INDIAN INSTITUTE OF TECHNOLOGY MADRAS

सिद्धिर्भवति कर्मजा

IIT Madras

ONLINE DEGREE

Mathematics for Data Science 1
Week 05 - Additional Lecture

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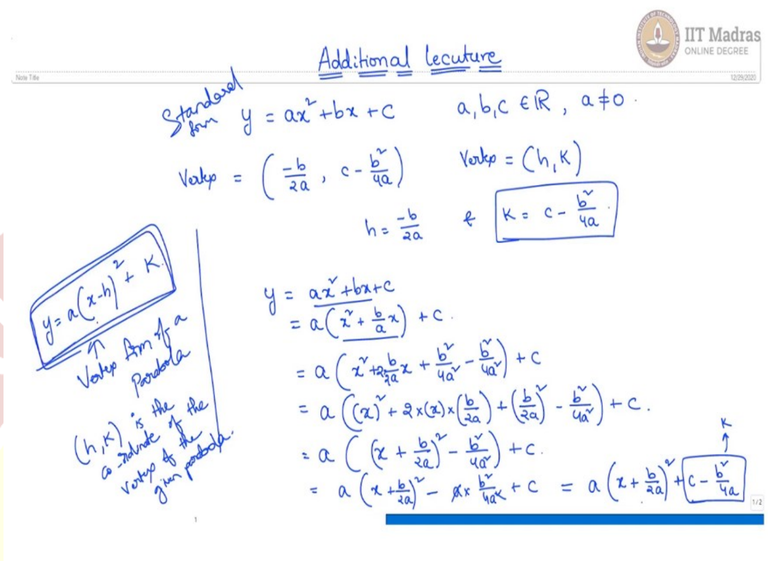

Hello, everyone, today we will discuss a small topic related to the forms of parabola. In other words we are going to see the relation between the standard form of a parabola and the vertex form of the parabola, we already know the standard form of a parabola which is given by $y = ax^2 + bx + c$ where these a, b, c belong to real and $a \neq 0$. From this standard form we will try to derive the vertex form of a parabola.

Now, from this equation we know that the coordinate of the vertex of the parabola is vertex will be x coordinate will be $\frac{-b}{2a}$ and y coordinate will be $c - \frac{b^2}{4a}$, this we already see in the previous lecture. Now, let us denote this coordinate of the vertex as (h , k). So, our h will be nothing but $\frac{-b}{2a}$ and k will be $c - \frac{b^2}{4a}$ .We have obtained the required data, now let us start the deriving.

We have $y = ax^2 + bx + c$, I will take a common from these two terms, I will get $a\left(x^2 + \frac{b}{a}x\right) + c$, also I will add and subtract $\frac{b^2}{4a^2}$ to this term, I will get $a(x^2 + \frac{b}{a}x + \frac{b^2}{4a^2} - \frac{b^2}{4a^2})$+c.

Now, also I will multiply with 2 and divide by 2 here, now I will rewrite this $a(x^2 + 2 \times x \times \frac{b}{2a} + (\frac{b}{2a})^2 - \frac{b^2}{4a^2})$+c. So, if we observe this $x^2 + 2 \times x \times \frac{b}{2a} + (\frac{b}{2a})^2$, this is in the form of $p^2 + 2pq + q^2$, we can write this as $(p + q)^2$.

So, writing like that, we get $(x+\frac{b}{2a})^2-\frac{b^2}{(2a)^2}+c$. Now, if I multiply a we get $a[\left(x+\frac{b}{2a}\right)^2-\frac{b^2}{4a^2}]$+c, a and this cancelled, finally I will obtain this will be equal to $a\left(x+\frac{b}{2a}\right)^2+c-\frac{b^2}{4a}$.

If we observe $c-\frac{b^2}{4a}$ is k here and $\frac{b}{2a}$ will be $-h$, so if we substituted h and k in this equation we get $y=a(x-h)^2+k$, this is the vertex form, vertex form of the form of a parabola, where this $(h\,,k)$ is the coordinate of the vertex of the given parabola, this is the standard form and from this standard form we derived the vertex form.

