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IIT Madras

ONLINE DEGREE

Mathematics for Data Science 1
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Lecture – 38
Graphs of Polynomials: Multiplicities

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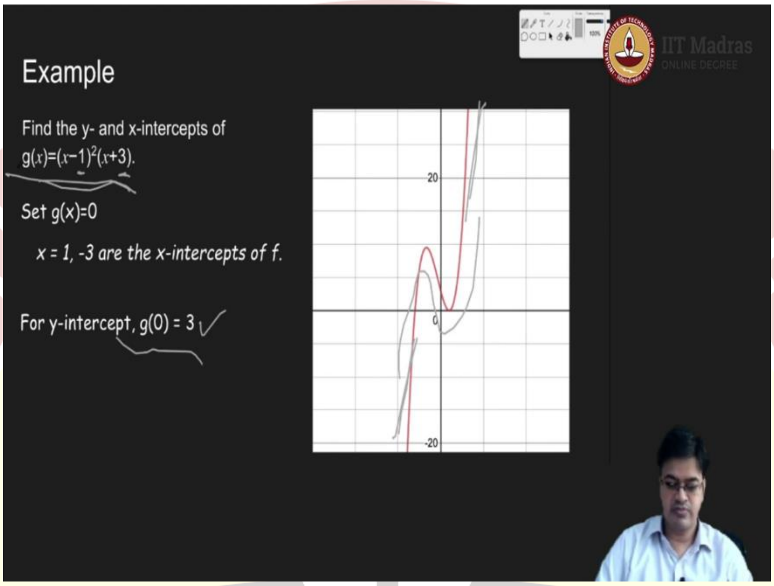

So, now, so far, we have mastered two skills; given a graph of a polynomial function, I know whether the given function is I can identify whether the given function is a polynomial function or not, ok. The second thing that we have seen is from algebraic expression of polynomial function whether it is in factored form or non-factored form, I have some set of rules or algorithm which will help me to identify, the roots of the polynomial or the zeros of the polynomial.

So, with this knowledge, can I explicitly write a polynomial function, or do I need to know something more about it? That is what the question that is troubling us. For example, the knowledge about x-intercepts in this case, and the knowledge of y-intercept, is this helping us to understand how the polynomial will look like?

For example, how will I decide the polynomial is going down from here, polynomial is going up from here, and it will stay going up forever, or when will this kind of shape come, the curve when will the rise and fall will happen, I do not know anything about this right now. What I know is simply the function should be smooth, yes, this is a smooth function.

But, from this graph can I write this equation? Seems to be difficult right now, but we will get the handle over it in due course of time.

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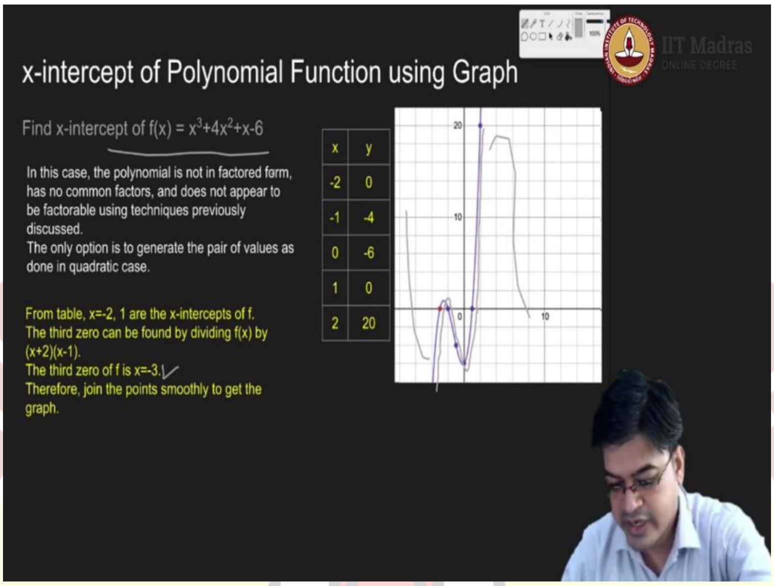

So, let us now look at the x-intercepts or identifying the x-intercepts using graph. So, you have been given a polynomial function. You have used a technology to identify the graph, but still you are not convinced.

