

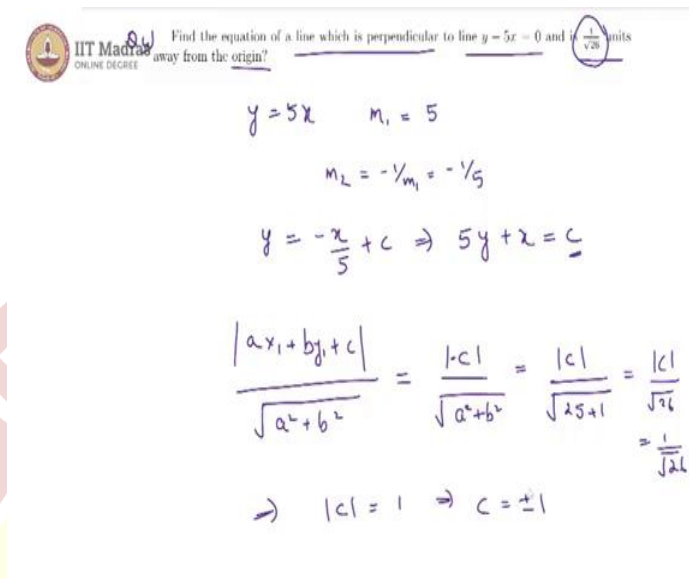


# IIT Madras

ONLINE DEGREE

**Mathematics for Data Science 1**  
**Week-03**  
**Tutorial-04**

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Find the equation of a line which is perpendicular to line  $y = 5x = 0$  and is  $\frac{1}{\sqrt{26}}$  units away from the origin?

$$y = 5x \quad m_1 = 5$$

$$m_2 = -1/m_1 = -1/5$$

$$y = -\frac{x}{5} + c \Rightarrow 5y + x = c$$

$$\frac{|ax_1 + by_1 + c|}{\sqrt{a^2 + b^2}} = \frac{|c|}{\sqrt{a^2 + b^2}} = \frac{|c|}{\sqrt{25 + 1}} = \frac{|c|}{\sqrt{26}}$$

$$\Rightarrow |c| = 1 \Rightarrow c = \pm 1$$

For our fourth question, we want the equation of a line which is perpendicular to this line, and is at this distance from the origin. So, from  $y - 5x = 0$ , we get  $y = 5x$ , so therefore, the slope  $m_1$  is 5. And if our line is perpendicular to it, then our line  $m_2$  must be  $-1/m_1$ , which is equal to  $-1/5$ . So, we know that our line is some  $y = -\frac{x}{5} + C$ .

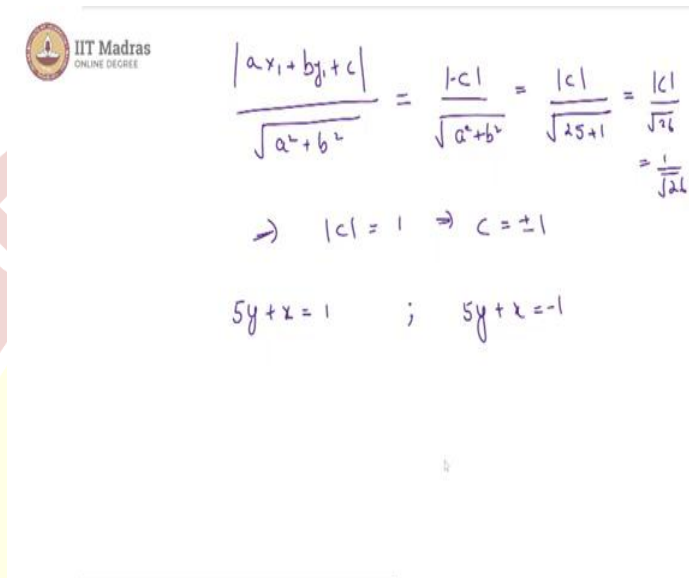
If we kind of simplify it, we are going to get  $5y + x = C$ , this C is not the same thing as the previous C, I have just used that as C because it is an arbitrary constant, which is yet to be determined, otherwise it should have been 5C. Anyway, now we have to find the value of this C in this equation. For that, we are going to use the next bit of information that is given to us, which is the distance from the origin.

