INDIAN INSTITUTE OF TECHNOLOGY MADRAS

सिद्धिर्भवति कर्मजा

IIT Madras

ONLINE DEGREE

Mathematics for Data Science 2

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Week 2 Tutorial

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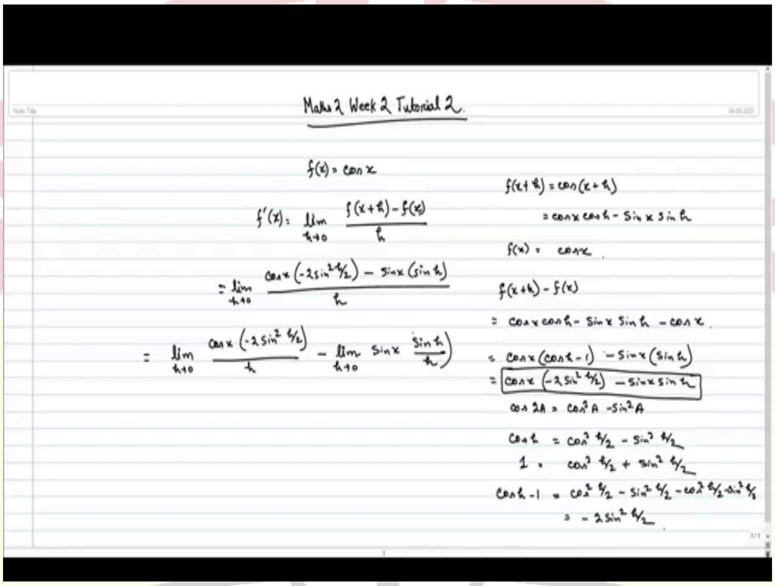

Hello everyone. Welcome to the second tutorial video of Math 2 week 2. So, let us consider the function $f(x) = cos\ x$. So, in the lecture you have seen how to calculate the derivative of sin x using the definition of derivative, so in this tutorial we will try to calculate the derivative of cos x using the definition. So, the definition of the derivative was $f'(x)$ is nothing but $\lim_{h \to 0} \frac{f(x+h)-f(x)}{h}$. So, if this limit exist then the derivative exist, otherwise the derivative does not exist.

So, let us try to find what is the derivative of $cos\ x$ in this case. So, what is $f(x+h)$? So, f of x plus h is cos of x plus h that is cos x cos h-sin x sin h. Now what is $f(x)$? $f(x) = cos\ x$, so $f(x+h) - f(x)$ that will give us $cos\ x\ cos\ h - \ sin\ x\ sin\ h - cos\ x$. So, this will give us $cos\ x$, we can take common, so $cos\ h - \ 1 - \ sin\ x\ sin\ h$, so we take this bracketed term, which are dependent on h, so $cos\ h\ minus$ 1, now this we can write further that, we know that $\cos 2a = cos^2 a - sin^2 a$.

So, in our high school trigonometry we knew this formula. So, here our $h = 2a$, so cos of h that can be written as $cos^2 \frac{h}{2} - sin^2 \frac{h}{2}$. And this 1 can be written as $cos^2 \frac{h}{2} + sin^2 \frac{h}{2}$, so $cos\ h -$

1, this can be written as $cos^2\frac{h}{2} - sin^2\frac{h}{2} - cos^2\frac{h}{2} - sin^2\frac{h}{2}$. So, this will give us $-2sin^2\frac{h}{2}$. So, this we can simplify to $\cos x - 2sin^2\frac{h}{2} - \sin x \sin h$. So, this is the thing we got in the numerator of this limit of this fraction.

So, we can write it as limit of x tending to 0 $\lim_{x\to 0} \cos x - 2sin^2\frac{h}{2} - sin\ x\ sin\ h$ by h and limit h tending to 0, so this we can separate in two limit, so we can write it as $\lim_{h\to 0} \cos x - 2sin^2\frac{h}{2}$ by h and this we can write as $\lim_{h\to 0} \frac{\sin x \sin h}{h}$.

