INDIAN INSTITUTE OF TECHNOLOGY MADRAS

सिद्धिर्भवति कर्मजा

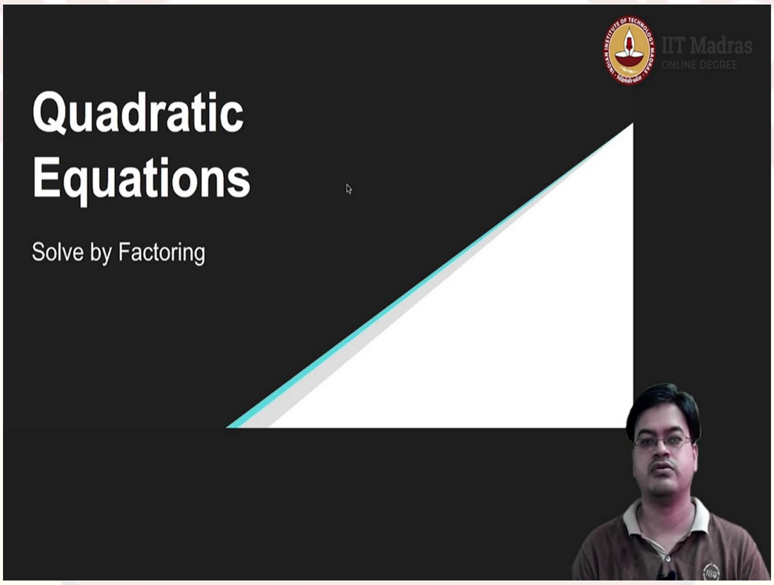
IIT Madras

ONLINE DEGREE

Mathematics for Data Science 1
Prof. Neelesh S Upadhye
Department of Mathematics
Indian Institute of Technology, Madras

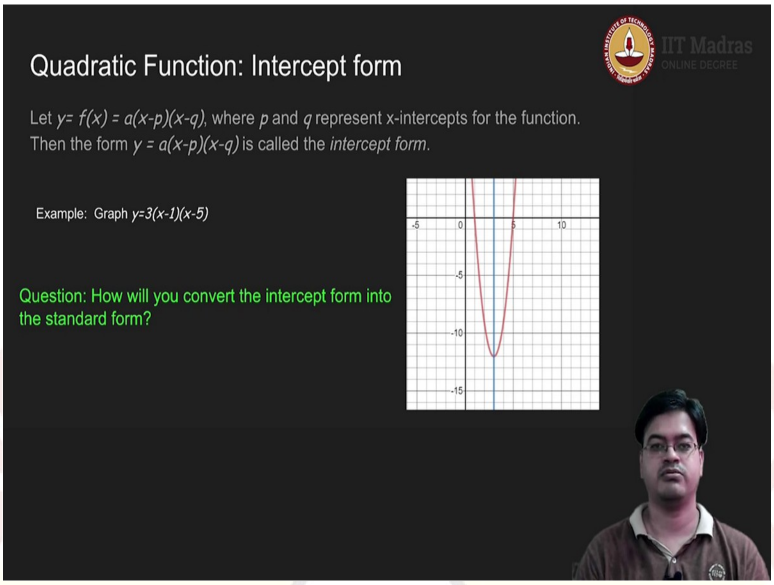
Lecture – 27
Solution of Quadratic Equation Using Factorization

(Refer Slide Time: 00:15)



So, in the last video, we have seen how to find the roots of a Quadratic Equation by graphing the associated quadratic function.

(Refer Slide Time: 00:37)



In this video, we will see how to find the solution of a quadratic equation by a well known method called factoring method. For that, we will define one new form of a quadratic function that is intercept form.

What is an intercept form? If $y=f(x)$ which is a quadratic function is written in this form , where and $(x-q)$ are called binomials ok. Where this p and q are nothing but x intercepts of the quadratic function.

So, essentially what you have done is, you have seen a graph of a function and you have located the two intercepts x intercepts of the function. Whenever the expression is possible in this form you are writing it. So, $y=a(x-p)(x-q)$, and this form is called the intercept form.

So, let us try to see one example of intercept form which is let us say you have been given this intercept form $y=3(x-1)(x-5)$. And you also know that these 1 and 5 are x intercepts of the quadratic function, that means, you have been given two values. Can you find the third value? The answer is yes.

