IIT Madras

ONLINE DEGREE

Mathematics for Data Science 1

Week-03 Tutorial - Point of Intersection of two lines

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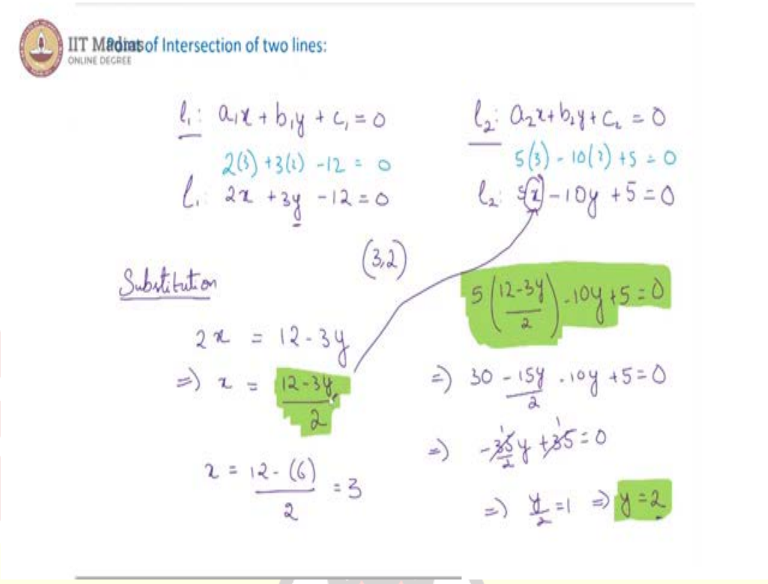

Hello mathematics students. In this tutorial, we are going to learn to find the point of intersection of two given lines. So, you have two-line equations given to you. Let us call one $a_1x + b_1y + c_1 = 0$, let this be line l1, and line l2 is a $a_2x + b_2y + c_2 = 0$. And we try to find out the point at which these two lines intersect. And that would basically be the solution the (x, y)which satisfies l1 and l2 as well. It is easier to observe this process with example. So, let us take 2 example lines and find out where they intersect.

So, for our examples, let us take l1 is $2x + 3y - 12 = 0$, whereas $5x - 10y + 5 = 0$. So, when we have these 2 line equations, how do we solve for x and y. So, the best thing to do is to eliminate one variable, either x or y and get a single equation in the other variable. So, what I mean by that, and this could be done in 2 ways. One way is called substitution. In substitution, in order to remove one variable, we basically express the other in terms of it.

For example, if I wanted to eliminate the y variable, what I do is I express x in terms of y. So, I get all externs on 1 side, so $2x$ is on one side, and the other terms non x terms on the other side, which will give me $12 - 3y$. This would then indicate that x is $\frac{12-3y}{2}$, and then I take this representation of x in terms of y, and substitute it into this equation. What that gives us is, suppose I substituted it, now I will get $5\left(\frac{12-3y}{2}\right) - 10y + 5 = 0$.

So we get $30 - \frac{15y}{2} - 10y + 5 = 0$. That is essentially taking the y common I am going to get $-\frac{15y}{2} + 35 = 0$, canceling off the 35, so I get 1 here, 1 here, that would indicate $\frac{y}{2} = 1$, this implies $y = 2$. So because we eliminated the x here, we got an equation which is entirely in y, which lets us solve for y, and we get the value of y.

Now, to obtain x, we simply have to substitute this value of y in this representation of x, so we will $x = \frac{12-(6)}{2} = 3$. Which means the solution for these 2 line equations is $(3, 2)$, $x = 3$ and $x = 2$. And we can verify this quite immediately by substituting these values into the equations, I will get $2(2) + 3(1) - 12 = 0$. Likewise, $5(3) - 10(2) + 5 = 0$. So it is fairly clear that $(3, 2)$ is the solution which satisfies both linear equations.

