INDIAN INSTITUTE OF TECHNOLOGY MADRAS

सिद्धिर्भवति कर्मजा

IIT Madras

ONLINE DEGREE

Statistics for Data Science-I
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Lecture – 5.6
Permutations and Combinations – Applications

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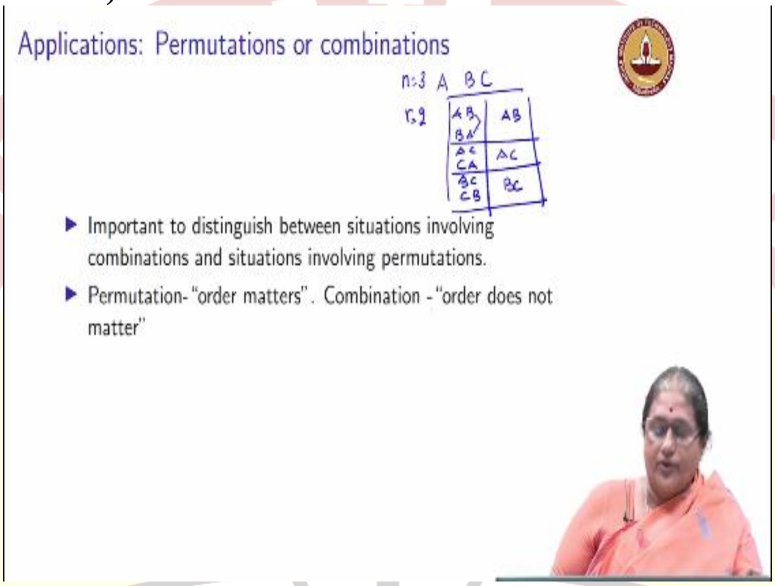

Now, we look at a very important application of whatever we have learned so far. So, given a situation it is very important for us to distinguish as to whether the situation under consideration requires us to answer in terms of permutation or in terms of combination. So, we need to distinguish the situations involving combinations or permutations that is the key understanding which we want to give now.

So, what is it? Remember when we talk about permutation the order matters, but when we talk about combination the order does not matter. Again recall that when I had three objects and I was choosing or arranging two objects or when my n was equal to 3 and I was arranging two objects I could do it in AB, BA, AC, CA, BC and CB, 6 ways.

Now, when I am talking about a combination these two are the same. So, I had an AB, AC and BC the order in which the objects are arranged did not matter. This was the key difference between a permutation and combination that is the absence of order in a combination and order mattering in a permutation.

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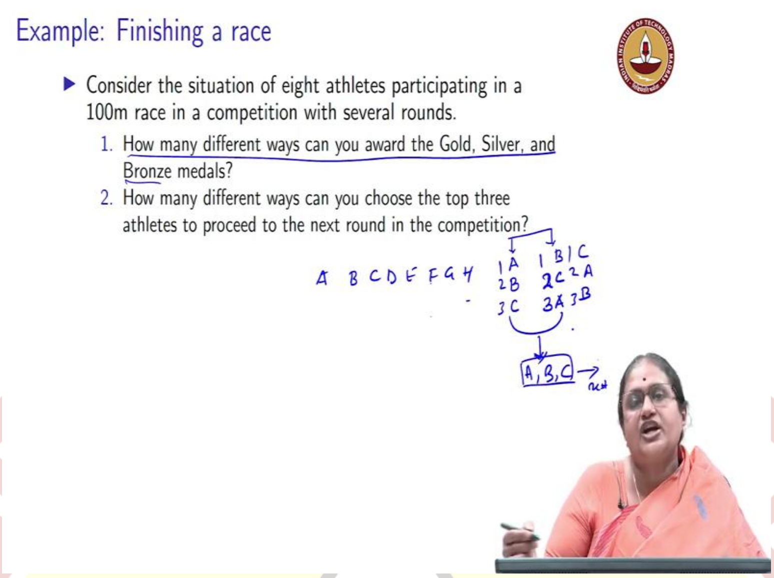

Now, let us look at a few examples to articulate this difference. For example, suppose you consider the situation of 8 athletes participating in a 100 meter race. Now, if the situation says that okay there are 8 people participating in a race and I want to award three medals, namely the gold medal, the silver medal and the bronze medal to these 8 people then if I have these 8 people who are A B, C, D, E, F, G and H.

