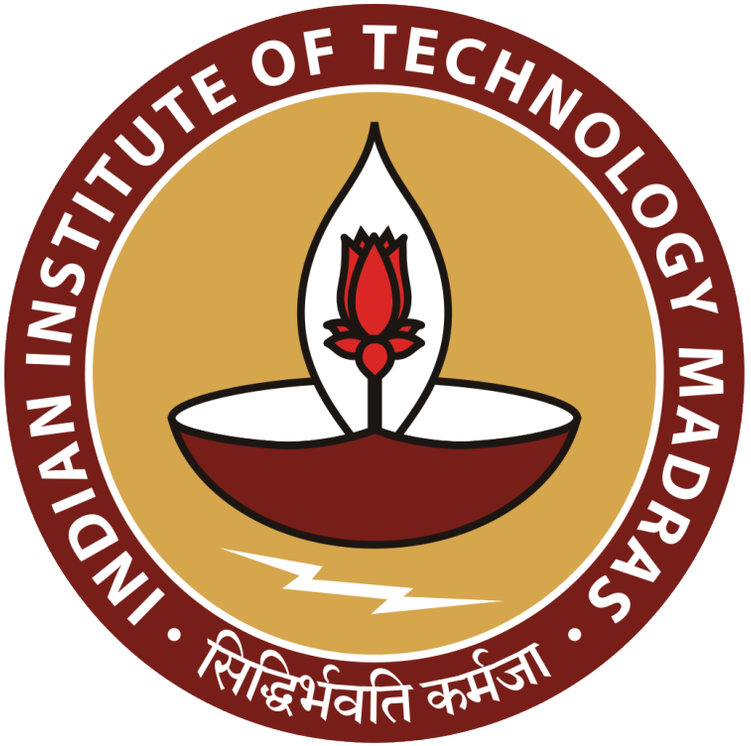

IIT Madras

ONLINE DEGREE

Sets

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Mathematics for Data Science 1
Week 1

Constructing subsets

Set comprehension

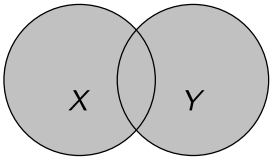
- The subset of even integers
  $\{x \mid x \in \mathbb{Z}, x \bmod 2 = 0\}$
  - Begin with an existing set, $\mathbb{Z}$
  - Apply a condition to each element in that set
    - $x \in \mathbb{Z}$ such that $x \bmod 2 = 0$
  - Collect all the elements that match the condition
- Examples
  - The set of perfect squares
    $\{m \mid m \in \mathbb{N}, \sqrt{m} \in \mathbb{N}\}$
  - The set of rationals in reduced form
    $\{p/q \mid p, q \in \mathbb{Z}, gcd(p, q) = 1\}$

Intervals

- Integers from -6 to $+6$
  $\{z \mid z \in \mathbb{Z}, -6 \leq z \leq 6\}$
- Real numbers between 0 and 1
- Closed interval $[0, 1]$
  – include endpoints
  $\{r \mid r \in \mathbb{R}, 0 \leq r \leq 1\}$
- Open interval $(0, 1)$
  – exclude endpoints
  $\{r \mid r \in \mathbb{R}, 0 < r < 1\}$
- Left open $(0, 1]$
  $\{r \mid r \in \mathbb{R}, 0 < r \leq 1\}$

Union, intersection, complement

- Union — combine X and Y, $X \cup Y$
  $\{a, b, c\} \cup \{c, d, e\} = \{a, b, c, d, e\}$



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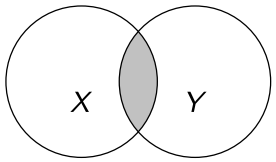
- Intersection — elements common to X and Y, $X \cap Y$
  $\{a, b, c, d\} \cap \{a, d, e, f\} = \{a, d\}$



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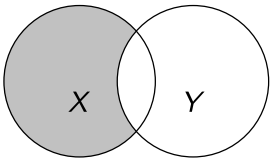
- Set difference — elements in X that are not in Y, $X \setminus Y$ or $X - Y$
  $\{a, b, c, d\} \setminus \{a, d, e, f\} = \{b, c\}$



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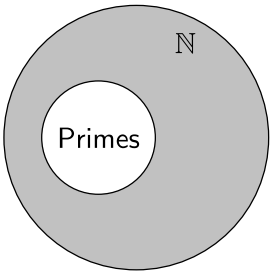
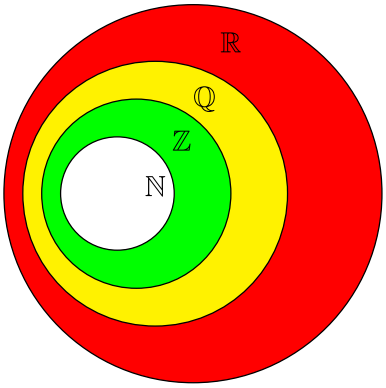
- Complement — elements not in X, $\overline{X}$ or X^c
  - Define complement relative to larger set, *universe*
  - Complement of prime numbers in $\mathbb{N}$ are composite numbers



Summary

- Sets are a standard way to represent collections of mathematical objects
- Sets may be finite or infinite
- Can carve out interesting subsets of sets
- Set operations: union, intersection, difference, complement