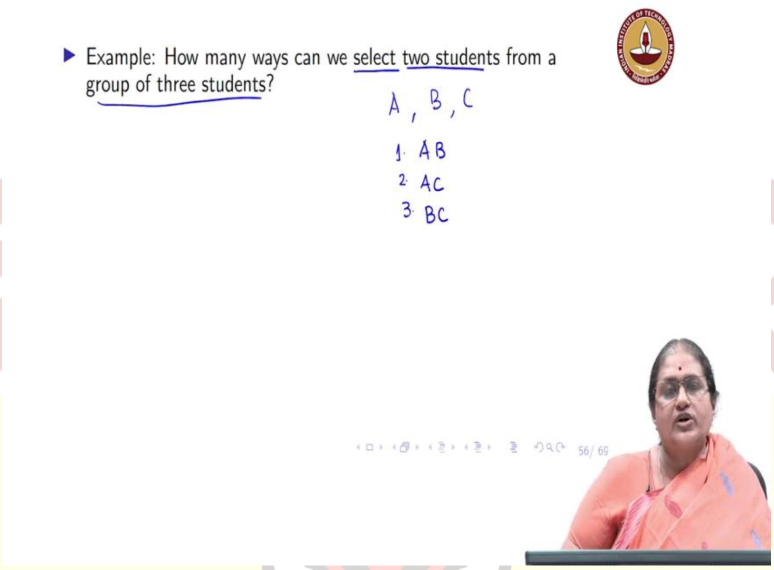
IIT Madras

ONLINE DEGREE

Statistics for Data Science – 1
Professor Usha Mohan
Department of Management Studies
Indian Institute of Technology, Madras
Lecture 5.5
Permutations and Combinations – Combinations

(Refer Slide Time: 00:28)



So, the next topic we are going to learn about is the topic of combinations, we have learned about permutation, when we talked about permutation remember we were focusing on the word ordered arrangement. So, now let us go and understand what is a combination. So, let us start with an example, suppose I am asked to select I am clearly telling I want to only select two students from a group of three students, here we are not emphasizing on any arrangement or any order, we are very clearly specifying that we only want to select two students or choose two students, I am not bothered about whether the first student or second student, I do not have any order here.

So, in this case, let me start by telling that let me have A, B, C as the three students, just I am assuming that the first student is called A, B, and C, now from these three students if I have to make groups of two I can either have AB as a group, I could have AC as a group or I could have BC as a group, I can see that I have 1, 2, 3, so there are three ways of selecting two students from the group of three students.

(Refer Slide Time: 01:48)

Introduction

AB BA
1 2
A B
B A

- Example: How many ways can we select two students from a group of three students?
  - Let A, B, and C be the three students.
  - We can chose AB, AC, or BC.
  - Note, when we talked of permutations, the order was important, i.e., AB was different from BA.
  - In this case, they are the same- order is not important.

Introduction

- Example: How many ways can we select two students from a group of three students?
  - Let A, B, and C be the three students.
  - We can chose AB, AC, or BC.
  - Note, when we talked of permutations, the order was important, i.e., AB was different from BA.
  - In this case, they are the same- order is not important.

Introduction

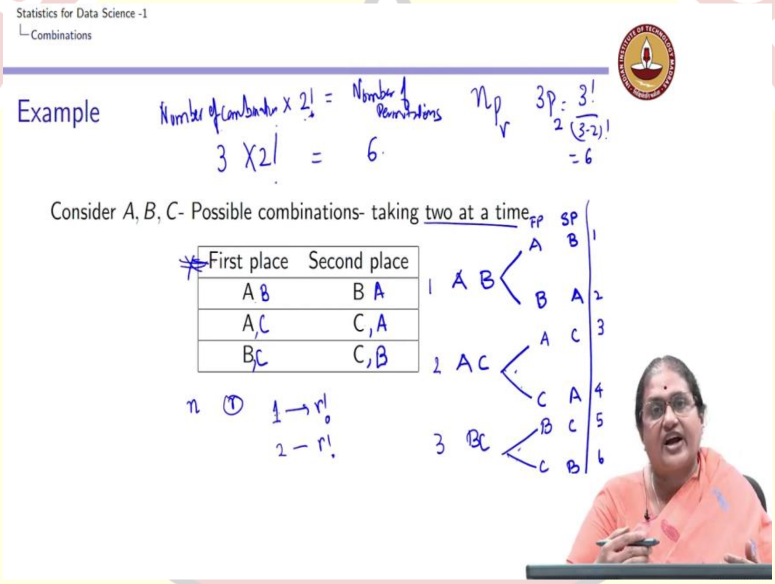
3 3 n AB COMBINATION
$\downarrow$ $\downarrow$ AC
2 r BC

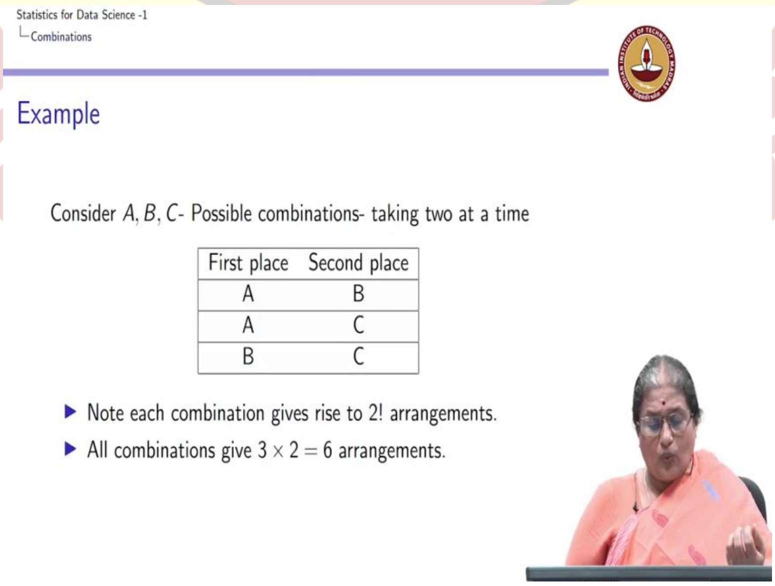
- Example: How many ways can we select two students from a group of three students?
  - Let A, B, and C be the three students.
  - We can chose AB, AC, or BC.
  - Note, when we talked of permutations, the order was important, i.e., AB was different from BA.
  - In this case, they are the same- order is not important.
- Each selection is called a combination of 3 different objects taken 2 at a time.

