IIT Madras

ONLINE DEGREE

Mathematics for Data Science 1
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Lecture – 30
Polynomials

Let us introduce Polynomials. So, today we are going to see how the polynomials look like, how they behave. So, let us start with polynomials. Let us go ahead and see what expressions do we call as polynomial.

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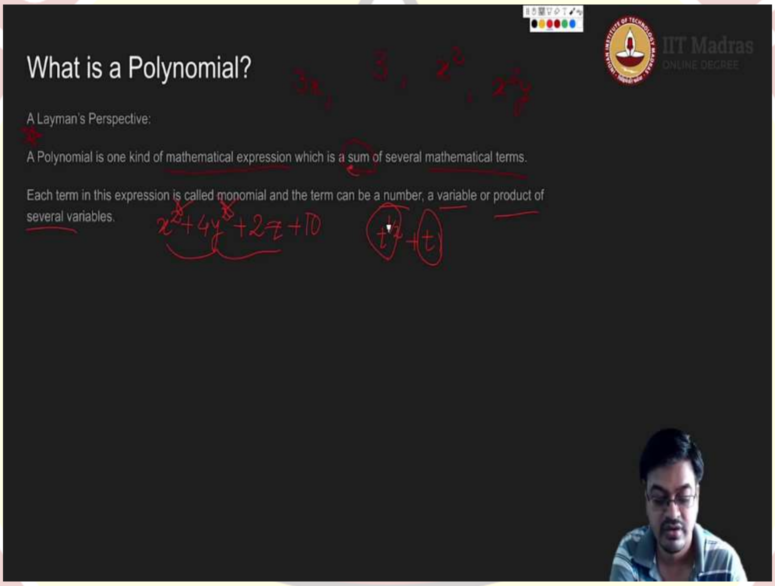

So, for that let us take the first point where we will take a Layman's perspective and, we will try to understand what a Layman will think of a polynomial. In order to do that let us first take a Layman's perspective and see what Layman will think. So, for a Layman, a polynomial it is some kind of mathematical expression which is a sum of several mathematical terms.

Then we asked Layman what do you mean by mathematical terms? The answer is each term in this expression, each term in this expression, that is mathematical term in this expression can be a number, a variable, or a product of several variables.

So, according to Layman each term this mathematical term can include a number, a variable, or product of several variables. These are the things that are allowed. So, basically then, let us take one example for this. And let us see if I have $3x$ this is a number and some variable. So, I have a constant 3, I have some number like x^2, I have some term like x^2y, all this contribute to something called polynomial ok.

Now, take a more significant number that is say $x^{2} + 4y^2 + 2z + 10$; will this contribute to be a polynomial? Yes, because it is sum of a number which is 10 here, a variable. There are many variables 1, 2, 3, there are three variables, and product of several variables; in particular here we have x^2 and here we have y^{2}. So, this also qualifies to be a polynomial.

Then according to this, suppose what I will write is here, let us say some expression of the form $t^{\frac{1}{2}} + t$; is this expression a polynomial? Layman will say yeah, it can be a polynomial because, $t^{\frac{1}{2}}$, if you square this number you will get this, correct. So, this is one variable this is another variable and therefore, we are actually having a polynomial.

So, then we went and asked mathematician, what is a mathematician's perspective of a polynomial?

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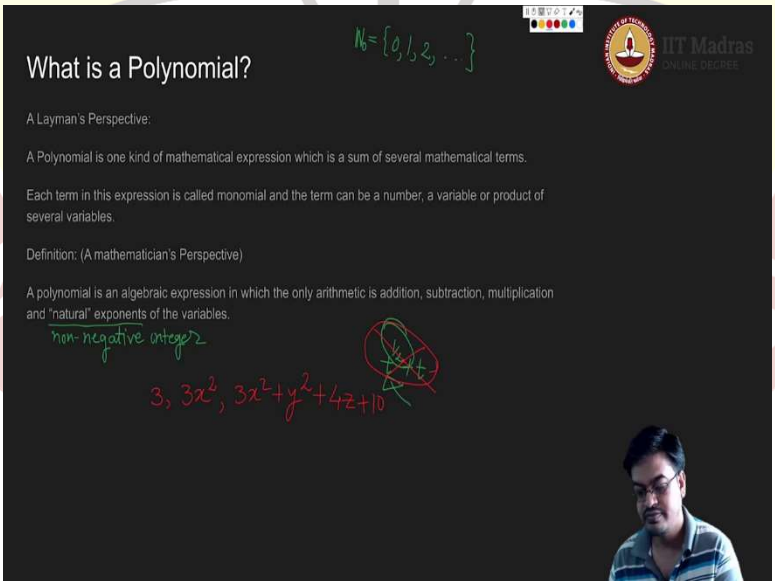

