INDIAN INSTITUTE OF TECHNOLOGY MADRAS
सिद्धिर्भवति कर्मजा

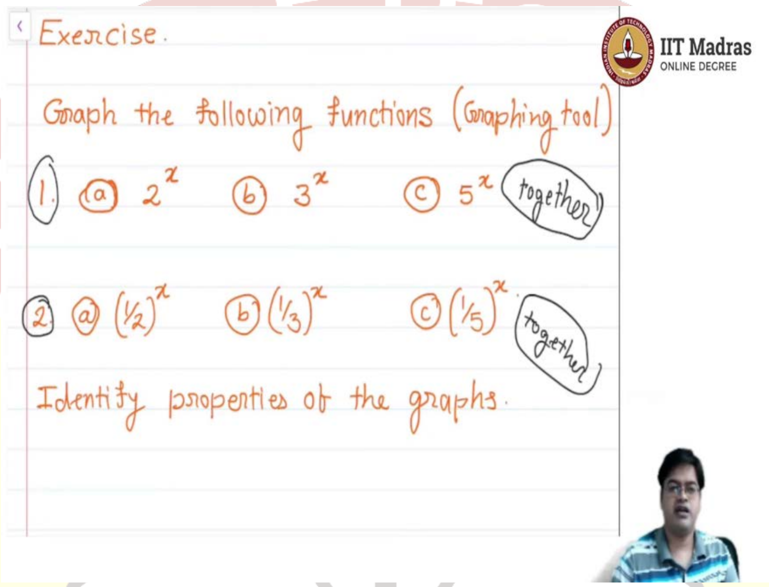
IIT Madras

ONLINE DEGREE

Mathematics for Data Science 1
Prof. Neelesh S Upadhye
Department of Mathematics
Indian Institute of Technology, Madras

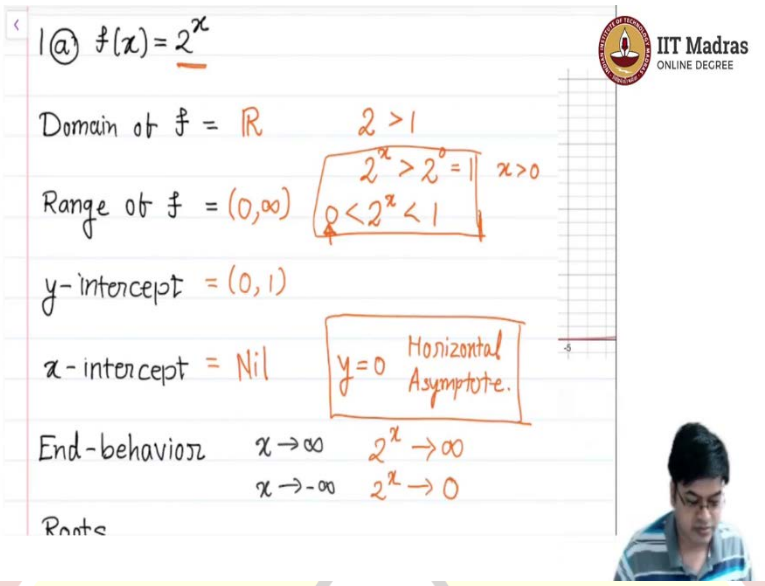
Lecture – 8.4
Exponential Functions: Graphing

(Refer Slide Time: 00:15)



Welcome back. So, I hope you must have done your exercises and you must have developed some understanding about the exponential functions. Let us try to collect recollect that understanding through 2 examples given here.

(Refer Slide Time: 00:31)



So, let us first take 1 a which is $f(x) = 2^x$. If you have used DESMOS, you must have got the figure of the function. But prior to receiving the figure of the function, let us see what should be the domain of a function.

We have already discussed in greater detail that the domain of a function can be a ℝ, entire real line. Now, if you look at this function which is 2^x, this $2 > 1$ and the $2^x > 2^0$ which is equal to 1, $2^x > 2^0$ whenever x is positive correct.

Now, because $x > 0$, then $2^x > 2^0$ ok. So, if $x < 0$, what will happen? 2^x, when $x < 0$ will always be less than 1. This is also possible. But when this 2 raised to; can this 2^x become negative? No. So, it is always greater than 0.

