IIT Madras

ONLINE DEGREE

Statistics for Data Science-1
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Lecture 7.5
Conditional Probability - Independent Events Examples

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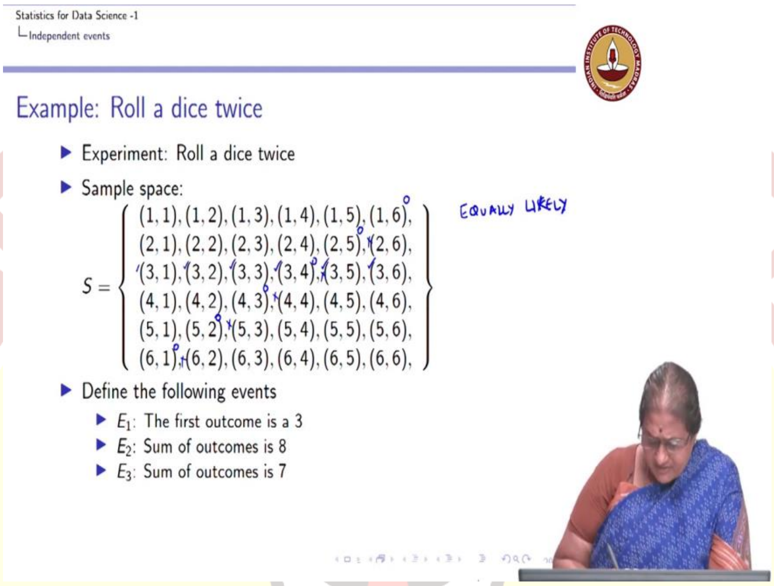

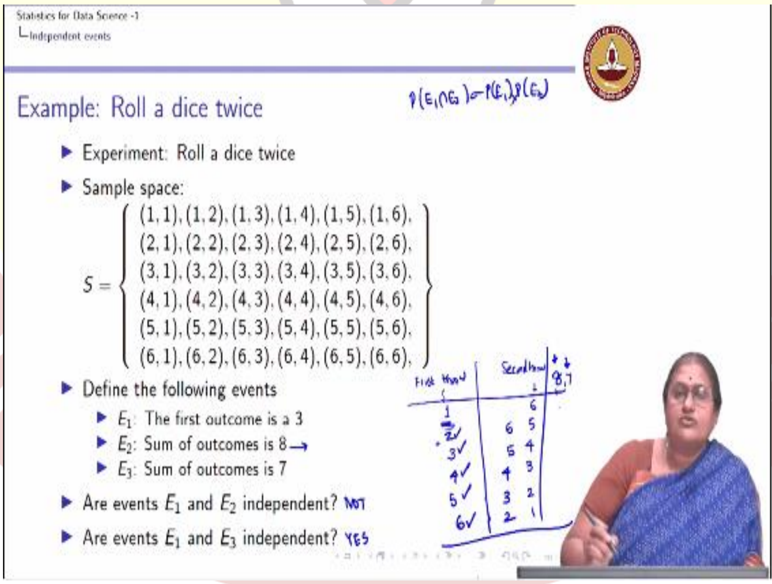

Now let us look at an example we have already established the sample space of the experiment of rolling a dice twice. We know that the sample space has 36 outcomes and these are the 36 outcomes that are listed in the sample space. Each one of them are equally likely that is another assumption we have and we also know it is a good assumption it is not a bad assumption to have.

So now let us define three events. E_1 is the event that the first outcome is the 3 so we can see that this outcome, this outcome, this outcome, this outcome, this outcome and this outcome namely (3,1), (3, 2), (3,3), (3, 4), (3, 5), (3, 6) are the outcomes in my event E_1. E_2 is the outcome, sum of outcomes is a 8. So now I can see that let me denote it by cross 5+3, 4+4, 3+5, 2+ 6 and 6+ 2.

These are the outcomes that satisfy this and they are in this event E_2. Now when I look at sum of outcomes as a 7 I see the outcomes I put a circle here (1,0), (2,5), (3,4), (4,3), (5,2) and (6,1) these are the outcomes in my event E_3, but now having defined this events the question we are asking is are E_1 and E_2 independent and are E_1 and E_3 independent. So before we go to verify whether E_1 and E_2 and E_1 and E_3 are independent events.

Let us look at the intuition behind independence. Now E_2 is an event where the sum of outcome is 8. Now let us write down the following the first throw of a dice and the second throw of a dice. Now to get a sum of 8 if the first row was a 1 I cannot get a sum of 8 because the maximum that I can get in a second throw is 6 and 1 plus 6 is 8. So if the first throw is a 1 I cannot get a sum of 8.

