IIT Madras

ONLINE DEGREE

Mathematics for Data Science 1
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Lecture 50
Composite Functions: Domain

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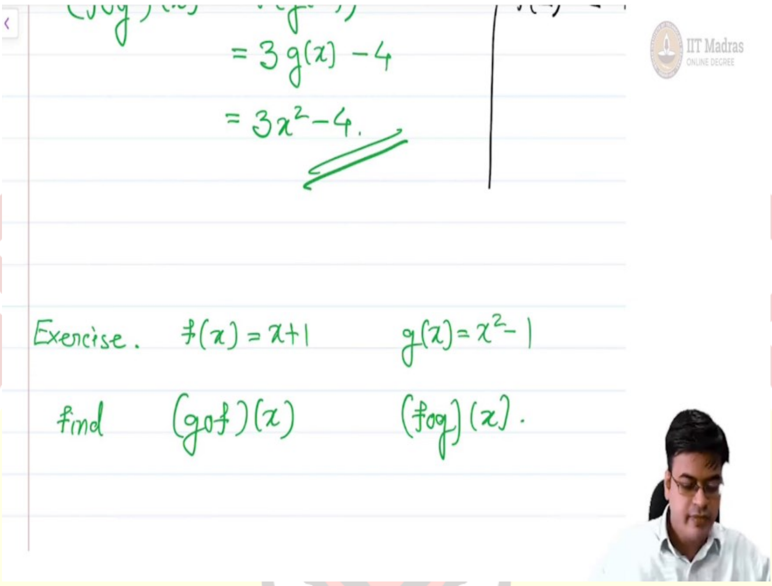

Let us now go further and talk about how to determine the domain of composite functions. So, this will be any important question that is determination of domain of a composite function.

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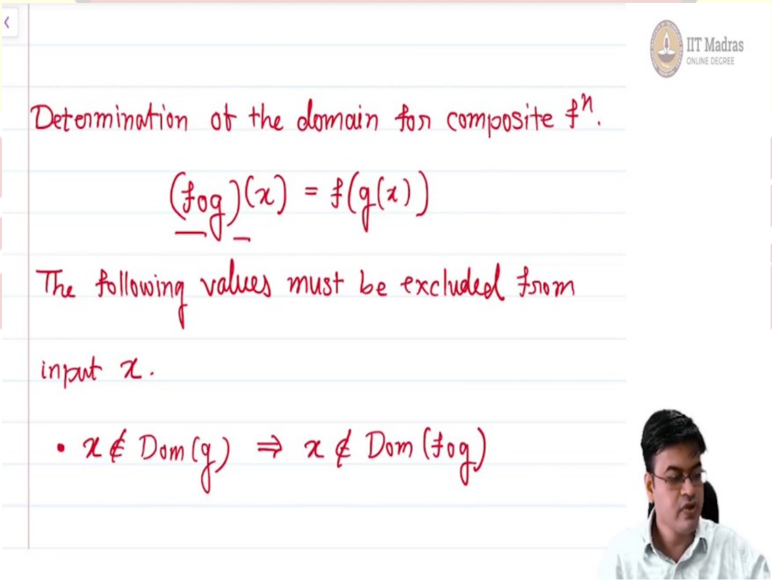