These are the 8 people then the order in which they are finishing. So, A could be first followed by D and F this could be one order. D could be first followed by A and third place F this could be another order so here A gets the gold, D the silver and F gets the bronze. In this case D gets the gold, A gets silver and F gets the bronze. So, the three people who are actually there are ADF in both this situation and in this situation.

But clearly they are different because the order of finishing is different. So, when I want to know how many different ways I can award the gold, silver and bronze. I need to look at permutations because here the order matters. So, whenever you are asked a question the first thing we need to do is to understand whether order matters or not. Now, the second question says that suppose my question I have again the 8 people who are participating in the race.

But here I am looking at a heat and I am just going to send the top three people to the next round. When I am going to send the top three people to the next round whether A came first, B came second and C came third or B came first, C came second and A came third in both these cases A, B, and C go to the next round. So, in this case I am not bothered or I am not concerned about the actual order in which A, B, C finish the race because I am just choosing the top three people to go into the next round.

So, in this second case A, B, C, B, C, A, C, A, B whatever has been their order of finishing are the same thing because they proceed to the next round. So, in the second case the order is not important to me when I choose the top three and I will be looking at a combination. So, this distinction is very important to us. So, when we encounter situations with the question we need to first see whether the order matters or the order does not matter.

When the order matters you will apply a permutation. When the order does not matter you will apply a combination.

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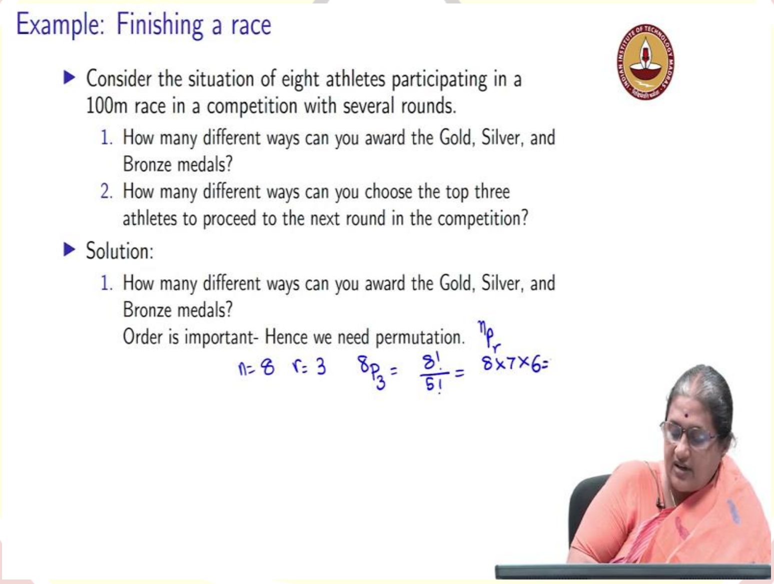

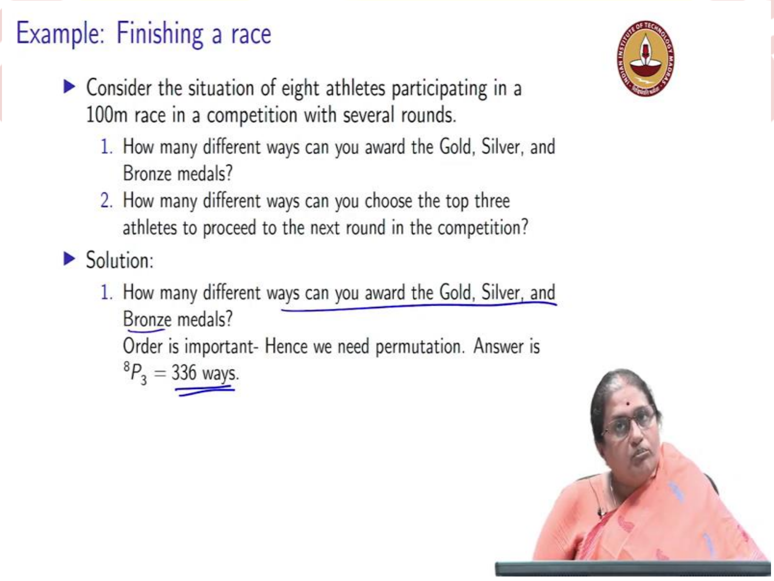

So to answer this question, in the first case we see that the total number of ways the order is important you go with the permutation. So, what is n here, so what is the permutation I am going to seek npr. In this case I have n equal to 8, r equal to 3 so I am looking at 8p3 which is $\frac{8!}{5!}$ which reduces to $8 \times 7 \times 6$. That is what I have which is 336 ways I can award the gold, silver and bronze medals.

