INDIAN INSTITUTE OF TECHNOLOGY MADRAS
सिद्धिर्भवति कर्मजा

IIT Madras
ONLINE DEGREE

Mathematics for Data Science 1
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Lecture No. 57
Logarithm Function: Properties -2

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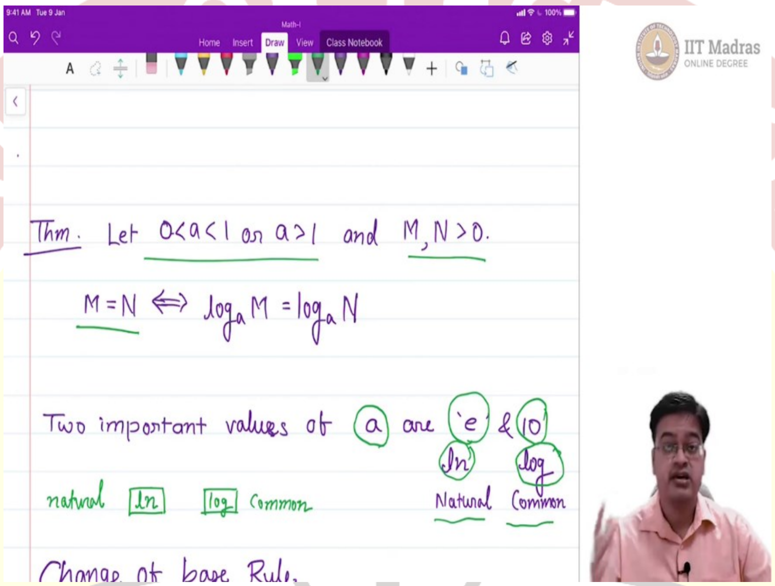

Now, we already know some facts, but it is better to revise them when we are actually handling the problems. So, here is one such theorem that says that for a between 0 and 1, open interval 0 and 1 and for a > 1 and M and N > 0. So, these are essentially taking care of the conditions that we are in the valid domain of logarithm.

Then, if M = N we know that $\log_a M = \log_a N$. How this is derived? Because if you recollect we have already proved that our function logarithmic function is one to one function that means for every element in the domain there exists a single element in the co-domain or the range.

So, based on that you can actually see that if M = N $\log_a M = \log_a N$. Then, another thing that we want to see is two important values of logarithms. That is log, whenever we are talking about log to the base a, this a is typically limited to or restricted to when we are solving any engineering problem or any any stuff that involves some kind of engineering discipline, we generally restrict to two numbers that are e that is Euler's constant, which we have mentioned and 10.

So, when whenever you are talking about these two numbers when you are talking about $\log_e M$ we will give a notation ln and when we are talking about log to the base 10, we will give a notation in log without any base. Why we are doing this? These two have special names also natural logarithm and common logarithm. Now, this natural logarithm is as I already mentioned, it comes naturally in the theory of calculus and therefore it has some special value and if you are using a scientific calculator, then you will identify this natural logarithm as ln.

So, they are there are specific provisions given in scientific calculator because it is a natural logarithm and then if you look at the scientific calculator you can open on your computer as well, you will have some symbol of this kind, log this is log to the base 10 which represents our decimal system. Because in decimal system, everything is given in powers of 10. So, therefore this is called a common logarithm because we commonly used our decimal system and this is called a natural logarithm.

So, these are the two important logarithms that we can study in the entire life. Now, the problem that comes is why only these two why not others. So, here is my bold claim that only these two logarithms will suffice for studying logarithms and why?

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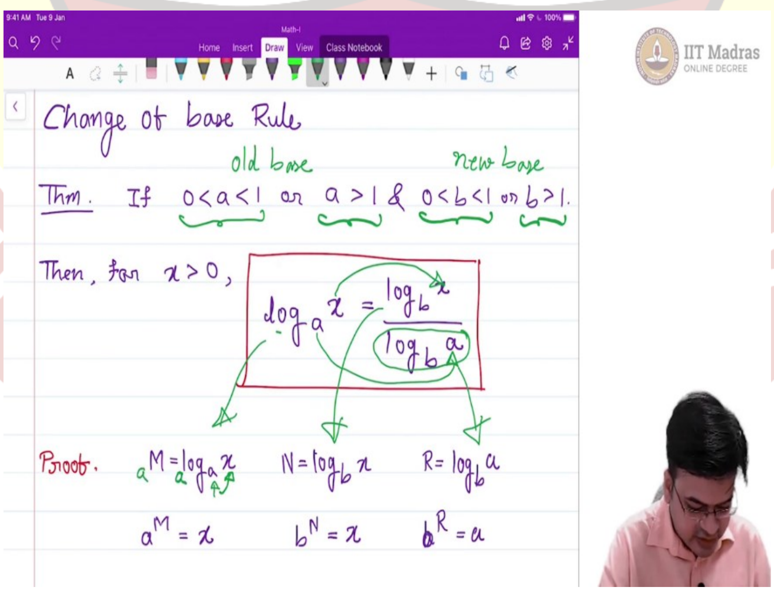

