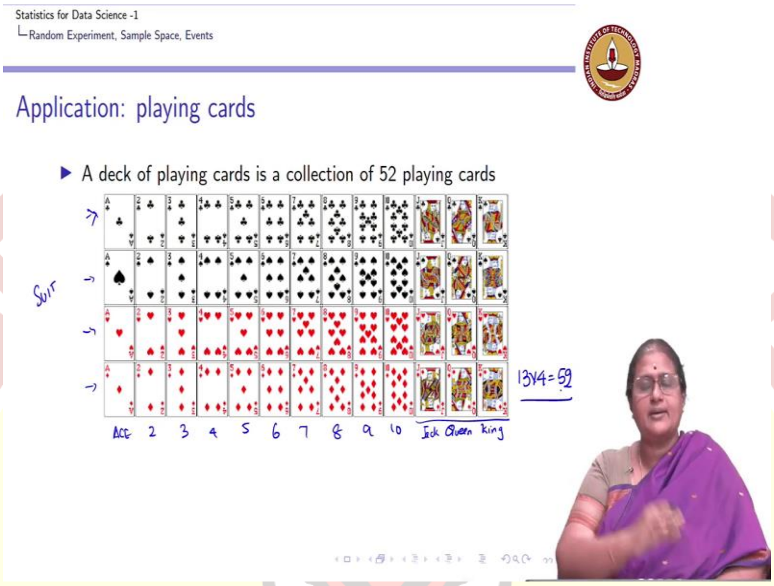
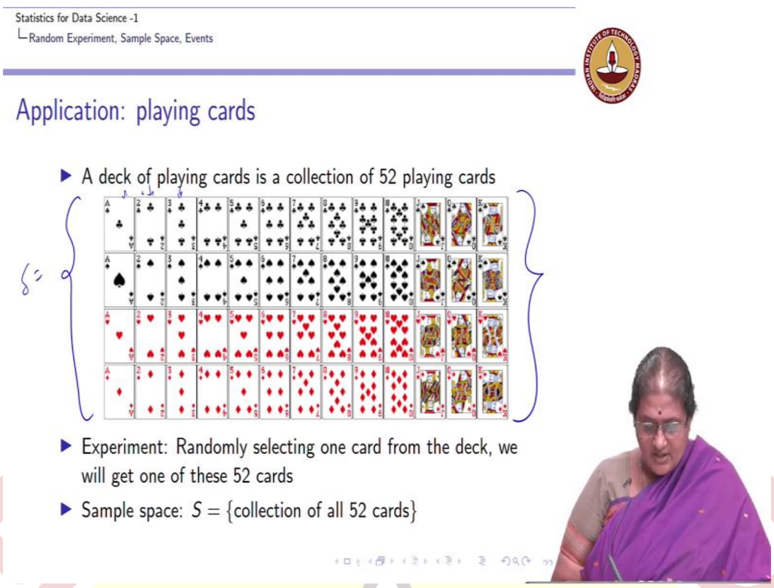
IIT Madras

ONLINE DEGREE

Statistics for Data Science 1
Professor Usha Mohan
Department of Management Studies
Indian Institute of Technology, Madras
Lecture 6.3
Probability- Random experiment, Sample space, events

(Refer Slide Time: 00:13)





So, let us learn the concepts you have learned so far to one example or application of decks of cards or playing of cards. Recall in the permutation and combination module, we introduced you all to what we mean by a deck or a playing card deck.

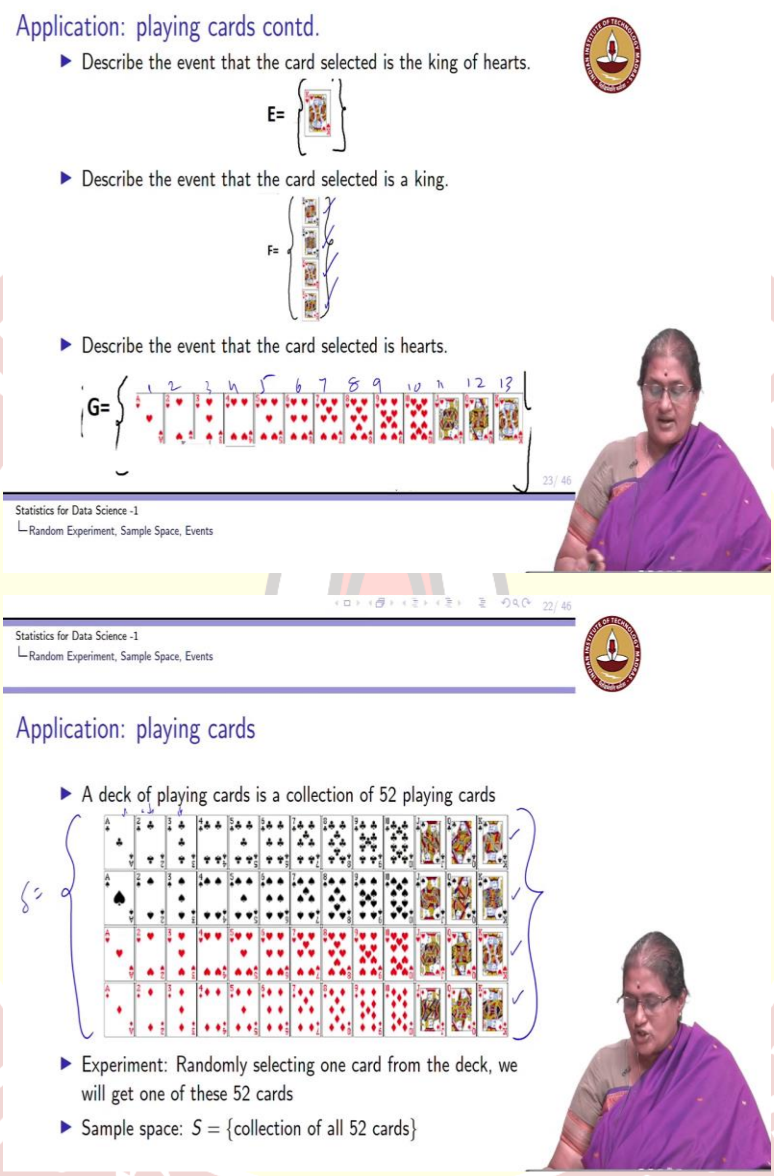
So, a deck of playing cards is actually this collection of these 52 playing cards. Now these clubs, the clubs, the spade, the heart and diamond is what we referred to as a suit of the card, it goes from Ace, this is an ace, this is a 2 3 4 5 6 7 8 9 10. Now these are referred to as face cards, you have this is a jack, this is a queen and this is a king.

