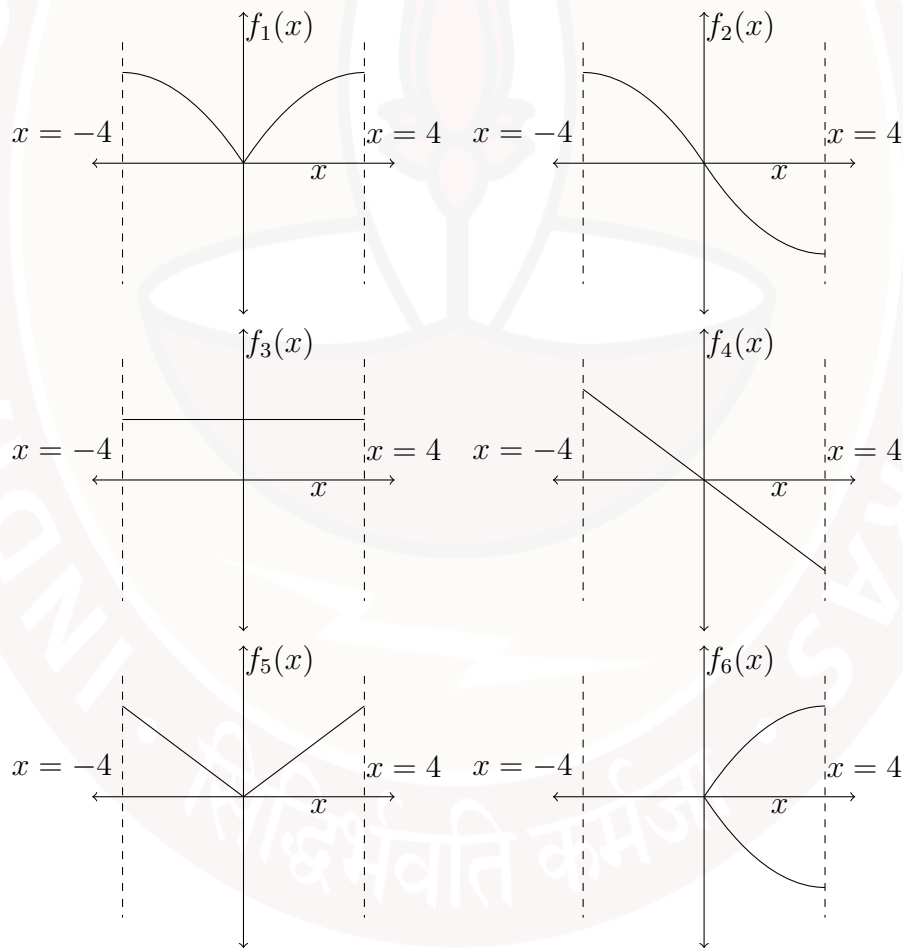| **Week - 7** |
| Practice Assignment |
| **Exponential Functions** |
| Mathematics for Data Science - 1 |

NOTE:
There are some questions which have functions with discrete valued domains (such as month or year). For simplicity, we treat them as continuous functions.

1   Multiple Choice Questions (MCQ):

Answer the questions 1, 2, and 3 based on the given graphs.

Domain for each one is $[-4, 4]$.

1. Choose the correct option.

   - ○ f_3 is not a function.
   - ○ f_6 is not a function.
   - ○ f_5 is not a function.
   - ○ All of the above are functions.

2. Choose the correct option.

   - ○ f_1 and f_3 are one-one functions in the given domain.
   - ○ f_2 and f_4 are one-one functions in the given domain.
   - ○ f_3 and f_5 are one-one functions in the given domain.
   - ○ f_5 is one-one function in the given domain.

3. Choose the correct option.

   - ○ f_1 and f_5 are strictly increasing functions in the given domain.
   - ○ f_2 and f_4 are strictly decreasing functions in the given domain.
   - ○ f_4 and f_5 are strictly decreasing functions in the given domain.
   - ○ f_5 is strictly increasing function in the given domain.

Use the following information for the questions 4 and 5.

Let N_0 be the number of atoms of a radioactive material at the initial stage i.e., at time $t = 0$, and $N(t)$ be the number of atoms of the same radioactive material at a given time t, which is given by the equation $N(t) = N_0 e^{-\lambda t}$, where λ is the decay constant.

4. If at time t_1, the number of atoms reduces to the half of N_0 and at the time t_2 the number of atoms reduces to the one fourth of N_0, then which one of the following equations is correct?

