Solving Exponential Equations

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Example 1. Solve for x. $2^{x+1} = 64$

$\log_a(a^x) = x$

$64 = 16 \times 4 = 2^4 \times 2^2 = 2^6$

$2^{x+1} = 2^6 \iff 2^x = 2^5$

$\iff \log_2 2^x = \log_2 2^5$

$\iff \boxed{x = 5}$

Example 2. Solve $e^{-x^2} = (e^x)^2 \frac{1}{e^3}$

$\ln$

$e^{-x^2} = e^{2x-3}$

$-x^2 = 2x - 3$

$0 = x^2 + 2x - 3 \iff 0 = (x+3)(x-1)$

$\boxed{x = -3, 1}$

Example 3. Solve $9^x - 2\,3^{x+1} - 27 = 0$

$9^x - 2\,3^{x+1} - 27 = 0$

$(3^2)^x \quad 6\,3^x$

$(3^x)^2$

$(3^x)^2 - 6(3^x) - 27 = 0$

$t^2 - 6t - 27 = 0$

$t^2 - 9t + 3t - 27 = 0 \iff (t-9)(t+3) = 0$

$(3^x - 9)(3^x + 3) = 0$

$\boxed{3^x = 9}$ or $\boxed{3^x = -3}$

$\boxed{x = 2}$

Example 4. Solve $5^{x-2} = 3^{3x+2}$

$\ln(5^{x-2}) = \ln(3^{3x+2})$

$$(x-2)\ln(5) = (3x+2)\ln(3)$$

$$-2(\ln(5)+\ln(3)) = 3x(\ln(3)) - x\ln(5)$$

$$-2(\ln(15)) = x[3\ln(3) - \ln(5)]$$

$$x = \frac{-2\ln(15)}{\ln(27)-\ln(5)} = \frac{\ln(1/225)}{\ln(27/5)}$$

Example 5. Solve $x + e^x = 2$

$$x + e^x = 2$$

$$e^x = 2 - x$$

$$x = \ln(2-x)$$

$$\ln(2-x) - x = 0$$

$$x \approx 0.443$$