INDIAN INSTITUTE OF TECHNOLOGY MADRAS

सिद्धिर्भवति कर्मजा

IIT Madras

ONLINE DEGREE

Mathematics for Data Science 1
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Lecture – 42
Graphs of Polynomials: Graphing and Polynomial creation

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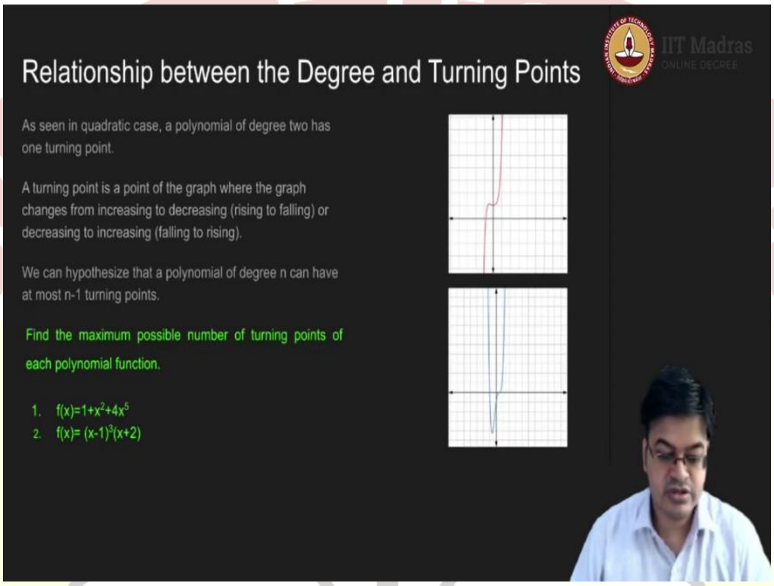

So, now I want to use all this knowledge to plot a Polynomial Function or graph a Polynomial Function.

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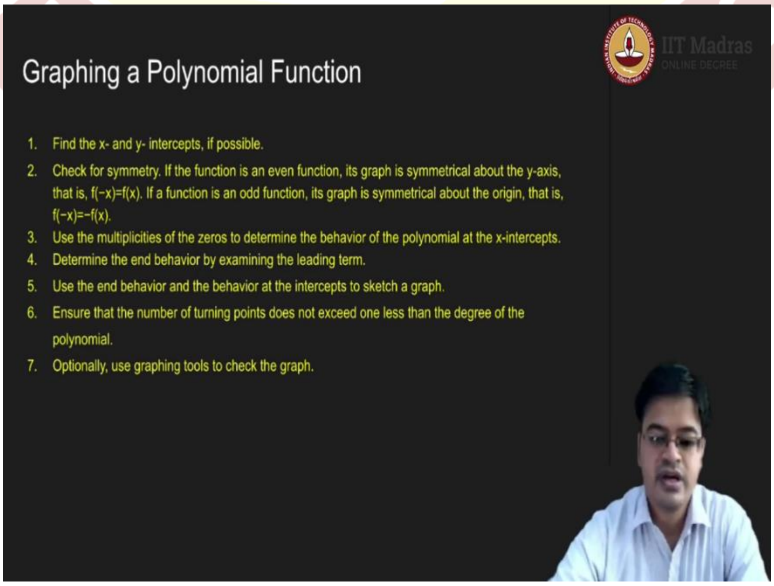

So, let us reiterate what are the things that we have seen. For graphing the polynomial function, one way is to find the tabular form and try to graph it as in a crude manner. More knowledgeable way is, you follow this algorithm that is find x intercept, y intercept if possible because it may happen that they do not have any real roots and you may not be able to get x-intercept, all the x-intercepts right.

Then for graphing it is helpful to check the symmetry that is; if $f(x)$ and $f(-x)$ are same if it is an even degree polynomial; that means, you have symmetry about y axis. If it is an odd function you can check whether they are symmetric about origin that is $f(-x) = -f(x)$. Typical case is the first symmetry is $y = x^2$, it is an even degree polynomial and it is symmetric. So, once you have drawn here for $-x$ you have to just keep the mirror image.