The first row is a 2 I need to have a 6, 3, 5, 4, 4, 5, 3 and 6, 2. In a sense to get a sum of 8 I need the first throw to be either 2, 3, 4, 5 or 6 to get a sum of 8. Now let us repeat the experiment with 7. So if I have sum 7 written here the first throw is a 1 then if the second throw is a 6 I get a sum of 7 5 I get a sum of 7 3 plus 4 is a 7, 4 plus 3 is a 7, 5 + 2 is a 7 and 6 + 1 is a 7.

So to get a sum of 7 I see that the first throw can be of these 6 values and I get a sum of 7 which is possible with this second throw whereas to get a sum of 8 we saw that if the first throw was a 1 it is impossible to get a sum of 8. In other words to get a sum of 8, the sum 8 is dependent on what was your first throw whereas the sum 7 was independent of what was your first throw so this is the intuition.

So sum to get an 8 I could not have got a sum of 8 if my first throw was a 1 whereas a sum 7 is independent even if it was a 1 and I throw a 6 I get a sum of 7 if it was a 2 and I throw a 5 so it is independent of whatever was my first throw there is a chance of me a sum of 7 whereas to get a sum of 8 I needed the first throw to be either a 2 or a 3 or a 4 or a 5 or a 6. So intuitively we can say that E_1 and E_2 are not independent.

Whereas E_1 and E_3 are independent events. Now let us apply the formula to actually verify it. Recall the formula sates that if E_1 and E_2 are independent then the probability of the intersection has to be the product of the probability so this is what we have to check.

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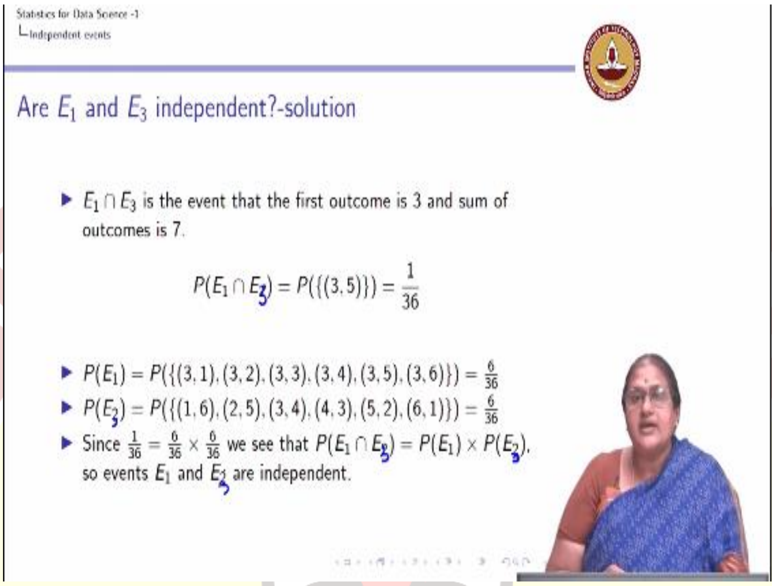

So let us go to the first thing E_1 and E_2 are independent. So I know $E_1 \cap E_2$ is a event that the first outcome is a 3 and the sum of outcomes as a 8 that is how I can articulate the event $E_1 \cap E_2$. I know that only this outcome and since all the outcomes are equally likely I know the probability of this outcome alone is equal to 1 by 36 and I know that is what I have as the probability of $E_1 \cap E_2$ which is $\frac{1}{36}$.

Now probability of E_1, E_1 is the event that the first outcome is a 3 again we saw that the outcomes the first outcomes is a 3 has these 6 outcomes where the outcome is a 3. So my E_1 event has the outcomes the outcomes of the E_1 event is going to be (3,1), (3,2), (3,3), (3,4), (3,5) and (3,6) again using that all of them are equally likely and the probability of events I get probability of E_1 is $\frac{6}{36}$.

Probability of E_2 where the sum of outcomes is a 8 we also saw that there are 5 outcomes which give an outcome of 8 hence probability of E_2 equal to is $\frac{5}{36}$. Now you can see that is $\frac{1}{36}$ is not equal to is $\frac{6}{36} \times \frac{5}{36}$. Hence, again using the if and only if condition we can conclude that probability of E_1 intersection E_2 is not equal to the product of probabilities hence E_1 and E_2 are not independent events.

