Statistics for Data Science -1

Continuous Random Variables-Continuous distributions

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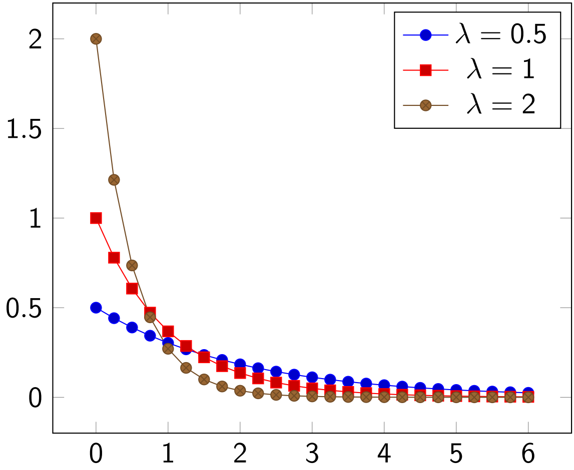
Exponential distribution

A continuous random variable whose probability density function is given, for some $\lambda > 0$, by

$$f(x) = \begin{cases} \lambda e^{-\lambda x} & x \geq 0 \\ 0 & \text{otherwise} \end{cases}$$

is said to be an exponential random variable (or, more simply, is said to be exponentially distributed) with parameter λ.

Graph of pdf for different values of λ



cdf of Exponential distribution

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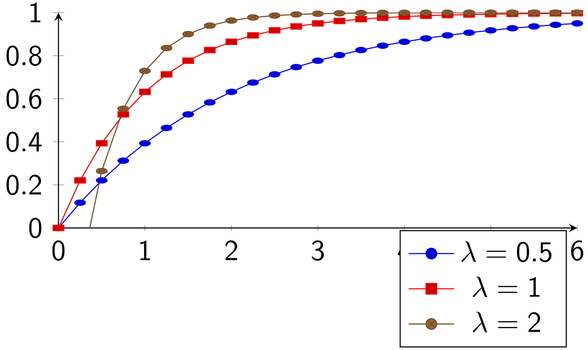

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Thus, the mean of the exponential is the reciprocal of its parameter λ, and the variance is the mean squared.

Application

In practice, the exponential distribution often arises as the distribution of the amount of time until some specific event occurs.

- Suppose that the length of a phone call in minutes is an exponential random variable with parameter $\lambda = 0.1$. If someone arrives immediately ahead of you at a public telephone booth, find the probability that you will have to wait
  - a more than 10 minutes
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Section summary

- Exponential distribution and its applications.