IIT Madras

ONLINE DEGREE

Mathematics for Data Science 1
Professor Neelesh S Upadhye
Department of Mathematics
Indian Institute of Technology, Madras
Lecture 4.1 C
Slope of a Quadratic Function

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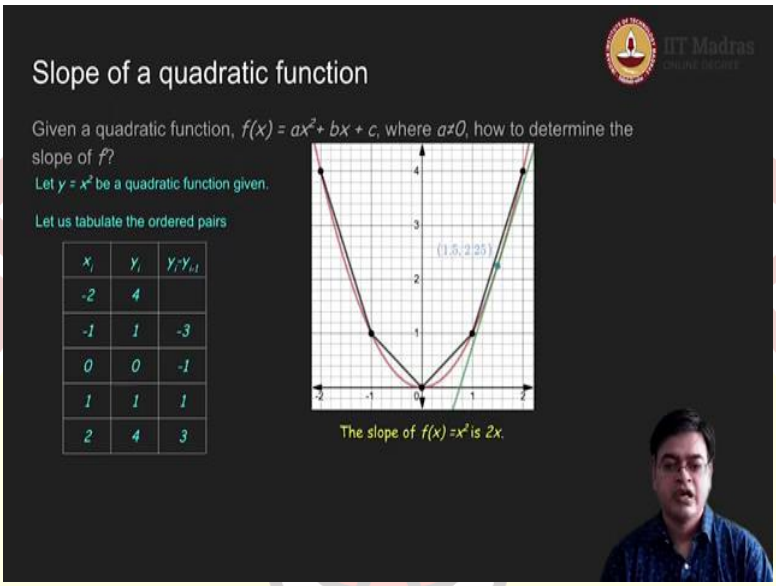

So, in this situation, I want to know what is the slope of this curve and is it a constant or which variable or what else? So, we want to answer this question. So, first, we need to plot this function, for plotting the function, we know what is the axis of symmetry for this b is 0. So, so $y - axis$ is the axis of symmetry and it will be symmetric about $y - axis$. Minimum value will be 0 as it can be seen.

So, I will take a symmetry about $y = 0$ that is, I have taken $-2, -1$, 0, 1, 2, these are the points then I have evaluated the ordered pairs, that is 4, 1, 0, 1, 4. The symmetry is clearly visible in these. Now, what is the definition of slope? It is change in y upon change in x.

So, if you look at the left-hand side, the first column, the change in x is constant. It is 1 all the time so I will use this notation, and I will go ahead and figure out what is the difference between y_i values because the denominator is always 1, it suffices to take the difference between y_i values.

So, the first value is -3, 1 -4 is -3, $y_i - y_{i-1}$. $0 - 1 = -1$, $1 - 0 = 1$,$4 - 1 = 3$, so I got the changes in y with respect to 1 unit change in x so this is the slope, but where does this slope

lie or at what point is this slope? Because if it is a straight line, I know the slope is constant. So, in order to understand this let us go to a figure and try to understand.

This is a curve, $y = x^2$. Now, when I consider these 2 points $y_i - y_{i-1}$, what I am actually doing is, I am assuming a straight line connecting these 2 points and I am calculating slope for it. So, I have assumed all these straight lines and I have calculated the slope for it. Is this a slope for a curve? No, basically not because it is a slope for that straight line.

So, now how will I identify this slope? So, if at all I want to decide what is the slope of the line if you look at our old definition the change in y by change in x also associated with $tan\ \theta$, the $tan\ \theta$ plays a crucial role, what is θ? θ is the angle of inclination. So, if I consider any point over here, and if I draw the inclination of, if I draw a line passing through that point and if I measure the inclination of that point with this positive $x - axis$ then I will get a slope because the definition of $tan\ \theta$ was not dependent on the line per se, it was dependent on that line on that particular inclination.

So, tan of that is still a slope of a line. So, let us try to use this idea and see what we can get. So now, I have identified 1 point let us say this point is actually (1.5, 2.25) because I am considering a curve, which is $y = x^2$. What will be the slope of a line at this point? We can ask this question but if you look at this line, this vertical line, this vertical line and if I slide this vertical line slightly for this point, then this is nothing but a tangent to this curve, it passes through it only once.

Let us try to actually plot that line. Yes so, once we have plotted this line, this is the tangent to that curve and the line is actually parallel to this line and the point is 1.5. This gives me a hint that this is something like you have -3, the point is, you have a slope between these 2 points as 3, you have a point which is 1.5 and if I divide this point, this particular difference by that point I am getting 2. Then let us look at these differences, what are these differences, the difference is $-1 - (-3) = 2, 1 - (-1) = 2$ so, all these differences are 2.

