IIT Madras
ONLINE DEGREE

Statistics for Data Science-1
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Lecture – 7.4
Conditional Probability - Independent events

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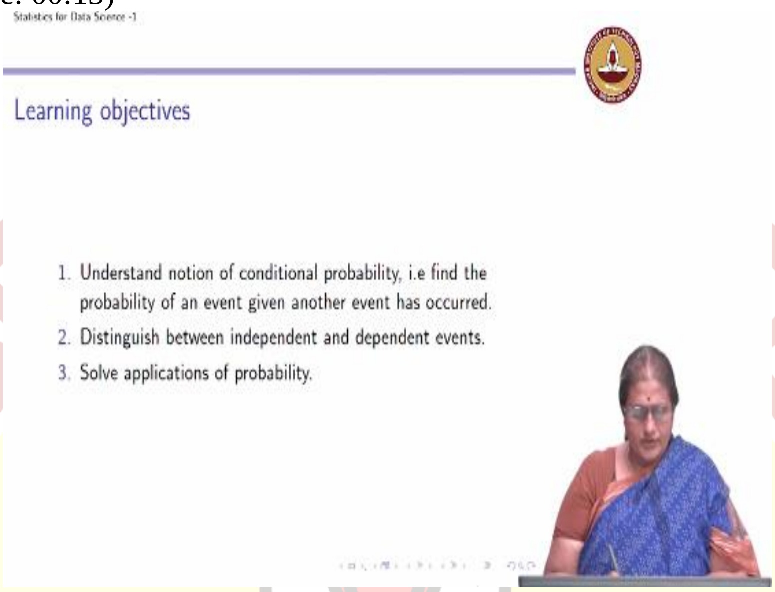

In this lecture, we are going to look at an extremely important concept which is called independence of events.

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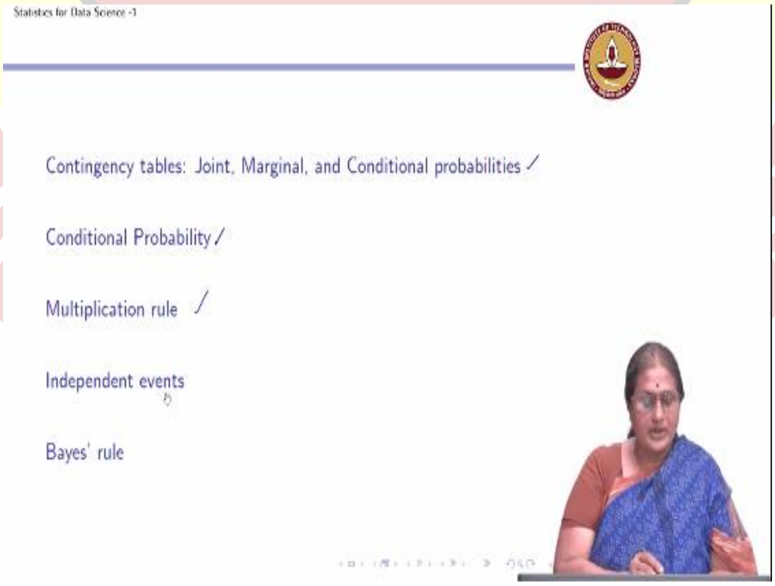

So, if you recall that we have already seen what are joint, marginal and conditional probabilities. We introduced the notion of conditional probabilities and the multiplication rules so far. So, today we are going to look at what do we mean by independent events?

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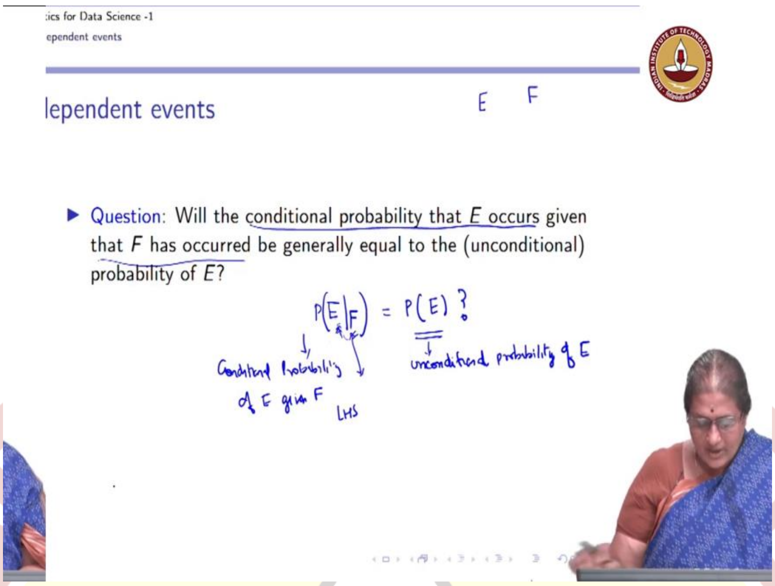

So, let us begin by asking a question, will the conditional probability of an event. So suppose I have two events let me call the events E and F. These are the two events defined on the same sample space. So the question we are asking is will the conditional probability that E occurs given that F has occurred and we have already seen that I am writing this as E occurs given F has occurred.

