IIT Madras

ONLINE DEGREE

Mathematics for Data Science 1
Indian Institute of Technology, Madras
Week 02 - Tutorial 07

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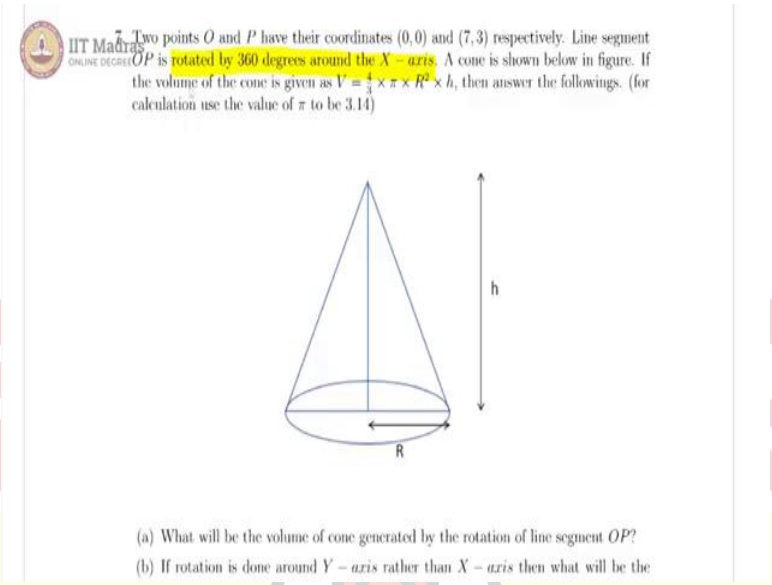

In the 7th question we have two points, one is the origin O, and some other point P (7, 3). And this line segment OP is being rotated. So first, lets mark out these points.

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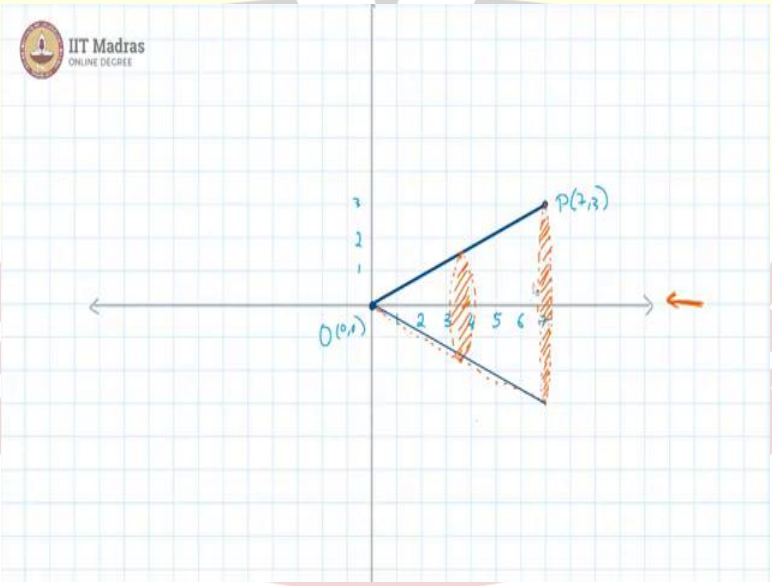

So first lets mark out these points, we have this is the origin and this is 1, 2, 3, 4, 5, 6, 7, this is 1, 2, 3. So our point P is here, this is P (7, 3) and this is the origin of course O. And we have this line segment OP given to us. Now, OP is being rotated by 360° about the x axis, lets see what that means. So every point on OP is going around the x axis in a circle, that is what rotation is, rotation is a combined circular motion of many particles, here we have this point let us take P, P goes around the x axis reaching this bottom point, then it circles back to itself.

So you would see the circle if you looked at it from the right. From the screen's perspective, this is what it will look like. And this is the case with every point on this line.

Suppose I took this point, this is just oppositely going to go till here in this circle and return back. So, every point is doing this circle, which means on this side we actually have the mirror image of OP with respect to the x axis which looks something like this. So, we have these circles being formed due to the rotation and as you can see, the final shape it appears to be a cone that is what has happened. Take a line segment and you rotate it about some central axis, you obtain a cone about that central axis.

