



IIT Madras
ONLINE DEGREE

Mathematics for Data Science -1
Weel 07-Tutorial 08

(Refer Slide Time: 0:14)

8. Given that $p(x) = (x^2 + kx + 4)(x - 5)(x - 3)$, and K is the set of values of k . Choose the correct option if $p(x)$ always have four distinct real roots.
- A. $K = \{z | z \in (-\infty, -4) \cup (4, \infty)\}$
 - B. $K = \{z | z \in (-\infty, -4) \cap (4, \infty)\}$
 - C. $K = \{z | z \in (-\infty, -5.8) \cup (-5.8, -\frac{52}{12}) \cup (-\frac{52}{12}, -4) \cup (4, \infty)\}$
 - D. None of the above.

$$k^2 - 16 = 0 \quad (\times)$$

$$k^2 - 16 > 0 \Rightarrow |k| > 4$$

$$(-\infty, -4) \cup (4, \infty)$$

Question 8 is very closely related to question 7, we again have the same, quadratic into monomial into monomial is the same polynomial and again the same set is given to us. Now we have to see the correct option for $p(x)$ to have four distinct real roots, that means a root should not be equal to each other and that is a catch.

So, we have already seen that $k^2 - 16$ the discriminant being equal to 0 will give us equal roots. So, this case is not done, this time the discriminant has to be greater than 0, so that would indicate $|k| > 4$, so you will have $(-\infty, -4) \cup (4, \infty)$ for the quadratic condition. The other condition here is that the roots for the quadratic should not be equal to 5 or 3.

(Refer Slide Time: 1:24)



$$k^2 - 16 = 0 \quad (*)$$

$$k^2 - 16 > 0 \Rightarrow |k| > 4$$

$$\frac{-k \pm \sqrt{k^2 - 16}}{2} \neq 5, 3$$

So, these routes which are $\frac{-k \pm \sqrt{k^2 - 16}}{2}$ this should not be equal to 5 or 3.

(Refer Slide Time: 1:46)



$$\frac{-k \pm \sqrt{k^2 - 16}}{2} \neq 5, 3$$

$$\frac{-k \pm \sqrt{k^2 - 16}}{2} = 5$$

$$\Rightarrow -k \pm \sqrt{k^2 - 16} = 10$$

$$\Rightarrow (10 + k)^2 = (\pm \sqrt{k^2 - 16})^2$$

$$\Rightarrow 100 + \cancel{k^2} + 20k = \cancel{k^2} - 16$$

$$\Rightarrow 20k = -116$$

$$\Rightarrow \boxed{k = -5.8}$$

So, for finding that condition let us start with $\frac{-k \pm \sqrt{k^2 - 16}}{2} = 5$ let us start with this and you get

$-k \pm \sqrt{k^2 - 16} = 10$ and that would mean $10 + k = \pm \sqrt{k^2 - 16}$.

Now if you square this we do not need to worry about the plus or minus, so let us square it and we will reach $100 + k^2 - 16$, k^2 and k^2 goes away. So we get $20k = -116$ and that would imply $k = -5.8$. So when $k = -5.8$ the root of the quadratic part will be equal to 5.

(Refer Slide Time: 2:52)

8. Given that $p(x) = (x^2 + kx + 4)(x - 5)(x - 3)$, and K is the set of values of k Choose the correct option if $p(x)$ always have four distinct real roots.

- A. $K = \{z | z \in (-\infty, -4) \cup (4, \infty)\}$
- B. $K = \{z | z \in (-\infty, -4) \cap (4, \infty)\}$
- C. $K = \{z | z \in (-\infty, -5.8) \cup (-5.8, -\frac{52}{12}) \cup (-\frac{52}{12}, -4) \cup (4, \infty)\}$
- D. None of the above.

$$k^2 - 16 = 0 \quad (*)$$

$$k^2 - 16 > 0 \Rightarrow |k| > 4 \quad k \neq -5.8$$

$$(-\infty, -4) \cup (4, \infty)$$

$$\frac{-k \pm \sqrt{k^2 - 16}}{2} \neq 5, 3$$

$$\frac{-k \pm \sqrt{k^2 - 16}}{2} = 5$$

So the root of this part will be equal to 5 and that is not allowed, so we should somehow eliminate 5.8 from this set, -5.8 from this set.

(Refer Slide Time: 3:08)



$$\Rightarrow k = -5.8$$

$$\frac{-k \pm \sqrt{k^2 - 16}}{2} = 3$$

$$\Rightarrow -k \pm \sqrt{k^2 - 16} = 6$$

$$\Rightarrow 6 + k = \pm \sqrt{k^2 - 16}$$

$$\Rightarrow 36 + k^2 + 12k = k^2 - 16$$

$$\Rightarrow 12k = -52$$

$$\Rightarrow k = \frac{-52}{12} = -\frac{13}{3}$$

And further let us check for three case where $\frac{-k \pm \sqrt{k^2 - 16}}{2} \neq 3$ so we first check when is it equal to 3 and that gives us $-k \pm \sqrt{k^2 - 16} = 6$ that gives us $\pm \sqrt{k^2 - 16} = 6 + k$ and that further gives us $36 + k^2 + 12k = k^2 - 16$. So k^2 and k^2 canceled off and that gives us $12k = -52$ this implies $k = \frac{-52}{12}$ which is essentially for 3 and 4 13, so $-\frac{13}{3}$.

(Refer Slide Time: 4:09)

8. Given that $p(x) = (x^2 + kx + 4)(x - 5)(x - 3)$, and K is the set of values of k . Choose the correct option if $p(x)$ always have four distinct real roots.
- A. $K = \{z | z \in (-\infty, -4) \cup (4, \infty)\}$
- B. $K = \{z | z \in (-\infty, -4) \cap (4, \infty)\}$
- ✓ C. $K = \{z | z \in (-\infty, -5.8) \cup (-5.8, -\frac{13}{3}) \cup (-\frac{13}{3}, -4) \cup (4, \infty)\}$
- D. None of the above.

$$k^2 - 16 = 0 \quad (\times)$$

$$k^2 - 16 > 0 \Rightarrow |k| > 4$$

$$(-\infty, -4) \cup (4, \infty)$$

$$k \neq -5.8$$

$$k \neq -\frac{13}{3}$$

$$\frac{-k \pm \sqrt{k^2 - 16}}{2} \neq 5, 3$$

$$\frac{-k \pm \sqrt{k^2 - 16}}{4} = 5$$

So, k should not be also be equal to $-\frac{13}{3}$, so which of these options does that is here we see option c is goes from $(-\infty, -5.8) \cup (-5.8, -\frac{13}{3})$ and keeping it open interval we are basically exploding -5.8 and similarly the open interval on the $-\frac{13}{3}$ side on in this and this is essentially excluding $-\frac{13}{3}$ and lastly we are doing the union with $4, \infty$. So, this is correct, we are excluding all values from -4 and 4 and also excluding -5.8 and also excluding $-\frac{13}{3}$.