

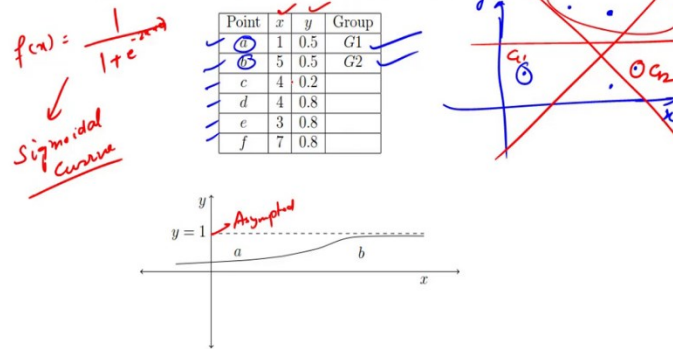


**IIT Madras**  
ONLINE DEGREE

**Mathematics for Data Science 1**  
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**Week 9 Tutorial 4**

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4. The table shows the different points on a coordinate plane. All the points can be divided in two groups  $G_1$  and  $G_2$ . It is sure that points  $a$  and  $b$  belong to groups  $G_1$  and  $G_2$  respectively. Anita found a way that if she can draw a curve which divides all the points in two groups as shown in figure. She somehow found the equation of the curve to be  $f(x) = \frac{1}{1+e^{-2x+9}}$ .
- (a) Find what was the logic to separate the points.  
 (b) Assign  $G_1$  or  $G_2$  to each point based on that particular logic.



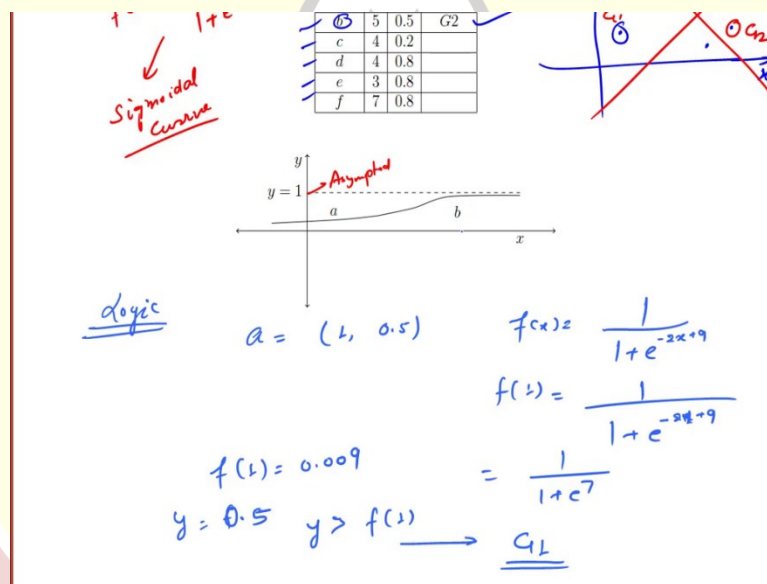
Our fourth question is about to categorize some points in two groups, one group is  $G_1$  and other group is  $G_2$ . So, it is saying that there are some points  $a, b, c, d, e, f$ , 5 points are given and 2 points are labelled as  $G_1$  and  $G_2$ . So, based on this logic we have to find where the  $c, d, e, f$  will lie, either in  $G_1$  or in  $G_2$ . For that purpose, Anita has found a way, so what does it mean, this is a point  $a$ ,  $a$  is  $(1, 0.5)$  let us say here.  $b$  is  $(5, 0.5)$  means here somewhere, if I say  $x$  and  $y$ ,  $(4, 0.2)$  means here and  $d$  is  $(4, 0.8)$  here,  $e$  is  $(3, 0.8)$ , so 3 and 0.8 will be here and  $f$  is  $(7, 0.8)$  which will be here somewhere.

