IIT Madras

ONLINE DEGREE

Mathematics for Data Science 1

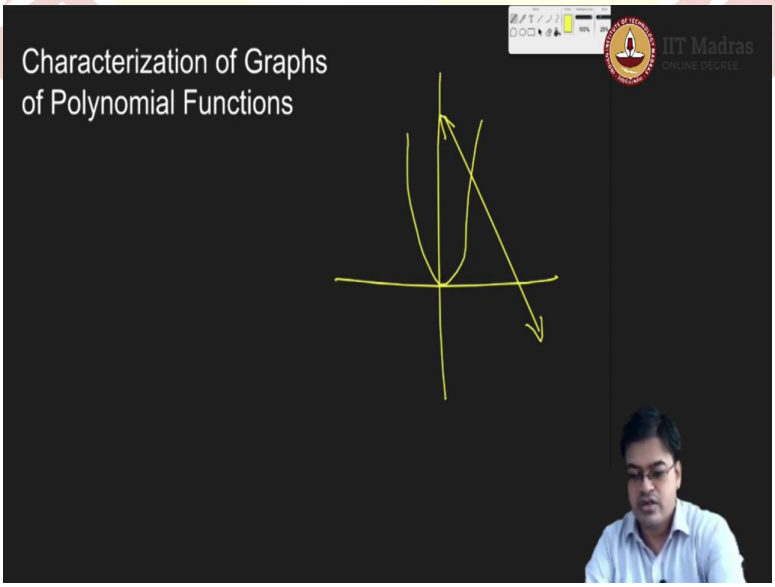
Prof. Neelesh S Upadhye
Department of Mathematics
Indian Institute of Technology, Madras

Lecture – 36
Graphs of Polynomials: Identification and Characterization

Hello friends, in this video, we will take up our next mission of about understanding the polynomials. This mission is given a graph of a function, whether we can identify the given function is a polynomial or not. If you have been given a polynomial equation, how will you put it on a graph paper?

So, the mission is twofold. First, If you have been given a graph of a function, you will identify whether this function is a polynomial or not. If yes, the, we will answer the second question that is can I derive the algebraic equation of this polynomial? The second part of the mission is we want to identify how the graph looks like if I have the equation of the polynomial. So, let us begin our journey about understanding the Graphs of the Polynomials.

(Refer Slide Time: 01:17)

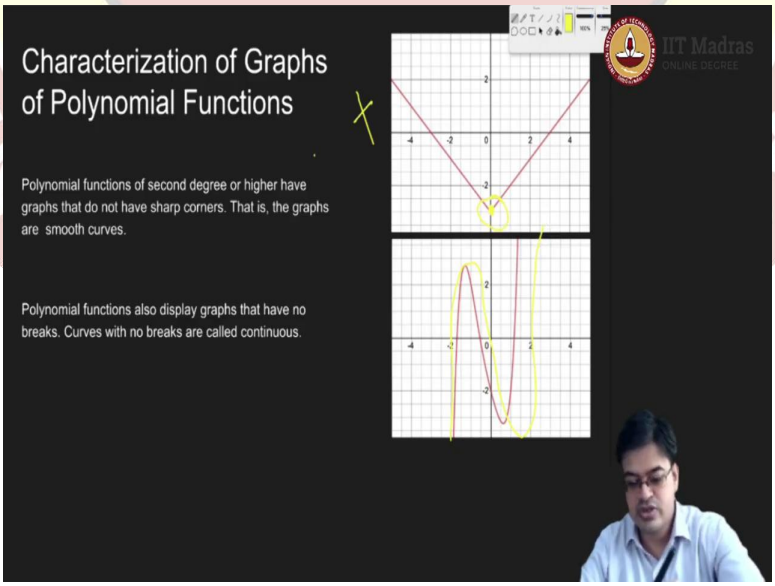


So, first of all let us recollect from our earlier experience that is linear functions and quadratic functions. If I am as linear functions and quadratic functions themselves are graphs of the functions. So, when I am plotting these two functions or when I am putting them on the graph paper, what happens? There you will never feel any abrupt jerk while drawing these functions. If you are trying to draw, you, for example, if you are trying to draw a line, then what you will do is you will simply draw a line, and then on graph paper. And there would not be any jerk for drawing the line.