So, they are totally 13 types. So, I have four shoots and I have 13 types. So, making it 13×4, which is 52 playing cards. Now, if I am laying down a deck of playing cards and I asked a person to choose one of the cards from this deck, I am laying, all these playing cards are laid face down. So, this is a face of a card. So, each one of them is laid facedown and I ask a person to choose one of the card.

Now, the random experiment in this case is to choose or randomly you are selecting one card from this deck of 52 cards. Why is it a random experiment? Because I am choosing one card. I know this card can be any one of these 52 cards, I know these are the possible outcomes of the experiment. But I really do not know what is the outcome, hence randomly selecting one card from these 52 cards will form my random experiment.

So, what are the outcomes? The outcomes are any one of these 52 cards, so the collection of all these 52 cards. So this, this was one of the outcome, these are my basic outcomes. So, a clubs is one of my basic outcome, this is the second basic outcome. So, I have these 52 outcomes. So, I can say my sample set is these 52 outcomes of the random experiment. So, that is my sample space.

(Refer Slide Time: 02:57)



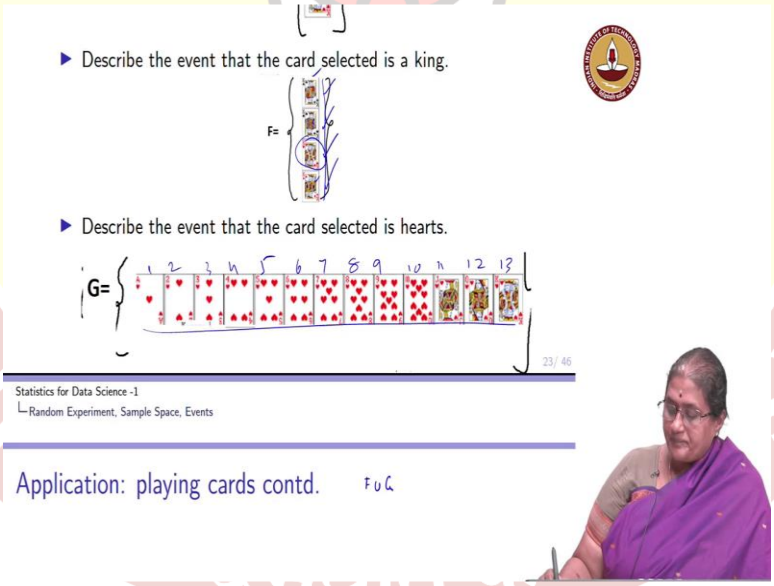
Now, let us go and look at describing different events. So, suppose my event is that the card selected is a king of hearts. So, if that is my event, I can go back and I can look at what is that event. So, if I go back here, I have 52 cards, describing an event which says that the card selected is a king of hearts, I can see that there is only one king of hearts in this entire 52 cards. So this event E which I am going to represent as the King of cards, this event has only one basic outcome and that is nothing but the king of hearts.

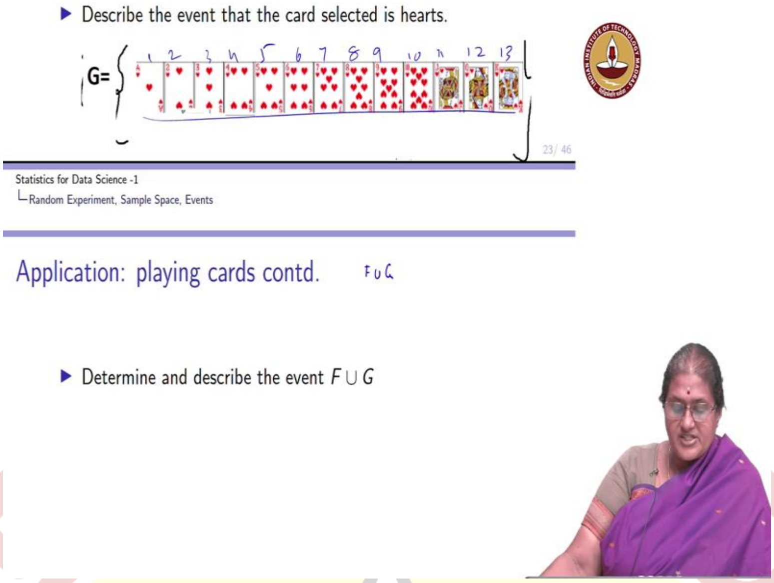
The card selected is a king again, let us go back and see that when we want to see what is the card selected is a king. Again, I go back to my earlier slide and I can see that I have king of clubs, it could be a king of spades, it could be a king of hearts or it could be a king of diamonds. So, the set of possible outcomes which are actually favouring the statement that the card selected is a king are any one of these four outcomes.

