

IIT Madras ONLINE DEGREE

Computational Thinking Professor Madhavan Mukund

Department of Computer Science

Chennai Mathematical Institute Professor G Venkatesh

Indian Institute of Madras Concept of Fair Teams

Professor Madhavan Mukund: So some of the students clearly are doing better than other

students. Maybe we should try and find the way where they can help each other, so that they can

pull each other up

Professor G. Venkatesh: Such students tend to study in groups, pairs so two people, two students

may decide that they can get together and study together, and usually when they study, when

they get together in pairs like that, they, when one student is able to help the other, one is better

than the other in something. So he is able to help the other student so...

Professor Madhavan Mukund: I think we should not get into situation where one student is

clearly better...

Professor G. Venkatesh: Better...

Professor Madhavan Mukund: Feels somewhat superior, so maybe...

Professor G. Venkatesh: Superior or just you know he feels does not getting anything out of it, so

he is spending all the time helping other person.

Professor Madhavan Mukund: Yeah

Professor G. Venkatesh: So it would be good if you find a pair of students in a study group

where one person, each person is able to help the other in something. Let us say, one student,

first student is able to help the second student in one subject. The second student is able to help

the first student in another subject.

Professor Madhavan Mukund: So you want to think, where say, one student has higher Maths

marks and the other have say...

Professor G. Venkatesh: Maths yeah

Professor Madhavan Mukund: For example, higher physics marks.

Professor G. Venkatesh: Physics marks, that might be a good one, so if...

Professor Madhavan Mukund: Then they can both get something out of it?

Professor G. Venkatesh: Both something, so first student can help the second student in Maths, second student can help the first student in Physics

Professor Madhavan Mukund: Okay.

Professor G. Venkatesh: That way they would help each other.

Professor Madhavan Mukund: So how do we find such pairs for example? So we want to find students who could be paired up, so we could suggest to them that if they work together they might be...

Professor G. Venkatesh: So we are actually looking for pairs of students where the Maths marks of the first student is higher than, significantly higher than the Maths marks of the second student. Wherein the Physics marks of the second student...

Professor Madhavan Mukund: Are significantly higher...

Professor G. Venkatesh: Higher than the Physics marks of the first student. We are starting with Maths and Physics. Of course we can do this for other subjects.

Professor Madhavan Mukund: Yeah, so let us say we do it for Maths and Physics. So we want this kind of opposite...

Professor G. Venkatesh: But you know what, if Maths marks of the first student is more than the Maths marks of the second, Physics marks of the second student is more than the Physics marks of the first then it is likely that the Maths plus Physics of both...

Professor Madhavan Mukund: Ah, so one is high, one is low in both cases. So they might have similar Maths plus Physics

Professor G. Venkatesh: Sums. Totals may be similar.

Professor Madhavan Mukund: Yeah, and also if we find people who have similar totals their

abilities are also probably like...

Professor G. Venkatesh: Roughly similar, right?

Professor Madhavan Mukund: Similar, so...

Professor G. Venkatesh: So should we look for that? Should we look for similar, so one, look for similar totals, so in fact actually if you are doing similar totals then it may, to pick a student we do not even need to compare two cards. So we can take one card with some total, let us say, 200 is a total, and if the person has a high Maths score must have a correspondingly lower Physics score, right? So just see, pick two people with similar totals in which one person has a high

Maths marks, other has a high Physics mark that should be enough I think.

Professor Madhavan Mukund: Yeah

Professor G. Venkatesh: Something like that. Looks like a heuristic.

Professor Madhavan Mukund: Yeah, so now we need to find pairs which have similar totals.

Professor G. Venkatesh: Similar totals how do we do that? Binning?

Professor Madhavan Mukund: Probably, yeah because otherwise because we will have to compare everybody's total with everybody others and then, so maybe we should put the totals in categories and then look into those categories. So we could probably decide, maybe...

Professor G. Venkatesh: 30 cards should we divide into say, 6 bins is that?

Professor Madhavan Mukund: Right from...

Professor G. Venkatesh: 5-6 bins?

Professor Madhavan Mukund: I think it is unlikely anybody has got less than 150 out of 300.

Professor G. Venkatesh: So 150 to 200

Professor Madhavan Mukund: Highest probably nobody has got more than about...

