INDIAN INSTITUTE OF TECHNOLOGY MADRAS

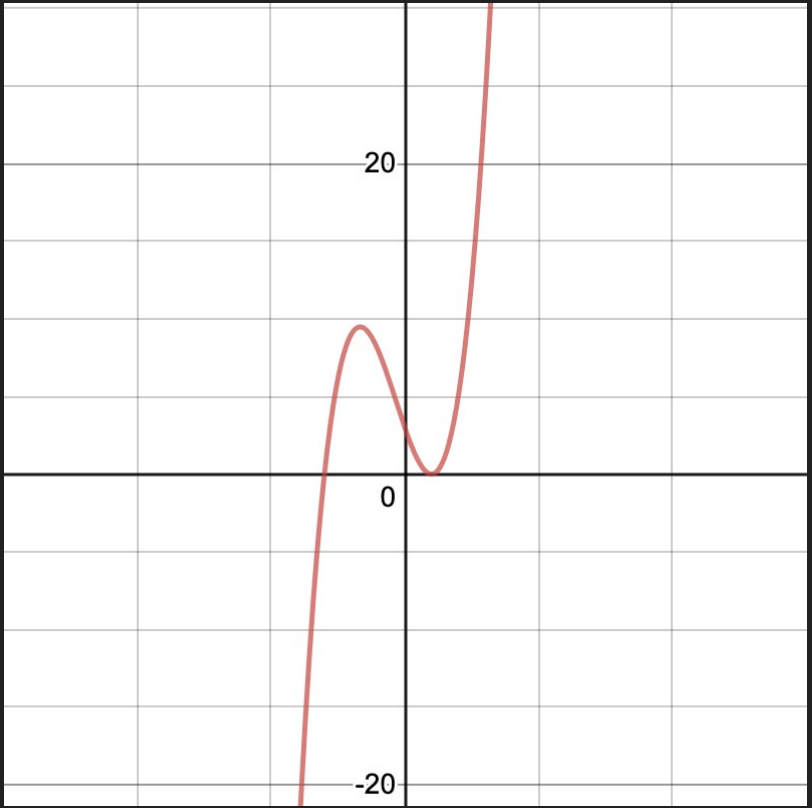
सिद्धिर्भवति कर्मजा

IIT Madras

ONLINE DEGREE

Example

Find the y- and x-intercepts of $g(x)=(x-1)^2(x+3)$.



Set $g(x)=0$

$x = 1, -3$ are the x-intercepts of f.

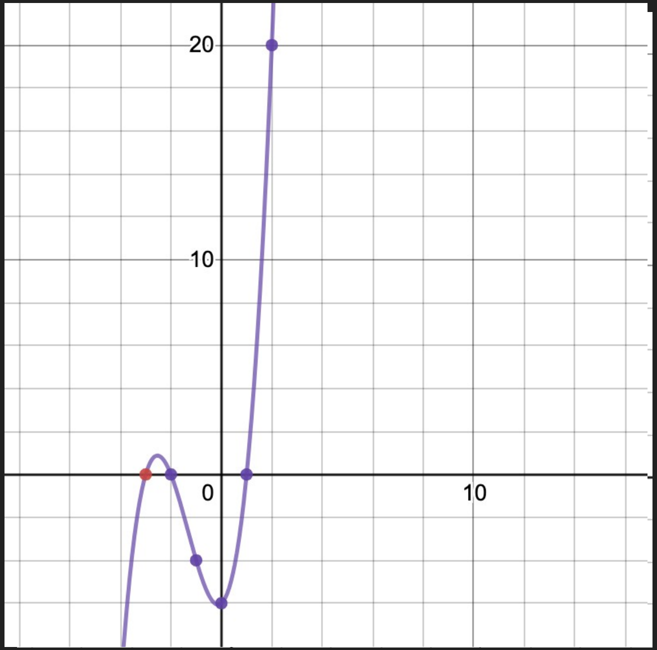
For y-intercept, $g(0) = 3$

x-intercept of Polynomial Function using Graph

Find x-intercept of $f(x) = x^3+4x^2+x-6$

In this case, the polynomial is not in factored form, has no common factors, and does not appear to be factorable using techniques previously discussed.
The only option is to generate the pair of values as done in quadratic case.

x	y
-2	0
-1	-4
0	-6
1	0
2	20



From table, x=-2, 1 are the x-intercepts of f.
The third zero can be found by dividing f(x) by (x+2)(x-1).
The third zero of f is x=-3.
Therefore, join the points smoothly to get the graph.