So, if you have this understanding, then you can easily write the function has a range which is $(0, \infty)$. So, there is a split from when you consider a point 1, there is something happening at point $(0, 1)$ right. What is $(0, 1)$? $(0, 1)$ actually is an y −intercept ok, something is happening at $(0, 1)$ because I have put 0 here for then it is I am getting 1.

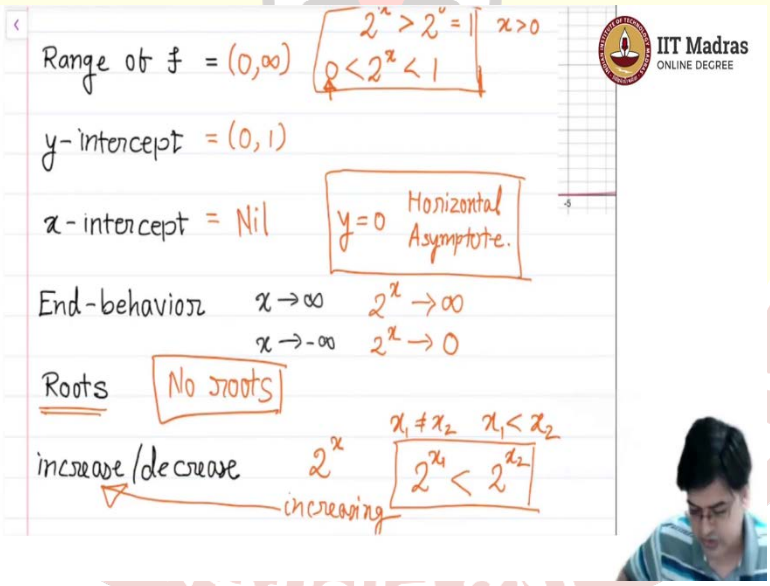
So, $(0, 1)$ is also y −intercept and there is something happening which is going below 0. Is going below 1, your graph is going below 1 and therefore, this particular thing is going down, but it never goes below 0. This is an interesting fact because if you consider 2^x, it never goes below 0.

It cannot go to a negative number. Therefore, will it touch the $X-$ axis? It will not touch $X-$ axis. In fact, $x-$ intercept is nil ok, but it is approaching 0. So, the something that is approaching 0, so $x-$intercept is actually it will never touch it; but it will actually go along that line. So, this $y=0$, it will touch at infinity ok. So, such a thing, we call as horizontal asymptote ok.

So, such a thing you call as horizontal asymptote. So, with this understanding, these are the things that I can make out directly without looking at the graph. So, let us now look at the graph ok, before going to that, let us see what happens to the end behavior. End behavior of a function as $x \to \infty$.

So, as 2^x, you consider a function 2^x as x increases, this also increases. In fact it increases at a rapid rate than x. So, this also should tend to infinity and as $x \to -\infty$, we have already figured out $y=0$ is the horizontal asymptote. So, 2^x will actually go to 0 ok.

(Refer Slide Time: 05:29)

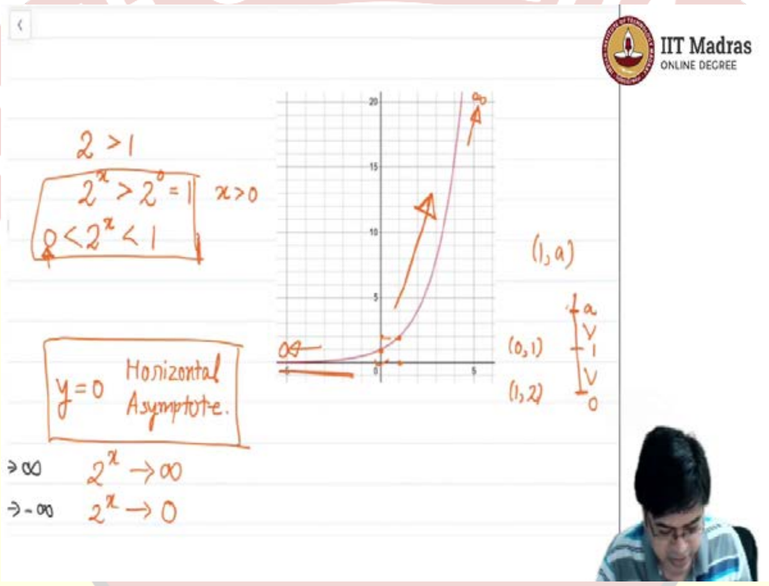


Then, the question that we used to quantify while considering the function, what are the roots of this function. So, do they have any roots? In fact, using graphical method, it is very clear that it never touches 0. So, there are no roots and the functions increase and decrease.