So, the question that is being asked is will the conditional probability of E given F has occurred be equal to the unconditional probability of E. So this is what we refer to as the unconditional probability of E and this is the conditional probability of E occurring given F occurring. So the question is will this situation happen generally or will it not happen so that is the question we are asking.

Now, let us understand what the left hand side means? It says that will the conditionality probability of E given F occurring be equal to the unconditional probability. So what we are asking is when will they if the conditional probability equals to the unconditional probability then it means that E does not depend on F. In other words, no matter what is this occurrence or non occurrence of this event F is unaffected because of it.

So, we are introducing a notion of whether there is any dependence of this event occurring given the information F has occurred or not. So, this is the question that we are going to answer and that is being answered by the following.

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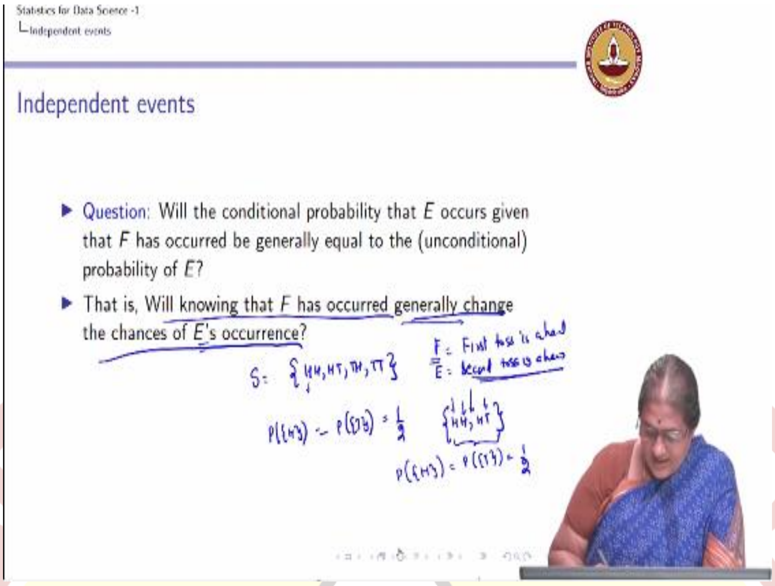

This is the question we are asking again is will knowing that F has occurred generally changed the chances of $E's$ occurrence. So, let us look at a simple example again I toss a coin twice I know my experiment is tossing a coin twice and this is my sample space. Now suppose I do not talk about probability or sample spaces I am just tossing a coin twice. Now the first toss is a head.

Now, the question we are asking is suppose I define an event E to be first toss is a head or let me define event F to be first toss is a head and event E to be second toss is a head. So the question we are asking is will the knowledge of the fact that the first toss is a head generally changed the chances of $E's$ occurrence in the sense that if I know that the first toss was a head will the chance of me getting a head or a tail in the second toss change.

So, we know that independently the probability of you getting a head or a probability of getting a tail in a fair coin is equal to half. Now if you look at this, if my toss is a head I know a head has happened. So this is my reduce sample space. Now in this sample space you again see the probability of getting a head or a tail is the same. In other words, you can see that the chance or the information knowing that I had a head in my first toss did not change the chance of a head in the second toss.

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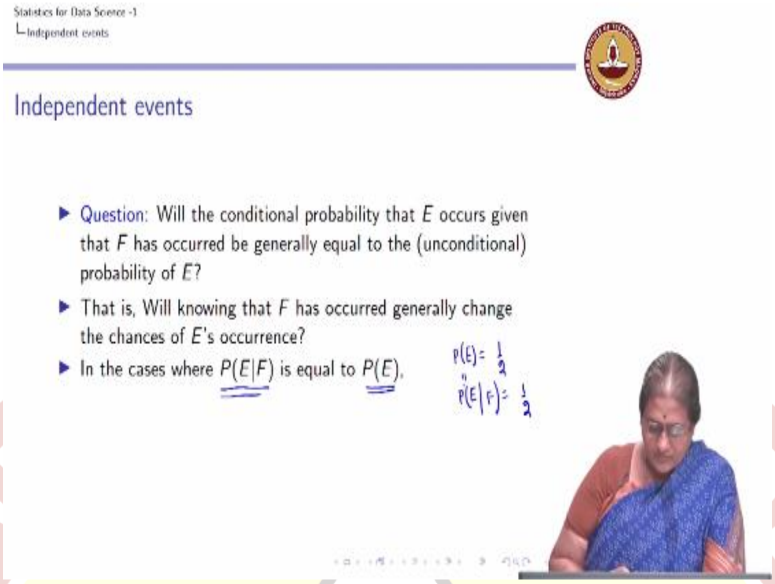

So, the question that is being asked is in general do we think that where are cases, are there cases where the conditional probability equals the unconditional probability. So we have just seen a case where probability of getting a tail or a head is equal to half and probability of getting head in the second toss given you have got a head in the first toss is also equal to half and these two are equal.

