IIT Madras
ONLINE DEGREE

Statistics for Data Science 1
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Lecture No. 8.2
Random Variables - Application

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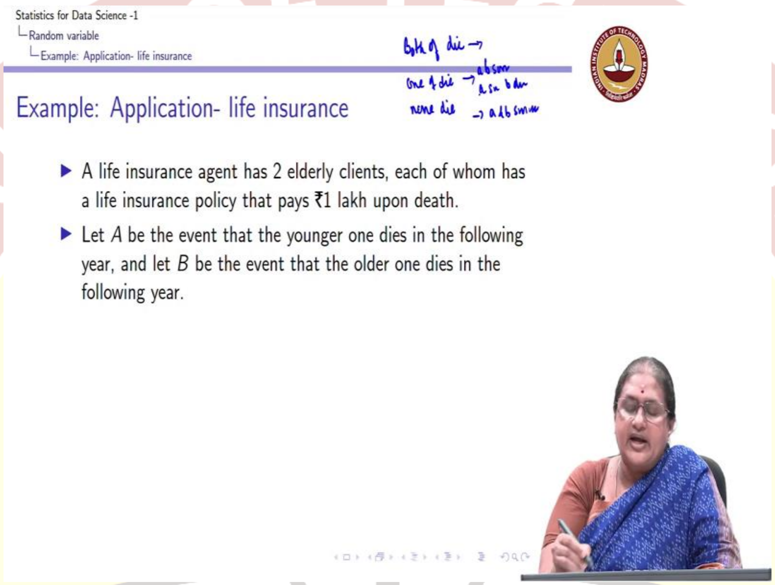

We just looked at a few generic examples of how we can define a random variable, we talked about mapping the random variable to a particular event. And talking also now, given the probability of an event we associated probability of the random variable taking a particular value. So now, let us look at an application wherein we define a random variable and we get the probability of these random variables through the events they correspond to the application is application from life insurance.

So, I have a life insurance agent who has two elderly clients. Both of them have a life insurance policy that pays 1 lakh upon death rupees 1 lakh upon death. So, this is a hypothetical case, I have two people, and both of them have insured life insurance and they will get a pay-out that is the people the survivor, the nominees should get a pay-out of 1 lakh upon the death. So, I am defining two events. Now, what are what is a random experiment here?

So, the random experiment is I want to know, in the next year, in the following year, what are the possible things that could happen, the possible things that could happen is both of them die in the following year, both of them die, or one of them die, or none of them die, these are the possible things that could happen in the following year. Now, when both of them die, I know both die one of them die. So, if I am calling A and B, A dies, B survives or A survives and B dies, or both A and B survive, these are the possible outcomes if I am talking of this in terms of a random experiment.

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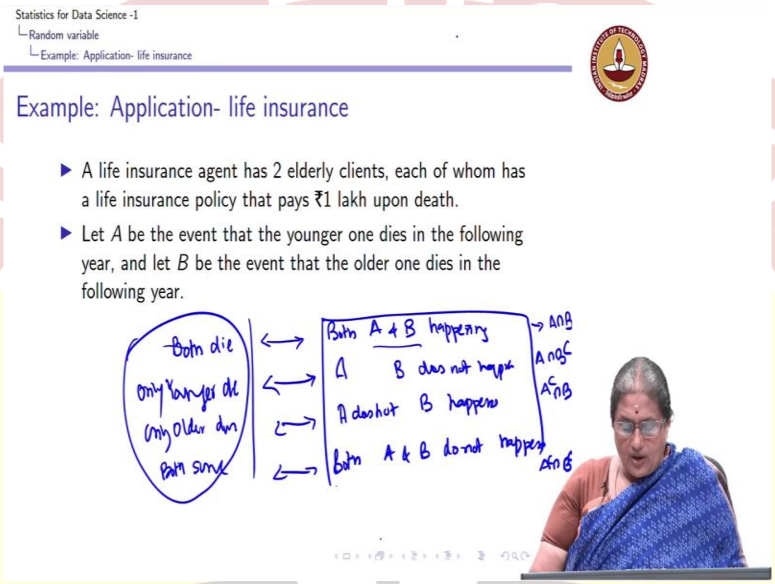

So now, let A be the event that the younger one dies. So, between the two of them I assume one is older, and one is younger, so I am calling A I am letting A to be the event that the younger one dies in the following year, and B to be the event that the older one dies in the following year. So, I said what are the outcome, the outcome is both die. So, this corresponds to both event A and B happening, younger one dies older one survives only A happens B does not happen. Older one dies only older one only younger one dies, only older one dies so, here A does not happen B happens both survive both A and B do not happen. I hope this is clear. So, I have just first these are the possible outcomes.