So, my event that the card selected is a king will have these four events which I have listed as king of clubs, king of spade, king of hearts and king of diamonds. So now let us look at the next event. Now the next event I am going to define is the event of the selected card selected as a hearts.

Again, let us go back to our deck of cards. You can see that the outcome which satisfies this even that the card selected is a heart can be any if I choose any one of these cards this is just ace hearts or 2 hearts, or a 3 or a 4 or 5, 6, 7, 8, 9, 10, jack hearts, queen hearts and a king hearts I have 13 outcomes, which satisfy this outcome that the card selected is a heart and I can define that or describe that event as G, which has ace, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10 11, 12, 13, these 13 outcomes are my event G. So, you can see that we can describe these events using whatever we have defined earlier. So, now let us look at set operations on this card sets.

(Refer Slide Time: 05:52)

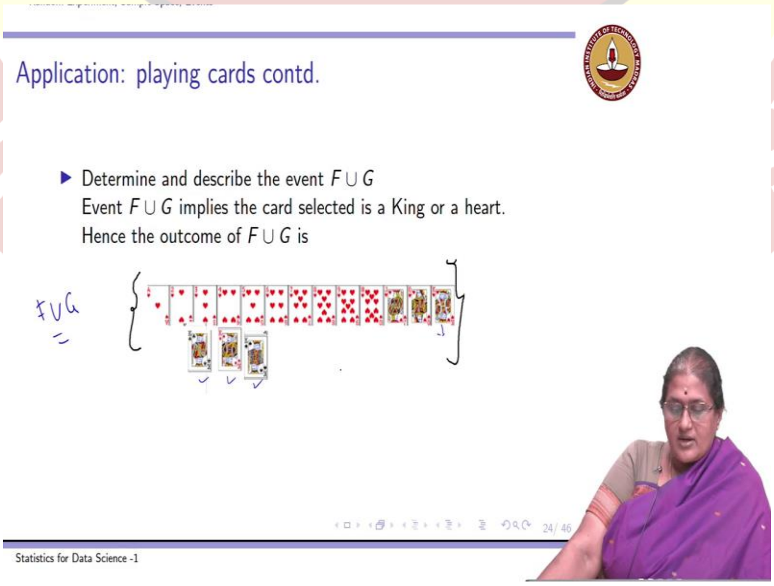




Now, suppose I want to define union of $F \cup G$, suppose I want to describe the event $F \cup G$. Now, F is the event that the card selected is a king, G is a event that the card selected is a heart. So the, by a definition the event $F \cup G$ is the event that card selected is either a heart or a king.

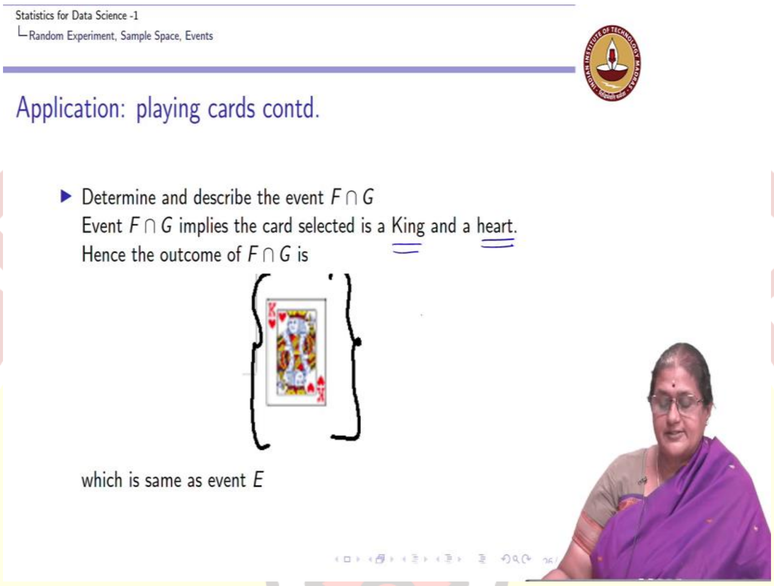
So, if you look at whatever is the, what are the cards that would satisfy it, these cards are hearts, these cards are king and I can see that this card that is king of hearts is actually common to both of them. So in addition to these 13 cards, I will have a king of spades, king of clubs and a king of diamonds, which would make 13 + 3, 16 possible outcomes in my set.

(Refer Slide Time: 07:07)



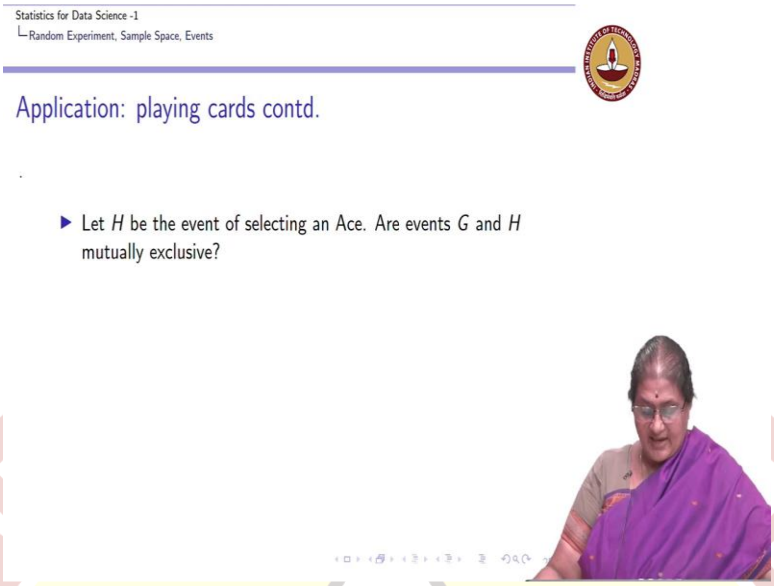
So, I can see that I can describe this event, $F \cup G$ to be this event here. Which has 16 possible outcomes, 13 which are from the hearts and three of the king, in inclusion with the king of hearts, I have these are my $F \cup G$. So, again to the deck of cards, I have applied the set operation $F \cup G$ and I have defined a new event which is my card is either a king or a heart. Now suppose I define an event that my card which I have taken is a king, and heart if that is the event I am going to define.

