IIT Madras

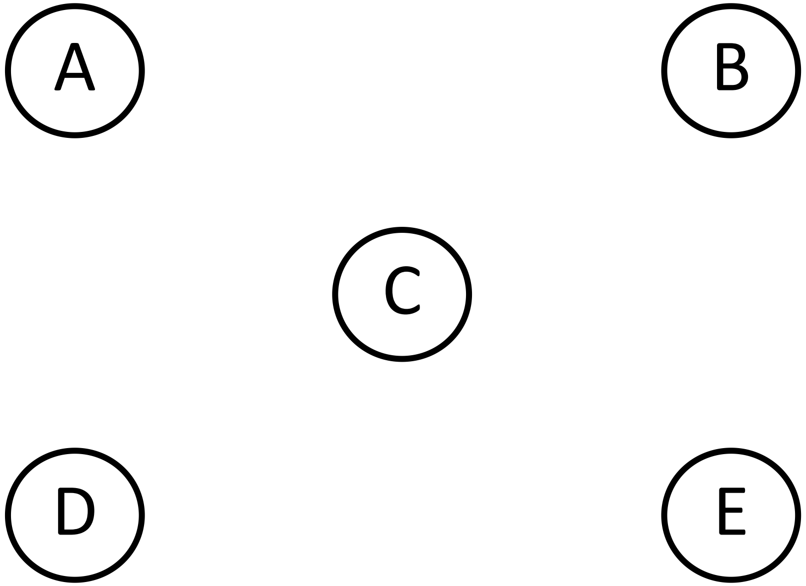
ONLINE DEGREE

Reducing number of comparisons

Reducing comparisons: what we observed

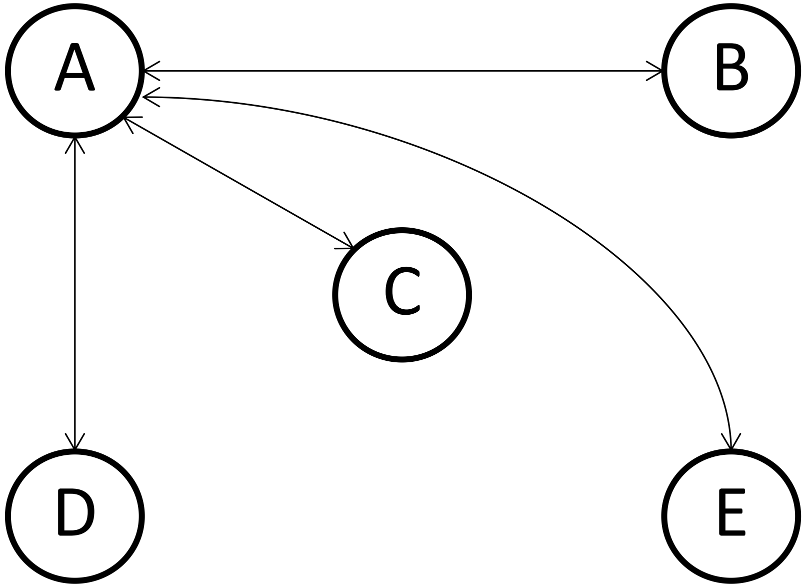
- Some computations seem to require comparisons of each card with all the other cards in the pile
  - for example, choosing a study partner for each student
  - the number of comparisons required can be very large
- We observed that if we can organise the cards into bins based on some heuristic:
  - then we only need to compare cards within one bin
  - this seems to significantly reduce the number of comparisons required
- Is there a formal way of determining the reduction in comparisons?
  - Calculate the number of comparisons without binning
  - Calculate the number of comparisons with binning
  - Use these calculations to determine the reduction factor

Comparing each element with all other elements



For 5 elements A, B, C, D, E:

Comparing each element with all other elements



For 5 elements A, B, C, D, E:
The comparisons required are:
A with B, A with C, A with D, A with E   (4)

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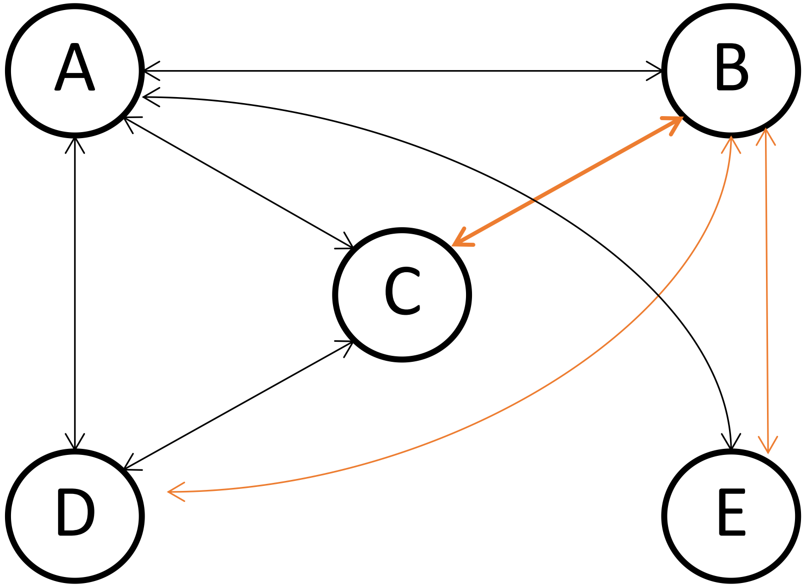