And, you want to try it by your hand. So, how will you do it? That is the question. So, if I want to find the x-intercept, the given polynomial is a cubic polynomial, fine. So, this polynomial is not given in a factored form; I cannot find greatest common factor that is not possible.

Then, if I want to find something which is like binomial thing that is $(x + 4)$, but the rest the other term is $(x - 6)$. So, I am not actually getting these two tricks done. So, there is no way in which, I can factor this polynomial. So, one crude way, if you do not know how to go about, is to plot the pair of values as we have done in quadratic case.

So, simply find out what are the function values at some points. So, these are some standard points, I have drawn them symmetrically 0, 1, 2, -1, and - 2. When I considered these two points, because the function is very nice, I accidentally came across two zeroes that are - 2 and 1, good.

So, -2 and 1 are the x-intercepts of f which is clear from the table. Now, can I use this knowledge to find the third zero? The answer is yes. And, we know the long division. So, what you do is you consider $(x+2)$ as one factor and $(x-1)$ as another factor. You multiply $(x+2)(x-1)$ and treat that as a divisor, and take $f(x)$ as a dividend, and do the long division.

If you do the long division; you will get, you may pause the video and try for yourself; otherwise you will get the third 0 to be equal to $x=-3$; that is $x+\ 3$ is another factor. And, this is a cubic polynomial, so it cannot have any other factor, can I have at most 3 roots. So, you got this $(x+\ 3)$, $(x+2)$, and $(x-\ 1)$ as the factored form of this equation. Then, it is easy to plot the equation along with this table of values.

So, what you will do is, you will simply put up the points, you will simply put up the point. So, over here I know something and I know I have figured out the third root to be equal to $x=-3$; therefore, I can put that point as well.

So, this is $x=-3$ and, how to draw a line passing through; so, the next step is joining the line. So, you can draw join a smooth line passing through these points, then at this point it will turn up; at this point it will turn up, but to connect to this point, it has to go down, and then I do not have any idea. So, right now I can draw only up to this, right.

We will analyze further and see the cubic polynomial cannot turn more than two times, so that we will that we will come later. But, right now I can draw it this way. So, let us see what our graphical tool gives us. Yes, where the previous image and this image are slightly perturb but they are exactly matching. So, this is how the behaviour of the function will be.

So, now we have slightly better edge over drawing the graph of a function, when we have been given a equation of this form, but still I do not know why this is not turning up or why this is not coming down, I need to understand these things in a better way by using some analytical tools. For a moment, we got the correct graph.

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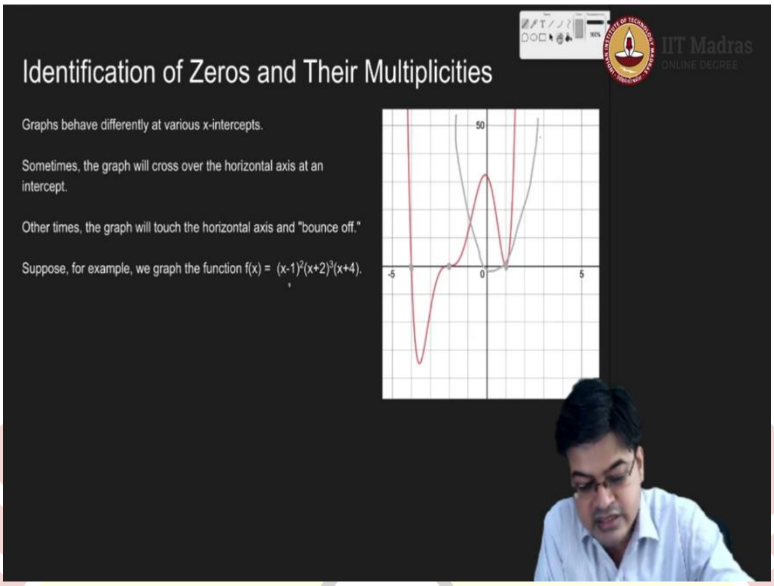

So, let us move ahead and try to see the behaviour of the graphs around the intercepts. For that, it is important to know the multiplicities of factor. So, in particular, graphs behave differently for at various x-intercepts.

