



IIT Madras
ONLINE DEGREE

Mathematics for Data Science 1
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Week 08 - Tutorial 04

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4. Given functions $f(x) = \frac{x^2 - 8x + 15}{x + 3}$, $g(y) = \sqrt{y^2 - 4}$, then answer the following questions.

(a) If the domain of $f(x)$ is $(-\infty, -m) \cup (-m, \infty)$, then find the value of m .

$f(x) = \frac{x^2 - 8x + 15}{x + 3}$

$x^2 - 8x + 15$

$x + 3$

$x + 3 = 0$

$x = -3$

$x < -3$ or $x > -3$

$(-\infty, -3) \cup (-3, \infty)$

$m = 3$

For our fourth question, we are given 2 functions a $f(x)$ and $g(y)$. And the question is asking the questions 2 questions are given a and b. And those are based on the given information. So, let us solve the first part a, it is saying that if the domain of $f(x)$ is $(-\infty, -m) \cup (-m, \infty)$, then find the value of m .

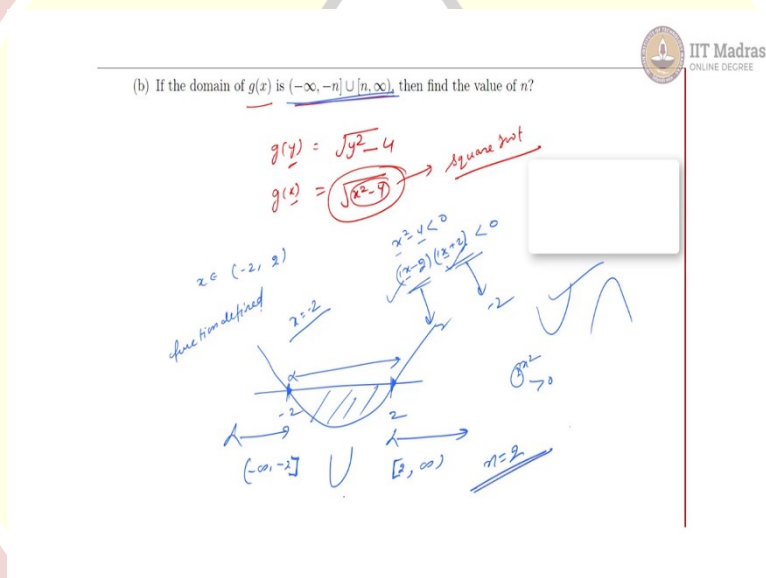
So, to find the value of m , first we will find the domain of $f(x)$. So, what $f(x)$ is given, $\frac{x^2 - 8x + 15}{x + 3}$, we will have real number the domain unless this function has any problem at any specific x . So, we will see this function in 3 parts first part, numerator, then denominator and then the whole fraction.

So, our numerator is actually $x^2 - 8x + 15$. This is our quadratic function, and we know that there is no problem it is a quadratic function and quadratic function has the domain real numbers or real numbers are the domain of quadratic function, so no problem with the numerator, the denominator has $x + 3$, is there any problem? No, if this is a function, then there is no problem with that, because it is a linear function and linear function has the domain from $-\infty$ to ∞ , where the problem is when we consider hole as a function, then we get that there could not be any function or any value which is 0 for denominator.

So, what does it mean we cannot allow a function when denominator is actually 0. So, when this is 0, $x = -3$, so the function is defined on real number except $x = -3$, what does it mean if I take the yellow line and this is 0, then our function is totally define and good till there, this will be 0 and this-3. So, till here and after here, so it is not defined at only this point, how can I write this?

So, if you consider either x is >-3 or x is <-3 , the function is defined this resembles this statement, and when we use the statement either or then it shows \cup . So, this actually is $-\infty$ to -3 and this part is actually -3 to ∞ and those will be open bracket because we are not including 3 and there those will be connected with sign \cup . And if you match these things with the given in the equation, will get that $m = 3$.

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For our second part, we have $g(x) =$ this, and if you remember the question is given $g(y) = \sqrt{y^2 - 4}$. So, first we need to find what is $g(x)$. So, $g(x)$ means what we will just replace y with x , so it will be x squared-4 and this is under square function q square root function and we properly know that the square root function is not defined when the any value or any function gives the value > 0 in square root, it means this value should not be > 0 .

So, will find when this value is > 0 , $x^2 - 4 > 0$, what does it mean? This will be open like $x-2$ and $x+2 > 0$, this is a quadratic function and this has 2 zeros and 2 zeros are -2 and 2 , this will give $+2$ and this will be -2 . Now, you know that parabola either will look like this or will look like this, it will dependent on the coefficient of x , if you multiply x with x I will get x^2 only,

there is 1 and 1. So, 1 and this is < 0 , which means open upward will be the correct representation of this parabola and when it is open upward this will be the representation.

Now, you can see where the value of function is negative this part and what is this part this is this. So, if x value is from -2 to 2, then the function will provide negative value and we cannot get negative values, other thing that what will happen if $x = -2$, then the function is giving value 0 and if it is 0, is 0 acceptable in square root? Yes. So, we can take these 2 critical points here -2 and 2 which means the function is not defined when x belongs to -2 to 2 in open upward.

How can I write in another way that the function is defined for this reason and then this reason, what is this reason, this is $-\infty$ to -2, as 2 is included open bracket here, as 2 is included I will close bracket as 2 is included. So, this is closed bracket and this is ∞ and how will I enter the join them using \cup . So, this is given here in the question, and if you see properly then $n = 2$ is our answer.