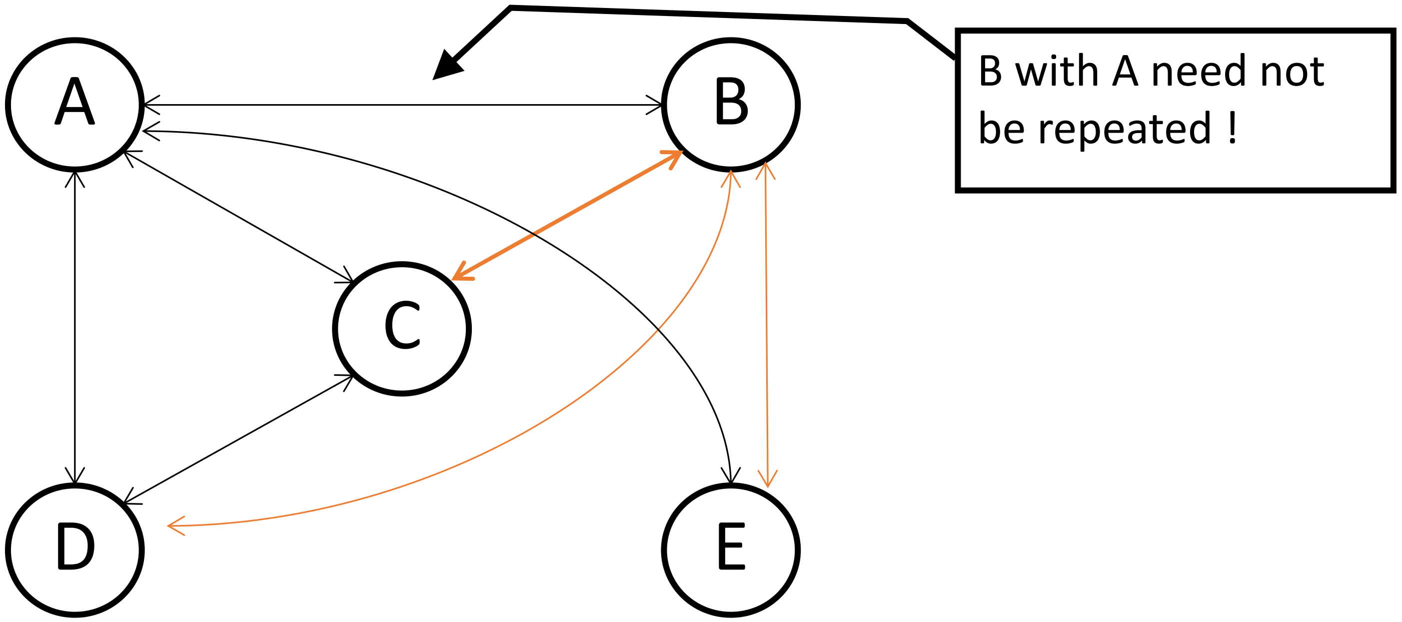
For 5 elements A, B, C, D, E:

The comparisons required are:

A with B, A with C, A with D, A with E (4)

B with C, B with D, B with E (3)

Comparing each element with all other elements



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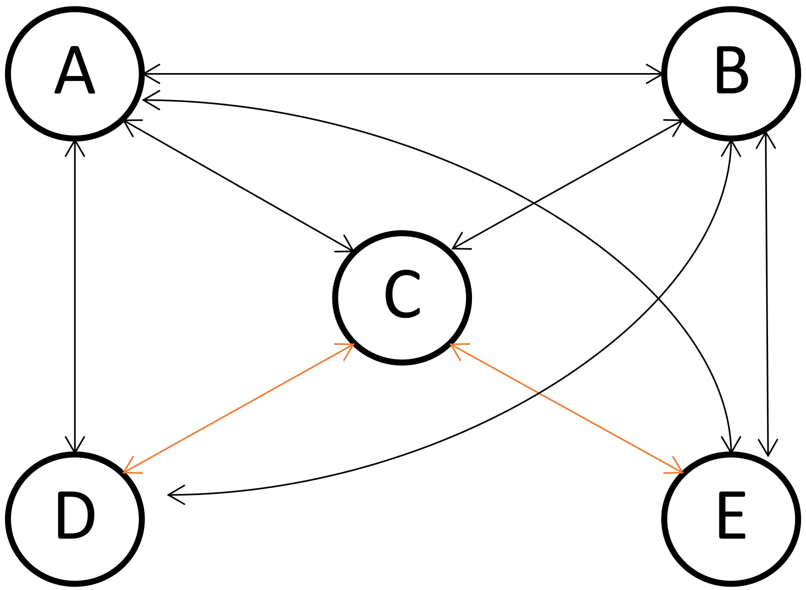


For 5 elements A, B, C, D, E:

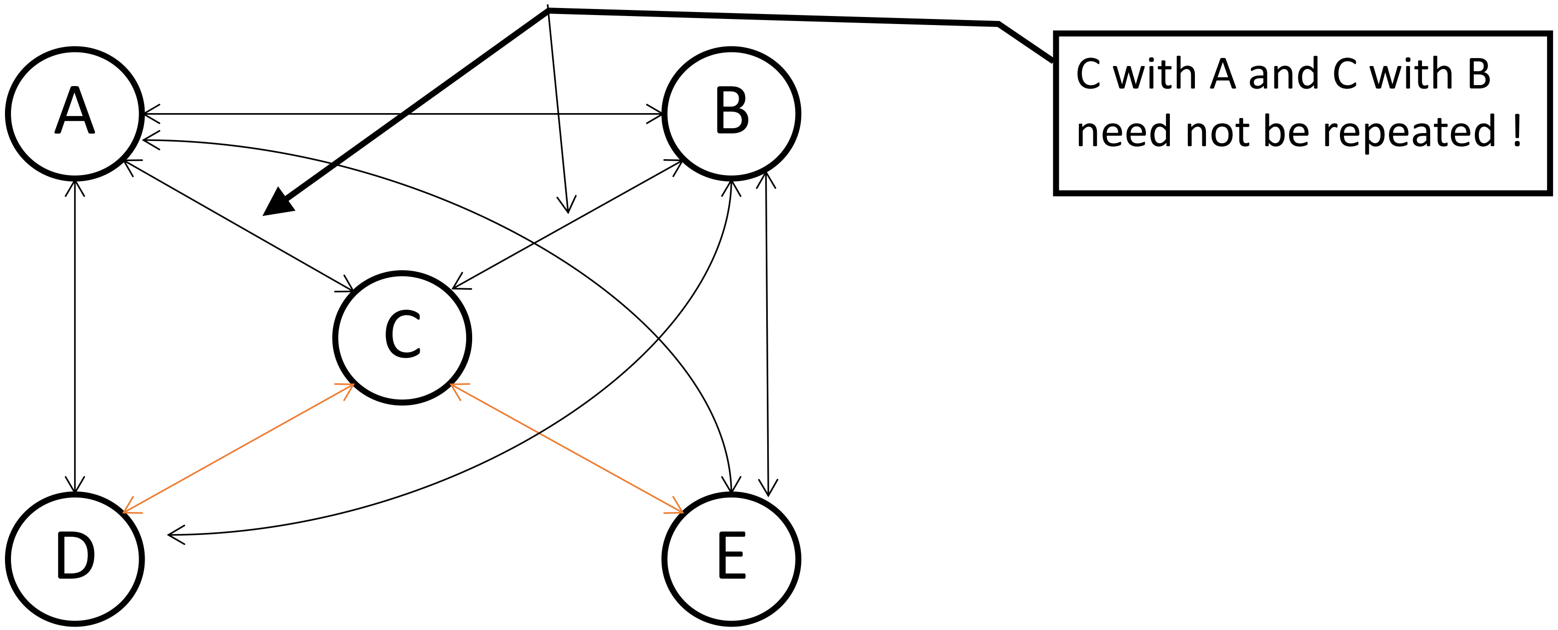
The comparisons required are:

A with B, A with C, A with D, A with E (4)

B with C, B with D, B with E (3)

C with D, C with E (2)

Comparing each element with all other elements



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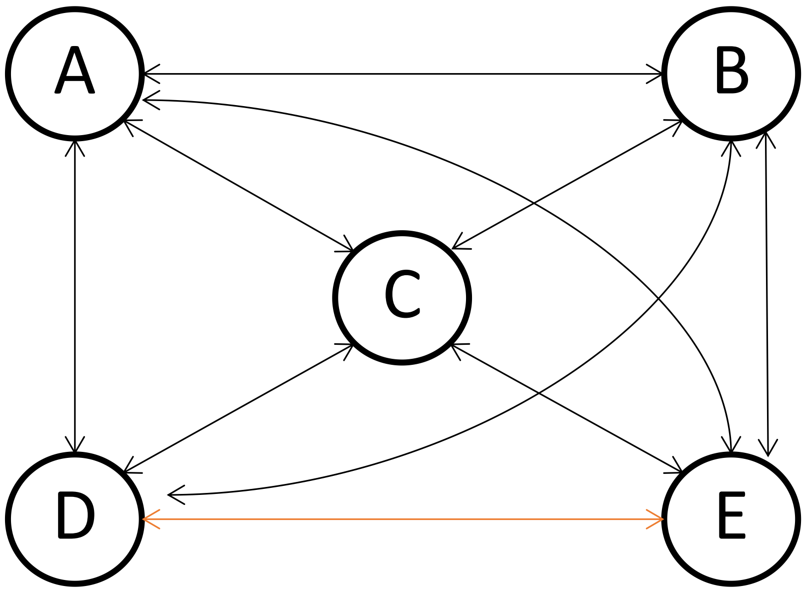

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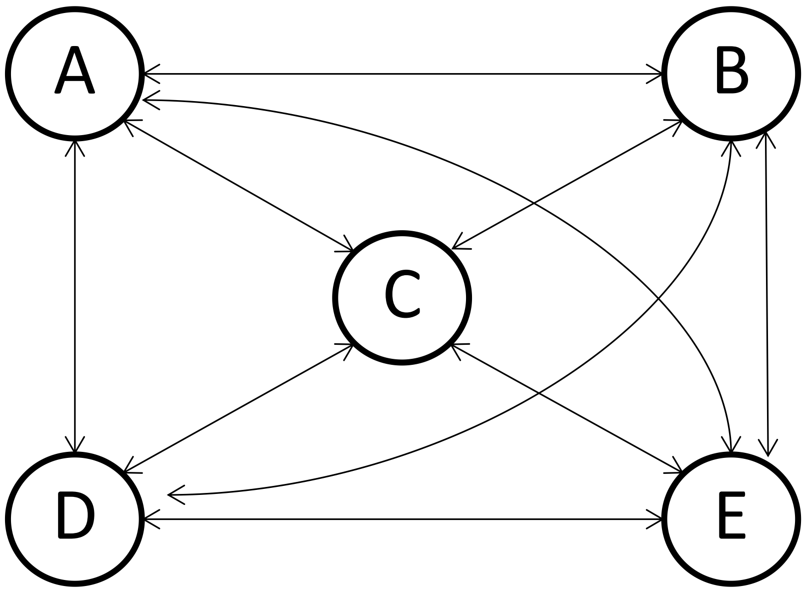
The comparisons required are:

A with B, A with C, A with D, A with E	(4)
B with C, B with D, B with E	(3)
C with D, C with E	(2)
D with E	(1)

