

IIT Madras
ONLINE DEGREE

Mathematics for Data Science 1
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Lecture – 31
Degree of Polynomials

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The Degree of the Polynomial $3x^3 + 4x^2y^2 + 10y + 1$

- The exponent on the variable in a term is called the degree of that variable in that term. $4x^2y^2$ $\deg(x)=2$, $\deg(y)=2$

So, in particular, if I want to tell something about a Polynomial, an important property is a degree of the polynomial. So, what is the degree of the polynomial? For demonstration purposes, let me take one example. Let us say my example is $3x^3 + 4x^2y^2 + 10y + 1$, this is my example.

Then, I say this is the example. So, if I want to decide the degree of the polynomial, we have already seen each term itself is a polynomial. So, $3x^3$ is one; $4x^2y^2$ is one; $10y$ is one and this 1 is one. So, I want to identify the degree of each term as well.

In each term, there are many variables. For example, you would look at this term, if you look at this term then there are 2 variables. So, I want to have a complete understanding. So, in order to define the degree of a polynomial, I will start with defining the degree of the variable. So, the exponent on the variable in a term, the exponent on the variable in a term is called degree of that variable in that term. So, for demonstration purposes, let us take the expression $4x^2y^2$.

So, in this particular expression or in this particular monomial, how many variables are involved? One variable is x , second variable is y . So, what I am saying is now in this term, the degree of x ; the degree of x let me abbreviate it as degree; degree of x is 2 and the degree of y , variable y is also 2, ok. So, this is how I will describe the degree of the variable.

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The Degree of the Polynomial $3x^3 + 4x^2y^2 + 10y + 1$

$\deg(4x^2y^2) = \deg(x) + \deg(y) = 2 + 2 = 4$

- The exponent on the variable in a term is called the degree of that variable in that term. $4x^2y^2$
- The degree of that term is the sum of the degrees of the variables in that term.
- The degree of the polynomial is the largest degree of any one of the terms with non-zero coefficients.

$3x^2 \leftarrow 2$
 $4x^2y^2 \leftarrow 4$
 $10y \leftarrow 1$
 $1 \leftarrow \deg(x)=0 \quad \deg(y)=0 \quad | x^0 y^0 = 1$

The slide also features the IIT Madras logo and a video inset of a man speaking in the bottom right corner.

Now, let us take this term as a term and say what is the degree of this term, right. So, let me erase this particular portion which is actually blocking our view ok. So, the degree of that term, this term, we have already seen the degree of x is something and degree of y is something.

Degree of x was 2 and degree of y was 2, the degree of the term is the sum of the degrees of those variables in the term. That means if I look at this expression which is $4x^2y^2$, then and I ask for the degree of this term, then it is essentially degree of x plus degree of y that is $2 + 2$ which is equal to 4.

So, degree of this term the second term in this expression is 4, fine. Now, we will answer the question, what is the degree of a polynomial? So, the degree of the polynomial is the largest degree of among these all the terms of any one of the terms with nonzero coefficients or terms will exist only when there are nonzero coefficients.

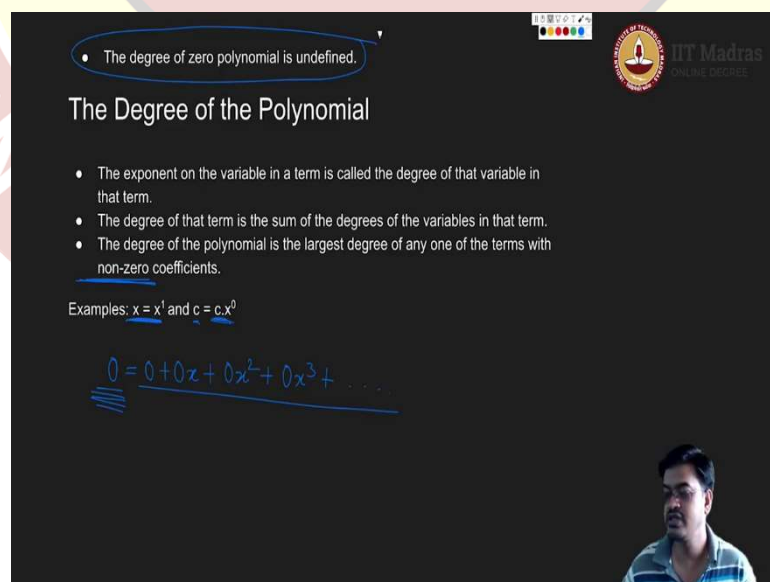
So, let us try to see how we can solve this problem. So, we will try to list all the degrees. So, if I take the first term that is $3x^2$, second term is $4x^2y^2$, then the next term is $10y$ and the last term is 1 which is the constant ok.

So, now, we will talk in terms of degrees. So, what is the degree of this particular term? It has only one variable x which is raised to the second power. So, the degree of this term is actually 2. What is the degree of this term? We have already seen here, the degree of this term is 4.

What is the degree of this term? The degree of this term is again y , means y^1 . So, the exponent is 1, Interesting. Now, what is the degree of this term? Now, remember this is an expression in two variables; x and y . So, what then, I will ask a question what is the degree of x and what is the degree of y ?

