IIT Madras

ONLINE DEGREE

Mathematics for Data Science 1
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Lecture – 8.2
One-to-one Function: Examples & Theorems

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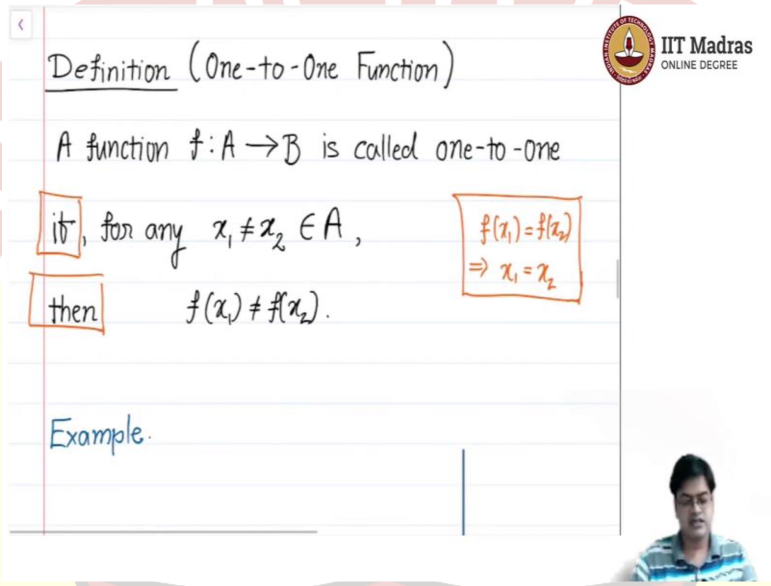

So, let discuss some examples of a functions that are one to one and not one to one. So, for this let us first take $f(x) = |x|$; is this function one to one?

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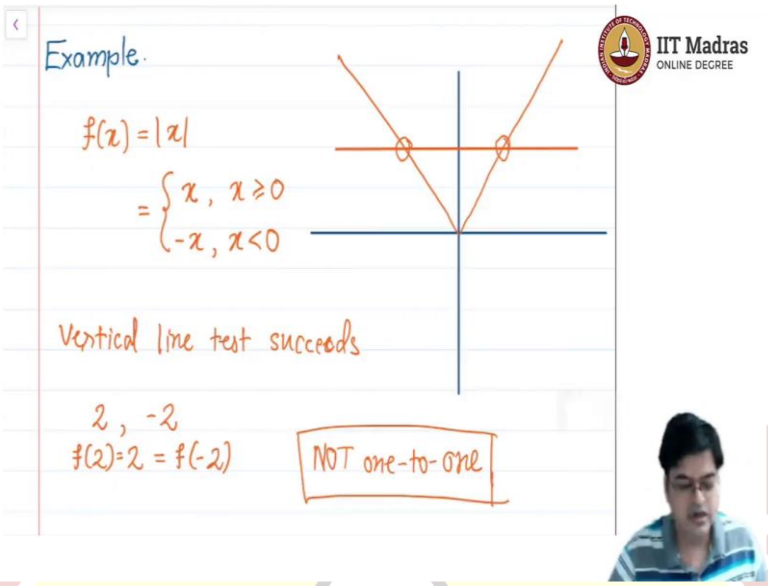

Try let us try to answer this question. So, let me write this function properly. So, if $f(x) = x$ for $x \geq 0$ and $-x$ for $x < 0$. So, it is actually a straight line on a passing through the origin like this 40 at a 45 degree angle and the $-x$ is this line ok. So, it is a V shape 90 degrees V; so, is this function one to one? First of all let us let us not take the argument, first of all is this a function $(x) = |x|$?

Pass a vertical line, take a vertical line and pass it through this; is there at if there is any point where two points more than one points pass through this function pass through that line then it is not a function. So, vertical line test is successful therefore, it is a function. Vertical line test says that it is a function succeeds and we know it is a function ok. Now, the question is the function one to one? Right. So, you pass a horizontal line. So, let me pass one horizontal line somewhere, let us take this horizontal line. Now, is the function one to one?

For $x_1 \neq x_2$, I got the same $f(x)$ ok. So, how will I prove it is not one to one? Let us take a value which is say 2 and -2; these are the two values, $f(2) = 2 = f(-2)$. Therefore, this is not going to be a one to one function. So, it is not one to one function. So, our conclusion is it is not one to one function. Then do we know functions that are one to one?

