Statistics for Data Science -1

Lecture 7.6: Conditional Probability: Independent events properties

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Learning objectives

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2. Distinguish between independent and dependent events.
3. Solve applications of probability.

Independence of E and F^c

Independence of three events

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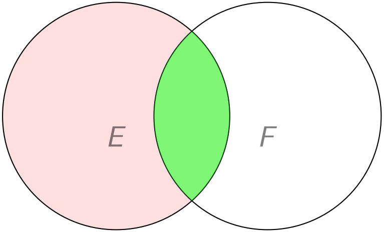

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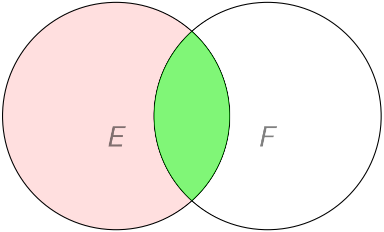

- E can be expressed as

$$E = (E \cap F) \cup (E \cap F^c)$$

Proof continued

- $$E = (E \cap F) \cup (E \cap F^c)$$

  $E \cap F$ and $E \cap F^c$ are mutually exclusive hence

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- Hence, $P(E \cap F^c) = P(E) \times P(F^c)$.

Thus, if E is independent of F, then the probability of E's occurrence is unchanged by information as to whether or not F has occurred.

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  - Let G denote the event that the second die equals 3.
- $F \cap G$ is the event of first throw is a 4 and second throw is a 3. Now $P(Sum = 7|\text{first throw is 4 and second throw is 3}) = 1$, i.e. $P(E|F \cap G) = 1$.

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For independent events, the probability that they all occur equals the product of their individual probabilities.

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Section summary

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   - Independence of E and F^c.
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