INDIAN INSTITUTE OF TECHNOLOGY MADRAS

सिद्धिर्भवति कर्मजा

IIT Madras

ONLINE DEGREE

Mathematics for Data Science
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Indian Institute of Technology, Madras
Lecture No. 56
Logarithmic Functions: Applications

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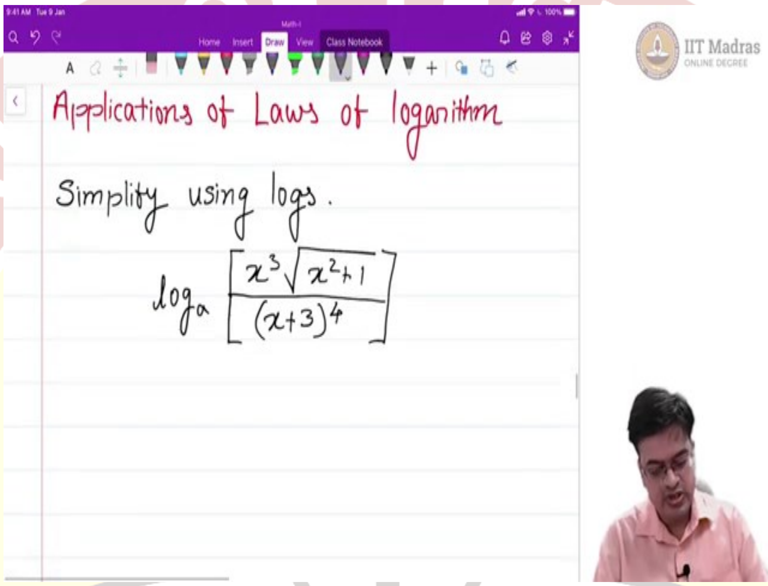

So, let us now go ahead and use these laws of logarithms and try to see some simple problems, how the problems can be simplified using logs or how the problems can be made complicated using logs.

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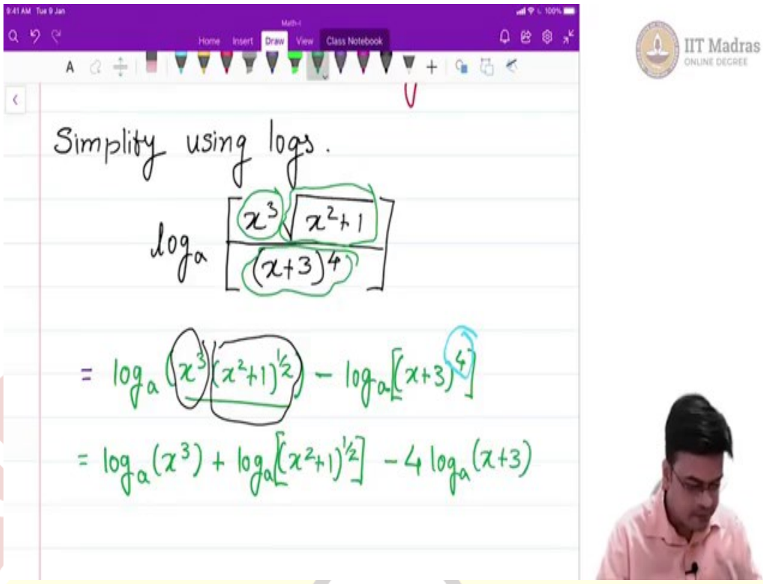

So, let us see the first problem, it looks an ugly sum or ugly product and we have taken log and how the process is simplified when you use the properties of logarithm that you have studied just now or the laws of logarithm that you have studied just now. So, let us go ahead and do that.

So, this particular thing let me write this to be equal to, so first you identify or isolate the terms that you identify can be separated. So, first term that I can separate out this is x cube, the second term that I can separate out is this square root and the third term is numerator. So, essentially this particular term if you look at can be split into 3 components, so I want end result that I want to get after simplification should have 3 components.

So, first I will apply the quotient rule that is $\log_a \frac{M}{N}$, so in this case this fetches me $\log_a x^3.(x^2+1)^{\frac{1}{2}} - \log_a (x+3)^4$. So, here I have not used any other rule so this is simply the quotient rule that I have used that is the second rule that we have derived.

