BSCMA1001 : Activity Questions
Week-7

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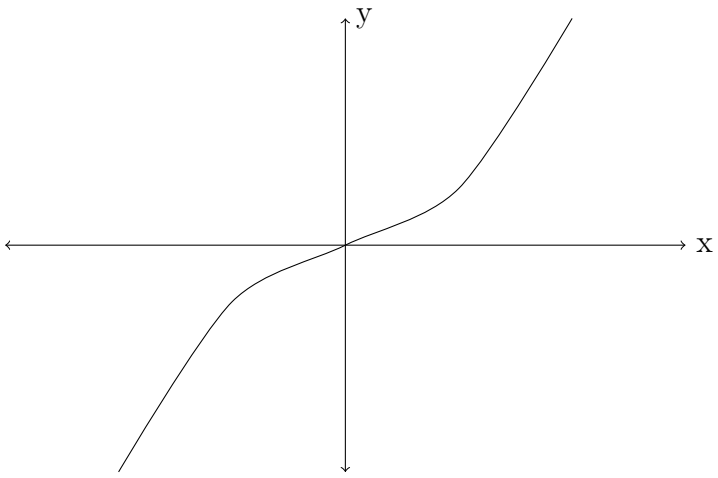
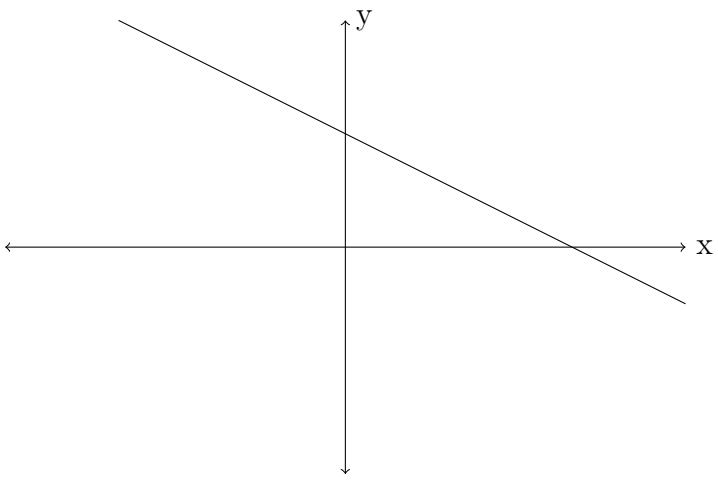
1 Lecture 1

One to One Function: Definition and Tests

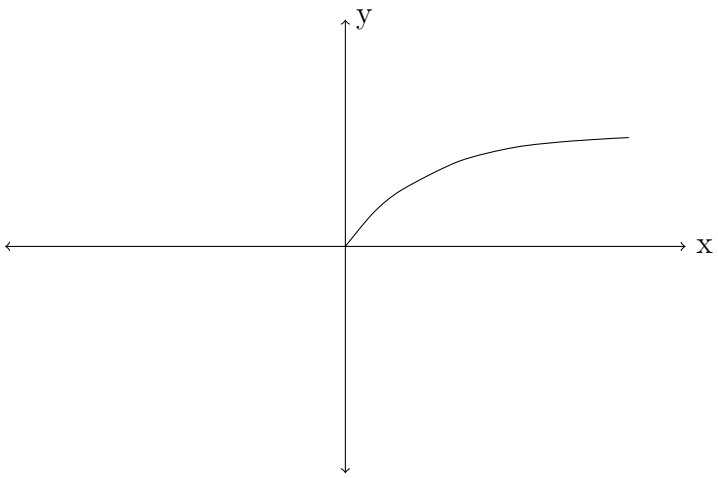
1. Which of the following statements is(are) TRUE?
   (a) The Vertical line test is used to find whether the given function is one-to-one or not.
   (b) The Horizontal line test is used to find whether the given function is one-to-one or not.
   (c) If for one value of x in domain gives more than one $f(x)$, then f is one-to-one function.
   (d) If for more than one value of x in domain gives one f(x), then f is one-to-one function.
2. Which of the following graph fails the Horizontal line test?