(Refer Slide Time: 07:57)



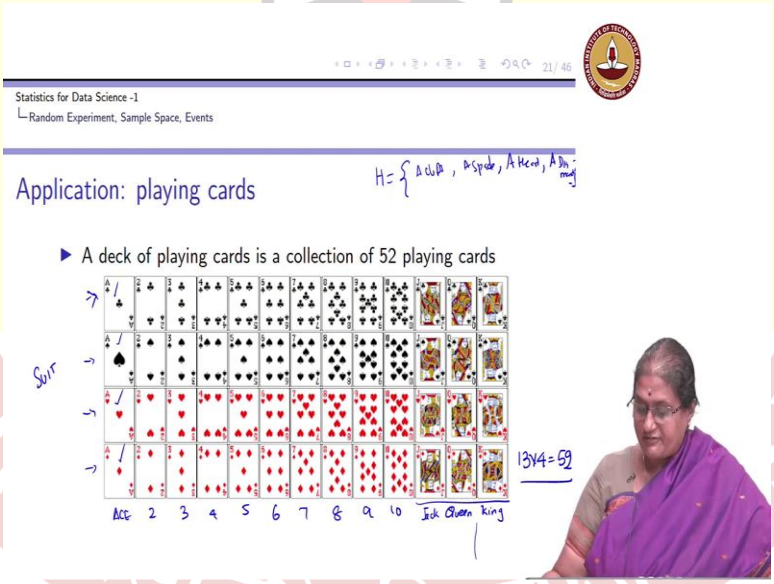
Now, suppose I have the event and I want to describe the event that my card selected as a king and a heart, recall F is a king and G is a this one, king and a heart, you can see that there is only one card that satisfies that it is both a king and a heart and you can actually notice that this is exactly the event E, where the event E was choosing a king of hearts.

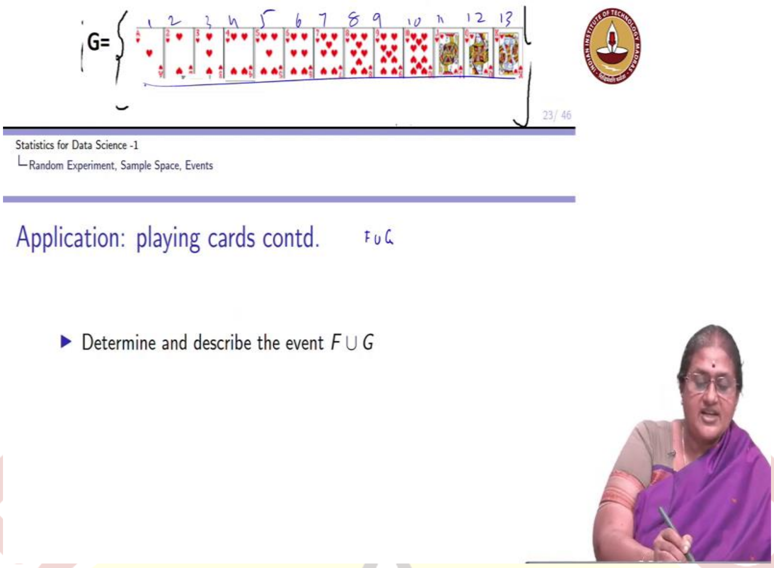
(Refer Slide Time: 08:30)

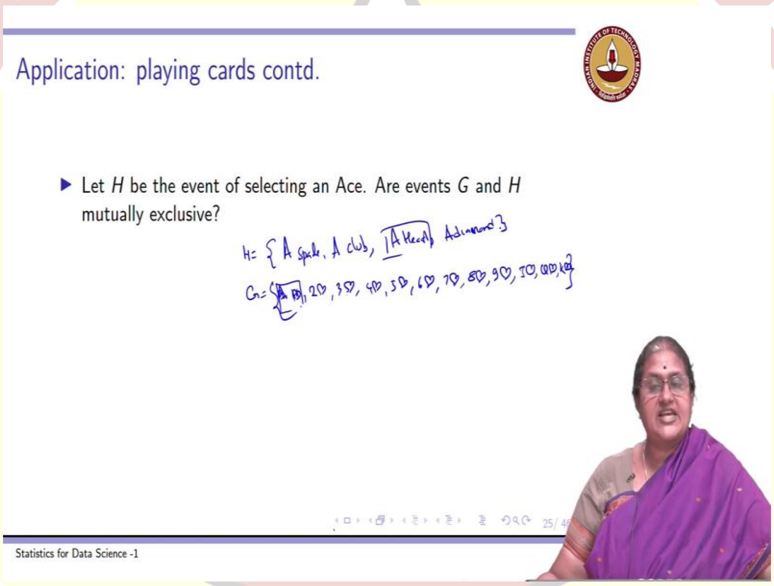


So, now let us continue with our application on playing cards. Suppose H is the event of selecting an ace. Again, let us go back to our deck of cards. So this is our deck of cards here.