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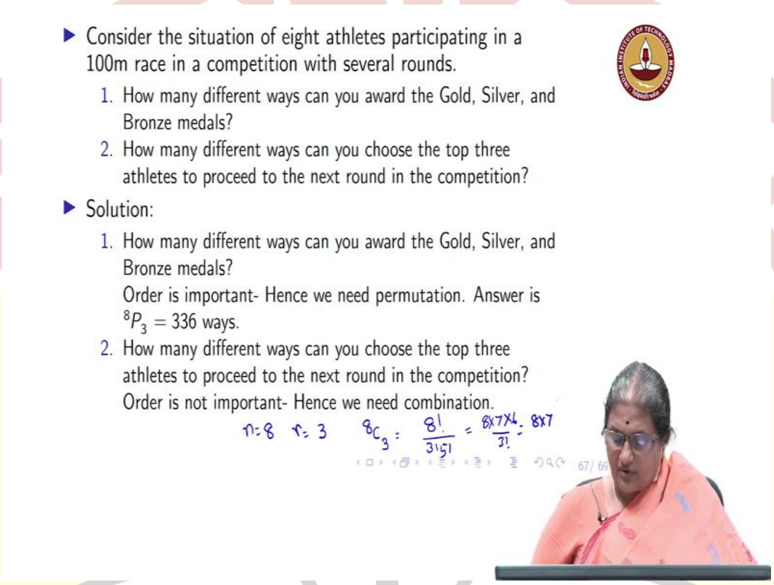

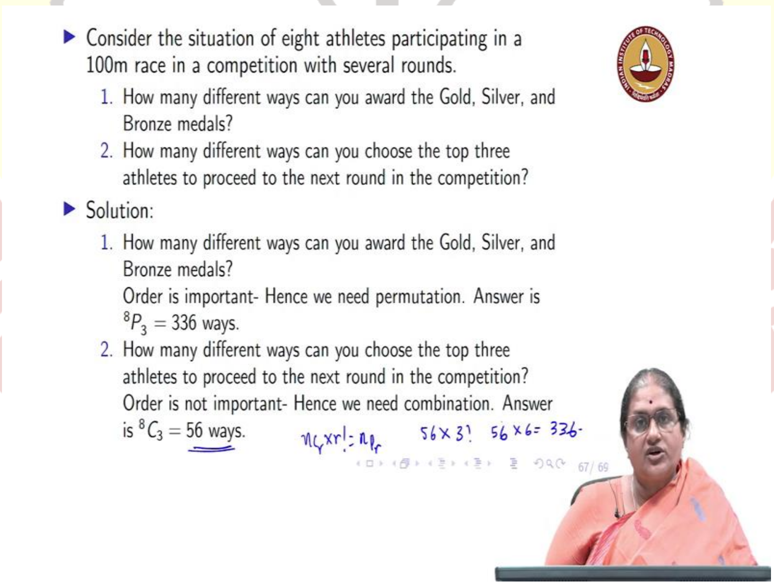

In the second way how many different ways can you choose the top 3? I again realize that the order is not important hence I proceed with combination again n equal to 8 r equal to 3 I look at 8 choose 3 which applying the formula is 8 factorial by 3 factorial which is $\frac{8!}{3! \times 5!}$.

So, which is $\frac{8!}{3!\times 5!}$ which I can see is $\frac{8\times 7\times 6}{3!}$ which is 8×7 is finally what I have the answer is just 56 ways of choosing the top 3 athletes to proceed into the next round of competition. You can see that $56\times 3!$ which is 56×6 is 336 which is my 8p3 this is the verification of your n choose r times r factorial is npr. We can check that n choose r into r factorial is your npr. Now, let us look at another few examples.

