Statistics for Data Science -1

Lecture 10.1: Bernoulli Distribution

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Learning objectives

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2. Understand the effect of parameters n and p on the shape of the Binomial distribution.
3. Expectation and variance of the binomial distribution.
4. To understand situations that can be modeled as a Binomial distribution.

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Non Bernoulli trial

- Experiment: Randomly choosing a person and asking their age.
- Not Bernoulli- Outcomes are not 2.

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- Variance of a Bernoulli random varaible:

$$V(X) = p - p^2 = p(1 - p)$$

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- In other words, the most uncertain Bernoulli trials, those with the largest variance, resemble tosses of a fair coin.

Section summary

- Bernoulli trial
- Bernoulli random variable