INDIAN INSTITUTE OF TECHNOLOGY MADRAS

सिद्धिर्भवति कर्मजा

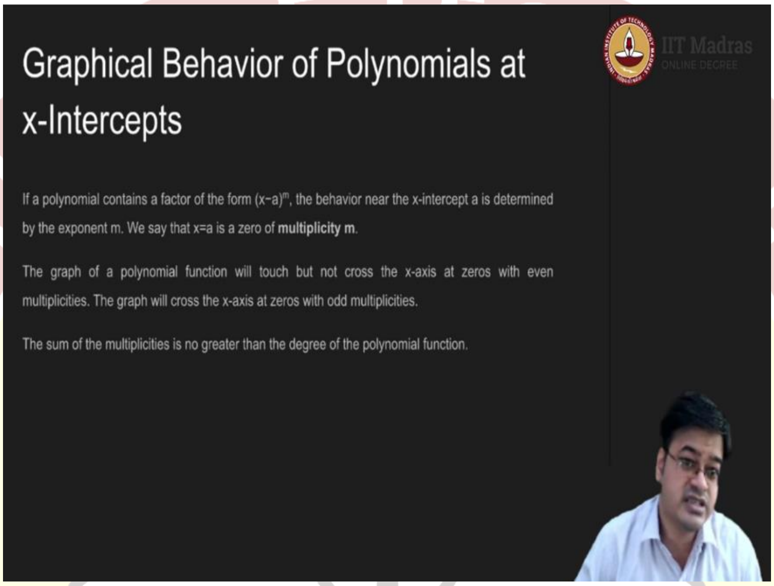
IIT Madras

ONLINE DEGREE

Mathematics for Data Science 1
Prof. Neelesh S Upadhye
Department of Mathematics
Indian Institute of Technology, Madras

Lecture – 39
Graphs of Polynomials: Behavior at X-intercepts

(Refer Slide Time: 00:14)



So, let us go ahead and look at the Graphical Behavior of Polynomials at X-Intercepts. So, in particular if the given polynomial has a factor of the form $(x-a)^m$, this m is called the multiplicity of the polynomial. And you will say $x = a$, is a zero of a polynomial f with multiplicity m. This is to fix the terminology.

Now, the graph of a polynomial function will touch, but not cross x axis at zeros with even multiplicities and the graph will cross x axis at zeros with odd multiplicity. We have iterated it enough number of times.

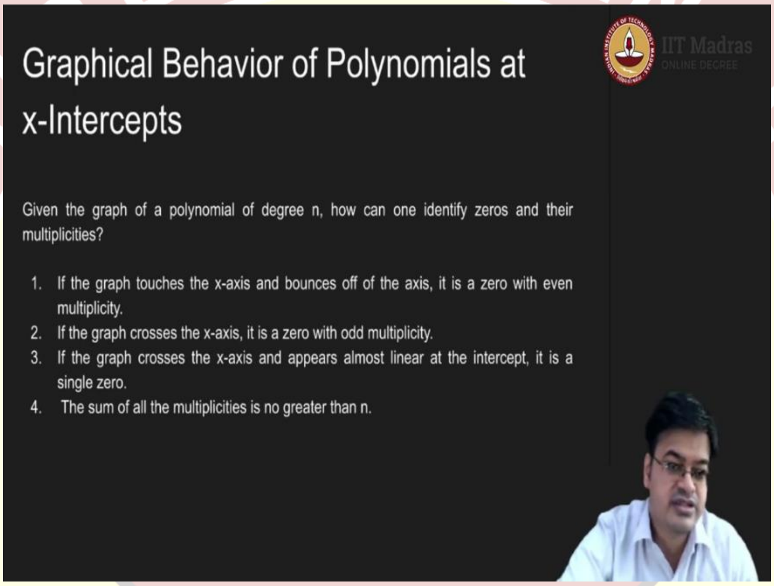
Also, one important thing is the degree of the polynomial cannot exceed the sum of the multiplicities, or the sum of the multiplicities is always less than or equal to the degree of the polynomial function, this is quite common sense right. If it exceeds, if it actually exceeds the polynomial degree then it is a polynomial of higher degree.