In a similar manner if you are asked to plot a quadratic curve, you will find a axis of symmetry, and around the axis of symmetry you will do something like this, this is let us say this is the graph right, that means, the curve that you are trying to draw is has always been smooth. So, that one feature we can record in our mind. And say that the, if I have been given a polynomial function, the polynomial function must be smooth that means, I should be able to join the points effortlessly without having any jerk.

If there is any corner or edge in the graph then it better not be a polynomial function. Another thing is you can draw these graphs without lifting your pen; you can draw these graphs without lifting your pen that means, these graphs always are continuous.

(Refer Slide Time: 03:04)



So, let us try to list these properties; first if you have polynomial of second degree or higher even a linear degree, the graphs do not have sharp corners that is the graphs are

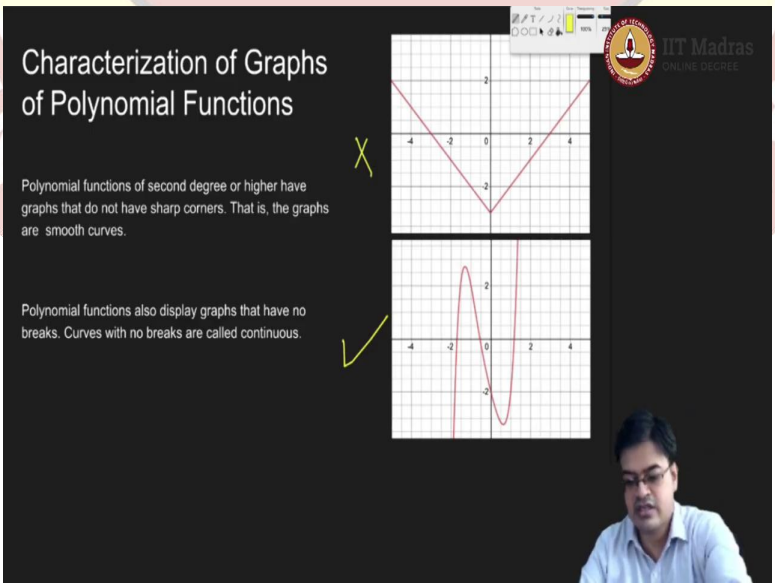
always smooth curves, this is a first feature that we will notice if I have been presented with graph of a function.

Second thing is polynomial functions always display graphs that have no breaks that is what I meant; the graph that I am drawing is always going to be continuous curve. Or you can say in better words, it is curves with no breaks are called continuous, and therefore the function itself will be continuous.

So, let us identify through two graphs. Let us have this graph. Now, is this a polynomial function? Does it satisfy the first criteria? That is it should be a smooth curve, is it a smooth curve? Yes, it is a smooth curve. But if you look at this point, then it had some sharp corner over here, this corner is very sharp. And therefore, I cannot qualify this as polynomial function; this is not a polynomial function.

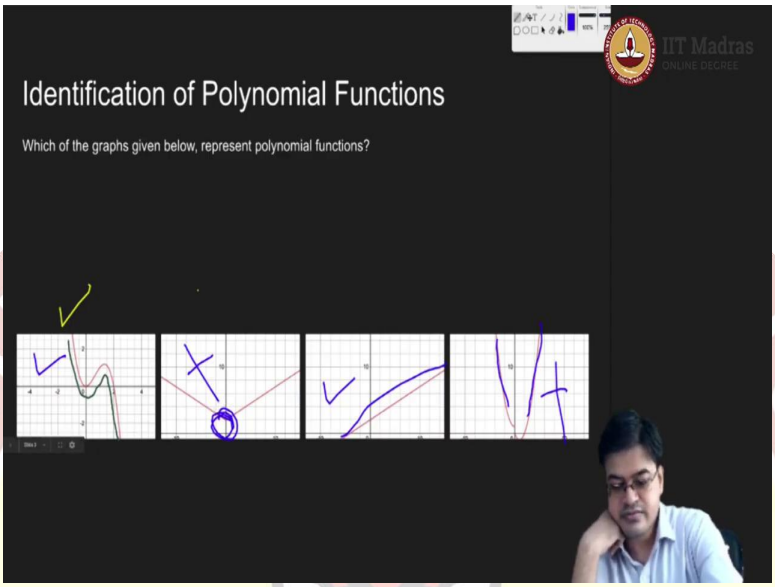
Let us have a look at the next graph which is this. Now, here I can use my free hand skill to draw a curve and I can actually find out how I can draw better curve. For example, if I start drawing this curve, then I can easily pass through this. So, you will all the transitions are very smooth, because the transition is very smooth I can easily identify this to be a polynomial function.