So, the domains of increase and decrease like polynomials, we studied domains of increase and decrease; but here, I think my claim is no need to identify the domains of increase and decrease. Why? Because you look at a function 2^x, let us take $x_1 \neq x_2$ or $x_1 < x_2$, without loss of generality, we can take this. Then, what can you say about 2^{x_1} and 2^{x_2}?

See $x_1 < x_2$, so naturally if it is raised to the power 2; 2^{x_1} and 2^{x_2}, this relation should hold. So, what I am saying is the function is actually an increasing function and increasing functions are 1 to 1. Therefore, I do not have any doubt that the increase and decrease, it is only increasing; throughout the real line, the function is only increasing.

(Refer Slide Time: 07:19)



So, let us look at the graph of a function $f(x) = 2^x$. Let us identify the points. So, here you can identify a point right. So, this point we have seen as $y -$ intercept and that point was (0, 1) right. Then, the one in this case, let us look at this point which is 1 and where will it go? It will actually tell you 2.

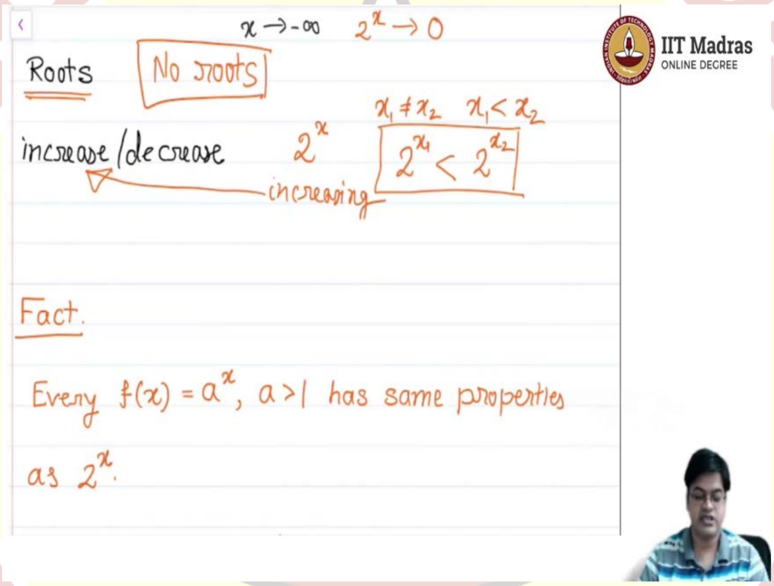
So, the point is (1, 2), the second point ok. So, these 2 points are very special points, they tell you something. So, in particular, had it not been 2^x, but a^x, then that point would have been $(1, a)$ and if you mimic this graph over here y x is over here ok, this is a point 1, this is the point 0 and this is the point which is a.

So, that says $a > 1$; this relation is there, is greater than 0 yeah and therefore, the graph was a point which lies here, which is here right. As $x \to \infty$, this graph actually goes to infinity; as $x \to -\infty$, this graph goes to 0. These two points is these two point and this is an increasing function.

As you come from left to right, it increases. So, this is an increasing function, $y = 0$ is the horizontal asymptote, that is very clear ok. The range of a function is 0 to infinity, that is also very clear. The domain of a function is entire real line, $\mathbb{R}$.

So, we have got all the details necessary for finding this. Now, what it so special about 2^x, if I replace this 2 with 3, still I will have y −intercept to be 0, 1 because 3^0 is also 1 and I will again have domain of f to be equal to $\mathbb{R}$; range of f to be equal to 0 infinity; no $x -$ intercept; $y = 0$ will be horizontal asymptote; $x \to \infty$, $a^x \to \infty$, $x \to -\infty$, $a^x \to 0$. There are no roots. The function is only increasing.

