Graphs

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Mathematics for Data Science 1
Week 10

Visualizing relations

- Cartesian product $A \times B$

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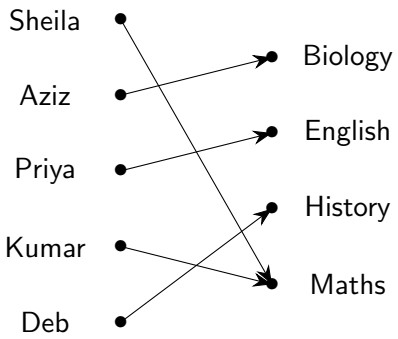
A relation as a graph



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- Introduce graphs formally

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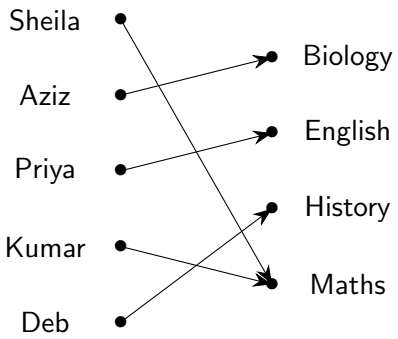
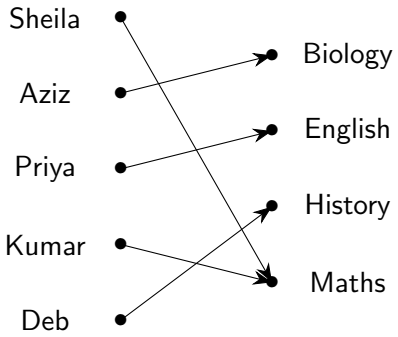


Graphs

- Graph: $G = (V, E)$
  - V is a set of vertices or nodes
    - One vertex, many vertices
  - E is a set of edges
  - $E \subseteq V \times V$ — binary relation

A relation as a graph



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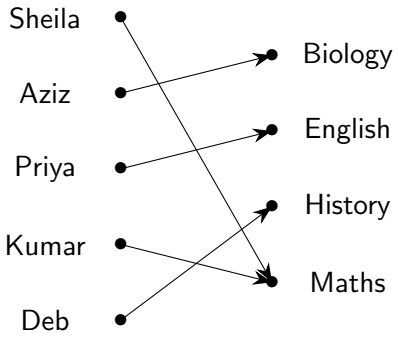

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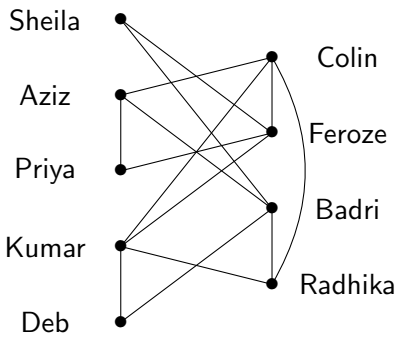
  - The teacher-course graph is directed
- Undirected graph
  - $(v, v') \in E$ iff $(v', v) \in E$
  - Effectively (v, v'), (v', v) are the same edge
  - Friendship relation

Friendship as a graph



Paths

- Priya needs some help that Radhika can provide. How will Priya come to know about this?

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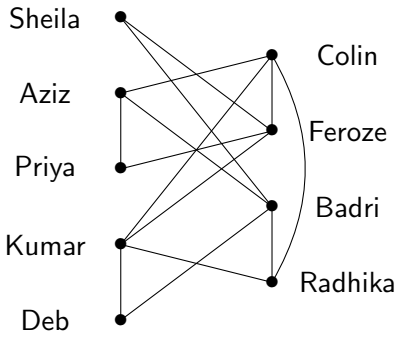

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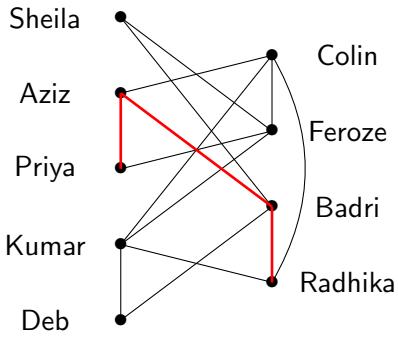

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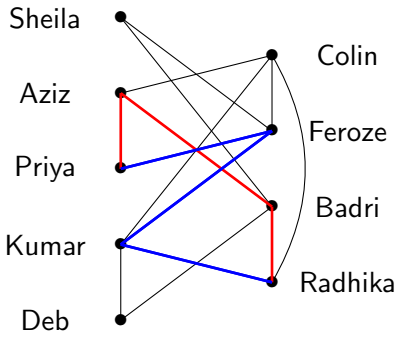

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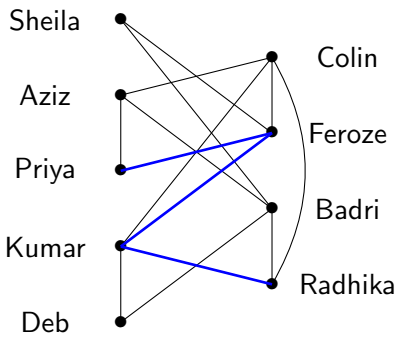

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  - For $1 \leq i < k$, $(v_i, v_{i+1}) \in E$
- Normally, a path does not visit a vertex twice
  - Kumar — Feroze — Colin — Aziz — Priya — Feroze — Sheila
  - Such a sequence is usually called a walk