That is how it helps in graphing. In a similar manner a $y = x^3$ is a odd degree polynomial and $f(-x) = -f(x)$. Therefore, whatever you got about origin if you reflect about origin then you will be able to retain the same shape; you do not have to compute explicitly. This is the way this checking of symmetry helps.

Next identify the zeros; x intercepts we have already identified. So, you have identified the zeros. Then you identify their multiplicities. If you identify the multiplicities of the polynomials you know the behavior of the polynomials at x intercept. You just recollect; multiplicity the sum of the multiplicities of all zeros cannot exceed the degree of the polynomial that you have to keep in mind. After identifying the multiplicity you know the behavior at the zeros of the polynomial function.

Now, you want to know the behavior beyond zeros of the polynomial function that is; the end behavior. So, end behavior you can use the leading term and you can identify the behavior. Remember the table that we have shown for identifying the end behavior.

And finally, you use the end behavior the behavior at intercepts to sketch the graph. Turning points - the number of turning points can be identified we may not be able to locate exactly where the turning point is. For that, you need the tools of calculus to identify the exact location of a turning point.

And when you identify those when you roughly estimate the turning points; kindly ensure that the number of turning points do not exceed one less than the degree of the polynomial.

So, if the degree of the polynomial is n, the number of turning point should not exceed n -1 ok. And finally, you can use technology to sketch the graph. So, use graphing tools like Desmos or some other tools for graphing the function ok.

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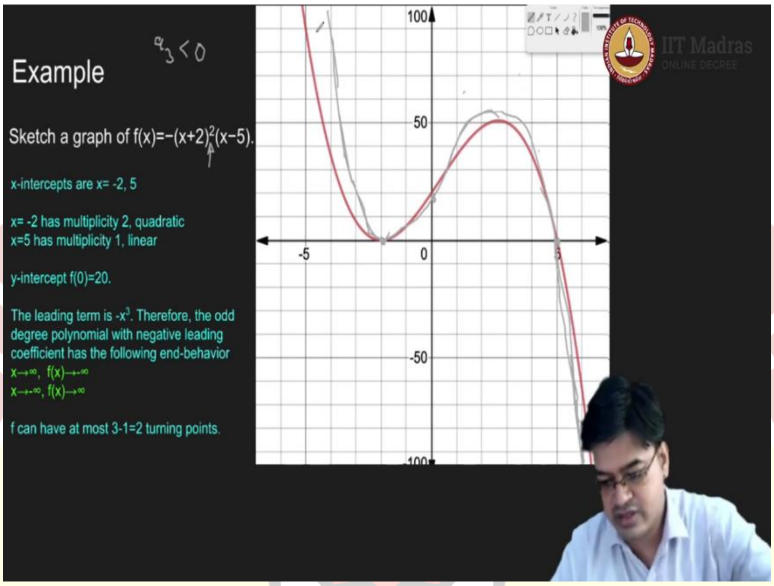

So, let us see this in action. So, here is an example I want to sketch a graph of this polynomial function; $-(x+2)^2(x-5)$. Obviously, I have figured out oh it is a $-(x+2)^2$. So, the first thing that we; so, I want to graph this function. So, the first thing that I want to find is the x intercept, because it is given in factored form it is no brainer; $x = -2$ which is this point $x = -2$ and then $(x - 5)$. So, $x = 5$ which is this point.

These two are the zeros of the polynomial functions; $x = -2$ has multiplicity 2 and it is an even degree polynomial. So, over here the behavior of function I am trying to sketch it will come from here, it will go from here. So, I know the behavior of the function is of this form, it will just pass through the axis.

And $x = -5$ I do not know the exact values, but roughly it will be more of the linear form and it will pass through the point -5, then it will come down over here and then it will pass through this point. So, up to this I am ok. Now, you look at this polynomial if you look at this polynomial, then the polynomial will be a cubic polynomial; it will have a negative term.