So, now this is AB, AC and BC. So, now here what you have to clearly understand is whether I write AB or BA is of no difference to me because the order in which I could choose A first and B first or B first and A first or both of them simultaneously there is no order in a combination. But when we talked about permutation whether A occupied the first block and B occupied the second or B occupied the first and A occupied the second there was a difference. So, you can see that when I am talking about combination the order is of no relevance to me, I do not have any importance to the order. Order is not important, that is something which we need to understand.

So, now each selection, so I have three selections, my selections are AB, AC and BC each selection is called a combination that is what we have written here. So, in general, here I had 3 objects and I selected 2 from this, so, I have from 3, I am selecting 2. In general, if I have n objects and I want to select r objects, I say I am choosing r from n or selecting r objects from n objects or I am choosing at a time, choosing r objects from n objects at the time.

(Refer Slide Time: 03:44)





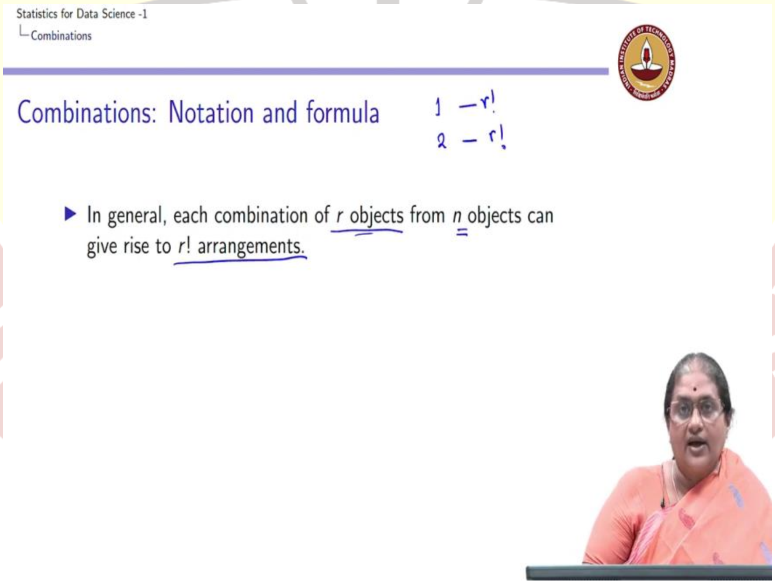
So, now going back to the example, I have suppose looking at the possible combinations taking two at a time, when we went back to our permutation, we had a first place and a second place here it really does not the order is of no importance here, so when I had the permutation, my first place could be an A or a B or it could be a B or A, the first place could be an A or second place could be a C or it could be the first place is a C and my second place is an A, first place is a B or a C or my second place is a C or a B.

These were the possible ways I could have had so you can see that if I have a AB combination, this has two arrangements one A in my first place, this is my second place and B in my second place or B in my first place and A in my second place, AC combination or AC selection I could have an A in my first place and C in my second place or C in my first place and A in my second place, BC combination could have a B in my first place, C in my second place or C in my first place and B in my second place. Recall, the number of permutations of 2 objects from 3, which was $^3P_2$ was $3! \times (3-2)!$ which was nothing but your 6, I have 1, 2, 3, 4, 5 and 6 possible permutations, the number of combination is 1, 2, 3.

So, from this example I have that the number of combinations which is equal to 3, each combination is giving rise to 2 permutation or 2! permutations is giving me the number of permutations, I repeat the number of combinations which is 3 each one is giving rise to 2! or 2 permutations, so finally I have the number of permutations which is 6, so number of combinations into 2! is giving me the number of permutations which was 6.

I can extend this logic to n thing, so if I have n objects and I am choosing r from these n objects, then I can say that if I have my first combination of n objects from r objects, each of these r objects can be so the one combination would give rise to $r!$ factorial arrangements, the second combination will also give rise to $r!$ arrangement, so every combination gives rise to $r!$ arrangements. Now, why is this important to us?

(Refer Slide Time: 07:26)



$1 \to r!$

Combinations: Notation and formula

$^nC_r \to \underbrace{{}^nC_r \cdot r!}_{\text{Total no. of Permut}}$

- In general, each combination of r objects from n objects can give rise to $r!$ arrangements.
- The number of possible combinations of r objects from a collection of n distinct objects is is denoted by ^nC_r and is given by

$1 \longrightarrow r!$

Combinations: Notation and formula

$^nC_r \times r! = {}^nP_r$

$^nC_r \times r! = \frac{n!}{(n-r)!}$

- In general, each combination of r objects from n objects can give rise to $r!$ arrangements.
- The number of possible combinations of r objects from a collection of n distinct objects is is denoted by ^nC_r and is given by

