INDIAN INSTITUTE OF TECHNOLOGY MADRAS

सिद्धिर्भवति कर्मजा

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Mathematics for Data Science 1
Week 07 Tutorial 02

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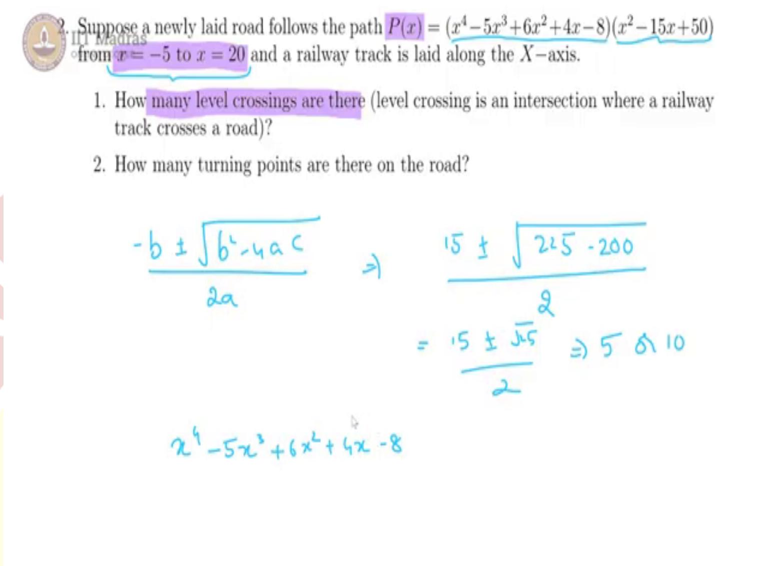

Now second question there is newly laid road which follows the path of this polynomial about some coordinate system, from $x = -5$ to $x = 20$. And railway track is laid along the x axis. So how many level crossings are there? So what we are interested in is; how many times does the x axis cut this polynomial? And for that we have to find the roots of this polynomial because roots give when the polynomial is touching or cutting the x axis.

Now this is of quartic forth degree polynomial multiplied with the quadratic polynomial, so the degree is 6, so at best we could have 6 roots but let us find out what these roots are. The easy way to start is to first find the roots of the quadratic, so that would be using $\frac{-b\pm\sqrt{b^2-4ac}}{2a}$. We get $\frac{-15\pm\sqrt{225-200}}{2}$, which is $\frac{-15\pm\sqrt{25}}{2}$ that is essentially 5 or 10.

So you get $\frac{10}{2}$ or $\frac{20}{2}$ so 5 or 10 those are the two roots and they are both within the given range. Anyway now we look at the other part, the quartic part. So here we have $x^4 - 5x^3 + 6x^2 + 4x - 8$. In this situations what is typically suggested is that we do a little bit of trial an error, we try out with the basic small integers and we see if we can find any roots at all.

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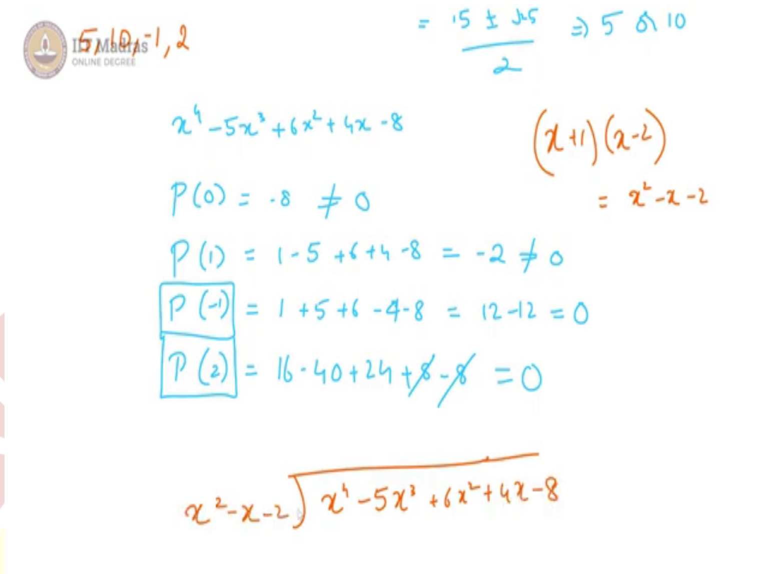

