Statistics for Data Science -1

Lecture 8.4: Discrete Random Variable: probability mass function

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Learning objectives

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4. Cumulative distribution function, graphs, and examples.
5. Expectation and variance of a random variable.

Probability mass function, graph, and examples

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- Represent it in tabular form

X	x_1	x_2	x_3	$\ldots$	$\ldots$	x_n
$P(X = x_i)$	$p(x_1)$	$p(x_2)$	$p(x_3)$	$\ldots$	$\ldots$	$p(x_n)$

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- Since X must take one of the values x_i, we have

$$\sum_{i=1}^{\infty} p(x_i) = 1$$

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- Verify that $\sum_{i=1}^{3} p(x_i) = \frac{1}{4} + \frac{1}{2} + \frac{1}{4} = 1$

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Let X be a random variable that takes values 1,2,3,4,5. Which of the following are probability mass functions?

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X	1	2	3	4	5
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  - Recall, $e^x = \sum_{i=0}^{\infty} \frac{x^i}{i!}$, hence $\mathrm{c}\sum_{i=0}^{\infty} \frac{\lambda^i}{i!} = ce^{\lambda}$
  - Hence, $\mathrm{c}\sum_{i=0}^{\infty} \frac{\lambda^i}{i!} = ce^{\lambda} = 1$ which gives $c = e^{-\lambda}$

Example: Rolling a dice twice

- $S = \left\{ \begin{array}{l} (1,1),(1,2),(1,3),(1,4),(1,5),(1,6),(2,1),(2,2),(2,3),(2,4),(2,5),(2,6), \\ (3,1),(3,2),(3,3),(3,4),(3,5),(3,6),(4,1),(4,2),(4,3),(4,4),(4,5),(4,6), \\ (5,1),(5,2),(5,3),(5,4),(5,5),(5,6),(6,1),(6,2),(6,3),(6,4),(6,5),(6,6), \end{array} \right\}$

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X	2	3	4	5	6	7	8	9	10	11	12
$P(X = x_i)$	$\frac{1}{36}$	$\frac{2}{36}$	$\frac{3}{36}$	$\frac{4}{36}$	$\frac{5}{36}$	$\frac{6}{36}$	$\frac{5}{36}$	$\frac{4}{36}$	$\frac{3}{36}$	$\frac{2}{36}$	$\frac{1}{36}$

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  - Verify: $\sum_{i=1}^{11} p(x_i) =$

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- X is the random variable which counts the number of heads in the tosses
  - Probability mass function

X	0	1	2	3
$P(X = x_i)$	$\frac{1}{8}$	$\frac{3}{8}$	$\frac{3}{8}$	$\frac{1}{8}$

  - Verify: $\sum_{i=1}^{4} p(x_i) = \frac{8}{8} = 1$
- Y is the random variable which counts the toss in which heads appears first
  - Probability mass function

Y	1	2	3	NIL
$P(Y = y_i)$	$\frac{4}{8}$	$\frac{2}{8}$	$\frac{1}{8}$	$\frac{1}{8}$

  - Verify: $\sum_{i=1}^{4} p(y_i) = \frac{8}{8} = 1$

Section summary

- Probability mass function.
- Properties of probability mass function.