IIT Madras

ONLINE DEGREE

Mathematics for Data Science 1
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Week - 04
Tutorial - 05

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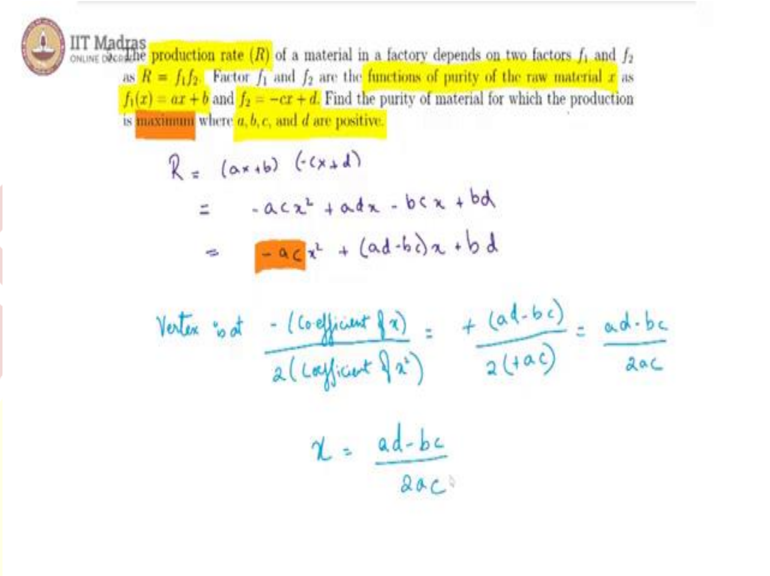

Our fifth problem looks a little complicated, but let us go one by one. And here we have the production rate of a material which is being made in a factory depends and it two factors f_1 and f_2 as $R = f_1 f_2$. And these two factors, they are the functions of the purity of the raw material. And that variable is x, x is the purity of the raw material. And both these functions are given to be linear $f_1(x) = ax + b$, $f_2(x) = -cx + d$. And it is given that a, b, c, d are all positive.

And it is asked find the purity of material, that is the value of x for which the production is maximum. So, let us understand what is being done here. We have two linear functions and the rate of production $R = f_1 f_2$, which will then $R = (ax + b)(-cx + d) = -acx^2 + adx - bcx + bd = -acx^2 + (ad - bc)x + bd$.

We are told that a, b, c, d are all positive, and that indicates the coefficient of x^2 is negative because the negative of ac and that means this is a quadratic function whose parabola is downturned, therefore, we will be able to get a maximum value at some point and this is going to be at the vertex, we know that this is going to be at the vertex. So, the vertex is at $\frac{-b}{2a}$, that is because here we have a, b, c, d already.

Let us write it down more carefully, that is the $\frac{-(coefficient\ of\ x)}{2(coefficient\ of\ x^2)} = -\frac{(ad-bc)}{2(-ac)} = \frac{ad-bc}{2ac}$, is where we will get the vertex. And since we know that the maximum is going to occur at this particular x, we get the $x = \frac{ad-bc}{2ac}$.