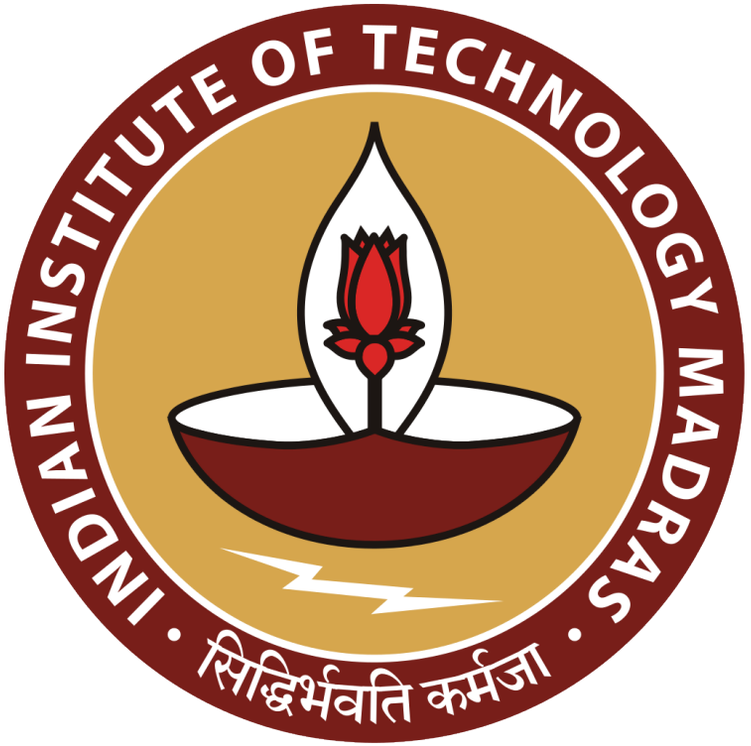


# IIT Madras

## ONLINE DEGREE

# Natural numbers and integers

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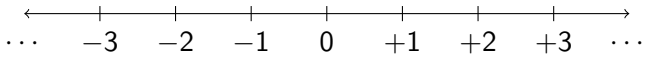
Mathematics for Data Science 1
Week 1

# Natural numbers

- Numbers keep a count of objects
  - 7 represents "seven"-ness
- $1, 2, 3, 4, \ldots$
- 0 to represent no objects at all
- Natural numbers: $\mathbb{N} = \{0, 1, 2, \ldots\}$
  - Sometimes $\mathbb{N}_0$ to emphasize 0 is included
- Addition, subtraction, multiplication, division
  - Which of these always produce a natural number as the answer?

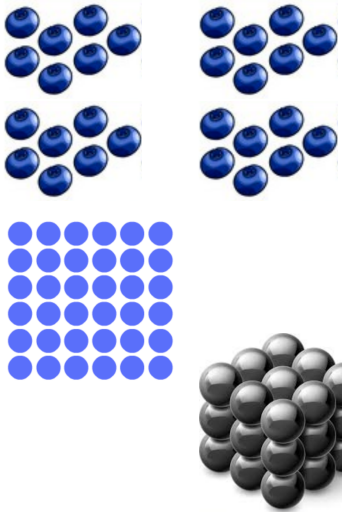
# Integers

- $5 - 6$ is not a natural number
- Extend the natural numbers with negative numbers
- $-1, -2, -3, \ldots$
- Integers: $\mathbb{Z} = \{\ldots, -3, -2, -1, 0, 1, 2, 3, \ldots\}$
- Number line



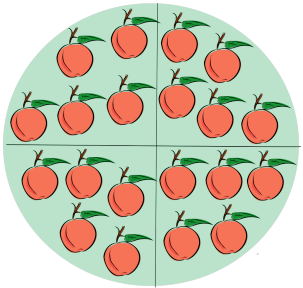
# Multiplication and exponentiation

- $7 \times 4$ — make 4 groups of 7
- $m \times n = \underbrace{m + m + \cdots + m}_{n \text{ times}}$
  - Notation: $m \times n$, $m \cdot n$, $mn$
  - Sign rule for multiplying negative numbers
    - $-m \times n = -(m \cdot n)$, $-m \times -n = m \cdot n$
- $m \times m = m^2$ — $m$ squared
- $m \times m \times m = m^3$ — $m$ cubed
- $m^k = \underbrace{m \times m \times \cdots \times m}_{k \text{ times}}$ — $m$ to the power $k$
- Multiplication is repeated addition
  Exponentiation is repeated multiplication