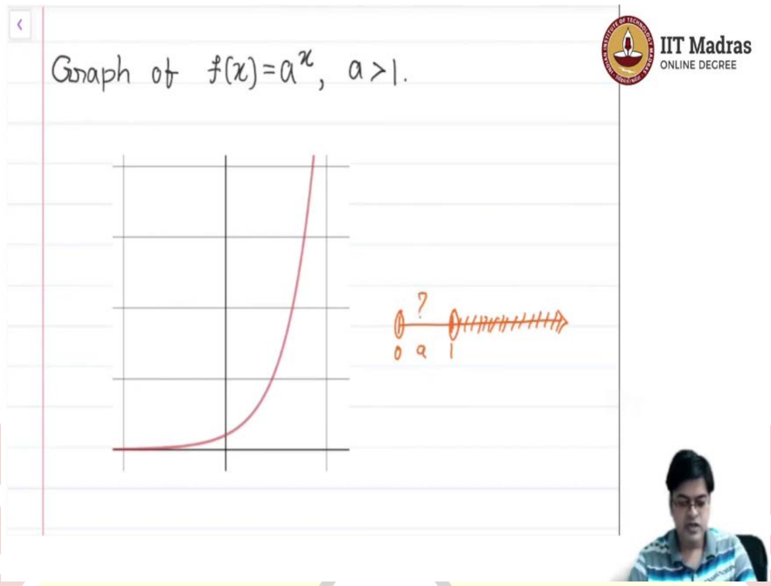
(Refer Slide Time: 10:19)



And therefore, I will state this as a fact that every $f(x) = a^x$, for $a > 1$ will have same properties as 2^x. So, I do not there is no need to draw different different values. The behavior is same only the values will change.

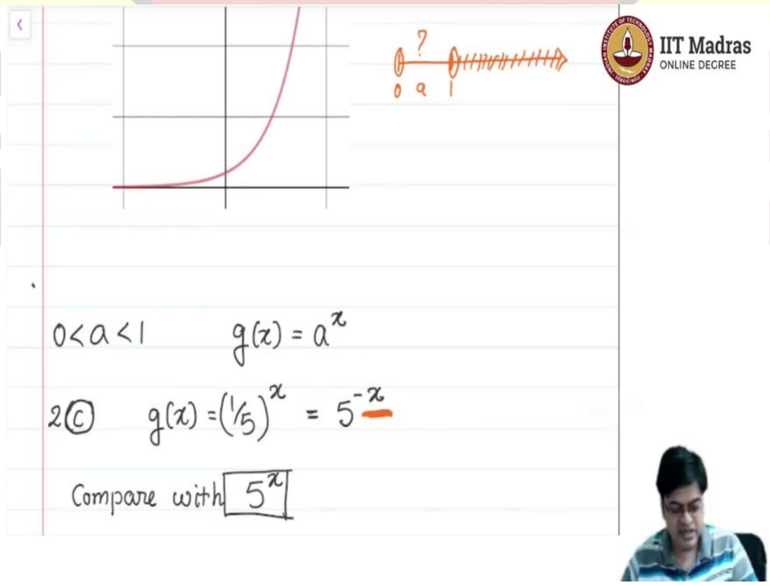
For example, in this case, where you have seen the graph of this (1, 2) is a point; (1, 2) is a point, suppose I consider 3^x, (1, 3) will be the point. So, only the values are changing; but the shape, the behavior, everything else that is listed here remains the same. Therefore, you do not have to draw a graph every time, only thing is you need to evaluate the values in general.

(Refer Slide Time: 11:15)



So, what is the graph of $f(x) = a^x$ in general? It is this way for $a > 1$. So, remember that line that we have drawn which is that the line for a, where we have eliminated these 2 points such as 0, this is 1, we have identified what is the case for $a > 1$. You have also identified the case, where $0 < a < 1$. So, let us go back and see what happens when $0 < a < 1$. So, if a lies here how is the behavior? So, you have already analyzed.

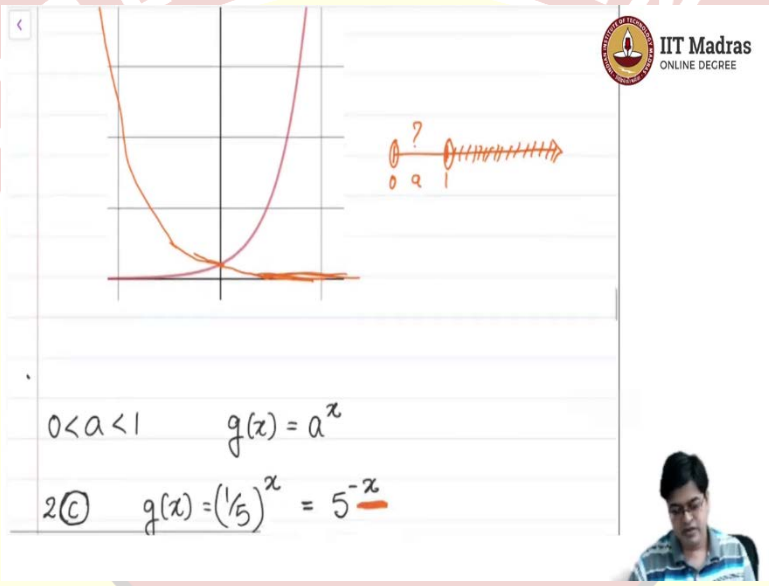
(Refer Slide Time: 11:59)



