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IIT Madras

ONLINE DEGREE

Statistics for Data Science 1
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Lecture 6.5
Probability - Applications

So, let us look at a few applications of this addition rule.

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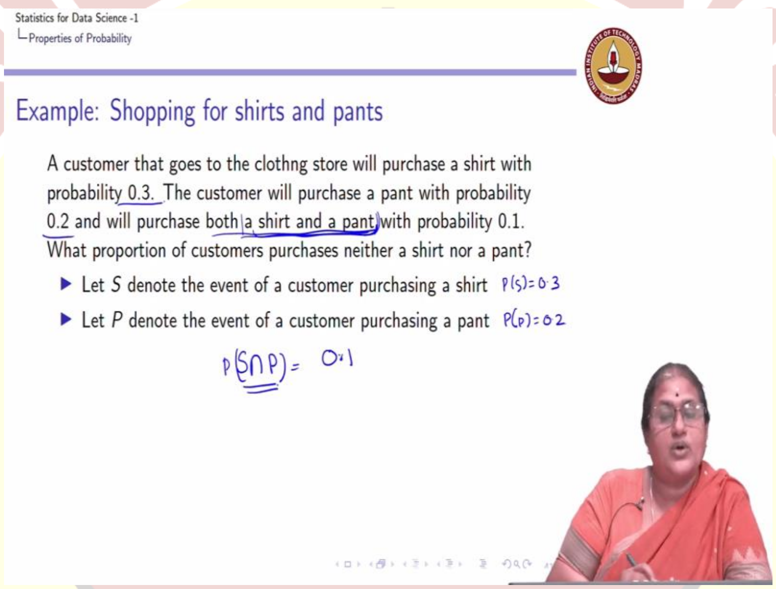

So, suppose we have a customer who goes to a clothing store, and we know that somebody gives us the following information that the person can purchase a shirt with the probability of 0.3, this is a clothing store, the customer will purchase a pant with a probability of 0.2 and will purchase both a shirt and a pant with probability 0.1. So, the question asked is what proportion of customers' purchases neither a shirt nor a pant? That is a question that is being asked.

So, if I am a shopkeeper, I would be interested in knowing the proportion of customers who visit me who will make no purchase. So, how do I translate this problem to what we have learned so far? So, first step is to identify the information as events. So let us identify the events. So, the first event is let me denote that event by S. So what is S? S is the event of a customer purchasing a shirt.

Now, if the shopkeeper is certain that every person who enters the shop will make a purchase there is no uncertainty here. But we all know that when a first customer enters a shop, they might buy a shirt. It is not will buy a shirt so there is that element of uncertainty. And this I am denoting by S is the event of a customer purchasing a shirt and what is given to us this the probability with which your customer buys a shirt is 0.3.

Similarly, the next event is the event of a customer purchasing a pant I know probability of S is 0.3. Similarly what is given to me is the probability a customer purchases a pant is given to me as 0.2. And some other information that is given to me is the probability that a customer would purchase both the shirt and a pant. Now this information a shirt and a pant can be captured by $S \cap P$.

Now, this is what is most important that is we are translating an information from a situation in real life, we are abstracting it to a mathematical expression. What is the mathematical expression $S \cap P$ is the event that a shirt and pant is being purchased and that probability is given to be 0.1. So, this is what is given to us.

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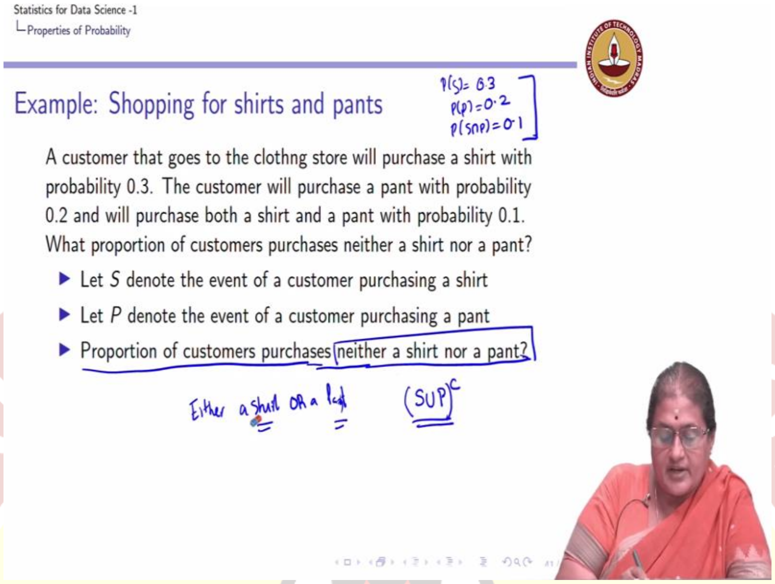

What is needed now, so, I am given $P(S) = 0.3$ $P, (P) = 0.2$ and $P(S \cap P) = 0.1$ this is the information that is given to us. We are interested in knowing the proportion of customers or probability that a customer does not make a purchase in other words neither a shirt or a pant. Now, what is that expression that can be used to capture this.