The answer to why is given by this particular theorem that is change of base rule. What let us, understand, what does this rule says, this is also called another law of logarithms. So, if you are handling x that is > 0 and if you are a is between 0 and 1 or if you are a is > 1. Remember this is a base of a logarithm. So, I am talking about change of base. So, I will talk about another base which is b, which is between 0 and 1 and b is > 1 this is called, this will be called as old base and this will be called as new base.

Now, if I want all the calculations to be done with respect to new base. If I want all the calculations to be done with respect to new base, then this theorem gives the answer, how should I go about this. So, the answer is for any $x > 0$ you consider log to the base a of x then what do you do, choice the new base that you want simply write the argument of the function over here in the numerator and the base in the denominator and write logs with appropriate base that you prefer.

So, in particular if you give me if this theorem is valid, we will prove this theorem is valid if this theorem is valid, then what we are actually talking about is you give me logarithm to any base, I will convert that logarithm into logarithm to the base 10 or logarithm to the base e and I will compute accordingly, that is the beauty of this theorem and therefore we have two important logarithms. So, we will stick to only those two important logarithms one is to the base e which is called natural logarithm. Another one is to the base 10, which is called common logarithm.

So, therefore, with this assumption, even our scientific calculators are designed to have only two keys that are ln and log they do not generally talk about log to the any base. If you have advanced scientific calculator, then it may talk about log to any base, but these two logarithms will suffice in general. Because what you are actually talking about is simple change in the basis is just a multiplication by a constant that log that constant will be given by this particular number, 1 upon this particular number. So, let us go and start proving this result.

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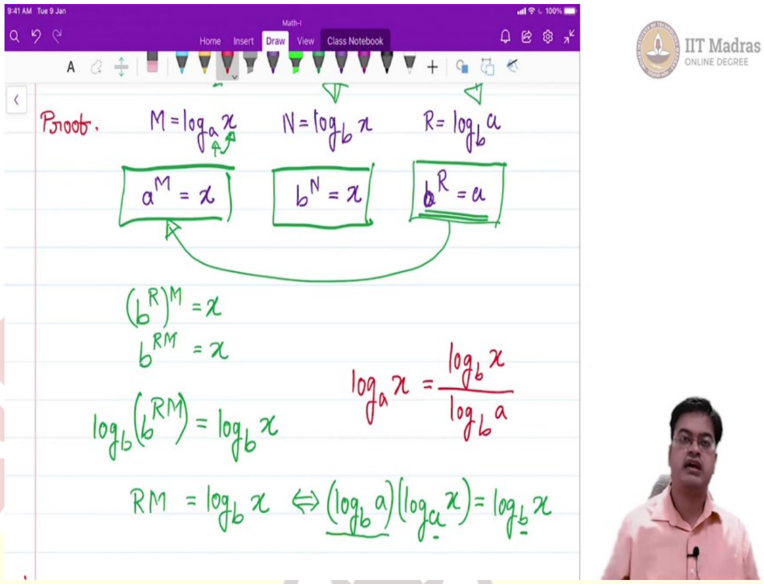

So, for proving it is very easy if you understand the basic logic in all these proofs for proving laws of all laws of logarithm, what we are using is a key factor that is exponential function is inverse of logarithmic function that is all. So, in particular if I want to prove this, let us say M = $\log_a x$ and N =$\log_b x$, R = $\log_a b$.

So, what I am doing is I am mapping all these terms in terms of M, N so this particular term is N and this particular term is R. Now, because it is this, you look at the other description of logarithmic function. So, when you are talking about logarithmic function you are basically asking a question, if I have been given a base a, to what power I should raise this base a so that I will get x this is the question that we ask and answer to that question is given by M. So, essentially this means if I apply an exponential function, which is a raise to M, $a^{\log_a x}$, then I should get back a raise to M = x this is what I should get back.

