IIT Madras

ONLINE DEGREE

Mathematics for Data Science 1
Prof. Neelesh S Upadhye
Department of Mathematics
Indian Institute of Technology, Madras

Lecture – 46
Graphs of Polynomials: End behavior

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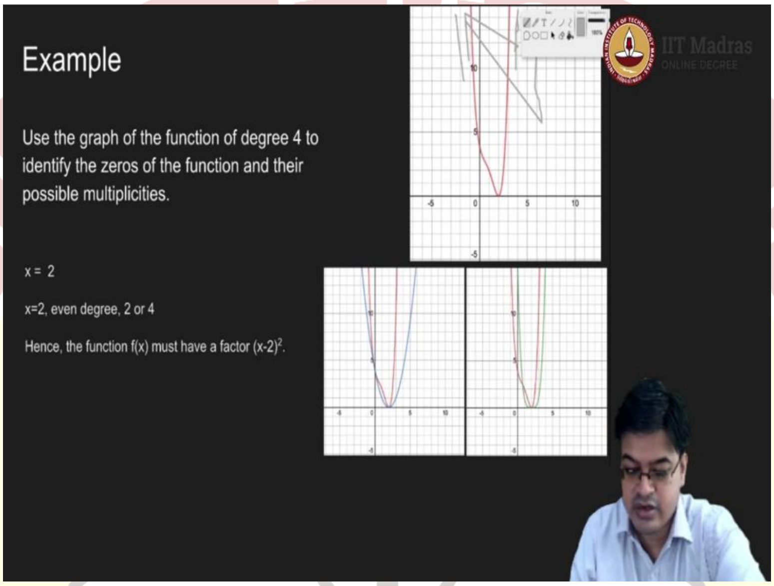

So, now we have understood how multiplicities affect the polynomial and how we are able to find the multiplicities of the polynomial functions with some factors, correct? Still we do not have an answer to a question that what why what is deciding this behavior that this function will go to infinity, this function will go up as usual, how this behavior is decided, we do not have any answer for that.

Let us try to understand that through end through what is called end behavior of the polynomials. So, the next slide is actually the end behavior of the polynomials ok. So, let us go to the next slide, it is end behavior of the polynomials.

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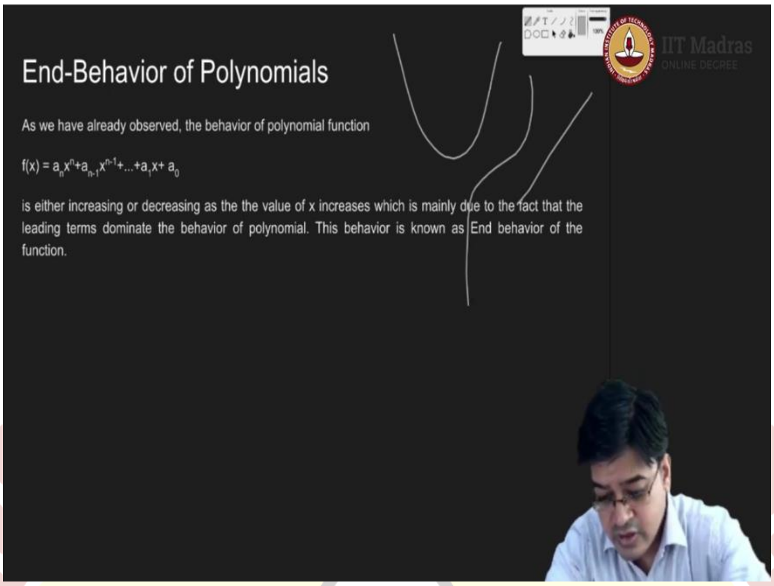

So, what is an end behavior of the polynomial? In order to understand end behavior, let us define an end behavior properly based on our understanding of quadratic equations. So, when we studied quadratic functions, we looked at the term of the form $a_2x^2 + a_1x + a_0$ right and then, we talked about a_2x^2 whether $a_2 > 0$ or $a_2 < 0$, then we decided the behavior of the function.