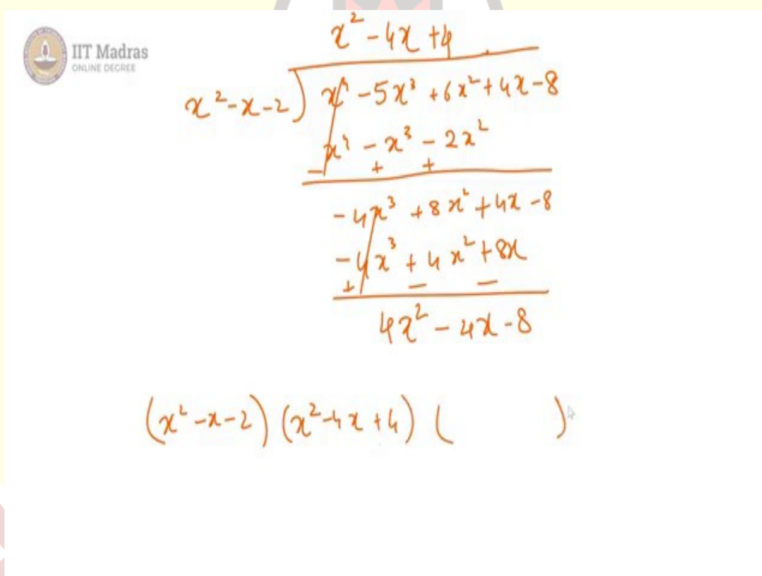

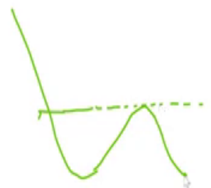

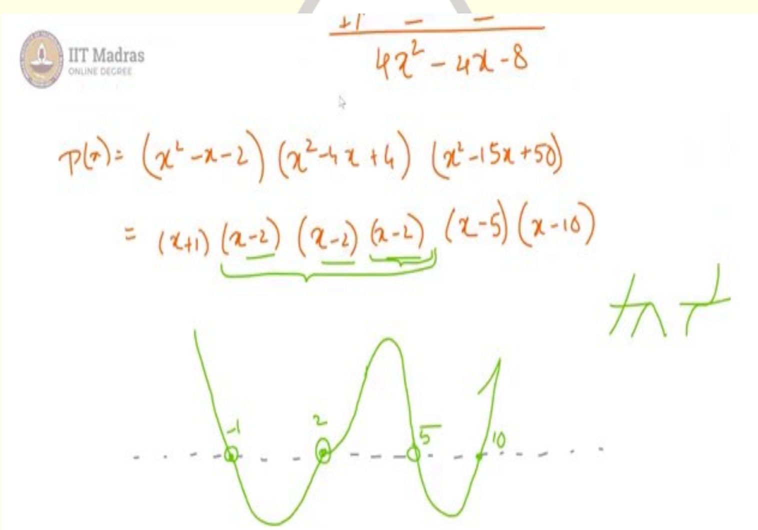

So let us start with $p(0)$, $p(0)$ is -8 which is clearly $\neq 0$, so 0 is not a root then we have $p\ (1)$ which is $1 - 5 + 6 + 4 - 8 = -2$ which is again $\neq 0$, so not a root. Then we try $p(-1)$ and we get $1 + 5 + 6 - 4 - 8$, so this is equal to $12 - 12 = 0$. So yes $p(-1)$ gives you 0 which means we have another root that is - 1.

So let us note down our roots that we have found here, roots we have found so far are 5, 10 and -1. Now going back to our trial and error let us try $p\ (2)$ and $p\ (2)$ gives us $16 - 8 \times 5 = 40 + 6 \times 4 = 24 + 8 - 8$. So we get $16 + 24 = 40$, $40 - 40 = 0$, so this is 0. So we have another root that we have found. So we now have two roots for our quartic and those two roots give us another quadratic which is $(x + 1)(x - 2)$ that is $x^2 - x - 2$.

So if we divide our quartic with quadratic we will get the other quadratic within it. So here we have $x^4 - 5x^3 + 6x^2 + 4x - 8$ and we divide it with $x^2 - x - 2$ so here go x^2 so $x^4 - x^3 + (m-2)x^2$ -,+ and + cancel this of you get $-4x^3 + 8x^2 + 4x - 8$.

And then we do $-4x$ times this, $-4x^3 + 4x^2 + 8x$, so + - and - cancel this and here we have $4x^2 - 4x - 8$. And that is just 4 times this, so + 4. So our quartic, so is basically x$(x^2 - x - 2)(x^2 - 4x + 4)$ and this gives the quartic and additionally we have to also multiply for our p of x we have to multiply the other quadratic which is $x^2 - 15x + 50$, this one.

So this is p of x totally, an if we further separate it out into all its roots we get this one as we know is x + 1 into x - 2 and this is if you notice x - 2 to the whole square, so $(x-2)(x-2)$ and then here this we have found the roots already which is $x - 5$ into what was the other root; the other root was 10, $x - 10$.

So these are our roots and the coefficient of x power 6 will be positive clearly. So therefore this is an even degree polynomial and thus if we have to sketch the graph it look something like this, it comes from infinity and what is the least lowest root here, the lowest root is - 1. So at - 1 if we draw this as the x axis at - 1 you have one root it crosses the x axis and then it goes around and it comes to 2.

But 2 is a triple root, so what happens with a root if it is a single root it crosses the x axis but if it is a double root it will just touch the x axis and come back but since it is a triple root it actually crosses the x axis. So here we do have a crossing and then afterwards at 5 and 10 we will have, so this will be for two this will be for - 1, this will be for 5 and this will be for 10. This is just a rough plotting of the graph.

The question was how many times does it intersect the x axis, so we have to draw this basic sketch and we find that the intersection are 4. If x - 2 was not a triple root, if it were a double root or a quadruple root like if it is there are 2 times or 4 times then the graph would be very different. It would be x - 1 would still be the same but at 2 you would not actually see a intersection, you will just see a touching. It would not be a cut.

So therefore we have to check how many times the $\sqrt{2}$ occurs. Since it is an odd number of times we can say it is actually cutting x axis and that gives us a number of level crossings is 4. And how many turning points are there? Now we can look at our graph and quickly tell; 1, 2 and 3; 3 turning points.