So, that we will define as a definition. So, a mathematician said a polynomial is nothing, but an algebraic expression in which only arithmetic is addition, subtraction, multiplication and this is interesting, he mentions it as natural exponents of the variables. Natural, by natural I mean the way we defined a set of natural numbers in our first week I mean natural means 0, 1, 2 and so on.

So, this is my set of natural numbers which include 0, if you want to emphasize it you can put it as N_0. Otherwise, you can call this set as set of whole numbers or set of non-negative integers. So, the definition can be twisted like this will have non-negative integer exponents. If you do not want any ambiguity, we can say that non-negative integer exponents of the variables.

So, then we if we go back to that earlier expression which is $t^{\frac{1}{2}} + t$, the all other expressions will qualify to be a variable, but this expression will not qualify to be a variable, why? Because this $t^{\frac{1}{2}}$ by definition is not a natural number, it is a rational number. We will come to it later, but this cannot qualify definitely so this cannot qualify as a polynomial if we go by this definition.

We have already seen many examples. Let us re-iterate them; one example was constant 3, another was $3x^2$, another one was $3x^2 + y^2 + 4z + 10$, all these are qualified to be polynomials. But this expression does not qualify to be a polynomial. So, this we will consider later as well and we will give our rational reason why it is not a polynomial.

So, we now we know what is polynomial. Now, it is time to see why is the name polynomial? Why do we call this as polynomial? That is what we will see now. So, let us go ahead and see something about the nomenclature. Why do we call them polynomials?

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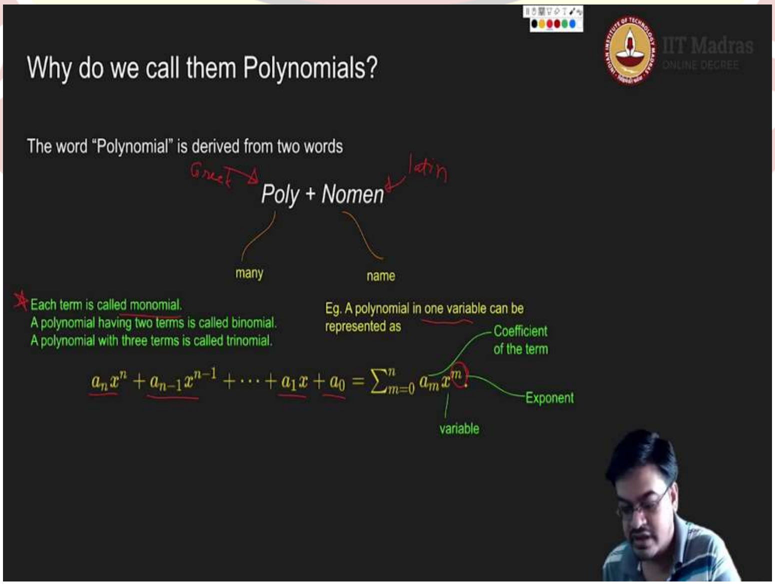

So, polynomials essentially is derived from two words one word is poly, second word is nomen. This is a Greek word this word and this has roots in Latin. The word poly essentially means many and the word nomen essentially means names or a terms. So, in our case it turns out to be terms. So, an expression having many terms is called polynomial ok.

Now, each term of the because it has many terms each term of the polynomial will be called as monomial, each term of a polynomial will be called as monomial. Then, if the polynomial has only two terms then you will call it as binomial. If the polynomial has only three terms then you will call it as trinomial.

So far, if you can label them you can label them, but in general we will treat them as polynomials. And remember that we will include this also; a monomial is also a polynomial for us. We will not distinguish between monomial and polynomial. Of course, monomial enjoy some different set of properties, but we will keep them with polynomials.

So, let us take one example. For example, a polynomial in one variable can be represented as $a_n x^n$ this is the highest term, $a_{n-1}x^{n-1}$, $a_1 x^1$, a_0 right. I am assuming that this a_n's not equal to 0. Otherwise the if they are 0, then the polynomial may extend to infinity. I do not want that. So, I am assuming that these $a_n{}'$s are not 0.