So, then now why it is not equal to, can be one question. The sum of multiplicities you will say will always be equal to the degree of the polynomial function. For that we need to understand that all the roots all the zeros of the polynomials or the only way we are identifying zeros of the polynomials is by identifying the x intercepts.

So, all x intercepts are real roots of the polynomials, but as in the quadratic case we have seen that some of the polynomials, some of the quadratic equations do not have real roots. In such cases the x intercepts are not visible.

I will demonstrate it further through some examples.

(Refer Slide Time: 02:35)

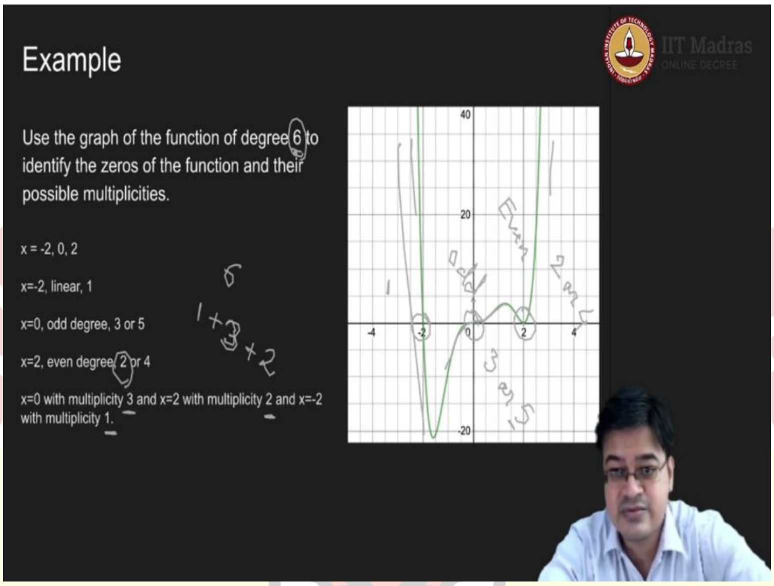


So, let us try to see. So, given a polynomial of degree n, that is a graph of a polynomial of a degree n. We want to identify zeros and their multiplicities; this is our goal. So, you are you have been told that this polynomial is of degree n and this is the graph of the polynomial, how, what will you do about it?

So, in that you will look at all the coordinates, where the graph touches x-axis, you take them. If the graph touches x-axis and bounces off the x-axis then, it is a zero with even multiplicity. If the graph actually crosses x axis, it is a zero with odd multiplicity. And finally, when you will conclude you have to take care that because the polynomial is of degree n, the sum of the multiplicities should never exceed the actual degree of the polynomial.

Another thing if the graph crosses x axis and appears almost linear at the intercept, then it is a single order; that means, it appeared only once; it is a linear function. And finally, that is what I explained the sum of the multiplicities is no greater than this fine.

(Refer Slide Time: 04:10)



So, let us go ahead and see some examples and let us see whether we can apply these principles in action. So, use the graph of a function of degree 6. So, it is a degree 6 polynomial, which is given to you and this is the graph, wonderful.

So, now you can easily see -2, 0 and 2. So, x intercepts are -2, 0 and 2 fine. Then if you, let us start from left; so, at x is equal to minus 2, how is the behavior? It is more like a straight line, it is more like a straight line. So, at minus 2, I feel the behavior is linear or it is a onetime event.

At 0, what is happening is; it is having this S shape, somewhat twisted S shape. And that is indicative of odd degree that is indicative of odd degree and degree can be 3 or 5 or 7 or 9 I do not know right, but I have been given that the polynomial is of order 6. So, at most it can have a degree 5 right, the multiplicity 5.

Now, look at this particular junction, it actually bounces off the x axis. So, this is a typical trait of even degree polynomial. So, what can be the degree? It can be 2 or 4 right; so it can be 2 or 4 ok. Now, we need to collate this information. So, x intercept $x = -2$ is linear, there is no doubt about it. $x = 0$, you have odd degree 3 or 5 is the degree and $x = 2$ it is even degree.