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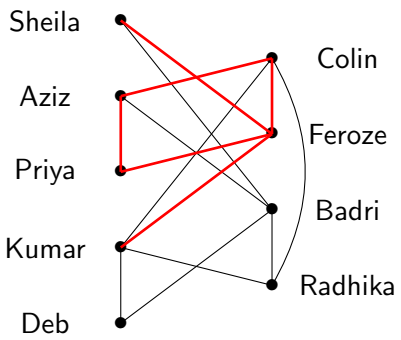

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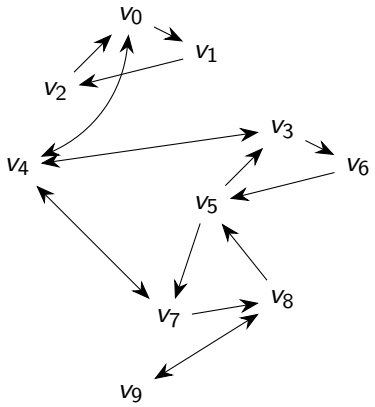
- How can I fly from Madurai to Delhi?

Airline routes



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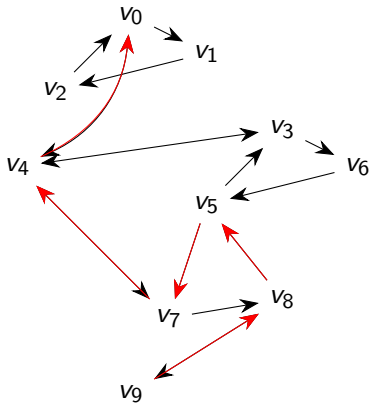

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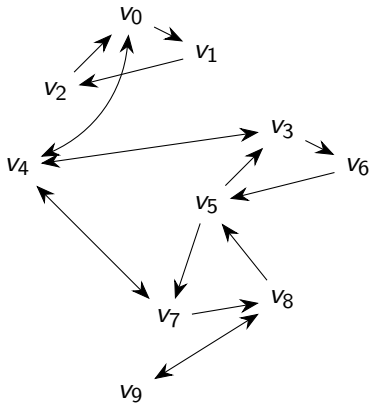

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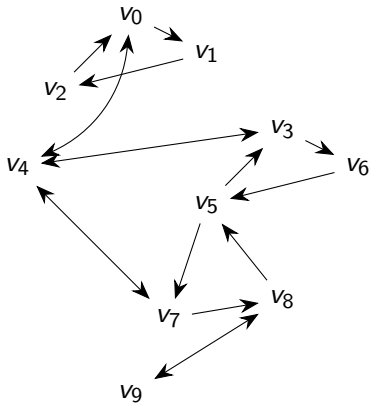

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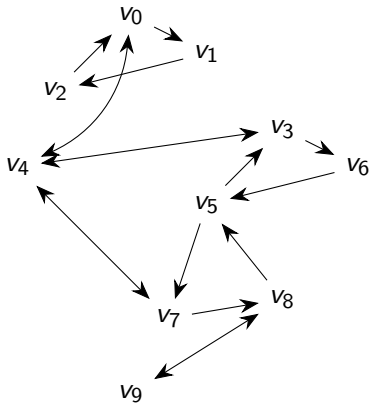

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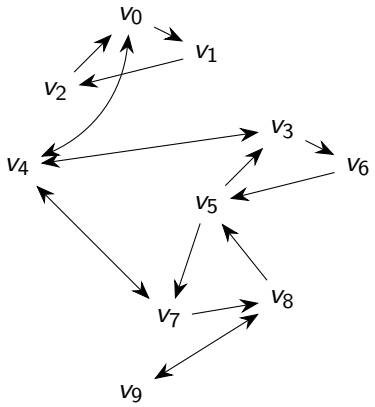

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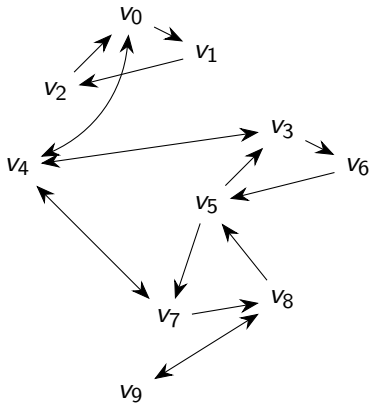

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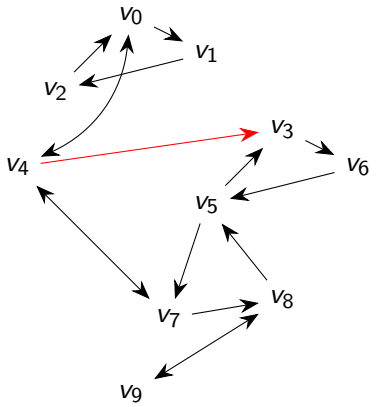

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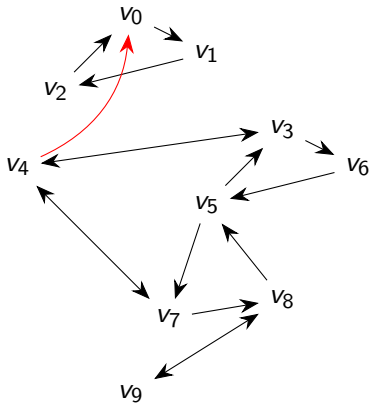

Summary

- A graph represents relationships between entities
  - Entities are vertices/nodes
  - Relationships are edges
- A graph may be directed or undirected
  - A is a parent of B — directed
  - A is a friend of B — undirected
- Paths are sequences of connected edges
- Reachability: is there a path from u to v?