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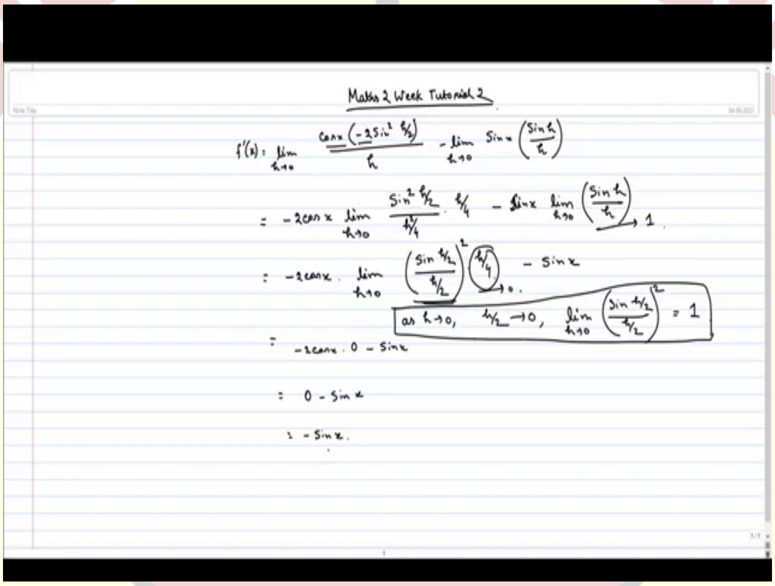

So, this is what we got in the earlier page. Now, what we will do? We will take this-2 and this $cos\ x$, these two are independent of h, so we will take that outside the limit, so this will give us $\lim_{h\to 0} sin^2\frac{h}{2}$ and $h/\ h$, so we can write it as $\frac{h^2}{4}$ and it will be $\frac{h}{4}$, so these will cancel out and it will, in the denominator there will be h, so we have written the same thing.

Now, why we have written this? We will see after a few minutes. So, in this side again we can take sin x outside as this is independent of h and then we got limit h tending to 0 sin h by h. Now, this is $-2\ cos\ x$ and limit h tending to 0, so this is sin h by 2 by h by 2 whole square into h by 4 and this is sin x and this is tending to 1, we know that limit of sin h by h, h tends to 0 this is 1, so this will give us $sin\ x$.

So, and in this portion as h is tending to 0, as h is tending to 0 h by 2 also tends to 0 and this will give us $\lim_{h\to 0}\frac{\sin\frac{h}{2}}{h/2}$, this whole square, this is tending to, this is basically 1 and, so this part is going to 1 and this part is going to 0. So, the product will go to 0. So, this is $(-2\, cos\, x)(0 - sin\, x)$. So, this I just separate out here. So, what we got? We got (0-sin x) that is $-sin\, x$, so the derivative of $cos\, x$, we got, we have derived from the definition and we got it is-of $sin\, x$ and now let us try to visualize this using graphs or using GeoGebra.

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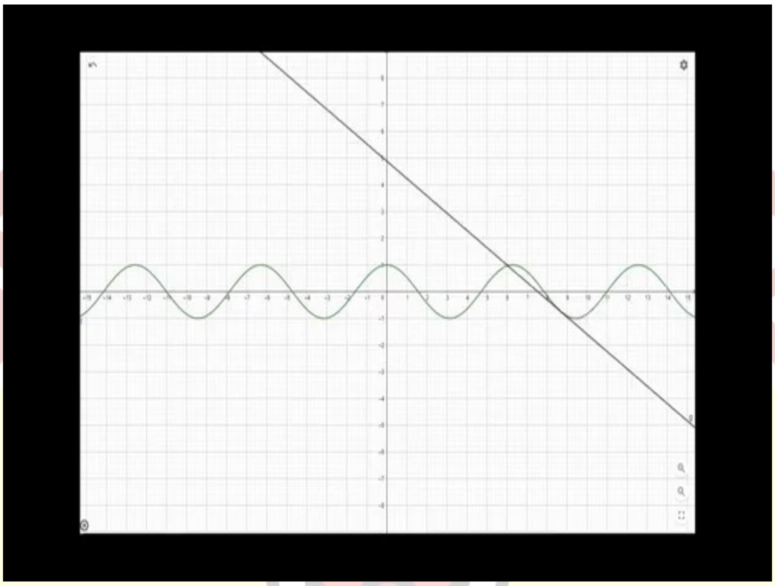

