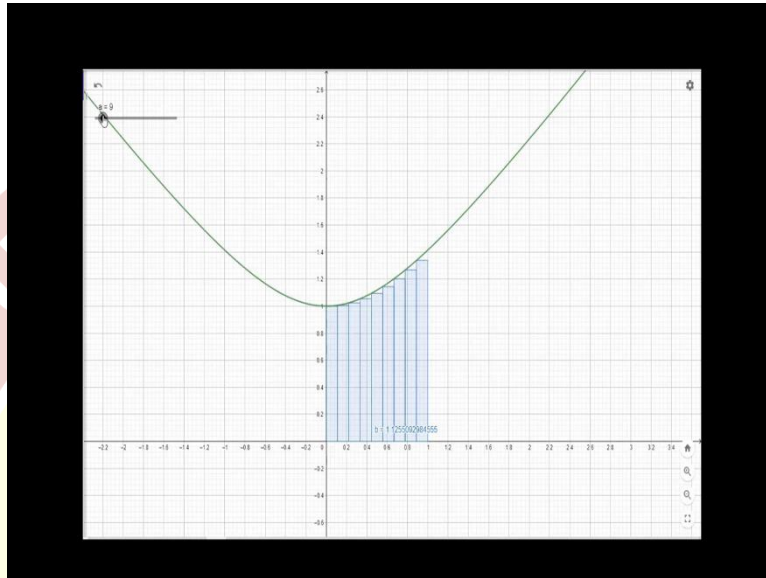


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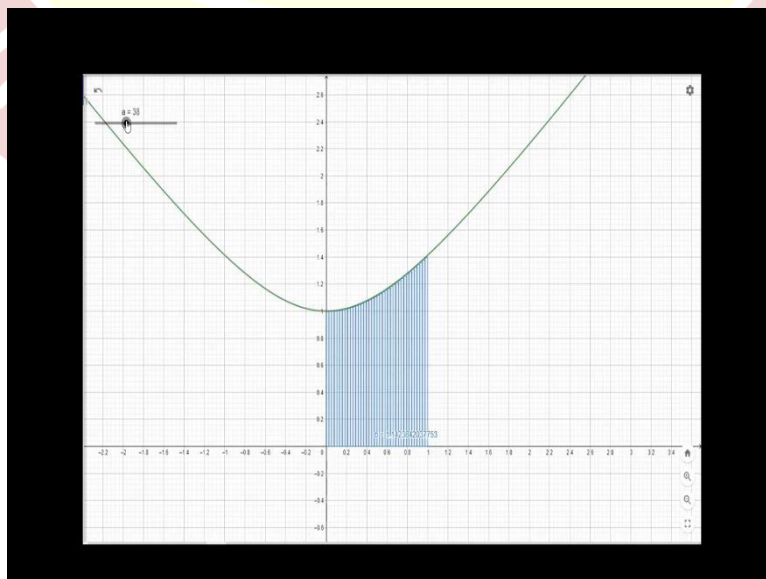
**Mathematics for Data Science 2**  
**Professor Sarang S. Sane**  
**Department of Mathematics**  
**Indian Institute of Technology Madras**  
**Week 03 - Tutorial 04**

