IIT Madras

ONLINE DEGREE

Mathematics for Data Science 1
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Lecture-1.7B
Function: Examples

So, let us take a closer look at functions now.

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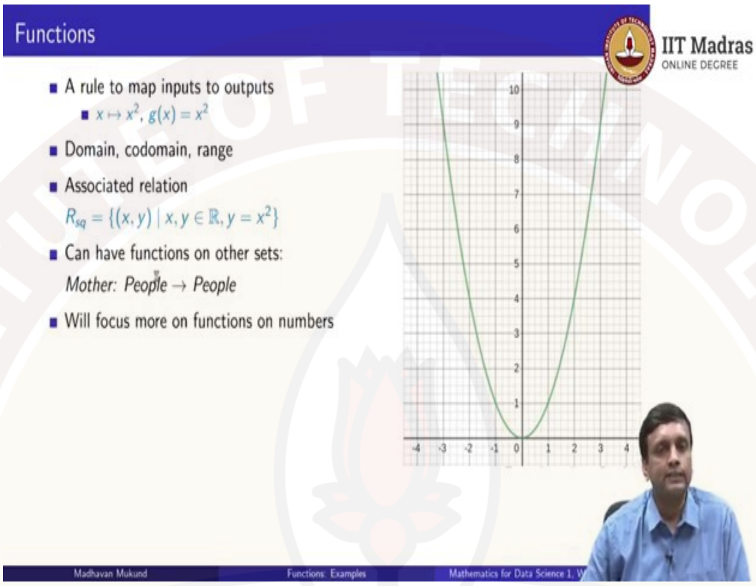

So, remember that a function is a rule that map's inputs to outputs. So for instance, if we are looking at numbers, a function could take an input x and map it to x^2, which can also write given a name saying $g(x)$ is equal to x^2, which says g is the name of a function, which when it takes an input of the form x produces an output of the form x^2.

And with such functions, we have a notion of a domain that is what are the inputs that are allowed, the set from which we take inputs. Codomain, what is the set to which the outputs belong and range which is the actual outputs that this input set generates for this given rule. So, for instance, we have for this function this relation associated with it, all pairs x comma y such that x and y are reals. So, the domain and the codomain are both reals, the rule is y equals x^2, so that is the filter that we put, we only want such pairs.

And if we plot all the points which belong to the relation, we get this graph on the right. And this actually tells us that the range of the function even though the codomain is all reals, the range of the function actually keeps this function above 0, so we only get non negative reals as outputs. Now, we are not restricted to looking at functions on numbers, we can also look at functions on other sets.

So, for instance, if we look at the set of all people in the universe, in the world, in the country, in any range of geographical regions, we can look for the function mother which says, given a person, this will map the person uniquely to the mother of that person. So, this is a function because every person has 1 mother. So, in this lecture, and in general, when we are talking about functions in this course, we will look more at functions on numbers. So, let us look at these a little more closely. What are the questions that we really want to ask about functions on numbers?

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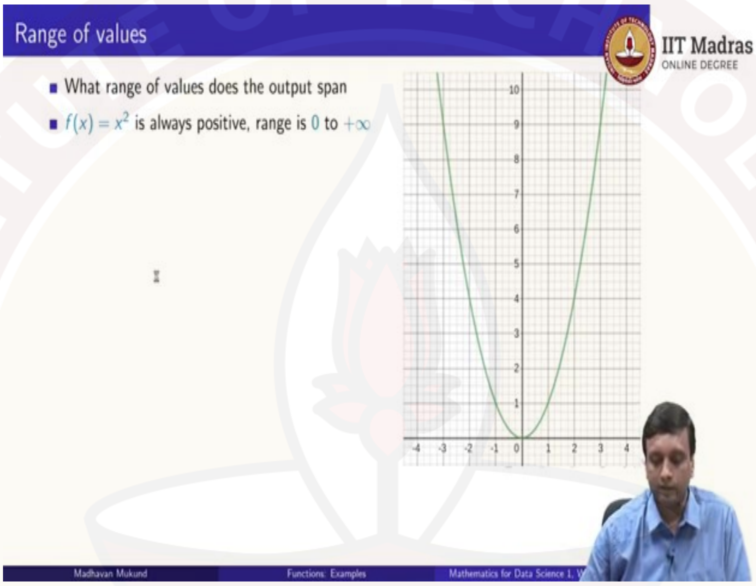

So, one of the basic questions is, what are the ranges of the values that we can get. So, in other words, we have a core domain. But what is the range of values that we can actually achieve through the function. So as we saw, this square function, $f(x)=x^2$ is always positive, so we always get something between 0 and $-\infty$, there is no upper bound, but we never get something which is negative.

