



IIT Madras
ONLINE DEGREE

Sets: Examples

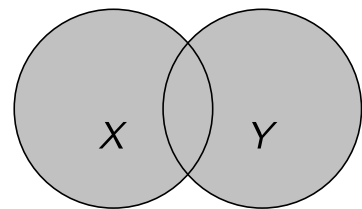
Madhavan Mukund

<https://www.cmi.ac.in/~madhavan>

Mathematics for Data Science 1
Week 1

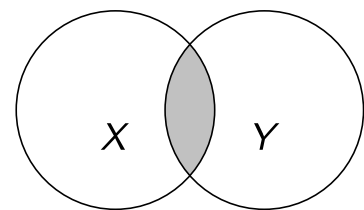
Union, intersection, complement

- **Union** — combine X and Y , $X \cup Y$
 $\{a, b, c\} \cup \{c, d, e\} = \{a, b, c, d, e\}$



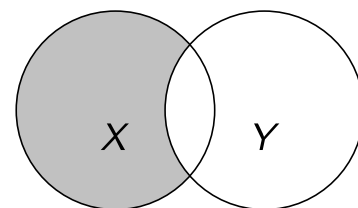
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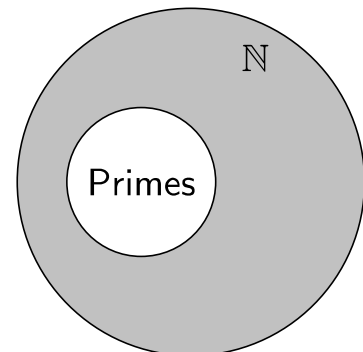
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- **Complement** — elements not in X , \overline{X} or X^c
 - Define complement relative to larger set, *universe*
 - Complement of prime numbers in \mathbb{N} are composite numbers

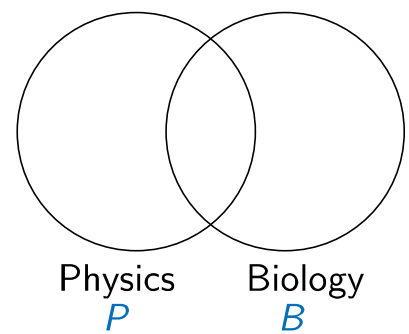


Counting problems

- In a class, 30 students took Physics, 25 took Biology and 10 took both, and 5 took neither. How many students are there in the class?

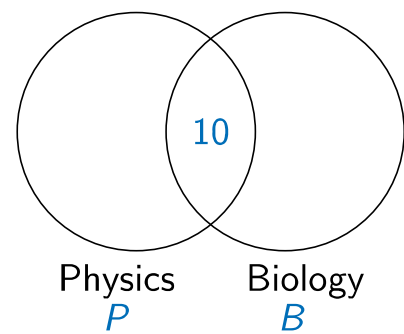
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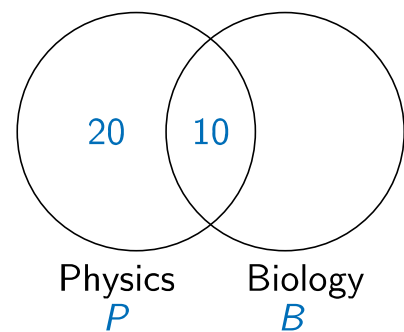
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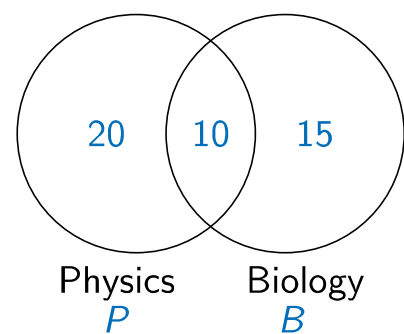
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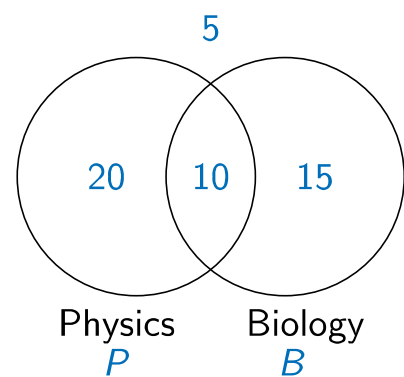
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 - Likewise 15 students in $B \setminus P$
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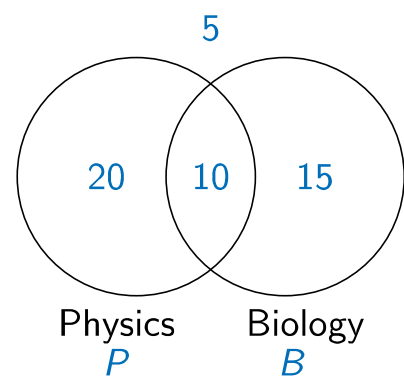
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In the class, but took neither Physics nor Biology
- Class strength: $5 + 20 + 10 + 15 = 50$