Professor G. Venkatesh: Somebody has got 280.

Professor Madhavan Mukund: Yeah 280 or so would be the highest.

Professor G. Venkatesh: So 150 to 280, so it is like 180 marks divide into 6 bins.

Professor Madhavan Mukund: 130 marks.

Professor G. Venkatesh: 130 marks.

Professor Madhavan Mukund: So maybe you can put in 20 mark bins.

Professor G. Venkatesh: 20 mark bins, will be 6 bins.

Professor Madhavan Mukund: Maybe we will start at the highest level, so 280?

Professor G. Venkatesh: 6 bins means each bin has 5 cards.

Professor Madhavan Mukund: So maybe we will start with 270, because I think very few students would have very high marks, so 270, 250, 230, 190 like that we can make.

Professor G. Venkatesh: You want to make it 5 bins because that way plus 6 cards, it is easier to pair, otherwise we will have one odd card in the bin.

Professor Madhavan Mukund: Fine, so what should we put as highest bin now in make 25 mark bins?

Professor G. Venkatesh: 25 mark bins.

Professor Madhavan Mukund: 150, 175, 200, 225, 250, 275. So let us start with 150 and make 25...

Professor G. Venkatesh: 25 mark bins.

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Professor Madhavan Mukund: So this is 250 to 275 bin.

Professor G. Venkatesh: Okay.

Professor Madhavan Mukund: This is a 220, 200 to 225 bin, this is 150 to 175 bin.

Professor G. Venkatesh: Okay.

Professor Madhavan Mukund: This is again in this 250 to 275 bin. This is a bin which comes in between these two, so we should probably shift it a little bit. So this is 175 to 200.

Professor G. Venkatesh: Right.

Professor Madhavan Mukund: This is 200 to 225, so this is as same as this one. 188 is in the same bin as 189. 187 also is in the same bin. 227 is above 225 so we should put it in this bin. 276 is above 275 so this is maybe we will keep it down here as in 6. So these are now 6 bins. I do not think we have any more new bins coming. This is below 225 so this should go in this bin. This is again between 200 and 225 so this goes in this bin. 247 is below 250 so it should go in this fourth bin. 281 is in the highest bin. Now 209 is in this, 200...

Professor G. Venkatesh: So I would say that, this particular bin which is the highest bin, no need to no need to...

Professor Madhavan Mukund: They must be getting...

Professor G. Venkatesh: Already doing well.

Professor Madhavan Mukund: An average of more than 90.

Professor G. Venkatesh: So we do not have to worry with that, yeah.

Professor Madhavan Mukund: 240 goes in the bin here, 244 is also in that bin. 254 goes in this bin. 219 goes in this bin. 216 goes in this bin. 227 goes in this bin. 174 is in the lowest bin. 224 is in the upper end of this bin. 181 is in the second bin. 196 in the second bin, 198 is in the second bin. 250 is at the borderline.

Professor G. Venkatesh: Borderline.

Professor Madhavan Mukund: But I think we had 251 probably, so we will put it here in this part.

Professor G. Venkatesh: Sure.

Professor Madhavan Mukund: 210 will go in this bin. 240 will go in this bin. And 261 will go in this bin. So it looks like these three bins have the biggest numbers. As you said, the top bin probably is not...

Professor G. Venkatesh: We do not need to pair.

Professor Madhavan Mukund: They do not require any help. They got almost 90 percent or more in everything.

Professor G. Venkatesh: So we will keep them aside.

Professor G. Venkatesh: Yeah.

Professor Madhavan Mukund: So let us just pick one of these. For example, let us pick this 198

Professor G. Venkatesh: Okay,

Professor Madhavan Mukund: So what we are looking for, let us keep these bins carefully so we do not disturb them. So what we are looking for in this are people who have, say high marks in one subject and...

Professor G. Venkatesh: So what we said, we said basically because the totals are roughly the same. Of course there are 3 subjects, we did not total Physics, (Chem) and Maths, so we do not know whether heuristic will be correct or not. But our guess is that, since the totals are roughly the same, if one person has high Physics marks, the other person will have high Maths marks

Professor Madhavan Mukund: Yeah.

Professor G. Venkatesh: Something like that.

