IIT Madras
ONLINE DEGREE

Mathematics for Data Science 1
Week-03
Tutorial-02

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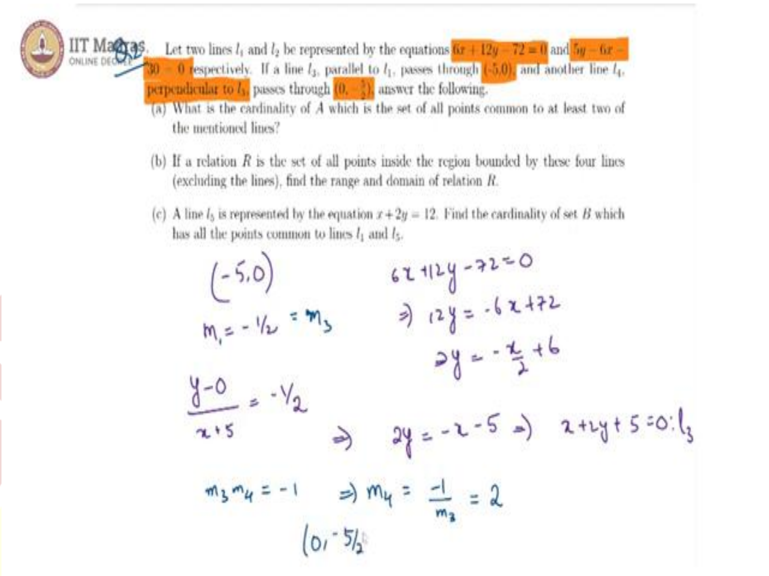

And for our second question, there are 2 lines, and these are the equations, which represent our lines. And a line l_3 is parallel to l_1, and passes through (-5, 0). Now we can find l_3, by using the point slope form, we already have the point, which is (-5, 0). And we can also find the slope from l_1 slope, we already have l_1. And we can write, so l_1 is this, $6x+12y-72=0,$ which tells us that $12y=-6x+72.$

And that gives us $y=-\frac{x}{2}+6$ so, the slope here is $-\frac{1}{2}$, because $y=mx+c.$ So, slope is $-\frac{1}{2}$. Now if we did point slope form on this, we would get $\frac{y-0}{x+5}=-\frac{1}{2}$ which indicates $2y=-x-5$. So therefore, $x+2y+5=0$ is basically our line l_3. And now if we look further, we have line l_4 which is passing through this point, and it is perpendicular to l_3.

So, if we took this to be $m_1=-\frac{1}{2}=m_3$ because m_1 and m_3 are the same slope. And let us consider the slope of l_4 to be m_4, so we can say $m3\times m4=-1$, because they are perpendicular, that would indicate $m_4=-\frac{1}{m_3}$, which is basically 2. So we now have the slope of l_4. And it also goes through this point.

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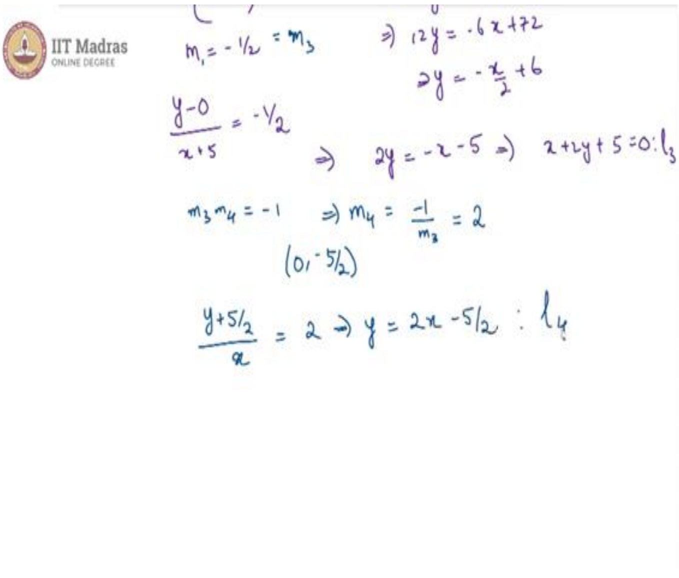

So again, using point slope form, we have $\frac{y+\frac{5}{2}}{x} = 2$, that would indicate $y = 2x - \frac{5}{2}.$ So this is our l_4.

