Integrals as anti-derivatives

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$$\int_a^b f(x)dx = \lim_{||P|| \to \infty} S(P) = \lim_{||P|| \to 0} \sum_{i=1}^{n} f(x_i^*)\Delta x_i.$$

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Example : An anti-derivative of $f(x) = x^7 + 2x^6 - \pi x^5 + 0.5x^4 - 9$ is $F(x) = \dfrac{x^8}{8} + 2\dfrac{x^7}{7} - \pi\dfrac{x^6}{6} + 0.5\dfrac{x^5}{5} - 9x$

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Fact : If F is an anti-derivative of f, then so is $F_1(x) = F(x) + c$ where c is any constant (i.e. a real number).

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Because of this, We will use the instead of an for the anti-derivative, since any two anti-derivatives only differ by a constant.

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Suppose f is continuous on the domain D which includes the interval $[a, b]$. Then an anti-derivative for f on (a, b) is given by

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Conversely, if f is continuous on the domain D which includes the interval $[a, b]$ and F is the (indefinite) integral of f, then the (definite) integral from a to b of f can be computed by

$$\int_a^b f(x)dx = F(b) - F(a).$$

Tables of derivatives and integrals

Function	Derivative	Integral
1	0	x
x^r	rx^{r-1}	$\frac{x^{r+1}}{r+1} \quad r \neq 1$
		$\frac{1}{ln \mid x \mid} \quad r = 1$
$sin(x)$	$cos(x)$	$-cos(x)$
$cos(x)$	$-sin(x)$	$sin(x)$
$tan(x)$	$sec^2(x)$	$ln \mid sec(x) \mid$
$sec(x)$	$sec(x)tan(x)$	$ln \mid sec(x) + tan(x) \mid$
$cot(x)$	$sec^2(x)$	$ln \mid sin(x) \mid$
$cosec(x)$	$sec^2(x)$	$ln \mid cosec(x) - cot(x) \mid$

can be obtained

Tables (contd.)

Function	Derivative	Integral
$e^{\lambda x}$	$\lambda e^{\lambda x}$	$\frac{e^{\lambda x}}{\lambda}$
a^x	$a^x ln(a)$	$\frac{a^x}{ln(a)}$
$ln(x)$	$\frac{1}{x}$	$xln(x) - x$
$\frac{1}{\sqrt{a^2 - x^2}}$		$sin^{-1}\left(\frac{x}{a}\right)$
$\frac{1}{\sqrt{a^2 + x^2}}$		$tan^{-1}\left(\frac{x}{a}\right)$
$\frac{1}{a^2 - x^2}$		$\frac{1}{2a} ln\left\|\frac{x+a}{x-a}\right\|$

Examples

$$\int_1^2 x dx = \left[\frac{x^2}{2}\right]_1^2 = \frac{2^2}{2} - \frac{1^2}{2}$$

$$= \frac{4}{2} - \frac{1}{2} = \frac{3}{2}.$$

$f(x) = x$

$F(x) = x^2/2$

$F(2) - F(1)$

$$\int_0^\pi sin(x)dx = \left[-\cos(x)\right]_0^\pi = (-\cos(\pi)) - (-\cos(0))$$

$$= (-(-1)) - (-1)$$

$$= 1 + 1 = 2.$$

$$\int_0^\infty e^{-2x} dx = \lim_{b \to \infty} \int_0^b e^{-2x} dx$$

$$= \lim_{b\to\infty} \left[\frac{e^{-2x}}{-2}\right]_0^b = \lim_{b\to\infty} \frac{-1}{2}\left(e^{-bx} - e^0\right)$$

$$= \lim_{b\to\infty} \frac{1}{2} - e^{-bx} = \frac{1}{2}.$$

$$\int_a^\infty f(x)\,dx = \lim_{b\to\infty} \int_a^b f(x)\,dx$$

Thank you