IIT Madras

ONLINE DEGREE

Mathematics for Data Science 1
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Lecture No. 8.1
Additional lecture Inverse function

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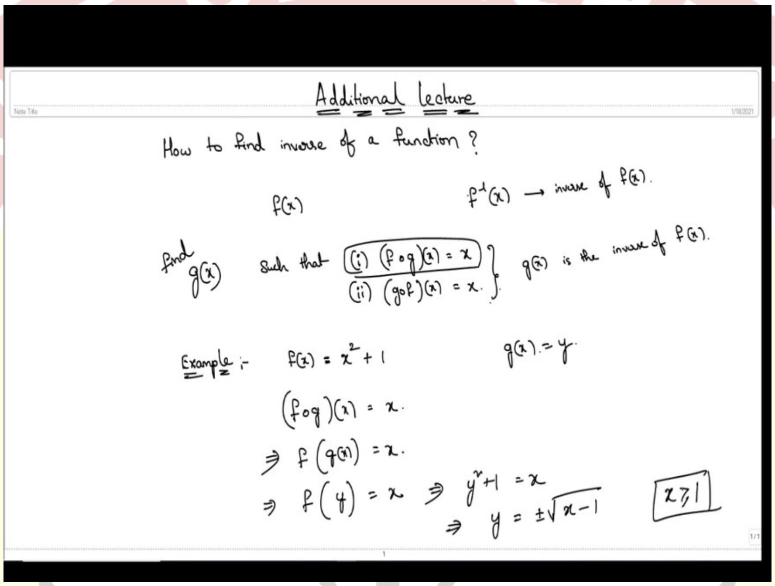

Hello everyone, today latest see how to find inverse of a function. Suppose $f(x)$ is a function, we try to find $f^{-1}(x)$ which is the inverse of $f(x)$, basically we try to find a function $g(x)$, we try to find $g(x)$ such that first $fog(x)$ will be x and also $gof(x)$ is x. So, when these two conditions are satisfied I can say, $g(x)$ is the inverse of $f(x)$. So, why do we need to check this two condition, why not only one is sufficient?

In order to find this let us take an example. Here is an example $f(x) = x^2 + 1$ we have to find the inverse of this function, let us say the inverse of this function is $g(x)$ let us assume. Now, we have this one condition, first condition is $fog(x) = x$, I will assume $g(x)$ =y. So, this implies $fog(x) = x$ and this imply f of if I substitute $g(x) = y$, $f(y) = x$ and I have $f(x) = x^2 + 1$.

So, this will imply $f(y) = y^2 + 1 = x$ if I take 1 to the right side and square root on both sides I will get finally $y = \pm\sqrt{x-1}$. So, in order to remove this complex ambiguities we take, we assume $x \geq 1$.

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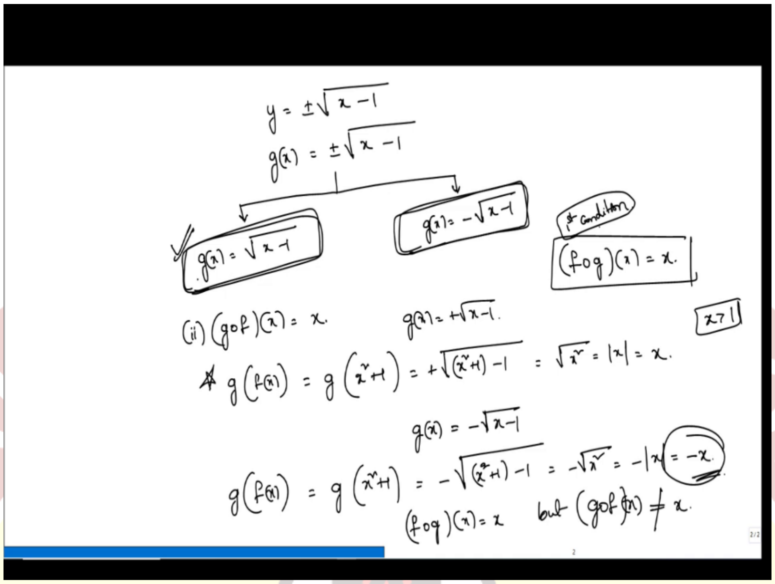

So, finally we have $y = \pm\sqrt{x-1}$ as we have assumed $y = g(x)$, so our finally $g(x)$ which is nothing but the inverse of the function $f(x) = \pm\sqrt{x-1}$,. So, I got two functions now, $g(x) = +\sqrt{x-1}$ and $g(x) = -\sqrt{x-1}$. So, we got these two functions that will satisfy the condition $fog(x) = x$ which is nothing but the first condition.

So, which one will be the inverse of $f(x)$? Whether $g(x) = \sqrt{x-1}$ or $g(x) = -\sqrt{x-1}$, that is why the second condition is also important which is our second condition is $gof(x) = x$. So, let us take two cases. Suppose my $gof(x) = -\sqrt{x-1}$. Now, $g\big(f(x)\big) = g$ of as I have $f(x) = x^2 + 1$that will be $g(x)^2 + 1 = \sqrt{x^2+1-1}$ which will be $\sqrt{x^2}$ which gives me $|x|$. As we have taken x>1, this will give me x. So, second condition is satisfied by this function $g(x)$.

