Outline

Sunday, 9 August 2020 9:51 AM

Exponential Functions

- One-to-One Functions.
- Exponential Function
- The Natural Exponential Function.

One-to-One Functions

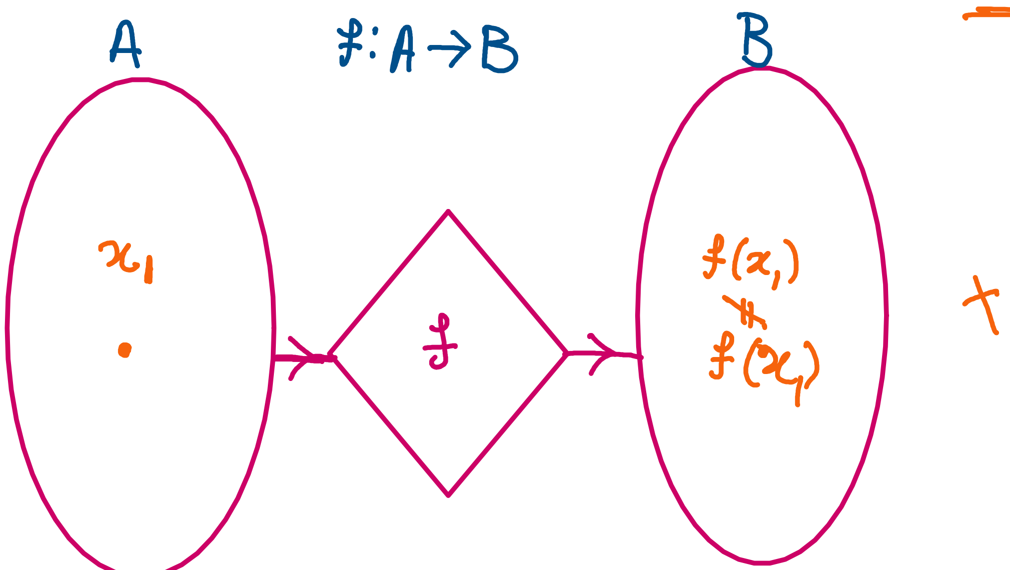
10 March 2020 08:28

$y = f(x)$ ✓

$f: A \to B$

$A, B \subseteq \mathbb{R}$

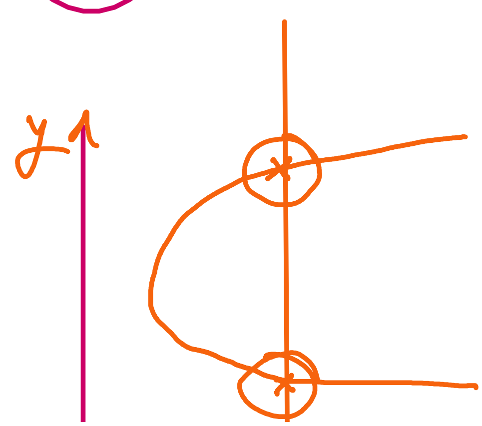
	Domain (A)	Codomain (B)	
✓1	One x	More than one $f(x)$	? Not a function
✓2	More than one x	One $f(x)$	? It is a function but it is not reversible
✓3	One x	One $f(x)$	? It is a function It is reversible

