Statistics for Data Science -1

Lecture 10.5: Expectation and Variance of Binomial distribution

Usha Mohan

Indian Institute of Technology Madras

Learning objectives

Learning objectives

1. Derive the formula for the probability mass function for Binomial distribution.

Learning objectives

1. Derive the formula for the probability mass function for Binomial distribution.

Learning objectives

1. Derive the formula for the probability mass function for Binomial distribution.
2. Understand the effect of parameters n and p on the shape of the Binomial distribution.

Learning objectives

1. Derive the formula for the probability mass function for Binomial distribution.
2. Understand the effect of parameters n and p on the shape of the Binomial distribution.
3. Expectation and variance of the binomial distribution.

Learning objectives

1. Derive the formula for the probability mass function for Binomial distribution.
2. Understand the effect of parameters n and p on the shape of the Binomial distribution.
3. Expectation and variance of the binomial distribution.
4. To understand situations that can be modeled as a Binomial distribution.

Expectation and Variance of Binomial Random Variable

Expectation and Variance of Binomial Random Variable

- A binomial random variable $X \sim Bin(n, p)$ is equal to the number of successes in n independent trials when each trial is a success with probability p.

Expectation and Variance of Binomial Random Variable

- A binomial random variable $X \sim Bin(n, p)$ is equal to the number of successes in n independent trials when each trial is a success with probability p.
- We can represent X as

$$X = X_1 + X_2 + \ldots + X_n$$

where X_i is equal to 1 if trial i is a success and is equal to 0 if trial i is a failure.

Expectation and Variance of Binomial Random Variable

- A binomial random variable $X \sim Bin(n, p)$ is equal to the number of successes in n independent trials when each trial is a success with probability p.
- We can represent X as

$$X = X_1 + X_2 + \ldots + X_n$$

  where X_i is equal to 1 if trial i is a success and is equal to 0 if trial i is a failure.
- $P(X_i = 1) = p$ and $\mathrm{P}(X_i = 0) = (1 - p)$

Expectation and Variance of Binomial Random Variable

Expectation and Variance of Binomial Random Variable

- $X = X_1 + X_2 + \ldots + X_n$

Expectation and Variance of Binomial Random Variable

- $X = X_1 + X_2 + \ldots + X_n$
- $E(X) = E(X_1 + X_2 + \ldots + X_n) = E(X_1) + E(X_2) + \ldots + E(X_n)$

Expectation and Variance of Binomial Random Variable

- $X = X_1 + X_2 + \ldots + X_n$
- $E(X) = E(X_1 + X_2 + \ldots + X_n) = E(X_1) + E(X_2) + \ldots + E(X_n)$
  - $E(X) = p + p + \ldots + p = np$

Expectation and Variance of Binomial Random Variable

- $X = X_1 + X_2 + \ldots + X_n$
- $E(X) = E(X_1 + X_2 + \ldots + X_n) = E(X_1) + E(X_2) + \ldots + E(X_n)$
  - $E(X) = p + p + \ldots + p = np$
- $V(X) = V(X_1 + X_2 + \ldots + X_n) = V(X_1) + V(X_2) + \ldots + V(X_n)$

Expectation and Variance of Binomial Random Variable

- $X = X_1 + X_2 + \ldots + X_n$
- $E(X) = E(X_1 + X_2 + \ldots + X_n) = E(X_1) + E(X_2) + \ldots + E(X_n)$
  - $E(X) = p + p + \ldots + p = np$
- $V(X) = V(X_1 + X_2 + \ldots + X_n) = V(X_1) + V(X_2) + \ldots + V(X_n)$
  - $V(X) = p(1-p) + p(1-p) + \ldots + p(1-p) = np(1-p)$