And these I have listed or mapped each one of the possible outcomes to in terms of the event, when both die I say both events A and B happens and I know mathematically I can express this as $A \cap B$ only the younger one dies, but the older one does not die, it is $A \cap B^c$. A does not die A^c happens, but and B happens $A^c \cap B$. Both A and B do not die, So, $A^c \cap B^c$. So, in terms of events, we can translate whatever we are expressing in plain English in the mathematical abstraction.

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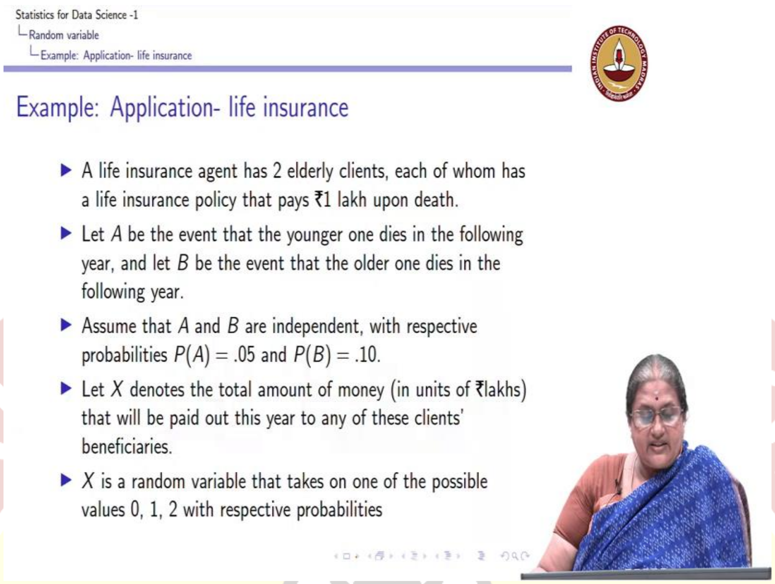

Now, we want to associate so now the next thing is, I assume A and B are independent. This is absolutely a very valid assumption because when I am talking about two people dying, it need not be the case that once death affects anybody so the events are independent and I am associating some subjective probabilities perhaps I am associating a probability A which is very low with the younger one and probability B which is point one with the older one.

Now, let X denote the total money that the company pays out in this year to its clients beneficiaries in amount of lakhs. So, what is that the company note will pay out money only if there is a death, let us assume that the company pays out the money only if there is a death. So, x is a total amount of money that is paid out to any one of the clients beneficiaries.

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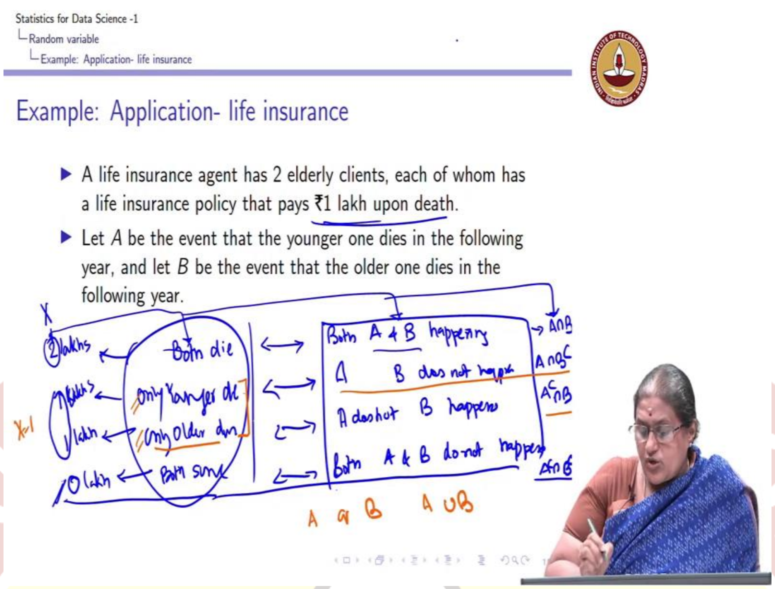

