INDIAN INSTITUTE OF TECHNOLOGY MADRAS

सिद्धिर्भवति कर्मजा

IIT Madras

ONLINE DEGREE

Mathematics for Data Science 1
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Lecture No. 53
Logarithmic Functions: Graphs

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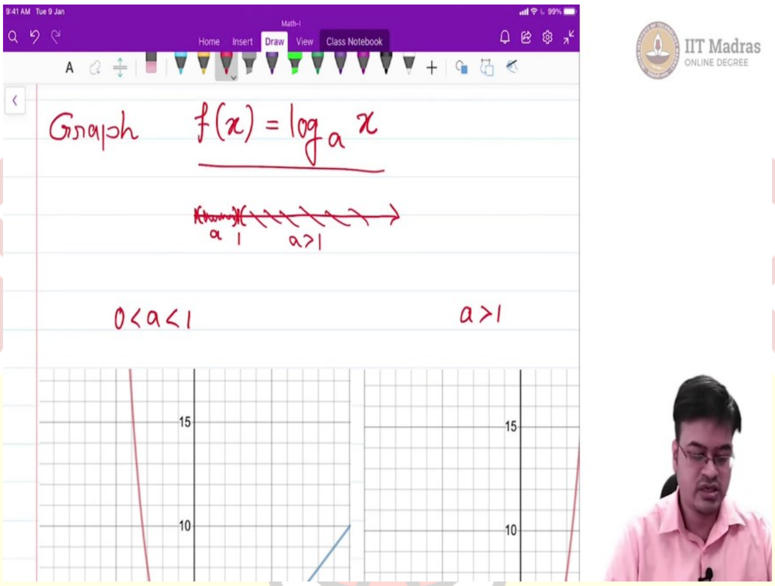

Hello friends, in the last video, we have seen how to graph a function $f(x) = \log_a x$. In particular, we have seen two kinds of graphs or two kinds of divisions, when our a is between 0 and 1 the graph has one form and when a was > 1 the graph has the other form.

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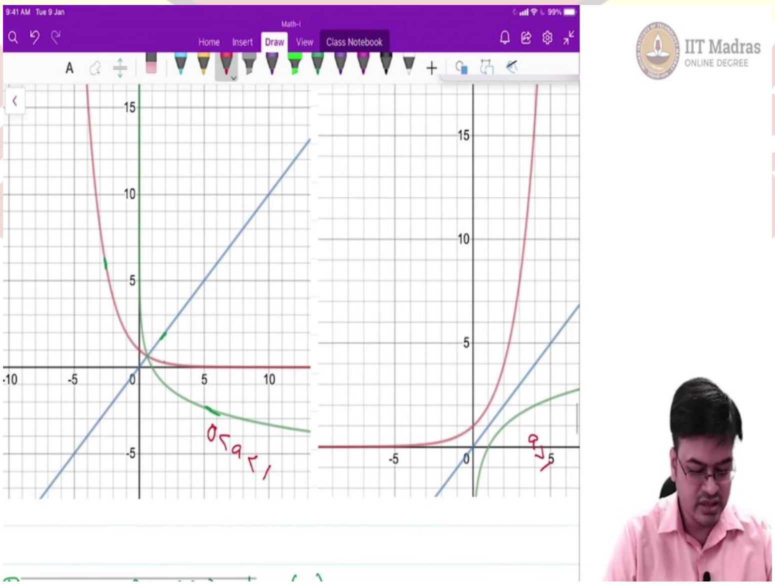

In particular, what we have seen is if the graph of exponential function for a < 1 there is the graph of a raise to x for a < 1 is given by a red line, then you can actually reflect this graph along the blue line which is y =x and get the corresponding graph for$\log_{10} x$ of x when 0 < a < 1.