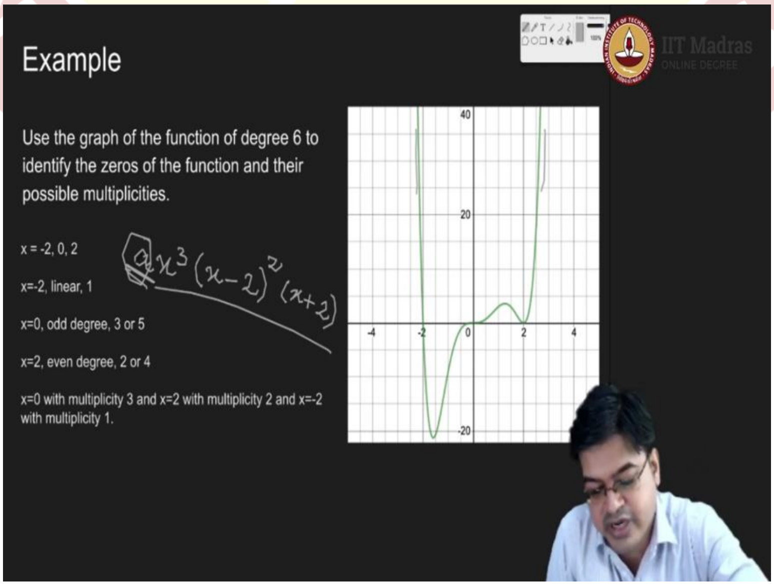
Now, together sum of the degree should be equal to 6, of which this 1 is fixed. So, now, I can assign 3 or 5. If I assign 5 then $1 + 5 = 6$ and if $1 + 5 = 6$ that essentially means; there is no root of the form $x = 2$ that is not possible. So, I have to assign 3 over here. Once I assign 3 over here, then I do not have any other choice, but to choose 2 over here.

So, therefore, I have identified the multiplicities of the factors like $x = -2$ will have a multiplicity of 1, $x = 2$ has multiplicity of 2 and $x = 0$ has multiplicity of 3. So, this is how we will identify the factors, and identify the zeros of the functions, and identify their multiplicities. This is much better for drawing the graph of a function.

Still, I have not answered a question that why this should go to infinity and why this also should go to infinity; those things are not clear, but we will come to them later. Right now, we have much better understanding about zeros of the polynomial functions and their multiplicities and how we can use them to understand the function.

Now, another interesting thing that you can ask yourself is ok, I have seen this function and I know the multiplicities of zeros and everything. Can I use this knowledge to actually tell what is the equation of the function or not. We can try our hand on it, but we may not be able to because $x = 0$ is with multiplicities 3.

(Refer Slide Time: 08:39)



That gives me x^3. $x = 2$ with multiplicity 2 that gives me $(x - 2)$ the whole square and $x = -2$ with multiplicity 1 which will give me $(x + 2)$ ok.

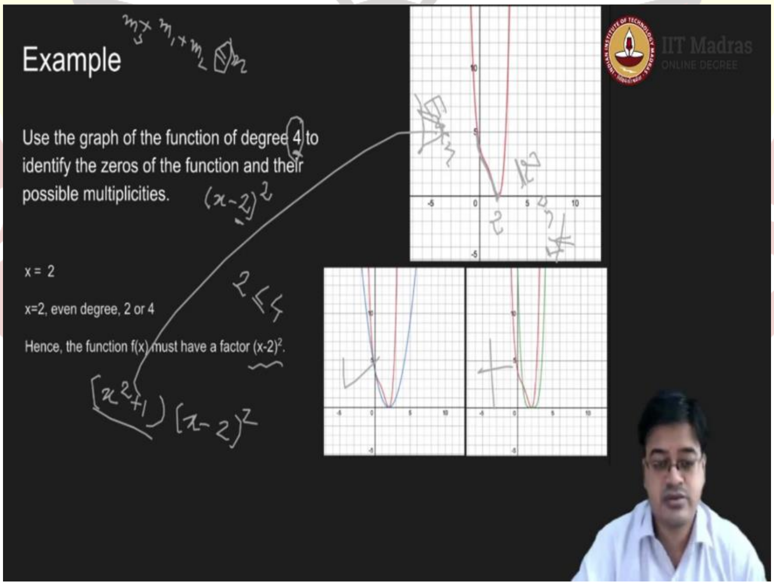
So, now I can say that this is the polynomial function that is graphed here, but how will I verify? So, for that I need something which is non-zero. So, you can choose some point and check whether this is there, but there is a catch over here.