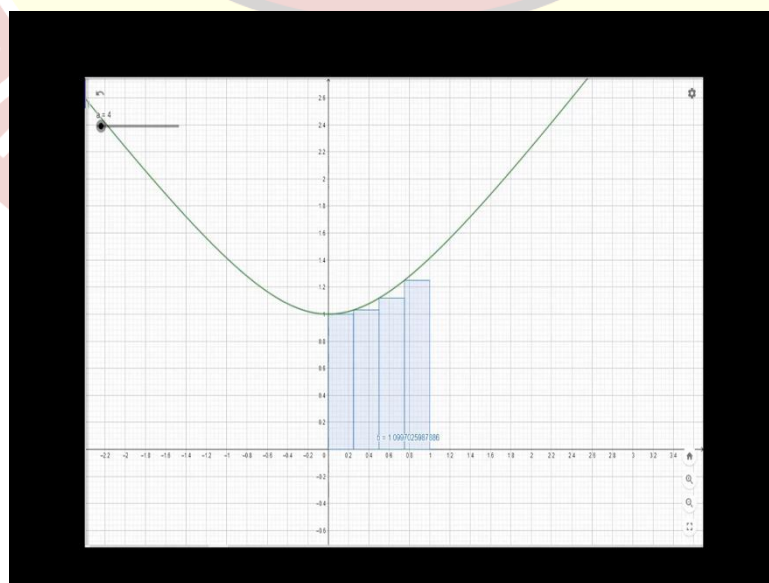
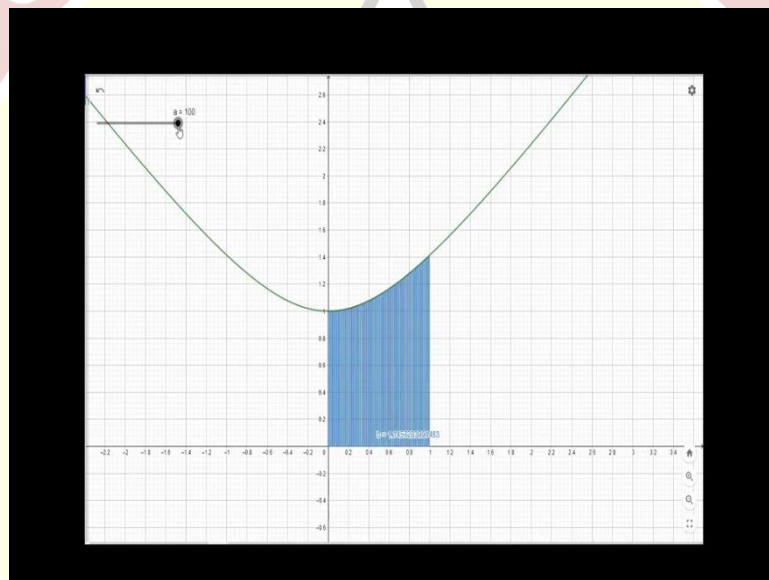
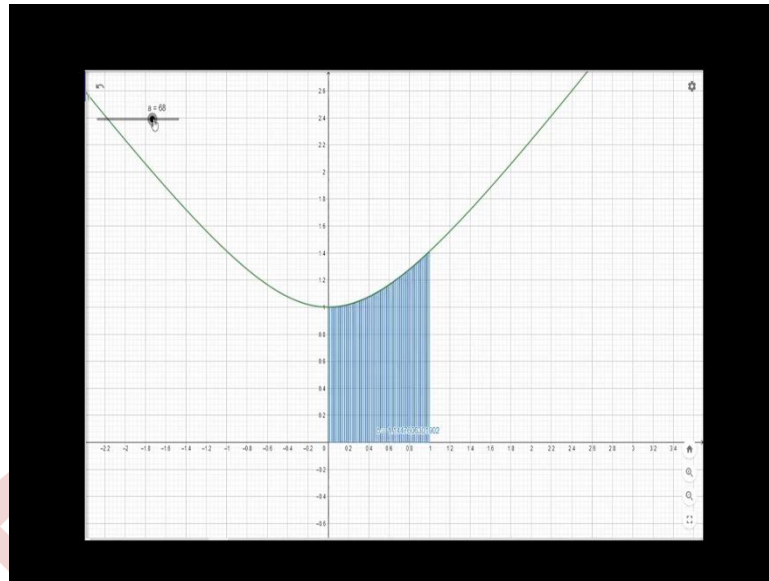
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Hello everyone, so this video, this tutorial video is only for a visualization of a particular function. So, here we can see that this function is basically a function  $\sqrt{x^2 + 1}$ . So, this is the curve representing that and we are trying to approximate the area under the curve in the interval 0 and 1. So, and this  $n$  denotes the number of sub interval, so this is the left sum as we have seen in the earlier two videos.