Comparing each element with all other elements



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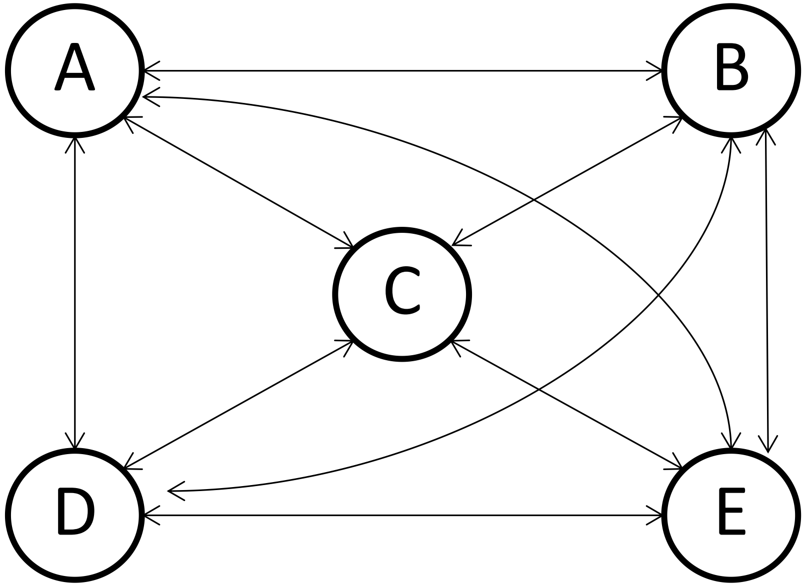
Number of comparisons: 4 + 3 + 2 + 1 = 10

Comparing each element with all other elements



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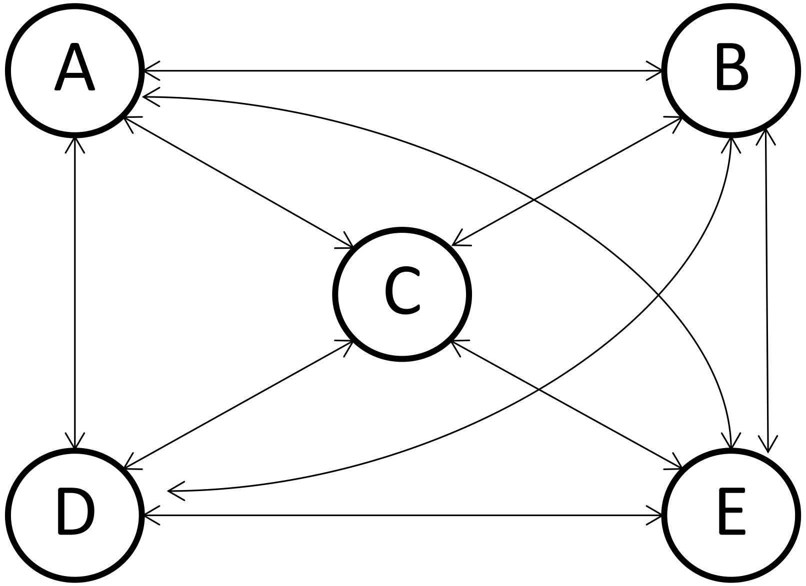
- For N objects, the number of comparisons required will be:
  - (N - 1) + (N - 2) + ... + 1
  - which is = $\frac{N \times (N - 1)}{2}$

Comparing each element with all other elements



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- For N objects, the number of comparisons required will be:
  - $(N - 1) + (N - 2) + ... + 1$
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  - ${}^{N}C_2 = \frac{N \times (N - 1)}{2}$

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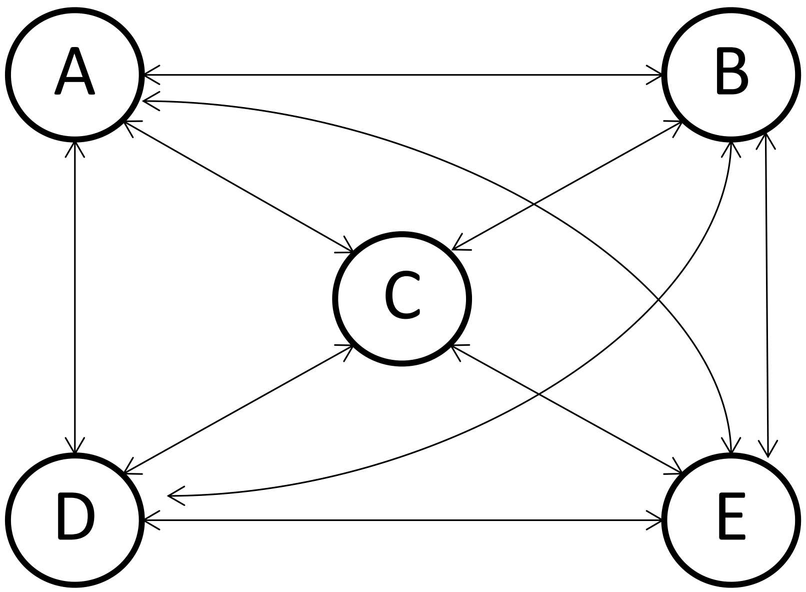

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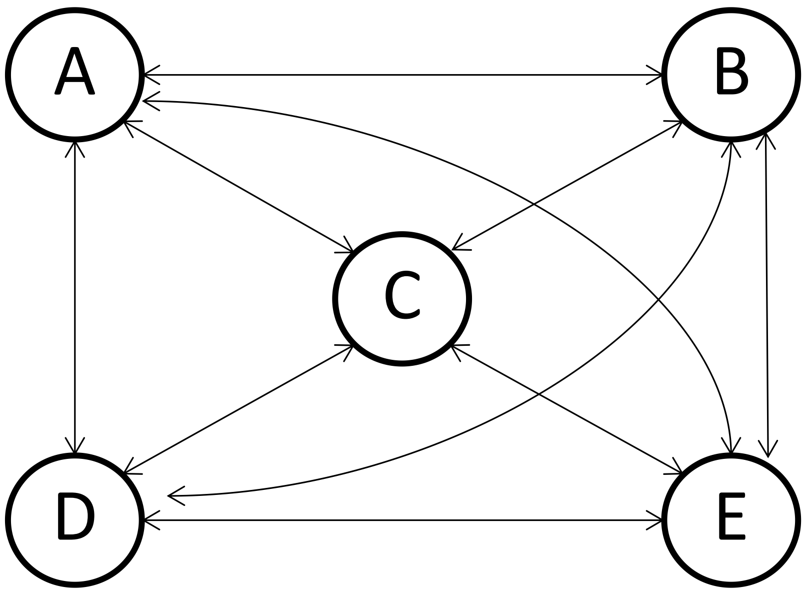
- This is the same as the number of ways of choosing 2 objects from N objects:
  - ${}^{N}C_2 = \frac{N \times (N - 1)}{2}$
- From first principles:
  - Total number of pairs is $N \times N$
  - From this reduce self comparisons (e.g. A with A). So number is reduced to: $N \times N - N$
  - which can be written as $N \times (N - 1)$
  - Comparing A with B is the same as comparing B with A, so we are double counting this comparison
  - So, reduce the count by half = $\frac{N \times (N - 1)}{2}$