And in a similar manner I will use this and I will I will use it for the second term and I will get b^N= x and I will get b^R = a. So, now what is happening is, if your number is a can be written in the form of b^R, if you are number a can be written in the form of b^R, then you can as well put this particular thing into this expression, that means I can the write a raise to M as b^{RM} = x that is justified.

Now, what you have actually done you actually started with what. Let us, simplify this and this is actually equal to b^{RM}= x. Now, based on our understanding you simply hit this function with

$\log_b b^{RM} = \log_b x$, no confusion in this but this is actually inverse of this log function and is the exponential unction.

So, I will get back RM= $\log_b x$. Let us, substitute what is R and M and then we will resolve. So, this will happen if and only if, what is R? R is $\log_b a$ and this is $\log_a M$ is log to the base a of x solve. Let me erase it and rewrite it again. Which = $\log_b x$. So, I am I have justified what I stated.

So, when I want to switch from log to the base a or log to the base b, it is simply a multiplication by a constant that is $\log_b a$and therefore, our result is actually proved and what is what is our result that my $\log_a x$ is actually $\log_b x$which goes in the numerator and $\log_b a$, which is a proportionality constant comes in the denominator, that is all. So, now you forget about all other bases and you simply try working with natural logarithm or common logarithm that is the key idea that we will follow. So, let us use this fact and try to prove certain things like this.

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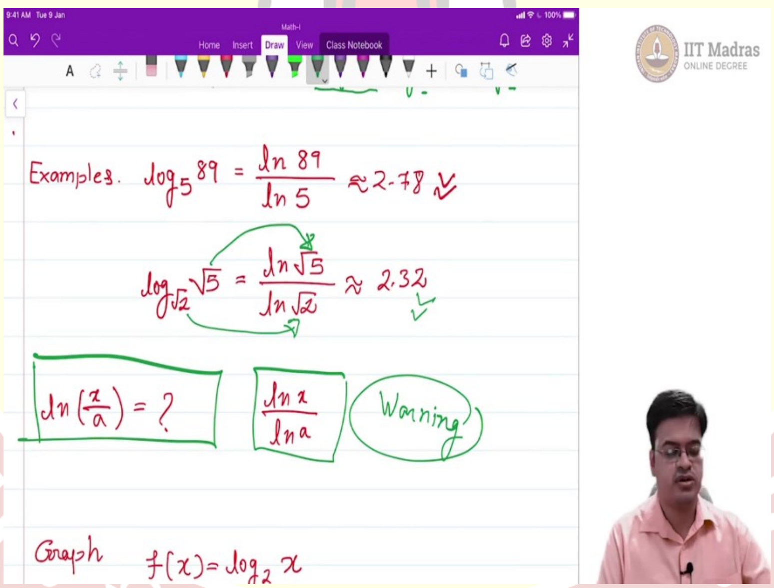

For example. Now, if you have been asked some question like, what is $\log_5 89$? So, you do not have to go into much detail that what is log to the base 5? You can simply use natural logarithm and use the change of base formula and get the answer to be $\frac{\ln 89}{\ln 5}$ use your calculator it has a natural key, which is ln, compute it you will get the answer to be equal to 2.78.

Somebody gives you any absurd number some irrational number $\log_{\sqrt{2}} \sqrt{5}$. Still you do not have to worry just apply $\ln \sqrt{5}$, $\ln \sqrt{2}$ change of base formula you will get $\frac{\ln \sqrt{5}}{\ln \sqrt{2}}$ that is this thing is going to the numerator argument is going to the numerator base is going to the denominator and you know ln is nothing but natural base log with natural base. So, this will become 2.32 that is all. So again, you can use scientific calculator and get the answer. So, this is how they were calculation simplifies no matter what base is given to you, you can easily solve all the problems.

Now, sometimes people confuse with this kind of identity $\ln\frac{x}{a}$ they simply write it as $\frac{\ln x}{\ln a}$, which is not true. So, just this is just a warning that I want to give in particular. So, this identity is not at all true. And therefore, you have to be careful while solving the problems. See in the verge of solving the problems you made tend to do these kind of mistakes, which will ruin your entire answer. So, in the next video, we will come up with better versions of graphing the logarithmic function to any base using natural logs, or some other variations. Right now we will stop here.