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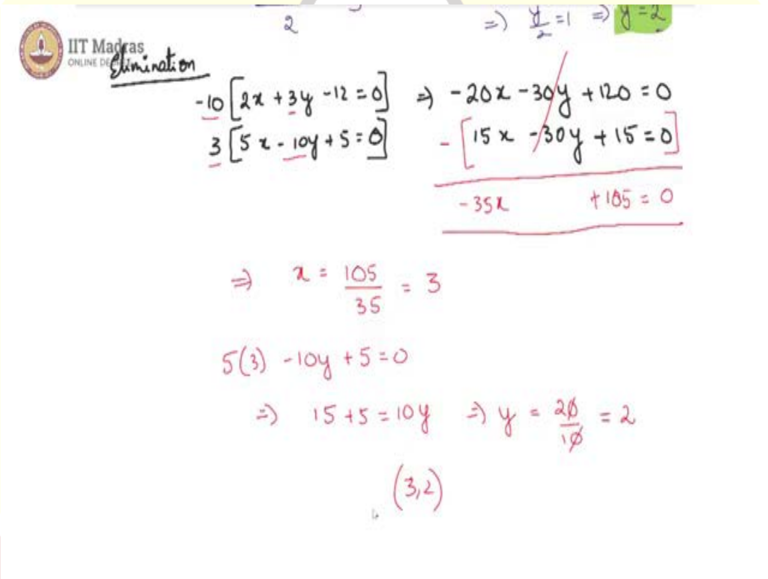

Another method of doing the same thing, which is to solve these 2 equations, we call it elimination. And in elimination, what we do is we again, take these 2 equations, which is$2x + 3y - 12 = 0$, and $5x - 10y + 5 = 0$. We again choose to eliminate either of these variables, because we earlier eliminated x and got an equation in y, now I am going to eliminate y and get an equation x. And for that, what we do is, we multiply this entire equation by the y coefficient in this equation, which is minus 10.

So, I am going to multiply this whole thing with minus 10. And we multiply this entire equation with the y coefficient here in the other equation, that is 3. What that will give us is this would give us minus $-20x - 30y + 120 = 0$. And this gives us $15x - 30y + 15 = 0$. And now what is to be observed is this is $-30y$ and this is also $-30y$, because here we multiply 3 with -10 and here we multiplied -10 with 3.

And that lets us cancel these off, if I subtracted this whole equation from the previous one now. So that will result in $-30y$ by $-30y$ getting canceled, and here, I will get $-35x + 108 = 0$. And this would indicate that $x = \frac{105}{35} = 3$. And now I can substitute $x = 3$ in either of those equations. If I substituted in the second one, I would get $5(3) - 10y + 5 = 0$, this indicates $15 + 5 = 10y$, which gives us $y = \frac{20}{10} = 2$. So, we got our value back, the point back, which is $(3, 2)$. This is the point of intersection of these 2 lines.

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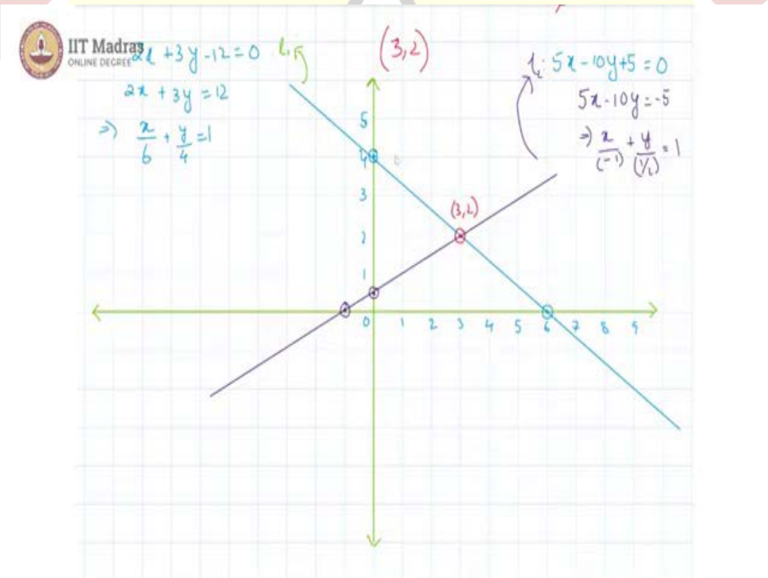

So, if we plotted these, these are our line equations, let us take the first one, I will reduce this to intercept form, which will have to be to $2x + 3y = 12$ is going to give us $\frac{x}{6} + \frac{y}{4} = 1$. So, x intercept is going to be 6, this and the y intercept is going to be 4, which is this and so our line is this is our l1. Now, if we try to plot the other equation, here, again, I will get $5x - 10y + 5 = 0, \frac{x}{-1} + \frac{y}{1/2} = 1$.

So, here we have this is the x intercept, whereas this is the y intercept 0.5 here. So, this is our line equation 2. And clearly the intersection is happening here at this point, which is you can see this is (3, 2). So, in this way, you can try to find the point of intersection of any 2 given lines. However, you are likely to run into a bit of trouble in 2 cases, and let us see those 2 cases.