Comparing each element with all other elements



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Number of comparisons can be written as: $\frac{1}{2} \times N \times (N - 1)$

The number of comparisons grows really fast

N	$\frac{N \times (N - 1)}{2}$
2	**1**
3	**3**
4	**6**
5	**10**
6	**15**
7	**21**
8	**28**
9	**36**
10	**45**
100	**49,500**
1000	**4,99,500**

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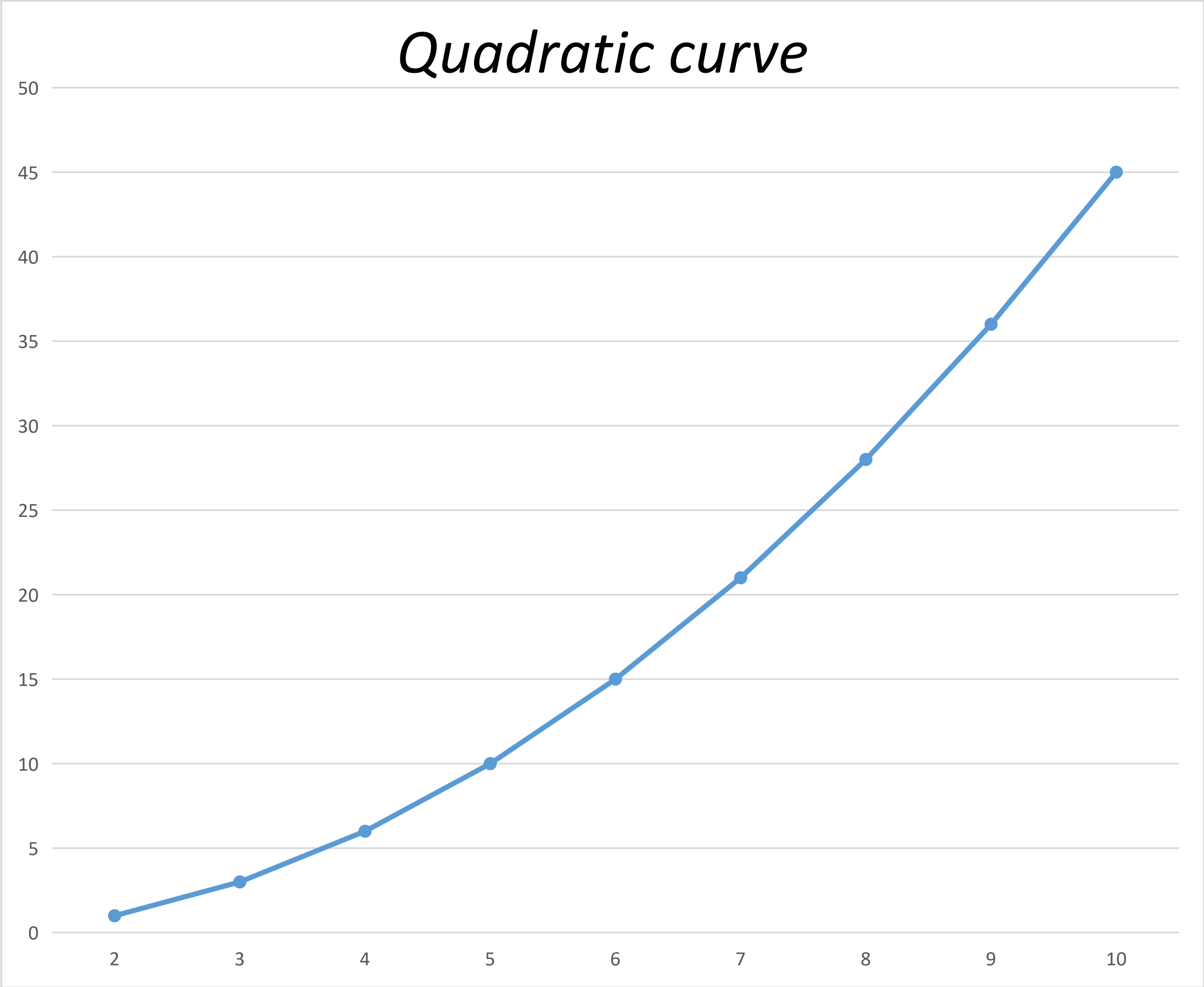
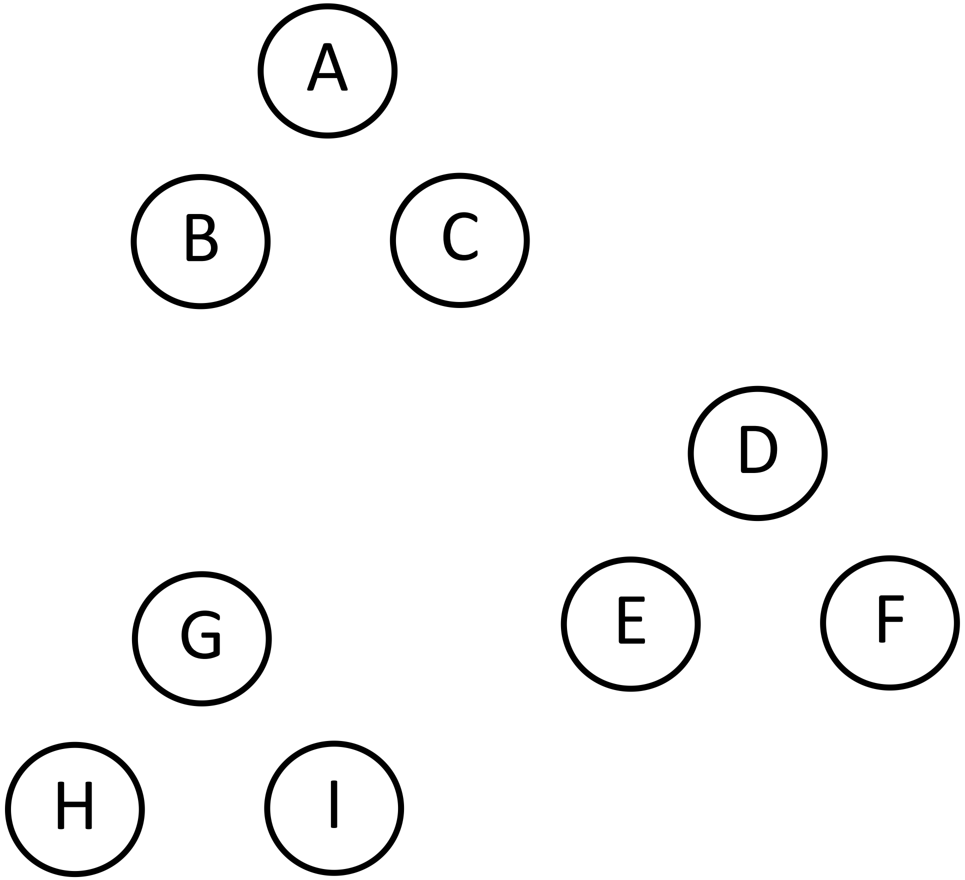