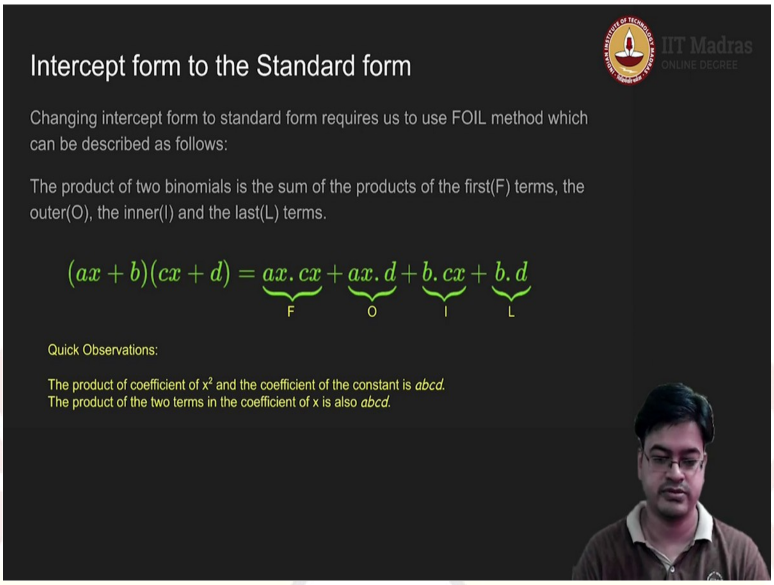
So, at point 1, when , the value is 0; at point 5, the value is 0. Now, using the logic that I gave you in the previous video, you can actually see that there will be some axis of symmetry between this 1 and 5, because 1 and 5 both take value 0 right. So, the axis of symmetry will be nothing but the distance between these two points divided by 2. So, $\frac{1+5}{2} = 3$. I am sorry it will not be a distance, it is just the sum of these two points divided by 2, average of these two points, that should be the correct terminology. So, it should be average of two points. So, $\frac{1+5}{2} = 3$

Now, $x = 3$ will be the axis of symmetry for this particular function if at all the this are the roots of the this are the x intercepts of the function right. So, now, I have given you $x = 3$ is the axis of symmetry. So, just substitute the value 3 in this particular graph in this particular equation, and you will get 3 - 1 = 2, and 3 – 5 = -2, that means, $2 \times -2 = -4, -4 \times 3 = -12$. So, you got three values. What are those three values? (1, 0), (5, 0), (3, -12).

So, based on this information, you can easily plot the graph which will look like this. As you can see this is the value -12 here, value -12 is here and 0, 0. So, you can easily plot this graph. You can connect the smooth curve using this.

Now, the main question is once given this kind of expression, how to convert this expression into a standard form? So, that is the question that I will post down. How will you convert the intercept form into the standard form? Just by multiplying the two binomials. So, for multiplying the two binomials, we have one rule which I will state in the next slide.

(Refer Slide Time: 04:17)



So, how the conversion from intercept form to standard form will happen using a method called FOIL method which is described below. So, the product of two binomials as I already mentioned that $(x-p)$ and $(x-q)$ these are the two binomials. So, the product of two binomials is sum of the product of first terms the outer, the inner and the last terms. So, let me make it more precise by demonstrating it.

So, let us consider this expression which is . Now, $(ax+b)(cx+d)$ what I will do is I will first take the first term of this expression, and the first term of this expression, and multiply them together that is the first term over here by the sum of the product of the first terms I mean this term.

Then I will take the inner term ok, then I will take the outer term, sorry, then I will take the outer term that is b is the outer term here, sorry, not b is not the outer term, b is the inner term. You have ax which is the outer term, and d which is the outer term. So, now you just multiply them together which gives me $ax.d$ which is the outer term product of the outer term.

