IIT Madras

ONLINE DEGREE

Mathematics for Data Science 1
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Indian Institute of Technology, Madras
Lecture 48
Composite Functions

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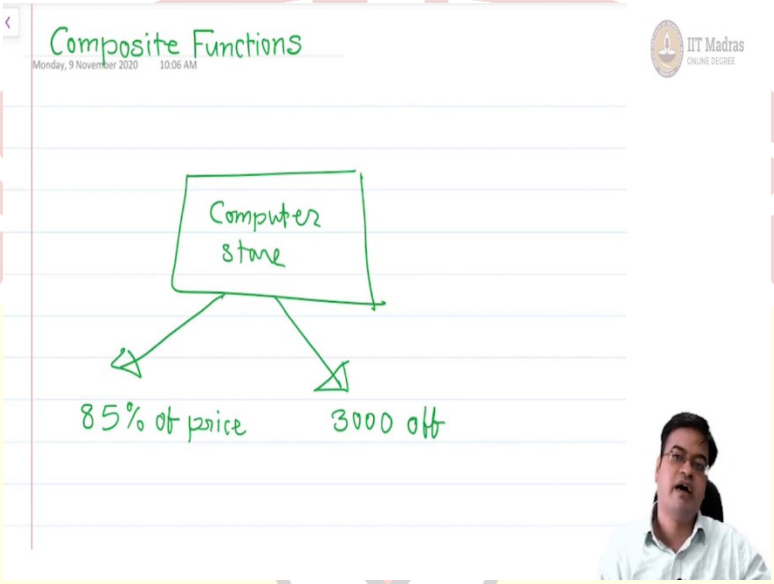

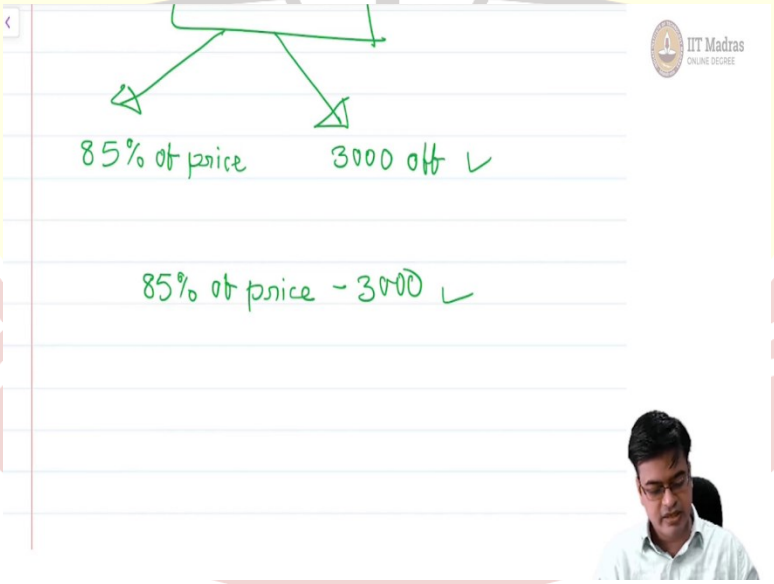

Hello students, today we are going to learn the concept of composite functions, what do you mean by a composite function? So, let me motivate this with an example. For example, it is known that you are a very good bargainer and your friend wants to buy a computer. So, your friend takes you to a computer store, so this is a computer store, in this computer store there are two offers available.

So, something is on sale, all items are on sale and there are two offers available, one offer is you will get 85% of the price, whatever you buy you will get the product at 85% of the price. And the other offer is you will get flat 3000 off on the MRP, the maximum retail price you will get 3000 off. So, these are the two offers that are available.

Obviously because you are a good bargainer, you bargain with a salesperson and you strike a deal that is the computer that you want to buy will be given to you at 85% of the price and of the amount, once the 85% of the price is decided further 3000 will be given you as a discount. So, there is a discount of rupees 3000 as well as you are getting 85% of the price.

Now, this kind of thing when we write mathematically can be considered as composite functions, you are in fact using these kind of tricks in a day-to-day life. So, let us see what happens when we put this mathematically and how composite functions arise. So, let us say the first draft that is 85% of the price. So, can I represent this as a function?

