IIT Madras

ONLINE DEGREE

Functions: Examples

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Mathematics for Data Science 1
Week 1

Functions

- A rule to map inputs to outputs
  - $x \mapsto x^2$, $g(x) = x^2$

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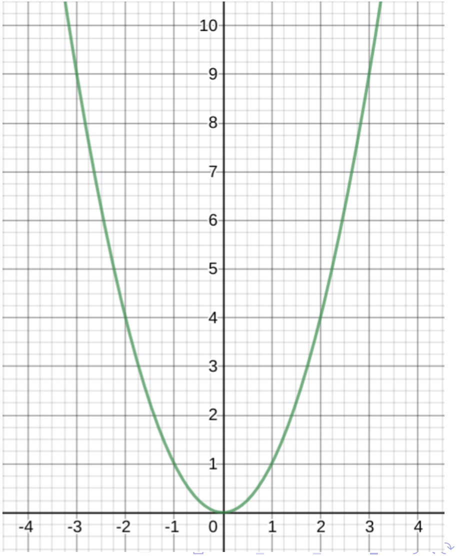

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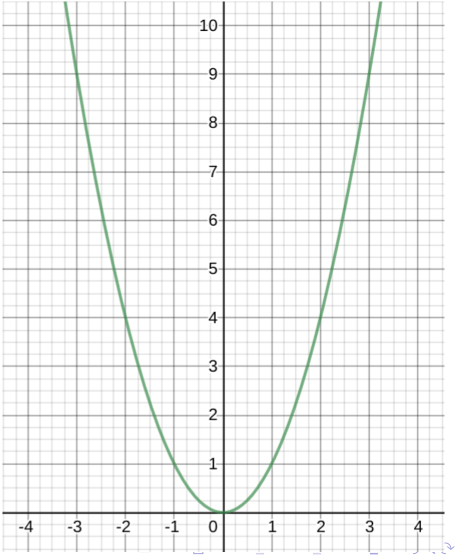

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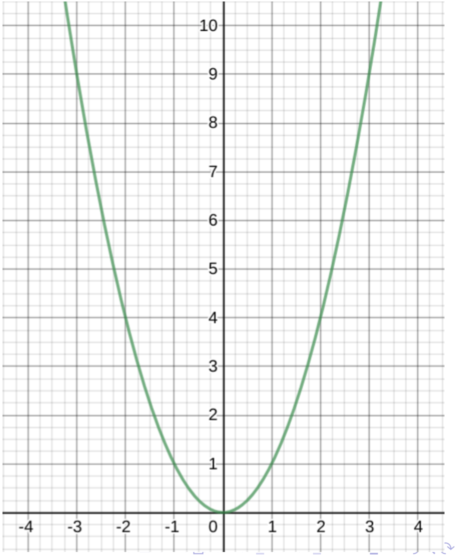

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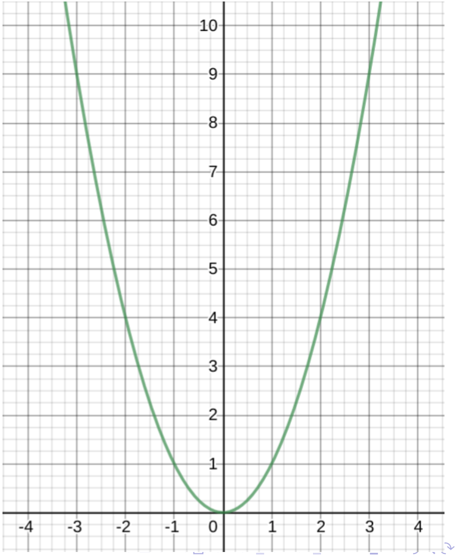
- What questions are we interested in?



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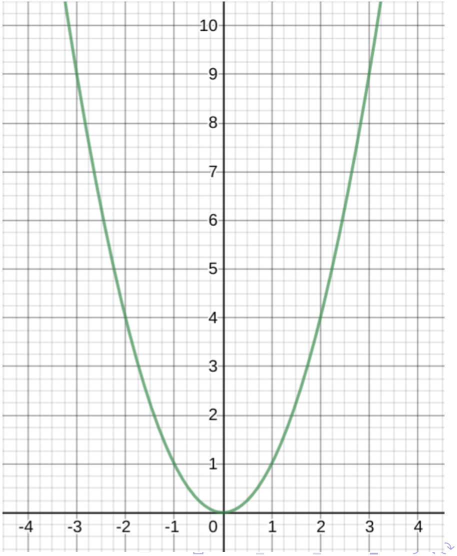

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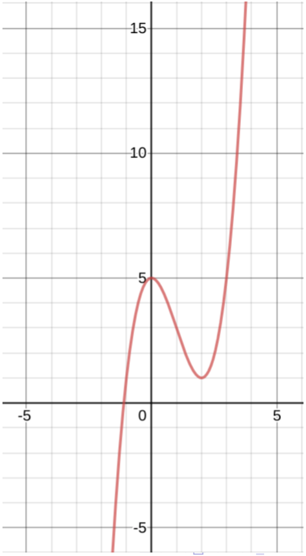