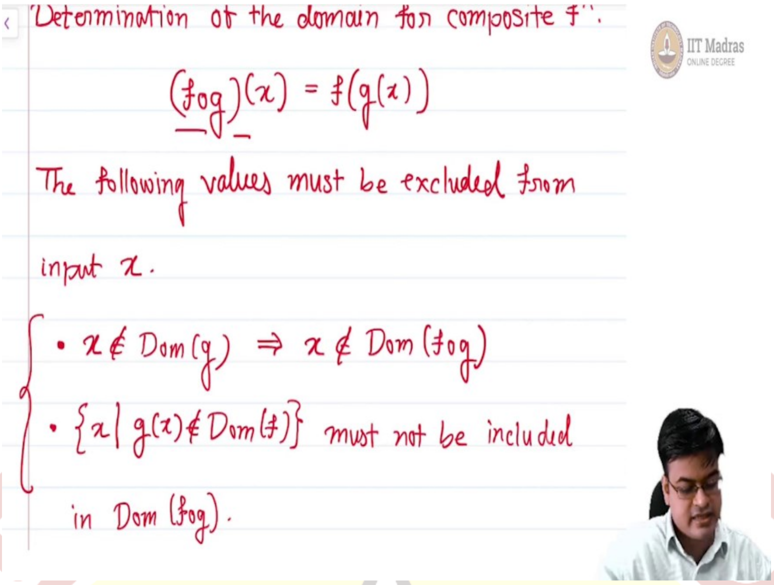

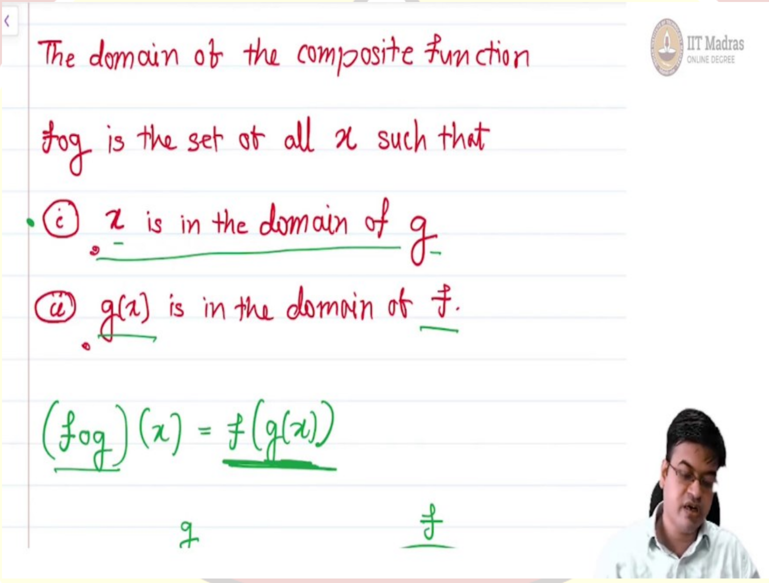

Determination of the domain let us say for domain for composite function, how will you determine this? So, I have let us say *f o g(x) = f(g(x)),* we are talking about all functions that are real value. So, in order to determine the domain there must be some rules that you should follow I will list the rules and that essentially says the following rules, the following rules must be followed and therefore, the following values must be excluded from input values of x.

So, this is again in concordance with what we have seen earlier that if you remember we have seen some conditions right, where *x* should be in the domain of *g* and *g(x)* should be in the domain of *f*. So, again what we are discussing now is in concordance with that, but here we were seeing what are the possible values.

Now, what we are seeing is what are the possible exclusions, that means, what value should be excluded from the input values. So, there are basically two rules the first rule which corresponds to the first rule of this that x should be in the domain of g that means, x if x is not in the domain of g then I cannot include it then x cannot be in the domain of the function $f \circ g$.

So, I am talking about $f\ o\ g$, when you talk about $g\ o\ f$, you will talk about the $x$belonging to domain of f implies. So, x does not belong to the domain of f implies x does not belong to domain of $g\ o\ f$. So, just remember the function the order in which they are taken it matters and in the similar manner, when I talked about $g(x)$ belonging to the domain of f. So, the set of all x's such that g $x$does not belong to the domain of f.

So, this is the set that you need to be careful about this set must not be included in domain of our function $f\ o\ g$ that is a composite function otherwise, we will have some ambiguity. So, in order to eliminate the ambiguity, we need to follow these two rules strictly very strictly. So, let me demonstrate how these rules can fail and then we will I will demonstrate it through an example and let me take that example as that is write it here.