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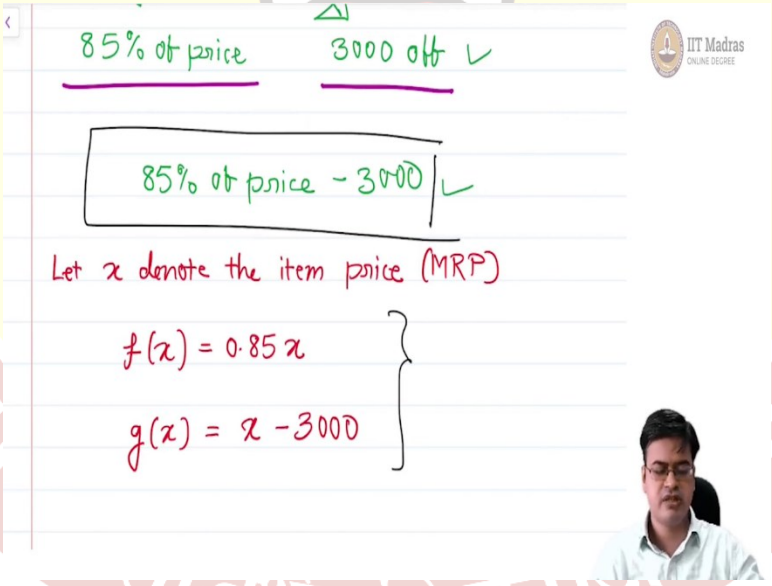

So, for cleanliness let us write let *x*denote the item price. So, let *x*denote the item price which is the MRP, you can write maximum retail price and on that you are getting 15% discount that is 85% of the price you are getting. So, I can write this particular offer as *f(x)* which is nothing but 0.85 times *x*.

Now, the other offer that is on in this particular computer store is this. So, I can write this as g of $x$to be equal to if $x$is the MRP I will subtract 3000 rupees from x, so these are the two offers that are available. Now, what we did is we want best of both the offers. Now, when a store is offering these two offers it is safe to assume that you may not have any item that is less than 3000 rupees, you may not have any item on sale which is less than 3000 rupees, so your $x$will always be greater than 3000.

Another thing that you can assume that because the store is offering you this kind of thing, that is 85% of the price -3000, the store has already taken care of that they do not have to pay back any money, that means after giving the 85% of the price, the price should be greater than 3000, so all these conditions are assumed implicitly, which we will deal with them in later when we will formulate a problem.

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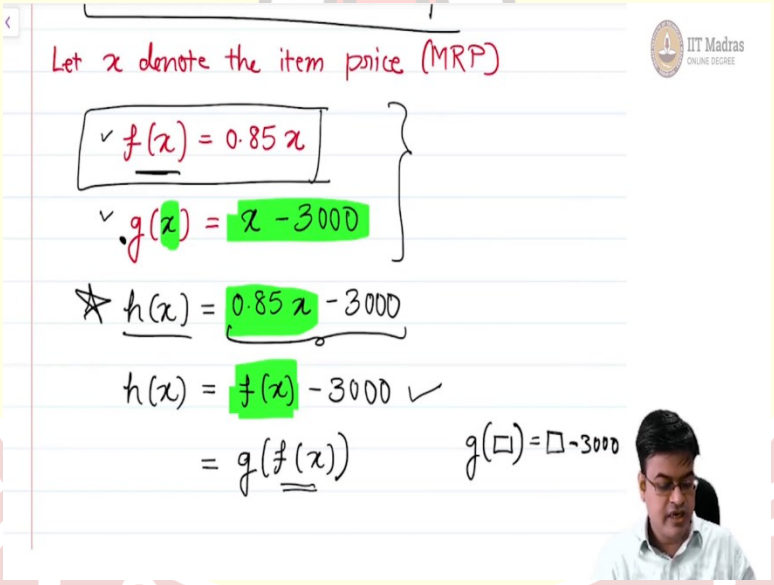