So, essentially $a_3 < 0$ ok. So, the end behavior of this polynomial because $a_3 < 0$ as $x \to \infty$ this function will tend to infinity. Yes, and as $x \to -\infty$ the function will naturally go up like this.

So, this is the vague understanding of the behavior. If I want to get more precise on what values this is roughly the shape of the function. If I want to get more precise on what values the function takes, I can consider the y intercept as well that is I will put $y;\ x = 0$. So, it will be 2^2 4 $2^2 4$ yes, and into -5 that will give me $-20 + 20$. So, this intercept that I have drawn is wrong. It should be somewhere here + 20. So, let me erase this and redraw the function again.

Let us take the eraser. So, it may not go this high as well. So, over here the font behavior of the function ok; so, let me again go back to the marker. And the function may cut here itself pass through this point and join this point. Yes, so, this bulge will not be there because this function is linear over here. It may be of this form. So, let me again erase this part yeah.

So, let us see. So, we have identified the end behavior ok, final check that number of turning points. The function is cubic, so it can have at most two turning points, there are only two turning points: one is here, one is here fine. So, let us see whether whatever we have said is correct or not. So, let me hide this first.

So, x intercept is -2 and 5, no problem. $x = -2$ has multiplicity 2. So, the quadratic behavior should be plotted there. Yes, $x = 5$ has multiplicity 1. So, a linear behavior is plotted here assume this is a line. So, linear behavior is a plot must be plotted here. And, then $f(y)$ intercept $f(0)$ is 20 which we corrected we were not correct in the initial stages and the leading term is $-x^3$.

So, therefore, the odd degree polynomial with negative leading coefficient has the following end behavior; as $x \to \infty$ $f(x) \to -\infty$, this is the behavior that we have plotted; $x \to \pm\infty$ $f(x) \to \infty$, this is also correct. And f can have at most $3 - 1 = 2$ turning points, this is the behavior right.

So, now, I was roughly ok in drawing the graph of a function. This is because I do not exactly know the behavior of the turning points. So, I will be roughly ok in drawing the graph of a function, but not exactly. If you want to be more precise you can actually tabulate the values around some critical points and then you can figure out. This is when the formula is given to you.

Now, the question can be asked that what if the formula is not given to you, but you have been given only a function. And from the graph you need to identify the polynomial.

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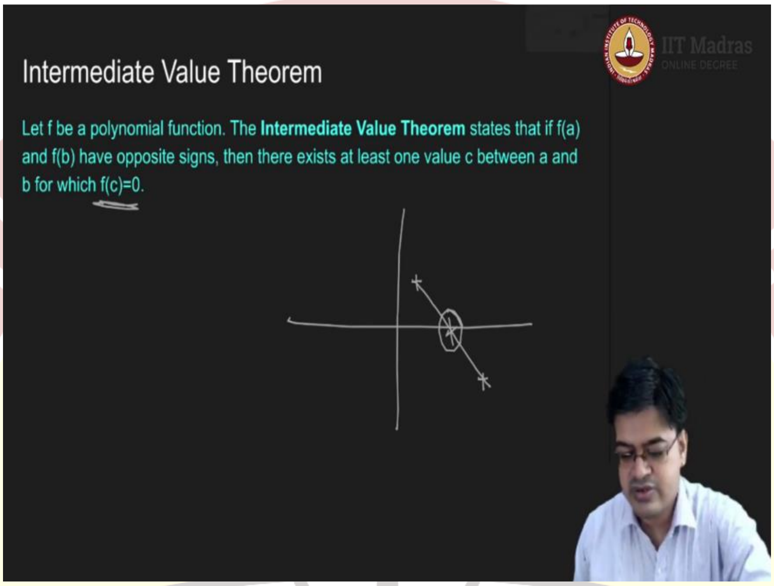

In such cases one theorem which will help you a lot, I will not use this theorem in a rigorous manner. But it will help you a lot, is intermediate value theorem because we are dealing with continuous functions. This intermediate value theorem is valid for all continuous functions.