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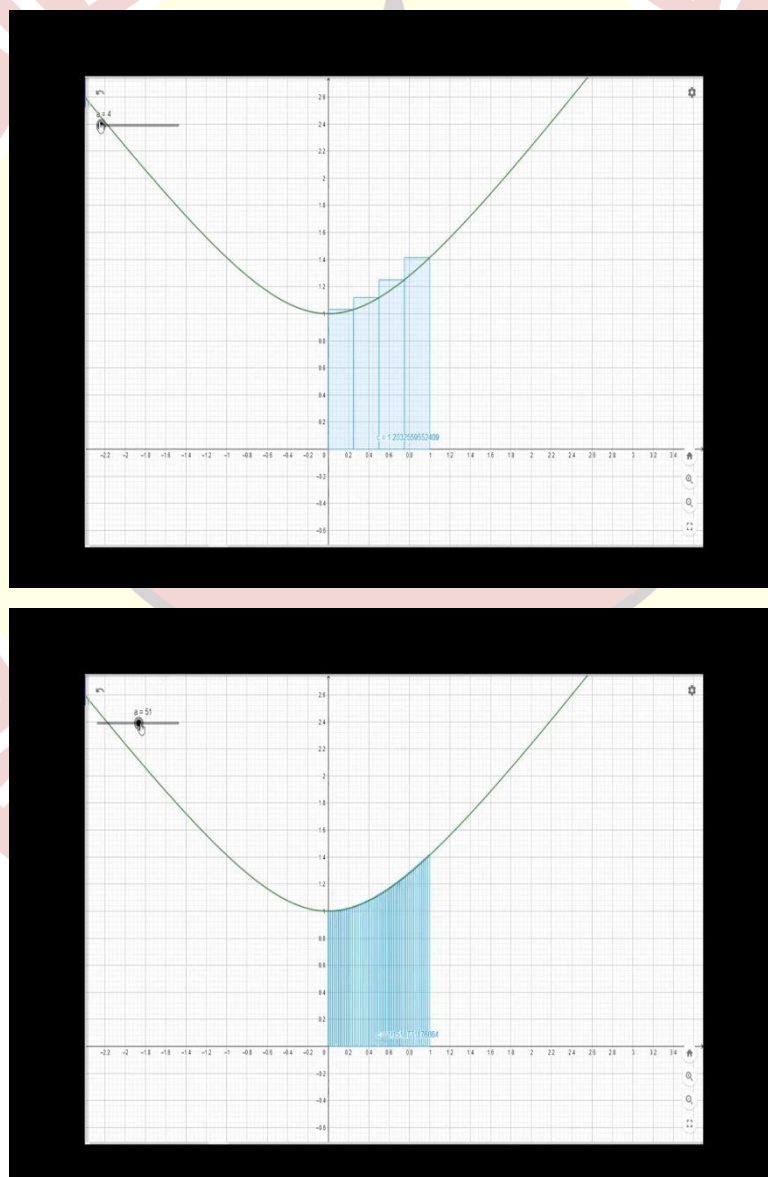


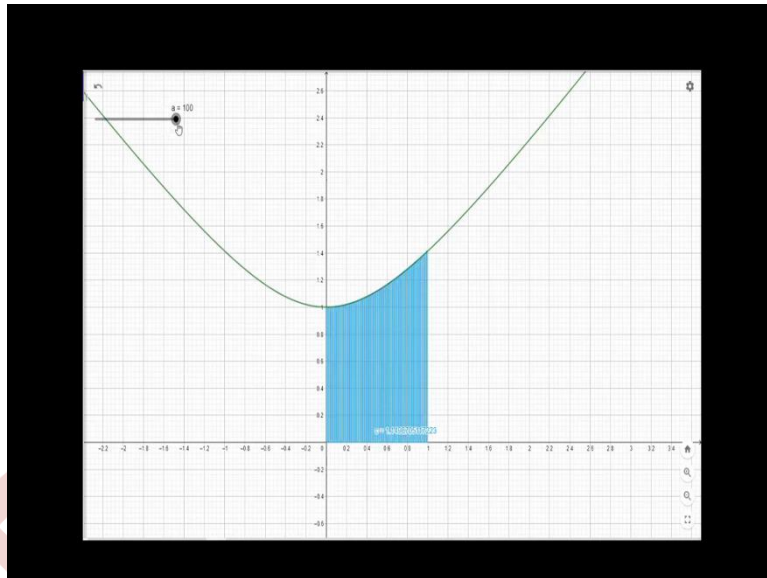


So, as the number of sub interval increases, you can see that the area under, that the area which we are calculated the Riemann sum that is actually approaching the actual value of the integration  $\int_0^1 f(x)dx$  in 0 and 1. So, that is the area under the curve bounded by  $x$  axis. As you can see as I am approaching, increasing the number of sub intervals, we are getting closer and closer to that value.