In a similar manner, when a is > 1 the matters are, matter is very easy. And you see there is because there is no intersection, you can simply reflect the red line along the blue line to get the green line and the final graphs will look like this. So, this is the this is the case where my a is > 1 and this is the case where 0 < a is < 1. In particular, we have already seen some something similar in the, when we studied exponential functions. In particular, we will try to list all the properties of this graph of exponential function.

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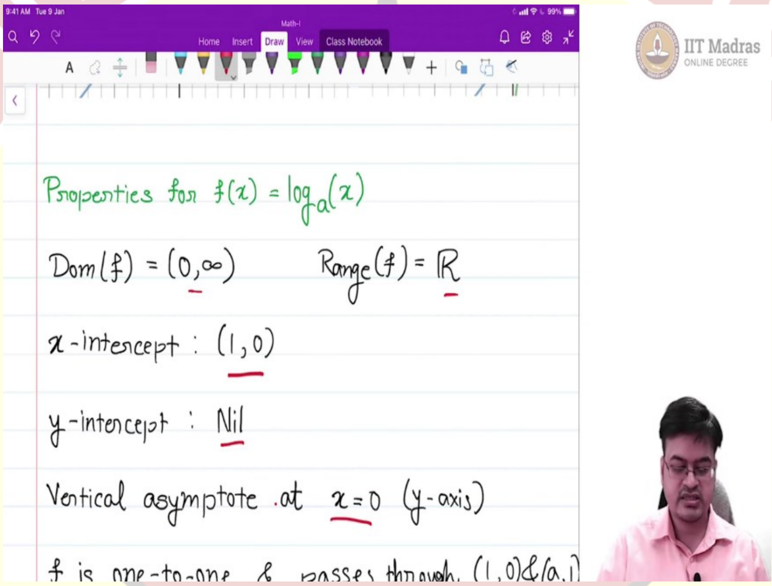

So, let us start our journey of listing the properties of graph of logarithmic function. So, first thing is the domain of the function we as it is an inverse function of exponential function is $(0, \infty)$ and the range of a function is real line, when you studied exponential function, the intercept was (0,1) here the x intercept is (1, 0) and there was no x intercept here, there is no y intercept, because simply because it is reflection along y =x line.

Then you had (vertic) in when you studied exponential function, you had a horizontal asymptote that is x axis was your asymptote in this case, you will have a vertical asymptote and that is x =0 is the line it is easily visible over here. For example, if you look at this green line, the vertical asymptote is towards the positive side of ∞ that is positive side of y axis. And if you look at this particular picture, it is towards negative side of y axis.

So, these are the typical features that you can you will understand when you look at the graph of a logarithmic function, then naturally this is the inverse function of a one to one function.

So, it is one to one and it passes through two points. If you recollect, the exponential function was passing through 0.01 and 1 a. So, naturally this function will pass through points 1 0 and a 1 all the time. So, these are the two static points whenever you consider a graph of a logarithmic function.

As it is visible from the graph for $0 < a < 1$ this green curve is actually a decreasing function. And for $a > 1$, this green curve is actually an increasing function. So, that those properties naturally boiled down to for $0 < a < 1$ the function is decreasing and for $a > 1$ the function is increasing.

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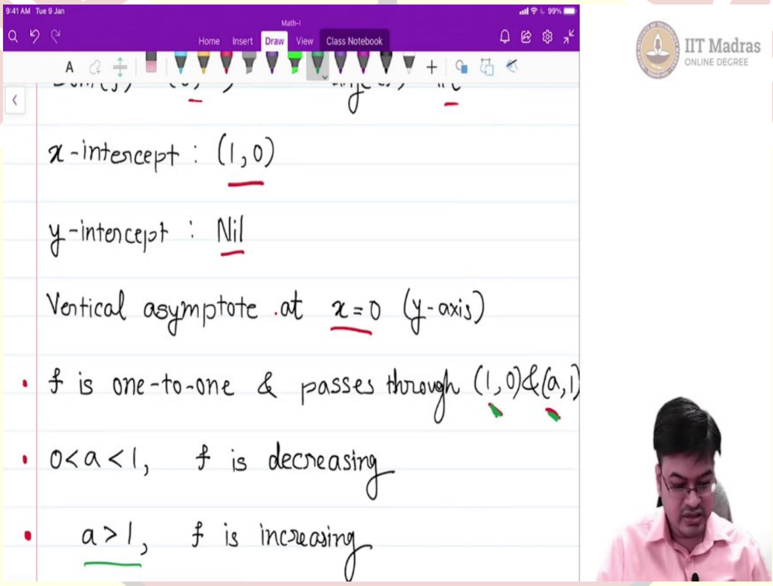