And let us take this function as $g(x)$ and take it to be $g(x) = a^x$ and this is $\left(\frac{1}{5}\right)^x$. Now, you do not really have to draw this graph, what you can do is ok. So, $g(x) = 5^{-x}$. So, here x is replaced by $-x$. So, what will be the change in the behavior?

So, when x is replaced by $-x$, you know its reflection across $Y-$ axis, you have solved many examples in the assignments. This $Y-$ axis, this is x; then when I put it as $-x$, it will be simply reflected along $Y-$ axis.

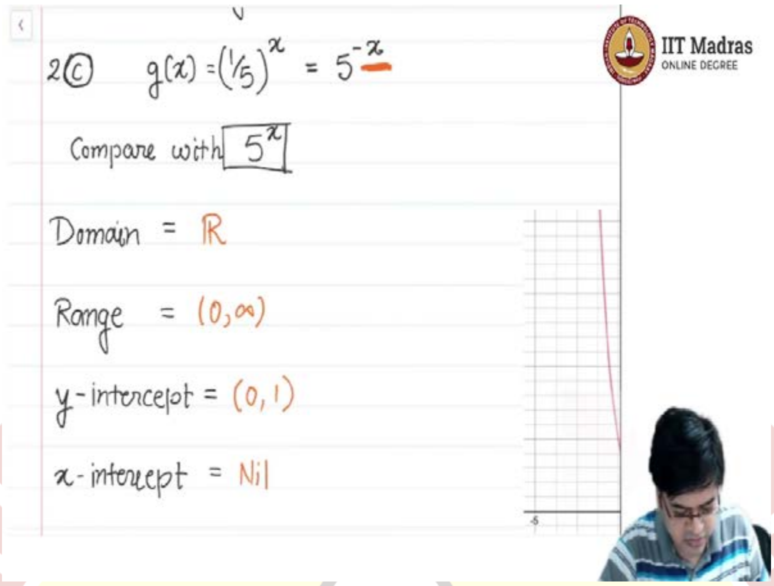
(Refer Slide Time: 12:53)



So, if you look at this graph and try to draw a graph of this function, then it should be something like coming from here going here, it should be something like this, it should actually look like a reflection along $Y-$ axis. So, let us try to show it as reflection ok. This will actually go very close, but never touch.

So, let me erase this ok. So, this is how it will look like. So, without actually thinking about anything else, you can simply draw a graph of $\left(\frac{1}{5}\right)^x$; but still let us try to do it in regular set up.

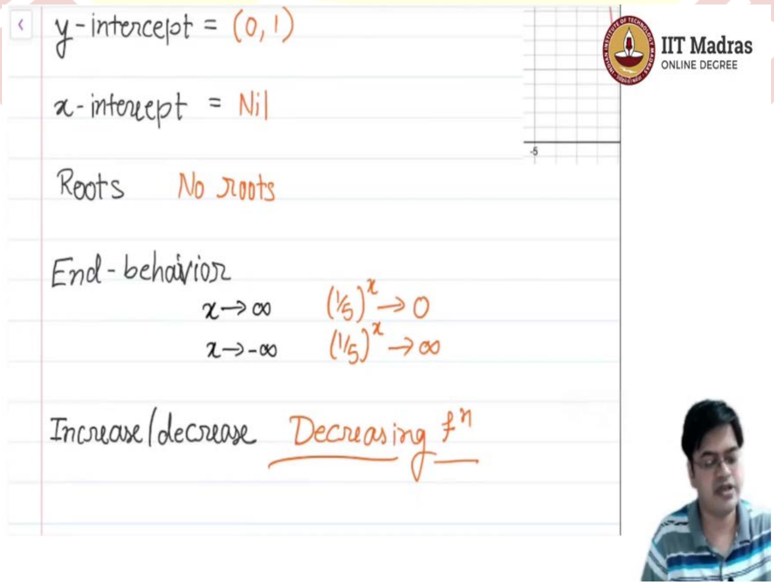
(Refer Slide Time: 13:43)



So, what will be the domain of this function? The domain of this function is very clear because we have used it several times, the domain of this function will be real line. Range, nothing changes; $(0, \infty)$ because it is a reflection across $Y-$ axis. So, let us look at this function.