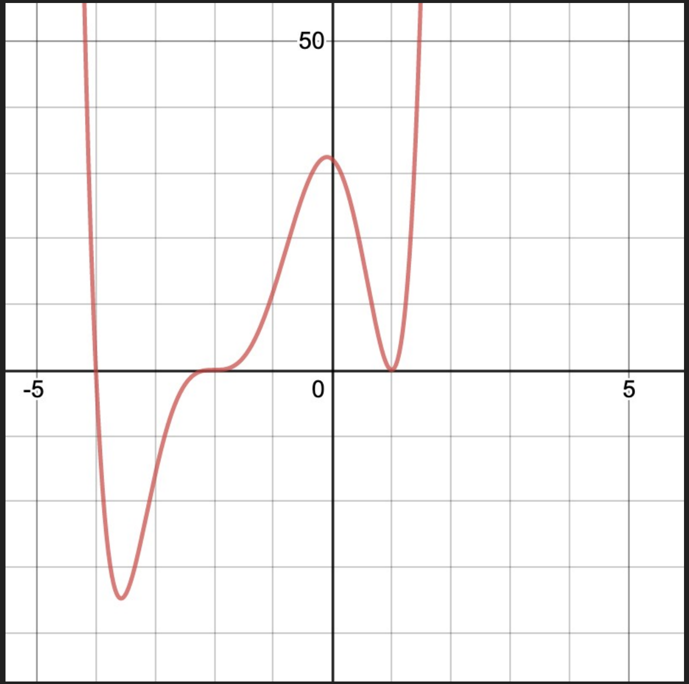
Identification of Zeros and Their Multiplicities

Graphs behave differently at various x-intercepts.

Sometimes, the graph will cross over the horizontal axis at an intercept.

Other times, the graph will touch the horizontal axis and "bounce off."

Suppose, for example, we graph the function $f(x) = (x-1)^2(x+2)^3(x+4)$.



Identifying Zeros and their Multiplicities

The x-intercept -4 is the solution of the equation $(x+4)=0$. The graph passes directly through the x-intercept at $x=-4$. The factor is linear (has a degree of 1), so the behavior near the intercept is like that of a line — it passes directly through the intercept. We call this a single zero because the zero corresponds to a single factor of the function.

The x-intercept 1 is the repeated solution of the equation $(x-1)^2=0$. The graph touches the axis at the intercept and changes direction. The factor is quadratic (degree 2), so the behavior near the intercept is like that of a quadratic — it bounces off of the horizontal axis at the intercept.

The x-intercept -2 is the repeated solution of the equation $(x+2)^3=0$.The graph passes through the axis at the intercept, but flattens out a bit first. This factor is cubic (degree 3), so the behavior near the intercept is like that of a cubic — with the same S-shape near the intercept as the toolkit function $f(x)=x^3$. We call this a triple zero, or a zero with multiplicity 3.

IDENTIFYING ZEROS AND THEIR MULTIPLICITIES

For **zeros** with even multiplicities, the graphs touch or are **tangent** to the x-axis.

For zeros with odd multiplicities, the graphs cross, or **intersect**, the x-axis.

For higher even powers, such as 4, 6, and 8, the graph will still touch and bounce off of the horizontal axis but, for each increasing even power, the graph will appear flatter as it approaches and leaves the x-axis.

For higher odd powers, such as 5, 7, and 9, the graph will still cross through the horizontal axis, but for each increasing odd power, the graph will appear flatter as it approaches and leaves the x-axis.