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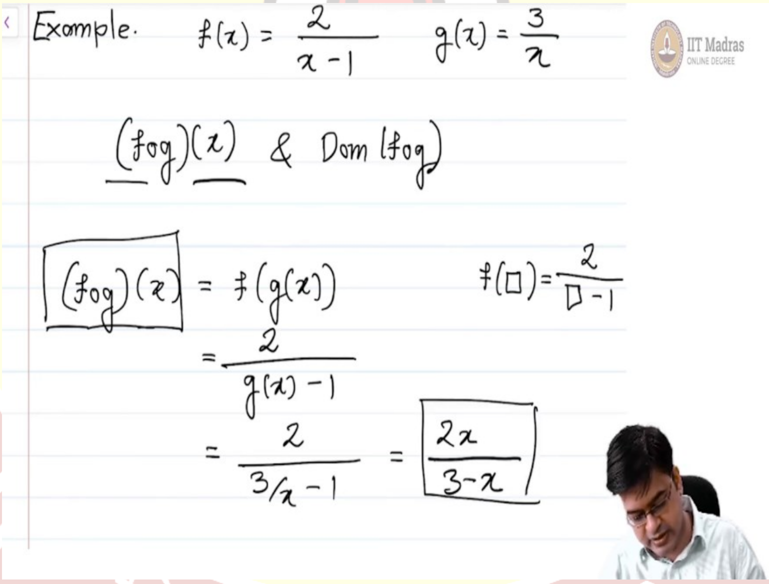

So example, so I have been given a function $f(x) = \frac{2}{x-1}$ and another function that is given to me is $g(x) = \frac{3}{x}$ and you want to find $f\ o\ g(x)$ and you also need to find a domain of this function $f\ o\ g$. What is the domain? Domain if you recollect from your week 1 it is nothing but the set of allowed values for which the function is well defined whatever input values you are fitting into the function, this function should be well defined this is the domain This is the notion of domain.

So, let us first see what is *f o g(x)?* And let us see if it gives you some hints about what can happen, correct? So, what is *f o g(x)?* Simply apply our definition it is *f(g(x)),* fine no confusion in this, then again you use that $f(\square) = \frac{2}{\square - 1}$. So, that gives me $\frac{2}{g(x)-1}$.

Now, what is *g(x),* it is $\frac{3}{x}$. So, substitute what is *g(x)?* So, it will be $\frac{3}{x-1}$, simplify this assume $x$is not equal to 0 and simplify this you will get $\frac{2x}{3-x}$. So, this is my *f o g(x).* Now, the question the second question that is asked is, so I have given answer what is a *f o g(x).*

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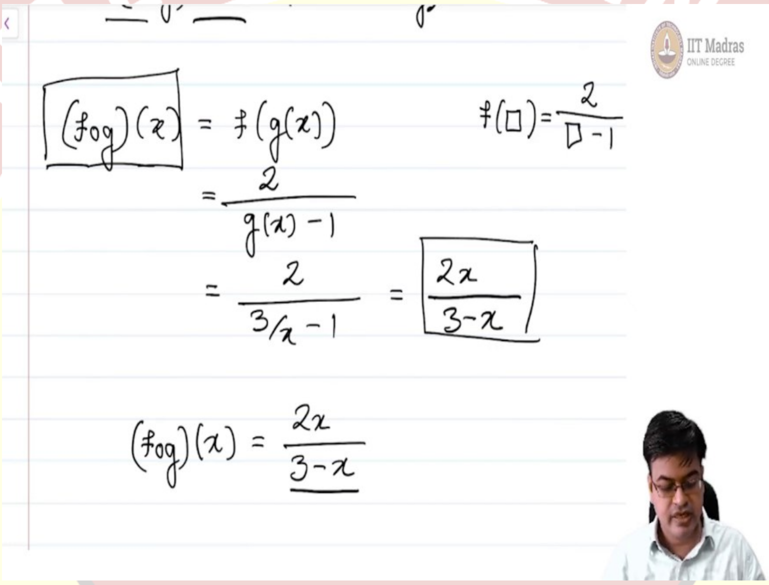

So, my $fog(x) = \frac{2x}{3-x}$. Now, if you look at this function, if you look at this function, you can simply see that at *x=3* this function is not defined, because the denominator is becoming 0. So, 6 / 0 is undefined. So, this function is not defined at *x=3*. So, the domain of this function must exclude 3 that is very well known.

But let us now see because of composition if I am eliminating any points, so here you look at this function which is *f(x).* And you look at this function which is *g(x)* and I am calculating *f o g(x).* So, if x does not belong to domain of *g*, then that function that particular value of x should not belong to domain of *f o g* that is the first rule that we have to implement.