Now, you can also see that $1x^0y^0 = 1$. So, degree of x is naturally equal to 0 and degree of y is also equal to 0, right. Therefore, I can write the degree of this expression is 0 ok. Now, which one is the largest among these four? 0, 1, 2, 4? 4 is the largest. So, the degree of this particular polynomial is 4. So, this degree is actually 4, then write it here 4. So, this is a polynomial of degree 4 ok.

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• The degree of zero polynomial is undefined.

The Degree of the Polynomial

- The exponent on the variable in a term is called the degree of that variable in that term.
- The degree of that term is the sum of the degrees of the variables in that term.
- The degree of the polynomial is the largest degree of any one of the terms with non-zero coefficients.

Examples: $x = x^1$ and $c = c \cdot x^0$

$$0 = 0 + 0x + 0x^2 + 0x^3 + \dots$$

So, in this contest in while finding the degree of this particular polynomial, we have seen two things. What are those two things? If the coefficient is if the variable is x , then this is

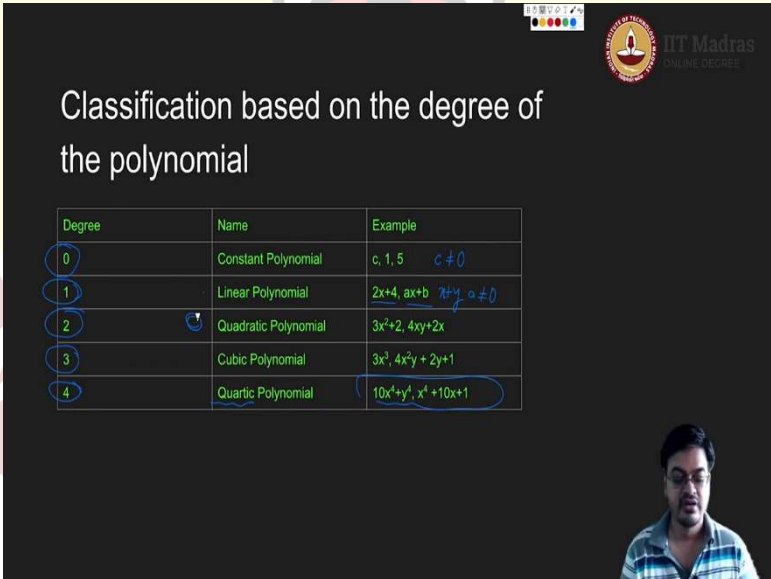
x^1 , if it is a constant, then $c \times x^0$. In our case because the polynomial was having 2 variables, it is cx^0y^0 .

Interesting question comes when we try to see polynomials at some other things. Let us say if I want to describe 0, when c is nonzero, it is ok, but if $c = 0$, then what? Then, you can see $0 = 0 + 0x + 0x^2 + 0x^3 \dots$, the matter is complicated further and so on right.

It will continue. So, if the point given, then this number is 0, then we will call this as 0 polynomial and we cannot define the degree of this polynomial because for a degree, we need a nonzero coefficient, just remember this in mind.

Therefore, the degree of 0 polynomial is always undefined this is an interesting fact which will be used when we use the division algorithm. The degree of 0 polynomial is undefined. So, you can use it in a more interesting manner that is what I can say. So, the degree; this is what? Degree of 0 polynomial is undefined.

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Degree	Name	Example
0	Constant Polynomial	$c, 1, 5$ $c \neq 0$
1	Linear Polynomial	$2x+4, ax+b$ $x, y, a \neq 0$
2	Quadratic Polynomial	$3x^2+2, 4xy+2x$
3	Cubic Polynomial	$3x^3, 4x^2y + 2y+1$
4	Quartic Polynomial	$10x^4+y^4, x^4+10x+1$

So, we have understood what is the degree of the polynomial. So, in particular, based on the degrees, now we have introduced one classification. So, based on the degrees, how the polynomials can be classified. So, if the polynomial has degree 0, then it is constant and this constant can never be equal to 0.

This is an important assumption. Then, if the polynomial is of degree 1, linear polynomial, then you will have a polynomial in this form. When I write this, then I should write $a \neq 0$, if I have a quadratic polynomial, if the polynomial ok.

So, here these are the polynomials in one variable, then I am considering a linear polynomials. If I am considering a polynomial of the form $x + y$, this is still a linear polynomial; but it is a polynomial in 2 variables. So, you can also encounter such polynomials in linear, but the crucial fact is degree is 1.

Second one is a quadratic polynomial which is of this form and here you can have polynomial in two variables, three variables or whatever way you want. Then, you will get a cubic polynomial which will have all terms containing degree 3, the highest term, highest monomial will have degree 3.

So, this is these are the examples of degree 3 polynomials. Similarly, degree 4 polynomials are called quartic polynomials and they will be given in this form and similarly, degree 5 polynomials are called quintic or quantic polynomials which will be represented with degree 5 polynomials, right.

So, and in general, you have a general term which is called polynomial. So, to be if you want to be specific, you can use this classification and say it is a quadratic polynomial, then you are giving more information about it.

This is what today's lecture meant to be. So, we have introduced the topic of polynomials.