And as the number of sub interval decreases, we can see the error increases that is, we are going further away from the area under the curve. So, at  $n = 4$ , you can see there are some errors, so this will give the area as 1.099 something. So, this is the left sum, left Riemann sum.

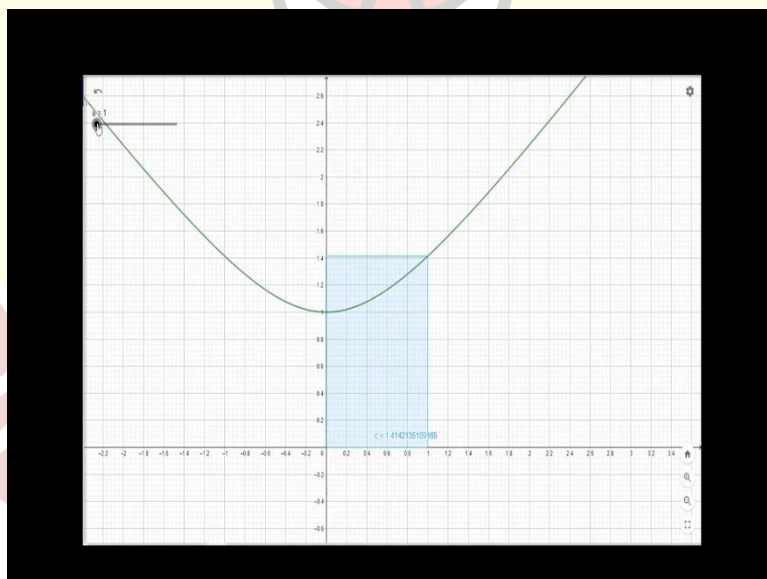
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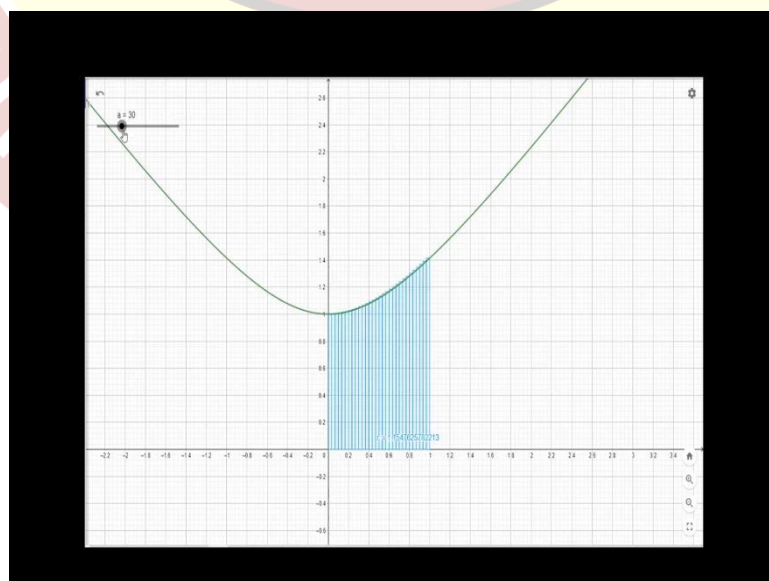
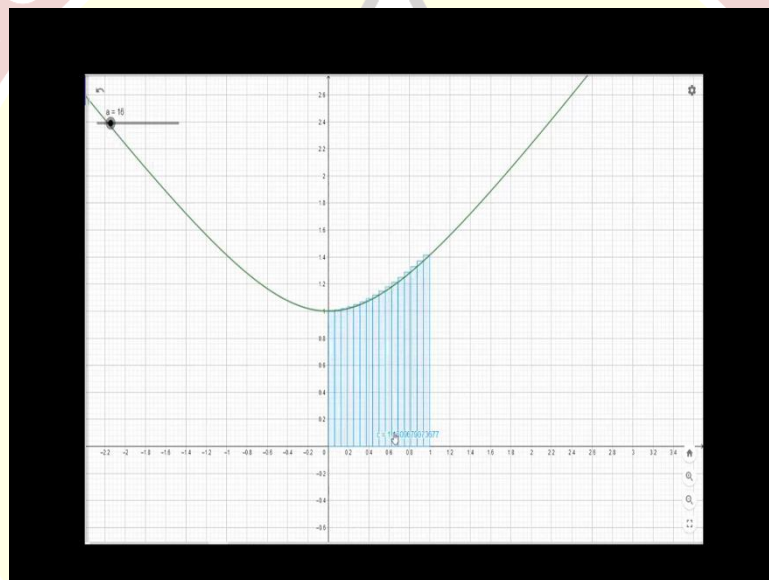
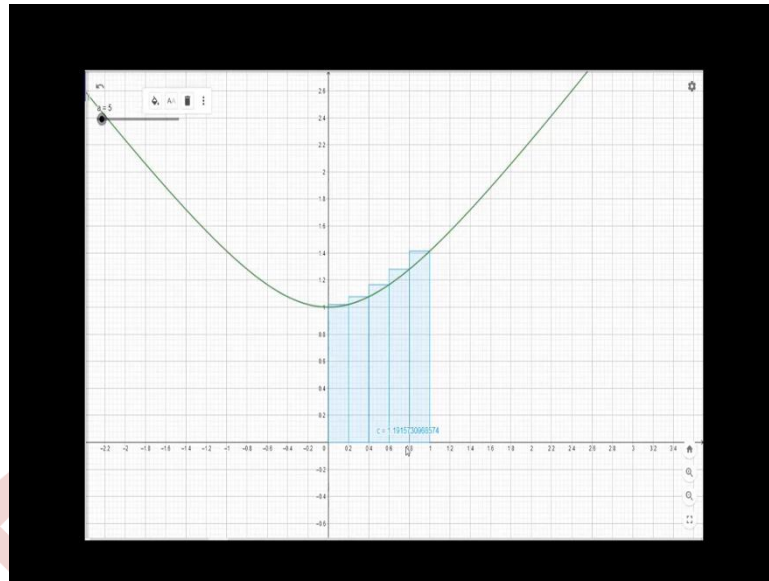




