



**IIT Madras**  
ONLINE DEGREE

## Mathematics for Data Science 1

### Week 06 - Tutorial 07

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7. A company is planning to produce a product  $A$  through three available processes. Cost of production through 1<sup>st</sup>, 2<sup>nd</sup> and 3<sup>rd</sup> processes are  $M_1(x) = 100x^3 + 20x^2 + 10$ ,  $M_2(x) = 20x^4 + 10x^2 - 20$  and  $M_3(x) = x^3 + 20$  and the waste management cost for each of the processes are  $W_1(x) = 0.01x^2 - 0.008x$ ,  $W_2(x) = 0.01x^4 - 0.001x^3 + 0.001x^2$  and  $W_3(x) = 0.01x^2$  respectively, where  $x$  is the cost of raw material per kg.
- What will be the effective manufacturing cost  $E_1(x)$ ,  $E_2(x)$ ,  $E_3(x)$  for each of the processes?
  - What will be the ratio of effective manufacturing cost of 1<sup>st</sup> and 3<sup>rd</sup> process when the cost of raw material per kg is ₹ 1?
  - Which of the processes among  $M_1$ ,  $M_2$ , and  $M_3$  should the company choose when the cost of raw material per Kg is ₹ 10.

$$E_1(x) = M_1(x) + W_1(x) = 100x^3 + 20 \cdot 0.01x^2 - 0.008x + 10$$

$$E_2(x) = M_2(x) + W_2(x) = 20 \cdot 0.01x^4 - 0.001x^3 + 10 \cdot 0.001x^2 - 20$$

$$E_3(x) = M_3(x) + W_3(x) = x^3 + 0.01x^2 + 20$$

In this question, we have a company which is producing a product  $A$  through 3 processes and the cost of production are given as  $M_1(x)$ ,  $M_2(x)$  and  $M_3(x)$ . These are the 3 cost of production. They have given us polynomials of  $x$ . So, what is  $x$ ?  $x$  is the cost of raw material per kilo. And now they are also giving us the waste management cost as  $W_1(x)$ ,  $W_2(x)$  and  $W_3(x)$ . What will be the effective manufacturing cost?

So, effective manufacturing cost simply has to be the sum of these. So,  $E_1(x) = M_1(x) + W_1(x)$ , so that is going to be so,  $M_1$  is here,  $W_1$  is here. So,  $M_1$  would be  $100x^3$  and there is no  $x^3$  term in  $W_1$ , so we first write down  $100x^3$ , then there is an  $x^2$  term here,  $20x^2$ , there is also an  $x^2$  term here,  $0.01$ . So, their sum will give us  $20.01x^2$  and then there is no  $x$  term in  $M_1$ . There is an  $x$  term here, so you get  $-0.008x$ . So, this is also done. And then lastly we have the constant term which is  $+10$ . So, this is the effective manufacturing cost for process 1.

Likewise, process 2 would be  $M_2(x) + W_2(x)$ . So, here this is  $M_2(x)$ , this starts with an  $x^4$  and  $W_2$  also has an  $x^4$  term. So, we have  $20.01x^4$  plus there is no  $x^3$  term in  $M_2$ , there is an  $x^3$  term here, so there is no, this is not plus, we write  $-0.001x^3$ , then the  $x^2$  term there is a  $10x^2$  here and a  $0.01x^2$  here. So,  $+10.001x^2$  and lastly there is a constant term and no constant term over there. So, this is  $-20$ . So, this is the  $E_2$ .

And then  $E_3$  is going to be  $M_3(x) + W_3(x)$  and in  $M_3$  there is just 2 terms, the  $x^3$  terms and a constant.  $W_3$ , there is only one term which is the  $x^2$  term. So, we just write all of them together,

$x^3 + 0.01x^2 + 20$ . So, this is the effective manufacturing cost for the third process and the three of them are given here.

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Handwritten notes showing the derivation of effective manufacturing costs:

$$E_1(x) = M_1(x) + W_1(x) = 100x^3 + 20.01x^2 - 0.008x + 10$$

$$E_2(x) = M_2(x) + W_2(x) = 20.01x^4 - 0.001x^3 + 10.001x^2 - 20$$

$$E_3(x) = M_3(x) + W_3(x) = x^3 + 0.01x^2 + 20$$

Calculations for  $x=1$  and  $x=10$ :

For  $x=1$ :

$$E_1(1) : E_3(1)$$

$$[100 + 20.01 - 0.008 + 10] : [1 + 0.01 + 20]$$

$$[130.002] : [21.01]$$

$$\frac{130.002}{21.01} \approx 6.18762$$

For  $x=10$ :

$$E_1(10) = 100000 + 2001 - 0.08 + 10 = 100010.92$$

$$E_2(10) = 200100$$

$$E_3(10) = 1000 + 1 + 20 = 1021$$

Comparison:  $E_2(10) > E_1(10)$  and  $E_3(10)$  is the minimum.

Now, what is the ratio of effective manufacturing cost of first and third processes when the cost of raw material per kg is rupees 1? So, basically we are looking for the ratio  $E_1(1):E_3(1)$  which is then we just substitute 1 in the  $E_1$  term, so we get  $[100 + 20.01 - 0.008 + 10]:[1 + 0.01 + 20]$  is to, so we get 130.002 is to 21.01. So, we have to put this down as a number 130.002 divided by 21.01 is roughly 6.18762.

Then we have the third question, third part of this question which says, which asks which of the processes  $M_1$ ,  $M_2$  and  $M_3$  should the company chose when the cost of raw material per kg is 10? So, the company should chose the cheapest process. So, we have to find out  $E_1(10)$ ,  $E_2(10)$  and  $E_3(10)$ . And then if we looked at that  $E_1(10) = 100x^3$  is 1 lakh plus  $20.01x^2$  is 2001  $- 0.08 + 10$ . This is then 102010.92.

Moving on then  $E_2(10) = 200100$ . So,  $x^4 = 10^4$  so, this is what we get and this is 200100, so  $x^4 = 10^4$ , so this is what we get and this is 200100. We see that the remaining smaller terms,  $x^3$ ,  $x^2$  and constant term have small coefficients as well, 0.001 and 10.001. So, they will not really impact the value very much. So, we know that this is already larger than  $E_1(10)$ , so we do not consider it.

Let us look at  $E_3(10)$  which is then  $1000 + 1 + 20$ , so this is just 1021 rupees. So,  $E_3(10)$  is the least which is why the company should chose the third process as their process when  $x$  is equal to 10 rupees per kilo.