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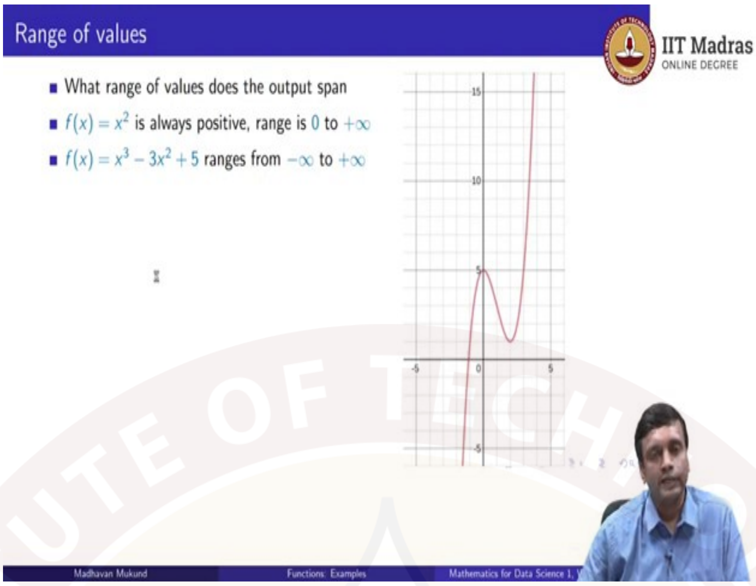

On the other hand, if we take a cubic function of this form $f(x)=x^3-3x^2+5$, then when x becomes very small, the x^3 becomes very small because the cube of a negative number is a negative number. So, cube have a large negative number, I mean magnitude, the $(-1000)*(-1000)*(-1000)=-10^{-9}$. So, as we go into negative, large negative values, we can at large negative outputs, same for large positive values. So, this has a range from minus $-\infty$ to $+\infty$.

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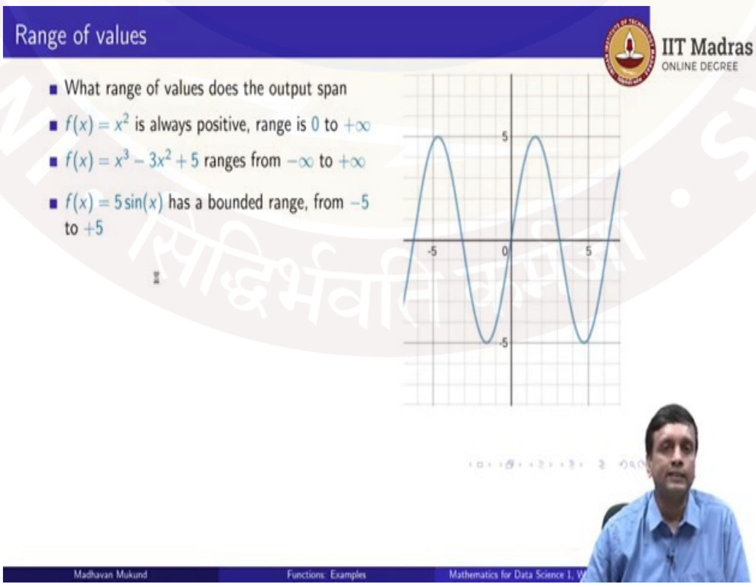

And then there are some functions like the trigonometric function $\sin x$, which oscillate between an upper bound and lower bound. So, if you take $\sin x$, usually it is between +1 and -

1. If we take $5\sin x$, then it will be between -5 and $+5$. So, this has a bounded range. Even though we consider all possible inputs, we never go outside this range from -5 and $+5$.

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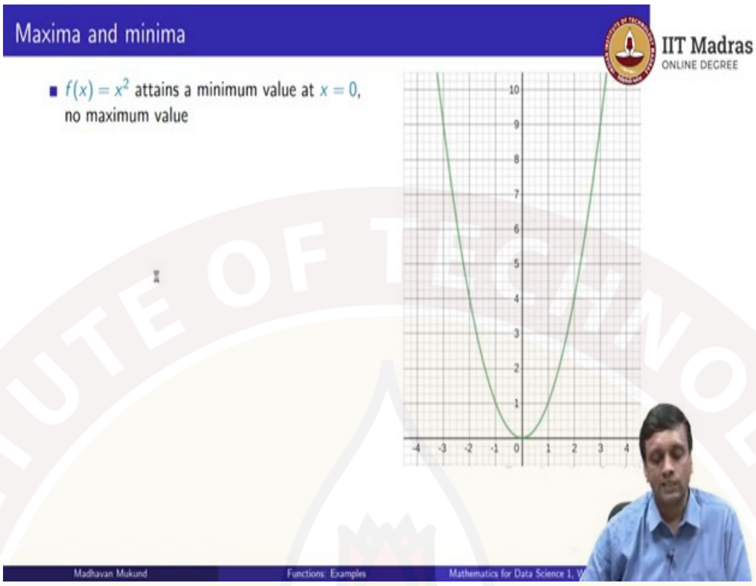