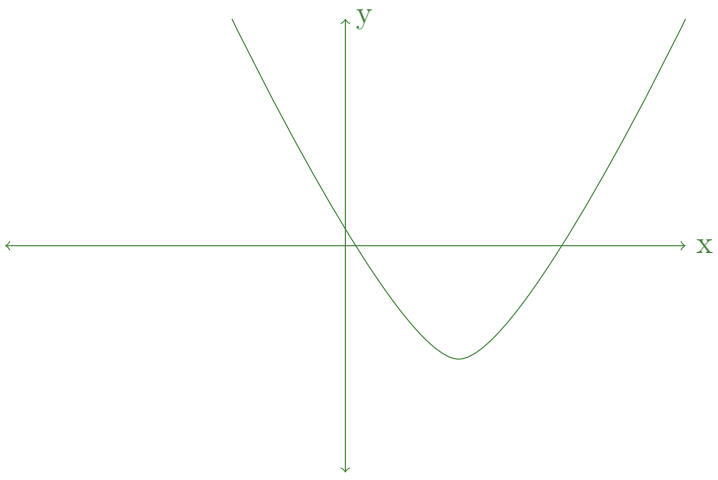
(a)



(b)



(c)



(d)

3. Which of the following statements is(are) TRUE?

   (a) One-to-one functions never fail the Horizontal line test.

(b) One-to-one functions may sometime fail the Horizontal line test.

(c) No function should fail the Horizontal line test.

(d) There are some functions that fail the Vertical line test.

4. Which of the following statements is(are) FALSE?

   (a) A function $f : X \longrightarrow Y$ is called one-to-one if, $f(x_1) = f(x_2) \in Y$, then $x_1 = x_2$.

   (b) One-to-one functions are not always reversible on their range.

   (c) If a function fails the Horizontal line test, then it is not reversible.

   (d) A function $f : X \longrightarrow Y$ is called one-to-one if, $f(x_1) \neq f(x_2) \in Y$, then $x_1 \neq x_2$.

5. Suppose $f : [0, \infty) \longrightarrow \mathbb{R}$. Which of the following is not a function?

   (a) $f(x) = 4x + 3$

   (b) $f(x) = \pm\sqrt{x + 1}$

   (c) $f(x) = x^2 + 4x + 30$

   (d) $f(x) = 100$

2 Lecture 2

One-to-one Functions: Examples & Theorems

1. Which of the following statements is(are) TRUE?

   (a) If any horizontal line intersects the graph of a function f in at most one point, then f is one-to-one.

   (b) If any horizontal line intersects the graph of a function f in at least one point, then f is one-to-one.

   (c) If f is a decreasing function, then f is not one-to-one.

   (d) If f is an increasing function, then f is one-to-one.

2. If $f : \mathbb{R} \longrightarrow \mathbb{R}$, then which of the following functions is(are) one-to-one?

   (a) $f(x) = |x + 1| - 20$

   (b) $f(x) = x^2 + 4x$

   (c) $f(x) = x^3 + 15$

   (d) $f(x) = x^3 - 5x^2 + 2x + 8$

3. Which of the following functions is(are) decreasing?

   (a) $f(x) = 4x^2 + 4x + 1$, for all $x \in \mathbb{R}$

   (b) $f(x) = 7 - 3x$, for all $x \in \mathbb{R}$

   (c) $f(x) = -2x^3$, for all $x \in \mathbb{R}$

   (d) $f(x) = \frac{1}{x}$, for all $x \in \mathbb{R} \setminus \{0\}$

4. Which of the following statements is(are) INCORRECT?

   (a) The function $f(x) = \frac{4x+3}{3x-5}$, for all $x \in \mathbb{R} \setminus \{\frac{5}{3}\}$ is one-to-one.

   (b) The function $f(x) = mx + c$, for all $x \in \mathbb{R}$ (where $m \in \mathbb{R} \setminus \{0\}$ and $c \in \mathbb{R}$) is one-to-one .

   (c) A quadratic function having two distinct roots is always one-to-one.

   (d) A quadratic function having equal roots can be one-to-one.

