Statistics for Data Science -1

Lecture 5.1: Basic Principles of counting

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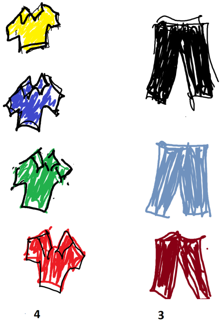
Learning objectives

1. Understand basic principles of counting.
2. Concept of factorials.
3. Understand differences between counting with order (permutation) and counting without regard to order (combination).
4. Use permutations and combinations to answer real life applications.

Example 1: Buying clothes

- You have a gift card from a major retailer which allows you to buy "one" item, either a shirt **or** a pant.
- The choices at the retailer are



- How many different ways can you use your card?

Solution

- There are four choices for buying a shirt
- There are three choices for buying a pant
- If you choose to buy a shirt (pant), you cannot buy a pant (shirt).
- Hence, the total choices available are $4 + 3 = 7$

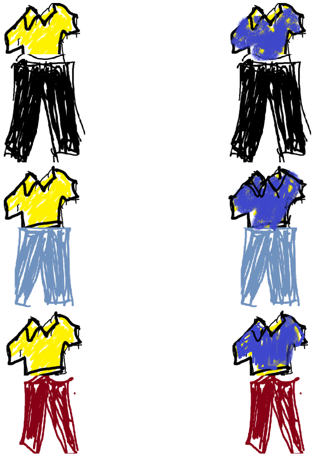
Addition rule of counting

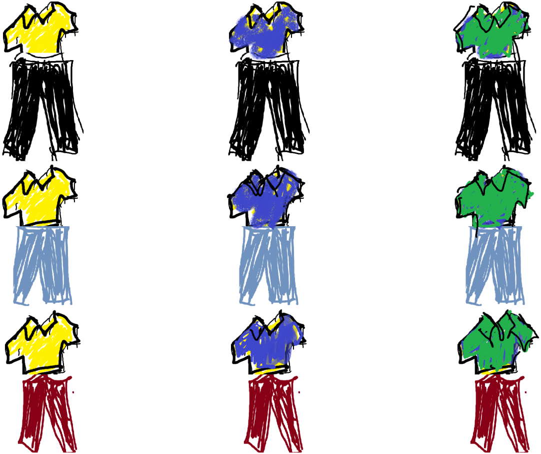
- If an action A can occur in n_1 different ways, another action B can occur in n_2 different ways, then the total number of occurrence of the actions A **or** B is $n_1 + n_2$.

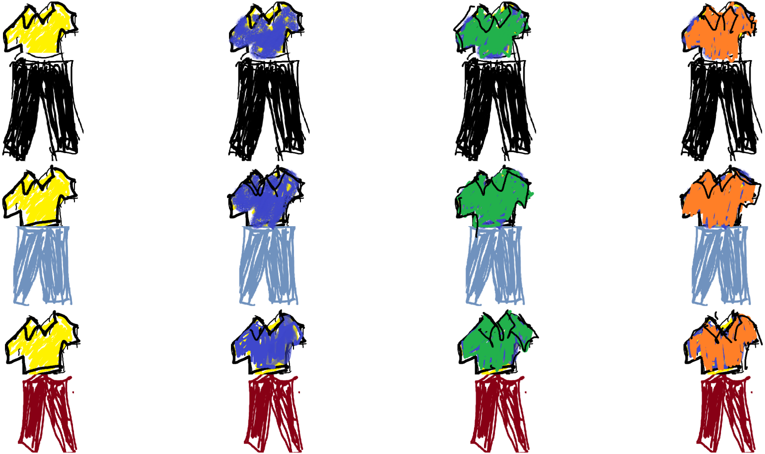
Example 2: Matching shirts and pants

- Suppose now your card allows you to buy one shirt **and** one pant- how many choices do you have?
- Suppose we have four shirts and three pants. How many sets can we make?