Now, if you can rewrite this using the notation of summation in this manner and therefore, this a_m will have a specific name and it will be called as coefficient of the term. Because this is a polynomial in one variable, x is the variable we are interested in x is the variable, and this m is the exponent of the variable.

Now, remember in order that this term to be a polynomial, this m should always be a natural exponent; by natural I mean the one that is non-negative integer. If it is not nonnegative integer then I cannot classify this as a polynomial ok. Let us go to the next step and see some examples of polynomials and try to identify whether the given expressions are polynomials or not.

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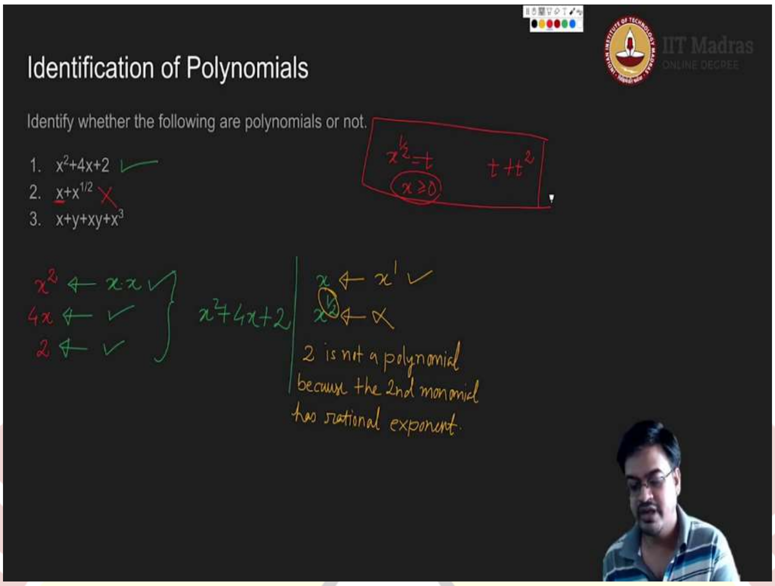

So, here is the question about identification of polynomials. Identify whether the followings are polynomials or not. The 1st one; $x^2 + 4x + 2$, what can you say about this? So, the first let us take term by term x^2, $4x$ and 2. So, the all these are monomials involved in this polynomial.

So, when I take x^2 it is nothing, but variable x multiplied by x. So, it is a product of two variables. So, this is ok. When I take $4x$ it is a number and a variable. So, this is also I do not have any and finally, this is just a number. So, together and the expression given it is sum of these that is; $x^2 + 4x + 2$, expression given it is sum of this. Therefore, this is a valid polynomial form.

Let us go ahead ok. Again, the same expression has come $x + x^{\frac{1}{2}}$. So, now, you look at the terms that are involved x and $x^{\frac{1}{2}}$. Now, if you look at the terms involved x and $x^{\frac{1}{2}}$, then this it is simply a variable raised to 1, x^1, right. So, I do not have any problem, this is a valid term because there is no issue with this; $x^{\frac{1}{2}}$ this term is not a valid term because, it has rational exponent. So, this is not correct.

So, this second expression 2 is not a polynomial, why? We need to justify we need to write a reason because, the 2nd monomial has rational exponent. This is an interesting observation. So, this does not qualify to be, to be a polynomial. So, I can erase this. This is not a polynomial.

Now, some people may say that what is a big deal? I can put $x^{\frac{1}{2}}$, let's say t and you can rewrite this expression as $t + t^2$, but remember when I am putting x raised to half as t, I am putting an explicit assumption on this x that is; this x should be greater than or equal to 0. So, I cannot define this polynomial on the entire real line.

So, we will refrain from doing such assumptions and therefore, it would not be a polynomial. Let us go to the next example, this example ok. So, let me erase the previous ones so that I will have some space.