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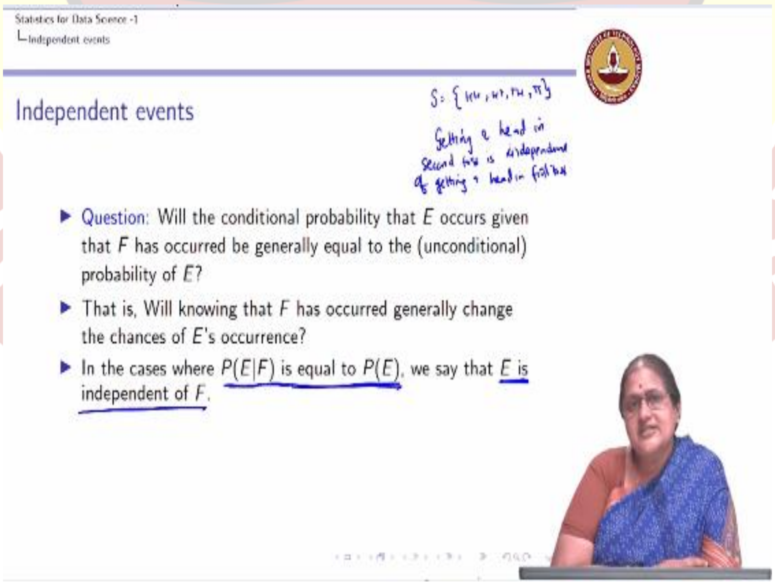

So, in cases where the conditional probability is equal to the unconditional probability we say that E is independent of F. I repeat in cases where the conditional probability is equal to the unconditional probability we say that E is independent of F. So in our coin tossing example you can see that getting a head in the second toss, getting a head in second toss is independent of getting a head in first toss.

In other words, whether I got a head or tail in the first toss would not affect my chances of getting a head or tail in the second toss. So in this case we say the events are independent events.

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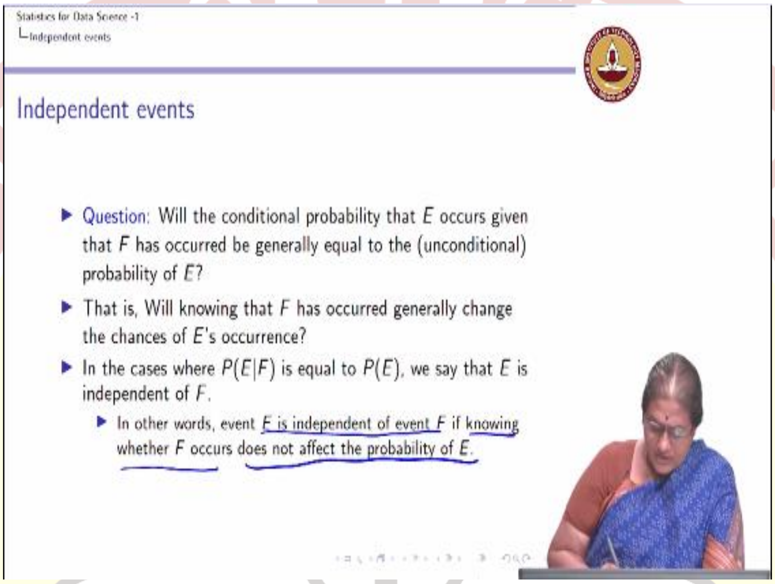

So, in other words we say that E is independent of an event if knowing whether F occurs does not affect the probability of E. So this is the notion of independence of events.