Now, within the range of values that it can take, we are often interested in specific points, in particular, where the value are a minimum and where they are maximum. So, for instance, this function that we have seen before $f(x)=x^2$, it is clear from the graph on the right that at 0 the output is 0 and at all other points is bigger than 0, so it attains its minimum value at 0. And because it keeps growing indefinitely in both sides, there is no maximum value.

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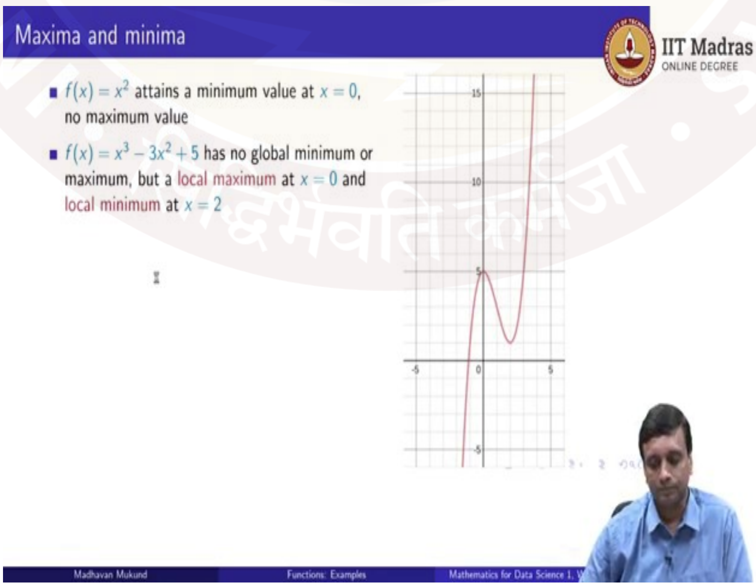

Now, the cubic function we said grows arbitrarily small as we go to the negative inputs and arbitrary large. So, there is actually no maximum and minimum, but it has an interesting behavior in between because it zigzags it goes up, comes down and goes up again. So, there is something called a local maximum and a local minimum. So, at $x=0$, it turns around, so it achieves a maximum value and starts falling briefly and then at $x=2$ it turns around again. So, it achieves a local minimum and goes up again. So, we are interested in finding out where these local maxima and minima are for various reasons.

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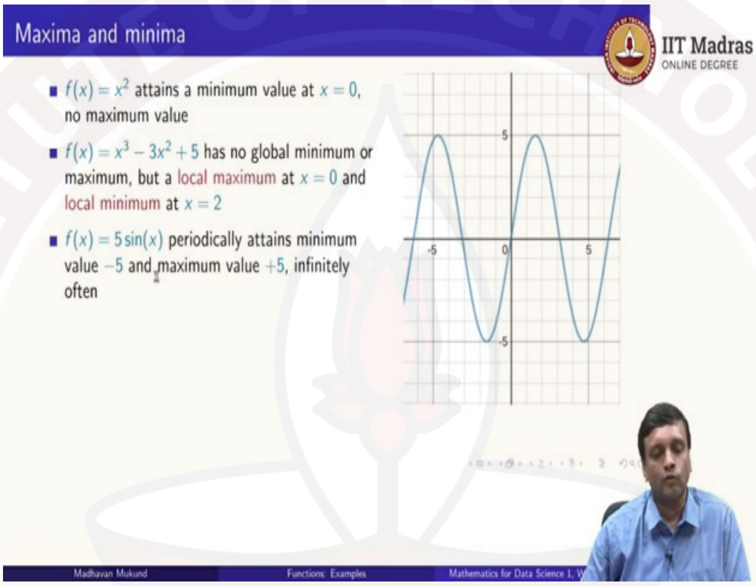

And similarly, if we look at something like $\sin x$, then it has, of course, local minima and maxima, $-¿5$ is a local minimum and $+¿5$ is a local maximum, it is also a global minimum and maximum because these are the maximum and minimum values that the function can ever attain. And now, these values are actually attained infinitely often periodically as we go from left to right.