   ○ $e^{\frac{t_1}{t_2}} = 2$

   ○ $e^{\frac{t_2}{t_1}} = 2$

   ○ $e^{\lambda(t_2 - t_1)} = 2$

   ○ $e^{\lambda(t_1 - t_2)} = 2$

5. If $N_{\frac{1}{\lambda}}$ is the number of atoms at the time $t = \frac{1}{\lambda}$, then what will be the ratio of N_0 to $N_{\frac{1}{\lambda}}$?

   ○ $1 : e$

   ○ $e : 1$

   ○ $1 : e^{-\lambda}$

   ○ $1 : e^{\lambda}$

3

2 Multiple Select Questions (MSQ):

6. Selvi deposits ₹P in a bank A which provides an interest rate of 10% per year. After 10 years, she withdraws the whole amount from bank A and deposits it in another bank B for n years which provides an interest rate of 12.5% per year. $M_A(x)$ represents the amount in Selvi's account after x years of depositing in bank A. $M_B(y)$ represents the amount in Selvi's account after y years of depositing in bank B. If the interests are compounded yearly, then choose the set of correct options.

○ $M_A(x)$ is an one-one function of x, for $x \in (0, 10)$.

○ $M_B(y)$ is an one-one function of y.

○ $M_A(12) = P \times 1.1^{12}$

○ $M_A(12) = 0$

○ $M_A(x)$ is a strictly increasing function of x, for $x \in (0, 10)$.

○ $M_B(y)$ is a decreasing function of y.

○ $M_B(n) = (P \times 1.1^{10}) \times (1.125)^n$

○ $M_B(n) = (P \times 1.1^n) \times (1.125)^{10}$

Use the following information for questions 7 and 8.

There are two offers in a shop. In the first offer, the discount in total payable amount is $M(n)\%$ if the number of products bought at a time is n. The second offer involves a discount of ₹1000 on the total payable amount. If Geeta shops of ₹15,000, then answer the following questions.

7. If the total payable amounts after applying the first and second offers (one at a time) are represented by the functions $f(n)$ and $g(n)$ respectively and the total payable amount after applying both the offers together is represented by $T(n)$, then choose the set of correct options.

   ○ $f(n) = (100 - M(n)) \times 15000$ and $g(n) = 14000$

   ○ $f(n) = (100 - M(n)) \times 1500$ and $g(n) = (100 - M(n)) \times 15000 - 1000$

   ○ $f(n) = (100 - M(n)) \times 150$ and $g(n) = 14000$

   ○ $T(n) = (100 - M(n)) \times 15000$ is the total payable amount when the first offer is applied after the second.

   ○ $T(n) = (100 - M(n)) \times 140$ is the total payable amount when the first offer is applied after the second.

   ○ $T(n) = (100 - M(n)) \times 150 - 1000$ is the total payable amount when the second offer is applied after the first.

8. If Geeta is allowed to use the offer in any sequence and $M(n) = -n^2 + 18n - 72$, where $n \in \{6, 7, 8, 9\}$, then choose the set of correct options which minimizes the total payable amount.

○ Total payable amount is same irrespective of the order in which the offers are applied.

○ She should choose offer one and then offer two i.e., $gof(M(n))$.

○ She should choose offer two and then offer one i.e. $gof(M(n))$.

○ If she chooses offer one and then offer two, the minimum payable amount will be ₹12650.

3   Numerical Answer Type (NAT):

Use the following information for questions 9-13.
Given two real valued functions $f(x) = \frac{5x+9}{2x}$, $g(y) = \sqrt{y^2 - 9}$. If $h(x) = f(g(x))$, then answer the following questions.

9. If the domain of $f(x)$ and $g(x)$ are $(-\infty, m) \cup (m, \infty)$ and $\mathbb{R} \setminus (-n, n)$ respectively, then find the value of $m + n$.

10. If the range of $f(x)$ and $g(x)$ are $(-\infty, m) \cup (m, \infty)$ and $[n, \infty)$ respectively, then find the value of $2(m+n)$.

11. If the domain of $h(x)$ is $(-\infty, -3) \cup (m, \infty)$, then find the value of m.

12. If the domain of $f^{-1}(x)$ is $(-\infty, m) \cup (m, \infty)$, then find the value of $2m$.

13. If $f^{-1}(5) = 9/m$, then find the value of m.