What this theorem says is, a polynomial function is a continuous function. So, let f be a polynomial function, then the intermediate value theorem states that; if $f(a)\ and\ f(b)$ have opposite signs; that means. So, let us say $f(a) > 0$, and $f(b) < 0$ and $a > b$, then there exist at least one c between a and b such that $f(c) = 0$; that is essentially the meaning.

For example, I have this coordinate plane my value of $f(a)$ is here, and $f(b)$ is here, and the function that is given to me is a continuous function right. So, finally, it has to pass through the x axis to reach the value here right. So, in such cases we will say that; this is the 0 of the polynomial that is what we are calling as c, $f(c)$.

So, you using this you when you are actually having trouble in finding the zeros of the function, you can actually evaluate two values any two values of opposite signs. And if you evaluate any two values of opposite signs, then you know that there is some root some 0 in between that will improve that you will gain a confidence by doing these things.

So, this is an important theorem in mathematics, intermediate value theorem. You can use this to find the roots of the polynomial when you are having difficulty in identifying the roots of the polynomial. So, you simply put $f(a)$ and $f(b)$ if they have opposite sign, then there is at least one root in between; that is the meaning. You can use this theorem to your advantage.

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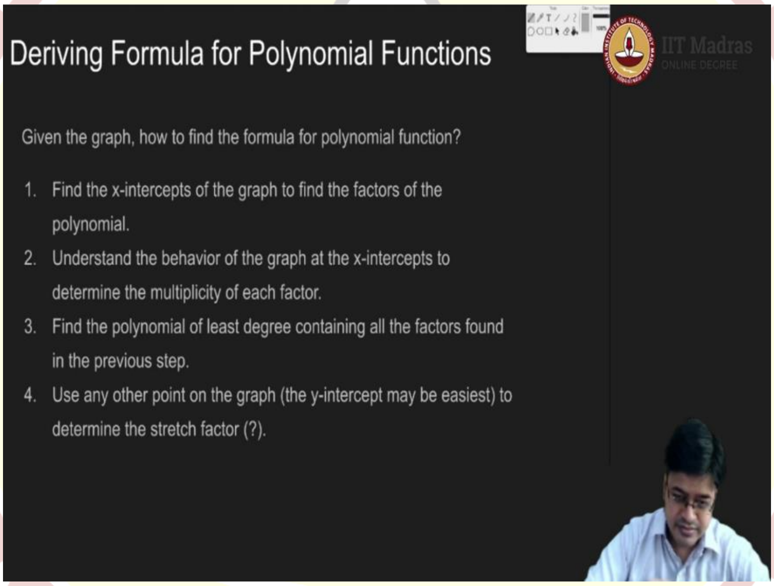

So, using this theorem we can actually derive a formula for polynomial function. You use this theorem to identify the zeros; rest of the methodology is similar. So, how to derive a formula for polynomial functions? So, given a graph of a polynomial, how to find with in coordinate axis? You have all the numbers attached to it, then the question can be asked as to how to find the polynomial function the algebraic expression of a polynomial function?

So, in that case our modus operandi is similar to what we have done. Find the x-intercepts from the graph. Find the factors of the polynomial, this we already know. Understand the behavior of x intercepts around x intercepts to get more understanding of the x intercepts that is zeros of the polynomial about their multiplicities.

So, you will find multiplicity of each factor. Once you have gained understanding identify the end behavior that also you have to do. Next, after doing that you find the least degree polynomial containing these factors. What are the factors? Those are x intercepts that you have figured out. You have also seen the end behavior, so the least degree polynomial which will give you that particular function behavior.

Once the least degree polynomial is figured out you use any point on the graph that is why the coordinate axis is important, the numbers are important. You use any point on the graph, in particular y intercept is the easiest and in that case you can determine the stretch factor.

The stretch factor over here is the unknown a that I have told you while figuring out the factors in one of the examples. So, that is the stretch factor. It will be more clear when we will solve the examples ok. So, this is our recipe for attacking the problem of deriving the formula given a graph.