So, these are important properties of graph of logarithmic function. So, while drawing the graphs of logarithmic function in a standard form, you should always remember whether you are satisfying these properties or not, that is a cross check whether your answer is correct or not.

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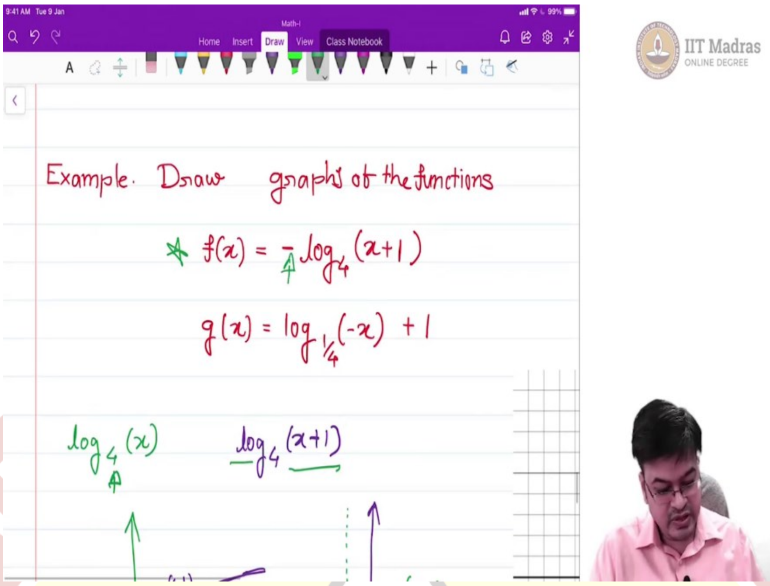

So, let us enhance our knowledge by taking an example of drawing a graphs drawing a graph is not correct, or drawing graphs of the functions that drawing the graphs of the functions, $f(x) = -\log_4(x+1)$ and $g(x) =\log_{\frac{1}{4}}(-x+1)$.

Now, you remember the domains the domains of the function. So, here if I want to draw a graph, let us take the function $f(x)$ here If I want to take a graph of $f(x)$, so, first let us understand how the graph of $\log_4 x$will look like in order to understand this let us go to the properties is a the base > 1 or < 1 this is the first concern, so my base is > 1.

So, the function should be increasing okay naturally the function is one to one and my curve should pass through (1,0) and (4,1) correct and I should have vertical asymptote at x =0 naturally the function is actually increasing that means, I will come from down to up and therefore, and obviously x intercept is (1,0) domain of x is 0 to ∞ range of f is ℝ.