$$^nC_r = \frac{n!}{r!(n-r)!}$$

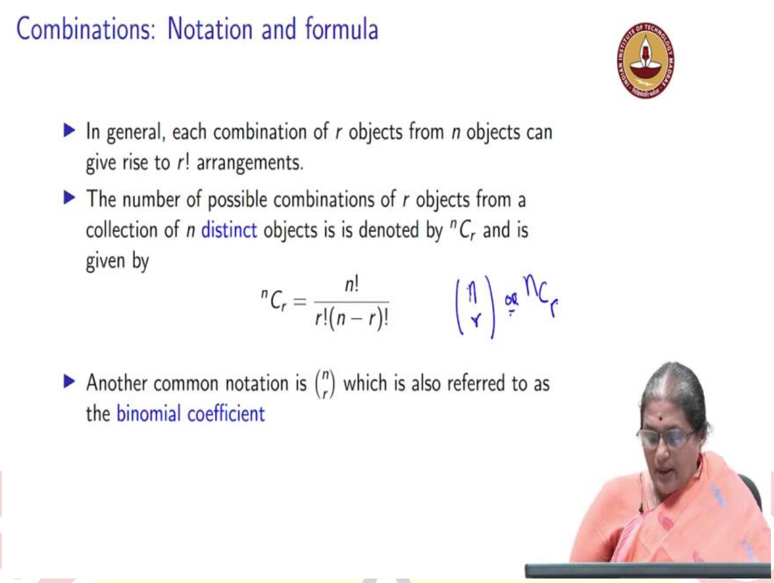
Combinations: Notation and formula

- In general, each combination of r objects from n objects can give rise to $r!$ arrangements.
- The number of possible combinations of r objects from a collection of n distinct objects is is denoted by ^nC_r and is given by

$$\boxed{^nC_r = \frac{n!}{r!(n-r)!}}$$

$$^3C_2 = \frac{3!}{2!\,1!} = 3.$$

AB
AC
BC



From this, we can actually set up what we are referring to as a combination, notation, and formula. So, in general each combination of r objects from n objects give rise to $r!$ arrangements so that is what we have, if I have one combination then after it will give rise to $r!$, the second combination another $r!$ arrangements. So, if I am going to denote the number of possible combinations of r objects from a collection of n distinct object, I repeat the number of possible combinations of r objects from a number collection of n distinct object is denoted by ${}^nC_r$, I am choosing r from n distinct objects.

So, this is the number of combinations, if I have from one combination I am going to have $r!$ permutations, so from ${}^nC_r$, I am going to have ${}^nC_r \times r!$ arrangements. Now, this ${}^nC_r \times r!$ is nothing but the total number of permutations, total number of permutations this is what we saw in our example, so in general what do I have? I can write this ${}^nP_r$.

Again, I repeat from one combination I get $r!$ permutations, so each combination because I have r objects these r objects among themselves can be arranged $r!$ ways if I have 2, I will have $2 \times r!$, ${}^nC_r$ is the number total number of combinations, so ${}^nC_r \times r!$ is going to give me the total number of permutations possible.

I already know ${}^nP_r$ is $\frac{n!}{(n-r)!}$, so I have ${}^nC_r \times r!$ is $\frac{n!}{(n-r)!}$ to give me the formula ${}^nC_r$ is $\frac{n!}{r!(n-r)!}$. So, this is the formula for ${}^nC_r$ and the way I express it is, the number of possible combinations of r objects from n distinct objects.

So, in earlier example I chose 2 objects from 3 distinct objects, so it is ${}^3C_2$ applying the formula it is 3! which is $n!$, $r!$ is $2!$, $(3-2)!$ is a $1!$, and I can see that this is equal to 3 which actually is the total number of combinations which I got namely AB, AC and BC, I had three combinations.

So, this formula is the number of possible combinations of r objects from n distinct objects. Another common notation which is used by many authors and book is $\binom{n}{r}$, this is also referred to as a binomial coefficient, some of the books use this either of these notations are fine $\binom{n}{r}$ or ${}^nC_r$ this is referred to also as a binomial coefficient.

(Refer Slide Time: 11:53)

Some useful results

${}^nC_r = \frac{n!}{r!(n-r)!} \longleftrightarrow \frac{n!}{(n-r)!\,r!} = {}^nC_{n-r}$

1. ${}^nC_r =$

Some useful results

1 2 . . n

r   n-r

1. ${}^nC_r = \frac{n!}{r!(n-r)!} = \frac{n!}{(n-r)!r!} = {}^nC_{(n-r)}$
   In other words, selecting r objects from n objects is the same as rejecting $n-r$ objects from n objects.

Some useful results

${}^nC_n = \frac{n!}{n!(n-n)!} = \frac{n!}{n!\,0!} = 1$

${}^nC_0 = {}^nC_{n-0} = {}^nC_n = 1$

$= \frac{n!}{0!(n-0)!} = \frac{n!}{n!} = 1$

1. ${}^nC_r = \frac{n!}{r!(n-r)!} = \frac{n!}{(n-r)!r!} = {}^nC_{(n-r)}$
   In other words, selecting r objects from n objects is the same as rejecting $n-r$ objects from n objects.
2. ${}^nC_n = 1$ and ${}^nC_0 = 1$ for all values of n

Some useful results

${}^nC_r = {}^{n-1}C_{r-1} + {}^{n-1}C_r$

1 2 3 4
A B C D E
A is in 3 chosen
A is not a part of 3

$n=5$
$r=3$

$$\boxed{\binom{5}{3} = \binom{4}{2} + \binom{4}{3}}$$

1. ${}^nC_r = \frac{n!}{r!(n-r)!} = \frac{n!}{(n-r)!r!} = {}^nC_{(n-r)}$
   In other words, selecting r objects from n objects is the same as rejecting $n-r$ objects from n objects.
2. ${}^nC_n = 1$ and ${}^nC_0 = 1$ for all values of n
3. ${}^nC_r = {}^{n-1}C_{r-1} + {}^{n-1}C_r; 1 \leq r \leq n$

Some useful results

${}^nC_r = {}^{n-1}C_{r-1} + {}^{n-1}C_r$

$n=5$
$r=3$

r has 'n'
r does not 'n'

${}^nC_r = \binom{n-1}{r-1} + \binom{n-1}{r}$

1. ${}^nC_r = \frac{n!}{r!(n-r)!} = \frac{n!}{(n-r)!r!} = {}^nC_{(n-r)}$
   In other words, selecting r objects from n objects is the same as rejecting $n-r$ objects from n objects.
2. ${}^nC_n = 1$ and ${}^nC_0 = 1$ for all values of n
3. ${}^nC_r = {}^{n-1}C_{r-1} + {}^{n-1}C_r; 1 \leq r \leq n$

