Statistics for Data Science -1

Lecture 6.2: Probability- Events

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Learning objectives

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7. Distinguish between independent and dependent events.
8. Solve applications of probability.

Random Experiment, Sample Space, Events

Events

Definition

An event E is a collection of basic outcomes.

- That is, an event is a subset of the sample space.
- We say an event has occurred if the outcome is contained in the subset.

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- Experiment: Measuring the lifetime (in hours) of a bulb
  Event: life time is less than or equal to four hours
  $E = \{x : 0 \leq x \leq 4\}$

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  - A comes first **or** B comes second.
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  Event:
  - head on the first toss $E_1 = \{HH, HT\}$
  - head on second toss $E_2 = \{HH, TH\}$
  - head on first or second toss $E_1 \cup E_2 = \{HH, HT, TH\}$

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  - A comes first **and** B comes second.
    $E_1 \cap E_2 = \{\mathbf{ABCDEF}, \mathbf{ABCDFE}, \mathbf{ABDCFE}, \ldots, \mathbf{ABDCFE}\}$

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  - head on the first toss $E_1 = \{HH, HT\}$
  - head on second toss $E_2 = \{HH, TH\}$
  - head on first and second toss $E_1 \cap E_2 = \{HH\}$

Null event and disjoint event

Definition

We call the event without any outcomes the null event, and designate it as Φ

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If the intersection of E *and* F *is the null event, then since* E *and* F *cannot simultaneously occur, we say that* E *and* F *are disjoint, or mutually exclusive.*

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  Event:
  - answer is A; $E_1 = \{A\}$
  - answer is B; $E_2 = \{B\}$
  - answer is A and B; $E_3 = E_1 \cap E_2 = \Phi$
  - We say events E_1 and E_2 are mutually exclusive or disjoint. Occurence of E_1 disallows occurence of E_2. In other words, if my $A(B)$ is my guess, then $B(A)$ cannot be my guess.

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- That is, E^c will occur if and only if E does not occur.
- The complement of the sample space is the null set, that is $S^c = \Phi$

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  - Event E_1: out come is head $E_1 = \{H\}$
  - Event E_2: out come is tail $E_2 = \{T\}$
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  - Sample space: $S = \{HH, HT, TH, TT\}$
  - Event: head on the first toss $E_1 = \{HH, HT\}$
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- Experiment: Tossing two coins and noting the outcomes
  - Sample space: $S = \{HH, HT, TH, TT\}$
  - Event: head on the first toss $E_1 = \{HH, HT\}$
  - $E_1^c = \{TH, TT\}$;tail on first toss

Subsets

Definition

For any two events E and F, if all of the outcomes in E are also in F, then we say that E is contained in F, or E is a subset of F, and denote it as $E \subset F$

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  - Sample space: $S = \{HH, HT, TH, TT\}$
  - Event: head on the first toss $F = \{HH, HT\}$
  - Event: head in both the tosses $E = \{HH\}$
  - $E \subset F$

Section summary

1. Notion of events
2. Union, intersection, complement of events
3. Null event and mutually exclusive (disjoint) events