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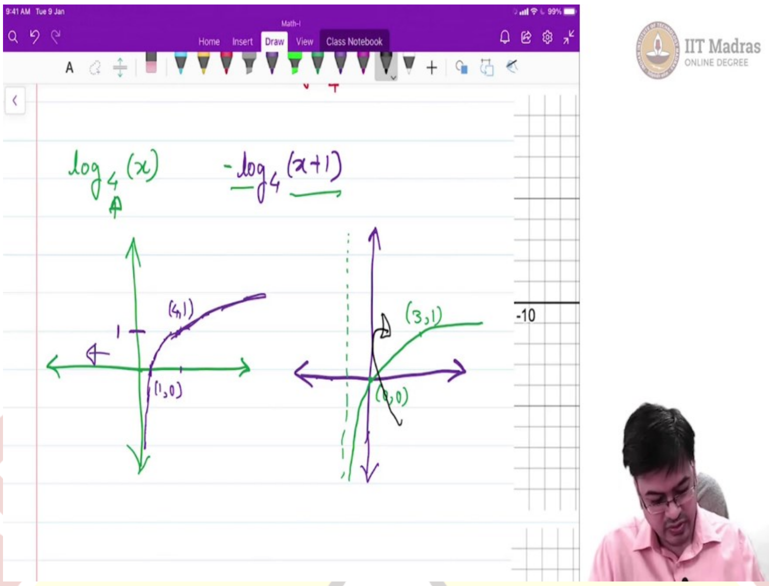

So, keeping all these things in mind, let me draw a quick sketch of this graph $\log_4(x)$. So, let me change the colour and it will be like this this is an asymptote, then it will pass through this point and you know what this point is and then it will be like this, this is the basic understanding it should pass through point (1,0) this is the point (1, 0) and (a, 1).

So, 4, 1 should be the point that it should pass through. So, naturally let us see this 1, 4 on x axis and this is 1 on y axis. So, as it passes through (4,1). So, this is these are 4 units this is 1 unit. So, this is 1 unit and that is 4 units on x axis this is 1 unit. So, this is how the graph will look like, but you have not been given a graph of $\log_4 x$.

So, what is happening it is going to x+1. So, now, $\log_4(x+1)$, how will it look like? That is the next question. So, what we are doing is we are shifting it on x axis. So, whatever value x was taking now, x+1 is taking that is a shift along this direction and shift along this direction of 1 unit. That means, that simply means, again you can easily quickly draw the graph, rough sketch of the graph whatever was happening for this particular thing, everything will shift and let me write use a green colour.

Now, instead of having $y = x$, instead of having this, this particular y axis as an intercept, I will have a new horizontal vertical asymptote which will be at 1 unit apart and my curve will pass through this this will be my new asymptote and my curve will pass through this and it will behave like this simple. So, instead of 1, 0, everything is translated by 1 unit. So, I will have 0, 0 the values on y axis will not change values on x axis will change. So, everything is translated by your 1 unit. So, I will have a point 0, 0 and this point where it intersects it will be 3,1. So, now this is my new graph of locked to the base 4 of x+1.

Now, the twist that is added is the-sign. So, that means it this is the graph of y =$\log_4(x+1)$. Now, if I add a-sign to this, the y will become the -y. So, now reflection along y axis so, the well what I have d1 is reflection along y axis. What I actually meant was reflection along x axis. So, the graph of this function should reflect along the x axis when I substitute $y = -y$.

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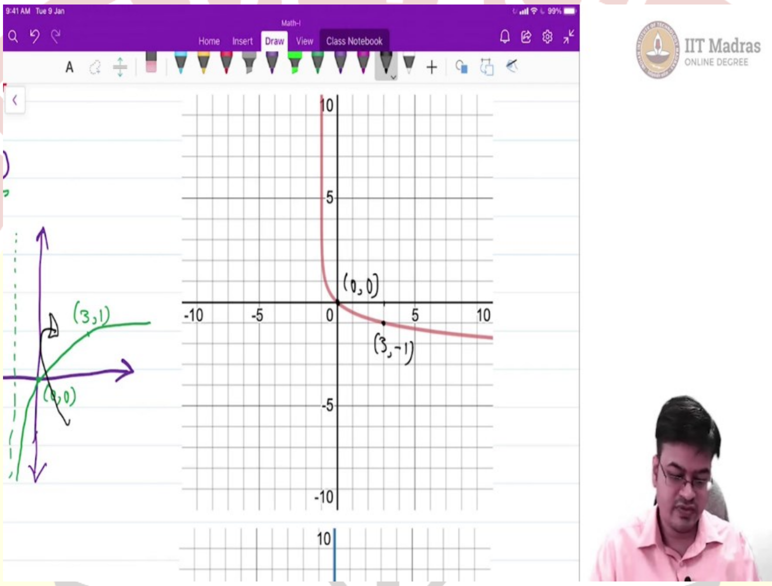