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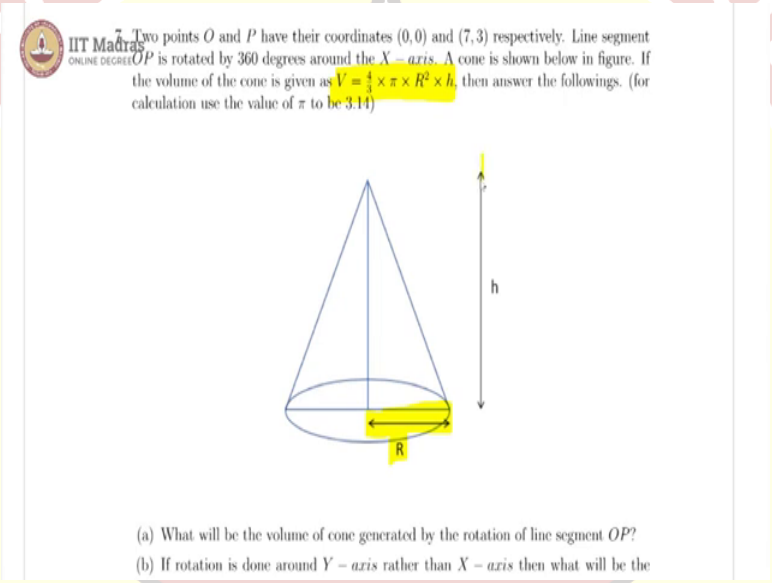

And they have given us the volume of a cone, volume of a cone is $(4/3)\pi R^2 h$, where R is the radius of the base circle and h is the height of the cone.

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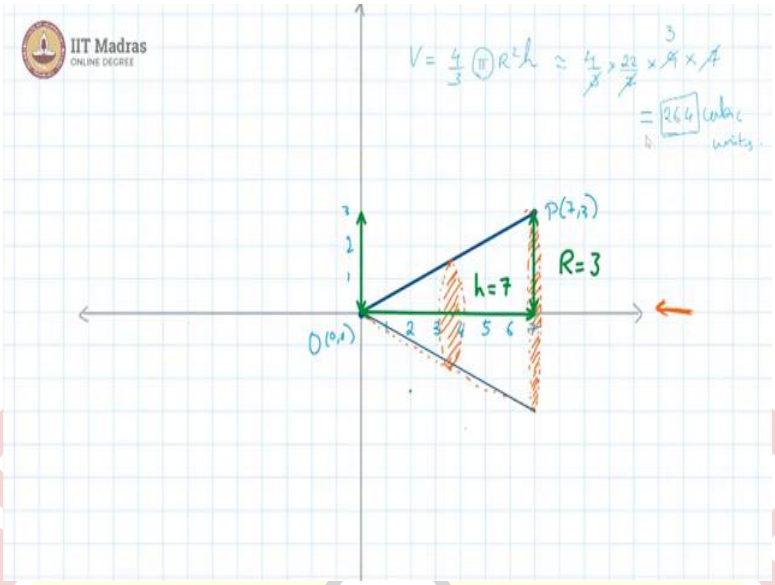

So in the cone we have obtained the radius, base radius is this quantity, which is R = 3 because it is a y coordinate of the point P that is a distance of point P from the x axis. And likewise, if you observe the height of this cone, that quantity h = 7 because the x coordinate of point. In this way, we can obtain the volume of our cone using the formula that is given $V= (4/3)\pi R^2 h$. We are going to approximate pi to be 3.14 or 22 by 7. So, this is roughly equal to (4/3)(22/7)(9)(7) so 7 and 7 cancels of, 3 and 9 gives us 3. So we get 264 cubic units, so this is our volume of the cone.

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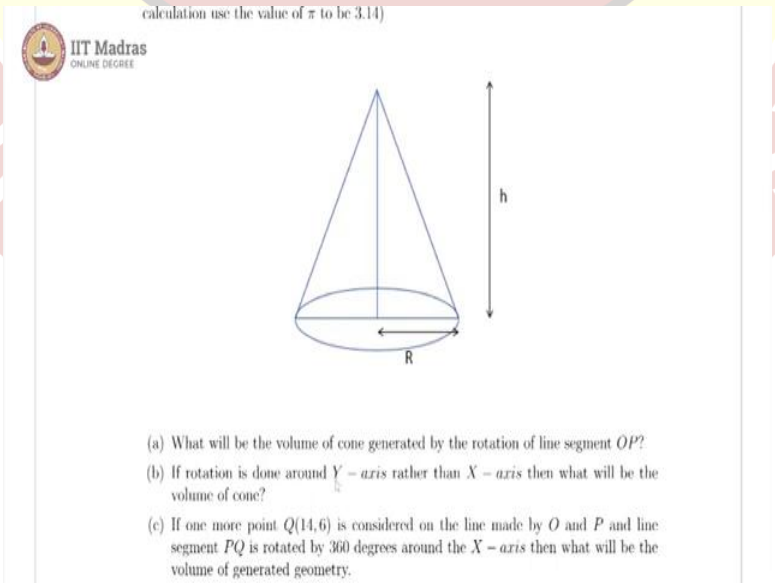