(Refer Slide Time: 08:51)







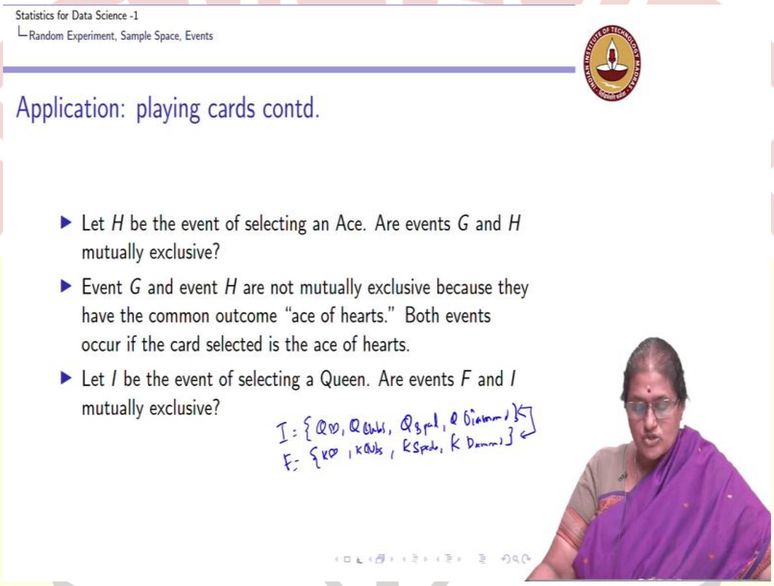
So, I have 52 cards and my event is selecting a ace. So, you can see that I have four aces, ace club ace, spade ace, heart ace and diamond ace. So, if I am selecting an ace and I am defining H to be the event of selecting an ace, it could be ace clubs or ace spade or a heart or a diamond.

Any one of these four outcomes that is my H. So if I have H is defined in that way and I want to see whether I am defining H as this is a collection of playing cards. So H, so my event H is A spade, A club, A heart and A diamond. Recall what is G? G is the event of having a heart that is how we define the event G.

If you recall G, go back to our event G, this is our event G, these are the outcomes in my event G we can see that the outcomes in my event G are a hearts, two hearts, we can see that a hearts is one of the outcomes in my event G and you can see that this a hearts is also an outcome in my, so a all heart. So, I have a heart, I have 2 hearts, I have 3 hearts, I have 4 hearts, 5 heart, 6 heart, 7 heart, 8 heart, 9 heart. So, this is a 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, jack heart, queen heart and king heart.

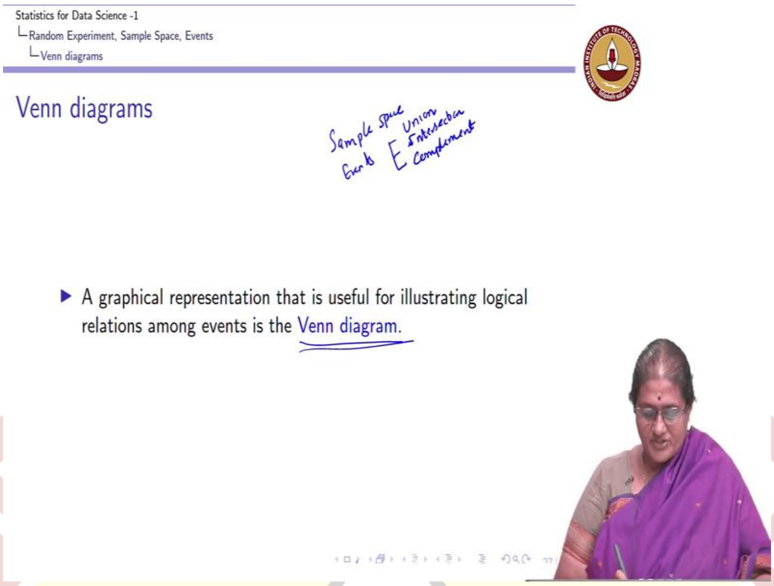
So, this a heart appears in both G and H. So, events G and H are not mutually exclusive. So, if H is the event of selecting an ace and G is an event of selecting an heart, we see that these events have H and G are not mutually exclusive.

(Refer Slide Time: 11:32)



However, G and H, if I is the event of selecting a queen again, Q is going to be queen of hearts, queen of clubs, queen of spades and queen of diamonds. My F is the event, this is event I my F is the event of king of hearts, king of clubs, king of spade and king of diamond, I see that there is no common outcome in both these events. So, the events F and I are obviously mutually exclusive that I can either select a queen card or a king card I cannot select both, a card which is both king and queen. So, the events F and I are mutually exclusive.