Professor Madhavan Mukund: So let us do one thing. So maybe we can just group it into those who are higher Maths and Physics, and higher Physics and Maths because there should be two groups.

Professor G. Venkatesh: Yeah

Professor Madhavan Mukund: So this person has higher Maths than Physics.

Professor G. Venkatesh: Okay.

Professor Madhavan Mukund: And this person has higher Physics than Maths. This person is actually equal. So this person has also higher Physics than Maths. This one is marginally higher in Maths. This one is significantly higher in Maths. So now since this student and one of these may be, these two students could possibly help each other.

Professor G. Venkatesh: So 74, 56; 78, 64, so looks like a good pair, right?

Professor Madhavan Mukund: Yeah.

Professor G. Venkatesh: Because it is good, significant difference in their Maths marks. So we could consider pairing these two. Consider.

Professor Madhavan Mukund: Here the difference is not much; there also the difference is not much. But maybe these two could.

Professor G. Venkatesh: Could be paired, 62, 52; 64; 45 looks good.

Professor Madhavan Mukund: So these are two reasonable pairs. Of course now what we are doing is we are putting students who are doing not so well but in a way...

Professor G. Venkatesh: It is all right I think because they will be....

Professor Madhavan Mukund: I think psychologically it might help

Professor G. Venkatesh: Help, correct. You know, even though they are not doing well, like Physics 78 is not bad.

Professor Madhavan Mukund: So we found two such things in this group.

Professor G. Venkatesh: Let us try the others. Let us keep this aside.

Professor Madhavan Mukund: These two, we can say, you know we can pair them up but it may not have any significant impact. So let us look at this one. So again we do the same thing. So we keep higher in Physics, higher in Physics, marginally higher in Maths, reasonably higher in Maths, higher in Maths, higher in Maths, higher in Maths, higher in Physics and higher in Maths.

Professor G. Venkatesh: There are more here.

Professor Madhavan Mukund: Yeah so now there are more people with high Maths marks but again if you look at this, there are two students who have a big difference between Physics and Maths, 80 plus and around 60. So we can look for people who have a big difference in the Maths marks and try to pair them up.

Professor Madhavan Mukund: So nobody has such a significant difference, but at least with a higher difference. So this is 9 marks, this is quite a big.

Professor G. Venkatesh: 76 and 58.

Professor Madhavan Mukund: This possibly can go with one of them. So this is a good pair. So we can pick up any of these ones.

Professor G. Venkatesh: So this is, why this is a good pair, because Maths 76, 62 and here Physics 81, 58.

Professor Madhavan Mukund: Yes.

Professor G. Venkatesh: So, very good pair.

Professor Madhavan Mukund: So roughly the total is the same, so they kind of are equally different in the two subjects. So now, in terms of that kind of a difference, possibly we have, so this comes from that pile actually. So is there a person who we can pair with 88 and 63, well?

Professor G. Venkatesh: This may be okay, 78, 63; 88, 69, seems like okay.

Professor Madhavan Mukund: So let us keep these two together.

Professor G. Venkatesh: 78, 63 is big difference. 88, 69 I think they will help each other, definitely, yeah.

Professor Madhavan Mukund: Now we have this person who has a marginal difference. Maybe we can take somebody who has a similar marginal difference. This is 72, that is 72 in Maths and Physics, and 64 in Maths.

Professor G. Venkatesh: 66 yeah.

Professor Madhavan Mukund: So these are fairly closely

Professor G. Venkatesh: Reasonably good match, yeah.

Professor Madhavan Mukund: And all these three now have higher Maths marks so we will keep them aside for now. And this is the other biggish pile. So let us see. So now here again, Physics is higher, Physics is marginally higher, Physics is higher, Maths is higher, Maths is higher, Physics is much higher, Maths is marginally higher. So here I think we can find some pairs. So you get...

Professor G. Venkatesh: 87, 64 yeah.

Professor Madhavan Mukund: So 89, 62 for example any of these could work. So 89, 62 could work, or 87, 64 could work with this.

Professor G. Venkatesh: Yeah let us put it.

Professor Madhavan Mukund: Take this pair.

Professor G. Venkatesh: So you see here, 92 can help 64, 87 can help 71.

Professor Madhavan Mukund: Yes.

Professor G. Venkatesh: Seems okay, actually.