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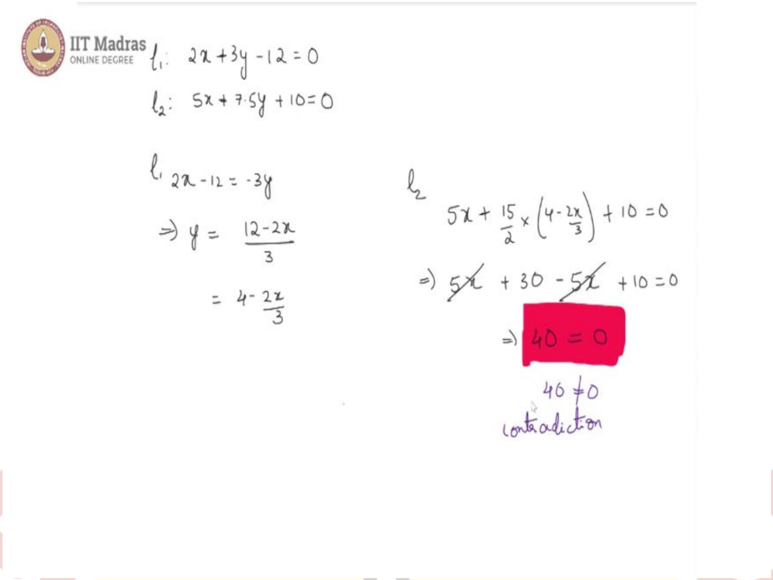

Consider these 2 line equations, 11 is still $2x + 3y - 12 = 0$, whereas $5x - 7.5y + 10 = 0$. If we try to solve this using the substitution method, for example, we would get, I would, let us say I try to eliminate the variable x in which case I should be doing to $2x - 12 = -3y$, which would indicate $y = \frac{12-2x}{3} = 4 - \frac{2x}{3}$. And substituting this in l2, I will get from l2, this is from l1.

And now in l2, if I substituted this, I would get $5x + \frac{15}{2}\left(4 - \frac{2x}{3}\right) + 10 = 0$. This gives us $5x + 30 - 5x + 10 = 0$. And you see that $5x$ and $-5x$ cancels and we come at the strange contradiction where $40 = 0$. And this is not okay right. We know that $40 \neq 0$. So, there is some contradiction we are arriving at.

And what does this contradiction indicate? It indicates that there is no point for which these 2 lines meet. So, you cannot find a point of intersection for these 2 lines. So why is that? That is because they are parallel. If we plotted these lines,

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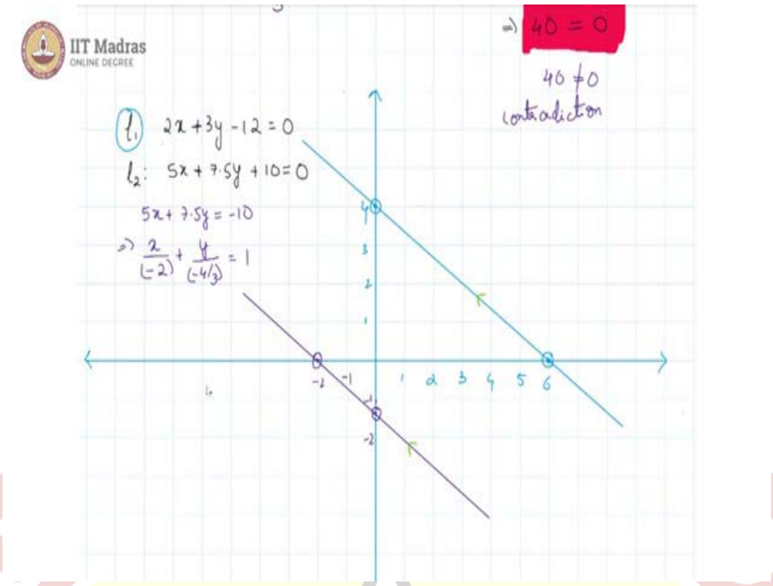

We know that for l1, the intercepts are 6 and 4, respectively. So this is 1, 2, 3, 4, 5, 6. So this is our intercept for l1, x intercept for l1 and y intercept for l1 is 4 1 ,2, 3 and 4. For l2, we have to see now for l2, we get $5x - 7.5y + 10$, which indicates $\frac{x}{(-2)} + \frac{y}{(-4/3)} = 1$.

So, in $x = -2$, so this would be our point and in $x = -\frac{4}{3}$is a little below -1, which is about one third the way from -1 and -2. So, this would be it. If we plotted these lines now we see that these are, in fact, parallel lines. They just do not meet anywhere, which is why when you try to solve for a point of intersection, you get a contradiction. So here, we can say that there is no solution for this system of linear equations.

