Statistics for Data Science -1

Lecture 10.3: Distribution of a Binomial random variable

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Learning objectives

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1. Derive the formula for the probability mass function for Binomial distribution.

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3. Expectation and variance of the binomial distribution.

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1. Derive the formula for the probability mass function for Binomial distribution.
2. Understand the effect of parameters n and p on the shape of the Binomial distribution.
3. Expectation and variance of the binomial distribution.
4. To understand situations that can be modeled as a Binomial distribution.

Binomial random variable

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Definition

X is a binomial random variable with parameters n and p that represents the number of successes in n independent Bernoulli trials, when each trial is a success with probability p. X takes values $0, 1, 2, \ldots, n$ with the probability

$$P(X = i) = \binom{n}{i} \times p^i \times (1 - p)^{(n-i)}$$

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  - Probability mass function

X	0	1	2	3
$P(X = x_i)$	$\binom{3}{0}\frac{1}{2}^0\frac{1}{2}^3$	$\binom{3}{1}\frac{1}{2}^1\frac{1}{2}^2$	$\binom{3}{2}\frac{1}{2}^2\frac{1}{2}^1$	$\binom{3}{3}\frac{1}{2}^3\frac{1}{2}^0$

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$P(X = x_i)$	$\frac{1}{8}$	$\frac{3}{8}$	$\frac{3}{8}$	$\frac{1}{8}$

Shape of the pmf for same n different p

A binomial distribution is

- right skewed if $p < 0.5$
- is symmetric if $p = 0.5$
- is left skewed if $p > 0.5$

. We demonstrate the same for $n = 4$ and different p

$n = 4, p = 0.3$, X = number of successes

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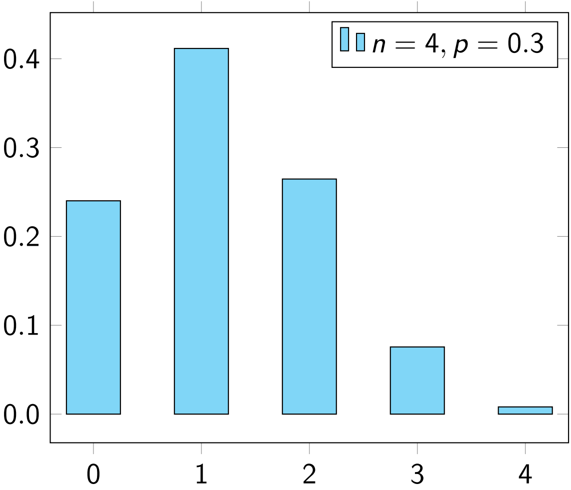
- The probability distribution of X

X	0	1	2	3	4
$P(X = i)$	0.2401	0.4116	0.2646	0.0756	0.0081

Graph of pmf of Binomial distribution- Right skewed



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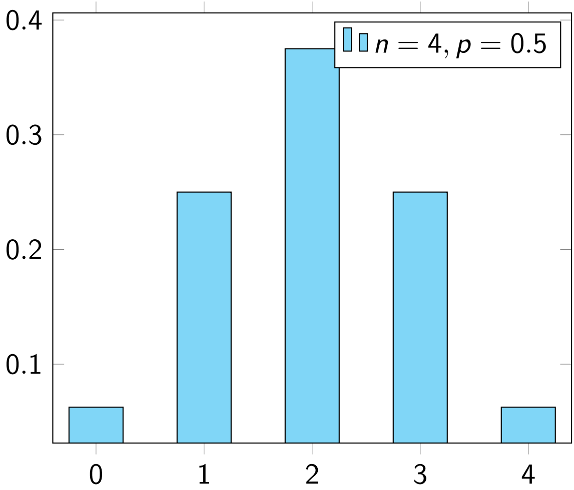
X	0	1	2	3	4
$P(X = i)$	0.0625	0.25	0.375	0.25	0.0625