Let us go ahead, and see what we can do with the first term that is this term. So, now you can see is simply see these terms can be characterize into two terms this is the first term and this is the second term. So, I can write this particular thing as$\log_a x^3 + \log_a (x^2+1)^{\frac{1}{2}}$. So, to identify the raise to half is inside the log I have put it this way then minus you look at this term again.

Now in this term something is raise to the power 4. Do I have any rule for indices of the law, indices within the argument of logarithm? Yes, we have just now proved it for set of natural numbers. So, you can use this that rule and say that this is equal to $-4\log_a(x+3)$.

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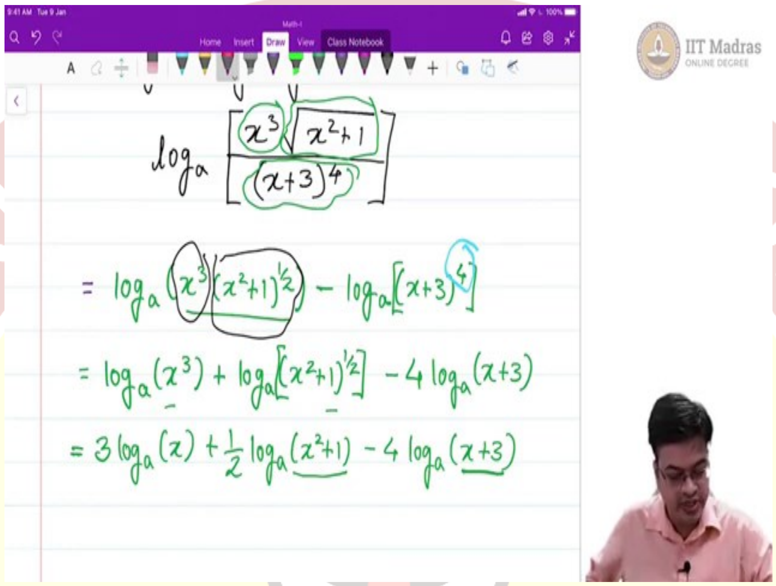

Let us go ahead and do a similar thing for the other two terms that are listed here then we will get the final answer that is $3\log_a(x+3)$. 3 times log to the base a of x plus half times here I am using it for rational numbers which I have not proved $0.5\log_a(x^2+1) - 4\log_a(x+3)$. So, this is how I can simplify. Now in general, when you study logarithms people generally get confused between the product becoming the sum. So, here we have some terms like this. Now these terms cannot be handled with log.

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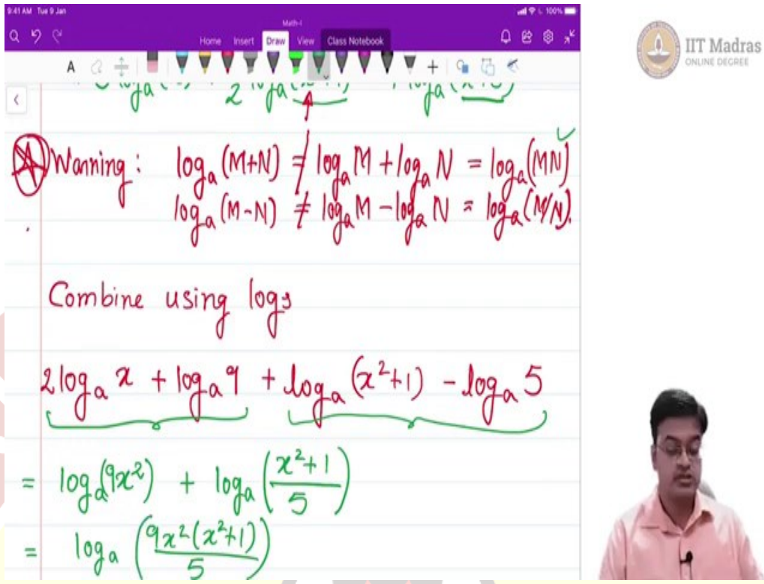

So, right now let me give you a note of caution or a warning so to speak that is let me write this as warning. Generally though it is obvious but while doing the calculation people used this rule; $\log_a M + N \neq \log_a M + \log_a N$. So, this is what people use and this try to solve the problem so that they think they will simplify but remember this is not equal to that, why? Because we have just now proved that this is nothing but $\log_a MN$. So, these two things are different.