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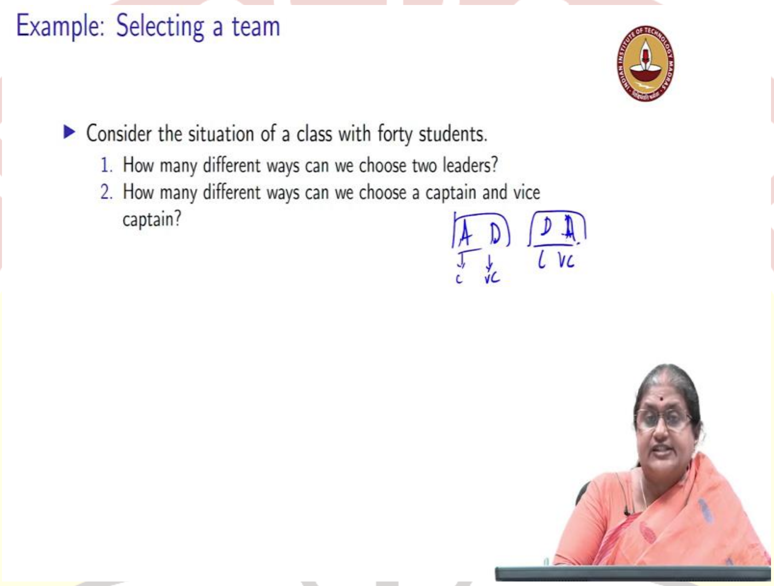

Now, looking at selecting a team. Consider a class of 40 students now the two questions I have here is. The first question is how many different ways can I choose two leaders or two representatives from this class. I can call them class representatives. I am not giving any order so I am not saying one of them is superior to the other or the other is superior.

So, I am just telling I just want to choose two class representatives from these 40 students. I want to know how many ways I can do it. The second question is different. I want to choose a captain and a vice captain from the 40 students. So, in the first case I am asking the question how many different ways can you choose two class representatives? In the second case I not only want to choose two students I am also asking the question I want to have one of them as a captain and another as a vice captain.

So, clearly when I am talking about the second question I am interested in having an order. For example, if I am choosing A and D in the first case choosing A and D as class representatives whether I choose A and D or D and A it is not going to make a difference, but in the second case if A is a captain and D is a vice captain and D is a captain and A is a vice captain these two are different in each other.

So, the second case needs a permutation to answer the question whereas the first case needs a combination to answer. Again it is extremely important for us to develop this idea or this skill of understanding which question for which question we need to apply a combination and for which question we need to apply a permutation.

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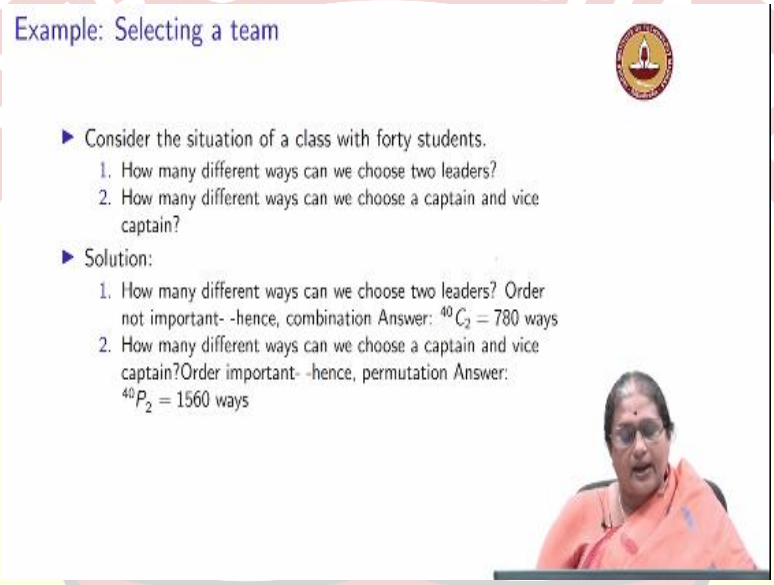

So, in this case I have how many different ways of choosing two leaders. I just have n equal to 40 when I have n equal to 40 I am going in for a combination because here the order is not important. My n equal to 40, r equal to 2, my n choose r is 40 choose 2 which gives me 780 ways of choosing a two class representatives or two leaders. Whereas in the second case, the different ways I should choose a captain and a vice captain from 40 students. I require a permutation because order is important again I will write that as 40P2 and 40P2 we know is 1560 ways of choosing a captain and a vice captain.