If $a_2 > 0$, then we said yes, if $a_2 > 0$, the function will take its minimum and therefore, it will go from both sides to infinity. If $a_2 < 0$, then the function will take its maximum and from both sides, it will go down and it will be unbounded. Now, from the graphs that you have seen in the earlier lectures as well as in this lecture, it is very clear; it is very clear that these functions, polynomial functions are either increasing or decreasing based on the way they wish right.

So, for example, it can be like this also. So, or it can go like this also or it can be a straight line as well, if it is linear or it can move like this. All these are polynomial functions. So, now, we want to have a better understanding. So, what is the behavior of a function after it has passed through all the roots is the question right, that is the term that was troubling us a lot.

So, for quadratic equations, we have decided that it is basically based on a 2 because quadratic equation the highest degree is 2. So, a 2 it is. So, now, in a similar manner, if I want to consider a polynomial function $f\ (x)$=$a_n x^n + a_{n-1}x^{n-1} + \cdots + a_0$. Then, the behavior should be decided by this term.

Why should I make this claim? Because if you look at x^n, what we are looking for is as the value of x increases or as the value of x decreases. Now, it is not in that zone, where it is passing through many roots. So, it has passed through all its possible roots and now, after that how the function will behave? There is no determining factor right.

So, in such case, the only determining factor is the term $a_n x^n$; why? Because for large values of x this term x^n will dominate all other terms corresponding to x; x^n raised to n will dominate x^{n-1}, and so on that is when x is becoming large. When x is becoming small that is x is tending to $-\infty$, the term x will be the small x^n will be the smallest possible term or if we that n is of even degree, still it will be the largest possible term.

In any case, the behavior of $a_n x^n$ will play a dominant role in identifying the behavior beyond roots of the polynomials or beyond zeros of the polynomials. This behavior we will call as end behavior of a function and for polynomial functions, it is determined by the leading terms that is $a_n x^n$.

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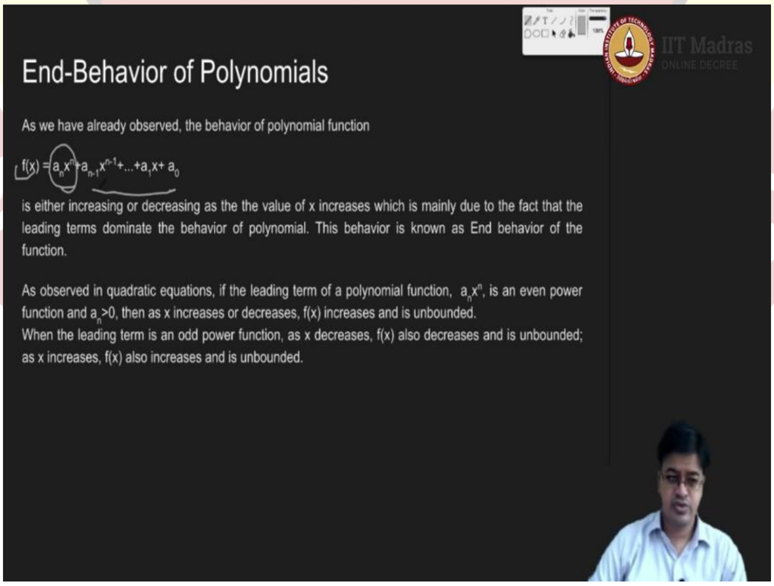

If this $a_n > 0$, and x^n that n is a even power exponent is even, then as x increases or decreases, it is very similar to quadratic. As x increases or decreases, $f(x)$ will always go to infinity. If $a_n < 0$ n is an even exponent, then whether x increases or decreases, $f\ (x)$

will go to $-\infty$. It will go on decreasing. Good. Then, what if $a_n x^n$ that n is the exponent which is of odd power or exponent is odd. What happens?

If $a_n > 0$, then as the function increases, $f(x)$ also increases. If $a_n > 0$ and it is of odd power as x increases, $f(x)$ also increases; as x decreases, $f(x)$ also decreases and both are going to infinity; one is going to ∞, another one is going to $-\infty$. They both are unbounded. Similar thing can be argued for $a_n < 0$. So, in order to improve our understanding, I have tabulated this zone; 1 minute, let me remove this part ok.