Now, recall either a shirt or a pant, either A or B is captured by $S \cup P$, either shirt or pant is captured by a operation union. So, neither a shirt nor a pant is the complement of this event.

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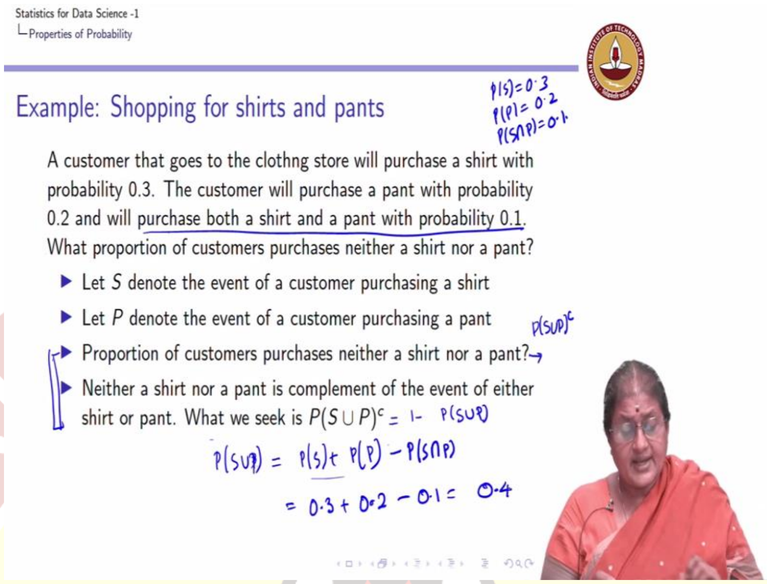

So, either a shirt or a pant is the union of this event, neither a shirt or a pant is the complement of the event either shirt or pant. So, what we seek to answer this is probability of $(S \cup P)^c$. Now, these are crucial observations and these are the crucial things that you need to know. Nobody is going to give us find out $P(S \cup P)^c$ directly. We need to understand that given the problem, this is what is being asked or this is what is being sought.

Once we pose a problem in this framework, then we apply the laws of probability or the axioms of probability to come up with the answer. So, what is that that we seek we seek $P(S \cup P)^c$. Now, I know the probability of A complement is 1 minus probability of S union P this is what we had just derived a few minutes before. So, what is $P(S \cup P)$? So, we first asked the question are S and P disjoint?

What do we mean by that? Is does a person who buy a shirt does not buy a pant on vice a versa? The answer is no because I have a probability of a person who could buy both a shirt and pants. So, the event of purchase of shirt and pant together can happen. Hence, they are not disjoint events. So, the next thing is if they are not just disjoint I know from the additive rule of probability, this is $P(S) + P(P) - P(S \cap P)$.

Now, I have all these 3 probabilities I know $P(S) = 0.3$, I know $P(P) = 0.2$, I know $P(S \cap P) = 0.1$. Hence, I can just plug in those values to get $0.3 + 0.2 - 0.1 = 0.4$.

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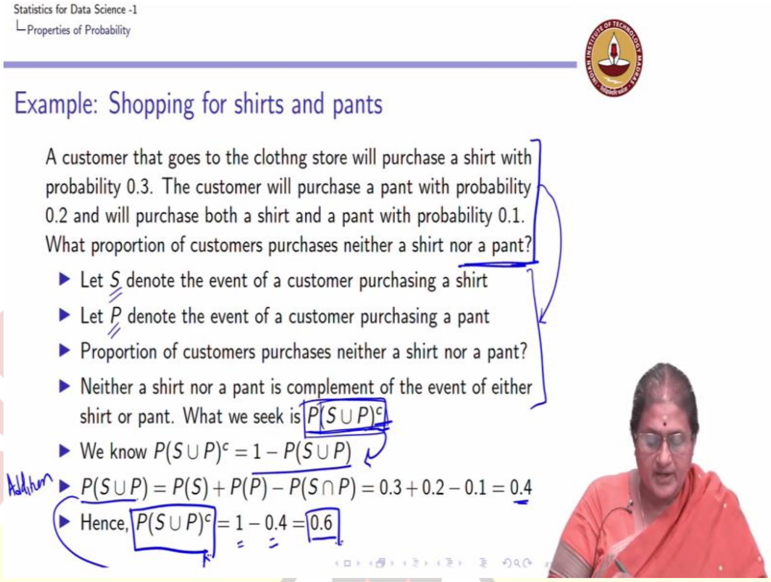