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$$= \frac{2}{3/x - 1} = \boxed{\frac{2x}{3-x}}$$

- $(f\circ g)(x) = \frac{2x}{3-x} = \frac{0}{3} = 0$

Rule 1. $x \notin \mathrm{Dom}(g) \Rightarrow x \notin \mathrm{Dom}(f\circ g)$

$g(x) = \frac{3}{x}, \quad x \neq 0$

$x = 0 \notin \mathrm{Dom}(g) \Rightarrow x = 0 \notin \mathrm{Dom}(f\circ g)$

$$\boxed{(f\circ g)(x)} = f(g(x)) \qquad f(\square) = \frac{2}{\square - 1}$$

$$= \frac{2}{g(x) - 1}$$

$$= \frac{2}{3/x - 1} = \boxed{\frac{2x}{3-x}}$$

$$(f\circ g)(x) = \frac{2x}{3-x}$$

$$= \frac{2}{g(x) - 1}$$

$$= \frac{2}{3/x - 1} = \boxed{\frac{2x}{3-x}}$$

- $(f\circ g)(x) = \boxed{\frac{2x}{3-x}} = \frac{0}{3} = 0$

Rule 1. $x \notin \mathrm{Dom}(g) \Rightarrow x \notin \mathrm{Dom}(f\circ g)$

$g(x) = \frac{3}{x}, \quad x \neq 0$

So, rule 1, what is the rule 1? If x does not belong to the domain of g that must imply x does not belong to the domain of f o g. So, what is that point? Let us look at what is *g(x)?* $g(x) = \frac{3}{x}$. So, this function is well defined only when $x \neq 0$. So, $x \neq 0$ not equal to 0. So, $x = 0$ cannot belong to domain of g. So, $x = 0$ do not belong to domain of g. So, naturally I will enforce that $x$equal to 0 should not belong to domain of $f\, o\, g$.

So, now, you may come up with some argument that when you look at this function, when you look at this function, if I substitute $x = 0$ if I substitute x=0, I am getting 0/3. Then this function is well defined because the answer is 0. That is what your argument will be. But no, why? I will tell you because when we when, when we were while we were coming to this particular form, what we were doing actually is we were multiplying a numerator and denominator by x or we are taking assuming $x$not equal to 0.

We are taking this x on the numerator on the numerator side and multiplying by $x$and that is where we have reached this point. If we had not assumed $x \neq 0$, then we would not have reached this point. Therefore, $x \neq 0$ is a valid condition still even when you cannot see anything visible over here, because I am composing the 2 functions where $x \neq 0$ is outside the domain.

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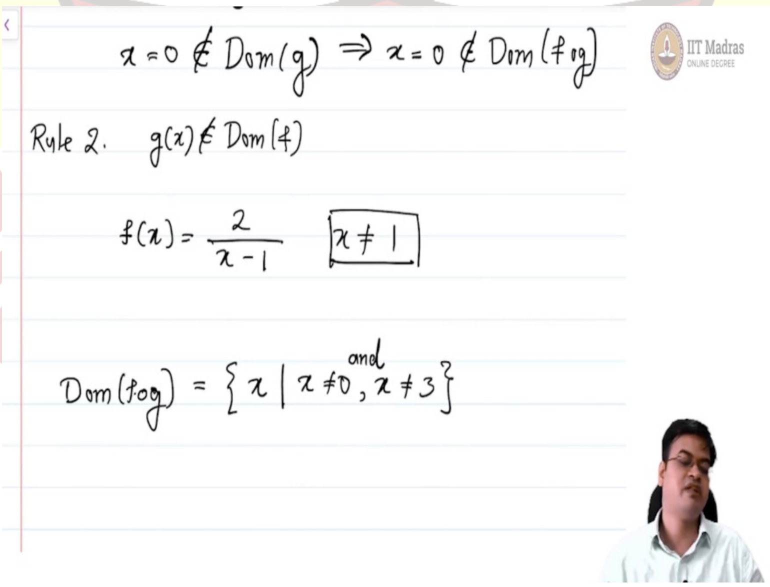