Some useful results

${}^nC_r = {}^{n-1}C_{r-1} + {}^{n-1}C_r$

$n=5$
$r=3$

r has 'n'
r does not 'n'

${}^nC_r = {}^{n-1}C_{r-1} + {}^{n-1}C_r$

1. ${}^nC_r = \frac{n!}{r!(n-r)!} = \frac{n!}{(n-r)!r!} = {}^nC_{(n-r)}$
   In other words, selecting r objects from n objects is the same as rejecting $n-r$ objects from n objects.
2. ${}^nC_n = 1$ and ${}^nC_0 = 1$ for all values of n
3. ${}^nC_r = {}^{n-1}C_{r-1} + {}^{n-1}C_r; 1 \leq r \leq n$

Some useful identities combinatorial identities which will be useful for us to understand more about the combinatorial formula. The first thing is ${}^{n}C_{r}$, so if I start with a ${}^{n}C_{r}$ by my formula I have this is $\frac{n!}{r!(n-r)!}$. Now, in the denominator I can write it as $\frac{n!}{(n-r)!r!}$, these two are equivalent, I am just changing the order in which I have written them.

So, what do I get now? Now, this if I am applying the same formula this is same as ${}^{n}C_{(n-r).}$ So, the first identity we have is ${}^{n}C_{r} = {}^{n}C_{(n-r)}$. In other words, what we mean is selecting ${}^{n}C_{r}$, I can express as selecting r objects from n objects, ${}^{n}C_{(n-r)}$ is same as rejecting $(n-r)$ objects from r object. So, if I have n objects or n people, if I select r of them, I am basically rejecting $(n-r)$ of them, so if I have some ways of selecting these selecting these r objects is equivalent to rejecting these $(n-r)$ of them and that can be established by this identity.

The next important identity which we have is ${}^{n}C_{n}$, again applying the formula I have ${}^{n}C_{n}$ is $\frac{n!}{n!(n-n)!}$, I have which is same as $\frac{n!}{n!0!}$, recall $0!$ is equal to 1, so I have ${}^{n}C_{n}$ equal to 1. Now, when we look at ${}^{n}C_{0}$ we can apply the earlier identity, I know ${}^{n}C_{0}$ is same as ${}^{n}C_{(n-0)}$ which is equal to ${}^{n}C_{n}$, I have already established ${}^{n}C_{n}$ equal to 1, so ${}^{n}C_{0}$ is also equal to 1.

I can prove this from first principle, again ${}^{n}C_{0}$ is $\frac{n!}{0!(n-0)!}$ which is $\frac{n!}{n!}$, again $0!$ is 1, which is equal to 1. So, these are two important combinatorial identities that we need to understand. Now the third identity ${}^{n}C_{r}$ equal to ${}^{(n-1)}C_{(r-1)} + {}^{(n-1)}C_{r}$, this is a very important and a useful combinatorial identity. I am not going to formally prove this, this can be proved, you can give a proof for this also, but let me give you the intuition behind this identity by using an example.

For example, if n equal to 5 objects and I have to choose 3 objects out of them, so let me assume that I have A, B, C, D, and E who are my 5 students or 5 people or my 5 objects and I have to choose 3 from them, so now I choose one of them, for example if I choose A, I am choosing A, so there are 2 ways of looking at my total choices, 1 way is that the three group that is the group of 3 has A in it and the second is A is not a part of the 3 that is chosen, one way is A is a part of the 3 that is chosen and A is not a part of the 3 that is chosen.

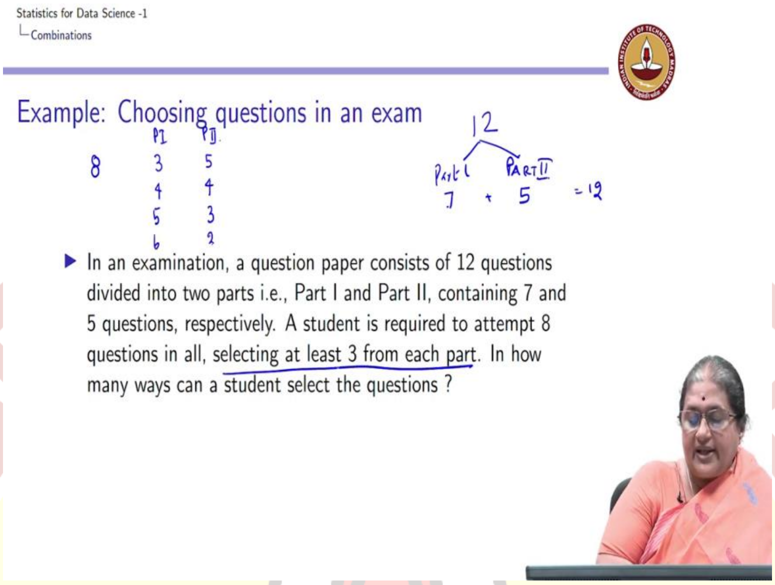
Now, if A is a part of the 3, so I am choosing 3 objects, if A is a part of the 3 that is chosen, I have to choose remaining 2, if A has already been chosen, I have to choose the remaining 2, for the remaining 2, if A is chosen I have 1, 2, 3, 4 available with me and I have to choose 2 and I can do that in ${}^{4}C_{2}$ ways, because A is already chosen I only have to choose 2 more objects, I have a remaining 4 objects to choose that from and that I can do in ${}^{4}C_{2}$ ways.

