Statistics for Data Science -1

Lecture 6.6: Probability- Equally likely outcomes

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Learning objectives

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- That is, $P(A) = \frac{\text{number of outcomes in } S \text{ that are in } A}{N}$

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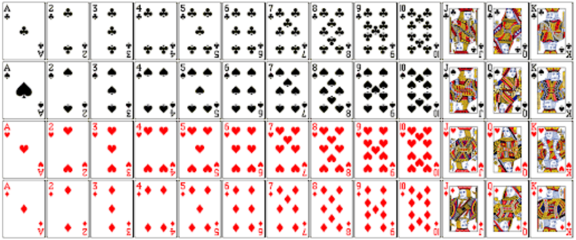
- Let B be the event that the outcome is greater than 4. $B = \{5, 6\}$ $P(B) = \frac{2}{6}$
- Let C be the event that the outcome is either odd or greater than 4. $P(C) = P(A \cup B) = P(A) + P(B) - P(A \cap B) = \frac{3}{6} + \frac{2}{6} - \frac{1}{6} = \frac{4}{6}$

Example: playing cards

When drawing a card from a standard deck of 52 playing cards, what is the probability that the card is either red or a queen?



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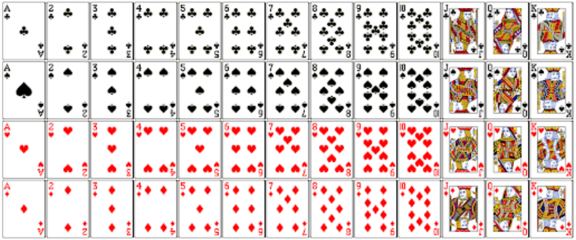

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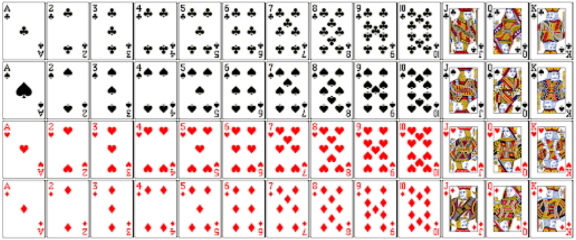

- Let R be the event that the card drawn is Red.
  $P(R) = \frac{26}{52} = \frac{1}{2}$
- Let Q be the event that the card drawn is Queen.
  $P(Q) = \frac{4}{52} = \frac{1}{13}$

Example: playing cards-contd.

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- $R \cap Q$ describes the event that the card drawn in a Red Queen. $P(R \cap Q) = \frac{2}{56}$
- Hence $P(R \cup Q) = \frac{26}{52} + \frac{4}{52} - \frac{2}{52} = \frac{28}{52} = \frac{7}{13}$

Topic summary

1. Interpretations of probability
2. Probability axioms
3. Addition rule of probability.
4. Equally likely outcomes.