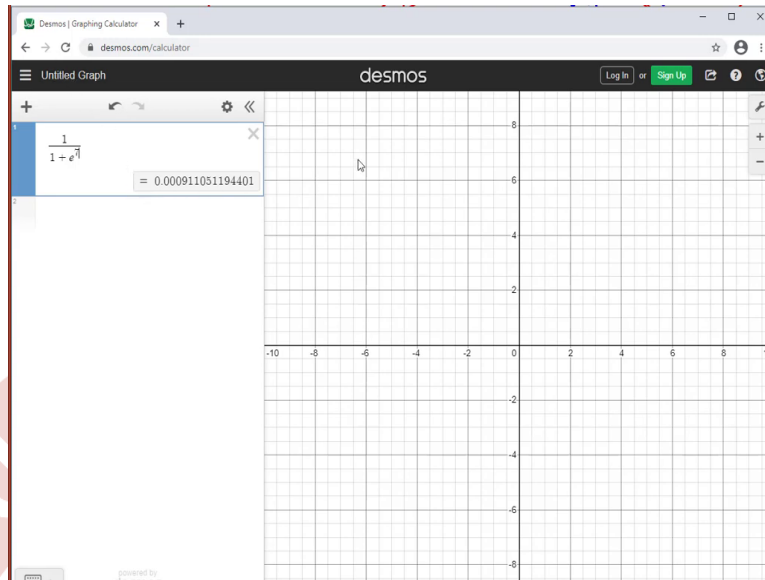
Now we need to categorize in which group it lies, so what she has done, she has found a function and based on that she categorized in  $G_1$  and  $G_2$ . So,  $a$  is  $G_1$  which means this is  $G_1$  and  $b$  is  $G_2$ ,  $b$  was 5 so this. So, this was  $G_2$ , how can we categorize them in two category? So, there are multiple ways we can find the line here, so it will categorize these 3 points and all, and these three points in the other way.

There could be this, there could be this which will categorize in different ways, but what she has done, she has applied some logics and found that a function  $f(x) = \frac{1}{1+e^{-2x+9}}$  if we apply, then we will get a curve like this. So, this is actually called a sigmoidal curve, sigmoidal curve is maximum limit is 1, so that is why  $y = 1$  is we having as a asymptote here. This is we have as a asymptote here.

Now the first question is that find what logic to separate the points, so we have two information, one information is  $f(x)$  and other information is the x-coordinate and the y-coordinate of the point a. And based on that level is  $G_1$  and similarly for b too, b. We need to find for the four other points. So, how will we do?

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So, part a we solve like, let us find the logic, what is a logic? For logic in I will take a which is 1, 0.5, if I put  $x$  in  $f(x)$  and get  $f(x) = \frac{1}{1+e^{-2x+9}}$  and if I take  $f(1) = \frac{1}{1+e^{-2 \times 1 + 9}}$ . Now  $x=1$ , so 2 into 1, it will become 1 by  $-2 + 9$ , 7,  $1 + 1$  by 7. If I calculate this, how will I calculate? We are habitual to use now Desmos, so we will use that directly, so let us see.

The value is  $\frac{1}{1+e^7}$  and we get the value 0.009, so  $f(x)$  value is 0.009. So, here  $f(1)$  is 0.009. What is the value,  $y$  here is 0.5, what does it mean? It means  $y > f(1)$ , so point is above the line and if the point is above the line, they has categorized as  $G_1$ . This will be clear once we get  $b$  is below the line, so we need to check the  $b$  also.