Hence, I can say that the probability of a customer or the proportion of customers who purchase neither a shirt nor a pant is, $1 - 0.4 = 0.6$. Because this is the probability I was seeking here. Hence this is the answer is 0.6. What I want you all to understand here is typically in real life situations, this is the situation that is given to us. From this situation, we abstracted and put it in a mathematical format or a framework by identifying what were the events and what is set in terms of the probability of which set are we seeking.

Once we post it in this format, we realize that this probability of complement is $1 - P(S \cup P)$, we applied the addition rule to find out the probability here and then we got the $P(S \cup P)^c$through the complement rule. So, this is what we need to train ourselves to first identify what is given in a problem and post it in the mathematical framework and then tell it in the language that is sought after find out what is being asked. That is extremely important.

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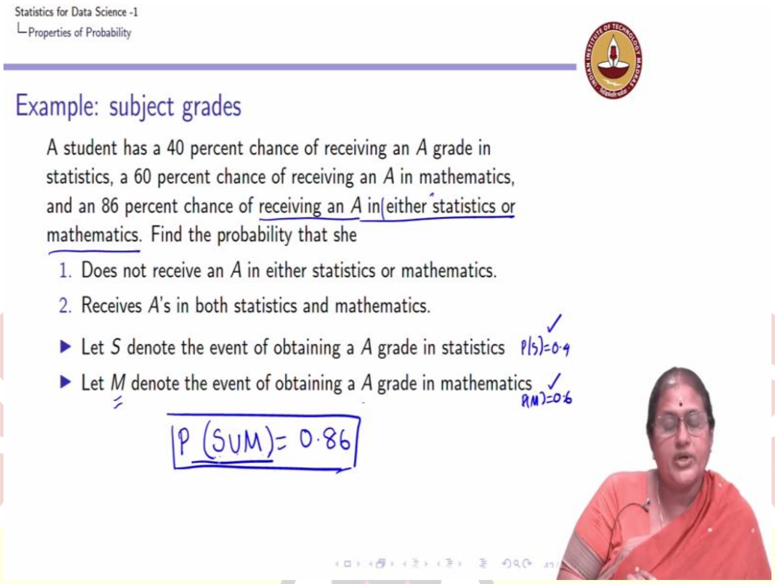

Now, let us look at another example. This is an example which students would relate to. So, I have a student who has a 40% chance of receiving an A grade in statistics, a 60% chance of receiving an A grade in mathematics and an 86% chance of receiving an A in either statistics or mathematics. I repeat, the student has a 40% chance of receiving an A grade in statistics, a 60% chance of receiving an A grade in mathematics, but an 86% chance of receiving an A in either statistics or mathematics.

What is being asked in this question? What is the probability that the student does not receive an A grade in either statistics or mathematics? The second question is receives an A grade in both statistics and mathematics. This is the question that is being asked. So, again, we start what is the sample, what is the experiment here? I have a student who has written an exam. So, the outcome is the grades the student obtains in statistics and in mathematics.

So, we can see that this I have a class and in this class, I can find out what is the grades obtained by different students in subjects or I given. So, first, what we do here is first we identify the events, how do we identify the events? So, again, I go back, I identify my event, let S denote the event of a student obtaining of the student obtaining A grade in statistics. So, the first thing we identify is what is the event. Let S denote the event of obtaining an A grade and statistics this probability is given to me and that is given to me by this 40%. I write that as 0.4.

Similarly, the next thing which is given to me, let me denote M to be the event of obtaining a, A grade in mathematics. So again, I have a probability of S is 0.4. I have probability of M is 0.6. Now, what does this mean receiving an A in either statistics or mathematics. Recall either statistics or mathematics is represented by the union of these two events and it is given probability of statistics or mathematics is 0.86.

So, the information given to us is probability of receiving a A grade in statistics is 0.4. Probability of receiving an A grade in mathematics is 0.6. And the probability of getting in either of the subjects is 0.86, which we recognize is the probability of the union of these two events. So, the first question that is being asked this, what is the probability that the student does not receive an A in either statistics or mathematics?