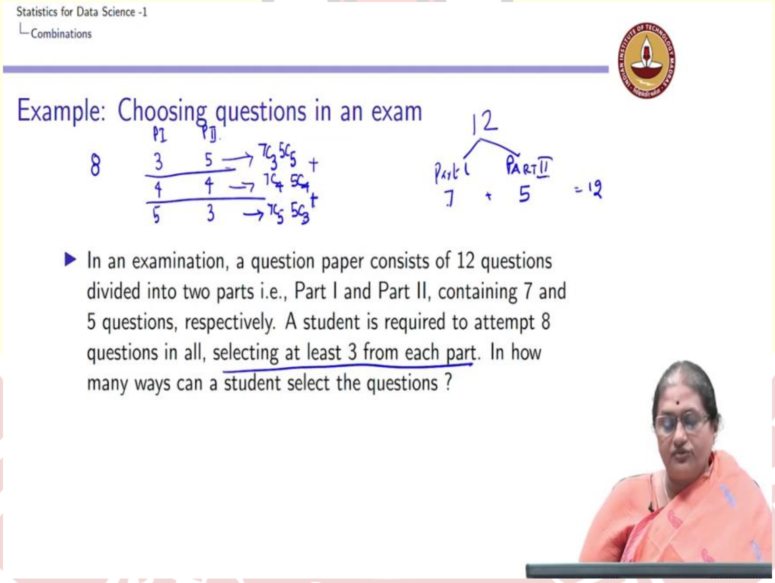
Now, suppose A is not a part of the 3 objects, then in that case I have 4 objects available with me, which is B, C, D, E and I have to choose all the 3 objects from this choice of 4 objects and that I can do in ${}^{4}C_{3}$ ways. So, the total number of ways I can choose 3 from this choice of A, B, C, D, E which is ${}^{5}C_{3} = {}^{4}C_{2} + {}^{4}C_{3}$, I can so you can see that n equal to 5, r equal to 3, (n-1) is 4, (r-1) is 2, plus (n-1) is 4, r is 3, so this is what we have got in this example. In general if I have n objects and I have to choose r from these n objects, I can fix, so the way I can understand this is I fix any one of the n objects, I fix any one of the n objects and I looked at the situation that the r objects has n and r does not have n.

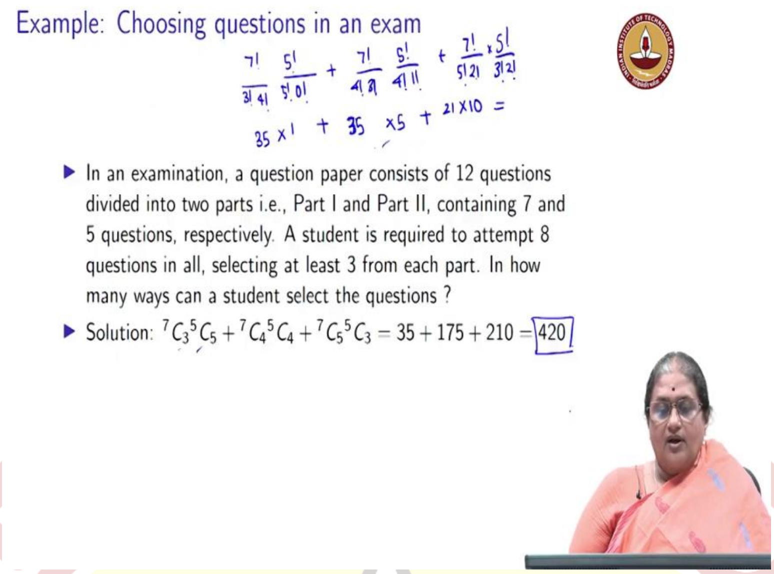
If r has n since I have already chosen one of the r objects, I only have to choose (r-1) of the objects from remaining (n-1) objects, here r does not have n, I have only remaining (n-1) because I am eliminating n, I choose r and these two put together give me ${}^{n}C_{r}$. I use this

$^{(n-1)}C_{(r-1)}$, $^{(n-1)}C_r$ and this is ^nC_r. So, this is an important combinatorial identity which we can use or apply and we will be applying such identities whenever we go to the application aspect.

(Refer Slide Time: 19:49)







So, now let us look at a few examples to apply the concepts of combination we have learned so far, one is choosing a question in an exam. So, consider a situation where a paper consists of 12 questions, now these 12 questions are in 2 parts, part 1 and part 2, part 1 has 7 questions, part 2 has 5 questions, so 7 plus 5 is 12 questions. Now, a student has to attempt 8 questions in all, how many questions should a student attempt? A student has to attempt 8 questions.

So, the condition is he has to choose at least 3 from each part, so I have a part 1, I have a part 2, I can either choose 3 from here, if I choose 3 from part 1, I have to choose 5 from part 2, if I choose 4 from part 1, I have to choose 4 from part 2, if I choose 5 from part 1, I have to choose 3 from part 2, I cannot be choosing 6 and 2 because this would be violating the at least 3 from each part, so if it is at least 3 from each part, these are the possible ways I can choose these 8 questions assuming that the student is choosing exactly 8 questions out of the 12 questions.

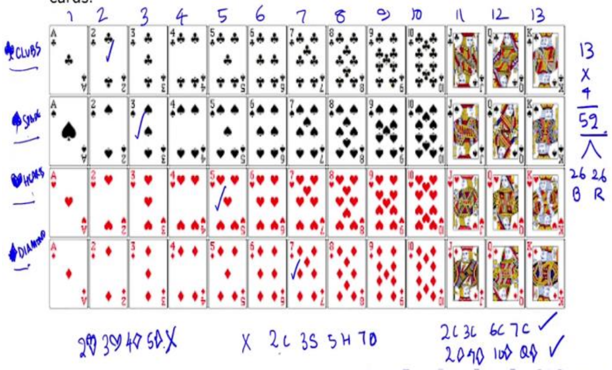
Now, how many ways can this be done? The number of ways this can be done is I have 7 questions in part 1, the number of ways I can choose 3 questions from 7 questions is ${}^7C_3$, the number of ways I can choose 5 questions from 5 questions is ${}^5C_5$, so I have a ${}^7C_3 \times {}^5C_5$ is a total number of ways I can choose 3 from part 1 and 5 from part 2. The number of ways I can do this is ${}^7C_4 \times {}^5C_4$, number of ways this can be done is ${}^7C_5 \times {}^5C_3$.