In a similar manner you can have a quotient rule that is $\log_a M - N \neq \log_a M - \log_a N$. Because this is actually equal to $\log_a \frac{M}{N}$. So, just remember this warning because generally in the when you are in the fighting spirit you are trying to solve the problem you tend to make these mistakes and which will ruin your entire answer. So, this is with extra star marked I am emphasizing that these two are not equal.

Now let us try to see how we can simplify, sorry we have here seen how we can simplify our life using logarithms where everything is now almost linear terms except for this quadratic term. Now the next question that can be asked is can you combine using logs? The answer is yes, so if you do not want to see such a big expression or you want to have a nice compact expression, the question is can you combine? The answer is yes, and now let us handle the terms one by one and merge the terms.

So, first term let us take these first two terms, what are these two terms? One is $2\log_a x$. So, you have already seen $\log_a x^r = r\log_a x$. Apply that in reverse so you will get this particular term as $\log_a x^2$, do not stop there you just apply the product rule now. $2\log_a x + \log_a 9$can actually be merged as we can use this rule and say this is equal to $9x^2$.

Now let us look at the next term which is this, next two terms in fact and there is negative sign so naturally a quotient rule will come and you will have something like $\log_a \frac{x^2+1}{5}$. Can you combine these two? Again apply the product rule and you will get this to be equal to $\log_a \frac{9x^2(x^2+1)}{5}$. So, this is how you can simplify your life while studying logarithms by giving a combined expression.

So, you can use this to simplify your life give a long expression with positive signs when it will help when you have lot of large numbers to be calculated. You can also combine the logarithmic terms and combine the expression in a compact form when this will help when you have lot of small-small terms that are unnecessarily occupying the space. So, when extremely large terms the simplification will help when extremely small terms will come the matter will be simplified by combining the terms. So, these are two simple avenues where you can actually do something.

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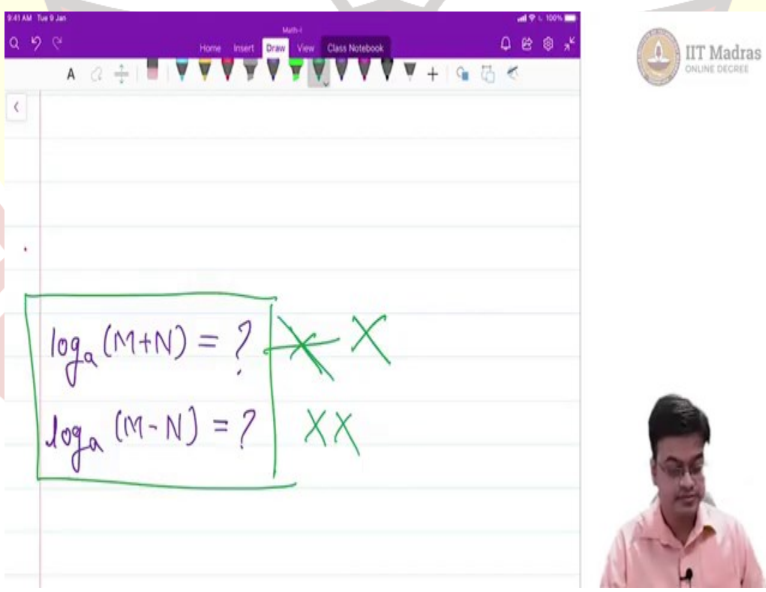

So, I have already warned you about this $\log_a M+N$, this is nothing you cannot get anything out of this, this is nothing. In a similar manner $\log_a M+N$ you cannot use reverse exponential or any other form to get something out of this while solving the problems of logarithms. So, just whenever these such terms come be aware and do not apply the things blindly.