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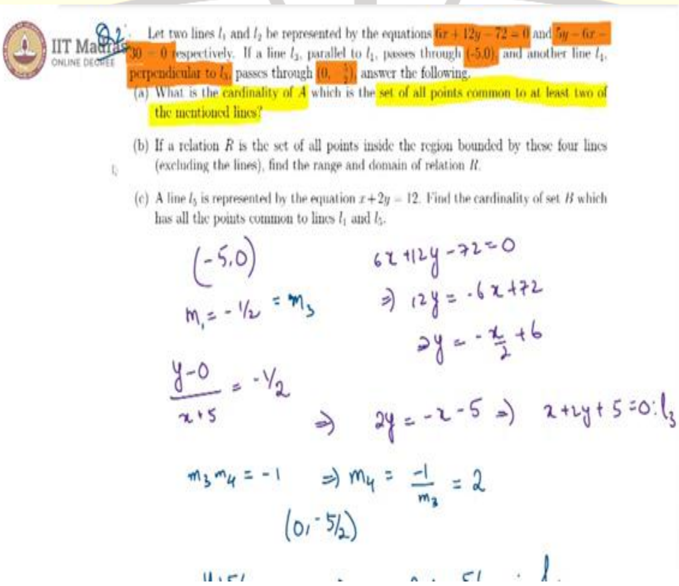

Now, the question is being asked is, what is the cardinality of A, which is a set of all points common to at least 2 of the mentioned lines.

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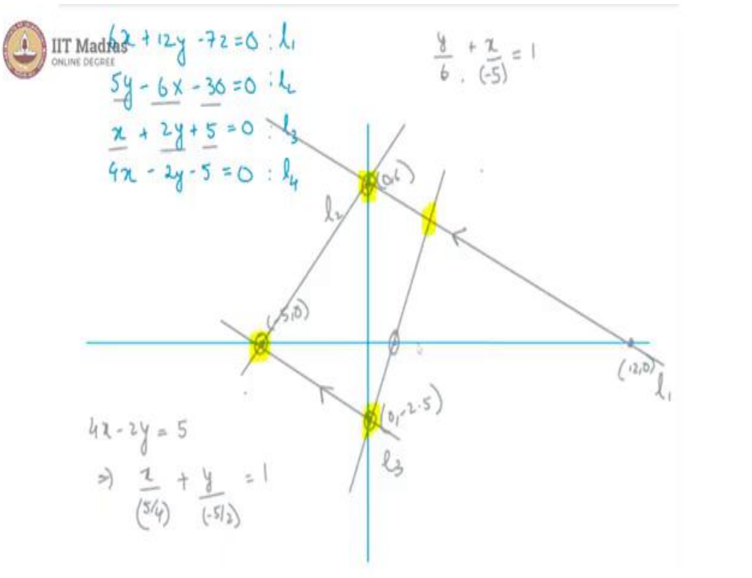

For that, let us try to draw our lines on the graph. $6x+12y-72=0$ would give us if $x=0$, it gives us $y=6$ which means some point let us call this here is (0,6), it goes through this point. And if $y=0$, you get $x=12$. So that would be some point here. So, our l_1 is this line. And now we know l_3 is parallel to this line. So l_3, if we, again did the same thing of putting $y=0$, x becomes -5, which is somewhere here.

So, as you can see, I am doing this on a rough estimate. I am not trying to be accurate, but even a rough estimate can work out here, because you might not always find graph paper when you require it. So often developing an intuition for the rough estimates is a good idea to solve problems. Now, this is one point and when $x=0$, y becomes -2.5, which is somewhere like this. So we have (0, -2.5). As you can probably see from our last rough estimate itself that these do appear to be parallel, they seem to be in the same direction.