So, the total number of ways student can select question is ${}^7C_3 \times {}^5C_5 + {}^7C_4 \times {}^5C_4 + {}^7C_5 \times {}^5C_3$, which I can see is equal to 35, because ${}^7C_3 \times {}^5C_5$ is $\frac{7!}{3!\times4!}$, ${}^5C_5$ is $\frac{5!}{5!\times0!}$, so this is equal to 1, $\frac{7!}{4!\times3!} \times \frac{5!}{4!\times1!} + \frac{7!}{5!\times2!} \times \frac{5!}{3!\times2!}$, so $\frac{5!}{5!\times0!} = 1$, so $\frac{7!}{3!\times4!} = 35$, so 7×5, I have a 35.

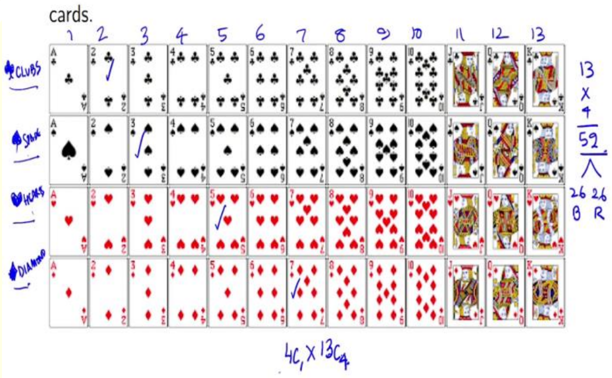
Similarly, here I will get a 5, so from here I have a 7 into this is again a 35, here I have a 7×3 which is $21 \times 5!$, 5×2 which is a 10, so this in total gives me 1, this is 175, so I get a 35 + 175 + 210 which gives me a total answer of 420.

(Refer Slide Time: 24:02)

Lets consider the case of choosing four cards from a deck of 52 cards.



Lets consider the case of choosing four cards from a deck of 52 cards.



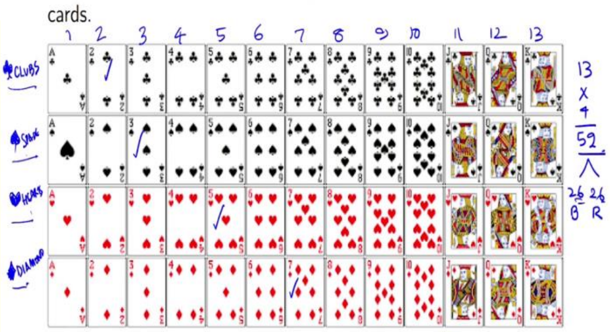
Example: Game of cards contd.

1. Total number of ways of choosing four cards from 52 cards = ${}^{52}C_4 = \frac{52!}{4!48!} = \boxed{2,70,725}$

Example: Game of cards contd.

1. Total number of ways of choosing four cards from 52 cards = $^{52}C_4 = \frac{52!}{4!48!} = 2,70,725$
2. All four cards are of the same suit
$^{4}C_1 \times {}^{13}C_4 = 4 \times \frac{13!}{4!9!} = \underline{2860}$

Lets consider the case of choosing four cards from a deck of 52 cards.



Example: Game of cards contd.

1. Total number of ways of choosing four cards from 52 cards = $^{52}C_4 = \frac{52!}{4!48!} = 2,70,725$
2. All four cards are of the same suit
$^{4}C_1 \times {}^{13}C_4 = 4 \times \frac{13!}{4!9!} = 2860$
3. Cards are of same colour $^{2}C_1 \times {}^{26}C_4 = 2 \times \frac{26!}{4!22!} = \boxed{2,99,00}$

So, now let us introduce you to a game of cards, the reason why I am introducing you to a game of cards is, when you look at problems and probability there are a lot of problems and probabilities which are based on a game of cards, so I will just introduce you to a game of cards, it is no gambling here, we are just is a nice way to understand the concept of probability that is why I have just introducing you to a formal deck of what we refer to as playing cards.

Now, when you look at playing cards these are called a suits, so I have clubs, a club is something of this kind this is what we refer to as a club, then I have what I refer to as a spade, then we have hearts and we have diamonds, a typical pack of cards has clubs, spade, hearts and, sorry this is a diamond.

So, now let me introduce you to a pack of cards, we are introducing you to a pack of cards because many problems in probability from many textbooks and whenever you learn probability, you will be introduced to questions which require you to understand something about a pack of cards.

So, when we look at a pack of cards, we have clubs, we have spade, we have hearts and we have diamond, now these are referred to as suits. So, typically I have 4 suits and what are the suits? They are clubs, they are spade, hearts and diamond, each of them have 13 denominations, that is I have A, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13. 13 denomination with 4 suits, which will give me a total of 52 cards and typically I will have of these 52, 26 are black and 26 are red, this is how a pack of cards is typically given to us.

Now, suppose I have to choose 4 cards from these 52 cards, I am not placing any other condition I am just telling choose 4 cards from this 52 cards, how many ways can we do it? The answer is pretty simple, it is just ${}^{52}C_4$ because I am not qualifying that what is it I require I am not saying choose 4 red cards, I am not saying choose for clubs, I am not saying choose 4 spades, I am just telling choose any 4 cards and those 4 cards could be a 2 clubs or a 3 spades or a 5 hearts or a 7 diamond, this is a perfect 4 choice where I have chosen these 4 cards. And how many ways can we choose 4 cards from 52 cards?

