



**IIT Madras**  
ONLINE DEGREE

## Mathematics for Data Science 1

### Week 05 - Tutorial 04

(Refer Slide Time: 0:14)

- $a$   $b$   $c$
4. If  $5x^2 + 8x + 1 = 0$ , then answer the following.
- Find the roots of above equation.
  - Calculate sum and product of roots.
  - Prove that sum and product of roots for any quadratic equation  $ax^2 + bx + c = 0$  will be  $-\frac{b}{a}$  and  $\frac{c}{a}$  respectively.

$$\begin{aligned} & \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \Rightarrow \frac{-8 \pm \sqrt{64 - 20}}{10} \\ & \frac{-8 + \sqrt{44}}{10} \quad \text{and} \quad \frac{-8 - \sqrt{44}}{10} \\ & = \frac{-4 + \sqrt{11}}{5} \quad \text{and} \quad \frac{-4 - \sqrt{11}}{5} \end{aligned}$$

This is the pretty straight forward question, we are given a quadratic equation and we are asked to find the roots and also calculate the sum and product of roots. So, the roots we are going to get from the formula again which is  $\frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$ . So, that gives us, here in this case  $a$  is 5,  $b$  is 8 and  $c$  is 1. So, we have  $\frac{-8 \pm \sqrt{64 - 4 \times 5}}{10}$ .

So, we have  $\frac{-8 + \sqrt{44}}{10}$  and  $\frac{-8 - \sqrt{44}}{10}$ . And if we simplify it taking 2 common out, you will get  $\frac{-4 + \sqrt{11}}{5}$  because the 4 comes out of the square root and becomes 2. And  $\frac{-4 - \sqrt{11}}{5}$ .

(Refer Slide Time: 1:44)

$$\begin{aligned}
 & \frac{-8 + \sqrt{44}}{10} \quad \text{and} \quad \frac{-8 - \sqrt{44}}{10} \\
 & = \frac{-4 + \sqrt{11}}{5} \quad \text{and} \quad \frac{-4 - \sqrt{11}}{5} \\
 & \frac{-4 + \sqrt{11}}{5} + \frac{-4 - \sqrt{11}}{5} = -\frac{8}{5} \quad (\text{Sum}) \\
 & \left(\frac{-4}{5}\right)^2 - \left(\frac{\sqrt{11}}{5}\right)^2 = \frac{16 - 11}{25} = \frac{1}{5} \quad (\text{Product})
 \end{aligned}$$

And the sum of these roots is if you just add them up you will get  $\frac{-4}{5} + \frac{\sqrt{11}}{5}$ . So, these get canceled so you get  $\frac{-8}{5}$  is the sum. And in terms of product you basically doing  $(a + b) \times (a - b)$  so you will get  $\left(\frac{-4}{5}\right)^2 - \left(\frac{\sqrt{11}}{5}\right)^2$ . So, that gives us  $\frac{16 - 11}{25} = \frac{1}{5}$ . So, this is the product of the roots.  $\frac{-8}{5}$  and  $\frac{1}{5}$  are just sum and product of the roots respectively.

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$$\begin{aligned}
 & \frac{-b + \sqrt{b^2 - 4ac}}{2a} \quad \text{and} \quad \frac{-b - \sqrt{b^2 - 4ac}}{2a} \\
 & = \frac{-2b}{2a} = -\frac{b}{a} \quad (\text{Sum of roots})
 \end{aligned}$$

$$\begin{aligned}
 & \left( \frac{-b}{2a} \right)^2 - \left( \frac{\sqrt{b^2 - 4ac}}{2a} \right)^2 \\
 &= \frac{b^2}{4a^2} - \frac{(b^2 - 4ac)}{4a^2} \\
 &= \frac{\cancel{b^2} - \cancel{b^2} + 4ac}{4a^2} = \frac{c}{a} \\
 & \quad \text{[Product]}
 \end{aligned}$$

The question is asking us to prove that the sum and product of roots for any quadratic equation will be this and this respectively.  $\frac{-b}{a}$  and  $\frac{c}{a}$  respectively. So, all we need to do for this is to sum  $\frac{-b}{2a} + \frac{\sqrt{b^2 - 4ac}}{2a}$ . This we are summing with  $\frac{-b}{2a} - \frac{\sqrt{b^2 - 4ac}}{2a}$ . So, clearly these two cancel off and you are left with  $\frac{-2b}{2a}$ , 2 and 2 cancel off and you have  $\frac{-b}{a}$  is sum of roots.

And when we do the product again it is in the  $(a + b)(a - b)$  form so we will get  $\left(\frac{-b}{2a}\right)^2 - \left(\frac{\sqrt{b^2 - 4ac}}{2a}\right)^2$  which gives us  $\frac{b^2}{4a^2} - \frac{(b^2 - 4ac)}{4a^2}$ . So, that gives us  $\frac{b^2 - (b^2 - 4ac)}{4a^2}$ ,  $b^2 - b^2$  cancels off then we have 4 4 going away *a and a* going away so you were left with  $\frac{c}{a}$ . So, this is the product of roots for a quadratic equation.