(Refer Slide Time: 05:06)



Therefore, this qualifies to be a polynomial function, whereas this does not qualified to be a polynomial function. Let us take a quick look at some other graphs and see whether they will qualify as polynomial function or not.

(Refer Slide Time: 05:22)



So, let us go ahead. And pose a question, which of the graphs given below represent polynomial functions? So, one by one I will unfold the graph, and we will argue for whether they are polynomial functions or not. This is the first graph. As I mentioned earlier, this also qualifies to be a polynomial function.

For example, if I want to draw a curve across this, I can easily draw a curve without lifting my pen and therefore, it qualifies to be continuous, and it has no breaks in between. So, therefore, it is continuous and it does not have any sharp edges, and the graph seems to be free hand. Therefore, it qualifies to be a polynomial function. So, my answer to this question is yes, this is a polynomial function ok.

Let us go ahead with the next graph ok. Now, what about this graph? Of course, we have argued for say similar graph in the earlier page that this graph does not seem to be a very neat graph, and it has a corner over here. This is the corner point of the graph. And therefore, this disqualifies to be a polynomial function; this is not a polynomial function. Again let me reiterate this was a valid polynomial function.

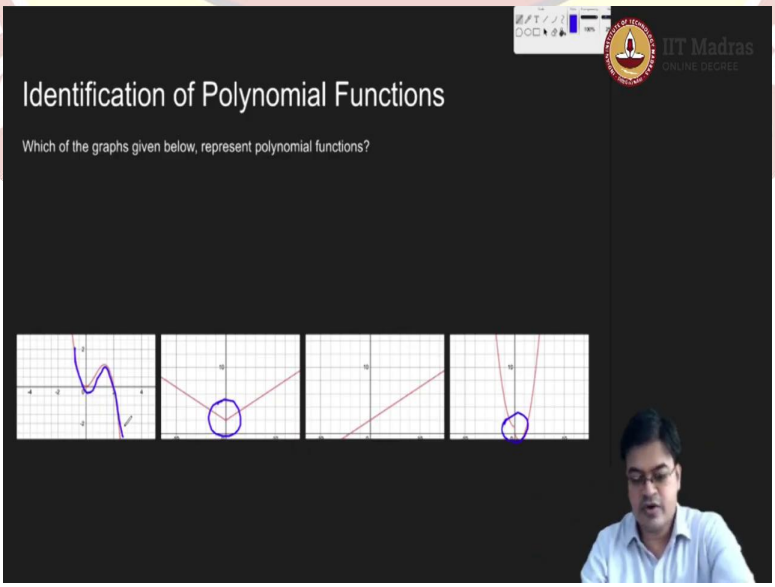
Next graph, let us look, let us try to see the next graph. This graph more or less seems to be a graph of a line, because it is a graph of a line. You can see this is also smooth. The transition is very smooth. So, again this will qualify as a polynomial function at least as far as the graph is visible. This is a graph of a line and it is a linear polynomial. So, it qualifies to be a polynomial function.

Let us go to the next graph ok. So, this graph is actually smooth. I can draw a curve over here, but at this point let me erase this graph; actually, you do it here. At this point, at this juncture, there is some problem. What is the problem? Over here, if I am drawing a curve over here, then I have to lift my pen come to a point 0 and then start drawing it.

So, this defeats the criteria that the graph should be continuous. Though the curves are very smooth, but at this point, this in this juncture, the problem is you cannot have a drawing without lifting your pen. Therefore, this will disqualify to be a polynomial function. This is not a polynomial function.

So, we have identified what is a polynomial function and what is not a polynomial function. Generally, whenever you have several ups and downs in the functions, we will estimate them or we will guess them to be a polynomial function if there is no corner as given in this graph, second graph to be precise.

(Refer Slide Time: 09:15)



And if there is no there should not be any corner of this kind and there should not be any discontinuity of this kind, ok. Then we can easily safely say that the given function is a polynomial function ok.

And if you are looking at the ups and downs, these up and down of a function, those are the typical features of polynomial functions. With this knowledge, we are ready to handle polynomial functions because now, if you have been given a function on a graph paper you can identify whether the given function is a polynomial function or not. The next important criteria for about polynomial functions like quadratic equations is our identity our ability to identify zeros; zeros of the function.