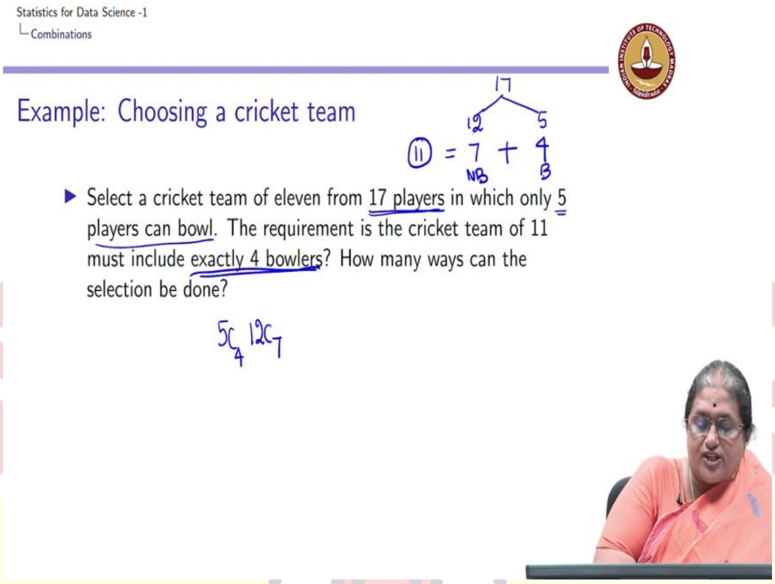
The ways I can choose 4 cards from 52 cards is just ${}^{52}C_4$ which is equal to this number which is a pretty large number. Now, suppose I am telling that all the 4 cards are of the same suit, so let us go back here, earlier I said that let us choose a 2 clubs, 3 spade, 5 hearts and 7 diamond, you can see that all these 4 cards or of different suits, I do not want this, this is not an acceptable selection for me, what is an acceptable selection? If I had a 2 clubs, 3 clubs, 6 clubs, and 7 clubs this is acceptable to me.

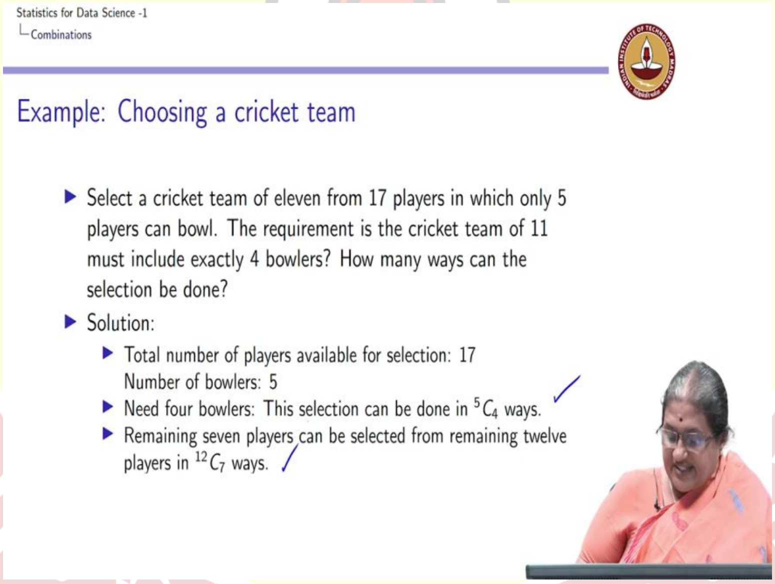
2 diamonds, 7 diamond, 10 diamond, and a queen diamond this is also acceptable to me, but a 2 hearts, a 3 hearts, a 4 diamond, and a 5 diamond it is not acceptable to me. So, how many ways can I choose this, again the requirement is choose 4 cards, but they should be of same suit. So, the way you can do it is, first let me choose a suit, how many ways can I do it? How many types of this? How many suits are there? There are 4, I need to choose 1 out of the 4 and how many ways can I do that? I can do that in ${}^{4}C_1$way.

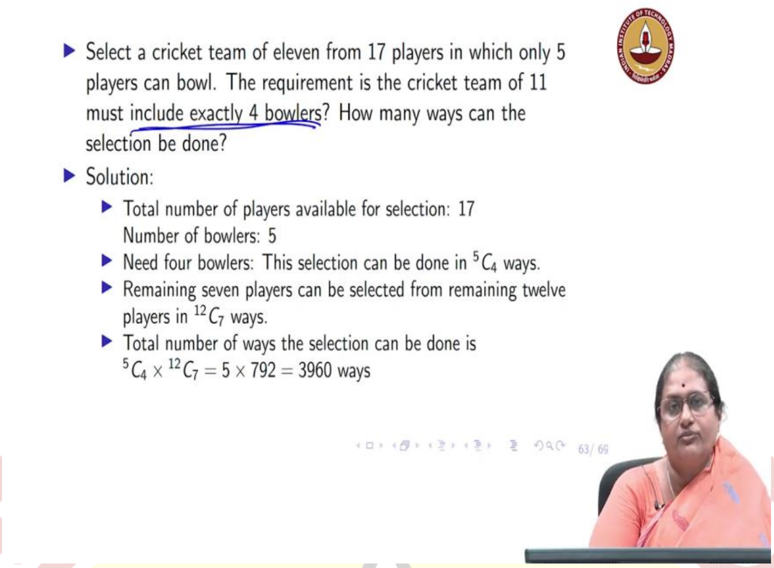
Now, within each suits I have 13 of each kind and I need to choose 4 from this 13 and that can be done in ${}^{13}C_4$ ways, so the total number of ways I can choose all the 4 cards from the same suits is ${}^{4}C_1\times {}^{13}C_4$ which is equal to 2,860 ways of doing it. Now, let us go to another problem, now I am asking, how do you choose the cards of the same colour? Now, again go back, how many colours do we have? You can see that we have 2 colours. How many ways can I choose 1 colour from 2 colours? Again the number of ways of choosing 1 colour from 2 colour is ${}^{2}C_1$.

Now, within each colour I have 26 cards that is what we have seen and I need to choose 4 from each colour so I can do that in ${}^{26}C_4$ ways, so the total number of ways I can choose 4 cards of the same colour are ${}^{2}C_1 \times {}^{26}C_4$ which we can see is 29,900 ways of doing it.

(Refer Slide Time: 30:54)







So, let us move on to the next example. Now, suppose all of us know how we choose cricket team, a cricket team has 11 players, of these 11 players we need to have some players who are batsman, some who are all-rounders, we need a wicket keeper, we also need bowlers, suppose I have 17 players available with me, I have 17 players available with me, of these 17 players only 5 players can bowl, so I have total 17, so recall I can put it in the framework of my question paper, of these 17 question players I have 5 who can bowl, so $17 - 5$ is 12 who cannot bowl.