If you look at the second differences of these points, there are 2 that means there is some relation, 3 and 1.5, 1.5 times 2 is 3. So, I can safely assume that this point 1.5 is actually a midpoint of 1 and 2 on the $x - axis$ and therefore, whatever value is given to it is actually the value of the slope of a curve. And in particular, if I go here, for example, if I go here, and if I talk about the point 0 and 1, then what I will get is a point 0.5, the midpoint of this. Again, I

can do a similar exercise, I can draw a line and the line again will be parallel to this line and at a point 0.5, I will get the line with a slope 1.

In a similar manner if I go here, I will get a line with a slope -1, in a similar manner here, I will get a line with a slope -3 and therefore, I can safely conclude that the slope of this particular curve is $2x$. How? I have computed it. So, let us now verify our hypothesis. So, let us take a point 0, consider any 2 points about 0, let us take symmetric points because I need a symmetry.

So, let us take the point (1, -1), what is the slope of this line? It is horizontal line, so the slope should be 0 and that is what this slope is. So, in particular, I can verify for all points if I consider a point, let us say a, a is used here. Let us say if I consider a point z then I will go z +u, z−u, I will consider those 2 things and I will assume their values, draw a straight line joining them and whatever is the value of the slope for that straight line will be the value of slope for my point. This is a beautiful idea that can be generalized to a general quadratic curve.

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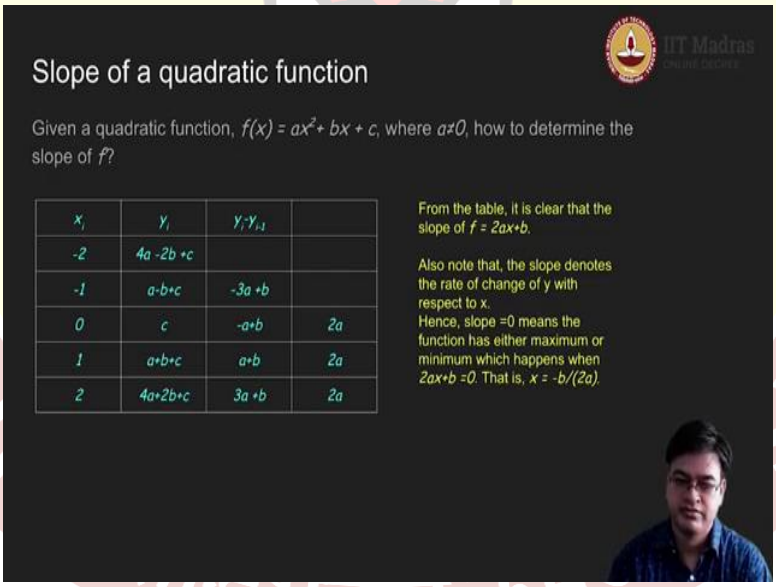

So, let us answer the general question that is, I want to find a slope of a quadratic function$ax^2 + ax^2 + bx + c$ where $a \neq 0$. So, we will simply take 5 set of points, standard 5 set of points, $-2, -1, 0, 1, 2$, I will just substitute these values in the function. So, I will get a corresponding values of $y_i's$, which are here, $4a - 2b + c$, and$a + b + c, c, 4a - 2b + c$. I will take the first differences of these two, those are given here and then I will take one more difference of these two, all these differences will turn out to be $2a$.

Now, if I look at the points which are here, and if I consider the midpoint of this midpoint of these 2 that is 1.5 so $-2a \times 1.5 + b$ will give me the answer to my question that what is the slope of that particular value, because if you look at this -3, -3 is actually 2 times 1.5. This -1 is actually 2 times -0.5, 1 is 2 times 0.5, 3 is again 2 times 1.5 so I am essentially getting the slope of all these values that means, my answer to the question that the slope of this curve quadratic function is $2ax + b$.

Now, from the table it is very clear the 2 way comes here, $ax + b$ I have derived it because this is a value containing 1.5 in the middle, so this is 2 times 1.5 So, that is ax so $2ax$ that is what this is $2ax + b$. Now, we can do some interesting observations, we have already seen around point 0 for $y = x^2$, the slope was flat it was 0. So, when will that happen? Right.

So, you can equate this $2ax + b = 0$, slope 0 means the function has reached its minimum or maximum, slope is 0. So, when will that happen? That is $x = \frac{-b}{2a}$. This is one of the reasons why $x = \frac{-b}{2a}$ is the value of the minimum or maximum, because the slope reaches the value 0. So, here what actually slope, calculates?

Slope actually calculate the rate of change with respect to x and a rate of change of y with respect to change in x. So, if the rate of change is becoming 0, that means the function has reached its minimum or maximum. So, this justifies the idea that why a quadratic function should have a minimum or maximum value at the point $x = \frac{-b}{2a}$.

Still, that point is pending where we want to find why the axis of symmetry is $x = \frac{-b}{2a}$ and we will come to it later. But as you can see here, the slope of a quadratic function is significantly different from slope of a line, slope of a line is constant, whereas the slope of a function quadratic function f is no longer a constant. In fact, it is variable that is $2ax + b$. It depends on a and b, not on the constant c, which is expected.