Counting problems

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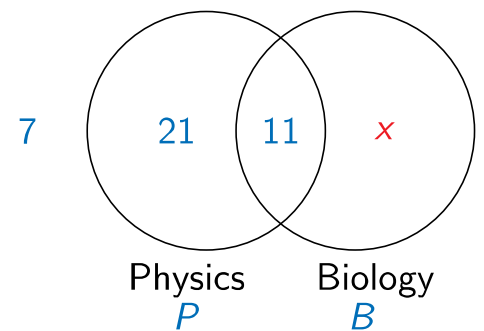
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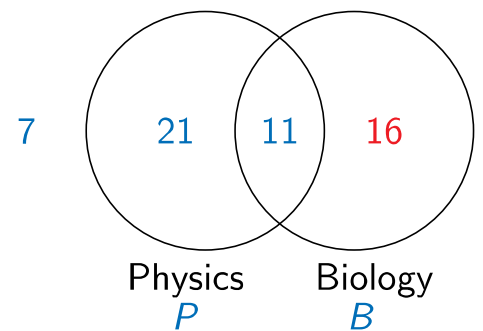


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- $7 + 21 + 11 + x = 55$
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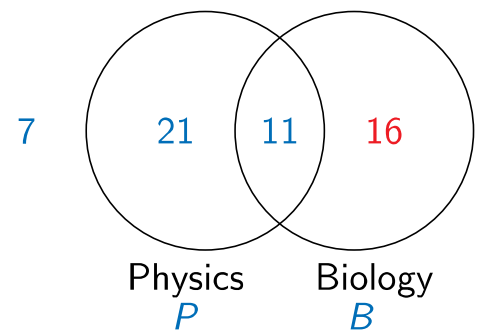
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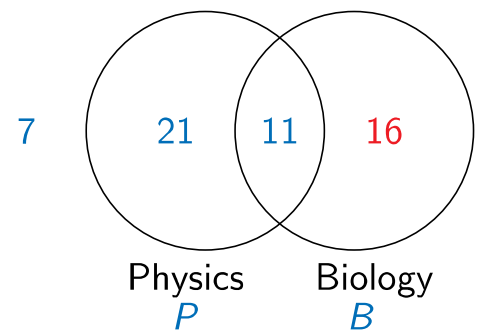
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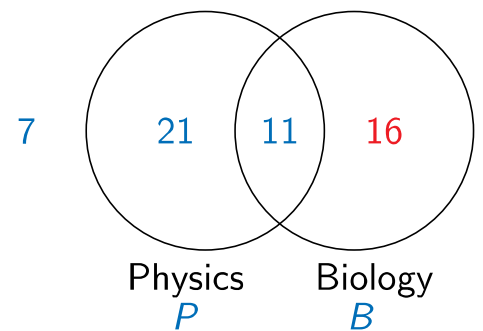
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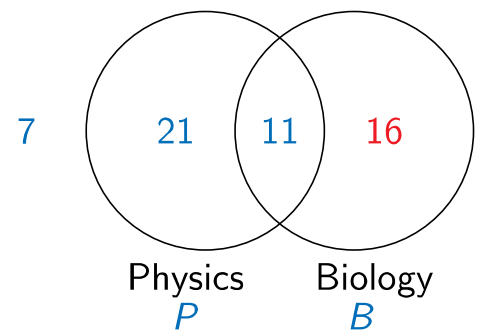
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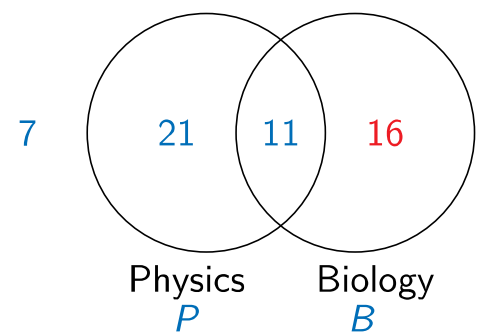
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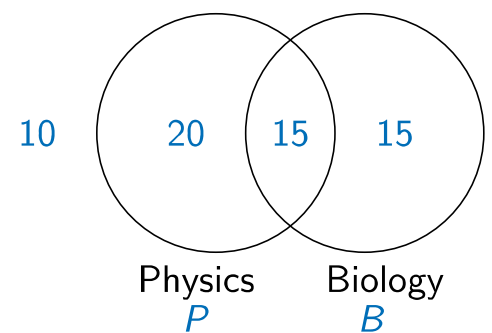
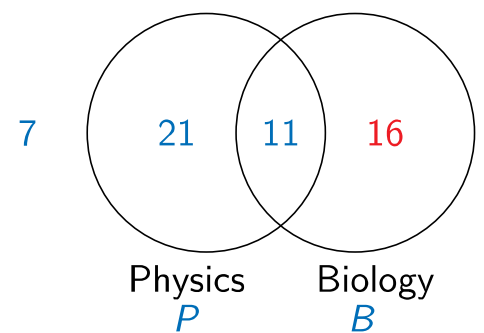
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- Venn diagrams can be useful to work out problems involving sets