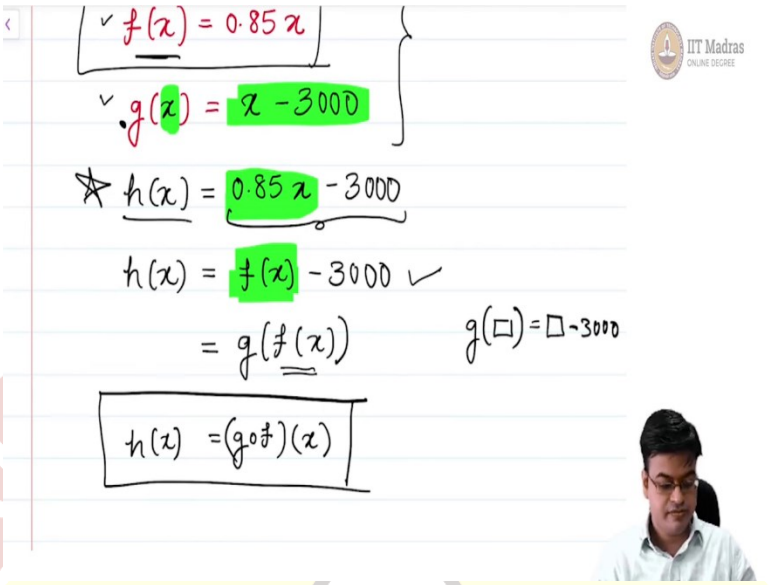

So, now the offer that you got if I want to write this offer mathematically I can write this as some function $h(x)$ which is equal to it is 85% of the price -3000. So, now when we are dealing with functions in mathematics it is good to see if I have some correspondence of the function h with these functions *f* and *g*, this is the question that we are trying to answer when we are studying composite functions.

So, let us first see what is being done over here, that is if I use this *f* then it is 0.85 *x*, so if I want to do something like this then I can write this as *h(x)* is equal to *f(x)*-3000 is that a safe assumption to do? Yes, of course because *f (x)* is 0.85 x so what I am essentially doing is, I am for this particular term I am substituting *f (x),* so it is a perfectly valid guess, fine.

Now, if you treat this *f*, if you treat this *f (x)* as one argument like $x$then what you are actually doing, you are actually saying it is $x$minus 3000 that means instead of this x had it been *f (x)* you would have written *f (x)* -3000. So, I will use that knowledge and I will try to do, I will try to rewrite this as, this is g times f(x). Is this acceptable? Let us redo the math.

For example, what is g times *f(x)*? So, if you look at g of, f of, *g(x)*, so whatever is x you will write that $x-$ 3000 or whatever, let me put it this way if g had some box inside it then I will write that box -3000. So, in particular, in that box right now *f(x)* is written, so I will substitute it as *f (x)* minus 3000, done?

And what is *f(x)?* Now, *f(x)* as you know is nothing but $0.85\times x$. Therefore, I can rewrite this function as *g(f(x)).* In mathematics you will rewrite this as *g(f(x))*, so my *h(x)* can also be written in terms of g and f in this fashion. So, this is the motivation for composition of two functions. So, in particular what we have seen is a practical example, we motivated it through a practical example of a computer store which is offering two kinds of sales, one is 85% of the price, another one is flat 3000 off on the MRP. So, after doing this you can easily guess that how will, how will I evaluate this function, how will I evaluate this function, that is what we have to see.

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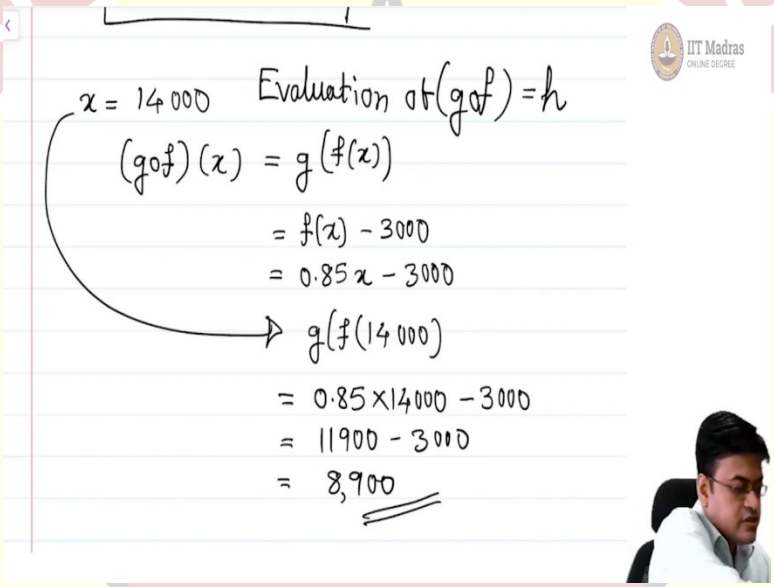

So, in particular let us say your *x*in this particular function is say you can take it to be 14000 let us say, 14000 is your *x*and you are asked to calculate *g o f(x).* So, how will you calculate? It is very simple, you will first insert g(f(x)). So, what is f(x)? f of *x* is nothing but point okay, let us follow the same notion the way we followed, so in particular in this case this is what will happen, this is going to be equal to $f(x) - 3000$.