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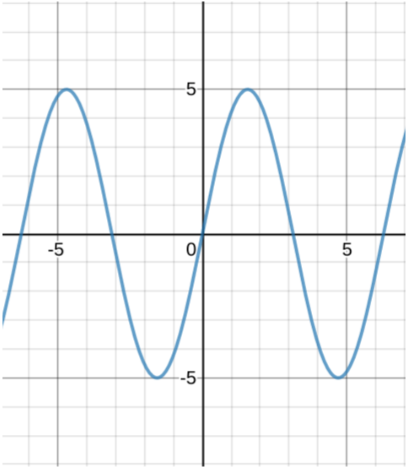
- $f(x) = 5\sin(x)$ has a bounded range, from -5 to $+5$



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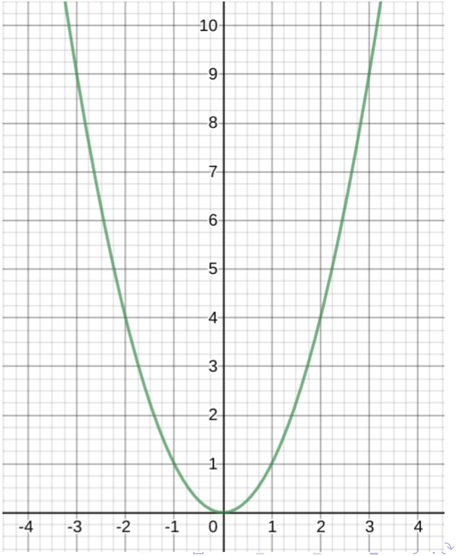

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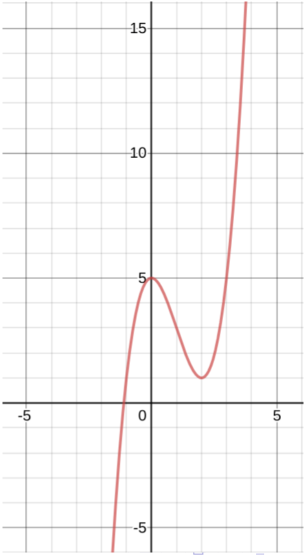

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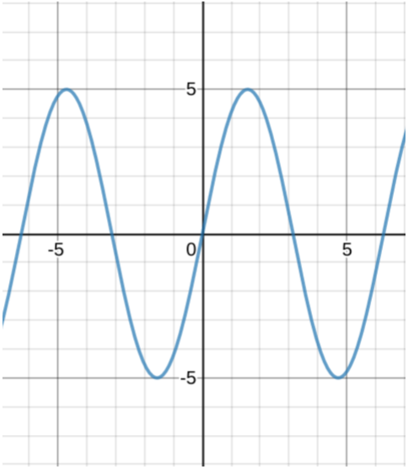
- $f(x) = 5\sin(x)$ periodically attains minimum value -5 and maximum value $+5$, infinitely often



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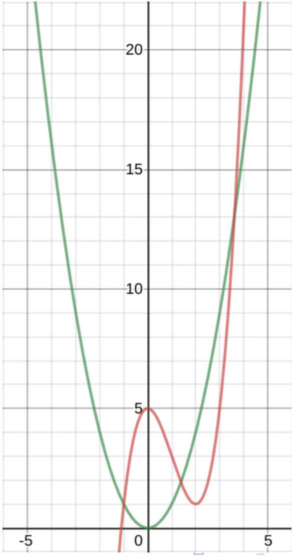

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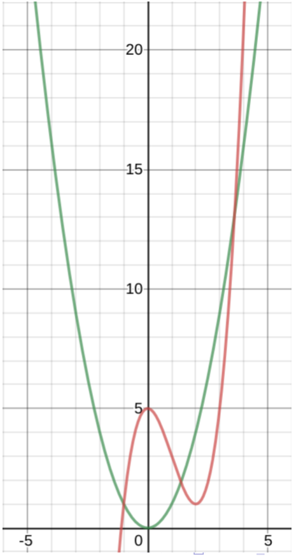

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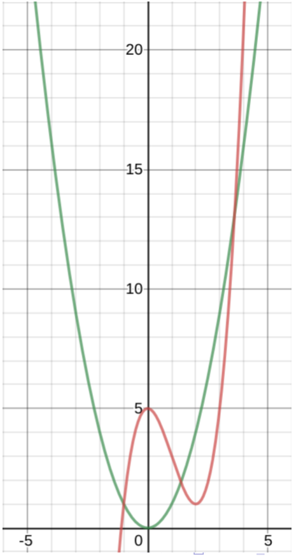

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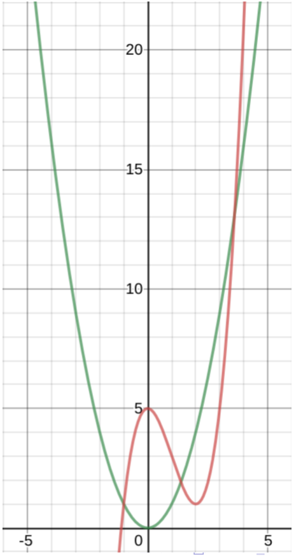

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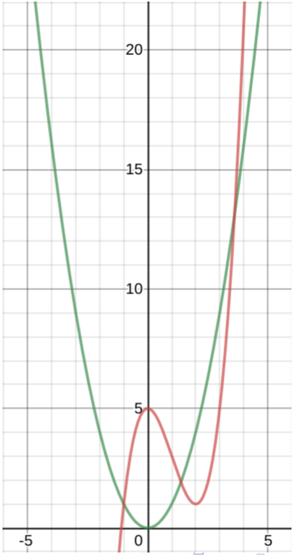
- If $J(y)$ grows faster than $G(y)$, more students will opt to study Data Science



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