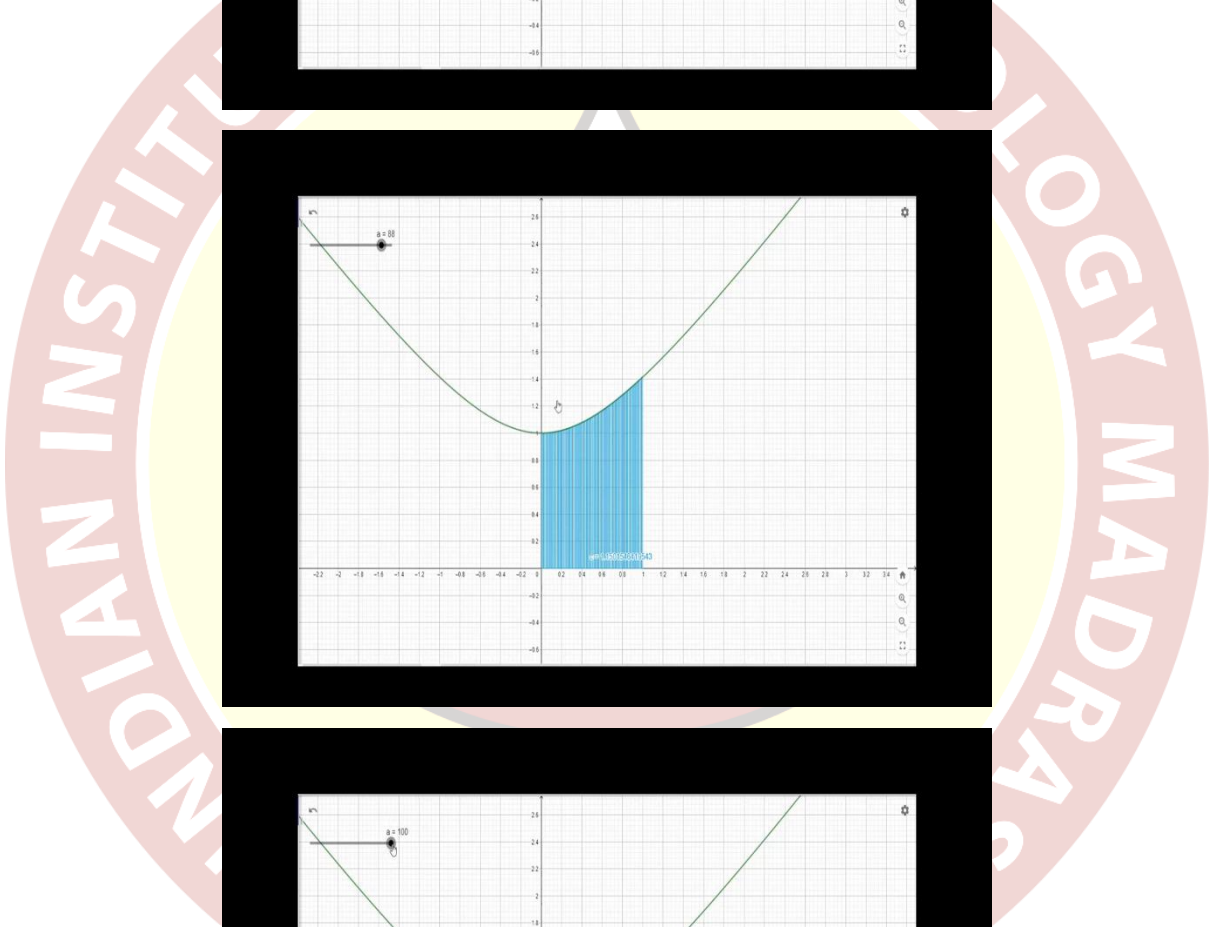
Now, if we consider the right Riemann sum for 4 sub intervals, you can see, we are getting more radius, so we are getting further away from the area under the curve and you are getting the value 1.203 and so on. And as we are approaching, so as we are increasing the number of sum intervals, we are approaching the actual values. That is the main point of this theorem.