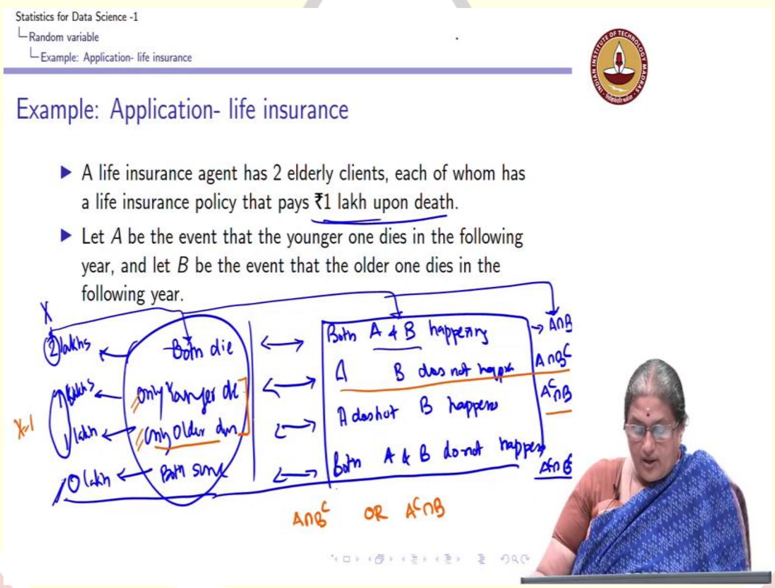

So again, let us go back to our earlier thing I had both dying. So, when both die, the company has to give money. When both die it gives money to both the beneficiaries, when the younger one dies it gives and both of them have insured for the same amount, so when both die both of them have insured for 1 lakh upon death, so I will have to pay 2 lakhs to both the beneficiaries only the younger one dies, I pay 1 lakh to the younger ones family, older one dies, I pay 1 lakh to the older one family, when both of them survive, I do not have to pay anything. So, my payment is 0 lakhs.

So, if you look at the value this X is taking, it is taking the value 0, it is taking the value 1, and it is taking the value 2. I repeat the value X is taking X is taking the value 0, X is taking the value

1, and X is taking the value 2. And you can see that X taking the value 2 is corresponding to the event that both of them die which again corresponds to both A and B happening which is written as $A \cap B$.

Similarly, the value of 0 corresponds to both surviving which is same as both A and B do not happen, which is $P(A^c \cap B^c)$. But you can see that the probability of X taking the value 1 can happen if only the younger one dies, or only the older one dies only the younger one dies corresponds to $A \cap B^c$, only the older one dies corresponds to $A^c \cap B$.

So, if one of these two events happen, I know X takes the value 1 we have already seen earlier, A or B happens is represented by $A \cup B$ here I am not talking about A or B, I am talking about $A \cap B^c$, or $A^c \cap B$ for the value of X to be equal to 1.

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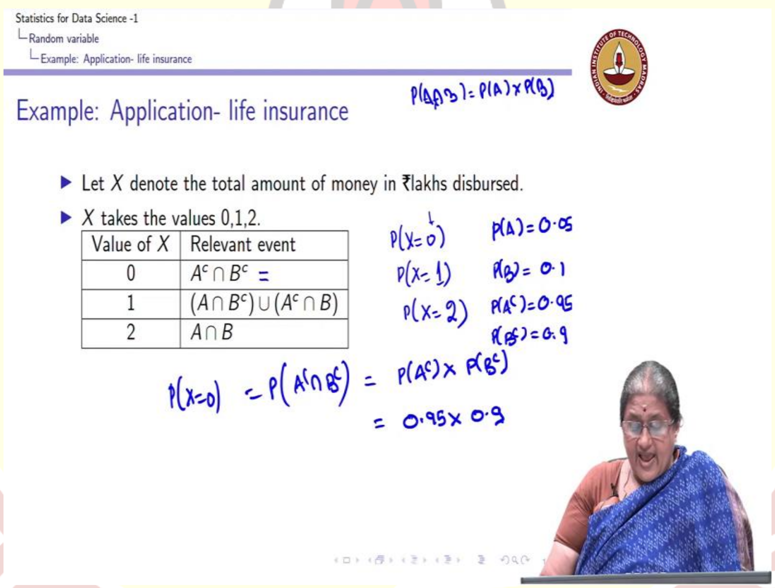