We can go back to the previous case, where everything is of linear order that is $x+\ 3$, $x+2$ and $x+1$, the graph was behaving like this. If you go further back; why would this graph behave like this? Over here, when $x=-3$ is a factor, the graph the graph was actually like a straight line and over here it turned around.

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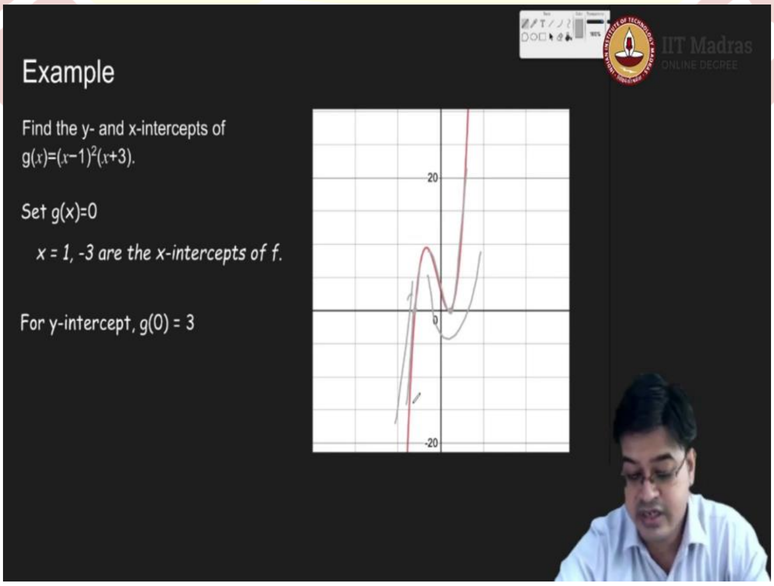

So, why should it turn around the factor? So, for example, $x+\ 3$ is one factor that is -3, and here $x-1$ is one factor. But, for this factor it turned around; and for this factor it crossed, it cross the x-axis. So, why is this happening? So, I need to have a deep understanding of this. For that, we will discuss the next that is what we will discuss in the next slide. So, is it related to the function being appearing the factor appearing multiple times? That is what we will try to see.

So, as mentioned as shown in the earlier slide, that the graph can cross over the horizontal axis or it may bounce off; that means, it will touch and go up that is tangential to that axis. So, why is this happening at x-intercept? That, so in that case let for that making the understanding clear we will write a polynomial in a factored form which is $(x-1)^2(x+2)^3(x+4)$, right. And, let us draw that polynomial using technology or graphing tool, ok.

So, now some crucial things, let us identify the factors first; $x-\ 1$ that is $x\ =\ 1$, this is the factor that we are talking about. Then, $x=-2$, this is the factor that we are talking about and $x+4$ this is the factor that we are talking about -4.

Now, at these points, what is happening, what exactly is happening at these points? So, when I consider the factor $(x-1)^2$, because it is quadratic and if I recollect the graph of a quadratic function, it behaved some it is not to the scale, but it behaved something like this, right. It will never cross x-axis.

So, a similar feature is visible over here, when I consider the graph of this function. So, if I consider a graph of this function, because $x-\ 1$ is coming twice; it is square $(x-1)^2$ is there; so, what I am getting is the behaviour is of the quadratic nature, ok.