Now, let us take the second case, where $g(x) = -\sqrt{x-1}$. Now, similarly I have $gof(x) = g(x)^2 + 1$ which is equals to $-\sqrt{x^2+1-1}$, which will be $-\sqrt{x^2}$, again the same thing $-|x| = -x$. So, if I take $g(x) = -\sqrt{x-1}$ my $fog(x)$will be x but my $gof(x) \neq x$. So, we can conclude that this function $g(x) = \sqrt{x-1}$ is the inverse of the function $f(x)$.

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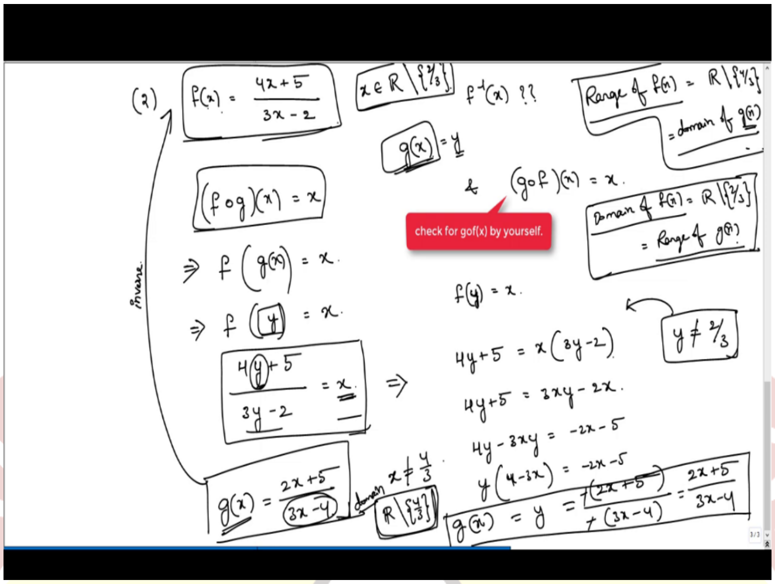

Now, let us take one more example $f(x) = \frac{(4x+5)}{(3x-2)}$. So, we have to find inverse of this function, so nothing but we have to find $f^{-1}(x)$, how do we find this? So, the same way let us assume there is a function $g(x)$ that is satisfying this condition $fog(x) = x$ and $gof(x) = x$.

Now, let us take this condition first, we have $f\big(g(x)\big) = x$. So, I will again assume $g(x) = y$, assume. So, this implies $f(y) = x$, so I have to find y such that my $f(y)$ giving me x. So, I have $f(x) = \frac{(4x+5)}{(3x-2)}$, so if I substitute now, I get $x = \frac{(4y+5)}{(3y-2)}$, so this is the condition I have and I have to find y in terms of x because y is a function of x, so y in terms of x and that y is nothing but $g(x)$ and that $g(x)$ is nothing but the inverse of this function.

So, if we solve this equation we get $4y + 5 = x(3y - 2)$. See if we are multiplying $3y - 2$ on both sides, that means we are assuming that $y \neq \frac{2}{3}$, so it is automatically comes in. Because the domain of this function f of x is x belongs to $\mathbb{R} - \frac{2}{3}$ so denominator should not be 0, so that is why the domain of this function will be $\mathbb{R} - \frac{2}{3}$.

So, now let us get back and solve this, we have $4y + 5 = 3xy - 2x$, so getting y terms on left side we get $4y - 3xy = -2x - 5$, so if I take $y(4 - 3x) = -(2x + 5)$, so ultimately we get $y = \frac{-(2x+5)}{-(3x-4)}$, so this get cancel and we get $\frac{2x+5}{3x-4}$ which will be the $g(x)$.

So, we got the function $g(x) = \frac{2x+5}{3x-4}$. So, this will be the inverse of the function $f(x)$, this is the inverse function. Now, if we see the domain of this $g(x)$, so if you see here the denominator if *3x -4* so it should not be equal to 0 that means x should not be equal to $\frac{4}{3}$, so the domain of $g(x)$will become set of real numbers $-\frac{4}{3}$, so this is the domain, this domain of $g(x)$ is nothing but the range of this function $f(x)$.

If we use this online graphing tools like the Desmos and put the function $f(x)$in that, we get an vertical asymptote at x is equals to $\frac{4}{3}$, that means $f(x)$ cannot take the value $\frac{4}{3}$. So, the range of this function $f(x)$, range of $f(x)$ will be set of $\mathbb{R} - \frac{4}{3}$ which will be the domain of $g(x)$ which is nothing but the inverse of the function $f(x)$. And we have one more thing this domain of $f(x) = \mathbb{R} - \frac{2}{3}$ which will be the range of $g(x)$.

So, finally this is how we find the inverse of any given function f of x and also the range of $f(x)$is equals to the domain of inverse of that function $f(x)$and the domain of that function $f(x)$ is equal to the range of the inverse of the function, of the function $f(x)$. Thank you.