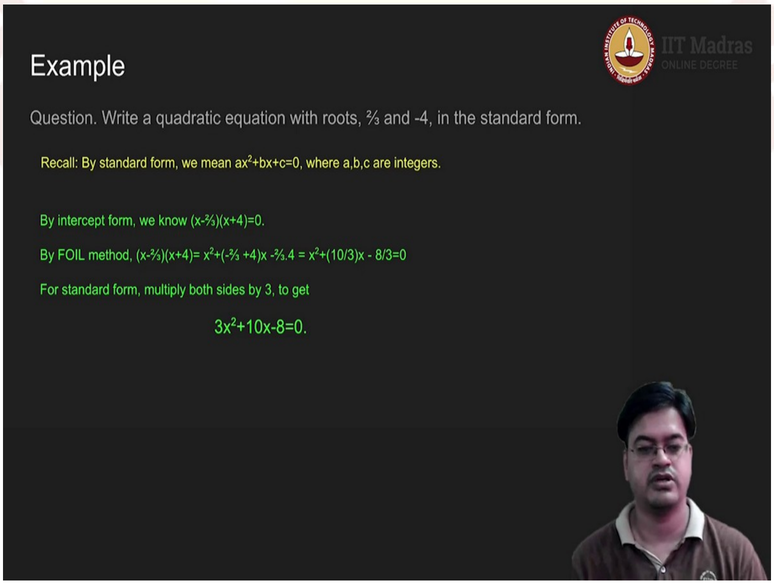
Then you take the inner terms that is b and cx. So, $b.cx$, this is the inner term. And $b.d$ are the last terms. First term, outer term, inner term and last term that way we will multiply these things together. That means, I will get the first term as acx^2 ; second term is $(ad+bc)x$; and the third term as bd .

Now, if you look at, so basically the ac is the term which is the coefficient of x^2, $ax+bc$ is the $ad+bc$ is the term which is the coefficient of x, and bd is the term which is the which is the constant term right.

So, now, a quick observation you can make is if you look at the product of this first term and the last term, what will you get $ac \times bd$, so it is $abcd$, the product is $abcd$ right. So, the product of the coefficient of x^2 and the coefficient of constant is $abcd$. In a similar manner, if you consider the coefficients of x, a d and b c are the coefficients of x, $ad+bc$, so if you take product of these two terms, again it will be $abcd$. This is a crucial observation which we will need while converting a standard form to intercept form and vice versa.

Just remember this the product of the coefficient of x^2 and the coefficient of constant is $abcd$. And the product of the two terms of the coefficients of x is $abcd$; both of them are $abcd$. So, this we will use to convert our expression into standard form, and convert our expression in intercept form in various ways. So, that observation is very crucial for us.

(Refer Slide Time: 08:03)



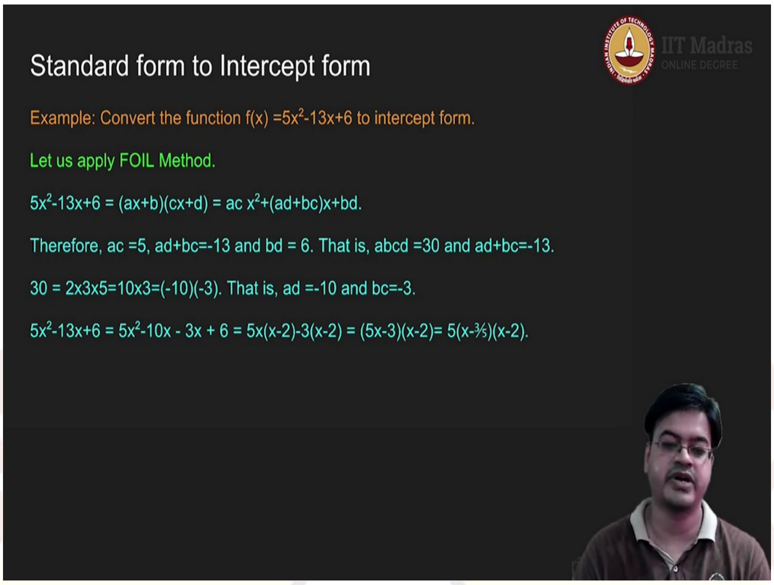
So, let us do take one example and see how we can apply our knowledge which we have gained in this particular video along with the previous videos to solve this problem. So, the question is the write a quadratic equation with roots 2, $\frac{2}{3}$ and -4 in the standard form ok. So, let us recollect what is a standard form. Standard form is of $ax^2+bx+c=0$; and a, b, c all are integers; and $a \neq 0$. This is the standard form; we have already seen that ok.