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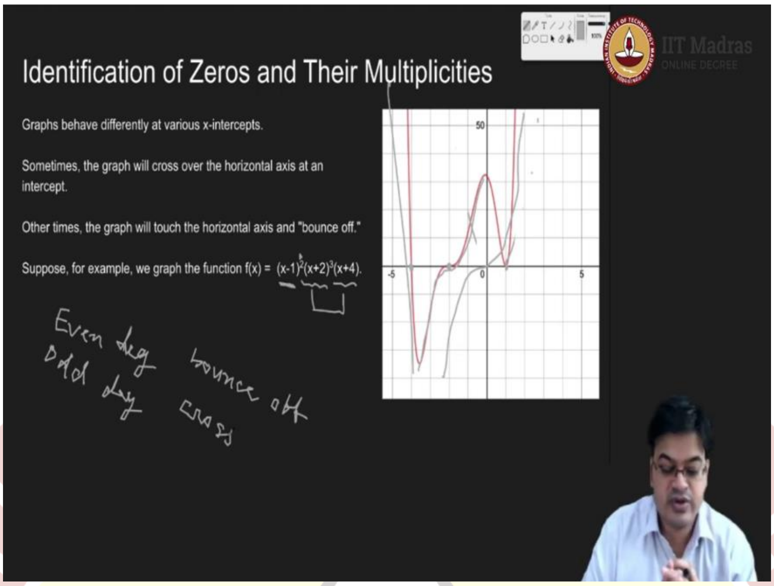

Now, let us look at that $(x+2)^3$. What is a graph of $(x)^3$? A graph of $(x)^3$ is somewhat like this; it crosses x-axis y is equal to x cube, it crosses x-axis. So, now that behaviour is evident when I consider that instead of x, I consider $(x+2)^3$ that behaviour is evident over here. It actually cuts and crosses x-axis.

And, if you look at the third factor that is $x+\ 4$ which is $x=-4$, it is behaving like a straight line that is also. So, what is happening here? I have two things; one and this one. So, in these both cases we have odd degree polynomials and, the odd degree polynomials as we know actually cross x-axis. And, in this case I have an even degree polynomial which is actually bouncing off the x-axis, this is a typical feature.

So, if I have even degree what we are saying is, if the polynomial is a, the factor is of even degree, then it will bounce off; that means, it will not cross x-axis but, if the polynomial as odd degree, then it will actually cross x-axis. So, these are the two typical features that we will employ while for plotting the functions which are of polynomial nature once we understand the factors.

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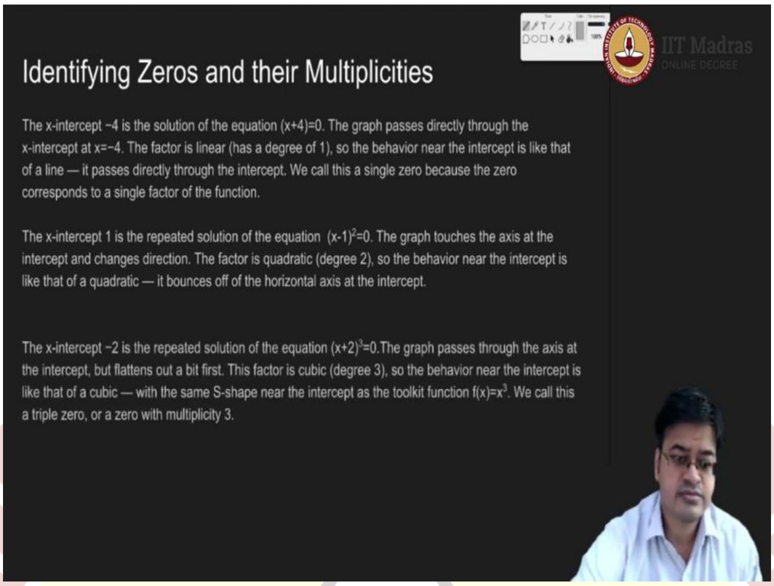

So, in the next slide, I have given a general description of these factors, you can go through these slides later, but it is essentially the same that I have said just now, ok.