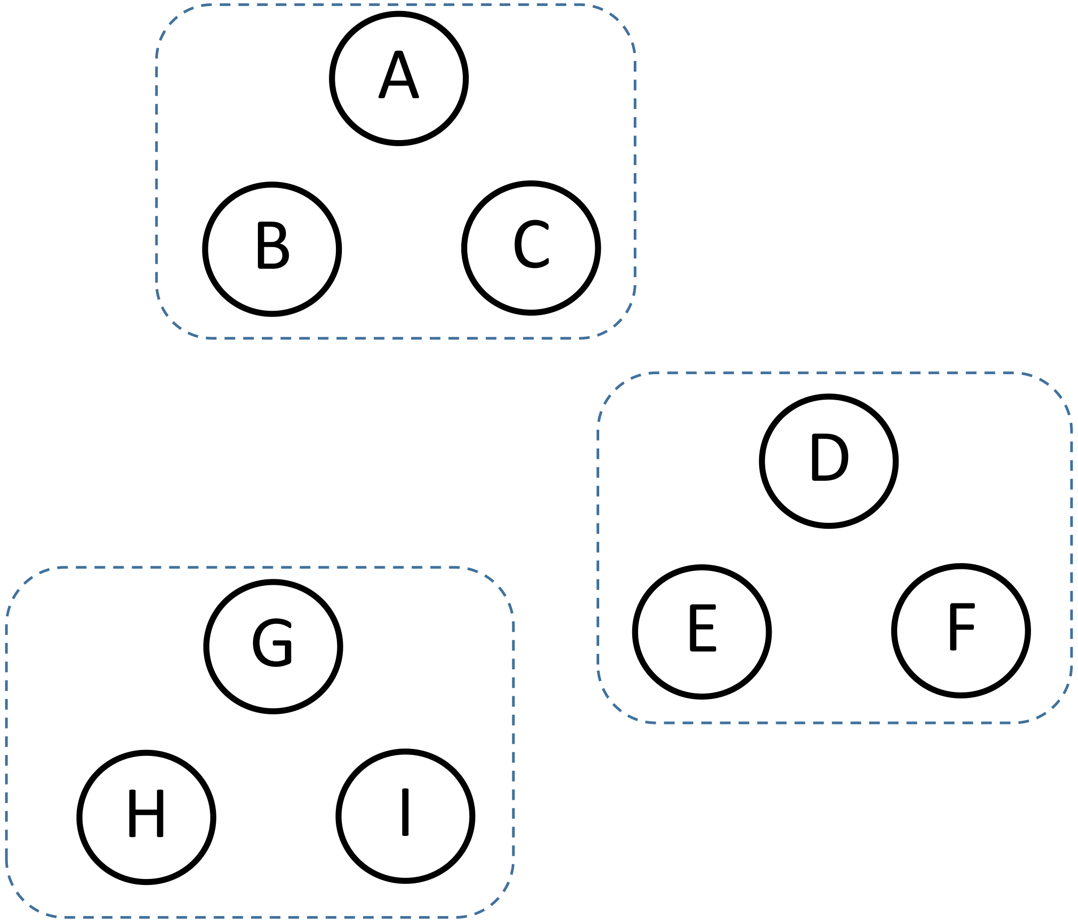
How do we reduce the number of comparisons?