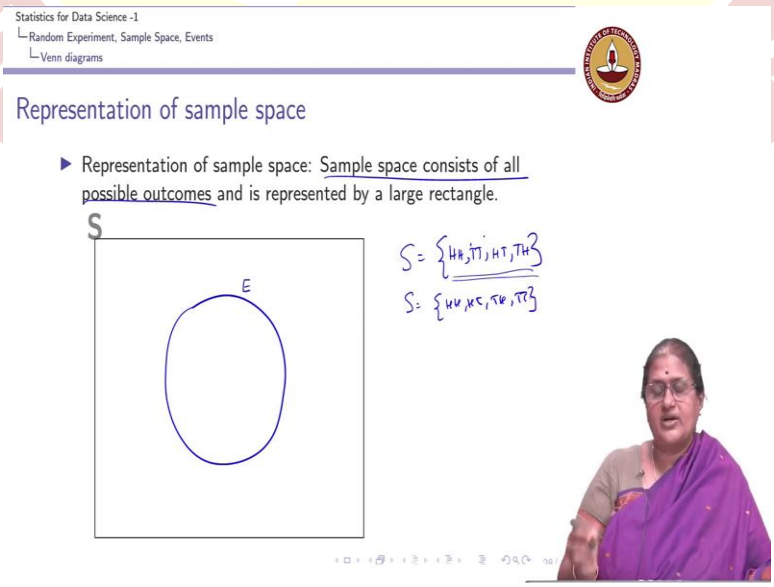
(Refer Slide Time: 12:42)

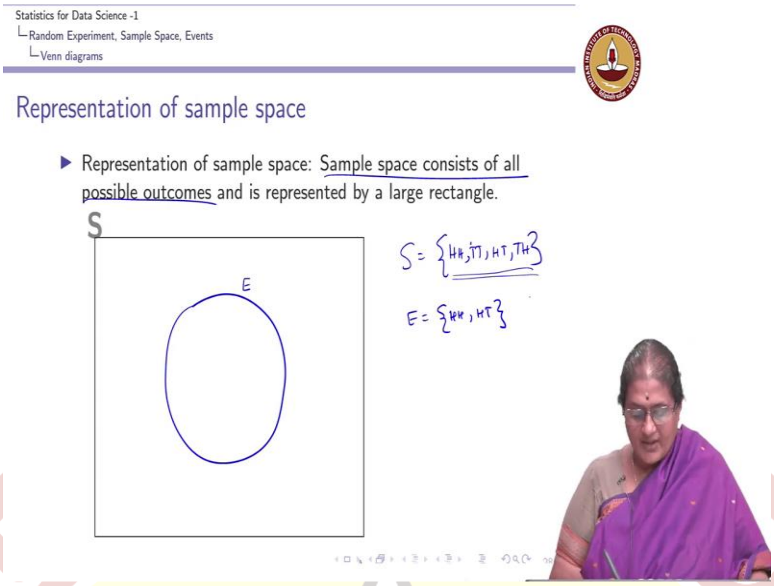


So, now we move forward to look at how to represent the events the sample space or how do we represent this in a useful way. A graphical representation that is very useful in illustrating logical relations, because we have seen what we have seen so far is, we have a sample space and we have events which are subsets of the sample space and we could define all set operations namely the union, intersection and complement on events.

So, if I can have a graphical representation, then logical relations between these events can be easily represented. Recall when we have sets, you have learned about what is called a Venn diagram. So similarly, in our probabilistic framework, we are going to represent these events and the sample spaces using our Venn diagrams.

(Refer Slide Time: 13:59)



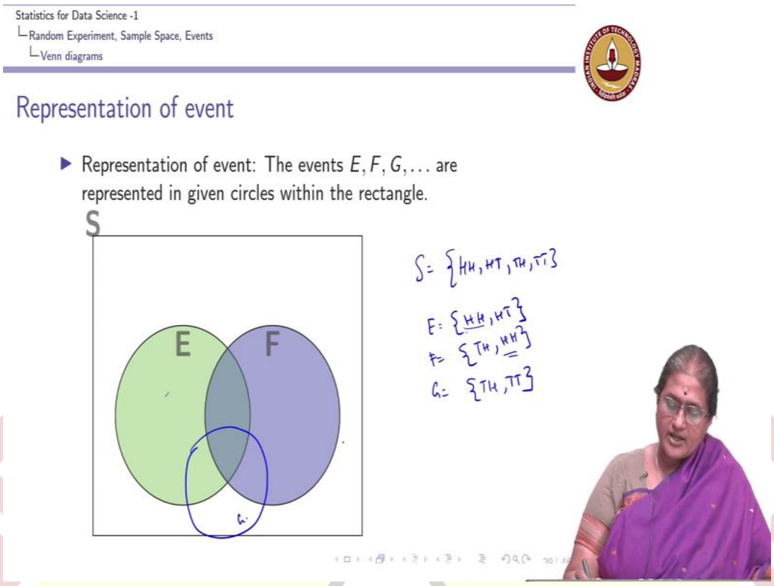


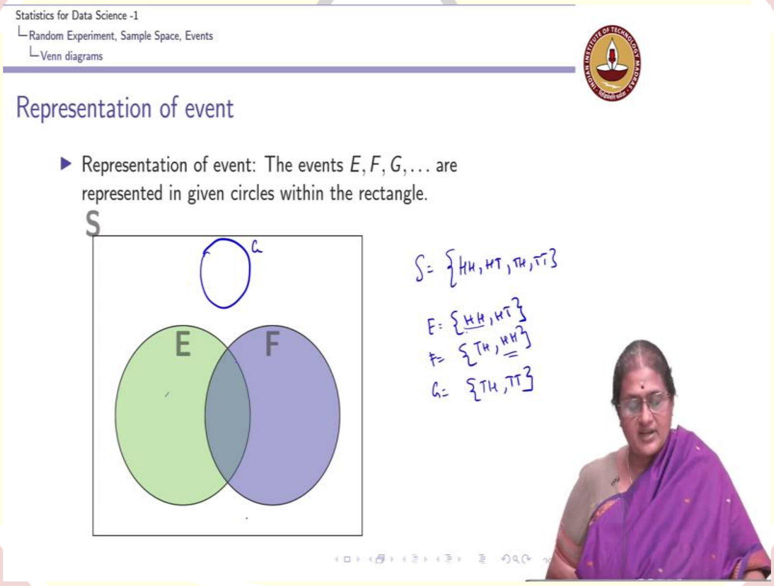
So, what is the convention? How do we represent a sample space, because sample space consists of all possible outcomes, it is just represented by a large rectangle. I am not listing all the outcomes, I am just representing the sample space by a large rectangle. Now, a event is a subset of this sample space. So, we represent a event using a circle.

