IIT Madras

ONLINE DEGREE

Statistics for Data Science – 1
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Week 6 - Tutorial 7

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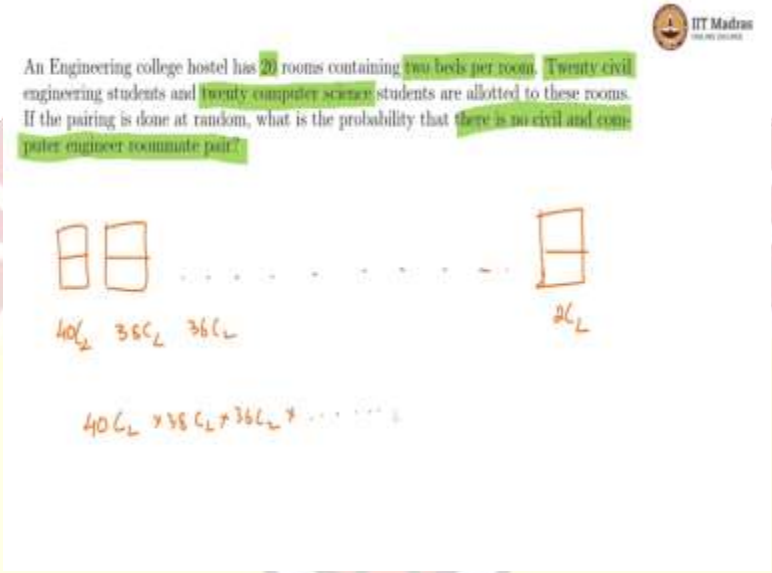

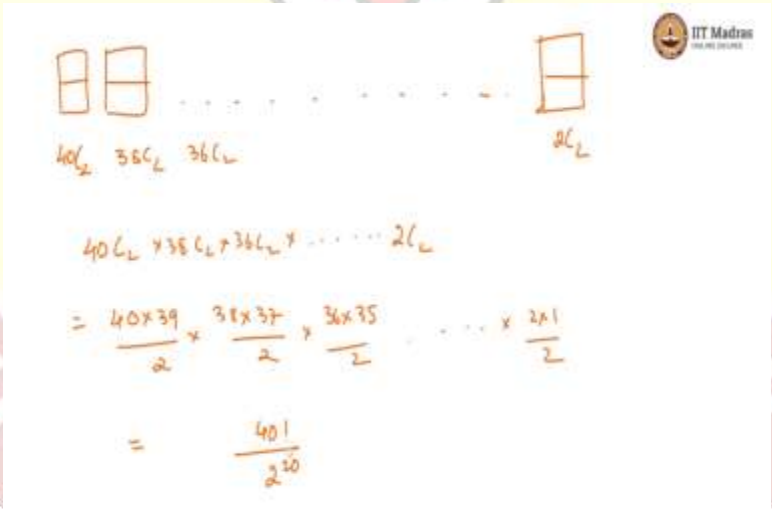

An engineering college hostel has 20 rooms and two beds per room. There are 20 civil engineering students, 20 computer science students who are allotted to these rooms. Pairing is done at random, what is the probability that there is no civil and computer engineer roommate pair?

So, all the civil engineering students are in 10 separate rooms and all the computer science students are in 10 separate rooms. So, for solving this problem let us look at the rooms which are like this, each room has two beds so on and there are 20 rooms all together and there are 40 students, there 20 in computer science and 20 in civil and we are looking at the total possibilities.

So, the first room gets $^{40}C_2$ ways of being filled. Then the next one has $^{38}C_2$ and the next one will have $^{36}C_2$ so on till we are left with $^{2}C_2$, so we have $^{4}C_2 \times {}^{38}C_2 \times {}^{36}C_2 \times \ldots\ldots {}^{2}C_2$ which is basically $\frac{40\times39}{2} \times \frac{38\times37}{2} \times \frac{36\times35}{2} \times \ldots\ldots\ldots \times \frac{2\times1}{2}$. So, this will give us $\frac{40!}{2^{20}}$. So, this is the total ways of being able to fill these rooms.