Professor Madhavan Mukund: And here we could take, may be this pair, so one has got 80 plus in one of the subjects and....

Professor G. Venkatesh: Yeah, 89, 74; 83, 62 again seems to be good pair, yeah.

Professor Madhavan Mukund: This is a bit marginal but we could, if we want, pair these two.

Professor G. Venkatesh: 74, 65; 76 yeah, okay.

Professor Madhavan Mukund: Physics is not really low but they are at least of similar, or we can just say we do not need to pair them.

Professor G. Venkatesh: Keep it, yeah.

Professor Madhavan Mukund: So these are the three big piles and we managed to find at least 5-6 pairs.

Professor G. Venkatesh: Lot of pairs, actually.

Professor Madhavan Mukund: 6-7 pairs. As we go higher up, again it is not here how much we need because somebody has got 260, 250 plus, then Maths, Physics, so Physics is higher there, Maths is higher there, Physics is higher here. These two have higher Physics and these two have higher Maths. So of course at this level of achievement, probably these two are a good pair, 92, 93; 72, 68. We could probably pair them up.

Professor G. Venkatesh: High achiever pair.

Professor Madhavan Mukund: High achiever pair. Here again it is not that they are doing badly but...

Professor G. Venkatesh: They could help each other.

Professor Madhavan Mukund: Significant difference between the two, so maybe we can pair.

Professor G. Venkatesh: May be they can pair, yeah.

Professor Madhavan Mukund: And now we said the highest one, we need not to bother

Professor G. Venkatesh: This one we do not need to.

Professor Madhavan Mukund: Lowest one we have two but both of them have done higher in Physics. So there is not much we can achieve there. But we could possibly look at across now if you want to do that.

Professor G. Venkatesh: Yeah.

Professor Madhavan Mukund: To the next group for instance. So this is the first group. And this was the people we left out...

Professor G. Venkatesh: They have roughly similar marks in Maths and Physics; we can look at Chemistry even that is another thing to do.

Professor Madhavan Mukund: Of course that is other thing, so we can see if, so here for instance Physics and Chemistry there is a natural pairing, because this is 78 and 72, 58 and 53.

Professor G. Venkatesh: Yeah.

Professor Madhavan Mukund: So these two can help each other.

Professor G. Venkatesh: Physics and Chemistry.

Professor Madhavan Mukund: Physics and Chemistry, and this also, for example this person can help in Physics and in both, I mean in Chemistry and the person on the left...

Professor G. Venkatesh: Maths and Physics

Professor Madhavan Mukund: Both Maths and Physics, so maybe that is a good pair we can put them together anyway. Now let us look at these three. Again if there is, so this person is very weak in Chemistry.

Professor G. Venkatesh: Chemistry.

Professor Madhavan Mukund: This person is good in Chemistry and this person is much better in Maths and Physics. So this may be a good pair. You can pair them up. And now we come to this

last, so let us see, among these four if we can find a good pair. So, so there is a very weak Chemistry mark here and this person actually has higher Maths marks than this.

Professor G. Venkatesh: So again these two we should be picking.

Professor Madhavan Mukund: But even here, this might also be okay because there is even bigger gap in the Maths marks.

Professor G. Venkatesh: Actually this is probably better.

Professor Madhavan Mukund: So probably we can choose that. And here actually this person is uniformly better so in a sense this is not a good pair from our perspective. So I guess this is inevitable that...

Professor G. Venkatesh: Not everything can be paired.

Professor Madhavan Mukund: Not everything can be paired in a good...

Professor G. Venkatesh: But we got almost, you know when, except for these two who were achievers, high achievers...

Professor Madhavan Mukund: Yeah.

Professor G. Venkatesh: Of course we could try one more iteration and see whether is a slightly better fit by doing some exchanges.

Professor Madhavan Mukund: Yeah because in the beginning we were not looking at chemistry at all, we could have looked at, among all three subjects which two to select, it would have been a little more complicated to pair up, but I think this idea of grouping them first by total...

Professor G. Venkatesh: Binning them by total marks.

Professor Madhavan Mukund: Of course, as you said we binned by the total and then looked at Maths, Physics may be if we had taken a subtotal of Maths and Physics we would have found a better pairing.

Professor G. Venkatesh: Better match, better matching.