Matching shirts and pants







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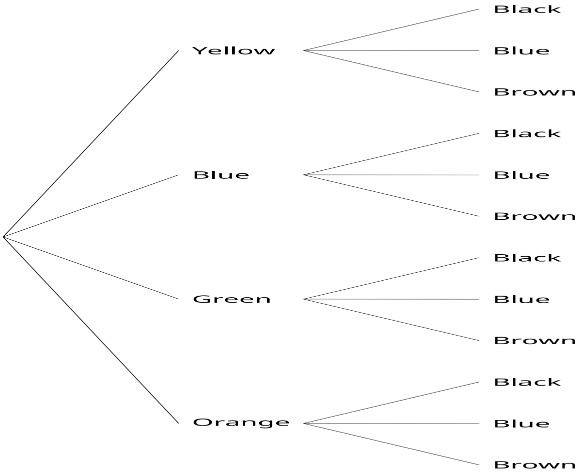
Tree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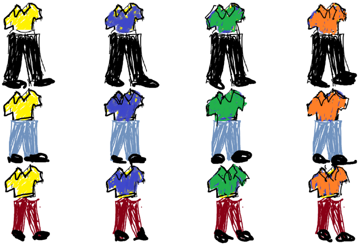
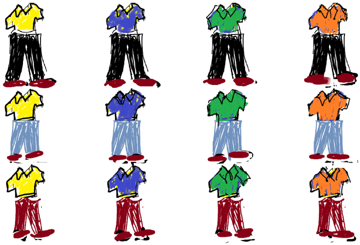
Matching shirts and pants and shoes



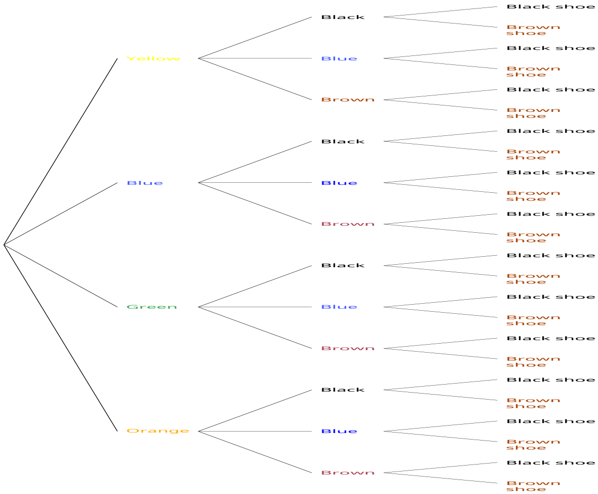
12 ways



12 ways

Total 12+12= 24 ways

Tree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Multiplication rule of counting

- If an action A can occur in n_1 different ways, another action B can occur in n_2 different ways, then the total number of occurrence of the actions A **and** B together is $n_1 \times n_2$.
- Suppose that r actions are to be performed in a definite order. Further suppose that there are n_1 possibilities for the first action and that corresponding to each of these possibilities are n_2 possibilities for the second action, and so on. Then there are $n_1 \times n_2 \times \ldots \times n_r$ possibilities altogether for the r actions.

Example 2: Application: Creating alpha-numeric code

- Suppose you are asked to create a six digit alpha-numeric password with the following requirement:
- The password should have first two letters followed by four numbers.
- Repetition allowed.
  - Number of ways- $26 \times 26 \times 10 \times 10 \times 10 \times 10 = 6,760,000$
- Repetition not allowed.
  - Number of ways- $26 \times 25 \times 10 \times 9 \times 8 \times 7 = 3,276,000$

Section summary

- Addition rule of counting.
- Multiplication rule of counting.

Example 3: Order of finishes in a race

- There are eight athletes who take part in a 100 m race. What are the possible ways the athletes can finish the race (assuming no ties)?

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- First place - any one of the 8 athletes; second - any one of the remaining 7, and so on, the seventh place - any one of the remaining 2, and finally the last place goes to the only one remaining.
- Hence the total number of ways = $8 \times 7 \times 6 \times 5 \times 4 \times 3 \times 2 \times 1 = 40,320$

Factorial

Definition

The product of the first n positive integers (counting numbers) is called n factorial and is denoted $n!$. In symbols,

$$n! = n \times (n-1) \times \ldots \times 1$$

Remark

By convention $0! = 1$

Example 4: Choosing shirts

1. 
2. 
3. 
4. 
5. 
6. 

Example 5

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   - In general, for $i \leq n$ we have,
   $$n! = n \times (n-1) \ldots \times (n-i+1) \times (n-i)!$$

Example 6: Simplifying expressions

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3. Express $25 \times 24 \times 23$ in terms of factorials-

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3. Express $25 \times 24 \times 23$ in terms of factorials-

$$\frac{25 \times 24 \times 23 \times 22 \times \ldots \times 1}{22 \times 21 \times \ldots \times 1} = \frac{25!}{22!}$$

Section summary

- Introduced factorial notation.
- Simplifying expressions.