IIT Madras

ONLINE DEGREE

Distance of a Point from a Line

R (0, -C/B)

P (x,y)

M

Q (-C/A, 0)

A(△PQR) = ½ |x$_1$(y$_2$- y$_3$) + x$_2$(y$_3$- y$_2$)+ x$_3$(y$_1$-y$_2$)|.
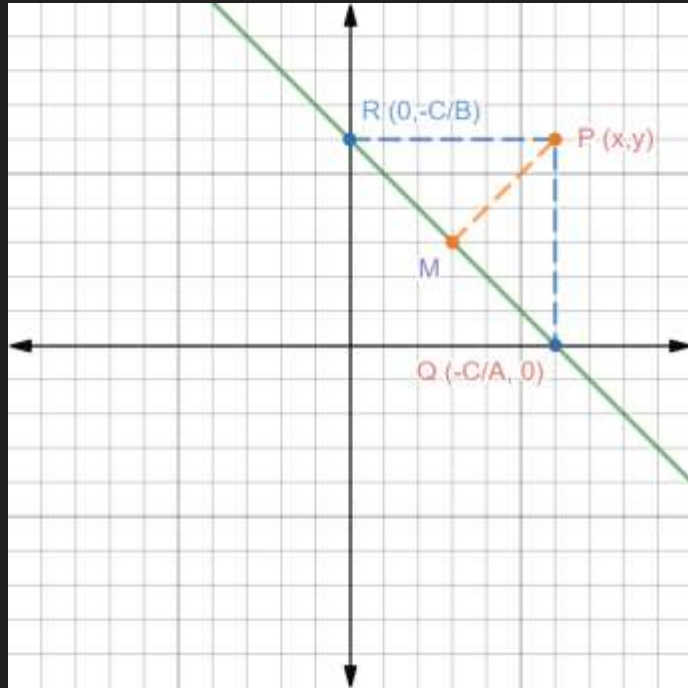
Goal. To find the distance of the point P(x_1,y_1) from the line l having equation Ax + By + C = 0.

For A, B ≠ 0, Using Intercept form,

x-intercept = - C/A and y-intercept = -C/B

A(△PQR) = ½ QRxPM. Hence, PM = 2 A(△PQR)/QR

$$A(\Delta PQR) = \frac{1}{2}|x_1(\frac{-C}{B}) - \frac{C}{A}(y_1 + \frac{C}{B})| = \frac{1}{2}\frac{|C|}{|AB|}|Ax_1 + By_1 + C|$$

$$QR = \sqrt{\frac{C^2}{A^2} + \frac{C^2}{B^2}} = \frac{|C|}{|AB|}\sqrt{A^2 + B^2}.$$

$$PM = \frac{2A(\Delta PQR)}{QR} = \frac{|Ax_1 + By_1 + C|}{\sqrt{A^2 + B^2}}.$$

Distance between two Parallel Lines

Let l_1 and l_2 be two parallel lines with slopes m.

l_1: $y = m x + c_1$. Comparing with general form, we get x-intercept at $(-c_1/m)$.

l_2: $y = m x + c_2$ Comparing with general form, we get $A = -m$, $B = 1$ and $C = -c_2$.

By using Distance of a point from a line formula, where point is $(-c_1/m, 0)$, we get

$$\frac{|A(-c_1/m)+B(0)+C|}{\sqrt{A^2+B^2}} = \frac{|c_1-c_2|}{\sqrt{1+m^2}}.$$

For general form, $m = - A/B$, $c_1 = -C_1/B$ and $c_2 = -C_2/B$, then

$$d = \frac{|C_1-C_2|}{\sqrt{A^2+B^2}}$$

Examples

Question. Find the distance of the point (3, -5) from the line 3x - 4y -26 =0.

Ax+By+C = 0 implies A = 3, B = -4 and C = -26.
Also (x_1, y_1) = (3, -5). Then

$$d = \frac{|3(3)-4(-5)-26|}{\sqrt{3^2+(-4)^2}} = \frac{3}{5}.$$

Question. Find the distance between parallel lines 3x -4y + 7 = 0 and 3x-4y+5=0.

Observe that A = 3, B = -4 and C_1 = 7, C_2= 5. Then

$$d = \frac{|C_1-C_2|}{\sqrt{A^2+B^2}}$$

$$d = \frac{|7-5|}{\sqrt{3^2+(-4)^2}} = \frac{2}{5}.$$