What is $f(x)$? *f(x)* is going to be $0.85 \times (x - 3000)$, so I will substitute the value 14000 over here which will give me, so since my *x* is 14000 I will plug this value in, so I am calculating g of f of 14000. What will be g of f of 14000? Again you have to do a similar calculation which will give me 0.85 multiplied with 14000 -3000 so this I think comes out to be 11900 just check if I am calculating it correctly -3000 which will give me 8900, 3000, 900 as it is 11 -3, 8, yes, so the final answer is 8900, this is what, this is actually, what I have just now shown is evaluation of a

composite function which is *g o f,* what is, which is actually *h*, there is nothing special in this, it is just a nomenclature that we are using.

But this kind of composition helps you in understanding lot of things. So, let me formally define what is the composition of a function and how we are going to handle them mathematically. Because composition of a function as you must have seen is again a function. So, natural questions about domain, range will arise and we will try to answer them as and when they come.

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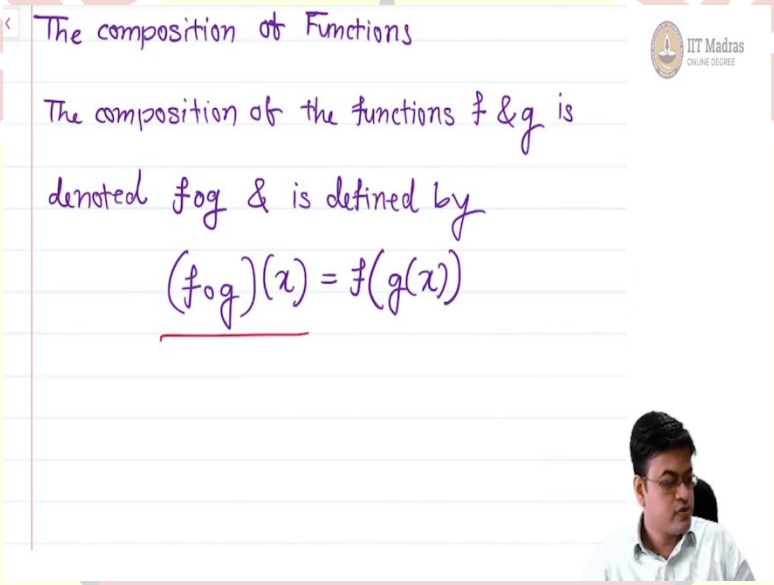

So, let me formally define the composition of functions. What is the composition of function? So, in particular we can write as the composition of functions f and g composition of the functions, there are two, at least two functions you need, functions f and g or we can write the composition of the function *f* with g that is also a valid terminology is denoted by, I have already defined this notation *f o g* and is defined by *f o g,* this is one function of *x*, so you can write this as *f(g(x))*.

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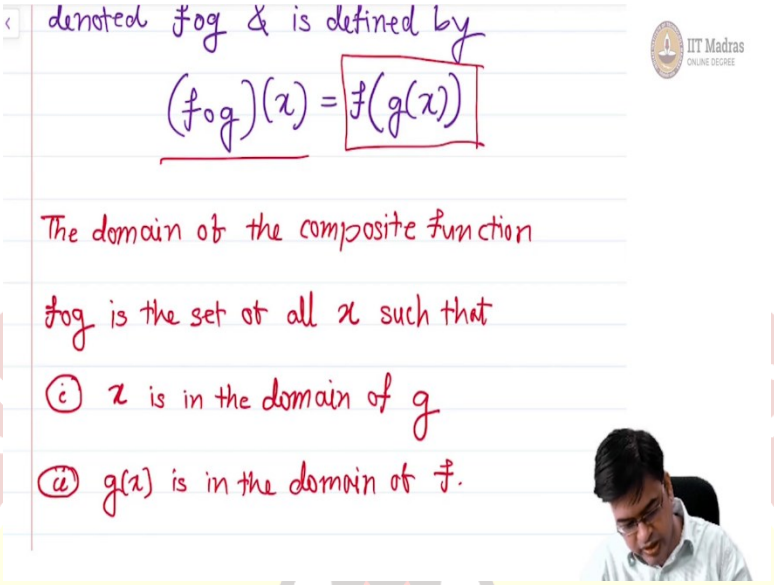

So, naturally the next question is what should be the domain of this function, so that we will answer as the domain of the composite function *f o g*, let me write it here, *f o g* is actually the set of, is the set of all *x*such that the two conditions we require and they are pretty evident, as we go further we will realize how these two conditions are evident.