Key idea: Use binning



- For 9 objects A,B,C,D,E,F,G,H,I:
  - The number of comparisons is $\frac{1}{2} \times 9 \times (9 - 1)$
    $= \frac{1}{2} \times 9 \times 8 = 9 \times 4 = 36$

Key idea: Use binning



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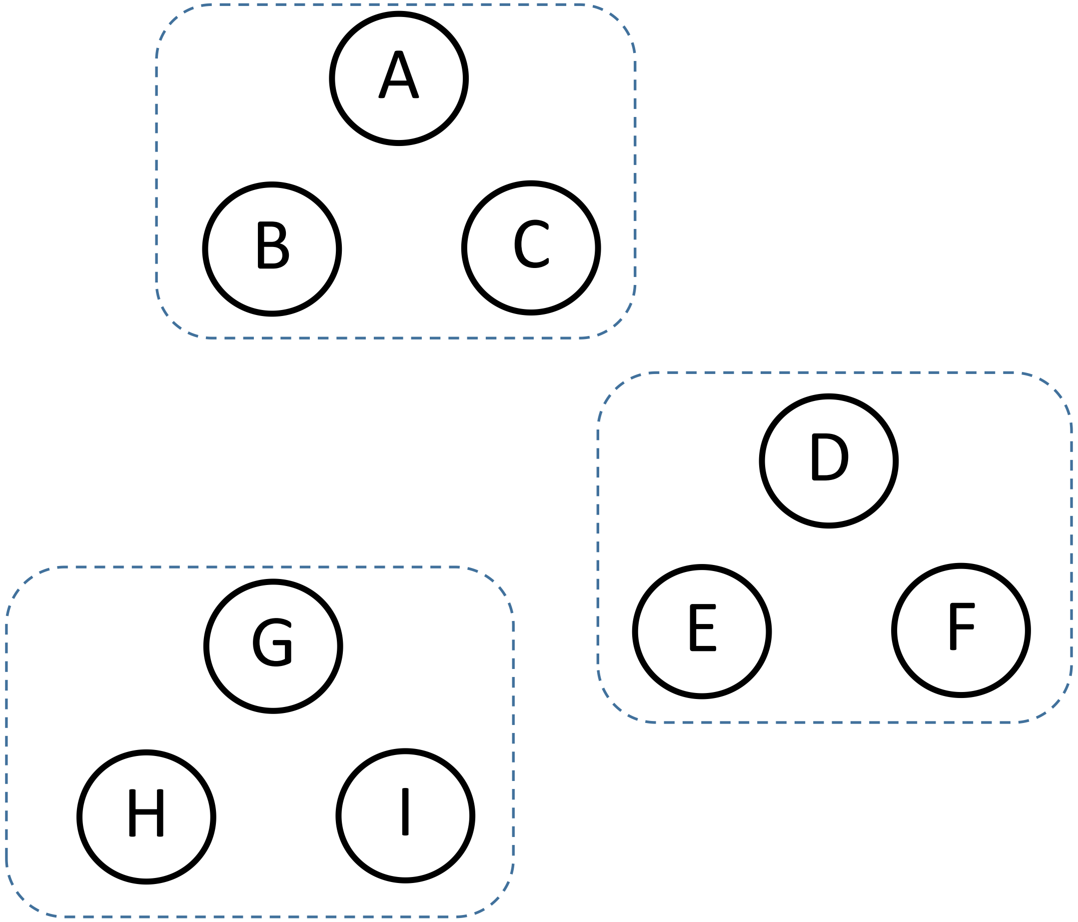

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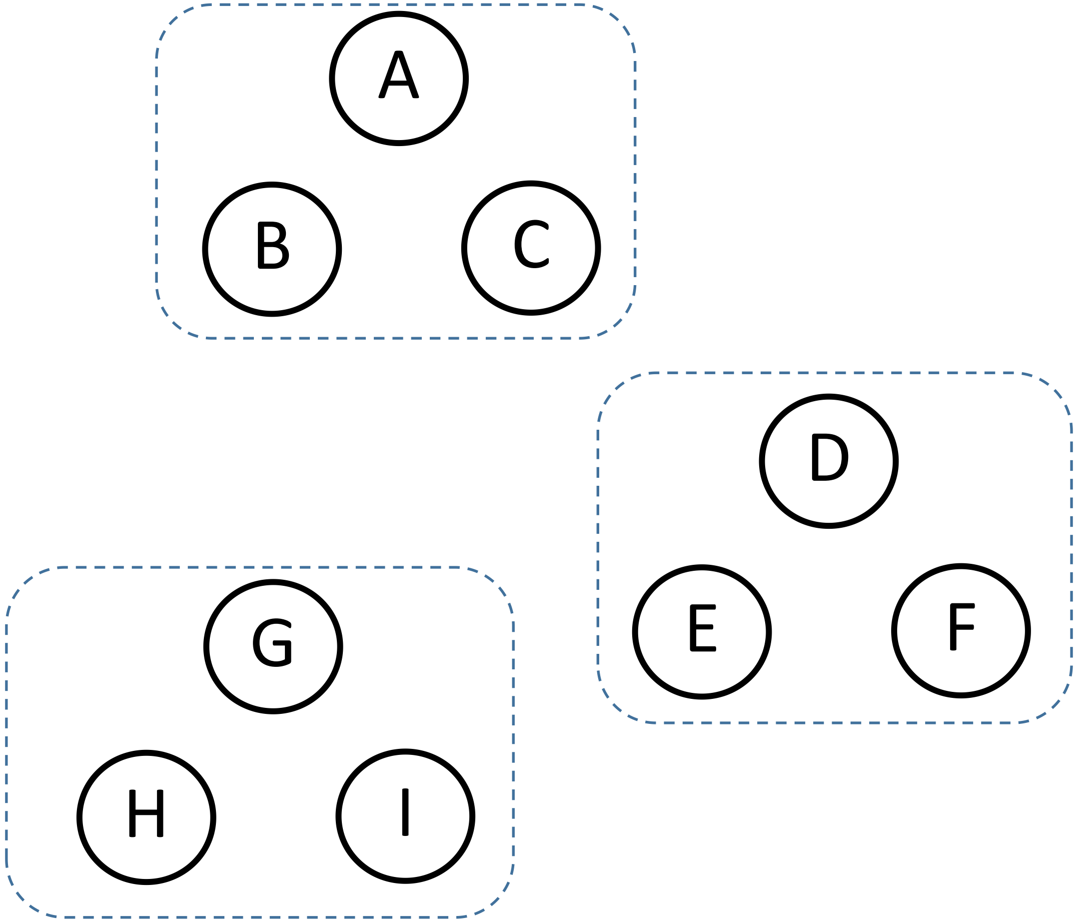
  - The number of comparisons is $\frac{1}{2} \times 9 \times (9 - 1)$
    $= \frac{1}{2} \times 9 \times 8 = 9 \times 4 = 36$
- If the objects can be binned into 3 bins of 3 each:
  - The number of comparisons per bin is:
    $\frac{1}{2} \times 3 \times (3 - 1) = \frac{1}{2} \times 3 \times 2 = 3$

