Statistics for Data Science -1

Lecture 5.4: Permuations formula: objects are not distinct & Circular permutations

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Learning objectives

1. Understand basic principles of counting.
2. Concept of factorials.
3. Understand differences between counting with order (permutation) and counting without regard to order (combination).
4. Use permutations and combinations to answer real life applications.

Permutations

Permutation when objects are not distinct

Circular permutations

Solving of n and r using permutation formula

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First place	Second place	Third place	Fourth place
A	D	T	A
A	D	A	T
A	T	D	A
A	T	A	D
A	A	D	T
A	A	T	D
D	A	T	A
D	A	A	T
D	T	A	A
T	A	D	A
T	A	A	D
T	D	A	A

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- If they are treated as distinct objects, then based on the earlier formula, total number of arrangements $= 4!$.
- Now A_1 and A_2 can be arranged among themselves in 2! ways.
- A_1 and A_2 are essentially the same. Hence, the total number of ways the letters in "DATA" can be arranged is $\frac{4!}{2!} = 12$

Permutation formula

- The number of permutations of n objects when p of them are of one kind and rest distinct is equal to

$$\frac{n!}{p!}$$

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- Total of ten letters of which there are five distinct letters : S,T,A,I,C.
- "S" appears 3 times; "T" appears 3 times, "A' once, "I" twice, and "C" once

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- Applying the above formula to the word " STATISTICS"; $n = 10, p_1 = 3, p_2 = 3, p_3 = 1, p_4 = 2, p_5 = 1.$ Hence, total number of ways =

$$\frac{10!}{3!3!1!2!1!} =$$

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$$\frac{10!}{3!3!1!2!1!} = 50,400$$

Section summary

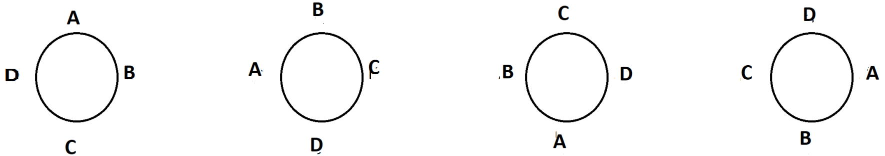
1. The number of permutations of n objects when p of them are of one kind and rest distinct is equal to $\frac{n!}{p!}$
2. The number of permutations of n objects where p_1 is of one kind, p_2 is of second kind, and so on p_k of k^{th} kind is given by $\frac{n!}{p_1!p_2!\dots p_k!}$

Example

- How many ways can four people sit in a round table?
- We consider two cases: each selection is called a combination of 3 different objects taken 2 at a time.
  - Clockwise and anticlockwise are different
  - Clockwise and anticlockwise are same.

Circular permutation: Clockwise and anticlockwise are different

- Consider the linear permutations of A, B, C and D
- The arrangements $ABCD$, $BCDA$, $CDAB$, and $DABC$ are different when the people are seated in a row.
- However, when they are seated in a circle as shown below:



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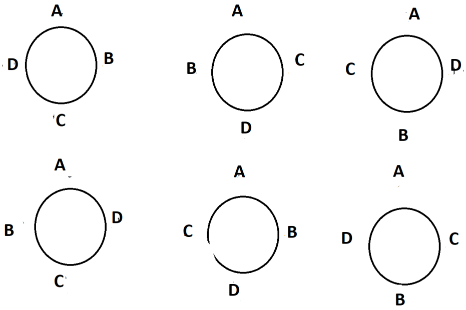
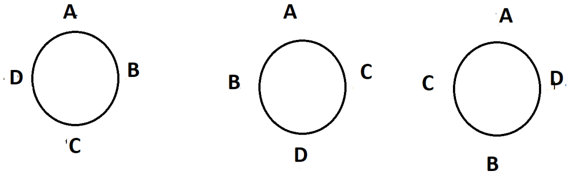


The number of ways n distinct objects can be arranged in a circle (clockwise and anticlockwise are different) is equal to $(n-1)!$

Circular permutation: Clockwise and anticlockwise are same



The number of ways n distinct objects can be arranged in a circle (clockwise and anticlockwise are same) is equal to $\frac{(n-1)!}{2}$

Example : Solving for n

- Find value of n if ${}^{n}P_4 = 20\,{}^{n}P_2$

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  Solving $(n-2) \times (n-3) = 20$, we get $n = -2$ or $n = 7$.
  Eliminating $n = -2$, we get $n = 7$.

Example : Solving for n

- $\dfrac{{}^{n}P_4}{{}^{n-1}P_4} = \dfrac{5}{3}$

  Answer: $\dfrac{n!}{(n-4)!} \times \dfrac{(n-5)!}{(n-1)!} = \dfrac{5}{3}$

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  $\frac{n}{(n-4)} = \frac{5}{3}$

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  Answer: $\frac{n!}{(n-4)!} \times \frac{(n-5)!}{(n-1)!} = \frac{5}{3}$
  $\frac{n}{(n-4)} = \frac{5}{3}$
  Solving for n gives us $n = 10$.

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  $\frac{5!}{(5-r)!} = 2.\frac{6!}{(7-r)(6-r)(5-r)!}$

Example : Solving for r

- Find r, if ${}^5P_r = 2.{}^6P_{r-1}$
  Answer: $\frac{5!}{(5-r)!} = 2.\frac{6!}{(7-r)!}$
  $\frac{5!}{(5-r)!} = 2.\frac{6!}{(7-r)(6-r)(5-r)!}$
  Solving $(7-r)(6-r) = 12$ gives $r = 10$ or $r = 3$.

Since $r \leq n$, the option $r = 10$ is eliminated and we get $r = 3$.

Topic summary

1. Permutations when objects are distinct
   1.1 repetitions not allowed.
   1.2 repetitions allowed.
2. Permutations when objects are not distinct.
3. Circular permutations:
   3.1 Clockwise and anticlockwise are different.
   3.2 Clockwise and anticlockwise are same.
4. Solving for r and n using the permutation formula.