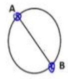
I need to choose 11 players, the requirement is, it should have exactly 4 bowlers, so if it should, the requirement is it should have exactly 4 players, I am not saying at least, I am saying exactly 4 players, if I have 4 bowlers, then I should have 7 non-bowlers in my team to make up to the 11 players, how many ways can the selection be done? The 4 bowlers can be chosen from the 5 bowlers in $^5C_4$ ways, which leads $^{12}C_7$ to be the number of ways I can choose the 7 remaining players from the 12 players to give me a total of $^5C_4$ and $^{12}C_7$ to give me a total number of ways as $^5C_4 \times {^{12}C_7}$, which I can simplify as 5×792 which is actually 3960 ways of selecting my team.

Here the qualifier was I should include exactly 4 bowlers and I can choose these 4 bowlers from 5 bowlers and the remaining 7 players from the remaining 12 non-bowlers. So, this is how I can have my team selection and apply the combinatorial or combination formula which we have stated earlier.

(Refer Slide Time: 33:39)

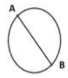
Example: Drawing lines in a circle

- Given n points on a circle, how many lines can be drawn connecting these points?
- $n = 2$ points, one line can be drawn connecting the points



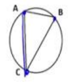
line segment: AB

- Given n points on a circle, how many lines can be drawn connecting these points?
- $n = 2$ points, one line can be drawn connecting the points



line segment: AB

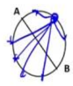
- $n = 3$ points, three line can be drawn connecting the points



AB
AC
BC

line segments: AB, AC, and BC

- Given n points on a circle, how many lines can be drawn connecting these points?
- $n = 2$ points, one line can be drawn connecting the points

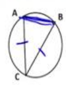


A, B → AB → 1

$${}^2C_2 = \frac{2!}{2!\,0!} = 1$$

line segment: AB

- $n = 3$ points, three line can be drawn connecting the points



A B C   AB AC BC   3

$${}^3C_2 = \frac{3!}{2!\,1!} = 3$$

line segments: AB, AC, and BC

- In general, given n points, number of line segments that can be drawn connecting the points is $\boxed{{}^nC_2}$

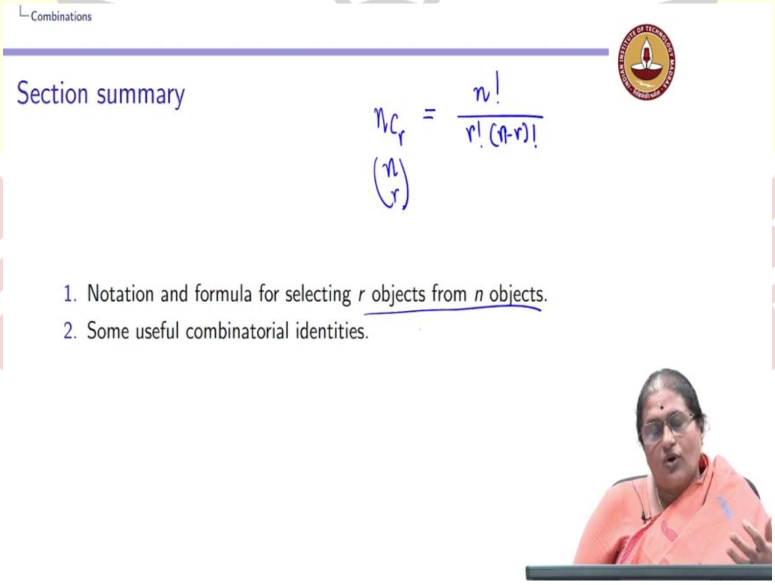
Now, let us look at another example, suppose I am given n points in a circle, the question is how many lines can be drawn connecting these points? So, now let us look at a very simple case of n equal to 2, let me turn the points A and B, so given A and B I know, I can draw only 1 line connecting A and B, so you can see that I am referring to that line segment as AB given 2 points on the circle, so n equal to 2, I can write I can connect it only with 2 points, sorry given 2 points I can connect it using 1 line.

Suppose I am given 3 points, so now you can see that I have A, B, and C are the 3 points I can connect AB with a line segment, I refer to that line segment as AB, I can connect AC with a line segment, I can also connect BC with a line segment. So, I can refer to my line segments as AB, AC, and BC.

So, I am given 3 points, what are the 3 points? My 3 points are so in the when n equal to 2, I had points A and B, my line segment was AB, I had 1 line, when I had points A, B, and C, I could form segments AB, AC, and BC, so I have 3 lines segments, so immediately you can notice that if I have to draw a line segment I need 2 points, I have 3, out of 3 points I can select 2 points at a time in $^3C_2$ ways and that $^3C_2$ we have already seen $^3C_2 = \frac{3!}{2!1!}$ which is 3, so I have 3 lines 1, 2, and 3.

Here $^2C_2$ is $\frac{2!}{2!0!}$ which is 1 and hence I have that number which is equal to 1. So, in general if I am given n points, in general if I have n points on the circle, to draw lines I need to choose any 2 points, that is if I fix this point I can draw lines between any 2 of these points, given n points I can choose these points in $^nC_2$ways. So, the number of lines segments that can be drawn connecting the n points is $^nC_2$. Again, here there is no direction in this line I just have a line which is AB, the order is not important because there I refer to BA or AB, it is the same here, so I am looking at a combination.

(Refer Slide Time: 37:11)



So, in summary we have started or we gave what is the notation and formula for selecting r objects from n objects, we represented that by ^nC_r, some books represented by $\binom{n}{r}$, the binomial coefficient, we saw ^nC_r is given by $\frac{n!}{r!(n-r)!}$, this is the important combinatorial formula which we gave and then we gave some useful combinatorial identities, we describe them and qualified it and we also gave a few applications of combinations, that is what we have done in this section.