Now, l_2 if we look into it with a similar logic, we can see that l_2 can be reduced to $\frac{y}{6}-\frac{x}{5}=1$. So in our intercept form, we can now tell that if I made this plus, this becomes -5, so the x intercept is -5, which is this point, again, and y intercept is 6, so that is this point. So, l_2, in fact, passes through these 2 points. So, this is our l_2. So, this was l_1 now, this is l_3 and this is l_2.

Lastly, let us reduce our l_4 into the intercept form, we get $4x - 2y = 5$, therefore, $\frac{x}{5/4} + \frac{y}{-5/2} = 1$. So, when we look at this then 5/4 is a quantity just a little greater than 1, so it is probably somewhere here and 5/2 is a 2 and a half basically. So, -2.5, so this and this plus we have something like this happening. So, overall there are four points, which are common to any pair of these four lines.

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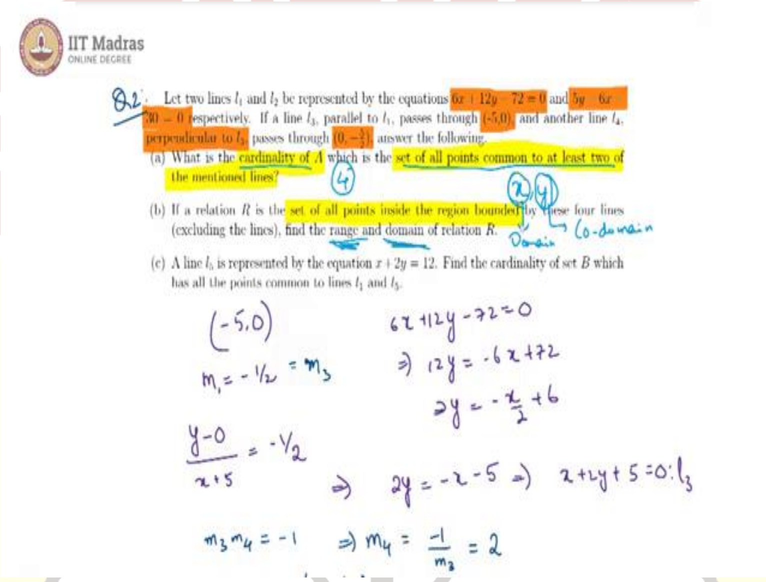

So, our question, the cardinality of A, where A is a set of all points common to at least 2 of the mentioned lines. So, that would be 4, there are 4 points of intersection here. Now, if R is a relation, and it is the set of all points inside the region bounded by these 4 lines. So, here we are, when we say relation, we are basically saying every point in the set when is taken as a ordered pair like this (x, y), then x would be from the domain of the relationship and y would be from the co-domain.

So, this is seen as a relation from the set of x values and to the set of y values. And now, we are asked to find the range and domain of relation R, which is to basically find when we say range, all the possible y values and the domain is all the possible x values.

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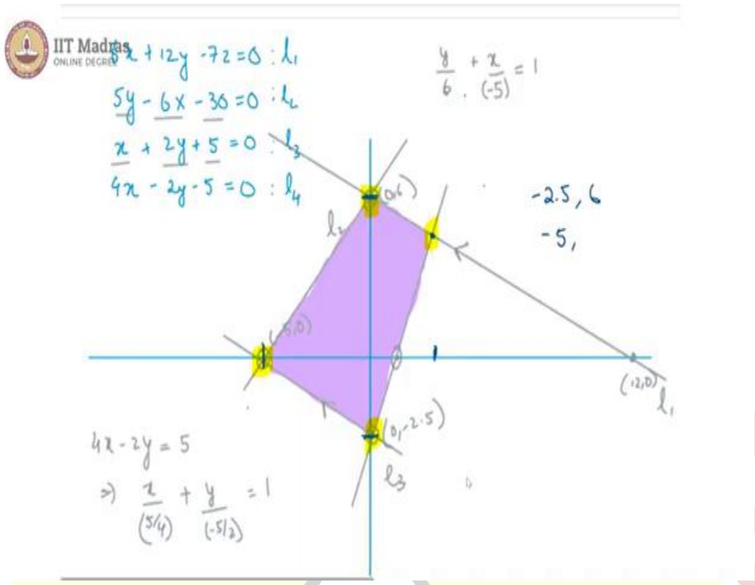

So, here in this region that we are looking at, the possible y values would be between this value and this value. So, all possible y values are between -2.5 and 6, whereas the possible x values are between this point and this point, that is between -5 to some particular quantity, which is the x coordinate of this point. And that point is the intersection of l_1 and l_4. So, let us try to solve l_1 and l_4 to find that point of intersection.