3 Lecture 3

Exponential Functions: Definitions

1. Which of the following options is(are) TRUE?
   (a) $a^s \times a^t = a^{s+t}$, for $s, t \in \mathbb{R}$ and $a \in \mathbb{R}$
   (b) $(a^s)^t = a^{s+t}$, for $s, t \in \mathbb{R}$ and $a > 0$
   (c) $(ab)^s = a^s \times b^s$, for $s \in \mathbb{R}$ and $a, b > 0$
   (d) $a^s \times b^s = (a+b)^s$, for $s \in \mathbb{R}$ and $a, b > 0$
   (e) $a^s \times a^t = a^{s+t}$, for $s, t \in \mathbb{R}$ and $a > 0$.
2. Which of the following equations is(are) CORRECT?
   (a) $2^3 \times 2^4 = 2^{12}$
   (b) $2^3 \times 2^4 = 2^7$
   (c) $3^5 \times 9^3 = 3^{11}$
   (d) $7^3 \times 5^3 = 35^9$
3. Simplify the expression $(2^2 \times 3^3)^5 \times 4^7 \times 5^3 \times (8 \times 25^3)^2$
   (a) 60^{15}
   (b) 120^{15}
   (c) 30^{30}
   (d) 30^{15}
4. Which of the following statements is(are) INCORRECT?
   (a) Every exponential function is a one-to-one function.
   (b) 0^0 is undefined.
   (c) $a^0 = 1$, for all $a \in \mathbb{R}$.
   (d) a is the exponent in the algebraic expression a^r.
5. Which of the following options is not an exponential function?
   (a) $f(x) = 3^{x/2}$
   (b) $f(x) = x^{\frac{9}{4}}$
   (c) $f(x) = \frac{15}{8^x}$
   (d) $f(x) = 20 \times 6^x$

4 Lecture 4

Exponential Functions: Graphing

1. Suppose $f : \mathbb{R} \longrightarrow \mathbb{R}$ is defined by $f(x) = 3^x + 20$. Which of the following is the domain and range of the function f respectively?

   (a) $\mathbb{R}$ and $(0, \infty)$

   (b) $\mathbb{R}$ and $\mathbb{R}$

   (c) $(0, \infty)$ and $(20, \infty)$

   (d) $\mathbb{R}$ and $(20, \infty)$

2. Suppose $f : \mathbb{R} \longrightarrow \mathbb{R}$ is defined by $f(x) = 2^x - 32$. Which of the following is(are) true about $f(x)$?

   (a) x-intercept is (5,0).

   (b) y-intercept is (0,-32).

   (c) $y = -31$ is the horizontal asymptote.

   (d) $y = -32$ is the horizontal asymptote.

3. Which of the following statements is(are) INCORRECT?

   (a) $f(x) = 8^{-x}$ is a decreasing function.

   (b) Every $f(x) = a^x$, $a > 1$ has same properties as $g(x) = 6174^x$.

   (c) $f(x) = 20^{-x}$ is an increasing function.

   (d) Every $f(x) = a^x$, $a < 1$ has same properties as $g(x) = 1729^x$.

4. Suppose $f : \mathbb{R} \longrightarrow \mathbb{R}$ is defined by $f(x) = \frac{1}{7^x} + 15$. Which of the following is true about $f(x)$?