For example, this is my event. So if I have a sample space, which is tossing a coin twice and the outcomes are HH, TT, HT, TH again notice the order in which I list the outcomes is not of any relevance. So, I could have, it is the same thing that if I listed as HH, HT, TH and TT this equivalent to this set.

So, the order in which I list the outcomes is not of any great importance, but they represent the sample space with the rectangle and suppose E is my event of head in my first toss. So, HH, HT are the outcomes which are in my event E, E is a subset which I represent using a circle.

(Refer Slide Time: 15:25)

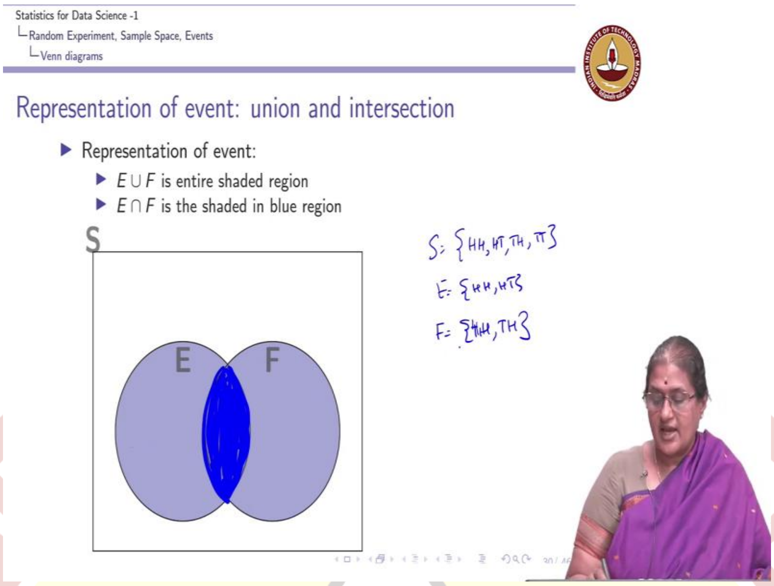


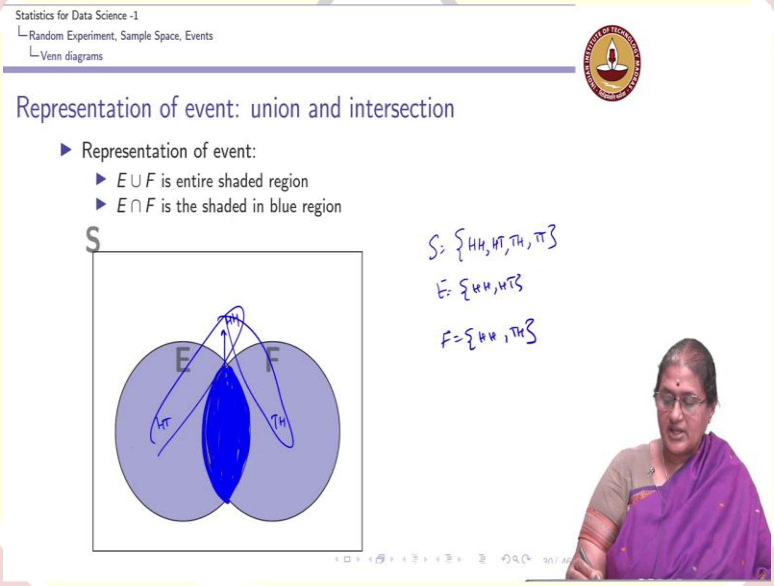


So, the first thing is we represent an event using a circle. So, I could have many events, so I have an event E, I have an event F, so I can represent, so for example, if S is my HH, HT, TH, TT that is represented by my rectangle. Event E could be head in my first toss which is HH, HT, event F could be head in the second toss which is TH, HH, what is common between these two is this outcome HH is common, I could have another event G which is tail in my first toss, which is TH and TT.

So, we can have all possible events and I could have another event G which is again listed here and you can see that there is nothing common to all the three events. So, G could be an event here, which is not common to this event E and F.

(Refer Slide Time: 16:37)



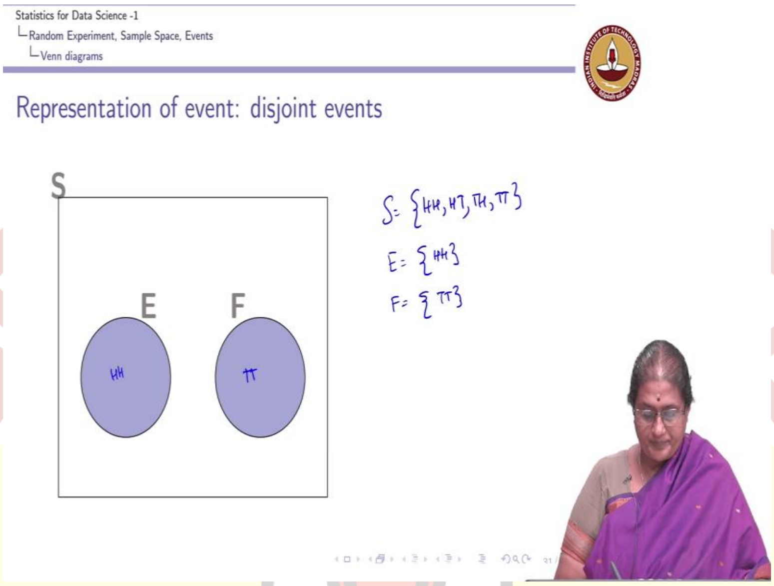


So, that is comes to us how we represent a union and intersection. So, this $E \cup F$ the entire shaded region is my $E \cup F$, whereas this region which is shaded in blue, it does not look like blue, but let me put a different colours here. So, if this is, so this region, which I am going to highlight now.