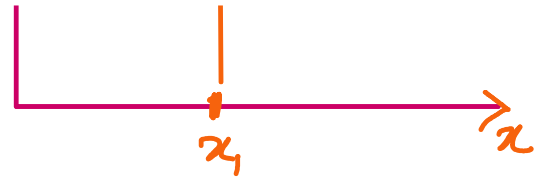


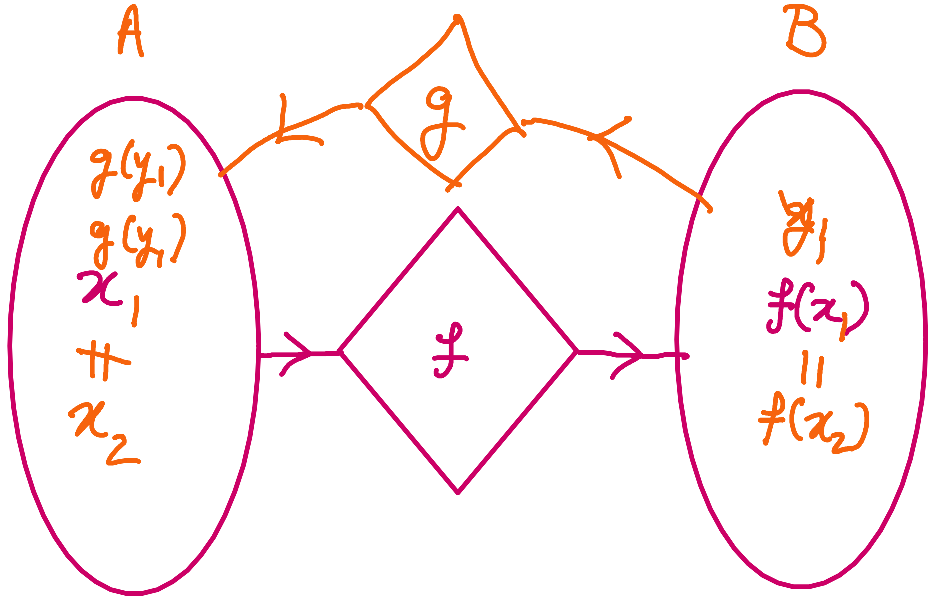
Vertical Line Test

$x = \text{const.}$

Vertical line test fails

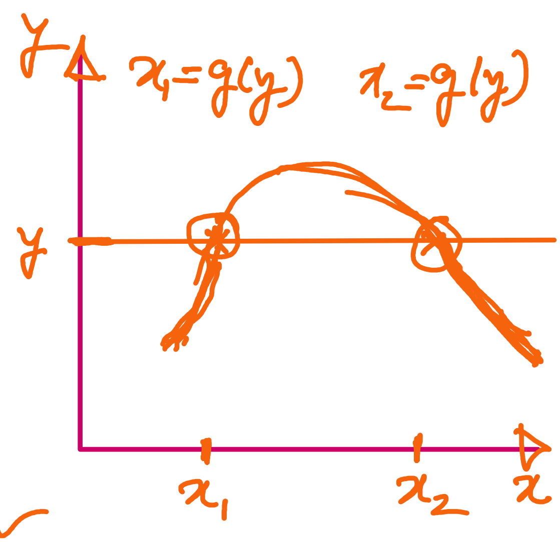






Horizontal Line Test

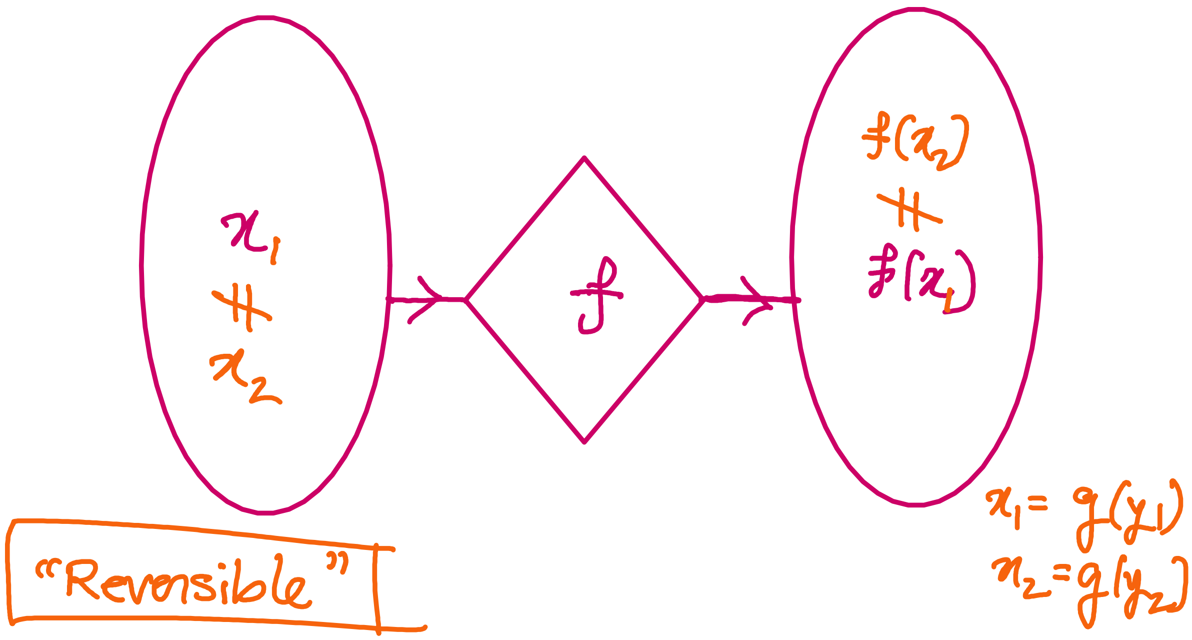
$$y = f(x) \not\Rightarrow x = f(y)$$

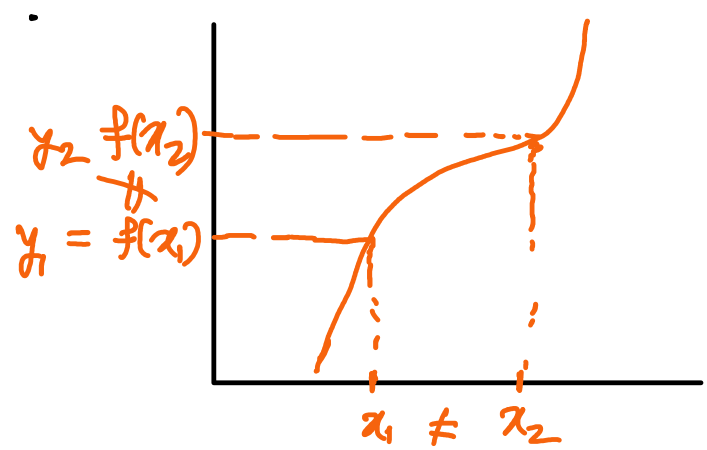


$\boxed{\text{Not 'Reversible'}}$ (?)

