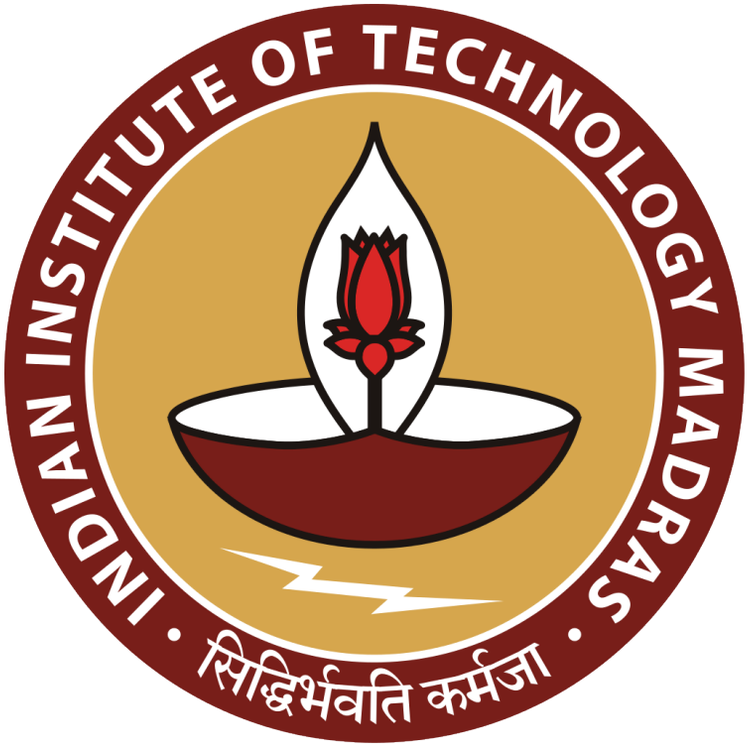


IIT Madras

ONLINE DEGREE

Example

Let $f(x) = x^2-6x+9$.

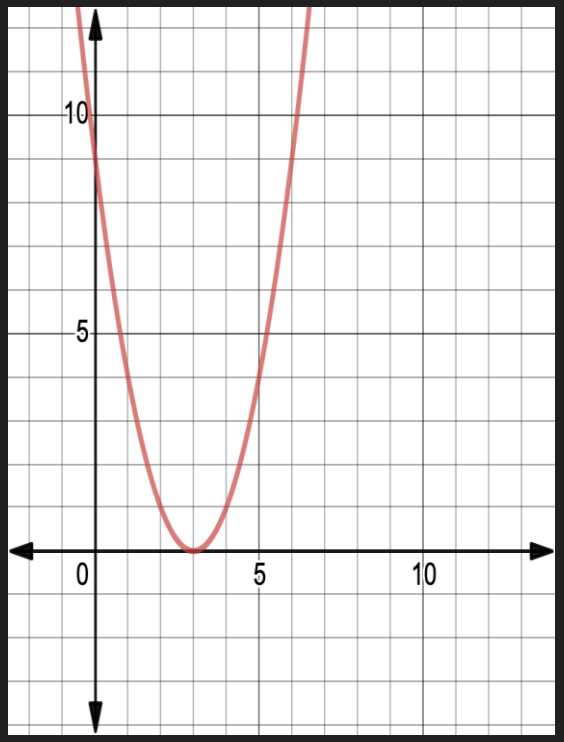
1. Determine whether f has minimum or maximum value. If so, what is the value?
2. State the domain and the range of f.

Observe that $a=1$, $b=-6$, and $c=9$.

Since, $a>0$, the function opens up and has the minimum value.

The minimum value is given by y-coordinate of the vertex. The x-coordinate of the vertex is $-b/(2a) = 3$. Therefore, the minimum value is $f(3) = 0$.

Domain = $\mathbf{R}$ and Range = $\mathbf{R} \cap \{f(x) | f(x) \geq 0\}$.



Example

A tour bus in Chennai serves 500 customers per day. The charge is ₹40/- per person. The owner of the bus service estimate that the company would lose 10 passengers per day for each ₹4/- fare hike.

How much should the fare be in order to maximize the income of the company?

Let x denote the number of ₹4/- fare hike. Then the price per passenger is $40+4x$, and the number of passengers is $(500-10x)$. Therefore, the income is

$$I(x) = (500-10x)(40+4x) = -40x^2+1600x+20000.$$

In this case, $a = -40$, $b = 1600$ and $c=20000$, and the maximum value attained will be $I(-b/(2a)) = I(20) =36000$.

This means the company should make 20 fare hikes of ₹4/- in order to maximize its income. That is the new fare = 40 + 4 x 20= ₹120/-

Slope of a quadratic function

Given a quadratic function, $f(x) = ax^2 + bx + c$, where $a \neq 0$, how to determine the slope of f?

Recall, for a linear function $y = g(x) = mx + c$, we have calculated the ratio of change in y and change in x and observed that it remains constant and is m. We also showed that $m = \tan\theta$, where θ is the inclination with positive X-axis.

Let us use similar analogy for a quadratic function and define slope of a quadratic function.

We now discuss the concept using a simple example.

Slope of a quadratic function

Given a quadratic function, $f(x) = ax^2 + bx + c$, where $a \neq 0$, how to determine the slope of f?

Let $y = x^2$ be a quadratic function given.