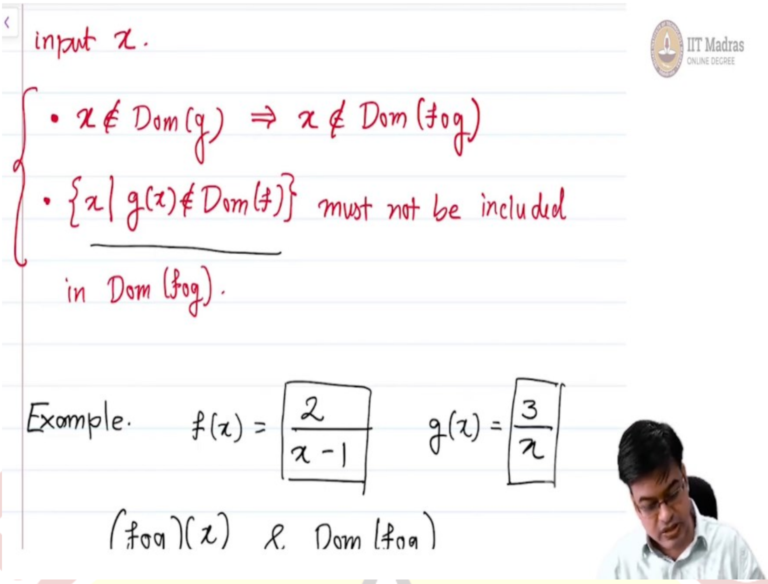

So, let us come to the next rule 2 that rule 2 was if *g(x)* does not belong to the domain of *f* then I am having a problem. So, that rule we have figured out like x says that *g(x)* does not belong to the domain of *f* must be excluded. So, let us look at our function *f* what is our function f it is 2 upon $x$minus 1 in this case $x=1$ I have a function where the denominator is 0.

So, x is equal. So, let me write for the sake of completeness $f(x) = \frac{2}{x-1}$ this is well defined when $x \neq 1$. So, so this also this point $x \neq 1$ should also be eliminated from the domain of *f o g*. So, what should be the domain of a *f o g*? All other points the function *f* and *g* are well defined. So, domain of *f o g* must be set of all x's belonging to real line such that $x \neq 0$ and $x \neq 3$, this comma means and or if you want me to be precise, I will write.

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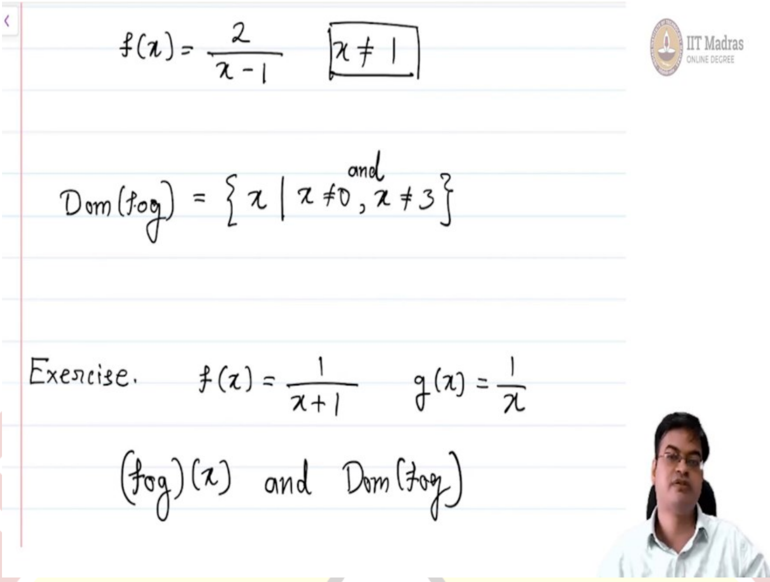

Another quick exercise that you can do in order to verify whether you have understood the concept of composition of function and the domain is you have been given 2 functions $f(x) = \frac{1}{x+1}$ and $g(x) = \frac{1}{x}$ and you are asked to find $f \circ g(x)$ and the domain of $f \circ g$. So, you can quickly solve this problem and check whether you have understood what we are supposed to understand. Thank you.