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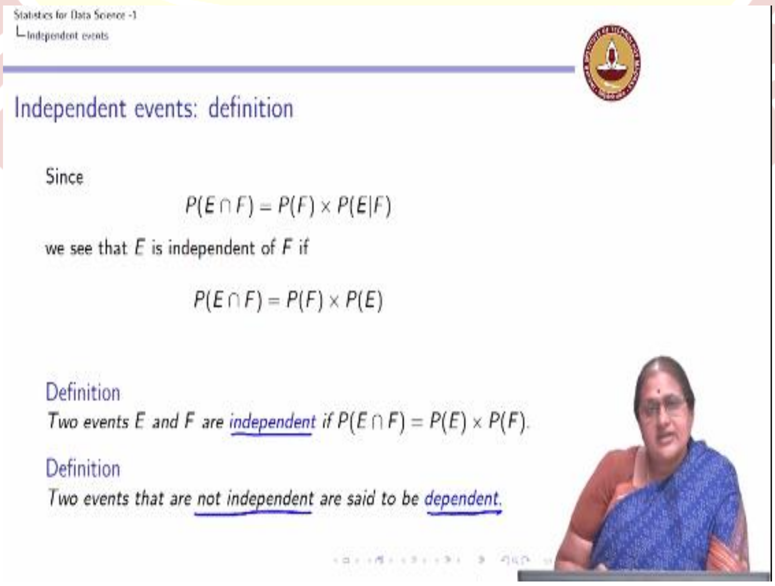

So, let us look at a formal definition to it. So since again from my multiplication rule I know probability given two events E and F I know the multiplication rule says that the probability of the intersection is $P(E) \times P(F)$. This is from my definition of conditional probability which says the $P(E|F) = \frac{P(E \cap F)}{P(F)}$ where $P(F) > 0$. I get $P(E \cap F) = P(F).P(E|F)$.

Now, E is independent of F, I said that the conditional probability is equal to the unconditional probability. Hence, we get $P(E \cap F) = P(F)P(E)$ which is nothing, but the product of the probabilities. Recall in the addition rule when I had disjoint a mutually exclusive events, recall in the addition rule when I had disjoint a mutually exclusive events the probability of the union was the sum of the probabilities.

Similarly, when I have independent events, probability of the intersection is product of the probabilities of the events. Hence I can pose my formal definition which says that two events E and F are independent if the probability of intersection is equal to the product of probabilities.

I repeat two events E and F are independent if the probability of the intersection of these two events is equal to the product of the probabilities. So once we know what is an independent event we are now in a position to define what is a dependent event. So two events that are not independent are said to be dependent.

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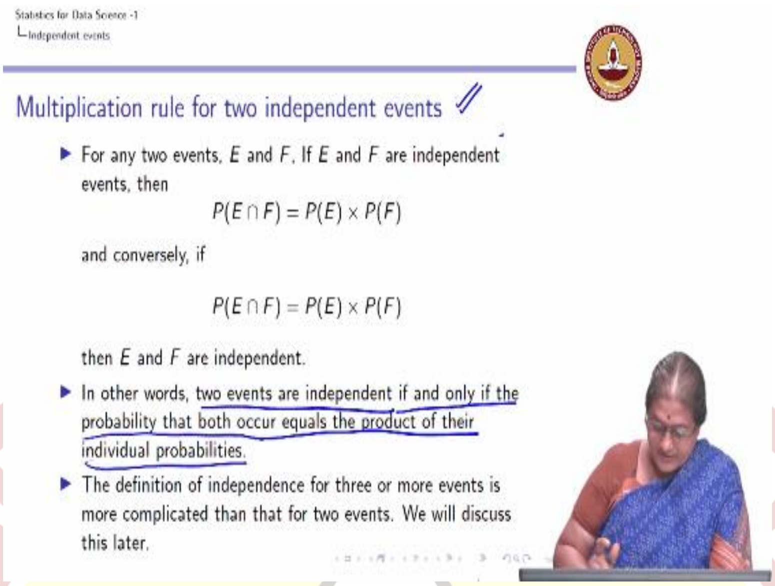

So, this notion of independence is extremely important. So recall the multiplication rule for two events are $P(E \cap F) = P(E|F)P(F)$ where $P(F)$>0. So now for two independent events I will have $P(E)P(F)$ so that is my multiplication rule says that for any two events if E and F are independent than probability of the intersection is equal to the product of the probabilities.

So, conversely it make sense for us to answer that if $P(E \cap F) = P(F)P(E)$ then can I say that E and F are independent that is the converse of the statement the answer is yes. Conversely, so this I have if E and F are independent then the probability of the intersection is product of probability. The converse is if probability of E intersection is product of the probabilities of events then E and F are independent.

So, hence I have a basic rule which is referred to as a multiplication rule for two independent events which states that two events are independent if and only if and the if and only if comes from this if E and F are independent then this happens. If probability of the intersection is equal to the product of probability then E and F are independent. Hence this is nothing but a necessary and sufficient condition, but the word of caution is I am looking at two events.

So, if the probability of both of them occurring together equals the product of the individual probabilities then we state that the two events are independent. To look at more than two events the definition of independence where more than two events that is three or more events is slightly more complicated we will look at it later, but for now I want you all to understand that if two events are independent if and only if the probability of the intersection. In other words, the probability that both occur together equals the product of their individual probabilities. So this is what we refer to as multiplication rule for independent events.