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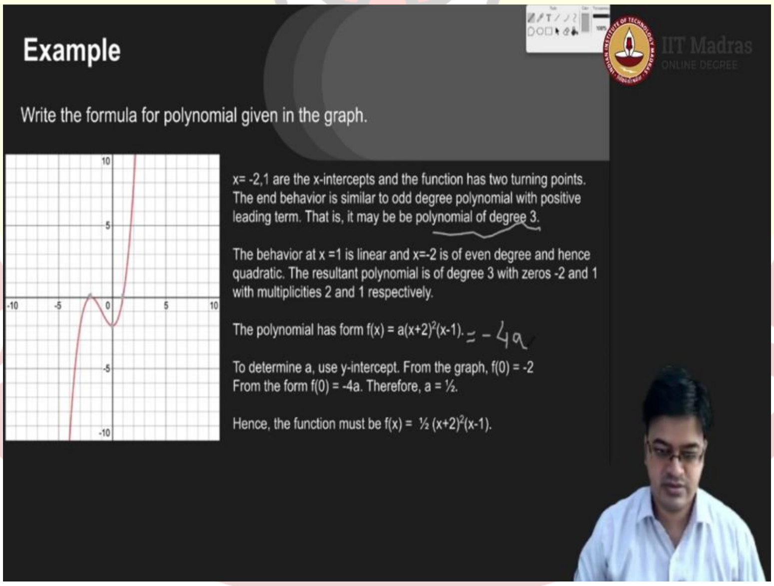

So, let us try to apply this recipe to one example. So, write the formula of a polynomial given in the graph, the graph is here ok. So, I will go around and try to find the x intercepts of this graph. So, one x intercept is here which I think is $x = 1$ and other x intercept is -2. So, -2 and 1 are the x intercepts; y intercept over here is-2, $0-2$ is the y intercept. So, we have identified x and y intercepts.

The graph actually seem to have two turning points. So, the least degree if it has two turning points the least degree polynomial will be because $n - 1 = 2$. So, the least degree polynomial should be cube degree 3 polynomial right ok. And since it is crossing over this end from end behavior also you will have some understanding that it is yeah, it should be an odd degree polynomial. So, therefore, the polynomial may be of degree 3, correct.

It should be an odd degree polynomial; it has only two turning points. So, the least degree of the polynomial is 3. Now what you will do next? Next I want to identify the multiplicities, that is $x = 1$ it the function more or less seems to be linear and at $x = -2$, the function more or less seems to be quadratic.

So, it is very easy in this case because, $x = -2$ is a even degree behavior, x is equal to because it is bouncing off. So, it is a even degree behavior and $x = 1$ is linear behavior and the polynomial is of degree 3 or more, but odd degree. So, the first instance is you guess the function to be of the form $(x + 2)^2(x - 1)$. So, now, I have not yet used the information that the intercept the y intercept is happening at -2, correct.

So, that information I have to use now because that is the function value that I have. These are the based-on factors we are basically equating to 0 right. So, the a may be missed out. So, where the non-zero value comes you should be able to figure out. You can you are free to choose any value, but for me it is better to choose y intercept.

So, y intercept is -2, we have already seen that, but if you put what is y intercept? It is $f\,(0)$. So, if you put this in the function form the value of 0 in the function form over here you will get actually this is to be equal to $-4a$. So, if you are getting this to be equal to $-4a$, then $-4a$ must be equal to -2; that means, $a = \frac{1}{2}$ great.

So, if $a = \frac{1}{2}$you substitute this value into the function. So, $f(x) = \frac{1}{2}(x + 2)^2(x - 1)$, fantastic. So, you have you got an algebraic expression. Now, to match this algebraic expression, you use the technology that is graphing tool to plot the function and you can verify the result for yourself that yes, this is the function that we have actually plotted ok.

So, this is the complete understanding of two-step mission that is; given an algebraic expression how to graph the polynomial function. Given the graph of a polynomial function, how to write an algebraic expression of a polynomial function. This ends our topic on polynomial functions.

Thank you.