   (a) End behaviour of $f(x)$ as $x \longrightarrow \infty$ is 0

   (b) End behaviour of $f(x)$ as $x \longrightarrow \infty$ is 15

   (c) End behaviour of $f(x)$ as $x \longrightarrow -\infty$ is 15

   (d) End behaviour of $f(x)$ as $x \longrightarrow -\infty$ is 0

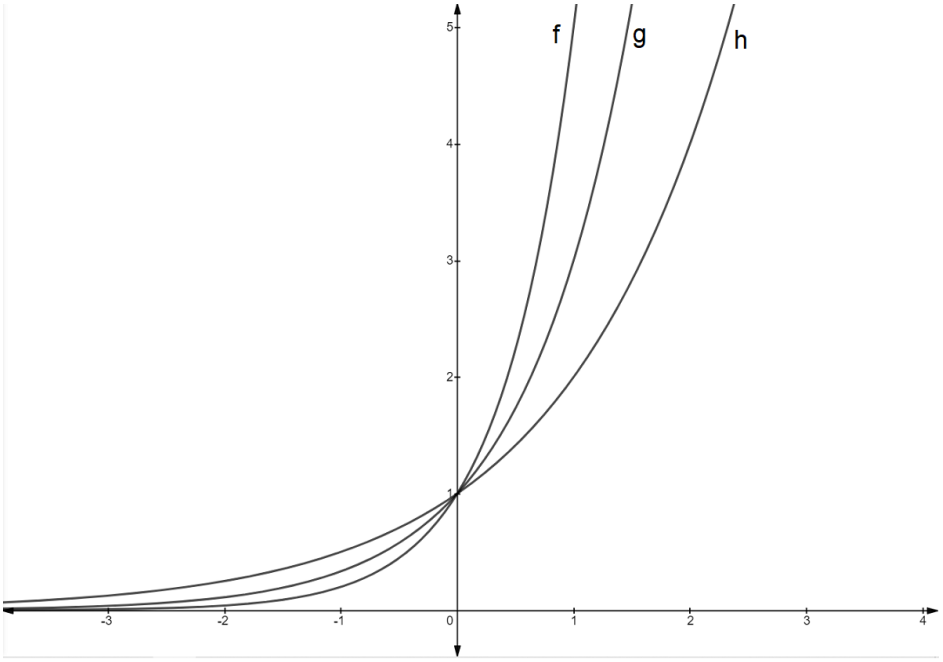


Figure 1

5. Observe Figure 1 and choose the correct set of options:

   (a) The curves f, g, h may represent $2^x, 3^x, 5^x$

   (b) The y- intercepts of f, g, h are 2,3,5 respectively.

   (c) The curves f, g, h may represent $5^x, 3^x, 2^x$.

   (d) The x- intercepts of f, g, h are 1,1,1 respectively.

5 Lecture 5

Natural Exponential Function:

1. Suppose that a stock's price continues to increase at the rate of 5% per year. If the value of one share of this stock is ₹1000 at present, then find the value(in ₹) of one share of this stock two years from now.
(Use the formula $S = a(1 + r)^n$, where r is the increase rate per year, n is the number of years from now, a is the present value of one share of the stock, and S is the value of one share of the stock after n years from now.)

   (a) 1157.625

   (b) 1000

   (c) 1050

   (d) 1102.5

2. Which of the following options yield the value 'e'?

   (a) The slope of the tangent line of the curve $f(x) = e^x$ at $x = 1$

   (b) The slope of the tangent line of the curve $f(x) = x^2 + ex$ at $x = 1$

   (c) The area under the curve $f(x) = 2e^x$, from $(-\infty, 1)$

   (d) The area under the curve $f(x) = e^x$ from $(-\infty, 1)$

3. Which of the following statements is(are) CORRECT?

   (a) If $n \longrightarrow \infty$, then $(1 + \frac{1}{n})^n \longrightarrow \infty$

   (b) If $n \longrightarrow \infty$, then $(1 + \frac{1}{n})^n \longrightarrow e$

   (c) Every $f(x) = a^x$, $a < 1$ has same properties as $g(x) = e^x$.

   (d) Every $f(x) = a^x$, $a > 1$ has same properties as $g(x) = e^x$.

4. In 1990, the population of blue whales in the world is approximately 3200. Let $F(t)$ be the approximate population of blue whales in the world after t years since 1990($t = 0$ corresponds to 1990), defined as $F(t) = 3200e^{-0.15t}$. What is the approximate population of blue whales in the world in the year 2020?

   (a) 360

   (b) 36

   (c) 12

   (d) 160

5. In 1990, the population of blue whales in the world is approximately 3200. Let $F(t)$ be the approximate population of blue whales in the world after t years from 1990($t = 0$ corresponds to 1990), defined as $F(t) = 3200e^{-0.15t}$. In which year will the population of blue whales approximately be 160? (HINT: Check from the given options)