Expectation and Variance of Binomial Random Variable

- $X = X_1 + X_2 + \ldots + X_n$
- $E(X) = E(X_1 + X_2 + \ldots + X_n) = E(X_1) + E(X_2) + \ldots + E(X_n)$
  - $E(X) = p + p + \ldots + p = np$
- $V(X) = V(X_1 + X_2 + \ldots + X_n) = V(X_1) + V(X_2) + \ldots + V(X_n)$
  - $V(X) = p(1-p) + p(1-p) + \ldots + p(1-p) = np(1-p)$

Result

Using the fact that the expectation of the sum of random variables is equal to the sum of their expectations, we see that Expectation of a Binomial random variable X is

$$E(X) = np$$

Also, since the variance of the sum of independent random variables is equal to the sum of their variances, we have variance of a Binomial random variable is

$$Var(X) = np(1-p)$$

Example: Tossing a coin 500 times

Example: Tossing a coin 500 times

If a fair coin is tossed 500 times, what is the standard deviation of the number of times that a head appears?

Example: Tossing a coin 500 times

If a fair coin is tossed 500 times, what is the standard deviation of the number of times that a head appears?

Let X = the number of heads in 500 tosses of a fair coin.

Example: Tossing a coin 500 times

If a fair coin is tossed 500 times, what is the standard deviation of the number of times that a head appears?

Let X = the number of heads in 500 tosses of a fair coin. Then $X \sim B(500, 1/2)$.

Example: Tossing a coin 500 times

If a fair coin is tossed 500 times, what is the standard deviation of the number of times that a head appears?

Let X = the number of heads in 500 tosses of a fair coin. Then $X \sim B(500, 1/2). V(X) = 125, SD(X) = \sqrt{125} = 11.1803$

Finding probability given expectation and n

The expected number of heads in a series of 10 tosses of a coin is 6. What is the probability there are 8 heads?

Finding probability given expectation and n

The expected number of heads in a series of 10 tosses of a coin is 6. What is the probability there are 8 heads?

Let X be number of heads.

Finding probability given expectation and n

The expected number of heads in a series of 10 tosses of a coin is 6. What is the probability there are 8 heads?

Let X be number of heads. $X \sim B(10, p)$.

Finding probability given expectation and n

The expected number of heads in a series of 10 tosses of a coin is 6. What is the probability there are 8 heads?

Let X be number of heads. $X \sim B(10, p)$.

1. Since $E(X) = np; 10p = 6$, hence $p = 0.6$

Finding probability given expectation and n

The expected number of heads in a series of 10 tosses of a coin is 6. What is the probability there are 8 heads?

Let X be number of heads. $X \sim B(10, p)$.

1. Since $E(X) = np; 10p = 6$, hence $p = 0.6$
2. Prob there are 8 heads; $P(X = 8) = 0.121$

Find pmf given Expectation and Variance

Find pmf given Expectation and Variance

If X is a binomial random variable with expected value 4.5 and variance 0.45, find

Find pmf given Expectation and Variance

If X is a binomial random variable with expected value 4.5 and variance 0.45, find

a $P(X = 3)$

Find pmf given Expectation and Variance

If X is a binomial random variable with expected value 4.5 and variance 0.45, find

a $P(X = 3)$

b $P(X \geq 4)$

Find pmf given Expectation and Variance

If X is a binomial random variable with expected value 4.5 and variance 0.45, find

a $P(X = 3)$

b $P(X \geq 4)$

- $X \sim B(n, p)$

Find pmf given Expectation and Variance

If X is a binomial random variable with expected value 4.5 and variance 0.45, find

a $P(X = 3)$

b $P(X \geq 4)$

- $X \sim B(n, p)$
- $np = 4.5, np(1 - p) = 0.45$.

Find pmf given Expectation and Variance

If X is a binomial random variable with expected value 4.5 and variance 0.45, find

a $P(X = 3)$

b $P(X \geq 4)$

- $X \sim B(n, p)$
- $np = 4.5, np(1 - p) = 0.45.$
- Solving gives $n = 5$ and $p = 0.9$

a $P(X = 3) = 0.0729$

b $P(X \geq 4) = 0.9185$

Section summary

- Expectation and variance of Binomial random variable
- Applications