Statistics for Data Science -1

Lecture 10.2: Independent and identically distributed Bernoulli trials

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Learning objectives

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1. Derive the formula for the probability mass function for Binomial distribution.

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3. Expectation and variance of the binomial distribution.

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1. Derive the formula for the probability mass function for Binomial distribution.
2. Understand the effect of parameters n and p on the shape of the Binomial distribution.
3. Expectation and variance of the binomial distribution.
4. To understand situations that can be modeled as a Binomial distribution.

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Definition

A collection of random variables is iid if the random variables are independent and share a common probability distribution

Non Independent trials- Example

- Consider the experiment where three balls are chosen without replacement from a bag containing 20 red balls and 40 black balls. The number of red balls drawn is recorded. Is this a binomial experiment?

Non Independent trials- Example

- Consider the experiment where three balls are chosen without replacement from a bag containing 20 red balls and 40 black balls. The number of red balls drawn is recorded. Is this a binomial experiment?
- NO!!- The balls are chosen without replacement. The colour of first ball will affect the chances of colour of the next balls- Independence criteria NOT satisfied.

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- Let X is the total number of successes that occur in n trials, then X is said to be a binomial random variable with parameters n and p.

Non Binomial experiment

- Consider the experiment of rolling a dice until a 6 appears- is it a Binomial experiment?

Non Binomial experiment

- Consider the experiment of rolling a dice until a 6 appears- is it a Binomial experiment?
- NO!!-Number of trials n is not fixed in this case

n=3 independent trials

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- The outcomes of the independent trials are

S.No	Outcome
1	(s,s,s)
2	(s,s,f)
3	(s,f,s)
4	(s,f,f)
5	(f,s,s)
6	(f,s,f)
7	(f,f,s)
8	(f,f,f)

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- The probabilities of outcomes of the independent trials are

S.No	Outcome	Probabilities
1	(s,s,s)	$p \times p \times p$
2	(s,s,f)	$p \times p \times (1-p)$
3	(s,f,s)	$p \times (1-p) \times p$
4	(s,f,f)	$p \times (1-p) \times (1-p)$
5	(f,s,s)	$(1-p) \times p \times p$
6	(f,s,f)	$(1-p) \times p \times (1-p)$
7	(f,f,s)	$(1-p) \times (1-p) \times p$
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- The probabilities of outcomes of the independent trials are

S.No	Outcome	Number of successes	Probabilities
1	(s,s,s)	3	$p \times p \times p$
2	(s,s,f)	2	$p \times p \times (1-p)$
3	(s,f,s)	2	$p \times (1-p) \times p$
4	(s,f,f)	1	$p \times (1-p) \times (1-p)$
5	(f,s,s)	2	$(1-p) \times p \times p$
6	(f,s,f)	1	$(1-p) \times p \times (1-p)$
7	(f,f,s)	1	$(1-p) \times (1-p) \times p$
8	(f,f,f)	0	$(1-p) \times (1-p) \times (1-p)$

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- Let X = number of successes in 3 independent trials.
- The probability distribution of X

X	0	1	2	3
$P(X = i)$	$(1-p)^3$	$3 \times p \times (1-p)^2$	$3 \times p^2 \times (1-p)$	p^3

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- $X=$ number of succeses in n independent trials.
- The probabilities of outcomes of the independent trials are

S.No	Outcome	Number of successes	Probabilities
1	(s,s,...,s)	n	$p \times p \times \ldots \times p$
2	(s,s,...,f)	$n-1$	$p \times p \times \ldots \times (1-p)$
3	(s,...,f,s)	$n-1$	$p \ldots \times p \times (1-p) \times p$
$\vdots$	$\vdots$	$\vdots$	$\vdots$
2^{n-2}	(f,...,s,f)	1	$(1-p) \times (1-p) \ldots \times p \times (1-p)$
2^{n-1}	(f,f,...,s)	1	$(1-p) \times (1-p) \ldots \times p$
2^n	(f,f,...,f)	0	$(1-p) \times (1-p) \ldots \times (1-p)$

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- There number of different outcomes that result in i successes and $(n-i)$ failures$=\binom{n}{i}$
- The probability of i successes in n trials is given by

$$P(X=i) = \binom{n}{i} \times p^i \times (1-p)^{(n-i)}$$

Section summary

- Independent and identically distributed distribution
- n independent trials
- Probability of i successes and $(n - i)$failures in n independent trials.