So, the first condition is *x* is in the domain of g and second condition is it will be about *x*so if g *x*is something that you are figuring out. Now, that *g(x)* should be in the domain of *f, g(x)* is in the domain of *f*. So, now why these two conditions are required that is what we need to figure out. For that you need to focus on this particular component *f (g(x))*. Let us use this particular component and try to answer the question.

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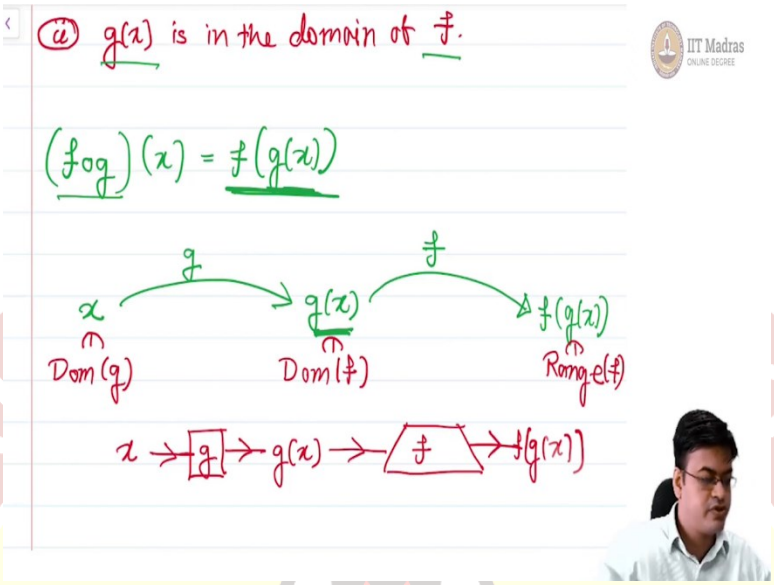

So, I have, if when I talk about *f o g(x),* what I am talking about is *f(g(x)).* Now, let us look at the first condition. If I want something to be in the domain of *f o g* that means it should be well defined, so when I input the value it should give me the output, if there is some ambiguity then it is not a properly defined function. So, let us say why this condition x is in the domain of g.

What if x is not in the domain of g? g of x is not defined, because g is defined only over domain of g, so g of $x$is not defined and therefore you need this condition that $x$should be in the domain of g. Now, when I am using this composite function, I am applying f to the value that is obtained by applying g, so it is g of $x$that is playing the part.

So, now if this g of x that is the value of $x$which is in the domain of g if that particular value g of $x$is not in the domain of f then again this f(g(x)) is not defined. Therefore, I need g of $x$also to be in the domain of f. So, in particular you can visualize it this way. So, if I have $x$then there is a map which maps everything that map is g and that maps it to a value called g of x.

Now, this *g(x)* should be in the domain of f because I will take this value to a function which is f(g(x)). So, this is another value and what is the application? *f* is the application, we are applying the function f to the value *g(x),* if this $g(x)$ is not, does not belong to domain of f then our function is not defined.

So, you can actually remember this diagram by using this particular, this belongs to, what it belongs to? It belongs to domain of g, this particular thing actually belongs to domain of f, this is my abbreviation for domain and this is nothing but the range of f, so this will be in the range of f but it can be smaller than the range of f because $g(x)$ may not cover the entire domain of f.

So, it can be smaller but this will belong to range of f or if you want to visualize it in a better manner there is something which is box, you feed an input to this box x, g is this box and it will throw out $g(x)$, so when you feed x, this will spit out $g(x)$. Now, for $g(x)$ to be fed into another let us say this is a trapezium and that trapezium is a function f, this is a machine, function machine that is $g(x)$.

If this $g(x)$ is not in the domain of f, then this machine is unable to produce the output, so I need this $g(x)$ to be in the domain of f in order to get the output which is $f(g(x))$. So, this is how you can always remember how to compute the composition of two functions and what are the necessary steps that are required to compute this composition.