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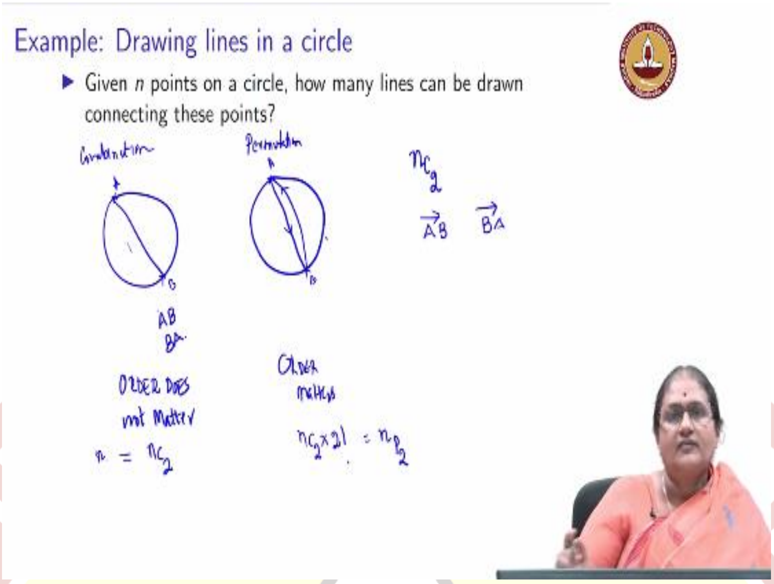

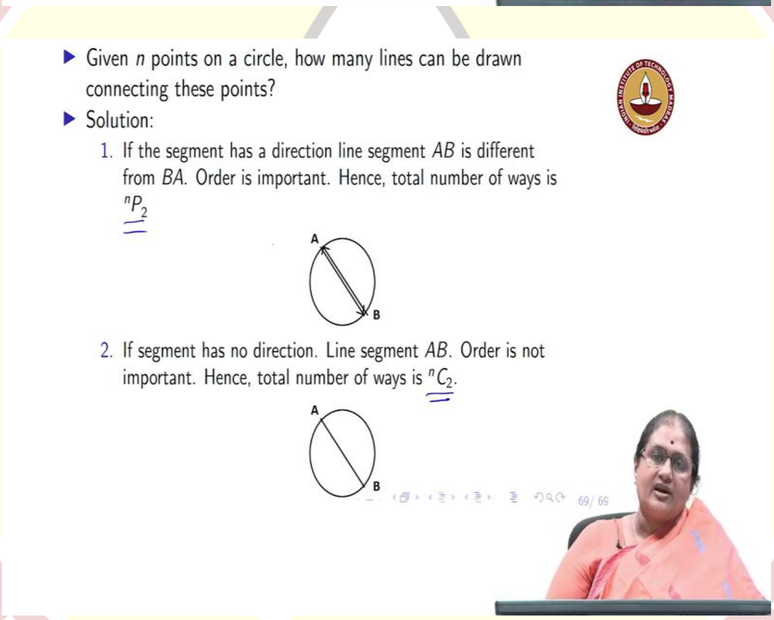

Now, let us go to another example. We have already seen how many ways we can draw line segments joining two points in a circle. So, if I am given a circle and I have two points say A and B, I know I can join it with a line drawn from A to B. So, suppose I am given a circle and I have two points A, B, I can join these two with a line. So, we have already seen that if we have given n points I can join them using n choose 2 is the total number of lines that can be drawn through n points which lie on a circle.

In other words I can have n choose 2 chords when if I have n points on the circle. Now, this chord is an undirected line. For example, if A and B were two locations on this point and I am interested in knowing from going from A to B to B to A. In that case A to B will be different from B to A because I have a direction from going from A to B to B to A which was different from the case when I just joined two points using a chord here the line segments AB was same as BA.