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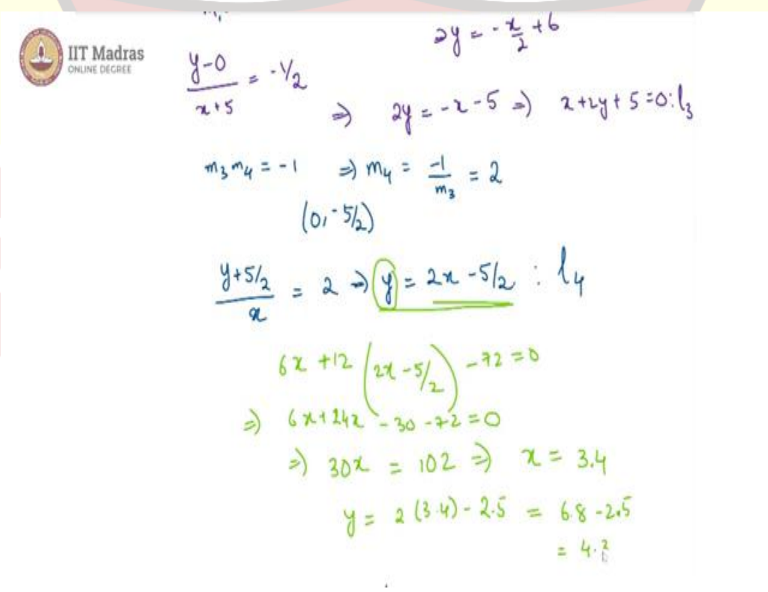

We know that this as l_1, and this is l_4 and from l_4, we know that y is basically $2x-\frac{5}{2}$. If we substituted this into l_1 we would get $6x+12(2x-5/2)-72=0$. This would give us

$6x+24x-30-72=0.$ That indicates $30x=102$ which indicates $x=3.4.$ Correspondingly, y would then be $2\times 3.4-2.5$, because 5/2 is 2.5, which gives us 6.8 - 2.5, which is equal to 4.3.

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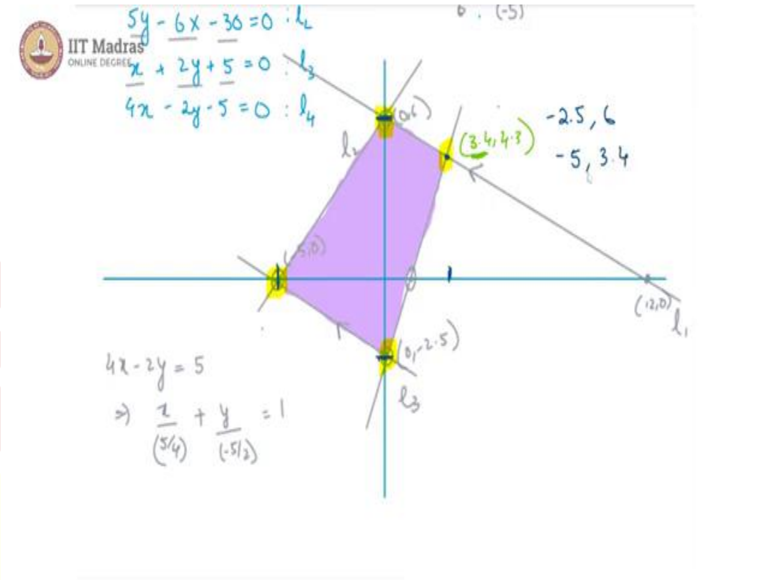

So this point here is (3.4, 4.3) and we only require the x value. So the x values range from -5 to 3.4.