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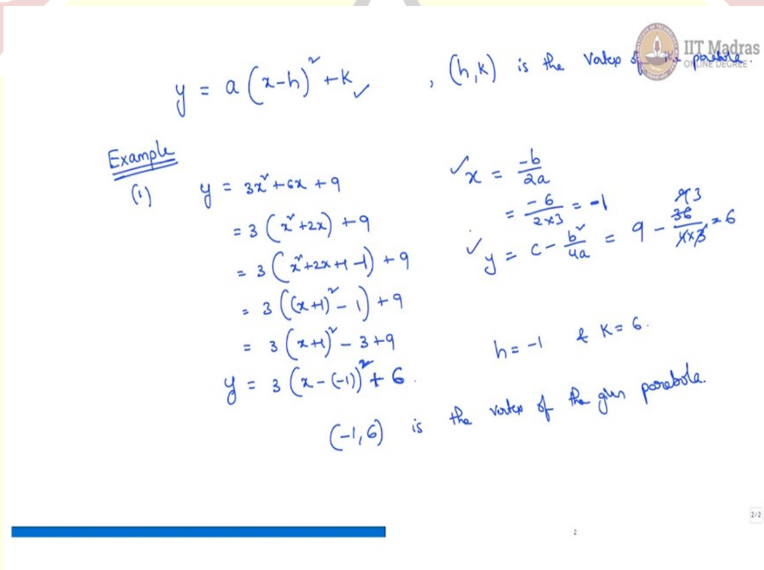

So, we have got the vertex form of the parabola which will be like this $a(x-h)^2+k$ where $(h\,,k)$ is the vertex of the parabola. Now, let us see one example to understand this vertex form clearly. So, suppose we have an equation of a parabola given like this $y=3x^2+6x+9$ now we try to write in vertex form, so I will take 3 common from the first two terms, I will get x^2+2x+9, so in order to make this a perfect square I will add 1 and subtract 1.

So, $3(x^2+2x+1-1)+9$, so 3 times this can be written as $3((x+1)^2-1)+9$, which gives us $(3(x-(-1))^2+6)$. So, we have got the equation $y=(3(x-(-1))^2+6)$. So, if we equate it with this vertex form we get $h=-1$ and $k=6$. So, our vertex will be at point $(-1\,,6)$ is the vertex of the given parabola.

So, we will just cross verify it, we know that if we have a standard form we can calculate the x coordinate of the vertex, so x coordinate of this vertex will be $x = \frac{-b}{2a}$, so here b is 6 and a is 3, so if I substitute that $-6\ by\ 2\ \times\ 3$ which I will get $x\ = -1$ and we know the y coordinate as $c - \frac{b^2}{4a}$ here we have c is $9\ -\ b$ is 6 so b^2 is 36 / $4a$ is 3 so 4 9's 9 3's, so I will get 6. So, my vertex point will be at $(-\ 1\ , 6)$, if we solve y, solve through standard form also.

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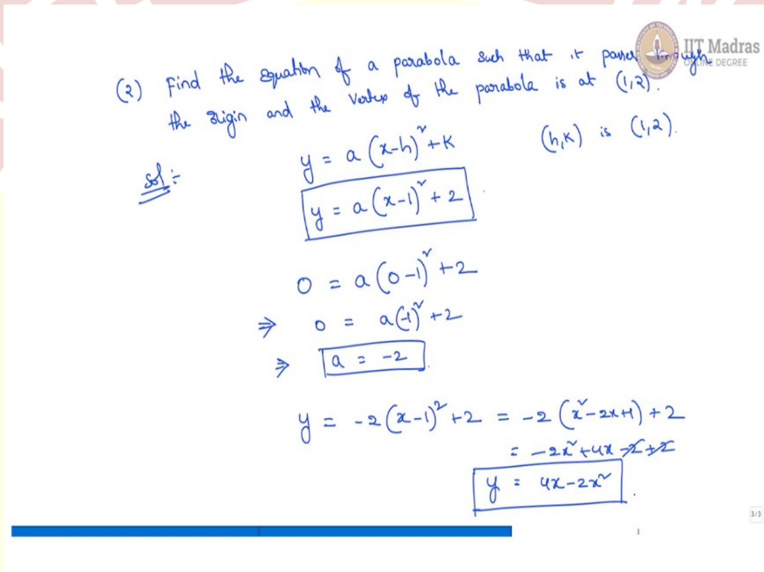

Now, let us see one more example, find the equation of a parabola such that it passes through the origin and the vertex of the parabola is at $(1\ , 2)$. So, as we know the vertex form given by y is equal to a times x minus h whole square plus k, here we have given that $(h\ , k)$ is nothing but (1 , 2).

So, if we substitute that our equation will be simplified to $a(x - 1)^2 + 2$, also it is given that this equation passes through the origin that means 0 , 0 should satisfy this equation. So, if we substitute it we get the value of a, so $0 = a(0 - 1)^2 + 2$, this implies $0 = a(-1)^2 + 2$, which implies again $a = -\ 2$.

So, our final equation of the parabola will be equal to $y = -2(x - 1)^2 + 2$, if you open that $y = -2x^2 + 4x -\ 2 +\ 2$, so this will be get cancelled and $4x - 2x^2$, so $y = 4x - 2x^2$ is the equation of the parabola that passes through the origin and the vertex of this parabola will be at $(1\ , 2)$. Thank you.