So, now, if I want to write this, we will use our knowledge about intercept form, and we can easily write this expression as $(x-\frac{2}{3})(x+4)=0$ because $\frac{2}{3}$ and -4 are the roots. Yes, but this equation is not in the standard form. So, now in the previous slide, we have seen that in order to convert this into a standard form, we will use a FOIL method. So, let us try to use a FOIL method. So, the what is a here? $ax+b$ that is a is 1, b is $\frac{-2}{3}$, c is 1, d is 4 ok.

Now, you use FOIL method that is first terms. So, first terms is 1×1, so it will retain $1x^2$, Then $ad+bc$, so $(\frac{-2}{3})(1)$ and a is 1, 4 $4.-\frac{2}{3}$ that is the product here $\frac{-2}{3}+4$ this is the term which have coefficient of x, and then $\frac{-2}{3}.4$ which is the term here. So, this is successful application of FOIL method.

Now, let us rewrite all these things that is you can sum this and write the sum that is x^2, so $4\times 3=12, 12-2=10$, so $x^2+\frac{10}{3}x-\frac{8}{3}$ right. Is this equation in the standard form? No, because for standard form a, b, c, all must be integers. So, what I will do is, I will multiply this equation with 3 on both sides. So, if I multiply on both sides with 3, then I get $3x^2+10x-8=0$ that is the solution to this question. So, the quadratic equation in standard form is $3x^2+10x-8=0$. So, we have solved.

(Refer Slide Time: 11:09)



Let us now go further and try to see how I will convert a standard form into an intercept form. Again we will use FOIL method, but in a reverse manner. So, I want to convert a function quadratic function which is given to me $5x^2-13x+6$ to intercept form, that means, I want to write $a(x-p)(x-q)$. So, how will I convert this?

So, what I will do is, I will take this particular function $5x^2-13x+6$, and apply FOIL method to it. How to apply FOIL method to it? I will equate this to be equal to $(ax+b)(cx+d)$. Then based on FOIL method, I have this expression which is ac^2 sorry $acx^2+(ad+bc)x+bd$. Now, remember we have done some observations that is the product of this and this is $abcd$ right.

So, now, I can equate this equation with this equation. So, term containing x^2 will be equated with term containing x^2. So, I will get $ac=5$, $ad+bc=-13$ and $bd=6$. Then from this expression I can also derive an information that is $abcd$ that is the product of the first and the last term and the product of the terms contained in the sum is 30. So, $5\times6=30$; and $ad+bc=-13$.

Now, my job becomes crucial. My job is to guess what those two terms will be ad and bc right. So, that their product is 30, and if you sum over them, then it must be -13. For that

I will use the prime factorization theorem that was introduced in week-1. So, if you look at this expression 30, I will get prime factors as $2\times3\times5$.

Now, I want the product to be equal to 30, and I want the sum to be equal to -13. So, based on this, what I can derive is if at all this, this term has to be negative, I should have some negative factors over here and both of them should be negative factors. In particular if I combine 5 and 2, I will get 10 and 3, and 10 + 3 =13, but it is not giving me -13.

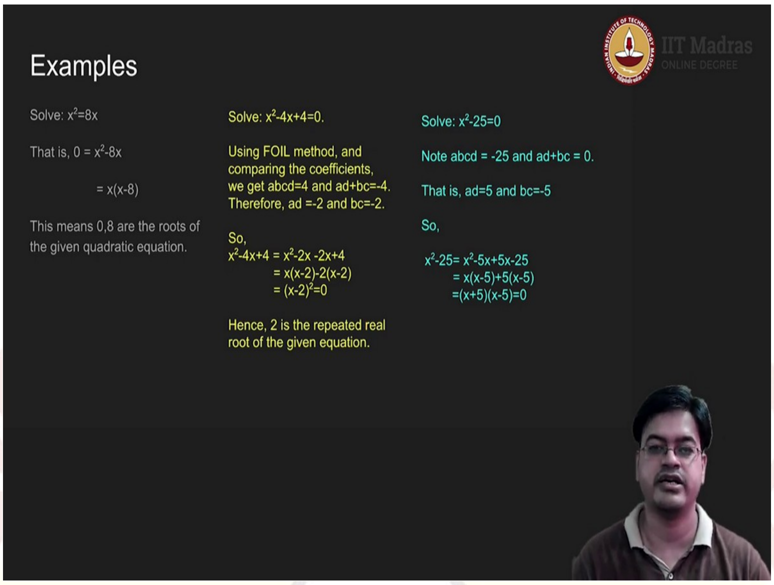
So, I will use a trick that multiplication of two negative numbers will become a positive number. So, it is $(-10)(-3)$ which will give me 30; at the same time, it will be the sum will be -13. So that means, my ad is -10; bc is -3. It does not matter, you can switch also. You can write bc as -3, and ad as -10 also, it does not matter.