So, you can see that when there is a direction involved A to B and B to A are different and the order matters in the second case whereas in the first case A to B and B to A are the same order does not matter here. Here order matters. So, in this case if I am given n points then I have n choose 2 is the total number of lines I can draw between these points here I can again have n choose 2.

But I know that should be multiplied by 2! because every line is giving 2 directed lines and this is nothing, but np2. Again this is an example of in the same situation one of the answers needs a combination which is an answer and the other needs a permutation which gives me the answer. So, this is about drawing circle.

So you can see that when I have a directed line or a directed arc or a directed quad I have the order which is important. Whereas when I am talking about a line segment the order is not important and I just have total number of ways n choose 2 in the earlier case when the direction was there it was np2.

So, in this section we have just seen that it is very important for us to distinguish between permutation and combination depending on the situation and the question that is asked on the situation, not all situations require us to answer in terms of permutations or combinations. So, it is very important for us to distinguish these two. There could be situations where neither of them are necessary we have not looked into that situations in this module. We also looked at a few examples to illustrate the point as to how we distinguish between whether permutation has to be used or combination has to be used.

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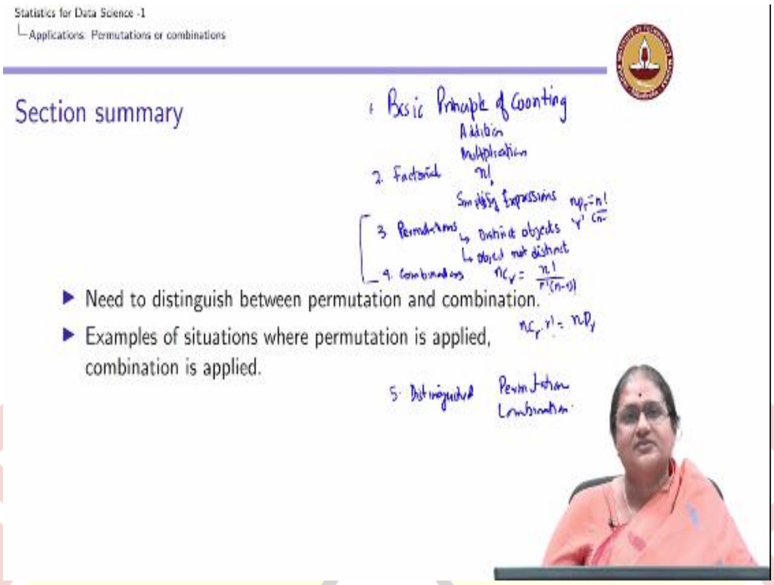

So, in summary what we have learned in this week is we started with the basic principle of counting. When we looked at the basic principle of counting we introduced both the addition principle of counting and the multiplication principle of counting.

The multiplication principle of counting helped us introduce what we call the factorial notation which is n! and this is a short hand for us to write the link the expression which we got from the multiplication rule for counting. Once we establish the factorial notation we looked at how do we simplify expressions using the factorial notation that is what we looked at.

Then we introduced permutations now when we looked at permutation we started with distinct objects and choose r objects from n distinct objects. We then extended it to objects are not distinct. So, first is I could have some of them which are of the same type. So, first we looked at distinct objects we looked at repetition not allowed, repetition allowed then we looked at objects were not distinct you could have p of one kind or p1 of one kind, p2 of the other kind and we saw how we can actually. So, objects are not distinct. We obtain formulae to come and understand how to do that.

The next thing we looked as combinations. We introduce and choose r, here we introduce what we was npr, npr was $\frac{n!}{(n-r)!}$. Here we introduce n choose r as $\frac{n!}{r!\times(n-r)!}$ by observing n choose r into r factorial is npr we get hold of this formula.

We looked at applications of combinations. The last thing we did was to actually look at examples where we distinguished whether we need a permutation or a combination we looked at a few examples wherein depending on the nature of the questioners you decide on whether to go with the permutation or with combination. Now, with this background we move forward to introduce you all to the probability module.

We start by again basics of probability namely we will introduce you to the concept of a random experiments, sample space, events, addition of events and all of that. So, that by that end you can compute probabilities of events. When you have to compute probabilities of events this notion of how you count becomes very useful. Hence this is a prerequisite for you to understand the probability module. Thank you.