IIT Madras

ONLINE DEGREE

Mathematics for Data Science 2
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Week 03 - Tutorial 01

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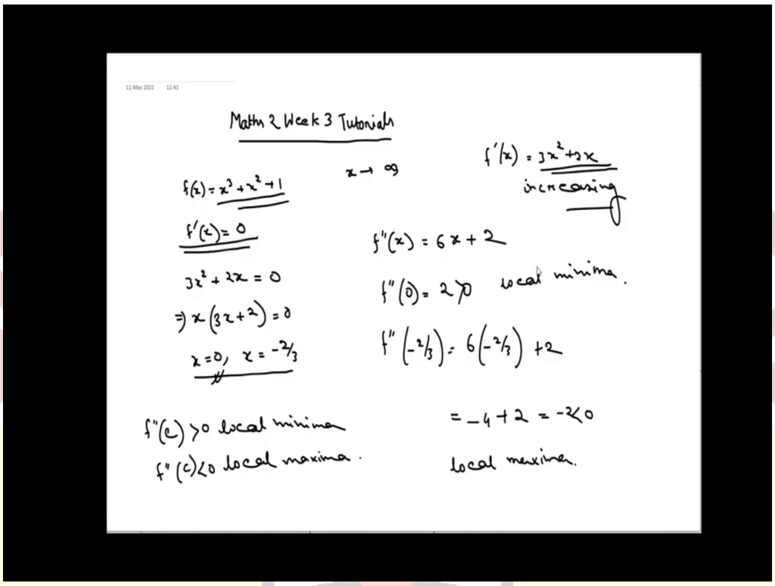

Hello everyone, welcome to Maths 2 Week 3 Tutorials. So, in this week, we have learned about the critical points where we have learned about maxima and minima and along with that we have learned about theory of integration. So, in the first tutorial video, we will take an example of a function and try to see its local maxima or minima. So, let us consider the function $f(x) = x^3 + x^2 + 1$.

So, you can see, this is the polynomial function of degree 3. Now to find local maxima, local minima, we have to find those points where $f'(x)$ is 0. So, what does it mean? It means that, at those points, at local maxima and local minima, at those points, the tangent of the function, basically the slope of the tangent of the function should be parallel to, I mean should be 0 that is, the tangent is parallel to x axis.

So, let us try to calculate f dash x. So, $f'(x)$ will be $3x^2 + 2x$. So, this is our $f'(x)$ and that should be 0. Now, let us see what are the critical points. So, the critical points if we try to solve this, this will be $3x + 2$. So, there are two critical points; one is $x = 0$ and the other one is x = - 2/3. So, we have found two critical points.

Now, how we can see that whether these are maxima or minima? We can check the second derivative, so this is called second derivative test. So, we will find out f double dash x. So, our, the second derivative of this function that will be 6x + 2. Now, if we put f double dash 0 here, that is x = 0, the first critical point, if we put it here, then we will get 2 which > 0. So, at x = 0, this function attains local minima.

And if we put x = - 2/ 3, so if we put it there, so $\frac{6-2}{3+2}$. So, it will give us - 4 + 2 that is - 2 which is less than 0. So, it is local maxima. So, what we get here? If f double dash c, where c is a critical point, if it > 0, it is nothing but the local minima. And if f double dash c is less than 0, then it is our local maxima.

So, remember that this local maxima or local minima can be global maxima or global minima, but it is not always. I mean, there can be many local maxima, local minima, but global maxima is the maximum value of the function and local minima is the minimum value of the function altogether, throughout the domain. So, as you can see in this example, as x tends to ∞, the function also tends to ∞, so basically this is an increasing function after x>, when x > 0, so this is an increasing function. We will see the graph of this function after sometime.

So, our $f''(x)$, what we have calculated, that is $3x^2 + 2x$. So, when x > 0, this is always> 0, so after x greater, after $x = 0$, the function is an increasing function. So, at 0 we have found it has local minima and after that the function will increase and it will, as x increases, x tends towards ∞, the function also increases and it tends towards ∞.

So, it does not have a global maxima. Actually, its global maxima is ∞, it is not a particular real number. Similarly, it has a global minima at - ∞. So, in this case, this local maxima and local minima are not actually the global maxima or global minima. Now, we try to visualize this function in the graph in GeoGebra to see how this local maxima and local minima behaves and what about the global maxima and global minima.

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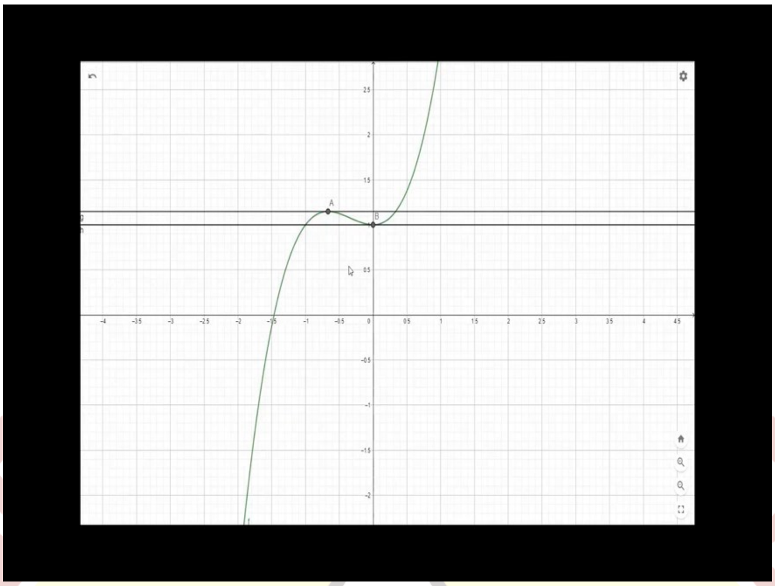

So, we are using GeoGebra to see the graph of this function. So, our function was x cube $x^3 + x^2 + 1$. So, if we see the graph will look like this. So, you can see that there are some turning points, 2 turning points basically. So, if we want to, what we have calculated in our calculation earlier, so we have found there is one local maxima and one local minima. So, this is our local maxima, the point A and at that point the tangent is basically parallel to x axis.

Similarly, at point B which is at $x = 0$, that is our local minima and at that point our tangent is parallel to x axis again. Now, you can see that these are not global maxima and minima because this is, at point A, this is our local maxima, but there are many points which are> that value and as we can see, that function is increasing after x = 0 and it tends towards ∞. As we go on towards $x = \infty$, we will see that the functional value also tends to ∞. So, its global maxima is at ∞ basically. So, it is not a particular real number.

Similarly, the function tends to - ∞ as x tends to - ∞ as you can see here because after, I mean, from the left-hand side of A, as we go towards the negative side, as we go towards x tending to - ∞, the functional value also tends to - ∞. So, here, this local maxima and local minima does not correspond to any global maxima or global minima.

So, I think that explains the difference between local maxima and local minima for this function and we can see that the characteristic of local maxima, local minima, which is one more important thing to remember that at those point, the tangent is parallel to x axis. Thank you.