So we can summarise the discussion by saying that X takes the values what are the values X takes? X takes the value 0, the relevant event is A compliment intersection B complement that is what we just established a couple of minutes before that this A X takes the value 0 is with A compliment intersection B complement takes the value 1 with $(A \cap B^c) \cup (A^c \cap B)$.

And X takes the value 2 with $P(A \cap B)$ it is equivalent to A intersection B so X takes the value 0 relevant event is $A^c \cap B^c$ 1 it is $(A \cap B^c) \cup (A^c \cap B)$, it takes the value 2 these are the only 3 values X can take is $(A \cap B)$. Again as in the previous case, I can associate probabilities to I want to know what is the probability with which X does not give out any money probability with it X gives out a pay out of 1 lakh probability with which X gives out a pay out of 2 lakhs. See these are the numerical values associated with the outcomes of the experiment.

So, you can see that probability X equal to 0 is same as $P(A^c \cap B^c)$, which I know if A and B are independent, A^c is also independent of B^c. And we also know$(A \cap B)$ is $P(A) \times P(B)$, this is a multiplication rule. So, I have this is $P(A^c \times B^c)$. What is given to us is $P(A)$ is 0.05. $P(B)$ is 0.1, which gives us $P(A^c)$ is 0.95 and $P(B^c)$ is 0.9. Hence, I have $P(A^c)$ is 0.95 into a 0.9.

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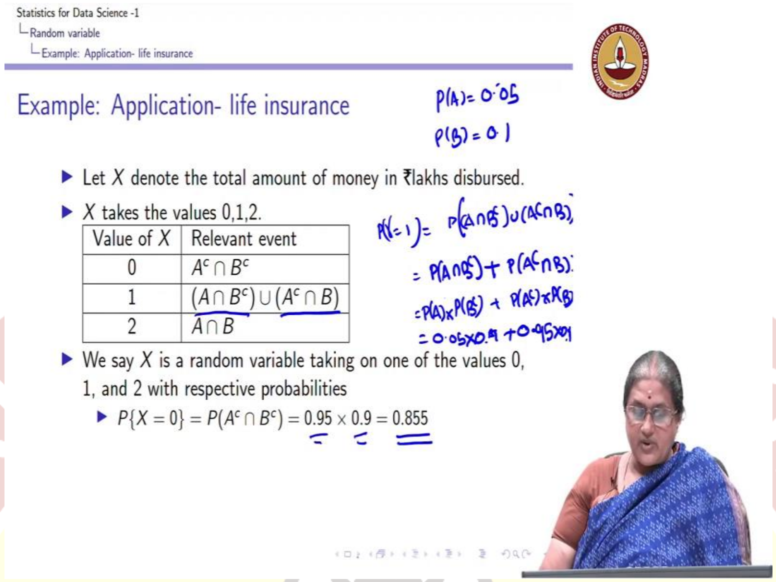

So, I have probability X takes the value 0 is 0.95 into 0.9, which is given by 0.855. Similarly, $P(X) = 1$, is again $P((A \cap B^c) \cup (A^c \cap B))$., we can check that these two are disjoint sets. And we know for any two disjoint sets $P(A \cup B)$ is $P(A) + P(B)$.

This is my addition rule for disjoint sets, I can use that I know probability X equal to 1 is $P((A \cap B^c) \cup (A^c \cap B))$, which is $P(A \cap B^c) + P(A^c \cap B)$. Again, if A and B are independent, A is independent of B^c. So, this would be $P(A) \times P(B^c) + P(A^c) \times P(B)$.