Horizontal line test fails. ✓

· Observe f is NOT "Reversible"





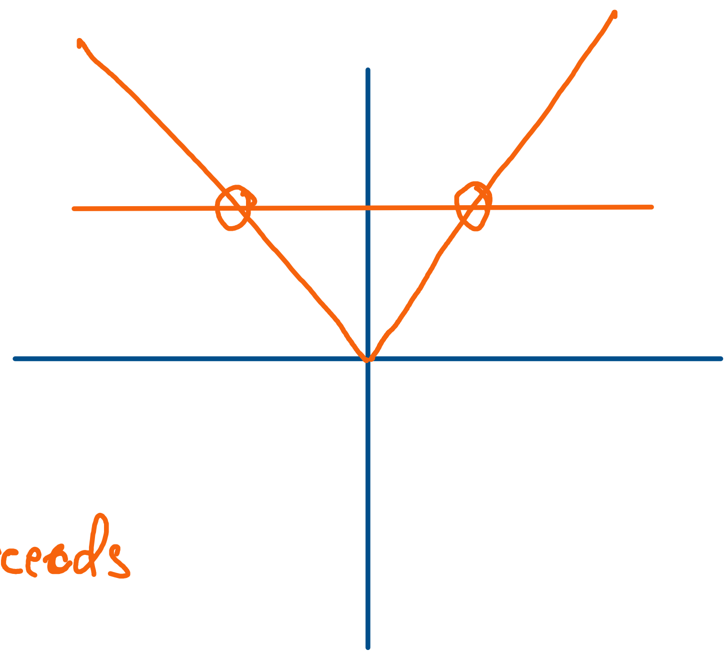
<u>Definition</u> (One-to-One Function)

A function $f: A \to B$ is called one-to-one $\boxed{\text{if}}$, for any $x_1 \neq x_2 \in A$, $\boxed{\text{then}}$ $f(x_1) \neq f(x_2)$.

$$\boxed{\begin{array}{l} f(x_1) = f(x_2) \\ \Rightarrow x_1 = x_2 \end{array}}$$

Example.