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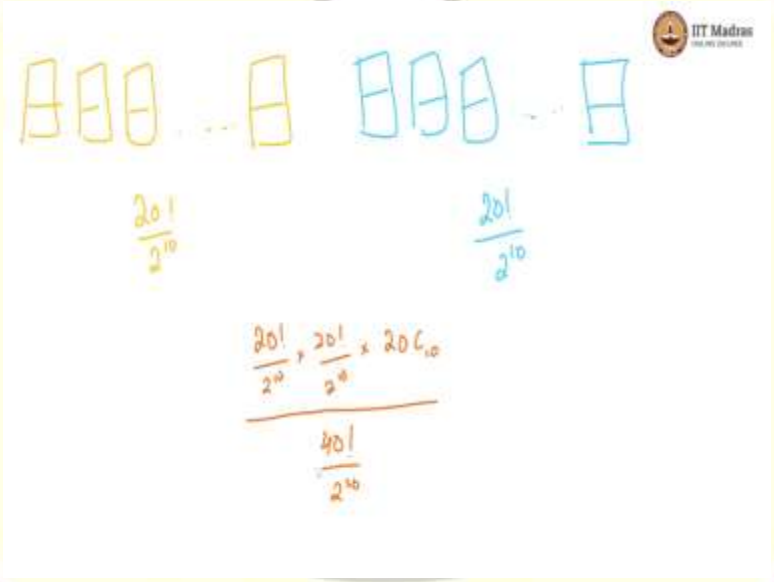

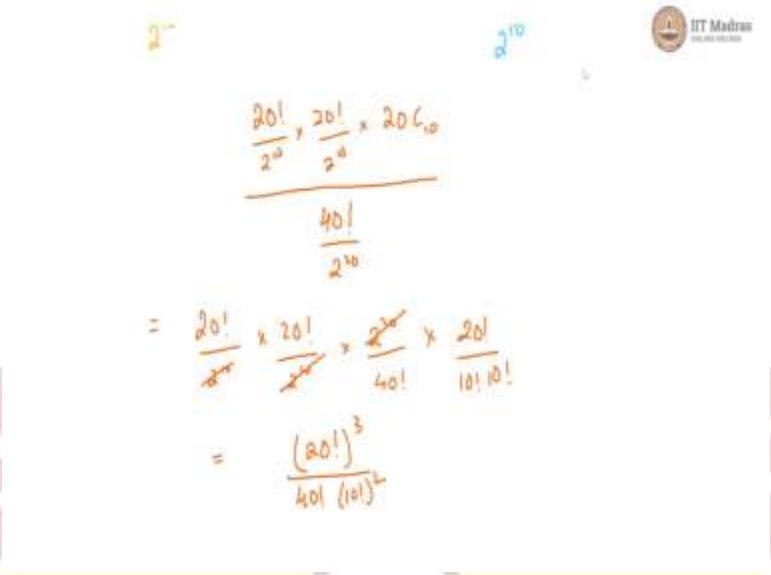

Now, imagine if we filled separately 10 rooms with just the civil engineering students and again separately 10 rooms with just the computer science students. Then we will get a very similar logic for both of these. You will get $\frac{20!}{2^{10}}$ for these and $\frac{20!}{2^{10}}$ for these as well.

However, how we choose the rooms also should matter. So, we will be getting $\frac{20!}{2^{10}} \times \frac{20!}{2^{10}} \times {}^{20}C_{10}$. After these rooms are chosen, these are the ways that they respectively get filled. And then this divided by the $\frac{40!}{2^{20}}$. This gives us the simplification would be $\frac{20!}{2^{10}} \times \frac{20!}{2^{10}} \times \frac{2^{20}}{40!} \times \frac{20!}{10!10!} = \frac{(20!)^3}{40!(10!)^2}$. Thank you.