Again, we know $P(A)$ is 0.05 $P(B)$ is 0.1. So, I have this as $0.05 \times 0.1 + 0.95 \times 0.9$ here 0.9 here into 0.1 0.05 into 0.9 plus 0.95 into 0.1 that is my probability X equal to 1 which is $0.05 \times 0.9 + 0.9 \times 0.1$ which is 0.140.

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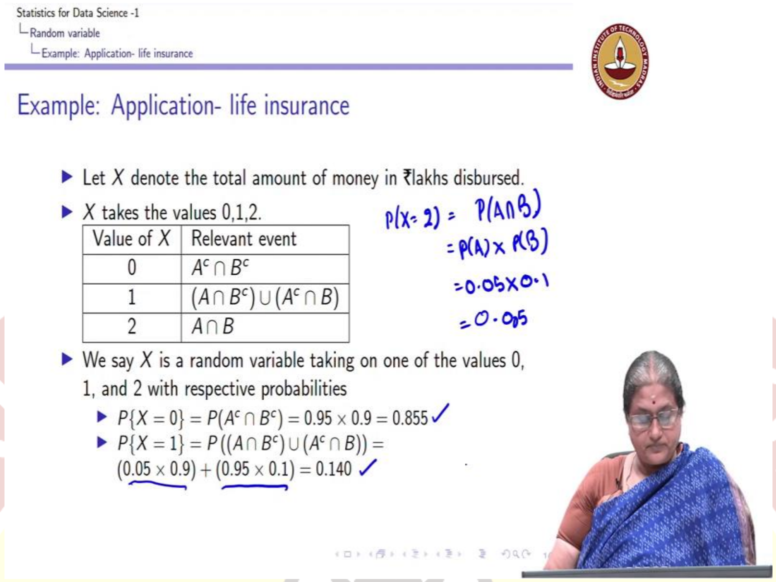

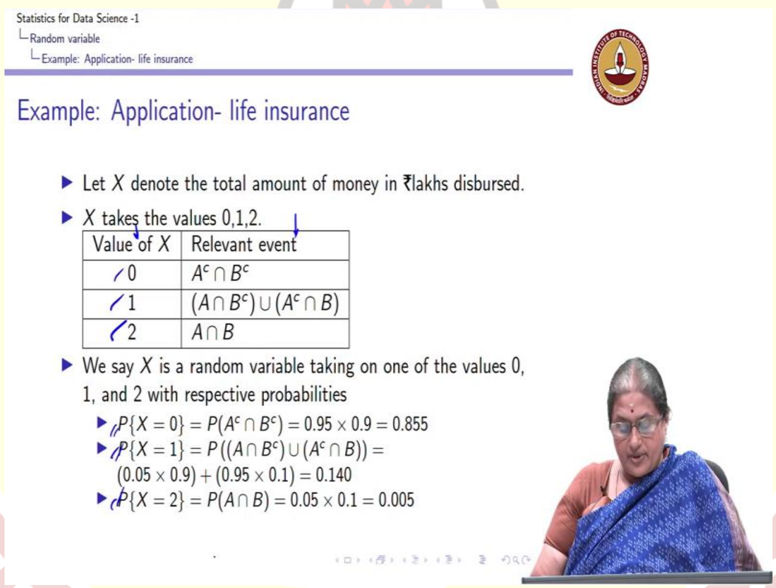

Finally, I want to know what is probability X equal to 2, this is $P(A \cap B)$ again A and B are independent. So, this is $P(A) \times P(B)$, we know $P(A)$ is 0.05 $P(B)$ is 0.1. So, I can get the probability of A X equal to 2 is 0.005. So, what we have seen is, I have from given the events A and B and the respective probabilities I am associating the relevant events to the values X can take to get what is the $P(X = 0)$, $P(X = 1)$, and $P(X = 2)$, . So, this is a simple application of how we define a random variable and associate probabilities to that random variable.