So, this line has this distance from the origin. So the distance from a point formula is  $\frac{|ax_1 + by_1 + c|}{\sqrt{a^2 + b^2}}$ , where  $(x_1, y_1)$  is the point from which we are measuring the distance for this

line. So, in our case,  $(x_1, y_1)$  is (0, 0) because we are doing from the origin. So in our case, we get modulus of  $\frac{|-c|}{\sqrt{a^2 + b^2}}$ , So, modulus of  $|-c|$  is just the same thing as  $|c|$ .

And root of  $\sqrt{a^2 + b^2}$ , in our case comes out to be  $\sqrt{25+1}$ , that is  $\sqrt{26}$ . So we have  $\frac{|c|}{\sqrt{26}}$ , this is given out to be  $\frac{1}{\sqrt{26}}$ , which would imply  $|c|=1$ , and that would imply  $c = \pm 1$ . So, we get two answers.

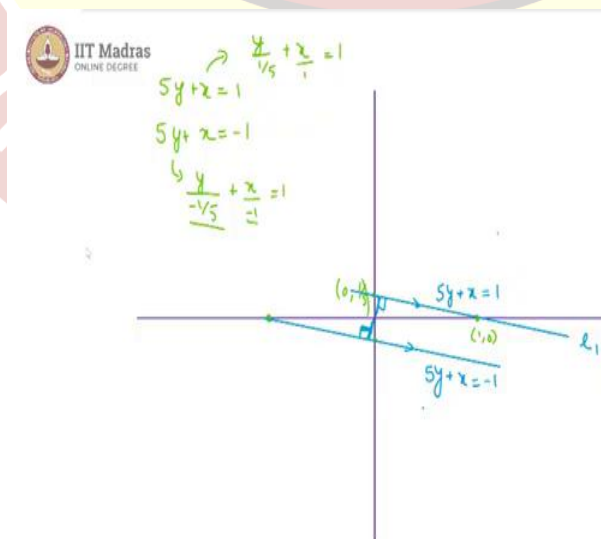
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The slide shows the derivation of the distance from a point  $(x_1, y_1)$  to a line  $ax + by + c = 0$ . The formula derived is  $\frac{|ax_1 + by_1 + c|}{\sqrt{a^2 + b^2}}$ . For the specific case of the line  $5y + x = 1$ , the distance is calculated as  $\frac{|c|}{\sqrt{25+1}} = \frac{|c|}{\sqrt{26}}$ . Setting this equal to  $\frac{1}{\sqrt{26}}$  gives  $|c| = 1$ , so  $c = \pm 1$ . The resulting two lines are  $5y + x = 1$  and  $5y + x = -1$ .

What are the two answers? One is for  $c$  being  $+1$ , we have  $5y + x = 1$ . And in the other case, we get  $5y + x = -1$  for the other choice. So, how does this happen, what is actually happening here to try to plot our lines?

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So, we have two lines, which we were looking at, which is  $5y + x = 1$  and  $5y + x = -1$  and from this we get the intercepts to be, the intercept form of this would be  $\frac{y}{1/5} + \frac{x}{1} = 1$ . And in this case, we get  $\frac{y}{-1/5} + \frac{x}{-1} = 1$ . So, in one case, we have a y intercept of  $1/5$ . So, let us assume this is  $1/5$ , then x intercept is 1 which is 5 times of that, so that so it must be somewhere here, so this would be  $(1, 0)$  and this is  $(0, 1/5)$ .

And our line is going through these two points, giving us something like this. Let us call this  $l_1$  and where do we get the  $\frac{1}{\sqrt{26}}$  distance from the origin, we get it when we measure it perpendicularly from the origin. Now, let us look at the other equation. So  $-1/5$ , so, this should be exactly below this this way and this is  $-1$ , so this would be exactly opposite in this way at the same distance.

So now we have these two points, so we can also construct this line, which goes this way. And as you can see, they are both parallel and exactly opposite to that  $\frac{1}{\sqrt{26}}$  you get this distance which is again perpendicular distance and it is also at  $\frac{1}{\sqrt{26}}$ . So, we have two lines which satisfy our requirements, one is  $5y + x = 1$ , the other is  $5y + x = -1$ .