IIT Madras

ONLINE DEGREE

Mathematics for Data Science 1
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Week 9 Tutorial 5

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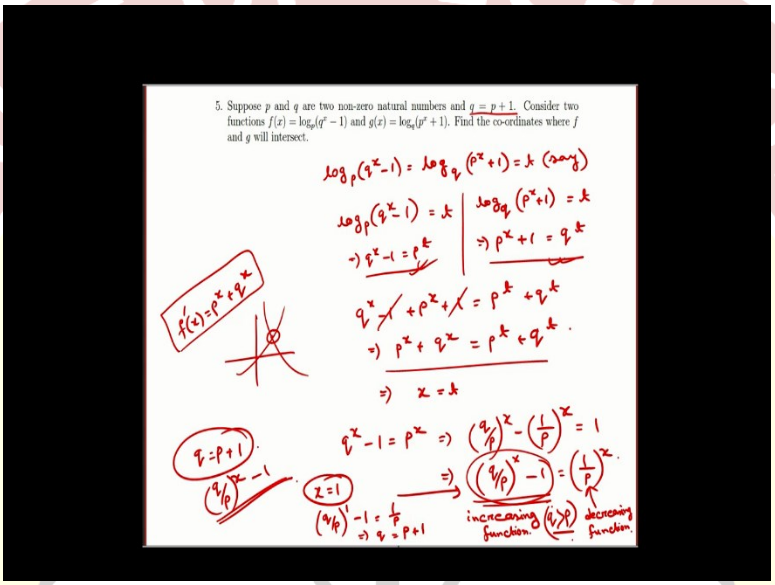

So, let us see what the fifth question is, so in the fifth question we are saying that p and q are two non-zero natural number together it a condition, $q = p + 1$,. And we are given two function $f(x)$ and $g(x)$ and we have to find a coordinates where f and g will intersect. So, f and g will intersect means at that point $f(x) = g(x)$. So, we have to find the solution of that equation.

So, let us write the equation, so the equation will be like this. So, let us take it as t, which we are going to solve. So, the first one if $\log_p q^x - 1 = t$, then $q^x - 1 = p^t$. And similarly, if we do it for the second term which is $\log_q p^x - 1 = t$, then $p^x + 1 = q^t$. So if we add this two equation, we will get this plus this equal to $p^t + q^t$ which will give us $p^x + q^x$because this 1 and minus 1 cancels out and we are getting this equation, $p^x + q^x = p^t + q^t$.

Now, this p and q are non-zero natural number. And you know that these are basically increasing function if we write $f(x) = p^x + q^x$. So, here are using f so let me write this as f'. So, our $f' = p^x + q^x$ this (()) (2:05) increasing function and clearly these are as we have seen in the lecture, these are one, one function. So, it gives us x equal to t because p and q both are natural number. So, this gives us $x = t$.

So, now substituting t in anyone of this two equation, we will get $q^x - 1 = p^x$. So, we can write it as $(\frac{q}{p})^x - 1 = (\frac{1}{p})^x$. Now we are given that $q = p + 1$. So, $(\frac{q}{p})^x - 1$ this is an increasing function, only this left hand side. So, the left hand side is an increasing function. So, let me write it here this is an increasing function, because $, q > p$. So, $q > p$ because we have $q = p + 1$.

And this part $(\frac{1}{p})^x$, this is a decreasing function. So, and those two are equal at for some x. Now one side is increasing function, the other side is a decreasing function, so it will intersect at most at one point. So, whatever the solution maybe it will be only one solution, the solution will be unique. So, if we substitute $x = 1$ in this equation, in this equation, we will get $(\frac{q}{p})^1 - 1 = (\frac{1}{p})^1$, which will give us $q = p + 1$, you can check it.

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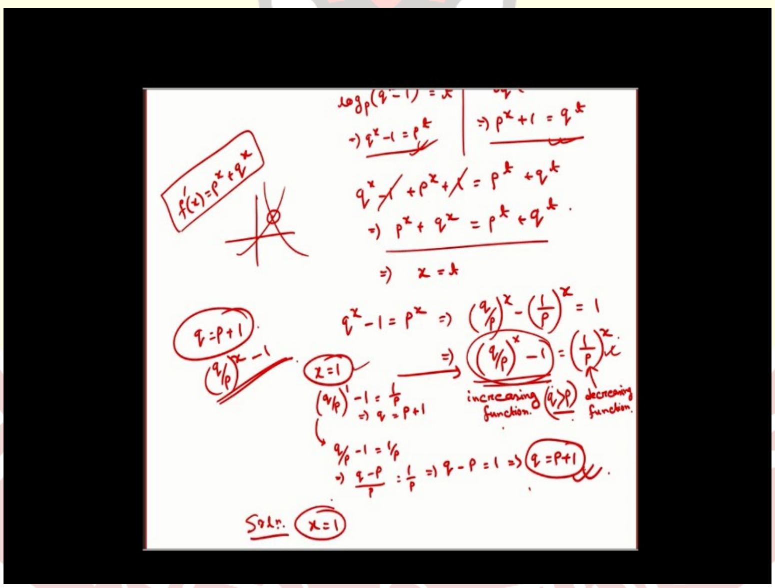

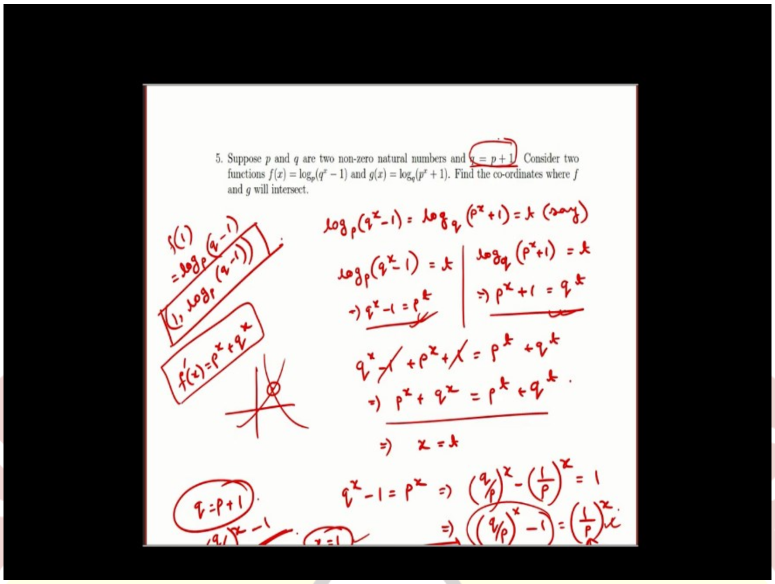

You can write it as $(\frac{q}{p})^1 - 1 = (\frac{1}{p})^1$ which will give us $\frac{q-p}{p} = \frac{1}{p}$ which will give us $q - p = 1$ and $q = p + 1$, which is true because this is the given condition, this is the given condition we have $q = p + 1$. So, $x = 1$ will be a solution of this equation which we have find out. And now I have already told that this one side is increasing function and the other side is decreasing function, so there will be a unique solution.

So, there cannot be any other solution other than $x = 1$, so our solution of this equation will be $x = 1$. So, let us see what we have to find, we have to find a coordinate where f and g will intersect. Basically, we have to find the value of x and the function f_1. So, f_1 will be $\log_p q - 1$. So this is f_1, hence the coordinate where f and g will intersect will be $(1, \log_p q - 1)$, this will be the coordinate.