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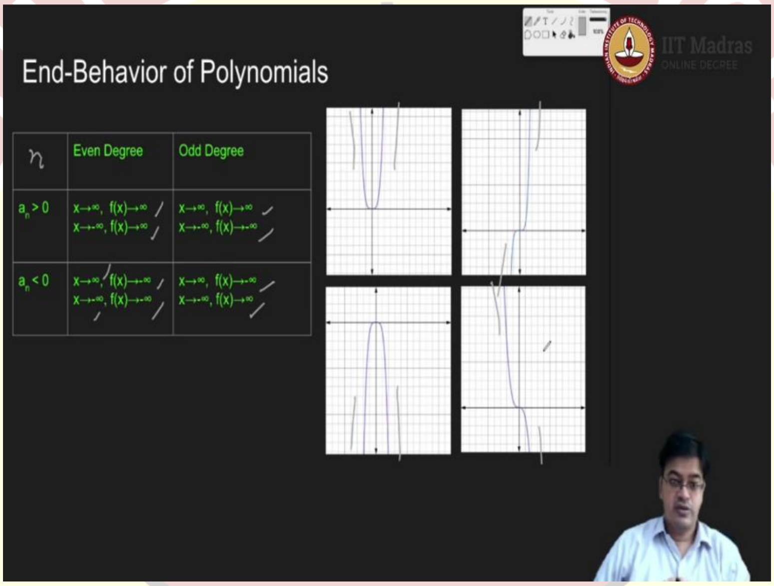

So, this is the better understanding. So, now, you look at the leading term $a_n x^n$ So, this is referring to n, n is of even degree, n is of odd degree. So, if n is of even degree and $a_n > 0$, x tending to ∞, x becoming larger and larger, $f(x)$ will become ∞; $f(x)$ will also increase. $a_n > 0$, x tending to $-\infty$; that means, x is becoming smaller and smaller and smaller; but because the polynomial is of even degree, it will again go to ∞.

In a similar manner, if $a_n > 0$ and the polynomial the leading exponent is of odd degree, then as x tends to ∞, $f(x)$ tends to ∞. You can imagine a function of the form x^3. Similarly, if $a_n > 0$, x tends to $-\infty$, $f(x)$ will also go to $-\infty$because $f(x)$ will also keep decreasing.

Remember polynomials of odd degree crossover x axis, if you link that point to this, then naturally it is very easy to and visualize the behavior of the polynomials. I will demonstrate these two graphs again, once again to reiterate the point. If $a_n < 0$, now $a_n < 0$; that means, x becoming larger, $f(x)$ the term, the leading term of $f(x)$ will be negative more and more negative.

So, $f(x)$ will tend to $-\infty$; but if x is becoming smaller and smaller, the exponent is of even degree, still $f(x)$ will again go to $-\infty$ because $a_n < 0$. Come back to odd degree, here the exact replica of what we have done for odd degree when $a_n > 0$ will happen.

So, in this case when $a_n > 0$, x tending to ∞, we will make bring this $f(x)$ to go to ∞; but in this case, it will bring it to $-\infty$ and similar case is true for the other part that is x tending to $-\infty$, $f(x)$ will tend to ∞. Let us visualize it through graphs.

Let us take this first block even degree $a_n > 0$. Imagine a function of the form x^2 or x^4 as x tends to ∞; both of them are going up. Just remember this figure that will clear this understanding.

Let us go to odd degree with $a_n > 0$, as x tends to ∞, here this is going up. This is going down right. Just imagine a figure of x^3 for the convenience. When $a_n > 0$, just imagine a figure of $-x^2$ or $-x^4$, both of them should naturally go down. That is what is written here as well. In a similar manner, just consider $-x^3$.

So, whatever was going down, will go up and whatever was going up, will go down that is what I meant when I said this. So, now we have much better hold over end behavior of polynomials. Now, you can look at the graph of a polynomial function and you by looking at the end behavior, you can say whether the polynomial, the leading term of the polynomial is of odd degree or even degree.

That is one more understanding, one more level of understanding that we have achieved through understanding this end behavior. But that is not over. We further need better understanding of the functions.