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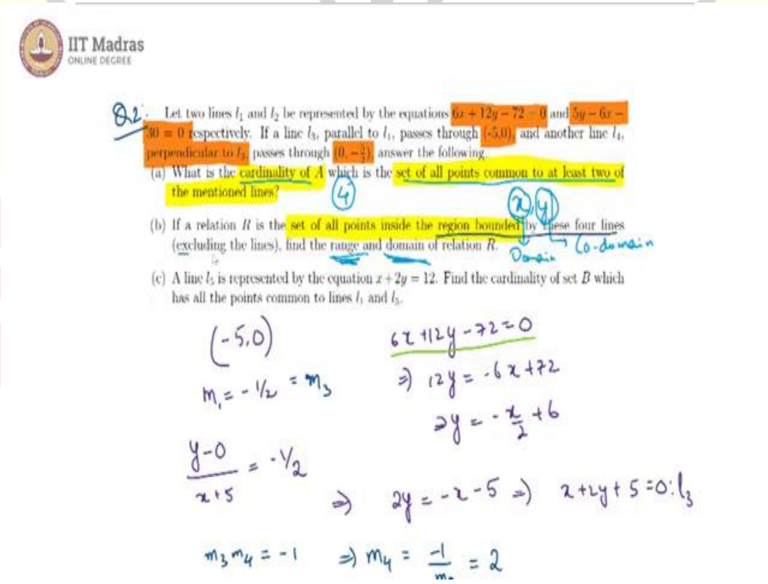

However, one important thing we need to look for here is the region bounded by these 4 lines, but excluding the lines themselves.

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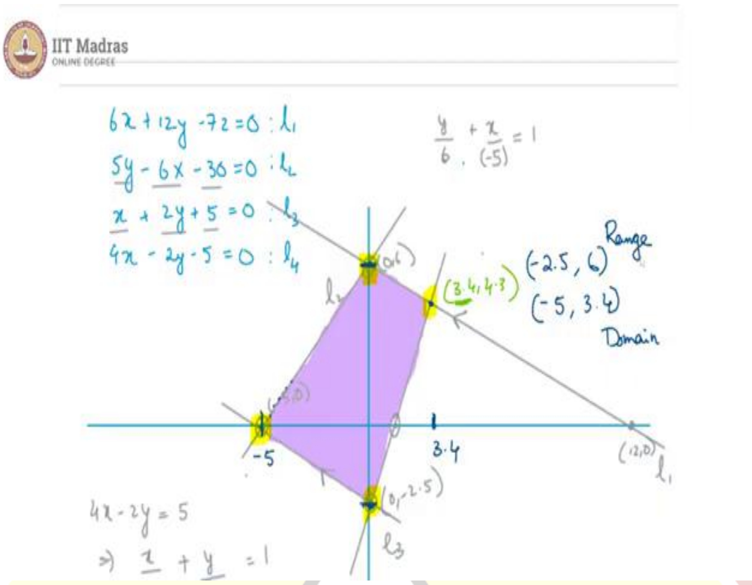

Which means -2.5 and 6 themselves do not fall into our domain because we are not interested in the points on the curve. So this point is on the curve, this point is on the curve, but it is not inside, similarly, for each of these, because they are the border points. So, -5 is not an x value inside the domain. Similarly, 3.4 is not a value inside the domain. So, our domain is the (5,3.4). Likewise, -2.5 is not a y value inside the range and 6 is also not a y value inside the range, so our range is (-2.5, 6).

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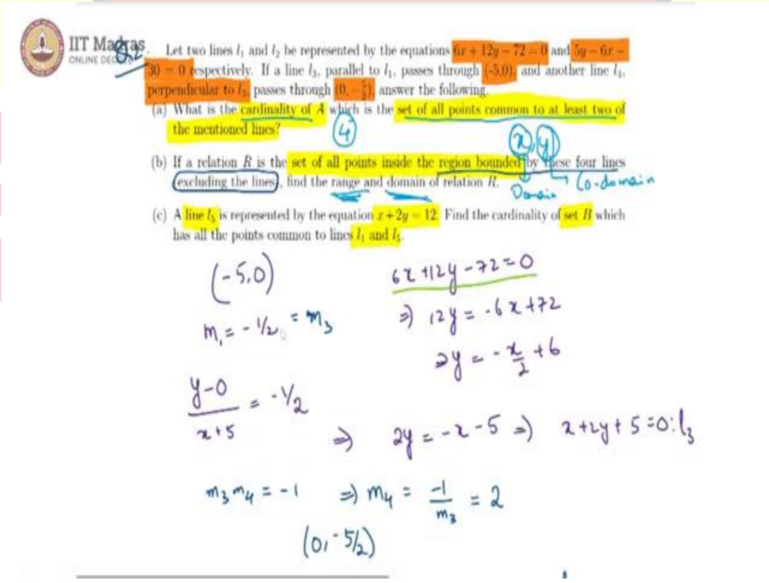

