Statistics for Data Science -1

Lecture 5.5: Combinations

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Learning objectives

1. Understand basic principles of counting.
2. Concept of factorials.
3. Understand differences between counting with order (permutation) and counting without regard to order (combination).
4. Use permutations and combinations to answer real life applications.

Combinations

Applications: Permutations or combinations

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  - In this case, they are the same- order is not important.
- Each selection is called a combination of 3 different objects taken 2 at a time.
- In this case, the concern is only which of the 2 objects are chosen and not in the order in which they are chosen.

Example

Consider A, B, C- Possible combinations- taking two at a time

First place	Second place
A	B
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- Number of combinations $\times$ 2! $=$ Number of permutations

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2. ${}^nC_n = 1$ and ${}^nC_0 = 1$ for all values of n
3. ${}^nC_r = {}^{n-1}C_{r-1} + {}^{n-1}C_r; 1 \leq r \leq n$

Example: Choosing questions in an exam

- In an examination, a question paper consists of 12 questions divided into two parts i.e., Part I and Part II, containing 7 and 5 questions, respectively. A student is required to attempt 8 questions in all, selecting at least 3 from each part. In how many ways can a student select the questions ?

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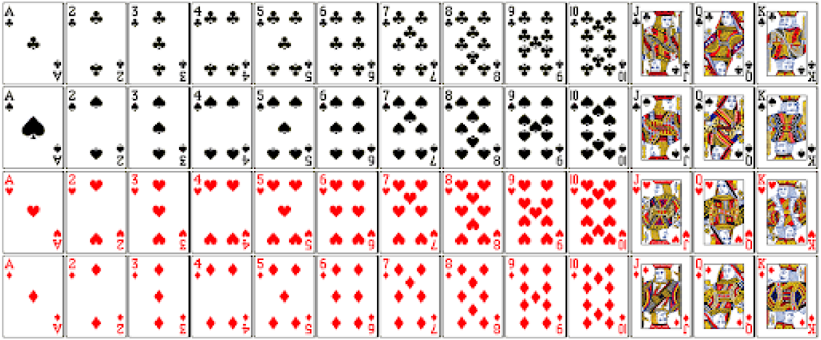
- Solution: ${}^7C_3{}^5C_5 + {}^7C_4{}^5C_4 + {}^7C_5{}^5C_3 = 35 + 175 + 210 = 420$

Example: Game of cards

Lets consider the case of choosing four cards from a deck of 52 cards.



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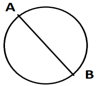

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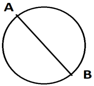

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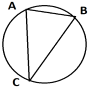

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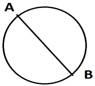

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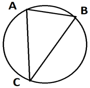

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- In general, given n points, number of line segments that can be drawn connecting the points is ${}^{n}C_2$

Section summary

1. Notation and formula for selecting r objects from n objects.
2. Some useful combinatorial identities.