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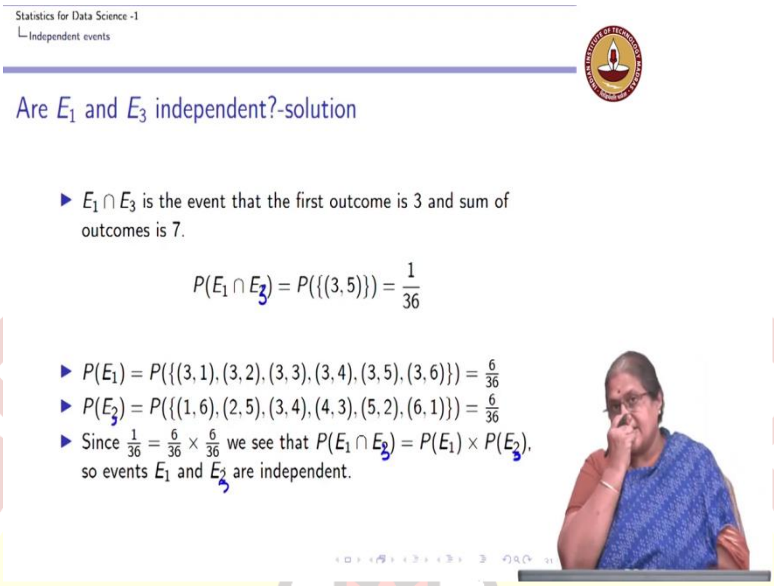

So the next question we are asking is are E_1 and E_3 independent? So E_1 intersection E_3 is the events that the first outcome is a 3 and sum of outcomes is a 4 so this is probability of 3, 4 which is is $\frac{1}{36}$. So I have probability of E_1 intersection E_3 is probability of 3,4 which is is $\frac{1}{36}$. So I have probability of E_1 as before which is equal to is $\frac{6}{36}$ I can see that this is equal to is $\frac{1}{6}$.

I have probability of E_3 is probability of (1,6), (2,5), (3,4), (4,3), (5,2) and (6,1) which is equal to is $\frac{6}{36}$ which is also equal to is $\frac{1}{6}$. Hence I have $P(E_1 \cap E_3)$ =$P(E_1)\times P(E_3)$ in this case which tells me that probability of, hence $E_1 \cap E_3$ is probability of E_1 into probability of E_3. Hence the events E_1 and E_3 are independent events.

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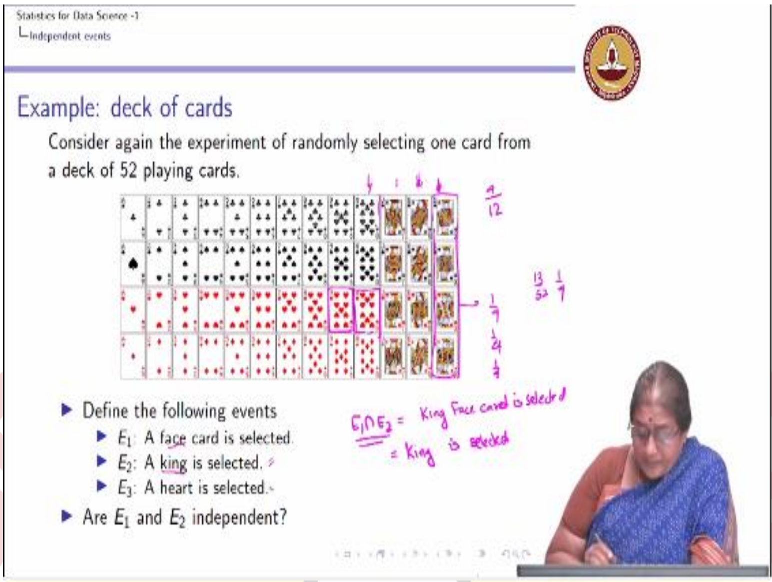

Now let us go and apply this notion of independence to our cards or deck of cards example. So again consider the experiment of randomly selecting one card from this deck of 52 playing cards. Let me define the following events E_1 is a face card is selected. What is a face card? Face card is a card which has a face on it so I have 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12 of these cards which are referred to as face cards.

These cards which has a face namely jack, queen or king are the cards that are referred to as a face card. So, a face card is selected you can see that I have 12 cards which gives me probability of E_1 is 12/52. Now probability of E_2 is the event that a king is selected again you can see that the event a king is selected I have 4 kings and the chance of choosing a king from 52 card is going to be 4/52.

A heart so you can see an event an heart is selected I have here 13 hearts so the probability or the chance that I am selecting a heart probability of E_3 is is $\frac{13}{52}$. So these are the actual probabilities which I can have. Now the question that is being asked is are E_1 and E_2 independent events. So now let us understand what is $E_1 \cap E_2$. Now E_1 is a face card E_2 is a king card.