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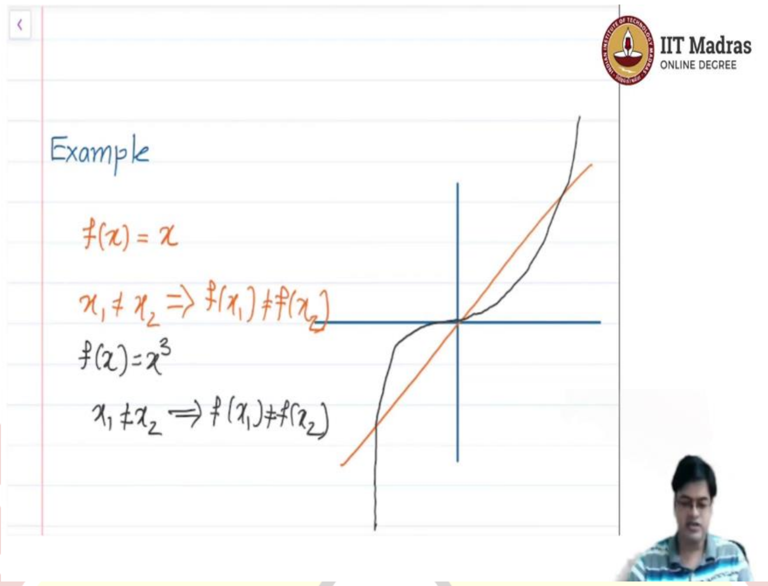

So, since we have taken $f(x) = |x|$, let us take a function $f(x) = x$; is this function one to one? It is a straight line passing through the origin, is this function one to one?

Let us take horizontal, first let us check whether this is a function, take a horizontal line, pass it through this pass it horizontally, the line parallel to x −axis. So, just drag x −axis up and down; do you see any point touching more than one point? No. So, it is a valid function; then, sorry yeah you have to pass the vertical line first ok.

Start with $f(x) = x$, take a vertical line which is y −axis, slide it to the left, slide it to the right. Do you see any where it has more than one points? No. So, it is a valid function. Then take a horizontal line, pass it from the top to bottom; see if you are getting any two points together for on that line; no. Therefore, this function is actually one to one because $x_1 \neq x_2$ implies $f(x_1) \neq f(x_2)$ which is more or less expected right.

Because $f(x) = x$ therefore, $x_1 \neq x_2$ will give $f(x_1) \neq f(x_2)$. So, what about it is an exercise then what about if you take a cubic functions? So, cubic function will pass like this sorry, it is not a correct diagram of a cubic function. So, cubic function let us change the color as well, cubic function will have something like this, symmetry will be retained and then this will go down.

So, if this function, now check whether this function is one to one or not. Again the exercise is very similar, pass a let the x − axis go up and down, see if you are finding any two points together. So, let us say this function is $f(x) = x^3$ and now you can easily make out that for $x_1 \neq x_2, f(x_1) \neq f(x_2)$. So, again through horizontal line test, I have detected that the function is one to one.

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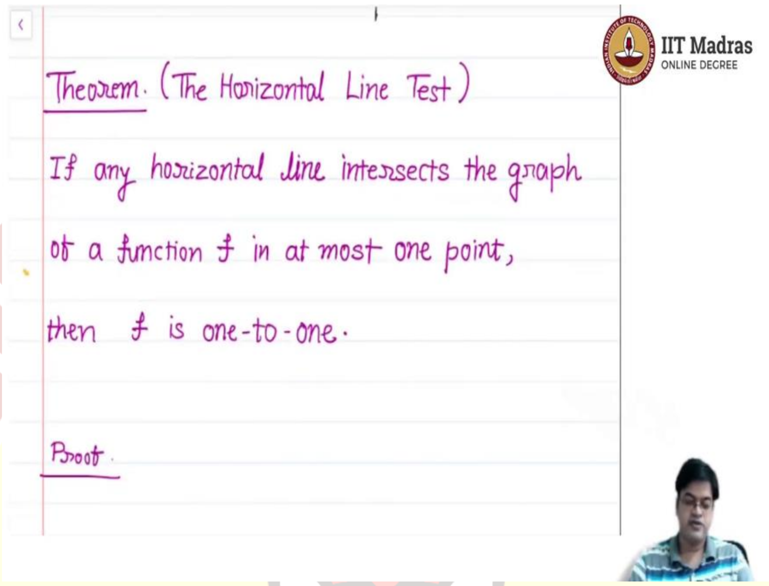

So, let us write this particular test as a theorem. If any horizontal line intersects the graph of a function in at most one point, then the function is one to one ok. So, then what we will show here, if you want the proof of this what we will show here is if the function is not one to one then it will intersect some horizontal line will intersect the graph of a function in more than one point ok. So, that is very easy to prove. So, I will prove it graphically.