So, the domain will be $\mathbb{R}$; range will be $(0, \infty)$. What will be the $y-$ intercept? Because it is a reflection, so $y-$ intercept would not change, so it will be 0, 1 only. $x-$ intercept will be nil, there would not be any $x-$ intercept.

(Refer Slide Time: 14:25)

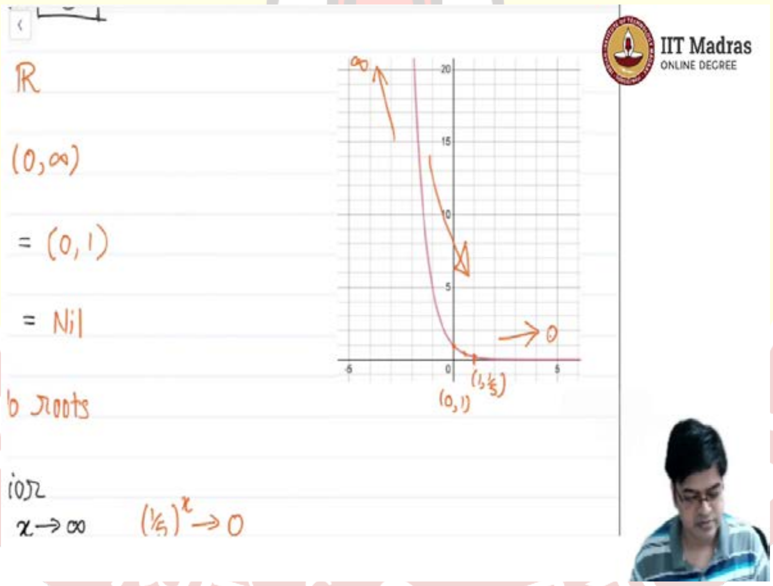


And therefore, no roots and what about the end behavior? End behavior is like $x \to \infty$, $x \to -\infty$. So, when $x \to \infty$, the end behavior will be because it is a reflection you see.

So, when $x \to \infty$ there, it was going to ∞. So, and $x \to -\infty$, function 5^x would have behaved, it will go to 0. So, that reflection will make this a^x or $\left(\frac{1}{5}\right)^x$ whatever is the function $\left(\frac{1}{5}\right)^x$, let me do it properly.

So, this will make $\left(\frac{1}{5}\right)^x$ to go to 0 and this function $\left(\frac{1}{5}\right)^x$ will go to infinity ok. Good. Then, because it is a reflection, the increasing thing will become decreasing. So, there is no intelligence here. So, this will be in fact a decreasing function wonderful. So, we have analyzed everything without taking much efforts. This is the beauty of once you understand the functions on graphical plane.

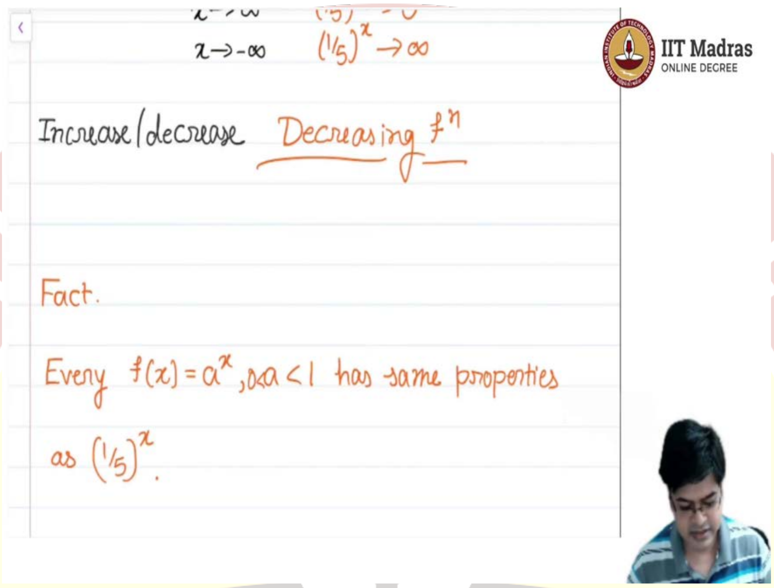
(Refer Slide Time: 16:03)



So, here is the graph of a function which is given to us $\left(\frac{1}{5}\right)^x$, you also might have plotted and naturally, the we will analyze whether it coincides with our thing. So, this is a point (0, 1), now it is $\frac{1}{5}$. So, your point will be somewhere here, sorry this is 5. So, the point 1 is here and this point is $\frac{1}{5}$.