Graph of pmf of Binomial distribution- symmetric



$n = 4, p = 0.8, X =$ number of successes

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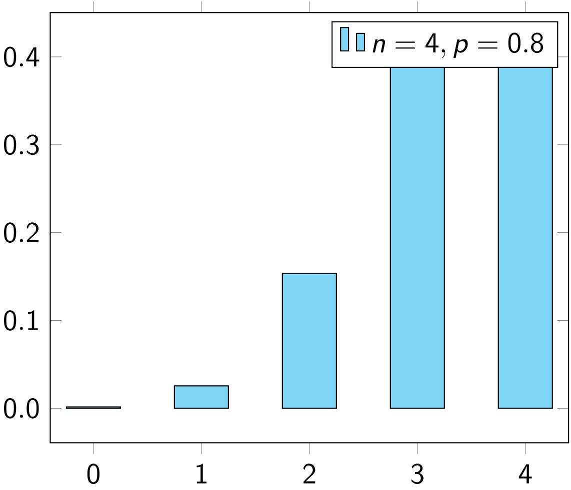
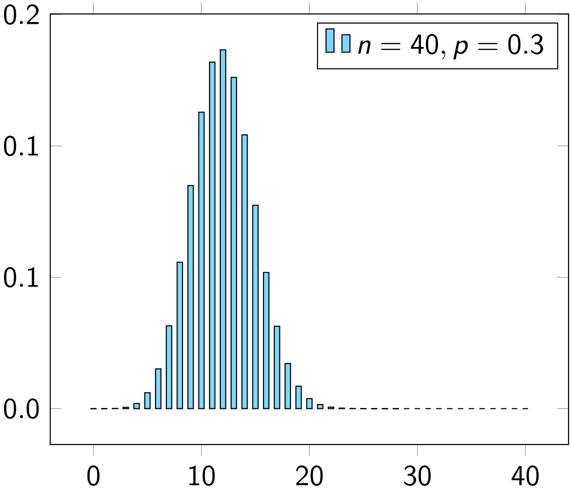
- The probability distribution of X

X	0	1	2	3	4
$P(X = i)$	0.0016	0.0256	0.1536	0.4096	0.4096

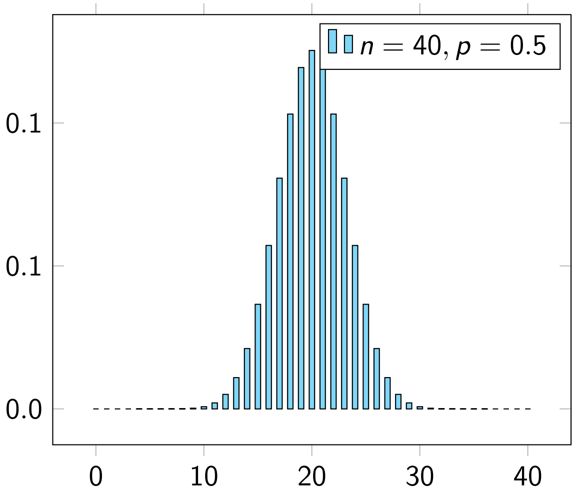
Graph of pmf of Binomial distribution- left skewed



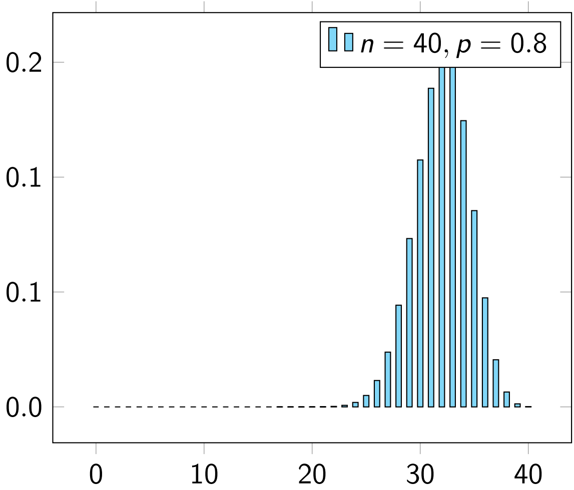
Graph of pmf of Binomial distribution- Right skewed- large n



Graph of pmf of Binomial distribution- symmetric-large n



Graph of pmf of Binomial distribution- left skewed- large n



Effect of n and p on shape of distribution

- small n, small p- right skewed
- small n, large p- left skewed
- small n $p = 0.5$- symmetric
- For large n, the binomial distribution approaches symmetry.

Section summary

- Introduced the Binomial random variable and its pmf.
- Studied effect of n and p on ths shape of the distribution.