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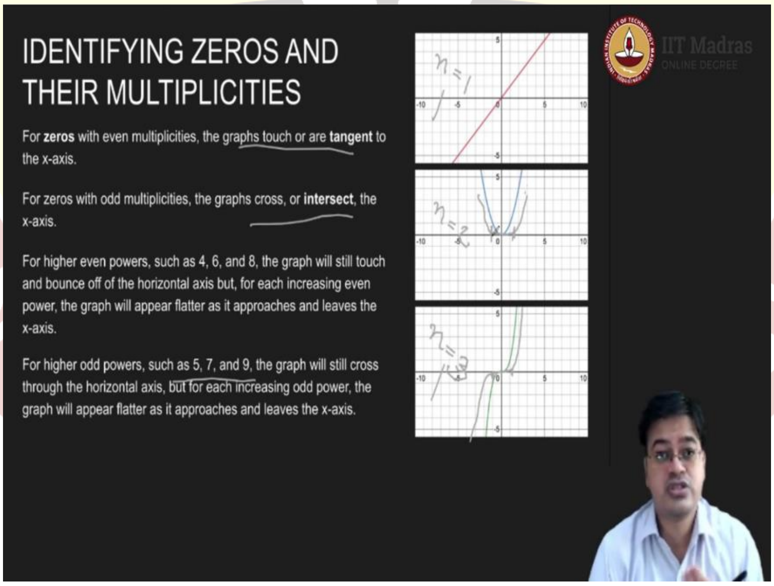

So, now for identifying zeros and their multiplicity. What do I mean by multiplicity? How often that factor is appearing. In the previous case the factor 1 was appearing twice, factor minus 2 was appearing thrice, and other factor was appearing only ones. So, if I want to identify the zeros and their multiplicities, I should look at the shapes of the curves. For example, if you look at the first graph, here the degree of the polynomial $n = 1$, here $n = 2$, $y = x^2$ this is, and here $n = 3$.

As mentioned earlier, it is more; it is more evident now, that when I have odd degrees, the curve actually passes through x-axis, when I have even degrees we can draw $y = x^4$, but this will be slightly broad and it will cut the x-axis, it will be somewhat like this. Let us not get into that. But, for odd degrees you will get something of this form, or even degrees you will get the bouncing off pattern and for odd degrees you will get a pattern which is actually crossing, one minute.

Let me reiterate this that; this is very as this is very important. If you have an odd degree polynomial, then you are almost sure you are sure to cross x-axis. If you have even degree polynomial, you will never cross x-axis at that point, you will simply bounce off from x-axis, ok. So, that gives us some more clarity. So, if the zeros of the polynomial or the factor has even multiplicities, the graph will touch or is tangent to x-axis or zeros with odd multiplicities, the graphs cross or intersect the x-axis.

Now, if you look at the even powers which are 4, 6 and 8, how will you guess, what is the strength of the power? So, in that case the graph will still touch x-axis it will bounce off, but which with each increasing even power it will appear to be flatter and flatter while approaching the zero and leaving from the zero. For example, the base will broaden; in this case, it will be something like this if it is x^4; x^6 further flattening.

If in a similar manner when you have odd powers like 5, 7 and then the graph will appear to be more flat over here, and while leaving also it will leave slowly and then it will decay at very fast rate. So, this is the typical feature from the bulge at these intervals you can actually guess the multiplicity of a polynomial. That is the importance of this slide.

So, now we have added one more weapon in our arsenal that is we will identify the multiplicities of the zeros. First we identify zeros. So, at the step zero is we identified given a function where whether it is a polynomial function or not. Then, we identified the x-intercept of the function that is zeros of the functions or roots of the functions.

After identifying roots of the functions, roots how many times repeated that is what, we have identified here in this by using the graphical tools. This is quite powerful. And, you can use it more often to understand the polynomial function. When you will actually solve some problems on identifying the polynomial functions, you will get a better hold of it.