities.

So, if this condition is satisfied, we can say that the events are independent and what is the condition here what is to be satisfied P(A) given A intersection B is both of them happening together. There is only one ace of spades overall. So that will give you 1 by 52 and P(A) is 1/13 P(B) is 1 by 4. So, their product is 1 by 52. So therefore, independence is satisfied.

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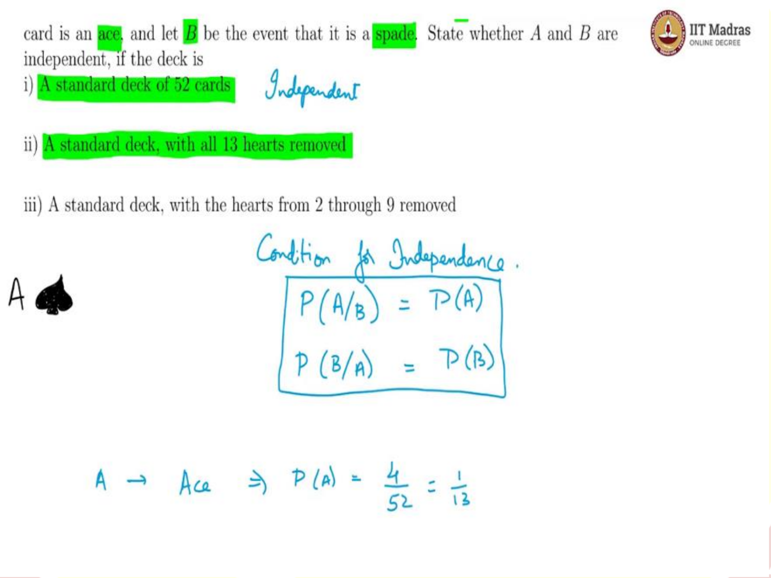

So, here standard deck when this happens, the two events are independent. Now, we look at a standard deck with all the 13 hearts removed. With all the 13 hearts removed, we will have to modify these let see if the conditions are still satisfied.

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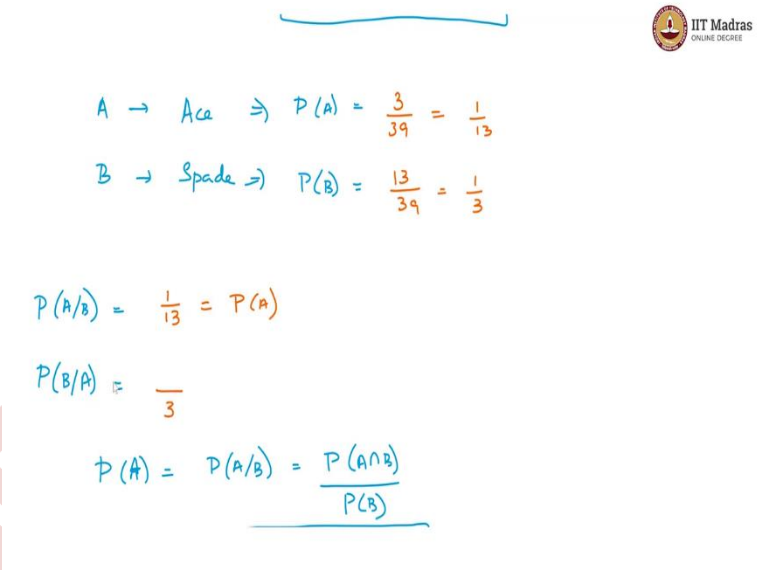