Therefore, the graph will simply twist itself along the file along the line y along x axis and therefore, the upside will go down and the downside will go up and therefore, the function will look like this, that is all. So, as you can easily see, the function will look like this the point 0, 0 will remain intact and then point 3,1. So, 3 on x will become (3,-1), rest of the things will remain intact. And naturally this is how the graph will look like that is all.

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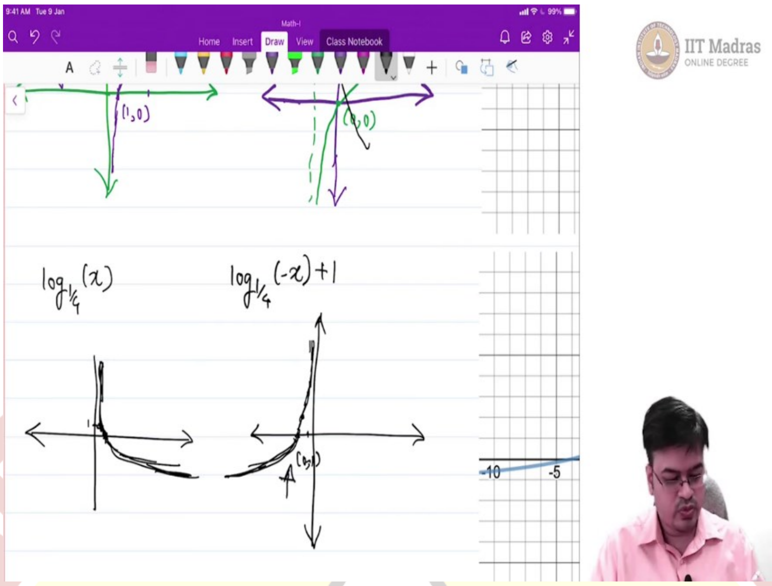

So, let us go to the next problem that is $g(x) = \log_{\frac{1}{4}}(-x+1)$ so, now we will first look at, can I draw a graph of $\log_{\frac{1}{4}}(x)$. The answer is yes I can draw and it will be simply this kind of graph. So, let me draw it a quick snapshot of that graph $\log_4(\frac{1}{x})$ by quick snapshot will give me something like this. So, this is the point and this will increase keep on increasing and it should pass through 1, 0 and a, 1. So, that point will be here somewhere. So, 1, 0 and a 1 so, $\frac{1}{4}$, 1 and 1, 0 correct. So, somewhere here it will be $\frac{1}{4}$ and 1 so this is a point 1 here, fine.

Now, the twist is this particular function is twisted with $\log_{\frac{1}{4}}(-x)$. So, wherever x is there, you are replacing the values with-x that simply changes the paradigm that is, you are taking a reflection along the y axis. So, now essentially when I substitute$-x$, this will be an asymptote this will still remain an asymptote and the graph will switch like this it is an exact mirror image of this along y axis.

So, naturally the point 0, 1 will become 0-1, 0-1 and that $\frac{1}{4}$, 11 will become $(\frac{-1}{4}, 1)$. So, that also will be there. So, this is how the graph will look like now, you are adding a twist to the problem by adding+1. So, what will this thing this operation do. So, earlier $y = \log_{\frac{1}{4}}(-x)$. was there. Now, adding+1 will simply shift these values along y axis in upward direction 1 level up.