Now in the second part of this question, it is being said that the rotation is done around the y axis instead of the x axis, so what will this look like?

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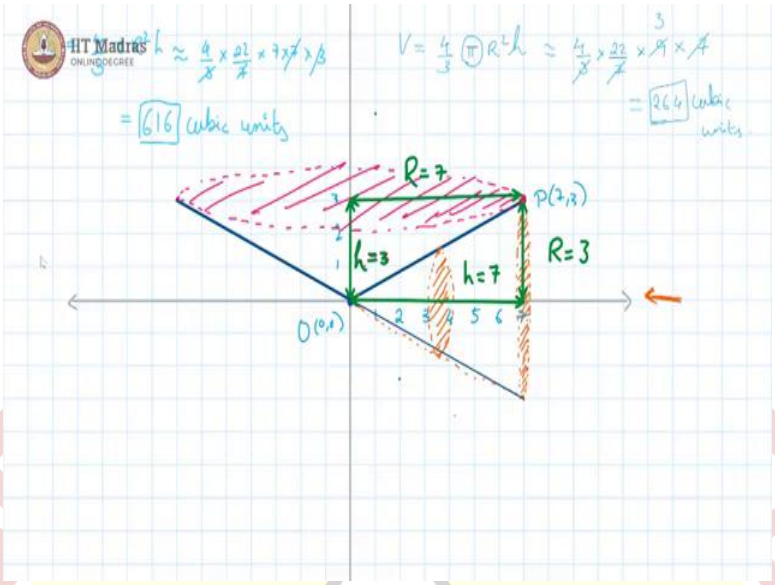

So, this OP is going to have a mirror image about the y axis which is going to look like this. So that means, our point P is going around in a circle to reach this opposite point here and it is coming back to itself. So this would become the base circle for our new cone which is obtained by rotation about the y axis. Now as you can see, this value is already the height so height is 3 now, whereas the radius is basically 7. So, our height is 3 and radius is 7 so these values have changed. So, if we call this quantity, this volume to be V_2, V_2 is $(4/3)\pi R^2 h$, which will be roughly equal to (4/3)(22/7)(7)(7)(3). So, 3 and 7 cancel off here and we get this is equal to 616 cubic units. So, this is the volume if OP is rotated about the y axis 360°, so that cone's volume is 616 cubic units.

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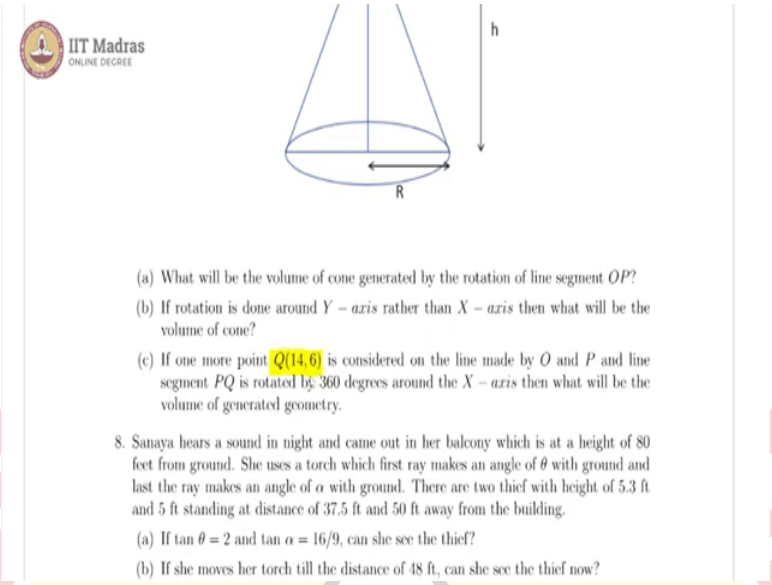

