Statistics for Data Science -1

Lecture 7.3: Conditional Probability: Multiplication rule

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Learning objectives

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2. Distinguish between independent and dependent events.
3. Solve applications of probability.

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- This rule states that the probability that both E and F occur is equal to the probability that F occurs multiplied by the conditional probability of E given that F occurs.
- It is often quite useful for computing the probability of an intersection.

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- In an introductory statistics class of forty students, the number of males is equal to 23 and number of females is equal to 17. Two students are selected at random from the class. The first student selected is not returned to the class for possible reselection; that is, the sampling is without replacement. Find the probability that the first student selected is female and the second is male.

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- Experiment: Selecting two students from forty students.
- Sample space: $S = \{M_1M_2, M_1F_2, F_1M_2, F_1F_2\}$; where M_1M_2 represents the outcome the first student is male and the second student is male. Other outcomes can be interpreted similarly.

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- Event: First student female and second is male.
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- $P(\text{First student is female}) = \frac{17}{40}$.

Example: application of multiplication rule-cont.

- Event: First student female and second is male.
- $P(F_1 M_2) =$
  $P(\text{First student is female} \cap \text{Second student is male})$
- $P(\text{First student is female}) = \frac{17}{40}$.
- Given sampling without replacement; first student selected is not returned to the class for reselection.

Example: application of multiplication rule-cont.

- Event: First student female and second is male.
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  $P(\text{First student is female} \cap \text{Second student is male})$
- $P(\text{First student is female}) = \frac{17}{40}$.
- Given sampling without replacement; first student selected is not returned to the class for reselection.
- Given that the first student selected is female, of the 39 students remaining in the class 23 are male, so
  $P(\text{Second student is male} | \text{First student is Female}) = \frac{23}{39}$

Example: application of multiplication rule-cont.

- Event: First student female and second is male.
- $P(F_1 M_2) =$
  $P(\text{First student is female} \cap \text{Second student is male})$
- $P(\text{First student is female}) = \frac{17}{40}$.
- Given sampling without replacement; first student selected is not returned to the class for reselection.
- Given that the first student selected is female, of the 39 students remaining in the class 23 are male, so
  $P(\text{Second student is male}|\text{First student is Female}) = \frac{23}{39}$
- Hence, $P(\text{First student female and second is male}) =$
  $P(\text{First student is female}) \times$
  $P(\text{Second student is male}|\text{First student is Female}) = \frac{17}{40}\frac{23}{39} =$
  0.251

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$P(E_1)P(E_2|E_1)P(E_3|E_1 \cap E_2) \ldots P(E_n|E_1 \cap E_2 \ldots \cap E_{n-1})$

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- Define events $E_i; i = 1, 2, 3, 4$ as follows
  1. $E_1 = \{\text{the ace of spades is in any one of the piles}\}$
  2. $E_2 =$
     $\{\text{the ace of spades and the ace of hearts are in different pile}\}$
  3. $E_3 =$
     $\{\text{the aces of spades, heart, and diamonds are in different piles}\}$
  4. $E_4 = \{\text{all four aces are in different piles}\}$

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- What we require is $P(E_1 \cap E_2 \cap E_3 \cap E_4)$

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2. $P(E_2|E_1) = \frac{39}{51}$
3. $P(E_3|E_1 \cap E_2) = \frac{26}{50}$
4. $P(E_4|E_1 \cap E_2 \cap E_3) = \frac{13}{49}$
5. $P(E_1 \cap E_2 \cap E_3 \cap E_4) = \frac{39}{51}\frac{26}{50}\frac{13}{49} \approx 0.105$

Section summary

1. Multiplication rule and its application to find probability of intersection of events.