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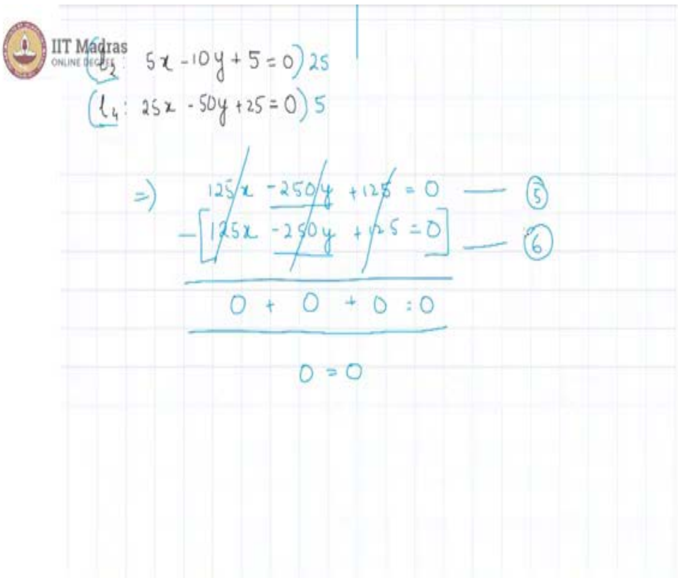

Now, in the third case, let us look at a line equation which is our l2 earlier that was $5x - 10y + 5 = 0$. And there is some other equation l4 let us call it, which is $25x - 50y + 25 = 0$. So, when we solve for these 2 equations, now let me try the elimination method. So, I am going to get 2 equations, then one is $125x - 250y + 125 = 0$. And here I am going to get another one, $125x - 250y + 125 = 0$.

We have the same coefficient for y. So if I attempted to subtract this equation entirely, I will get 0. So, I have this statement, which is always true. Unlike the previous case where it was never true, 40 was never going to be equal to 0, here I get a statement, which is always true, which is $0 = 0$, independent of the coordinates of x and y.

And this means something similar to the previous case, but not exactly the same. What is happening here is since this is always true, it means there are infinite solutions for these 2 equations. If you observe what is actually happening is l2 and l4 are the same line, which is why we got this entirely identical equations, both of these, let us call this equation 5 and let us call this equation 6. And we see that equation 5 and equation 6 are the same, there is no difference, which means our 2 original lines are coinciding.

If they are the same line, then we will get infinitely many points which satisfy both of them. So we have infinitely many solutions for these 2 lines. So whatever x you take, you are going to get a solution for that x. So in the graph, this is what is going to look like.

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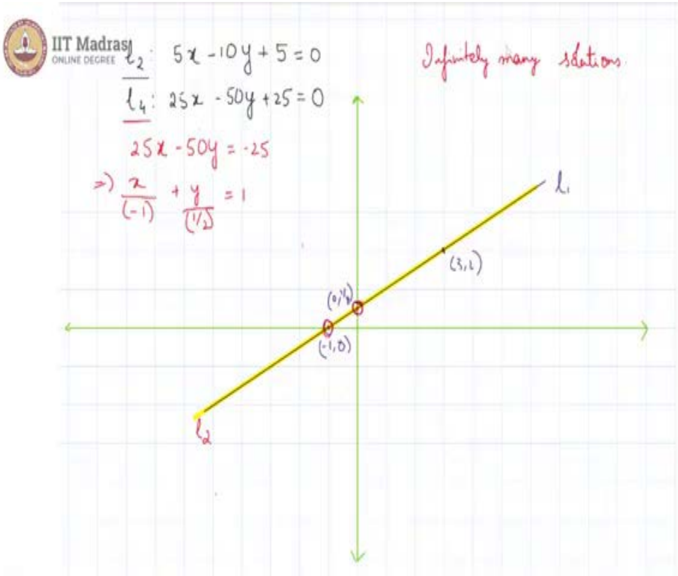

We know the intercepts of our l2, which is -1 and y intercept was half, so this would be our l1, it is passing through $(-1, 0)$, and also $(0,\ 1/2)$. And as we had found earlier, it is passing through $(3, 2)$ as well. Now let us consider the other equation. Now let us consider the other equation which is l4, and we will have $25x - 50y = -25$. This gives us $\frac{x}{(-1)} + \frac{y}{(1/2)} = 1$.

So, again we get the same intercepts. Thus, l2 will have to coincide entirely with l1. And that is what is happening, they are the same line. So, we get infinitely many solutions when we get a true statement, an always true statement independent of x and y in case of the same line, that is both line equations are representing the same line.