So $E_1 \cap E_2$ is the event that a king face card is selected which is equivalent to the probability a king is selected. Similarly, E_2 and E_3 what does the event $E_2 \cap E_3$? E_2 is a king E_3 is a heart so $E_2 \cap E_3$ is the event a king and heart. In other words, a king of hearts is selected. So now the question we are asking first question are E_1 and E_2 independent events?

So now let us look at the first question which is asking about $E_1 \cap E_2$ that is knowing that a face card is selected will that affect my chance of having a king. So you can see that there are 12 face cards. Now if I know that a face card is selected then I know that the chance of getting a king from these 12 cards is is $\frac{4}{12}$ intuitive explanation, but whereas if I know that a king is selected the chance of you having a heart is just is $\frac{1}{4}$.

Now the chance of you having a heart from these 52 cards is also is $\frac{13}{52}$ which is is $\frac{1}{4}$. Now this whether you choose a king or a queen again a chance of you getting a heart if you had selected a queen is also is $\frac{1}{4}$ whether jack is also is $\frac{1}{4}$, 10 also the chance of getting a heart is is $\frac{1}{4}$, 9 is $\frac{1}{4}$. So you see that the chance of you getting a heart is not dependent on whatever has been the card.

Whereas the chance of you getting a king changes if you know the information that what you had taken was a face card.

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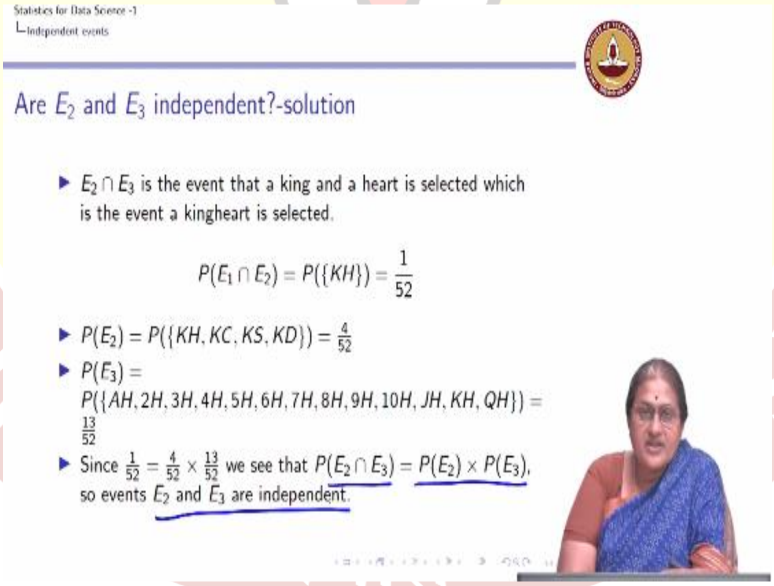

So now us move forward and verify this. So $E_1 \cap E_2$ is the event that a face card and a king is selected. So we know that this probability is nothing, but the probability of having a king. We know there are 4 king card which is king of hearts, king of clubs, king of spades and king of diamonds. Hence, the probability of a face card and the king is the same as the probability of a king which is 4/52. So what is the probability of a face card?

Again I know there are 12 face cards, jack heart, jack clubs, jack spades, jack diamond, king hearts, king clubs, king space, king diamonds, queen hearts, queen clubs, queen spades and queen diamonds. Hence, the probability of this event E_1 that is getting a face card is 12/52, probability of getting a king is 4/52 you can verify $\frac{4}{52} \neq \frac{12}{52} \times \frac{4}{52}$.

Hence, my $P(E_1 \cap E_2) \neq P(E_1) \times P(E_2)$. So, E_1 and E_2 are not independent of each other something which we could explain intuitively. Now let us look at the case of E_2 and E_3. Now what is event E_2? E_2 is the event that there is a king, E_3 is the event of a heart so $E_2 \cap E_3$ is the event of a king and a heart. In other words you are selecting a king of hearts card.

So probability of $E_1 \cap E_2$ is just the probability of choosing the king of heart cards which is 1/52 considering all of them are equally likely events. Now probability of choosing a heart. So probability of choosing a king is 4/52, probability of choosing a heart is I can have a 2, 3,4, 5, 6, 7, 8, 9, 10 jack king or queen so 13 possible choices it is 13/52. We observe $\frac{1}{52} = \frac{4}{52} \times \frac{13}{52}$.

So this is 13 so I get this is 1/52 hence I can say that since $P(E_2 \cap E_3) = P(E_2) \times P(E_3)$. E_2 and E_3 are indeed independent events. This is something which we even checked intuitively.