It need not match the values that are given here. So, what we will do is though the factors are correct the polynomial is of degree 6 here the degree is 6, we will put some unknown a over here and we will determine this a by putting the actual values.

We will come to it later when we have better understanding about the behavior of this kind, but right now you keep this in this point in mind; that we do not know this a. So, we cannot actually give the exact equation of the polynomial with reference to these points.

Though we know the form of the polynomial, but we do not have accuracy up to the exact matching on the coordinate plane with numbers. Just remember this point and this juncture and let us go ahead and do some other problems.

(Refer Slide Time: 10:16)



So, in the next problem is of similar nature. I have a polynomial of degree 4 and I have been projected with a graph of this polynomial function.

Now, this is an interesting example because, you have a polynomial of degree 4. In earlier case we had a polynomial of degree 6 and all 6 roots were actually visible. In this case I have a polynomial of degree 4, but there is only one root that is visible or one 0 that is visible.

So, what is that number? It is 2 ok. And based on our understanding of the algorithms what we know is the graph actually bounces off; the graph actually bounces off the x axis which is 2. So, this is a typical trait of an even degree polynomial, or even degree even multiplicity. So, in this case I can say it can have a multiplicity of 2 or 4. I cannot exceed beyond 4 because the given polynomial has degree 4 only ok.

So, now because the given polynomial has degree 4, it is safe to assume that this polynomial is of degree 4 right. But if this polynomial is of degree 4 you can see there is a perturbation of the shape over here. It is not the shape of a polynomial of degree 4, for example; y=x^4 will not be in this form. So, I can rule out the degree 4 constraint.

Therefore, I do not have any other choice, but to say that the polynomial is of degree 2, the multiplicity of this particular factor 0 is 2; that means, I have a factor of the form $(x-2)^2$ the whole square that is all I can say in this case. Yeah, $x = 2$, is of even degree 2 or 4 and hence the function based on the reasoning that I have given it must have a factor of $(x-2)^4$.

Let us understand this graphically as well. This is a function, the blue line over here, is actually $(x-2)^2$. As you can see it passes very closely to through the function and the other line is $(x-2)^4$, other graph the green line is $(x-2)^4$.

Now, if you look at graph of $(x-2)^4$ closely there is no possibility of changing the shape. You can scale with that unknown a, but you cannot change the shape of the function. So, this graph is actually ruled out. So, graphically we have understood why we are ruling out and this graph is not ruled out. So, this is somewhat familiar. So, it will have some factor of this form.

Now, one exercise for you is this graph actually though it is a polynomial of degree 4, the way we have constructed is we have multiplied (x^2+1) with $(x-2)^2$.

And if you look at this particular factor x square plus 1 because you, now what you can do is you can actually consider this $(x-2)^2$ as one factor. And you can see whether this x^2 you will get the same graph by multiplying this. The beauty of this example is that this (x^2+1) has no real roots.

Therefore, the degree though the degree of the polynomial is 4, I was not able to find the two missing roots; those are not in the real domain. They are in the complex domain. So, those things are there that is why the question that the sum of the multiplicities will always be less than or equal to the degree of the polynomial.

So, let us say there are three factors m_1, m_2, m_3 are the multiplicities. Then $m_1 + m_2 + m_3 \leq n$; that answers this question. So, in this case I was actually able to find for these real roots only 2. So, 2 is less than or equal to 4. So, that is why the; this is the example, why I cannot say that the sum of the multiplicities is always equal to the degree of the polynomial.