Statistics for Data Science -1

Continuous Random Variables-Uniform distribution

Usha Mohan

Indian Institute of Technology Madras

Learning objectives

Learning objectives

1. Define what is a continuous random variable.

Learning objectives

1. Define what is a continuous random variable.
2. Probability distribution function and examples

Learning objectives

1. Define what is a continuous random variable.
2. Probability distribution function and examples
3. Cumulative distribution function, graphs, and examples.

Learning objectives

1. Define what is a continuous random variable.
2. Probability distribution function and examples
3. Cumulative distribution function, graphs, and examples.
4. Expectation and variance of random variables.

Uniform distribution-pdf

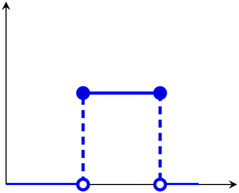
Standard uniform distribution

cdf of Uniform distribution

Expectation and variance of Uniform distribution

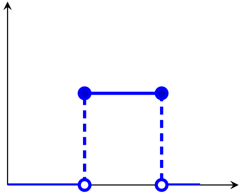
Uniform distribution $U(a, b)$

- A continuous random variable has a uniform distribution, denoted $X \sim U(a, b)$,



Uniform distribution $U(a, b)$

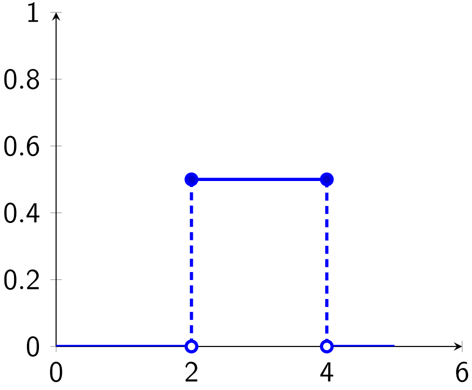
- A continuous random variable has a uniform distribution, denoted $X \sim U(a, b)$,



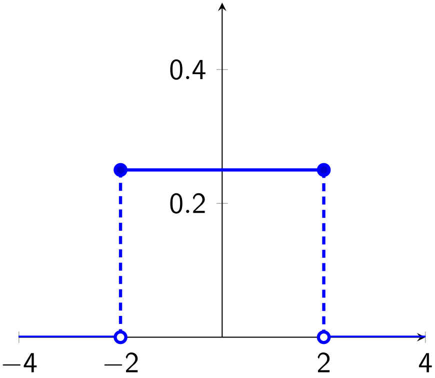
probability density function is:

$$f(x) = \begin{cases} \frac{1}{(b-a)} & a < x < b \\ 0 & \text{otherwise} \end{cases}$$

Graph of pdf of a Uniform distribution $U(2, 4)$



Graph of pdf of a Uniform distribution $U(-2, 2)$



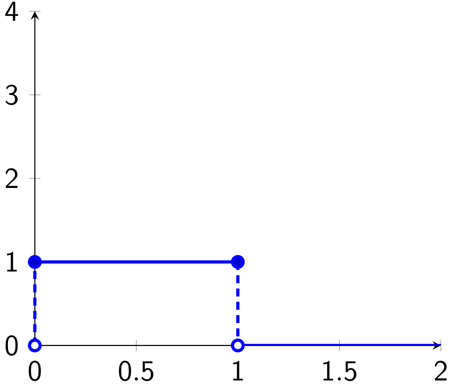
Standard uniform distribution

- A random variable has the standard uniform distribution with minimum 0 and maximum 1 if its probability density function is given by

$$f(x) = \begin{cases} 1 & 0 < x < 1 \\ 0 & \text{otherwise} \end{cases}$$

  The standard uniform distribution plays an important role in random variate generation.
- Verify $f(x)$ is a pdf
  - $f(x) \geq 0$, for $0 < x < 1$
  - $\int_{-\infty}^{\infty} f(x)dx = \int_0^1 f(x)dx = 1$

Graph of pdf of a Standard uniform distribution $U(0,1)$

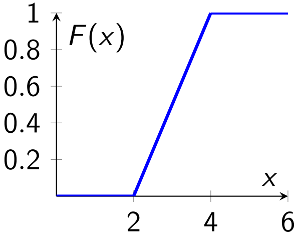


Cumulative distribution of Uniform distribution

For $X \sim U(a, b)$

$$F(x) = \begin{cases} 0 & \text{for } x < a \\ \frac{x-a}{b-a} & \text{for } x \in [a, b) \\ 1 & \text{for } x \geq b \end{cases}$$