Key idea: Use binning



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  - Total number of comparisons for all 3 bins is:
    $3 \times 3 = 9$

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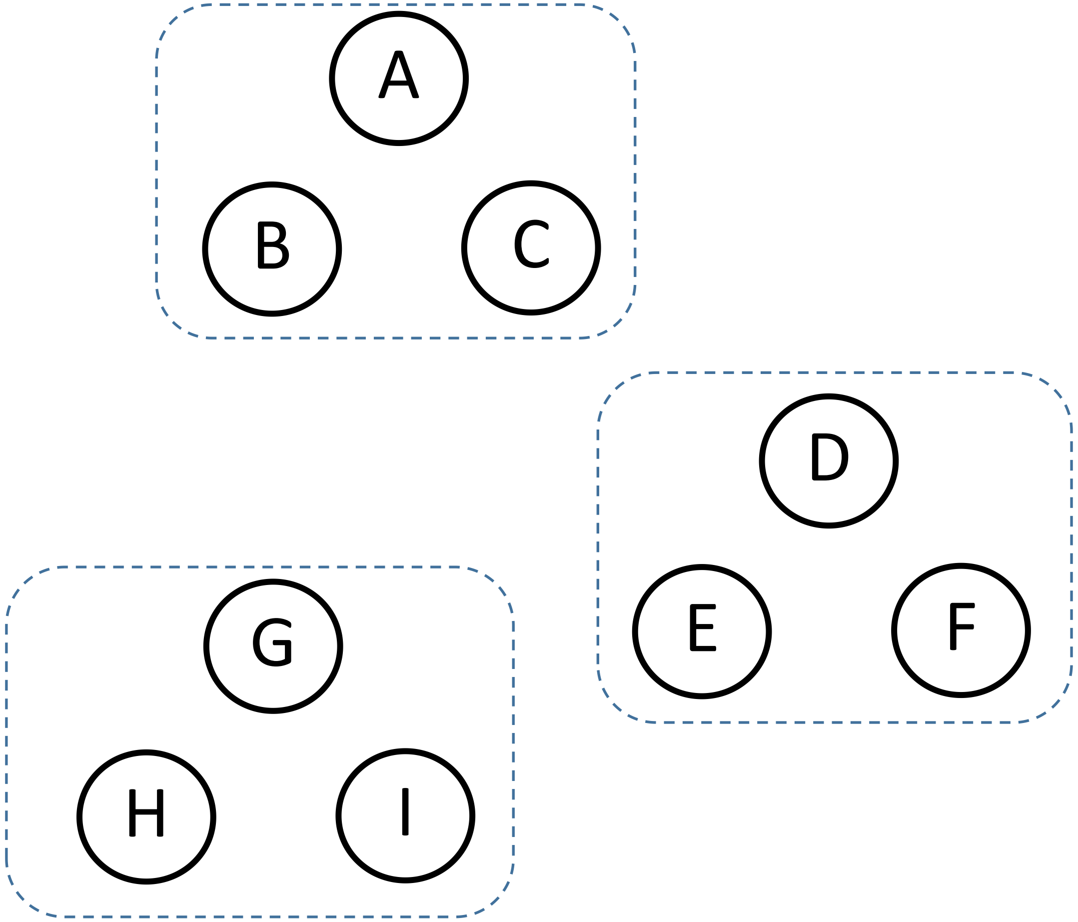

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  - The number of comparisons per bin is:
    $\frac{1}{2} \times 3 \times (3 - 1) = \frac{1}{2} \times 3 \times 2 = 3$
  - Total number of comparisons for all 3 bins is:
    $3 \times 3 = 9$
- So, the number of comparisons reduces from 36 to 9 !
  - *Reduced by a factor of 4 times.*

Calculation of reduction due to binning

- For N items:
- Number of comparisons without binning is: $\frac{1}{2} \times N \times (N - 1)$

- If we use K bins of equal size, number of items in each bin is: N/K
- Number of comparisons per bin is: $\frac{1}{2} \times N/K \times (N/K - 1)$
- Total number of comparisons is:
  $$K \times \frac{1}{2} \times N/K \times (N/K - 1) = \frac{1}{2} \times N \times (N/K - 1)$$

- Factor of reduction is: $[\frac{1}{2} \times N \times (N - 1)] / [\frac{1}{2} \times N \times (N/K - 1)]$
  $= (N - 1) / (N/K - 1)$
- For $N = 9$ and $K = 3$, this is $(9 - 1) / (3 - 1) = 4$
  - So reduction is by a factor of 4 times.

Summary

- The number of comparisons between all pairs of items grows quadratically, i.e. quite fast
- The formula of number of comparisons for N items is: $\frac{1}{2} \times N \times (N - 1)$

- Sometimes, it is possible to find a heuristic that allows us to put the items into bins and compare only items within the bins
- If there are N items put into K bins each of equal size, then the number of comparisons reduces to: $\frac{1}{2} \times N \times (N/K - 1)$

- The factor of reduction is: $(N - 1) / (N/K - 1)$