$$f(x) = |x|$$
$$= \begin{cases} x, & x \geq 0 \\ -x, & x < 0 \end{cases}$$



Vertical line test succeeds

$2, -2$

$f(2) = 2 = f(-2)$

$\boxed{\text{NOT one-to-one}}$

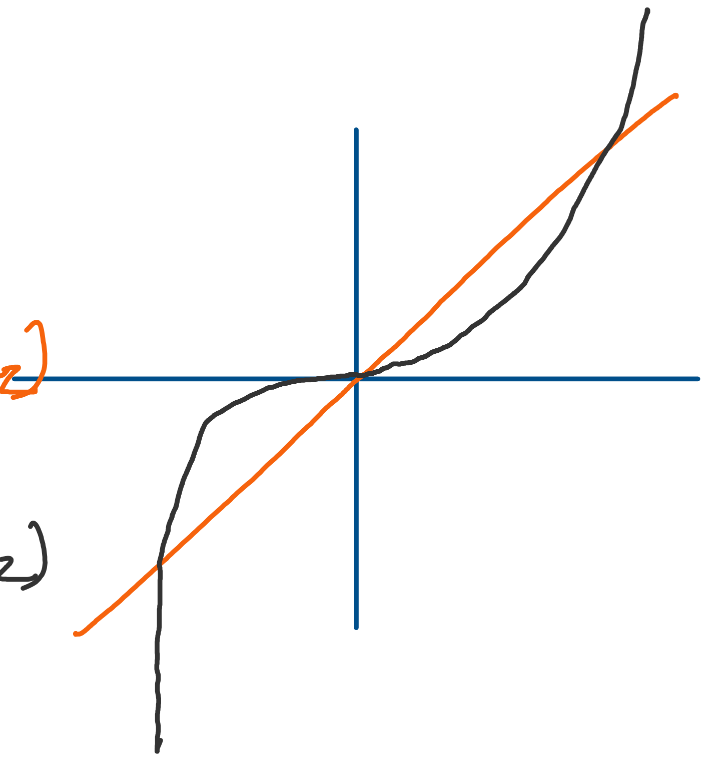
Example

$f(x) = x$

$x_1 \neq x_2 \Rightarrow f(x_1) \neq f(x_2)$

$f(x) = x^3$

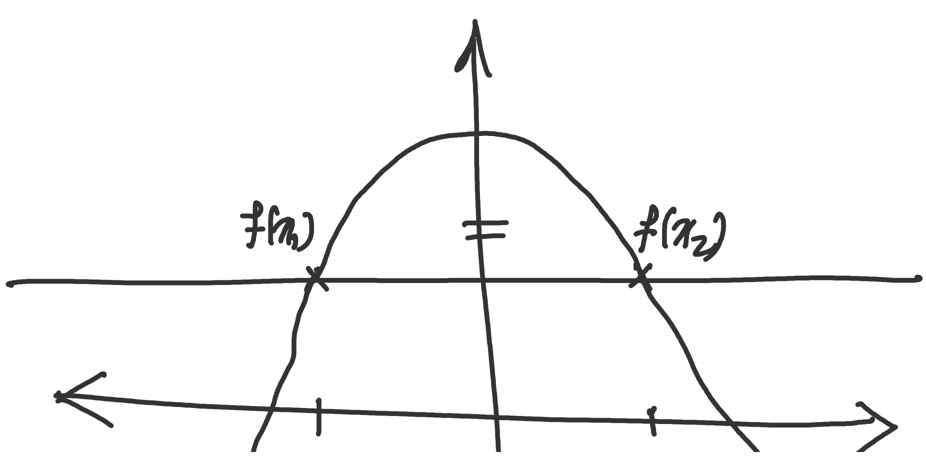
$x_1 \neq x_2 \Rightarrow f(x_1) \neq f(x_2)$



Theorem. (The Horizontal Line Test)

If any horizontal line intersects the graph of a function f in at most one point, then f is one-to-one.

Proof.

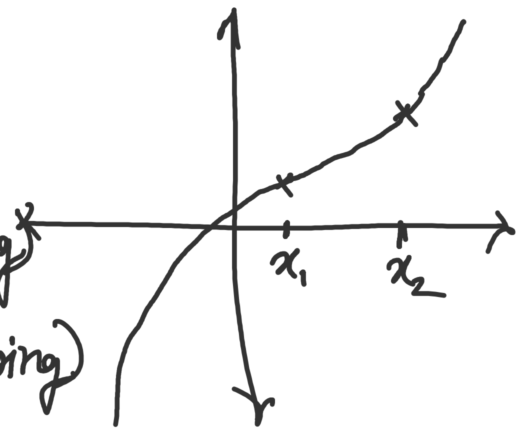


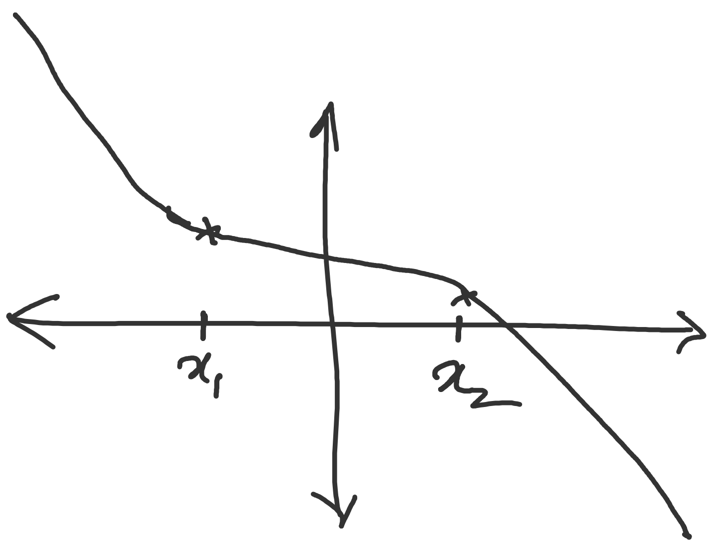
$\boxed{\text{Q.}}$ Can we identify the class of functions that are one-to-one?

For every $x_1, x_2 \in A$,

$x_1 \lneq x_2 \Rightarrow f(x_1) \lneq f(x_2)$ (increasing)

$x_1 < x_2 \Rightarrow \underline{f(x_1) > f(x_2)}$ (decreasing)





Theorem.

$\boxed{\text{If}}$ f is an increasing or decreasing function then f is one-to-one.