   (a) 2000

   (b) 2005

   (c) 2010

   (d) 2020

6. Suppose the population of bacteria in a culture is growing exponentially. At today 2:00 pm, 100 bacteria were present and by 5:00 pm, 448 bacteria were present. Find an exponential function $y = ae^{kt}$ that models this growth, where t is the number of hours since 2:00 pm($t = 0$ corresponds to 2:00 pm), y is the population of bacteria at t hours, and a, k are arbitrary constants.(HINT: Check from the given options)

   (a) $y = 100e^{0.5t}$

   (b) $y = 448e^{0.5t}$

   (c) $y = 100e^{2t}$

   (d) $y = 448e^{2t}$

6 Lecture 6

Composite Functions:

1. Let $f(x)$ and $g(x)$ be two functions. Which of the following options is(are) INCORRECT?

   (a) $(f \circ g)(x) = f\big(g(x)\big)$

   (b) $(g \circ f)(x) = g\big(f(x)\big)$

   (c) $(f \circ g)(x) = g\big(f(x)\big)$

   (d) $(g \circ f)(x) = f\big(g(x)\big)$

2. Suppose $f(x)$ and $g(x)$ are well defined functions. Which of the following statements is(are) CORRECT?

   (a) For any given functions $f(x)$ and $g(x)$, $(f \circ g)(x) = (g \circ f)(x)$.

   (b) The domain of a composite function $(f \circ g)(x)$ is always a subset of the domain of the function $g(x)$.

   (c) The domain of a composite function $(f \circ g)(x)$ is always equal to the domain of the function $g(x)$.

   (d) The range of a composite function $(f \circ g)(x)$ is always a subset of the range of the function $f(x)$.

3. Suppose $f(x) = 8x$, $g(x) = 50 - 8x$, $h(x) = 50$, and $k(x) = 50 - 64x$ are functions. Which of the following options is true?

   (a) $(f \circ g)(x) = k(x)$

   (b) $(g \circ f)(x) = k(x)$

   (c) $(g \circ f)(x) = h(x)$

   (d) $(f \circ g)(x) = h(x)$

7 Lecture 7

Composite Functions: Examples

1. If $f(x) = 10(x-5)^2 + 100x - 225$ and $g(x) = \sqrt{2x+5}$, then the composite functions $f\big(g(x)\big)$ and $g\big(f(x)\big)$ are respectively

   (a) $20x + 75$ and $\sqrt{20x^2+5}$

   (b) $\sqrt{20x+55}$ and $20x^2 + 25$

   (c) $20x + 75$ and $\sqrt{20x+55}$

   (d) $20x + 75$ and $\sqrt{20x^2+55}$

2. Suppose $f(x) = 3x + 10$ and $g(x) = \sqrt{x+11}$ are two functions. Find the value of $f\big(g(5)\big)$.
   ANSWER: 22

3. Suppose $f(x) = 3x + 10$ and $g(x) = \sqrt{x+11}$ are two functions. Find the value of $g\big(f(5)\big)$.
   ANSWER: 6

4. Which of the following pairs of functions $f(x)$ and $g(x)$ satisfies the equation $f\big(g(x)\big) = g\big(f(x)\big)$?