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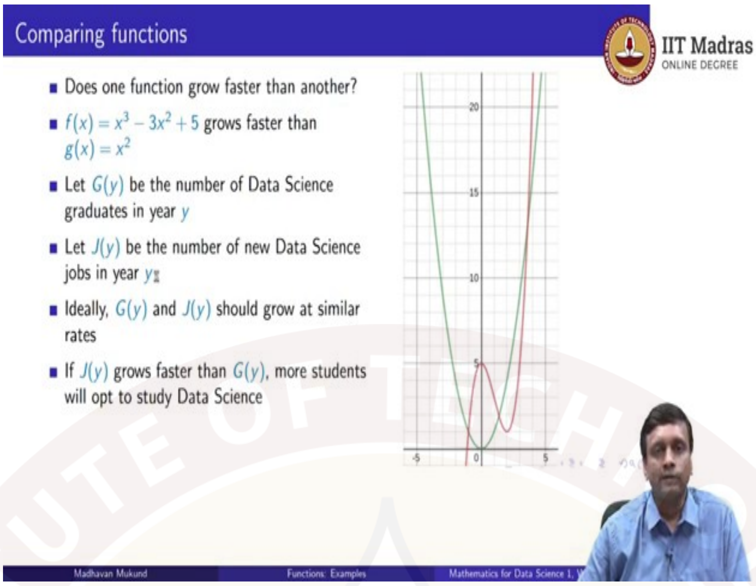

Another thing which we are interested in about functions is how fast they grow. Thus one function grow faster than another. So, if you look at our 2 functions, $f(x)=x^2$, and $f(x)=x^3-3x^2+5$, and we look at their 2 graphs, then it is very clear that the red line, although initially on the right, it is below the green line, it overtakes it, and after that, it is never going to be below the green line. So, in this way, the cubic function grows faster than the square function.

Now, why is this interesting? Well, we often see this informally stated in various contexts. So, let us look at a context which is relevant for you. So, let $G(y)$ be the number of data science students graduating in a year y. So, as the year increases, so we go from 2020 to 2021, and so on, the value G takes a certain number and hopefully because courses are growing, this number is increasing.

At the same time, there are jobs being created in data science. So, let $J(y)$ be the number of new data science jobs in a year . Now, ideally, you would like that these 2 are comparable, that the jobs are growing because the number graduates is growing and vice versa. If the number of jobs increases more than the number of graduates then there is a demand for graduates and of course, more graduates will opt to study data science. So, you would expect a demand for this kind of course.

Of course, the unfortunate case might happen the other way around, if suddenly there is a slump in demand, then people who graduate with a degree in data science will not be employable and then there will be a reverse trend. So, these are some of the reasons why when we look at data, we are interested in comparing the growth rate of functions and we will look at this in the context of the functions that we study mathematically.

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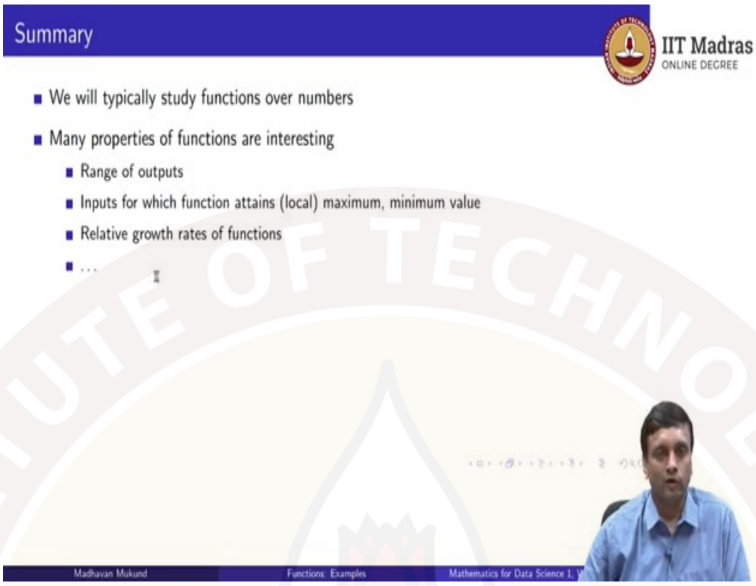

So, to summarize, we will typically study functions over numbers. And we are looking at many properties of these functions which are interesting to us, for instance, the range of outputs, where these functions attain local minima and local maxima and what are their relative growth rates and many other things which we will come across as we go along. Thank you.