Cumulative distribution of Uniform distribution

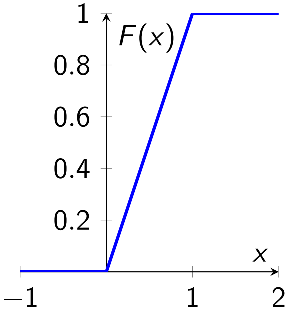


- When $x < a$ $F(x) = 0$
- When $x > b$ F(x) =1
- $a < x < b$ The slope of the line between and is $\frac{1}{(b-a)}$.

Cumulative distribution of standard uniform distribution

For $X \sim U(0,1)$

$$F(x) = \begin{cases} 0 & \text{for } x < 0 \\ x & \text{for } x \in [0,1) \\ 1 & \text{for } x \geq 1 \end{cases}$$



Expectation of $X \sim U(a, b)$

- $X \sim U(a, b)$;

$$E(X) = \frac{a+b}{2}$$

Expectation of $X \sim U(a, b)$

- $X \sim U(a, b)$;
$$E(X) = \frac{a+b}{2}$$

-
$$\begin{aligned} E(X) &= \int_a^b xf(x), dx \\ &= \int_a^b x\frac{1}{b-a}dx \\ &= \frac{1}{b-a}\frac{x^2}{2}\bigg|_a^b \\ &= \frac{b+a}{2} \end{aligned}$$

Variance of $X \sim U(a, b)$

- $X \sim U(a, b)$;

$$Var(X) = \frac{(b-a)^2}{12}$$

Variance of $X \sim U(a, b)$

- $X \sim U(a, b)$;
$$Var(X) = \frac{(b-a)^2}{12}$$
-
$$\begin{aligned} E(X^2) &= \int_a^b x^2 f(x), dx &&= \int_a^b x^2 \frac{1}{b-a} dx \\ &= \frac{1}{b-a} \frac{x^3}{3} \Bigg|_a^b &&= \frac{b^2+a^2+ab}{3} \end{aligned}$$

Variance of $X \sim U(a, b)$

- $X \sim U(a, b)$;
$$Var(X) = \frac{(b-a)^2}{12}$$

- $$\begin{aligned} E(X^2) &= \int_a^b x^2 f(x), dx &&= \int_a^b x^2 \frac{1}{b-a} dx \\ &= \frac{1}{b-a}\frac{x^3}{3}\Big|_a^b &&= \frac{b^2+a^2+ab}{3} \end{aligned}$$

- $$Var(X) = E(X^2) - [E(X)]^2$$

Variance of $X \sim U(a, b)$

- $X \sim U(a, b)$;
$$Var(X) = \frac{(b-a)^2}{12}$$

-
$$\begin{aligned} E(X^2) &= \int_a^b x^2 f(x), dx &= \int_a^b x^2 \frac{1}{b-a} dx \\ &= \frac{1}{b-a} \frac{x^3}{3} \Bigg|_a^b &= \frac{b^2+a^2+ab}{3} \end{aligned}$$

-
$$\begin{aligned} Var(X) &= E(X^2) - [E(X)]^2 \\ &= \frac{b^2+a^2+ab}{3} - \left(\frac{b+a}{2}\right)^2 \end{aligned}$$

Variance of $X \sim U(a, b)$

- $X \sim U(a, b)$;

$$Var(X) = \frac{(b-a)^2}{12}$$

- $$\begin{aligned} E(X^2) &= \int_a^b x^2 f(x), dx &&= \int_a^b x^2 \frac{1}{b-a} dx \\ &= \frac{1}{b-a} \frac{x^3}{3} \Bigg|_a^b &&= \frac{b^2+a^2+ab}{3} \end{aligned}$$

- $$\begin{aligned} Var(X) &= E(X^2) - [E(X)]^2 \\ &= \frac{b^2+a^2+ab}{3} - \left(\frac{b+a}{2}\right)^2 \\ &= \frac{(b-a^2)}{12} \end{aligned}$$