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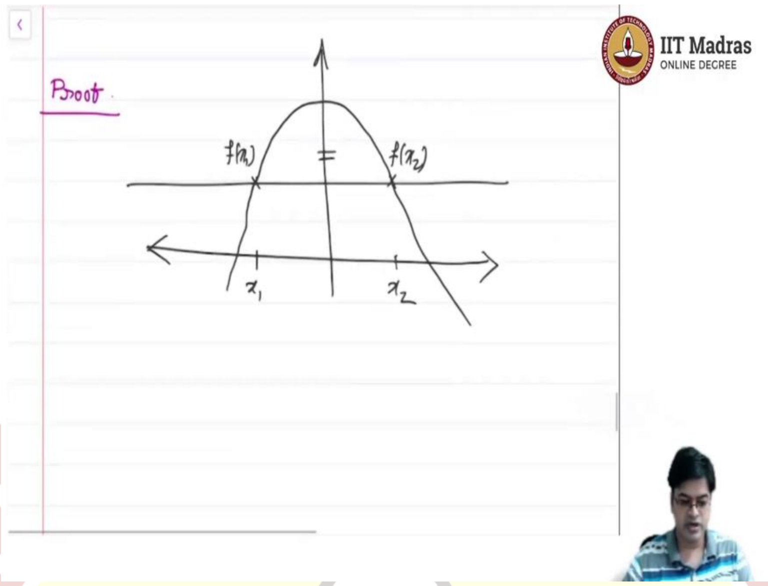

So, if the function is not one to one, let us say this is x −axis, this is y −axis. If the function is not one to one, I can take this point and call this as x_1 and I can take this point as call this as x_2. This is how I can make function not one to one and then pass a curve passing through these two points and pass the horizontal line over here which we have done several times now by now.

And therefore, $f(x_1)$ and $f(x_2)$ are same, they both are same. Therefore, the function is not one to one, that essentially proves the point that if a horizontal line intersects the graph of a function in at most one point then f is one to one good.

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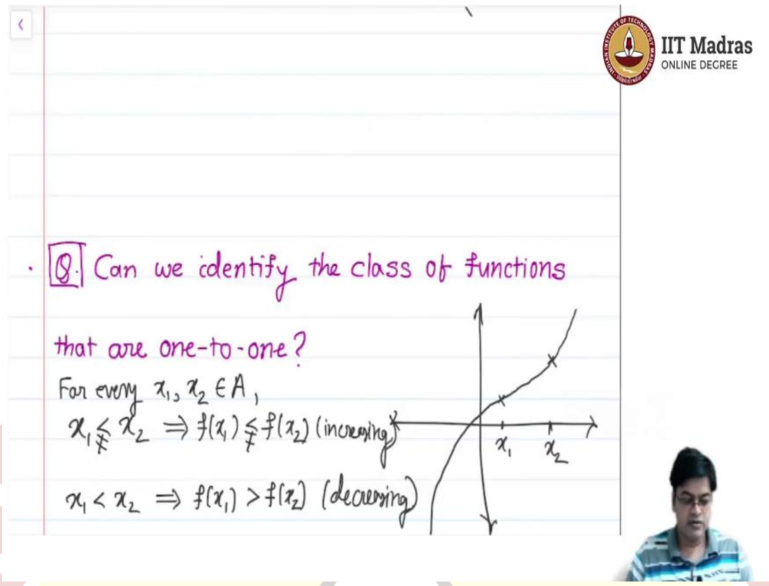

So, we are good to go now. Next thing that we will come is can we identify the class of functions that are one to one? So, what class of functions can you immediately think are one to one? For example, we have also seen some functions like if $x_1 \leq x_2$ then $f(x_1) \leq f(x_2)$ or let us not put this strict equality; let us put this way strictly increasing.

So, what does what do I mean by ok; let us can we question is can we identify the class of functions that are not one to one? So, I can; obviously, think of function of this form $x_1 < x_2$, $f(x_1) < f(x_2)$. Let me plot it and then the my imagination will work fine. So, this function is something like if x_1 is to the left of x_2 then $f(x_1)$ should always be less to the left of $f(x_2)$

Or, if you are plotting it on the y −axis then $f(x_1)$ below $f(x_2)$, this is the intuition and you can draw line joining these two points. Let it go ahead and this is true for every x_1, x_2 belonging to A this is true; then I am done.

