INDIAN INSTITUTE OF TECHNOLOGY MADRAS

सिद्धिर्भवति कर्मजा

IIT Madras

ONLINE DEGREE
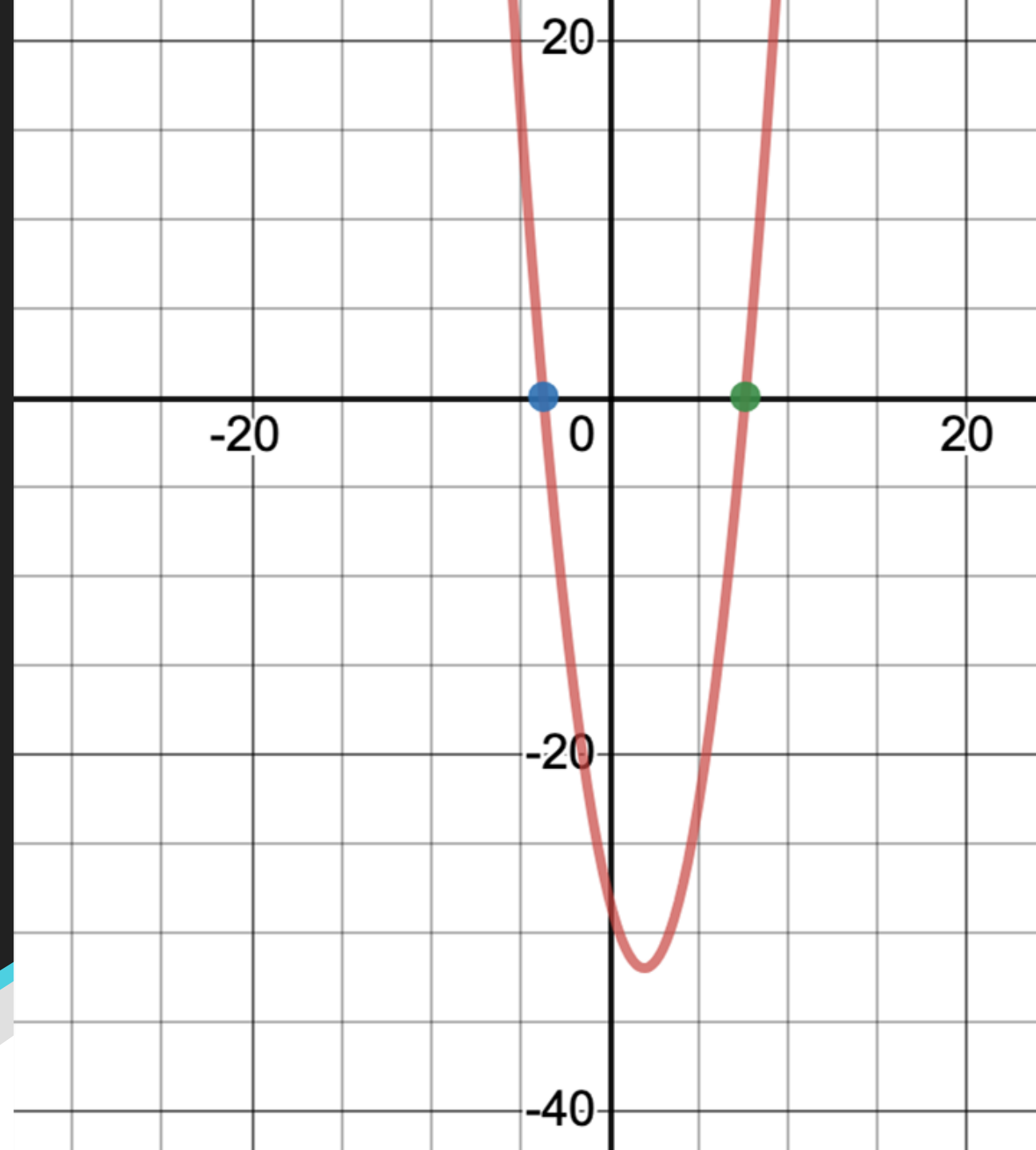
Quadratic Equations with Irrational Roots

Solve: $x^2-4x+4=32$

It can be easily seen that $(x-2)^2=32$.