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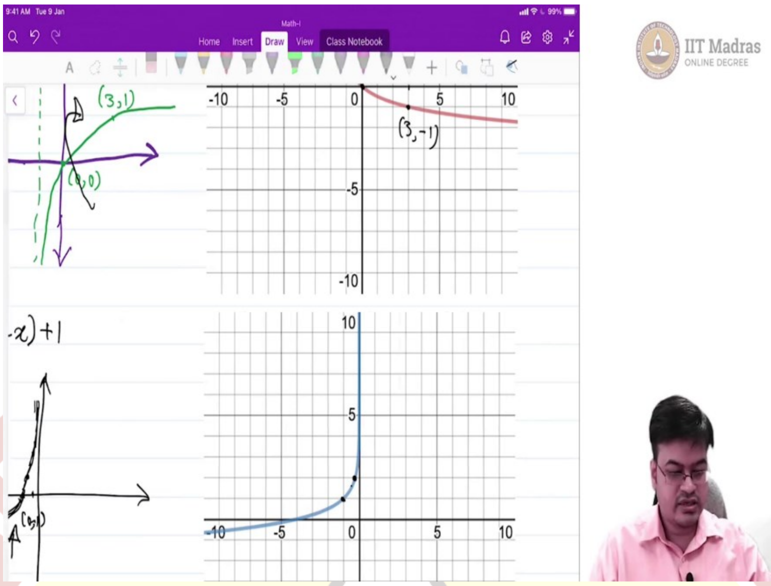

So, naturally the graph will look something like this. So, the point 1,0 will now be shifted to 1,1 that is this point and $\frac{1}{4}$,1 will be shifted to $\frac{1}{4}$,2 that is this point. So, these are the two points. So, we are able to map those two points and therefore, the verification of graph is complete and this is the current graph.

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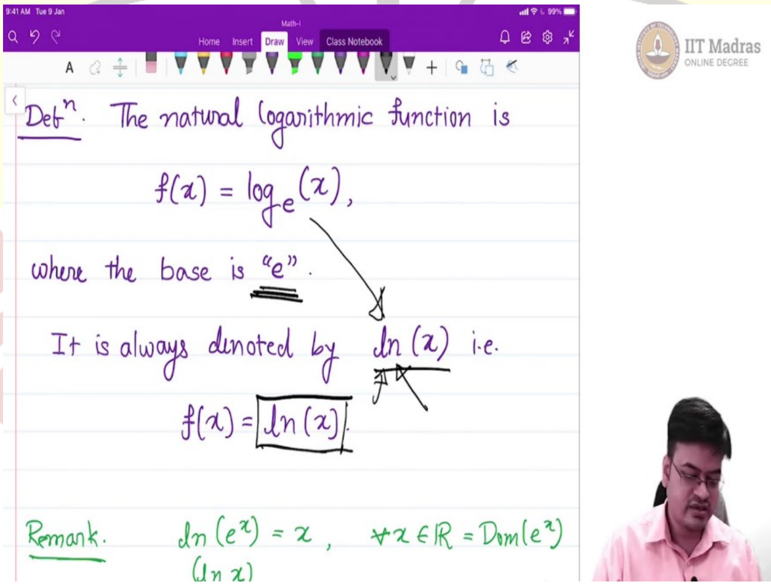

So, now in the next thing that we want to introduce to you is like logarithmic function to the general base a we have some special logarithmic function that is called natural logarithmic function which is which involves the notion of Euler's constant that is e and this is very special as the natural exponential function is special.

So, this function is defined in a separate way as the natural logarithmic function and it is defined as $\log_e x$, where the base is e we have already seen the importance of e in past few videos. But, to add to the speciality, we have some special notation for this $\log_e x$ it is always denoted by $\ln x$, where l stands for logarithm and n stands for natural base.