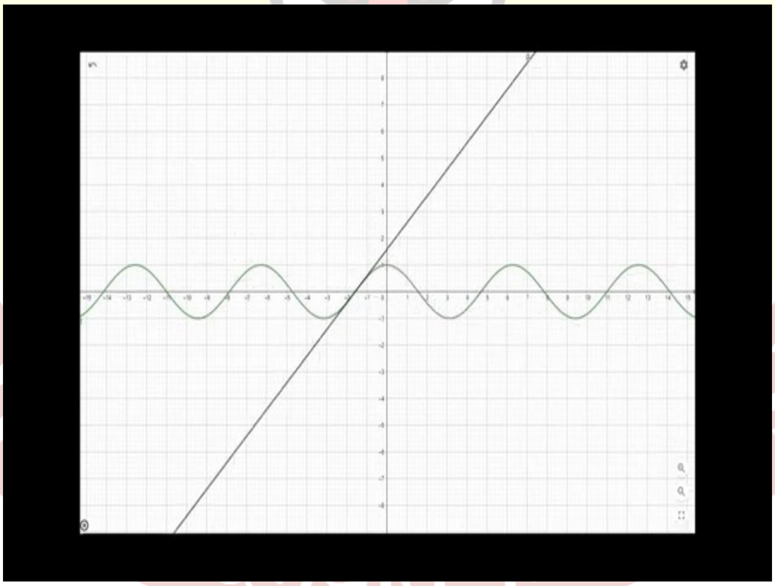

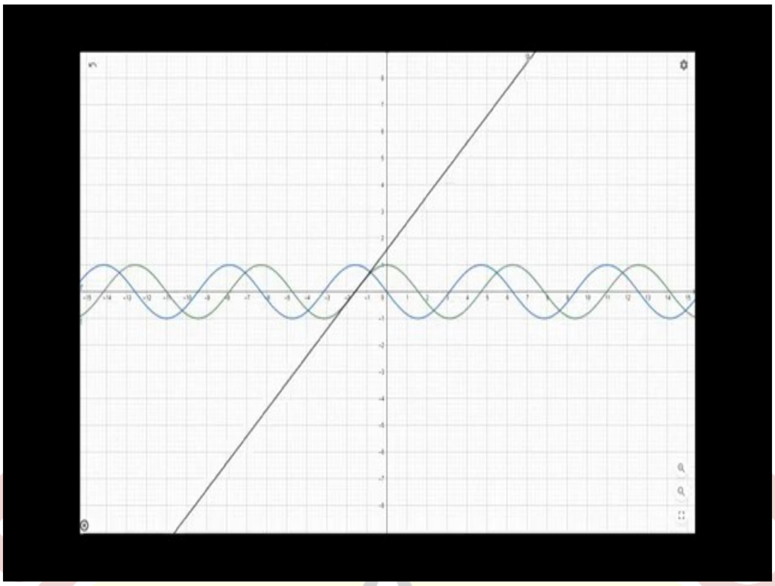

So, in GeoGebra we can see that the graph of the cos x look like this, it is an oscillatory function, so it look like this, at 0 basically it is going to 1. Now, if we want to see the tangent, that is how the slope of the tangent changes along this x-axis, along the points on this curve, so we can see that, that is how the slope of the tangent changes. So, the slope is changing in an oscillatory function as you can see in this animation.

Now, so as we have calculated the derivative of this function is nothing but-of sin x that is this new curve which we can see, so if I remove the curve $cos\ x,$ so this is the curve-sin x, which is the derivative of cos x and you can see that the slope of the tangent is basically changing by this function-of $sin\ x.$ so, this is the derivative of the function cos x. Thank you.