Hence, $(x-2) = \pm\sqrt{32} = \pm4\sqrt{2}$.

Thus, $x=2\pm4\sqrt{2}$ are the roots of the quadratic equation.

Quadratic Formula

b^2-4ac	roots
>0	2
<0	0
=0	1

$$ax^2 + bx + c = 0$$

$$x^2 + \frac{b}{a}x + \frac{c}{a} = 0$$

$$x^2 + \frac{b}{a}x + \frac{b^2}{4a^2} = +\frac{b^2}{4a^2} - \frac{c}{a}$$

$$(x + \frac{b}{2a})^2 = \frac{b^2 - 4ac}{2a}$$
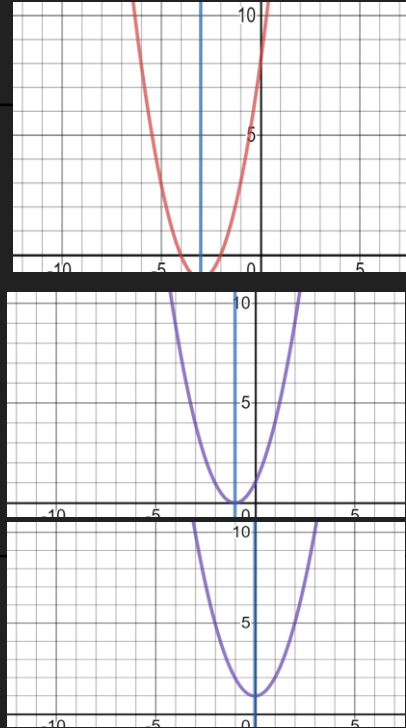
$$(x + \frac{b}{2a}) = \pm\frac{\sqrt{b^2 - 4ac}}{2a}$$

$$x = -\frac{b}{2a} \pm \frac{\sqrt{b^2 - 4ac}}{2a}$$

The above formula is known as quadratic formula.

The quantity in the square root is known as discriminant.

Summary of Quadratic Formula

Value of the discriminant	Type and number of roots	Example
$b^2-4ac >0$ perfect square	2 real, rational roots.	
$b^2-4ac >0$, no perfect square	2 real, irrational roots.	
$b^2-4ac =0$	1 real, rational root.	
Consider $ax^2+bx+c=0$, where a,b, and c are rational numbers.		
$b^2-4ac <0$	No real root.	

Examples

Find the value of the discriminant for each equation and then describe the number and type of the roots for the equation.

1. $9x^2-12x +4 =0$
2. $2x^2+16x+33=0$

1. $b^2-4ac = (-12)^2-4(9)(4) = 144 -144 =0$ so, it has one rational root.
2. $b^2-4ac = (16)^2-4(2)(33)= 256-264=-8$ so, it has no real roots.

Axis of Symmetry

Why *x=-b/2a* is the axis of symmetry?

$$f(x) = ax^2 + bx + c$$
$$= a(x^2 + (b/a)x + c/a)$$
$$= a(x^2 + (b/a)x + b^2/(4a^2) - b^2/(4a^2) + c/a)$$
$$= a(x + b/2a)^2 + (c - b^2/(4a))$$

Therefore, the symmetry is about *x=-b/(2a) which is the axis of symmetry.*

Summary of Concepts

Method	Can be used	When preferred
Graphing	Occasionally	Best used to verify the answer found algebraically
Factoring	Occasionally	If constant term is zero or factors are easy to find.
Completing the square	Always	Use when b is even.
Quadratic	Always	Use when