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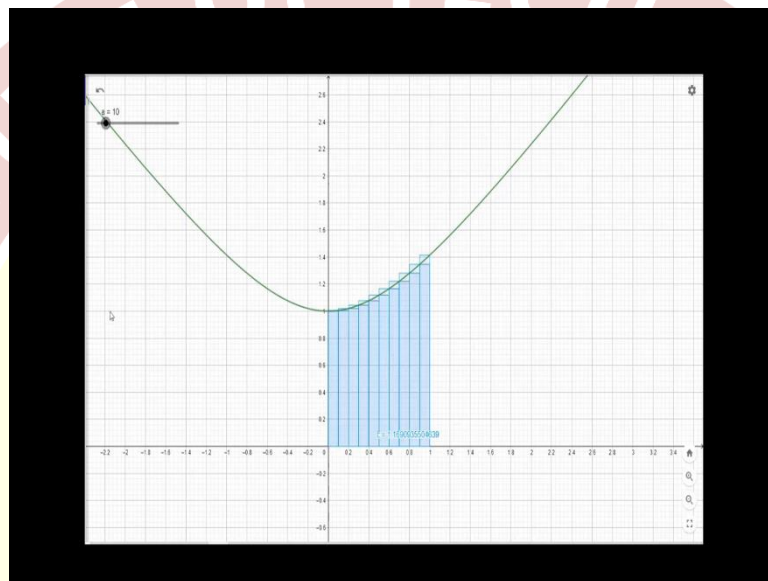




So, let us, let me show you the visualization. This is for one sub interval itself. So, you get the area 1.414. For  $a = 5$  you can see that comes towards 1.19. For  $a = 16$ , it has come 1.16,  $a = 30$ ; 1.15,  $a = 48$ ; 1.1521. So, here come, coming closer to closer to the actual values. So,  $a = 88$  it is giving 1.1501 and  $a = 100$ , it is giving us 1.14.

So, we can see the animation now how we are approaching to the actual value. This is for the right sum or the upper sum. Now, for the left sum if you we can see, this is for the left sum.

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And suppose for  $a = 10$ , we are seeing both the sum, so you can see the below things are for left sum and the upper thing which are coming from the upper section of the curve that is the upper sum of the upper Riemann sum. So, for  $a = 10$ , we can see this two together here. Thank you.