But, this function have a name that is they are called increasing functions ok. In a similar manner, if I multiply this function with minus sign. Then I will get a function which is decreasing function and that can be written as $x_1 < x_2$ employs $f(x_1) > f(x_2)$ and this is called decreasing function.

Now, you look at any increasing function and apply your horizontal line test. What is the horizontal line test? Just now we have seen that if you take the horizontal line, roll it across the axis across $y-$ axis and there should not be more than one point intersecting that line at any given point in time ok. So, this increasing function and decreasing function will satisfy this phenomena.

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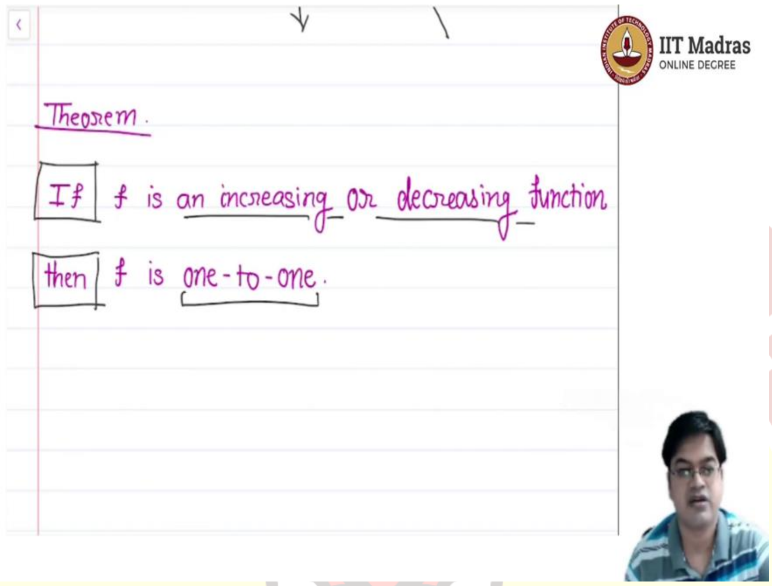

And therefore, we can easily write this as through horizontal line test that, if f is an increasing function or a decreasing function then f is one to one.

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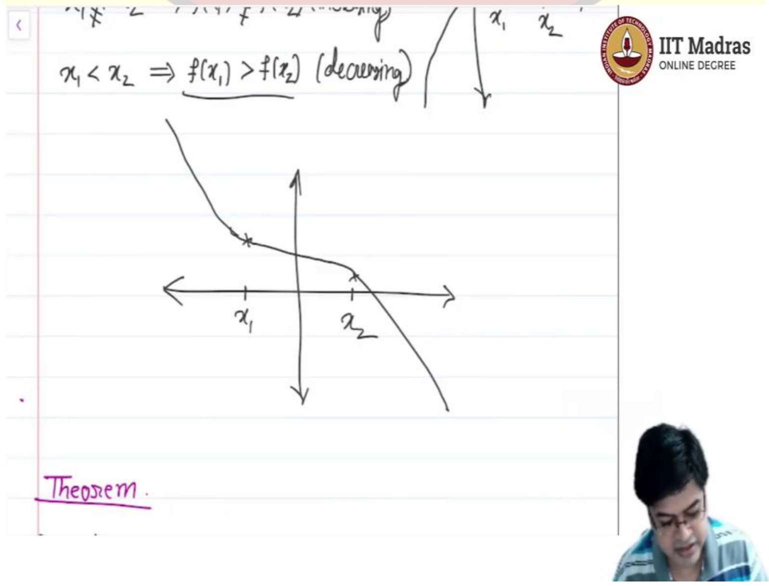

Let us see one decreasing function as well. What happens when the function is decreasing? As I go from left to right there is a x_1 is here, x_2 is here. As I go from left to right, I get x_1 here and now according to the condition $f(x_1) > f(x_2)$. So, it will be somewhere here and I can have a curve passing through this point in this manner ok.

This is true for every x_1 and x_2 belonging to the domain. And therefore, using our line test, horizontal line test we can easily see that the function whether it is increasing or decreasing, they are one to one. This gives us a big class of functions that are one to one. So, one to one functions are not rare to find, one to one functions are abundant in nature and they are reversible as well.

With this insight, we will go to our next topic which is exponential functions.