So, now I will substitute these values into this expression, essentially I will rewrite this expression. So, I will write this expression as $5x^2-10x-3x+6$ ok. Then what I will do is I will look at the first two terms; first two terms, and I will take the greatest common factor from these two terms that is $5x$. So, I will take $5x$, whatever is remaining I will put in a bracket that is $x-2$.

Here also I will do take the greatest common factor out that is -3, so $-3(x-2)$. Now, you can see these $(x-2)$ s are same. So, essentially this expression will come if I have $(5x-3)(x-2)$. Now, is this in the intercept form? No, still it is not in the intercept form. What is the intercept form? It is $a(x-p)(x-q)$. So, I will just divide everything by 5 in this expression and take the 5 out. So, $5(x-\frac{3}{5})(x-2)$ this is the intercept form.

So, using FOIL method, I have converted this expression into an intercept form. An expression was given to me in standard form; I have converted it into intercept form. Let us see few more examples as this concept is quite intricate. You may need some practice, you solve as many problems as possible, but I will give you some demo cases, so that it will be easy to distinguish for you.

(Refer Slide Time: 16:05)



So, let us take let us say you have you have been asked to solve this equation; $x^2 = 8x$. Now, here you do not need, you do not really need a FOIL method. What you need is, just rearrange $x^2 = 8x$ and you just take out the greatest common factor which is x. So, x this will give you $x(x-8)$. So, if I want to solve this, I know $x-0$ and $x-8$ are the things. So, 0 and 8 are the roots of this given quadratic equation. Simple, this solves our problem for such a simple case, where the constant term is absent right.

Now, let us take another example, $x^2 - 4x + 4 = 0$. Now, in this case, you will use FOIL method obviously but the essence of FOIL method reduces to that the coefficients of x are of the form $ad + bc$, and the product of this and this is $abcd$. So, the product $abcd$ is 4, and $ad + bc$ is -4, this is what it reduces to ok.

So, if abcd is 4, and ad + bc is -4, is there any other way out 4 can be factorized only in one way that is 2×2; 2×2. And ad + bc is -4, that means, both of them should be negative -2×-2. So, ad is -2, bc is -2. Substitute it in the master equation where you can write $x^2 - 2x - 2x + 4$. So, you have substituted it in master equation.

Now, you go ahead and take out the greatest common factors out, the first expression will have x out, the second expression will have 2 out. Then again these are product of binomials. So, it will be $(x-2)^2$ = 0 given in the expression. So, what is the root of this equation? 2; 2 is the real repeated root of this equation.

Let us go ahead and solve one more example. $x^2-25=0$. This is quite interesting 25, you can see is a perfect square 5, and I want to find the root of this equation. Again I will use FOIL method, abcd is -25, and ad + bc is 0 right. 5 is a perfect square. So, 5×5 is the factorization, but -25 is there. So, one will be, one 5 will be with a positive sign, another 5 will be with a negative sign. So, ad can be +5, and bc can be -5 or vice versa, it does not matter.

Substitute this substitute this knowledge into this expression. So, you take $x^2-25-5x+5x$, take out the greatest common factors that is x and 5 respectively, you will get this kind of expression. And then you just rewrite them as product of two numbers that is $(x+5)(x-5)=0$, and you have solved. Remember in all these expressions we have written this in intercept form.

So, all these expressions are written in intercept form. Once you write an expression in intercept form, it is very easy to find the roots of the equation, or in fact once you write in the intercept form you have already figured out the roots of the equation.