Professor Madhavan Mukund: Yeah.

Professor G. Venkatesh: So what did we do? Just let us reflect on what is the procedure? What method is this? Is this an algorithm? We want to make study groups. So what are we doing?

Professor Madhavan Mukund: So we are, yeah so we are trying to find good combinations within a group of, in this case 30 could be any number.

Professor G. Venkatesh: Good pairs, fine.

Professor Madhavan Mukund: Kind of good pairs.

Professor G. Venkatesh: But we are not blindly searching for all pairs because if you take 30, again as we saw last time....

Professor Madhavan Mukund: Yeah.

Professor G. Venkatesh: If we have to make all pairs that is 30 into 30, 900, and divided by 2 because A and B, A compared to B and B compared to A here are the same. So 900 by 2 will give you 450.

Professor Madhavan Mukund: Yeah.

Professor G. Venkatesh: 450 pairs is a lot of pairs to look at. We are not doing that.

Professor Madhavan Mukund: Yeah so we are initially grouping them so that we are only looking at pairs which are...

Professor G. Venkatesh: Within totals bins...

Professor Madhavan Mukund: Yeah within a close range of each other and this case...

Professor G. Venkatesh: Binning seems to be a generic method. It seems to be a method of reducing the comparisons because otherwise you get too many comparisons. So last time for date we decided to bin by month. So here we are binning by range of total marks.

Professor Madhavan Mukund: Yeah, and we could have used binning in a different way. See for instance, we could have really decided, so we argued that it is good for students to help each

other mutually. But we could also have taken the bins and said that, take the students with the highest totals and pair them with the students of the lowest totals.

Professor G. Venkatesh: Totals.

Professor Madhavan Mukund: Because they will get the maximum benefit

Professor G. Venkatesh: The lowest total will get the benefit.

Professor Madhavan Mukund: Yeah so still we are pairing across two bins but in a limited way and not across everything.

Professor G. Venkatesh: Yeah.

Professor Madhavan Mukund: So once we bin, I think we have different strategies we can use as to how to use those bins. And we also had some flexibility about choosing the bins. I mean, we decided on these 6 bins, but we could have chosen 8 bins or 4 bins, whatever...

Professor G. Venkatesh: 4 bins. So we said okay so we do not want to make all pairs because that looks really, I mean it looks that we are doing without planning, without thought. So if we apply a little bit of thought then we come to the idea that if you bin it then the number of comparisons reduces. So we binned it and after we got, we got 6 bins and then in each bin we took all the cards and then again we did not compare, we did something, even inside the bin we did something.

Professor Madhavan Mukund: Yeah so we knew that we wanted to pair across 2 subjects so we separated into the groups where one is higher and the other is higher. So we had...

Professor G. Venkatesh: One is high in Maths and one is high in Physics.

Professor Madhavan Mukund: So then we had to only look at...

Professor G. Venkatesh: Pairs between the two.

Professor Madhavan Mukund: Between these two.

Professor G. Venkatesh: So if we had 8, so let us say you had 6 or 8 in one bin, again you split it into 4 each.

Professor Madhavan Mukund: Correct.

Professor G. Venkatesh: So now you have to do only 4 into 4 comparisons, 16.

Professor Madhavan Mukund: Yeah instead of 8 into 8, 64 or something.

Professor G. Venkatesh: So that way you bring the, this is the way to bring the number of comparisons down, is it not? So you are trying to, the idea is to try and bring down the number of comparisons by doing something which is slightly bit more smart. Looking at...

Professor Madhavan Mukund: And of course this is also an example I guess, of a problem where there is no correct answer. It is not like adding up 20 numbers and getting the total where you know the total like, I mean...

Professor G. Venkatesh: Yeah, that is true. Here any pairing, I mean, so long as the pairing is...

Professor Madhavan Mukund: Reasonable.

Professor G. Venkatesh: Reasonable, there could be many choices, many solutions for this. There are many possible...

Professor Madhavan Mukund: First, decide what is the reasonable choice, and then according to that work towards the solution which is close to that, achieving that criterion.

Professor G. Venkatesh: Of course there could be, see if you try to measure whether the pairing is a good pairing or not, there could be an absolute best pairing in this also.

