



**IIT Madras**  
ONLINE DEGREE

**Mathematics for Data Science 2**  
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**Week 2 Tutorial**

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The image shows a handwritten derivation of the derivative of  $f(x) = x^3$  using the limit definition. The steps are as follows:

$$f(x) = x^3$$

$$f'(x) = \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h}$$

$$f(x+h) = (x+h)^3 = x^3 + 3x^2h + 3xh^2 + h^3$$

$$f(x) = x^3$$

$$f(x+h) - f(x) = x^3 + 3x^2h + 3xh^2 + h^3 - x^3 = 3x^2h + 3xh^2 + h^3$$

$$\frac{f(x+h) - f(x)}{h} = \frac{3x^2h + 3xh^2 + h^3}{h} = 3x^2 + 3xh + h^2$$

$$\lim_{h \rightarrow 0} (3x^2 + 3xh + h^2) = 3x^2 + 3x(0) + 0 = 3x^2$$

$$f'(x) = 3x^2$$

Hello everyone. Welcome to Math 2 week 2 tutorial. So, in this week we have learnt about differentiation and derivative. So, and we find, we have also learned how we can calculate the derivative of a function. So, in this tutorial video let us take a function  $f(x) = x^3$ , which is quite well known function, polynomial of degree 3 maybe, so and if we want to calculate its derivative that is divided by  $f'(x)$ , it is nothing but the  $\lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h}$ .

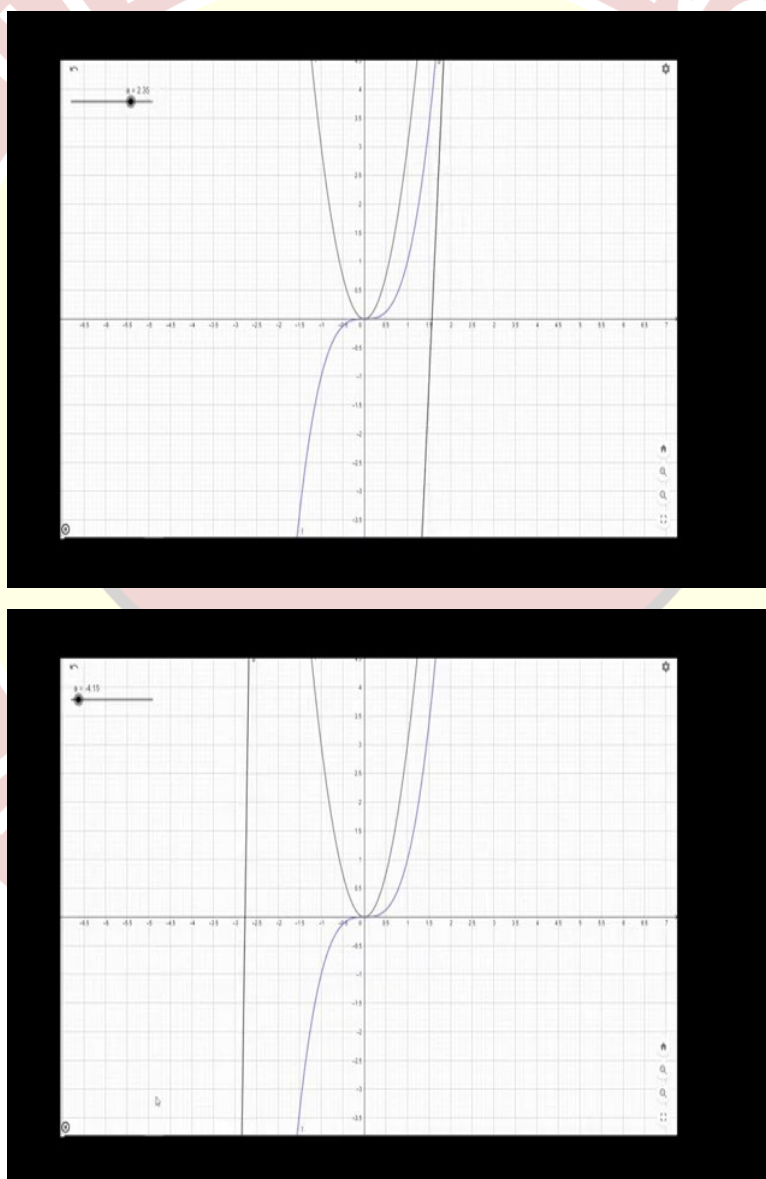
So, whenever this limit exists then this derivative of this function exist, rather we can say that the function is differentiable, so let us try to compute this limit. So, what is  $f(x+h)$ , here is  $(x+h)^3$ , which is  $x^3 + h^3 + 3x^2h + 3xh^2$  And what is  $f(x)$ ? This is given as  $x^3$ . So,  $f(x+h) - f(x)$  that is the difference between the functional values for some small change as  $h$  is tending to 0.

Now, it is given as, so we can write the whole thing down here, this  $x$  cube cancels off and it gives us  $h^3 + 3x^2h + 3xh^2$ , so this is the function we are getting here, this is the difference we are getting here, so let us put this value in this fraction. So, it will give us  $\frac{h^3 + 3x^2h + 3xh^2}{h}$ . So, we can take it as  $h^2 + 3x^2 \frac{h}{h} + 3xh^1$ .

So, as we have seen in week one that  $\lim_{h \rightarrow a} (f + g)$ , a limit  $f$   $h$  tending to  $a$  plus limit  $g$   $h$  tending to  $a$ , so we can apply this thing here, so we will get  $\lim_{h \rightarrow a} h^2$ ,  $3x^2h$  and  $h$  cancel out, so this term is basically independent of  $h$  and here we can write it as  $3xh$ , so this term, the first term, this is going to 0 as  $h$  tending to 0, the second term is independent of  $h$ .

So, it is nothing but  $3x^2$  and the third term is again dependent on  $h$  and as  $h$  is tending to 0, this will go to 0. So, it will give us  $3x^2$ , so our  $f'(x) = 3x^2$  when our  $f(x) = x^3$ . Now, let us try to visualize this thing in graph using GeoGebra.

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So, graph of  $f(x) = x^3$  will look like this and now if we want to see that the how slope of the tangent behaves for this curve, so we can see this, so as we are approaching to 0 the tangent

will, the slope of the tangent will become 0 and then again it will become positive, so whether  $x$  is negative or positive, the slope of the tangent always remains positive, as we have seen in the derivative of the function that is  $3x^2$ .

So, whatever the value of  $x$ , the derivative remains positive, so the slope as we can see here, the slope of this tangent is always positive and at the origin it becomes 0, which is nothing but the  $x$ -axis. So, this will be our, the derivative, the function, the function which represent the derivative of this curve that is  $3x^2$ . Thank you.