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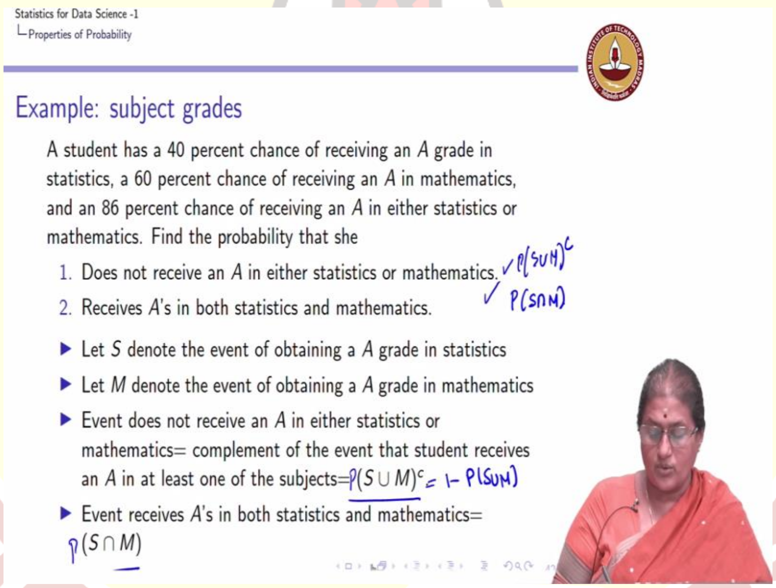

Again, like we have done in the earlier problem, the event that she does not receive an A is the complement of the event that the student receives A in at least one of the subjects she does not receive in either of the subjects is same as the compliment that she receives in at least one of the subjects, which is again S union M complement. And again, we apply the probability loss of probability to get that.

So, the first thing we understand that this is S union M complement, and the second is receives an A in both statistics and mathematics is we know, it is very simple, it is nothing but the intersection of these two events. So, let us apply our probability properties or axioms to find out the probability. So, I have to find out what is the probability of S union M complement and probability of S intersection M that is what we need. So, this is probability S union M complement this is probability of S intersection M. Now, $P(SUM)^c = 1 - P(S \cup M)$. This is what we know.

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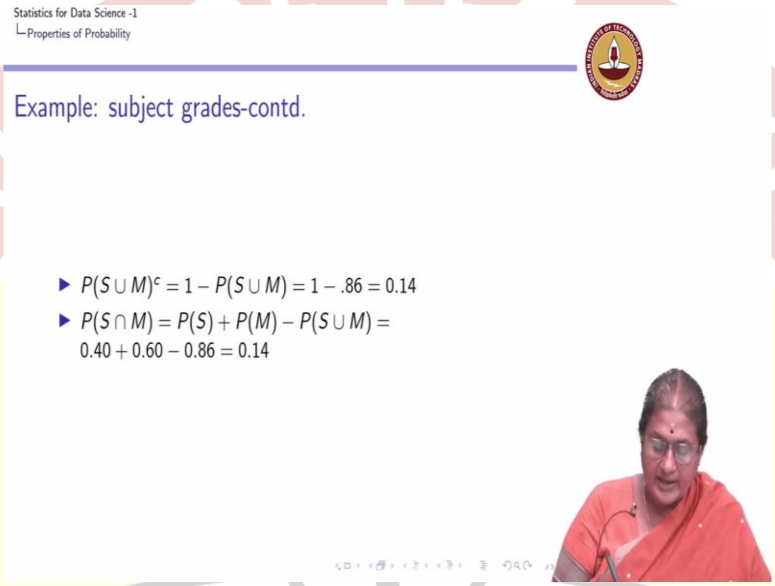

So, we need to understand what is $P(S \cup M)^c$ this I know is equal to $1 - P(S \cup M)$. But what is $P(S \cup M)$ that is given to be 0.86. Hence, I have the probability of S union M complement is nothing but probability 1 minus so that I have an 86% chance of either statistics or mathematics. So, apply whatever I have to find out that probability of S union M complement is 0.14.

Now, what is the next probability, the next probability is $P(S \cap M)$. Now, again recall S and M are not disjoint. Whatever addition rule says is $P(S \cup M) = P(S) + P(M) - P(S \cap M)$. And this is what my addition rule says, I have this probability I have this probability I have this probability I need to find out what this is.

So, what I do is I can rewrite this addition rule as $P(S \cap M) = P(S) + P(M) - P(S \cup M)$ and I have just rewritten my addition rule. I know probability of S is 0.4, I know probability of M is equal 0.6, I know probability of S union M is 0.86 to give me a 0.14 probability of the candidate receiving A grade in both these subjects together, there is a 0.4 or a 14% chance that the candidate, there is a 0.4 or a 14% chance that the candidate will get A grade in both the subjects.

So, these were examples of how you apply the addition rule. What we need to understand is, we need to first identify the events given the scenario and then apply the addition rule.