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Logic

$a = (1, 0.5)$  ✓

$f(x) = \frac{1}{1+e^{-2x+9}}$

$f(1) = \frac{1}{1+e^{-2+9}}$

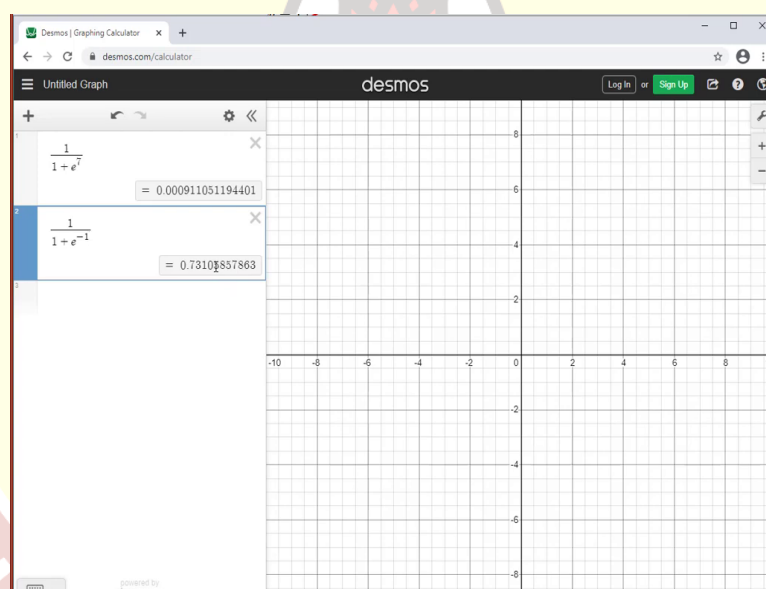
$f(1) = 0.009$

$y = 0.5$   $y > f(1) \rightarrow \underline{G_1}$

$b = (5, 0.5)$  ✓

$f(5) = \frac{1}{1+e^{-1}}$

$= 0.731 > y$



For b, the coordinates are 5, 0.5, so 5, 0.5. So,  $f(5) = \frac{1}{1+e^{-2 \times 5 + 9}} = \frac{1}{1+e^{-1}}$ . Similarly, again we will use Desmos for the calculation, so this will be  $\frac{1}{1+e^{-1}}$  and we are getting 0.731, 0.731.

So, around 0.731 which  $> y$ , if greater than  $y$  means, the point which is b is lower here this is actually this value is suppose this is a b point, then I will this is 5 and point is 0.5 and we are getting  $f(x)$  is 0.7, so it should be above the point, yeah it is above the point. So, b is lower the line, b is the lower to the I mean curve, so if b is lower to the curve it is categorized as  $G_2$ .

So, same thing we will follow up and we will see the results if the  $y$  coordinate of the point  $>$  the function value which is  $f(x)$ , will the point will be above the line curve and if the point is above the curve it will be categorized as  $G_1$  and if the case is other that is if  $y < f$  value it means it is below the curve and if the point is below to the curve it will be labelled as  $G_2$ . So, we will calculate for all the points like this and categorize.

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$b \equiv (5, 0.5)$ 
 $f(5) = \frac{1}{1+e^{-1}}$   
 $= 0.731 > y$

	$f(x)$	Observation	Label
a	(1, 0.5)	$y > f$	$G_1$
b	(5, 0.5)	$y < f$	$G_2$
c	(4, 0.2)	$y < f$	$G_2$
d	(4, 0.8)	$y > f$	$G_1$
e	(3, 0.8)	$y > f$	$G_1$
f	(7, 0.8)	$y < f$	$G_2$

So, using the Desmos a table is created and find out the  $f(x)$  values using  $x$  coordinate and it is compared with the  $y$ . So, for a we have seen here that  $y > f$ , so it was labelled as  $G_1$ . b we saw that  $y < f$ ,  $f$  is a  $f$  value at that particular  $x$ , so it was labelled as  $G_2$ . Similarly, you will follow the same procedure for others, so here  $f$  value is 0.27 which  $>$  0.2, it means  $y < f$ , if  $y < f$  will it go in  $G_1$  or  $G_2$  we can see b point showed the property if  $y < f$ , then it will go in  $G_2$ , so  $G_2$ . Similarly,  $G_1$  and  $G_2$ .