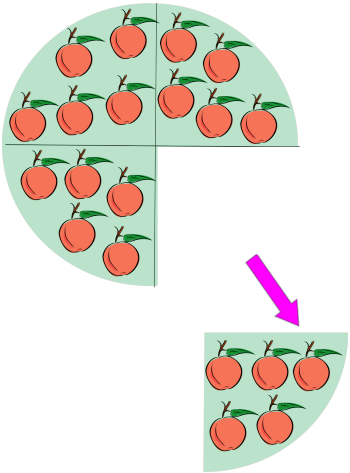
# Division

- You have 20 mangos to distribute to 5 friends. How many do you give to each of them?



# Division

- You have $20$ mangos to distribute to $5$ friends. How many do you give to each of them?
  - Give them $1$ each. You have $20 - 5 = 15$ left.
  - Another round. You have $15 - 5 = 10$ left.
  - Third round. You have $10 - 5 = 5$ left.
  - Fourth round. You have $5 - 5 = 0$ left.
  - $20 \div 5 = 4$
- Division is repeated subtraction
- What if you had only $19$ mangos to start with?
  - After distributing $3$ to each, you have $4$ left
  - Cannot distribute another round
  - The quotient of $19 \div 5$ is $3$
  - The remainder of $19 \div 5$ is $4$
    - $19 \bmod 5 = 4$

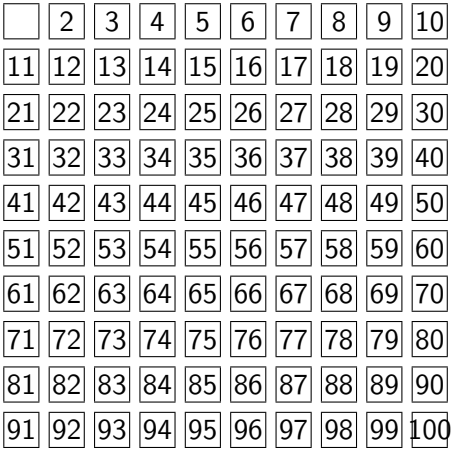


# Factors

- $a$ divides $b$ if $b \bmod a = 0$
  - $a \mid b$
  - $b$ is a multiple of $a$
- $4 \mid 20$, $7 \mid 63$, $32 \mid 1024$, ...
- $4 \nmid 19$, $9 \nmid 100$, ...
- $a$ is a factor of $b$ if $a \mid b$
- Factors occur in pairs — factors of $12$ are $\{1, 12\}$, $\{2, 6\}$, $\{3, 4\}$
- ... unless the number is a perfect square — factors of $36$ : $\{1, 36\}, \{2, 18\}, \{3, 12\}, \{4, 9\}, \{6\}$

# Prime numbers

- $p$ is prime if it has only two factors $\{1, p\}$
  - 1 is not a prime — only one factor
- Prime numbers are $2, 3, 5, 7, 11, 13, \ldots$
  - Sieve of Eratosthenes — remove multiples of $p$
- Every number can be decomposed into prime factors
  - $12 = 2 \cdot 2 \cdot 3 = 2^2 \cdot 3$
  - $126 = 2 \cdot 3 \cdot 3 \cdot 7 = 2 \cdot 3^2 \cdot 7$
- This decomposition is unique — prime factorization



# Summary

- $\mathbb{N}$: natural numbers $\{0, 1, 2, \ldots\}$
- $\mathbb{Z}$: integers $= \{\ldots, -3, -2, -1, 0, 1, 2, 3, \ldots\}$
- Arithmetic operations: $+, -, \times, \div, m^n$
- Quotient, remainder, $a$ mod $b$
- Divisibility, $a \mid b$
- Factors
- Prime numbers
- Prime factorization