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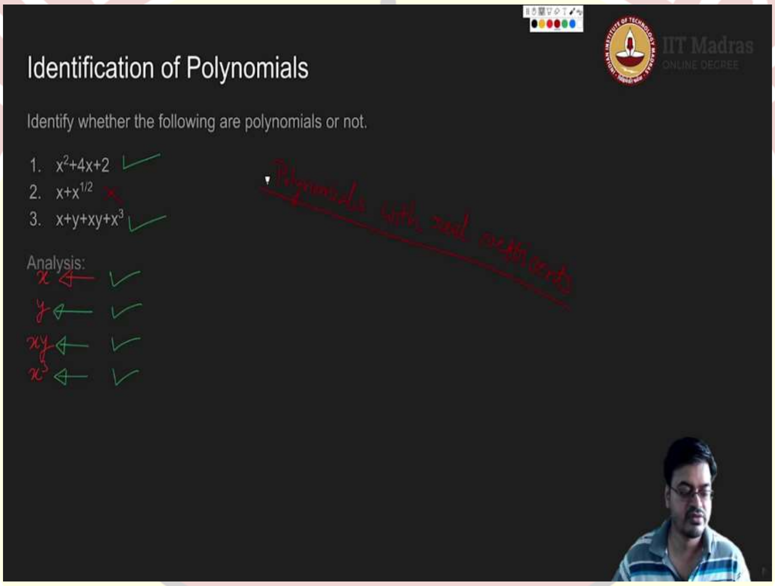

We will write all the terms as usual $x,\ y,\ xy,\ and\ x^3$. We will analyse the each monomial one by one, is this valid? Let me change the colour. Is this valid? Yes, it is valid because it is just a variable y? Yes, it is valid just a variable; product of several variables; product exponent natural exponent of single variable? Yes, it is valid.

So, according to me this and this qualify as a polynomial. And this do not qualify as a polynomial. So, we have identified what are the polynomials and how they look like. So, our identification part is complete. In particular we are dealing with polynomials having real coefficients because, all the numbers that I am giving you are real numbers.

So, just remember this fact we are only handling polynomials with real coefficients. If you go to the further branches of mathematics you may have polynomials with simply integer coefficients, you may have polynomials with complex coefficients, we are not dealing with them. So, this is how we will identify whether a polynomial whether a given expression is a polynomial or not.

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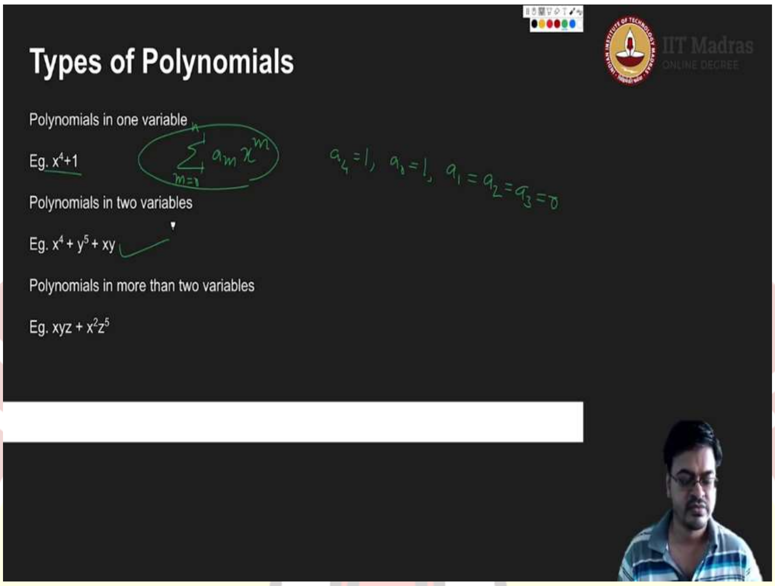

Let us go ahead and try to describe what are the types of polynomials, that we can encounter. We have already seen them; we are just enlisting them for the sake of completeness. So, there can be polynomials in one variable which will typically look like $\sum_{m=0}^{n} a_m x^m$. So, that let me rewrite it.

So, that was our expression; summation over $\sum_{m=0}^{n} a_m x^m$. So, this particular thing falls into that category. What is what will be here in this particular case $a_4 = 1$, $a_0 = 1$ and all others like a_1, a_2 and a_3 , all of them are 0. So, this is how we will describe the polynomial. So, this is a polynomial in one variable.

You can encounter a polynomial in two variables. For example, we have already seen some examples this is a polynomial in two variables and you can have similar expression, but now, you will have $a_m b_m$ and a_{mn} or something of that sort, to indicate the powers of these exponents. So, we will not indulge into a mathematical representation of this, but you can have polynomials in two variables.

In a similar manner you can have polynomials in three variables or more than two variables. So, here is an example of a polynomial in more than two variables. And these are the types of polynomials that you may encounter with real coefficients. So, this summarizes the topic of representation of polynomials.

Now, let us go ahead and see some further properties of these polynomials.