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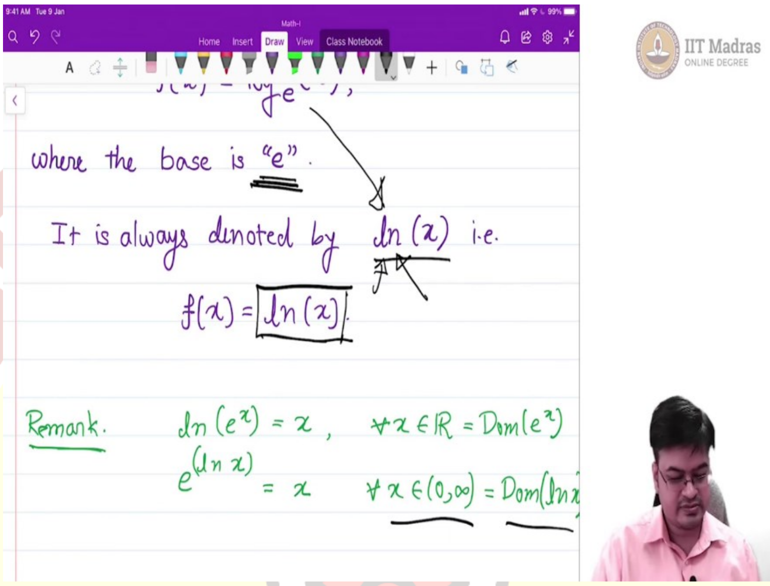

And whenever we write this as ln of x that simply means, I am talking about the natural logarithm of x that is $\log_e x$. So, hence forth whenever we discuss about natural logarithmic function, we have to use this notation ln of x it is quite standard simple verification, you can actually check ln of e raise to x is x for x belonging to $\mathbb{R}$ which is the domain of e^x and $e^{\ln x}$ is x for x belonging to positive real line, which is the domain of $\ln x$.

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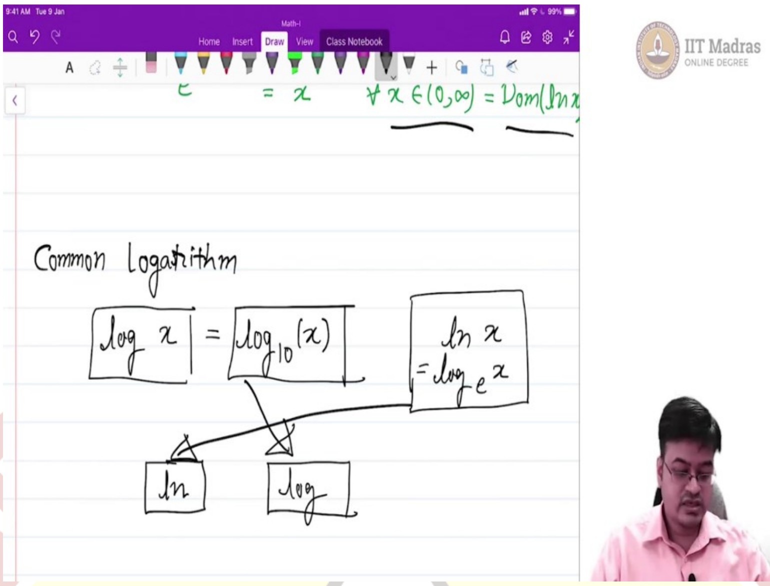

In a similar manner, there is some other log there are some other logarithms, what is called this called natural log something else is called common logarithm, which has to do with our decimal representation and common logarithm is actually denoted as log without any base. So, that means, it is $\log_{10} x = \log x$.

So, in general, the common in common terminology, you may consider when there is no mention of a log, you may consider this log is to the base 10 and if something like $\ln x$ is written, that is $\log_e x$. So, this is what you these are the important things you need to remember. In olden days when we used to use calculators, there were two separate keys associated with this 1 key was ln and another key was log.

So, in these keys, they were commonly referring to $\log_{10} x$if I am talking about log and log to the natural base if I am talking about ln, so just remember these are two commonly used logarithms, which we will use quite often in our daily practice. Calculators distinguish them with ln and log and we prefer that you also distinguish them with ln and log. This finishes the topic of natural logarithm, natural logarithm nothing is special. It is just a way of taking the logs with e as a base.