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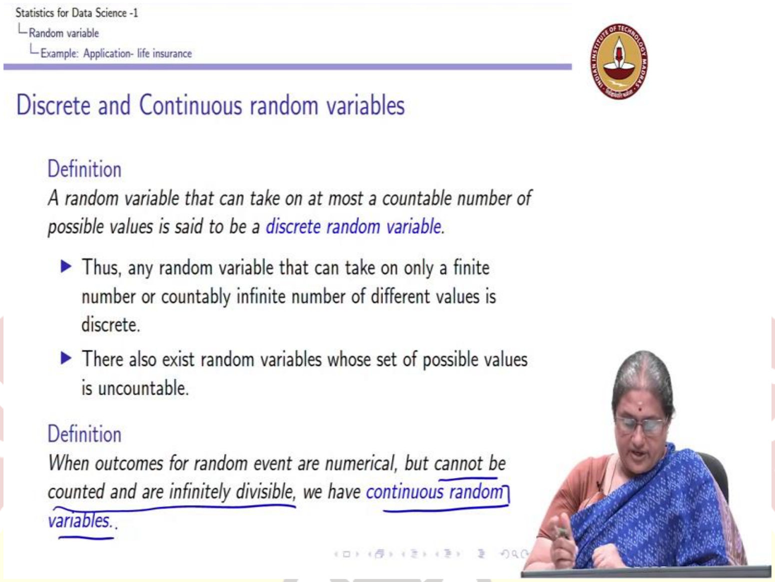

So, what is next, so if you look at what we are going to do in the first portion of the next few weeks is we are going to look at discrete random variables. So, what is a discrete random variable, a discrete random variable can be formally defined as a random variable that can take on at most a countable number of possible values. In other words, we will look at examples where a random variable can take on a finite number of values.

For example, X takes the value 0, 1, 2, 3 finite number of values or X could take a value x 1, 2, 3, 4. So, for countably infinite but it has to be countable. You have already learned about what is countable sets or what is properties of countability in your mathematics courses, but what I want you to understand is a random variable which can take on a finite number or a countably infinite number is referred to as a discrete random variable.

Now, there could be random variables whose set of values are uncountable. Now, I am not going to formally define a continuous random variable, but then, when the outcomes of a random variable are numerical, but cannot be counted on and infinitely divisible, we have continuous random variables. The formal definition of a continuous random variable is slightly more involved. I will come to it when we specifically look at the continuous random variable case. But for now, we are going to focus on discrete random variables.

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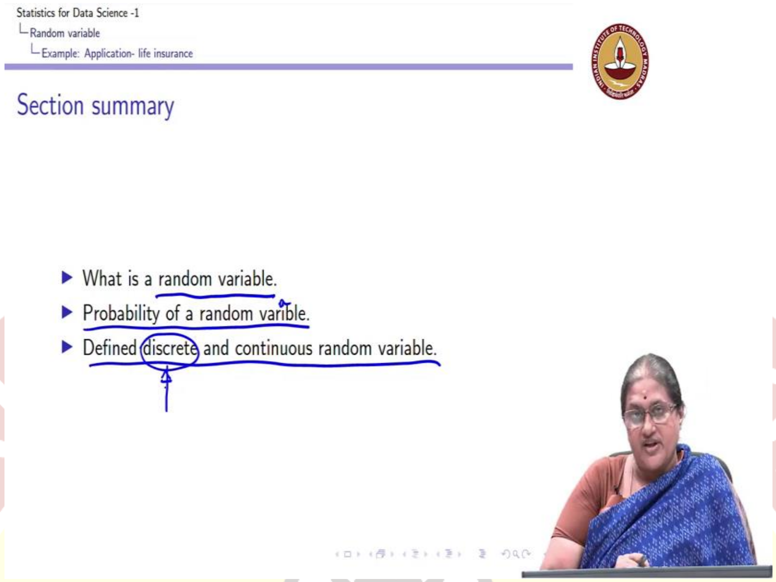

So, in summary, what we are learned in this module is, we introduce the extremely important concept of what is a random variable, namely a numerical quantity associated with outcomes of a sample space. And then through actually mapping it to relevant events from the sample space, we computed what was the probability of a random variable, this is what we did. Then, we define what are discrete and continuous random variables, we did not give a very rigorous definition of a continuous random variable. But whenever a random variable takes countable number of values, we typically refer to them as discrete otherwise, and we refer to them as continuous random variables.

So, moving forward, we would focus on a discrete random variable. And how do we describe these discrete random variables, in terms of its distribution is what is going to be of interest next.