So, this region, which is in blue, fluorescent blue, is my $E \cap F$ event. So, if we are going back to our example. If my sample space is H tossing of two coins, the outcomes of tossing of two coins and my F is head in my first toss. So, this are my outcomes HH and HT, HT. F is head in my second toss.

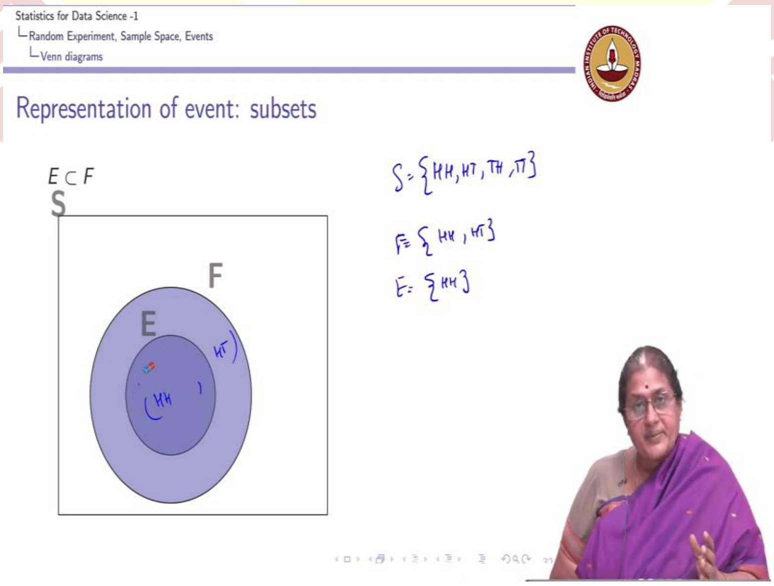
So, this has, suppose my E is head in my first toss, F is. Suppose, E is head in my first toss, let F be head in the second toss. So, you can see that the outcome HH is in my shaded region, HT would be here and TH which would be here. So, these two outcomes are in my F these two outcomes are in my T and this outcome HH is common to both E and F which will appear in the shaded region

(Refer Slide Time: 18:48)



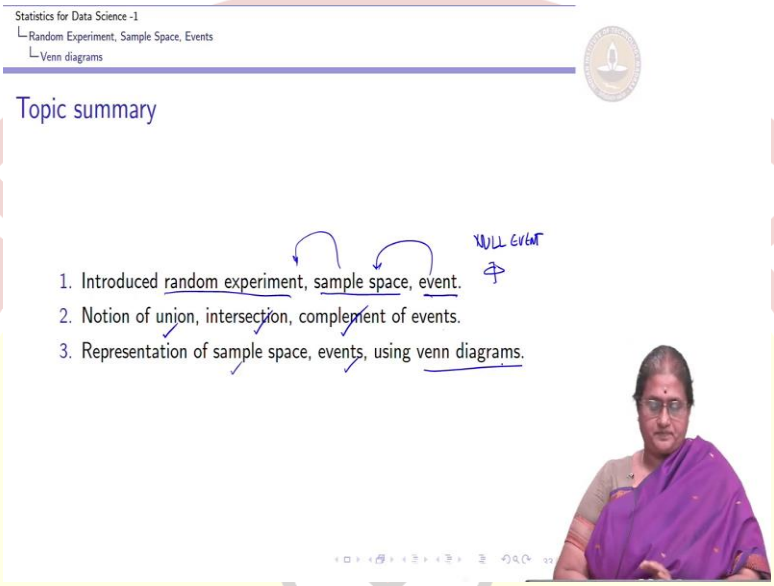
Now, if two events are disjoint, for example, again go back to our tossing a coin twice, I have HH, HT, TH, TT, I define an event E which is head in my first, head in both the tosses together it is HH. F is an event where I define tail in both the tosses. So, you can see that HH is an event here, TT is an event here, there is nothing common in these two or they are mutually exclusive that is head in both the tosses and tail in both the tosses are mutually exclusive of each other and they are disjoint events and I can represent that event in this way.

(Refer Slide Time: 19:32)



Now, suppose I again go back I have a sample space HH, HT, TH, and TT head in the first toss is my let F be in my event, head in my first toss, so that would be HH and HT. Let E be my event head in my first and second toss, so that is going to be HH, we can see that E which is only HH is a subset of F which has both HH and HT and this is a way I can visually represent subset events where E is a subset of F.

(Refer Slide Time: 20:13)



So, what we have learned in this lecture is, we have introduced the notion of a random experiment, sample space and event. Sample space is a set of all possible basic outcomes of the random experiment, event as a subset of a sample space. We also introduced a notion of what we call a null event.

Since these are sets we can introduce a notion of union intersection and complement of events. We have seen how to define these events, given a basic event and finally, we looked at how do we represent a sample space and event using Venn diagrams, because Venn diagrams is a powerful graphical representation of sets, since my sample space and events are sets, we have looked at how to represent these events using Venn diagrams. The next thing we are going to learn is how we assign probabilities to these events.