

**IIT Madras**  
ONLINE DEGREE

**Mathematics for Data Science 1**  
**Professor. Neelesh S Upadhye**  
**Department of Mathematics**  
**Indian Institute of Technology, Madras**  
**Week - 04**  
**Tutorial - 08**

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8. An advertiser is analysing the growth of likes for their new ad on YouTube. She analyzed that the increase in likes in a given second is equal to  $4t_{av}$  where  $t_{av}$  is midpoint of the time interval. For example, the increase in likes from 3 seconds to 4 seconds is equal to  $4 \times 3.5$ . Answer the following questions.

(a) If the total likes follow the path as  $l(t) = at^2 + bt + c$  then what is the value of  $b$ ?

(b) Find the total likes at the end of 60 seconds.

(c) If the domain of the function  $l$  is  $[k, \infty]$ , what is the value of  $k$ ?

Handwritten derivations:

$$t, t+1$$

$$l(t+1) - l(t) = 4 \times \frac{(t+1)+t}{2} = 4t+2$$

$$l(t+1) = a(t+1)^2 + b(t+1) + c$$

$$l(t) = at^2 + bt + c$$

$$l(t+1) - l(t) = at^2 + 2at + a + bt + b + c - at^2 - bt - c = 2at + a + b$$

For our eighth question we have an advertiser who is analyzing the growth of likes for their new ad on YouTube. She analyzed that the increase in likes in a given second is equal to 4 times  $t_{av}$ , where  $t_{av}$  is the midpoint of the time interval, that is the average time in that time interval. And so we are given an example to explain what this is. The increase in likes from 3 seconds to 4 seconds. So, from the time  $t = 3$  to the time  $t = 4$ , there is a number of increase in likes, which is equal to  $4 \times 3.5$  and 3.5 is the midpoint of 3 and 4.

So, one way to write this is, let us look at time  $t$  seconds and the time  $t + 1$  seconds. Then it is given to us that the likes at time  $t + 1$ , so, number of likes is a function of time. So,  $l(t + 1) - l(t) = 4 \times t_{av} = 4 \times \frac{(t+1)+t}{2} = 4t + 2$ , this is the difference in the likes from time  $t$  seconds to  $t + 1$  seconds.

Now, it is further given to us that this particular function is a quadratic function. So,  $l(t + 1) = a(t + 1)^2 + b(t + 1) + c$  and  $l(t) = at^2 + bt + c$ . Then  $l(t + 1) - l(t) = at^2 + 2at + a + bt + b + c - (at^2 + bt + c) = 2at + a + b$

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$$l(t+1) - l(t) = \frac{2}{2} [t+t+1] = 2[2t+1] = 4t+2$$

$$l(t+1) = a(t+1)^2 + b(t+1) + c$$

$$l(t) = at^2 + bt + c$$

$$l(t+1) - l(t) = \frac{\cancel{a}t^2 + 2at + a + \cancel{b}t + b + \cancel{c}}{-\cancel{a}t^2 - \cancel{b}t - \cancel{c}} = 2at + a + b$$

$$2at + a + b = 4t + 2$$


$$2at = 4t ; a+b=2$$

$$a=2 ; b=2-a=0$$

This quantity is basically equal to  $2t + 4$ . So, we are saying that  $2at + a + b = 2t + 4$ . Now, what are we supposed to acknowledge here is that the term with the  $t$  in it, that is the time dependent term is going to be same on both sides. Whereas the term which is constant is going to be same on both sides.

Thus, we are saying  $2at = 4t$  and  $a + b = 2$ . This gives us 2 times  $t$  and  $t$  cancelled. So, we know  $a = 2$  and that would imply  $b = 2 - a = 0$ .

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 IIT Madras advertiser is analysing the growth of likes for their new ad on YouTube. She analyzed that the increase in likes in a given second is equal to  $4t$ , where  $t$  is midpoint of the time interval. For example, the increase in likes from 3 seconds to 4 seconds is equal to  $4 \times 3.5$ . Answer the following questions.

(a) If the total likes follow the path as  $l(t) = at^2 + bt + c$  then what is the value of  $b$ ?  $\Rightarrow 0$


(b) Find the total likes at the end of 60 seconds.

(c) If the domain of the function  $l$  is  $[k, \infty)$ , what is the value of  $k$ ?

$$\begin{aligned}
 & t, t+1 \\
 & l(t+1) - l(t) = \frac{2}{2} [t+t+1] = 2[2t+1] = 4t+2 \\
 & l(t+1) = a(t+1)^2 + b(t+1) + c \\
 & l(t) = at^2 + bt + c \\
 & l(t+1) - l(t) = \frac{at^2 + 2at + a + bt + b + c}{-at^2 \quad -bt \quad c}
 \end{aligned}$$

And our question is asking us what is the value of  $b$ . So, we know this is equal to 0. Second question, the second part of the question is asking what is the total number of likes at the end of 60 seconds.

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$$\begin{aligned}
 l(t) &= at^2 + c \\
 &= 2t^2 + c \\
 l(60) &= 2(60)^2 + c \\
 \text{If } l(0) &= 0, \text{ then } c = 0 \\
 \Rightarrow l(t) &= 2t^2 \\
 l(60) &= 2 \times 60 \times 60 = 7200 \text{ likes}
 \end{aligned}$$

That would be impossible to calculate because we have the values of  $a$  and  $b$ , so we know that our  $l(t)$ , in this case we want  $l$  of 60.  $l(t) = at^2 + 0t + c = 2t^2 + c$ . But we do not know what  $c$  is. So,  $l(60) = 2 \times 60^2 + c$ . Now, if we made further interpretations that there were 0 likes at time  $t = 0$ . So, if  $l(0) = 0$  then  $c = 0$ . So, this is a particular assumption we are making, we are assuming that the timer started when the likes were 0 and that would imply your  $l(t) = 2t^2$ .

So  $l(60) = 2 \times 60 \times 60 = 7200$ , that is 7200 likes at the end of 1 minute.

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$$c) \quad l(t) = 2t^2 + c$$

$$l(t) \geq 0 \rightarrow 2t^2 + c \geq 0$$

$$l(0) = c \geq 0$$

$$2t^2 + c \geq 0$$



$$[0, \infty)$$

And lastly, for Part C, we are being asked the domain of the function is  $[k, \infty)$ , what is the value of  $k$ . We know that  $l(t) = 2t^2 + c$ . Now only real requirement we have is that our likes be greater than or equal to 0. So,  $l(t) \geq 0 \rightarrow 2t^2 + c \geq 0$ . Another thing we have is clearly that  $l(0) = c \geq 0$ , because at 0 time, it is not like you can have negative likes. So,  $c \geq 0$ .

Now we know that  $t^2 \geq 0$  and now we also found that  $c \geq 0$ . So,  $2t^2 + c \geq 0$ , which means any time that is 0 or greater than 0. So, we are looking at the timer being started at a particular time and from there on, if this is 0 from there on your function is well defined and the number of likes will be greater than or equal to 0. So, the domain will be all the time from 0 seconds to  $\infty$ .