



IIT Madras
ONLINE DEGREE

Example: Real-World

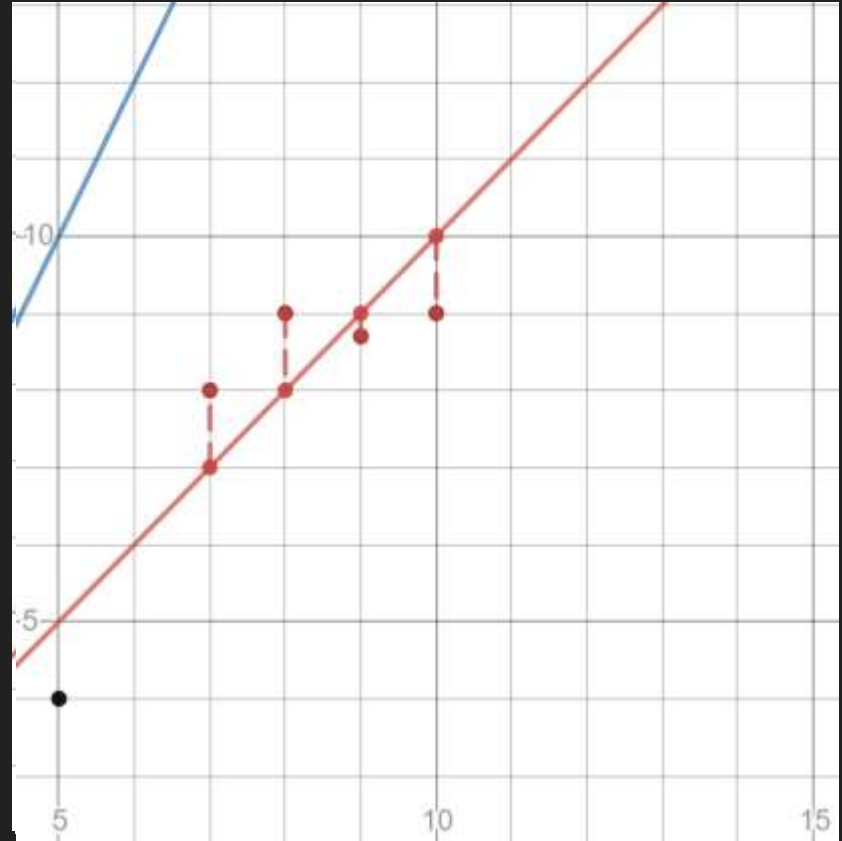
It is known that $V=IR$.

That is, Voltage= current x Resistance

In our context, this represents a line passing through the origin.

That is, $y = m x$, where y is the voltage, x is the current and m is the resistance.

You have been asked to perform an experiment to verify this phenomenon.



Example: Real-World (Contd)

How to say mathematically which line is better?

Let the equation of two lines be $y = x$ and $y = 2x$.

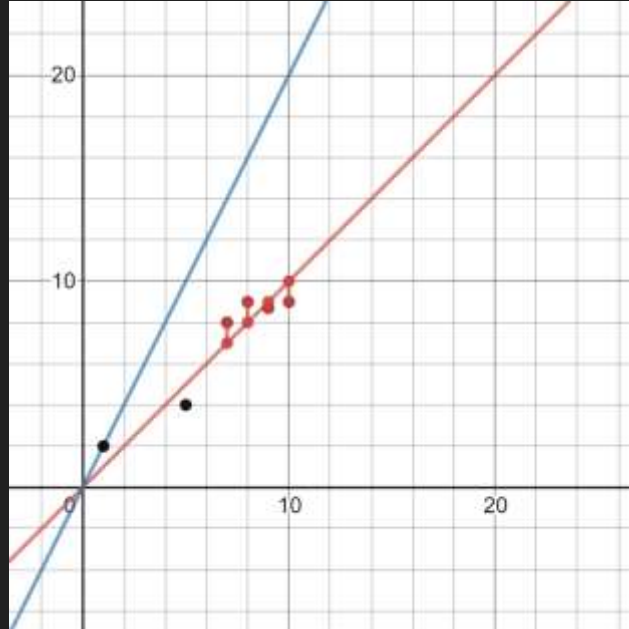
From the set of observations, (x_i, y_i) ,
 $i=1,2,3,4,5,6$.

We can consider the square of the differences

$$\sum_{i=1}^6 (y_i - x_i)^2 \text{ and } \sum_{i=1}^6 (y_i - 2x_i)^2$$

The first difference is 5.09 and the second difference is 328.49.

Therefore, the first line is better than the second line.



x_i	y_i
1	2
5	4
7	8
8	9
9	8.7
10	9

Distance of a Set of Points from a Line

Apart from perpendicular distance, we can also talk about the distance which is parallel to Y-axis.

Consider the set of points $\{(x_i, y_i) \mid i=1, 2, \dots, n\}$ and a line with equation $y = mx + c$.

Then the **squared sum** of the distance of set of points from the line is defined as

$$SSE = \sum_{i=1}^n (y_i - mx_i - c)^2.$$

Least Squares Motivation

- In general, this raises the following question
- Given a set of points, how to find the line that fits the given set of points?
- In other words, what is the equation of the best fit line for given set of points?

In other words, if I need to find the equation of line $y = m x + c$, then the question can be reframed into two questions.

- What is the value of m and c that best fits the given set of points.
- What is a meaning of best fit?

Best Fit: Given a set of n points, $\{(x_i, y_i) \mid i = 1, 2, \dots, n\}$, define

$$SSE = \sum_{i=1}^n (y_i - mx_i - c)^2.$$

Find the value of m and c that minimizes SSE .