   (a) $f(x) = 2x$ and $g(x) = 10x + 3$

   (b) $f(x) = 7x + 6$ and $g(x) = 4x + 3$

   (c) $f(x) = e^x$ and $g(x) = x$

   (d) $f(x) = 4e^x$ and $g(x) = 4x$

5. If $f(x) = 2\sqrt{x+9}$, $g(x) = 8x^2$, and $h(x) = 5x + 7$, then which of the following options are CORRECT?

   (a) $f\big(g(0)\big) < g\big(f(0)\big)$

   (b) $h\big(f(0)\big) > f\big(h(0)\big)$

   (c) $f\big(g(x)\big) < g\big(f(x)\big)$, for all $x \in (-\infty, +\infty)$

   (d) $f\big(h(x)\big) < h\big(f(x)\big)$, for all $x \in [0, +\infty)$

   (e) $h\big(g(1)\big) > g\big(h(1)\big)$

8 Lecture 8

Composite Functions: Domain

1. If $f(x) = \frac{x+5}{x-4}$ and $g(x) = \frac{1}{x+1}$, then find $(f \circ g)(x)$.

   (a) $\frac{x-4}{2x+1}$

   (b) $\frac{2x+1}{x-4}$

   (c) $\frac{-5x-6}{4x+3}$

   (d) $\frac{5x+6}{4x-3}$

2. If $f(x) = \frac{x+5}{x-4}$ and $g(x) = \frac{1}{x+1}$, then find the domain of the function $(f \circ g)(x)$.

   (a) $\mathbb{R} \setminus \{\frac{-3}{4}\}$

   (b) $\mathbb{R} \setminus \{-1, \frac{-3}{4}\}$

   (c) $\mathbb{R} \setminus \{-1, \frac{-3}{4}, 4\}$

   (d) $\mathbb{R} \setminus \{-1, \frac{3}{4}, 4\}$

3. Which of the following statements is(are) CORRECT?

   (a) If there exists $x \in \mathbb{R}$ that is not in the domain of a function f, then that x will not be in the domain of some composite function $(f \circ g)$.

   (b) The domain of a composite function $(f \circ g)$ is the set of all x such that x is in the domain of a function f and $f(x)$ is in the domain of a function g.

   (c) If there exists $x \in \mathbb{R}$ that is not in the domain of a function g, then that x will not be in the domain of some composite function $(f \circ g)$.

   (d) The domain of a composite function $(f \circ g)$ is the set of all x such that x is in the domain of a function g and $g(x)$ is in the domain of a function f.

4. If the domain of a composite function $(f \circ g)$ is $(-2, \infty)$ and $f(x) = \frac{1}{x^2}$, then which of the following options can be $g(x)$?

   (a) $\frac{\sqrt{x+2}}{10}$

   (b) $\frac{\sqrt{x+2}}{5x}$

   (c) $\frac{3}{\sqrt{x+2}}$

   (d) $\sqrt{2-x}$

9 Lecture 9

Inverse Functions:

1. Which of the following options is(are) CORRECT for a reversible function f?
   (a) For any function f, $f^{-1}(x) = \frac{1}{f(x)}$
   (b) The domain of a function f is always equal to the range of f^{-1} function.
   (c) For any $x = a$ in the domain of a function f, if $(a, f(a))$ is on the graph of f, then $(f(a), a)$ is on the graph of f^{-1}
   (d) For any function f, $(f \circ f^{-1})(x) = x$.

2. If $f(x) = \sqrt{x-5}$, then the domain and the range of the function f^{-1} are respectively
   (a) $[0, \infty)$ and $[5, \infty)$
   (b) $[5, \infty)$ and $[0, \infty)$
   (c) $\mathbb{R}$ and $[5, \infty)$
   (d) $[5, \infty)$ and $\mathbb{R}$

3. Which of the following functions satisfy the condition $f(x) = f^{-1}(x)$?
   (a) $f(x) = \frac{1}{x^3}$
   (b) $f(x) = \frac{1}{x}$
   (c) $f(x) = \frac{2x-1}{3x-2}$
   (d) $f(x) = 9 - 5x$
   (e) $f(x) = -x$

4. Which of the following statements is INCORRECT?
   (a) The inverse function(g) of a function $f(x) = \frac{1}{x^3}$ is $g(x) = \frac{1}{\sqrt[3]{x}}$
   (b) If $f(x) = \frac{1}{x-11}$ and $g(x) = 11 - \frac{1}{x}$, then $f^(x) = g^{-1}(x)$ for all $x \in \mathbb{R}$
   (c) $(f \circ f^{-1})(x) = x = (f^{-1} \circ f)(x)$, for all x in the domain of the function f.
   (d) A function f is symmetric to function f^{-1} about the line $y = x$.

5. If $f(x) = x^3 - 5$, $g(x) = \sqrt[3]{x} + 5$, $h(x) = (x-5)^3$ and $k(x) = \sqrt[3]{x+5}$ are functions, then which of the following options is(are) true?
   (a) $k(x) = f^{-1}(x)$, for all $x \in \mathbb{R}$
   (b) $f(x) = h^{-1}(x)$, for all $x \in \mathbb{R}$
   (c) $g(x) = k^{-1}(x)$, for all $x \in \mathbb{R}$
   (d) $h(x) = g^{-1}(x)$, for all $x \in \mathbb{R}$