Statistics for Data Science -1
Lecture 5.2: Factorials

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Learning objectives

1. Understand basic principles of counting.
2. Concept of factorials.
3. Understand differences between counting with order (permutation) and counting without regard to order (combination).
4. Use permutations and combinations to answer real life applications.

Factorial

Example 3: Order of finishes in a race

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▶ Hence the total number of ways $=$
$8 \times 7 \times 6 \times 5 \times 4 \times 3 \times 2 \times 1 = 40,320$

Factorial

Definition

The product of the first n positive integers (counting numbers) is called n factorial and is denoted n!. In symbols,

$$n! = n \times (n-1) \times \ldots \times 1$$

Remark

By convention $0! = 1$

Example 4: Choosing shirts

1.
2.
3.
4.
5.
6.

Example 5

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   ▶ In general, for $i \leq n$ we have,
   $$n! = n \times (n-1) \ldots \times (n-i+1) \times (n-i)!$$

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3. Express $25 \times 24 \times 23$ in terms of factorials-

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$$\frac{25 \times 24 \times 23 \times 22 \times \ldots \times 1}{22 \times 21 \times \ldots \times 1} = \frac{25!}{22!}$$

Section summary

- ▶ Introduced factorial notation.
- ▶ Simplifying expressions.