This problem gets progressively more complex, we are now adding the new point (14, 6) which is along the line segment OP, it is on the extension of OP, and then PQ is rotated about x axis by 360°

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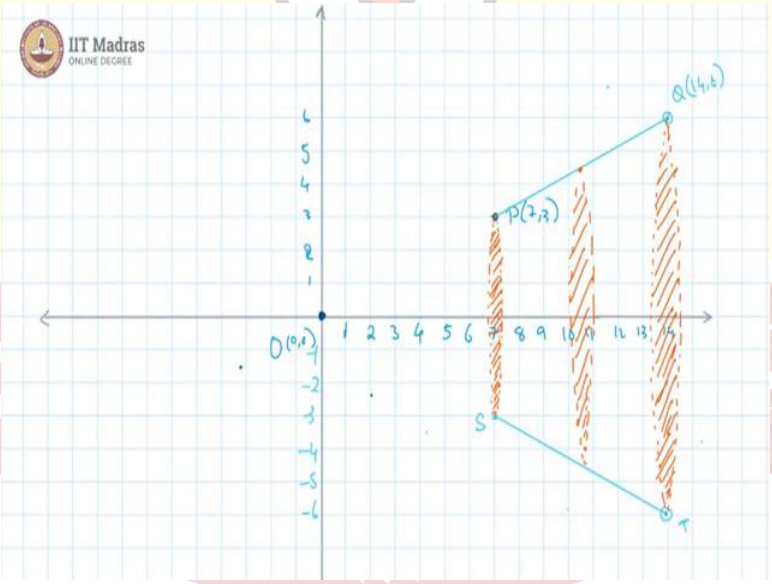

So, lets see what is happening here, so our point (14, 6) Q, (14, 6) is here, which gives us PQ as this line segment. And now, they are saying that PQ is being rotated about the x axis which will result in the mirror image in this way. This is -1, -2, -3, -4, -5, and -6. So, we are here now, I think we can call this point T and this point is S. So, we have ST in this way again, so what we see here the rotated geometry. So, for reference we are going to take one more point here, which kind of moves around. So, what we are seeing here this is what is called the

frustum of a cone. This is a cut-off portion of a larger cone which would be QO rotated about the x axis.

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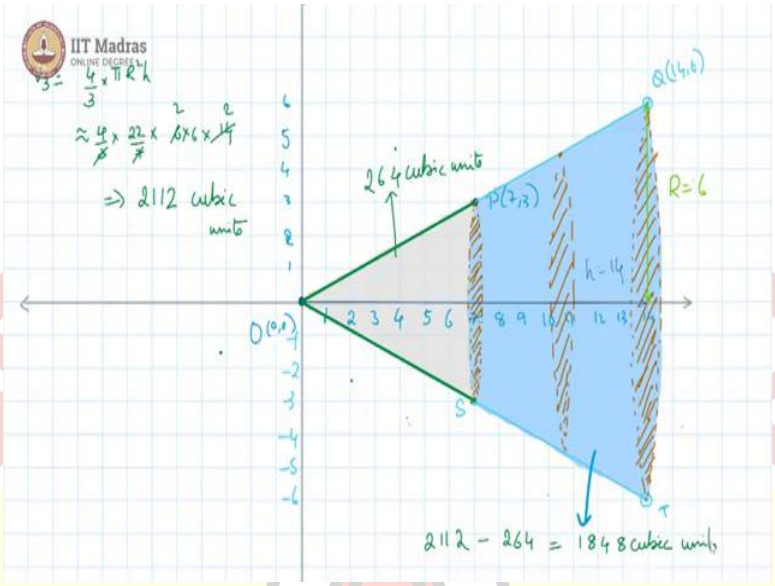

So, thus as you can see, the volume we require is the frustum of the cone which is this region and this volume is the result of subtracting this volume from the total cone. So, we already know this volume OP as that cone's volume to be 264 cubic units. So, the blue shaded region that would be the volume of the large cone that is of OQ rotating about x axis and that we can calculate as $V_3 = (4/3)\pi R^2 h$, where this is approximately equal to (4/3)(22/7)(6)(6)(14). So, 3 one's 3 two's, 7 one's 7 two's, so we have 2,112 cubic units. So the volume we require is going to be 2,112 – 264 = 1,848 cubic units.