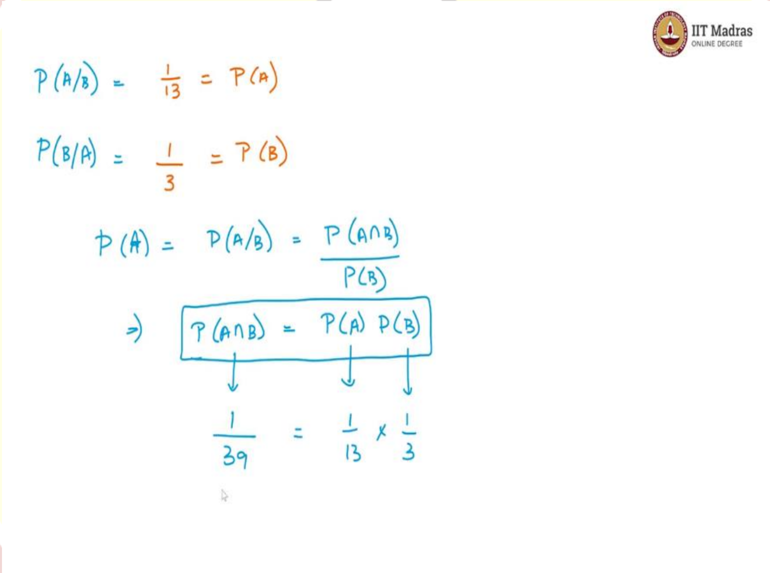

All the hearts are removed, which means probability of an ace coming up will be 3, but there are only 39 cards left. So, this is equal to 1/13 again. And what is the probability of spades there are now 13 spades out of 39. So that will give us 1/ 3 is P(B), then P(A) given B is basically given that it is a spade how many aces.

So, that would again be 1/13 which is equal to P(A). And what is P(B) given A this would be there are only 3 aces now, and how many of them would be spared only 1. So, this is equal to P(B), and again our condition is satisfied, except in this case, P(A) intersection B is going to be 1/39 there is one ace of spades in the 39 cards and P(A) is 1/13 and P(B) is 1/3 and again 1/13 and 1/3, 1/39. So, the condition is satisfied.

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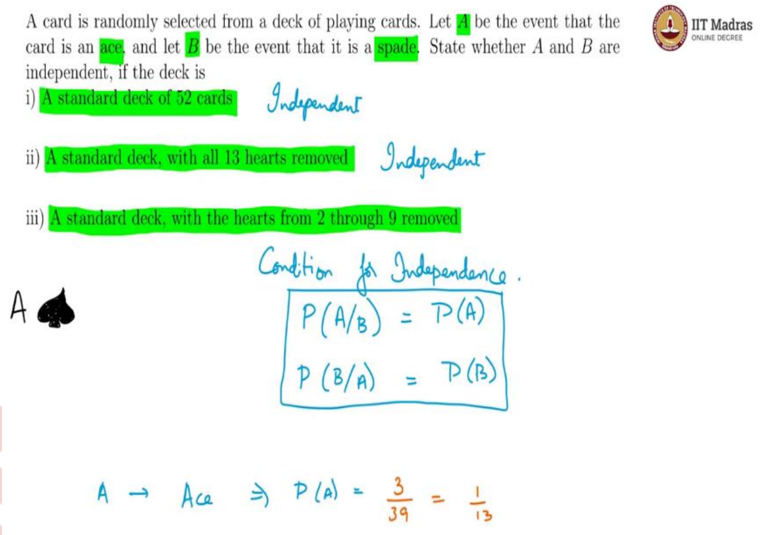

So, we can again claim that the second case also these two events will be independent. Now, let us look at the third case a standard deck with hearts from 2 through 9 removed that is heart 2, heart 3, heart 4, hearth 5, heart 6, heart 7, heart 8 and heart 9 are remote, which means 8 cards are removed.

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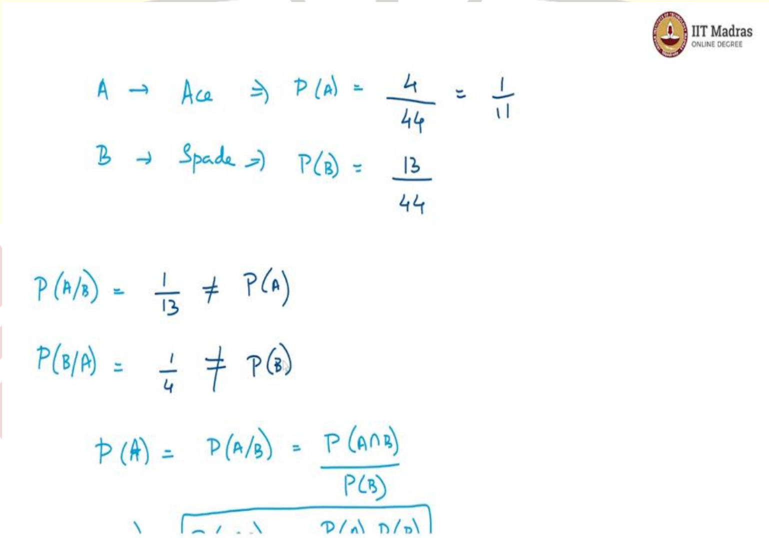

