IIT Madras

ONLINE DEGREE

Sets

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Mathematics for Data Science 1
Week 1

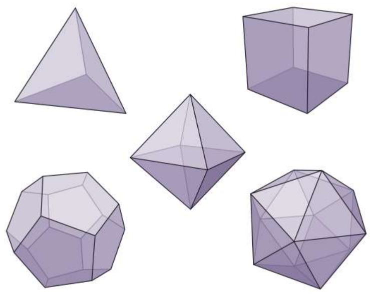
Sets

- A set is a collection of items
  - Days of the week: {Sun,Mon,Tue,Wed,Thu,Fri,Sat}
  - Factors of 24: {1,2,3,4,6,8,12,24}
  - Primes below 15: {2,3,5,7,11,13}
- Sets may be infinite
  - Different types of numbers: $\mathbb{N}$, $\mathbb{Z}$, $\mathbb{Q}$, $\mathbb{R}$
- No requirement that members of a set have uniform type
  - Set of objects in a painting
  - Spot the dog!

Three Musicians, Pablo Picasso
MOMA, New York

Order, duplicates, cardinality

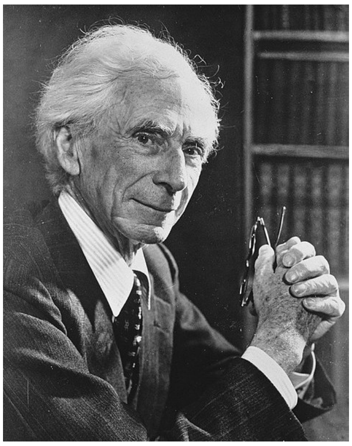
- Sets are unordered
  - {Kohli, Dhoni, Pujara}
  - {Pujara, Kohli, Dhoni}
- Duplicates don't matter (unfortunately?)
  - {Kohli, Dhoni, Pujara, Kohli}
- Cardinality: number of items in a set
  - For finite sets, count the items
    - {1,2,3,4,6,8,12,24} has cardinality 8
    - May not be obvious that a set is finite
  - What about infinite sets?
    - Is $\mathbb{Q}$ bigger than $\mathbb{Z}$?
    - Is $\mathbb{R}$ bigger than $\mathbb{Q}$?
    - Separate discussion



The Platonic solids
Set of cardinality 5
Wikimedia

Describing sets, membership

- Finite sets can be listed out explicitly
  - {Kohli, Dhoni, Pujara}
  - {1,2,3,4,6,8,12,24}
- Infinite sets cannot be listed out
  - $\mathbb{N} = \{0, 1, 2, \ldots\}$ is not formal notation
- Not every collection of items is a set
  - Collection of all sets is not a set
  - Russell's Paradox: Separate discussion
- Items in a set are called elements
  - Membership: $x \in X$, x is an element of X
  - $5 \in \mathbb{Z}$, $\sqrt{2} \notin \mathbb{Q}$



Bertrand Russell
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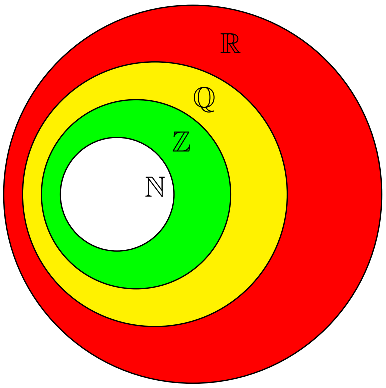
Subsets

- X is a subset of Y
  Every element of X is also an element of Y
- Notation: $X \subseteq Y$
- Examples
  - $\{$Kolhi,Pujara$\} \subseteq \{$Kohli,Dhoni,Pujara$\}$
  - Primes $\subseteq \mathbb{N}$, $\mathbb{N} \subseteq \mathbb{Z}$, $\mathbb{Z} \subseteq \mathbb{Q}$, $\mathbb{Q} \subseteq \mathbb{R}$
- Every set is a subset of itself: $X \subseteq X$
  - $X = Y$ if and only if $X \subseteq Y$ and $Y \subseteq X$
- Proper subset: $X \subseteq Y$ but $X \neq Y$
  - Notation: $X \subset Y$, $X \subsetneq Y$
  - $\mathbb{N} \subsetneq \mathbb{Z}$, $\mathbb{Z} \subsetneq \mathbb{Q}$, $\mathbb{Q} \subsetneq \mathbb{R}$,

Venn Diagram



The empty set and the powerset

- The empty set has no elements — $\emptyset$
- $\emptyset \subseteq X$ for every set X
  - Every element of $\emptyset$ is also in X
- A set can contain other sets
- Powerset — set of subsets of a set
  - $X = \{a, b\}$
  - Powerset is $\{\emptyset, \{a\}, \{b\}, \{a, b\}\}$
- Set with n elements has 2^n subsets
  - $X = \{x_1, x_2, \ldots, x_n\}$
  - In a subset, either include or exclude each x_i
  - 2 choices per element, $\underbrace{2 \cdot 2 \cdots 2}_{n \text{ times}} = 2^n$ subsets

Subsets and binary numbers

- $X = \{x_1, x_2, \ldots, x_n\}$
- n bit binary numbers
  - 3 bits: 000, 001, 010, 011, 100, 101, 110, 111
- Digit i represents whether x_i is included in a subset
  - $X = \{a, b, c, d\}$
  - 0101 is $\{b, d\}$
  - 0000 is $\emptyset$, 1111 is X
- 2^n n bit numbers