Professor Madhavan Mukund: Yes, we could for instance have asked that, you know the total should be as close to each other as possible and the difference in the marks should also be as close to each other as possible.

Professor G. Venkatesh: So you could find the best possible pairing among all pairings.

Professor Madhavan Mukund: Yes.

Professor G. Venkatesh: But that is a, looks like a very complicated problem.

Professor Madhavan Mukund: That could have required us to possibly look at all the possible pairings.

Professor G. Venkatesh: All possible pairings.

Professor Madhavan Mukund: Because even here there is no guarantee, we might have taken a 20 mark bin and everybody might have been in the same bin. So binning is not a guarantee that you will get a smaller number of comparisons but it is reasonable to expect when you have natural data like this that they will be spread across possible range. The other thing I think we should have realized is that we saw that there was grouping in the center.

Professor G. Venkatesh: There are more cards in the middle. So it is a kind of...

Professor Madhavan Mukund: So we need not have had a, we need not have even made the bins equal.

Professor G. Venkatesh: 25, in fact.

Professor Madhavan Mukund: We could have said you know...

Professor G. Venkatesh: At the edges...

Professor Madhavan Mukund: Yeah, at edges we could have had 50 mark bin and may be had 10 mark bins in the middle. That could have been a finer thing.

Professor G. Venkatesh: Typically, I mean, in statistics we learn that any distribution of marks will follow a kind of a Gaussian. Some curve like this.

Professor Madhavan Mukund: Towards the average it will be bunched and there will be fewer people at the extremes.

Professor G. Venkatesh: Extremes, so extremes you make the bins the little wider and...

Professor Madhavan Mukund: Yeah so if you want be bins of equal size...

Professor G. Venkatesh: Equal in terms of number...

Professor Madhavan Mukund: Width of the bins should be of unequal size.

Professor G. Venkatesh: Unequal size.

Professor Madhavan Mukund: They must be narrower as you come to the populated part and wider, in the extremes you are not so fussy in some sense about.

Professor G. Venkatesh: But you could do that dynamically also. You could start with assuming, first you make into 4 bins let us say, and you find that some bin is becoming too large, split it into 2 bins. Then it will dynamically come.

Professor Madhavan Mukund: You could do that also. That is right,

Professor G. Venkatesh: Right?

Professor Madhavan Mukund: Yes

Professor G. Venkatesh: So let us say our overall target is to get to about 6 bins.

Professor Madhavan Mukund: Yeah and then you suddenly find, for instance that half of them are fitting in one bin. Then you could take that bin and subdivide that into, say 5 mark bins or 10 mark bins or something.

Professor G. Venkatesh: You could usually start with 3 bins or 2 bins even, 3 bins and then after that you start dividing those bins and large you start dividing it further.

Professor Madhavan Mukund: And good thing about binning is you can do a binning in one pass. You do not have to do this pair wise comparison. So you have to do, 3 times you repeat it; you are doing only a 3.

Professor G. Venkatesh: 3 times you are doing.

Professor Madhavan Mukund: Regardless whether you have 30 cards or 300 cards, it is only 3 passes.

Professor G. Venkatesh: 3 passes. But within a bin, separating again into high marks and high marks again is 1 pass.

Professor Madhavan Mukund: Yes.

Professor G. Venkatesh: But when you compare you are comparing, so for that comparison between the two....

Professor Madhavan Mukund: Yeah finally you have to do...

Professor G. Venkatesh: A nested iteration.

Professor Madhavan Mukund: But we want to cut that number of pairings that we have to check explicitly down to a minimum.

Professor G. Venkatesh: Very small number. So here what are we doing, so basically we are doing one pass preparing data, some kind of preparation we have done, once you have prepared then after that we do, in the final round we are doing a nested iteration.

Professor Madhavan Mukund: Yes.

Professor G. Venkatesh: So we are doing, because to compare you need 2 iterations, one inside the other.

Professor Madhavan Mukund: So we are only doing two sets of preparation. So we do one overall binning preparation; within each bin we do again preparation of separating into a which is higher, so if you combine this, there is one round of preparation.

Professor G. Venkatesh: Preparation.

Professor Madhavan Mukund: And then we, yeah within each such group we then do a pair wise.

Professor G. Venkatesh: Pair wise.

Professor Madhavan Mukund: Good. So that is something worth remembering for the future.