Now, let us see what happens to these probabilities. 8 cards are removed which means there are now 44 cards in the deck and among them how many aces are there are 4 aces, because the ace of hearts is not removed. So, you will get 1 by 11 is P(A). Now, what is P(B) of these 44 cards how many spades are there 13. So, 13 by 44 is P(B).

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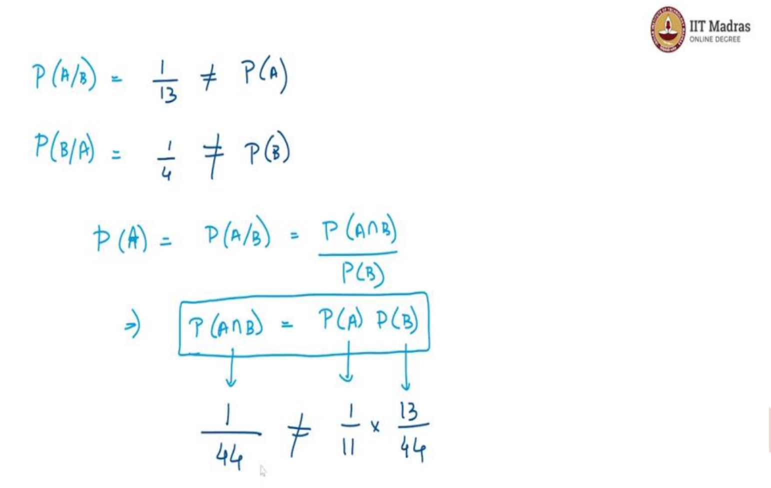

Now let us see what happens with P(A) given. So given that the card is a spade, how many of them is an ace. So, you have 1/13 here directly, but 1/13 is not equal to P(A) likewise P(B) given A. So, if it is an ace what is the possibility probability that it is a spade, that is again 1 by 4 because there are 4 aces and only 1 spade among them. And this is again not equal to P(B). If you check for this condition now P(A) intersection B is basically one card out of that is spade… aces spades is one card out of your 44 whereas, P(A) is basically 1/11 and P(B) is 13/44 and this product is not equal to 1/44.

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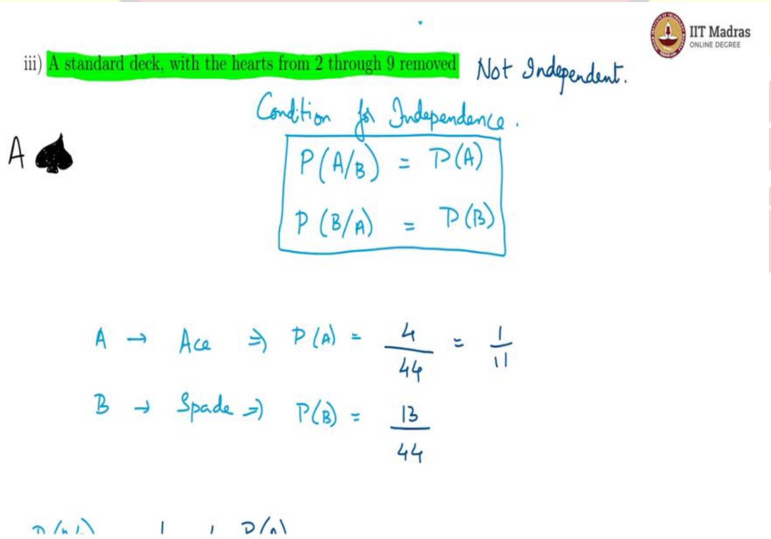

Therefore, in this case the two events are not independent.