Lastly, there is a line l_5 represented by this equation given to us find the cardinality of set B, which has all the points common to l_1 and l_5.

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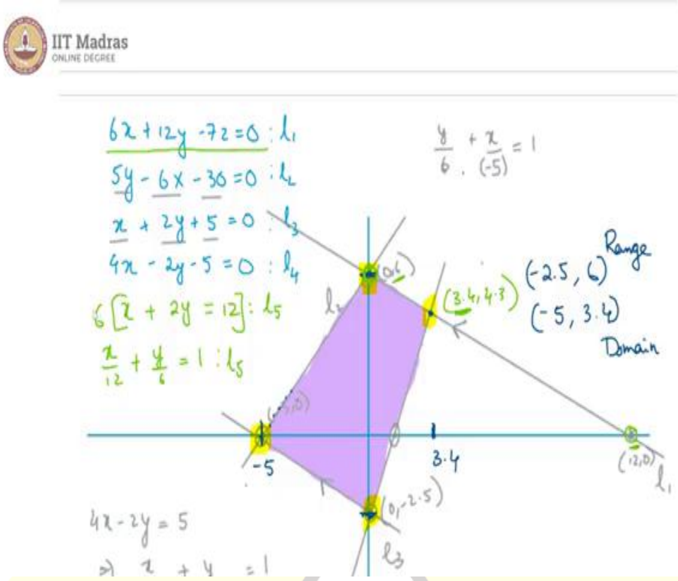

Let us look at l_1 and l_5. l_5 is given as $x+2y=12$. Now, if we applied our intercept form again, we would get $x/12+y/6=1$. Let us look at that $x/12$ indicates x intercept of l_2, $y/6$ indicates y intercept of 6. So, we see that l_5 is basically the same line as l_1, indeed if you multiply this whole equation with 6, you will just get the form of l_1. Therefore, l_1 and l_5 are the same lines.

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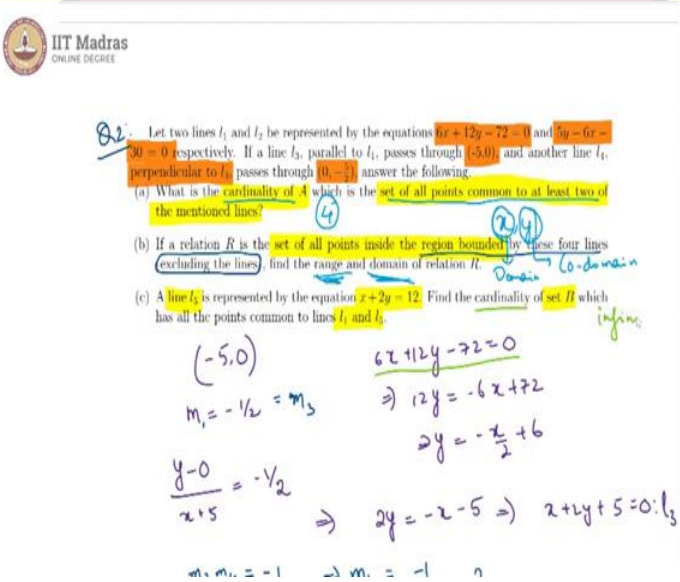

Then, the question is asking, find the cardinality of set B, which has all the points common to the lines l_1 and l_5. There are infinite points because they are the same line. So, the cardinality of set B is infinite.