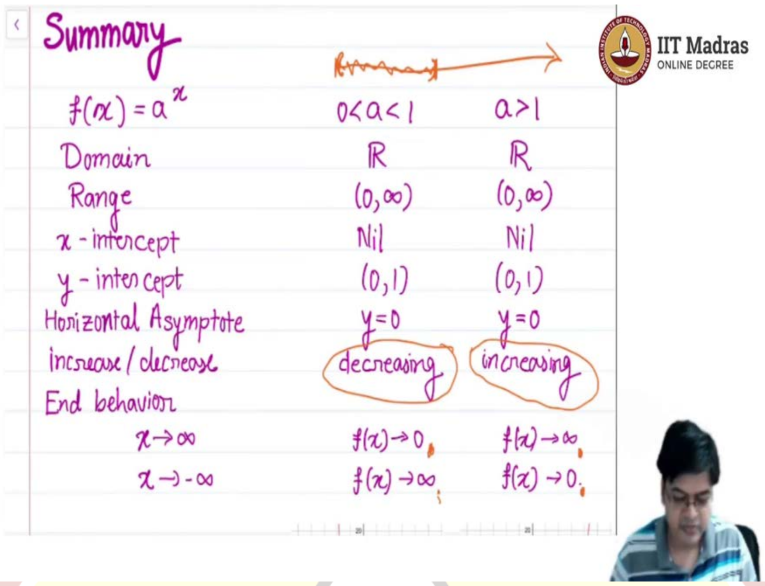
So, $\left(1, \frac{1}{5}\right)$, this is done. Then, as $x \to \infty$, this function goes to 0. As $x \to -\infty$ that is this way, this function actually goes to ∞ and this function is decreasing. From left to right if you come, you are actually coming down. So, it is a decreasing function. So, this completely gives us an understanding of what the graph of a function will look like.

(Refer Slide Time: 17:17)



Also, the same fact is true that every $f(x) = a^x$, where $0 < a < 1$ has same properties as $\left(\frac{1}{5}\right)^x$. Therefore, it is a representative class. So, you do not have to worry about the because it is a representative class, you have to worry about all other functions. All other functions will have a similar behavior.

(Refer Slide Time: 17:47)

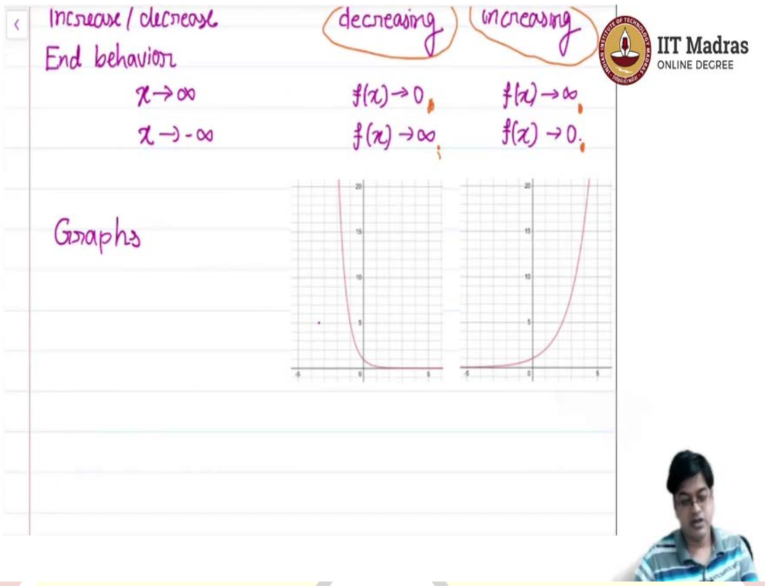


So, we have done a lot, let us summarize these things in a neat table which is this. So, this is the summary of the table. So, if I have been given a function $f(x) = a^x$, then to be more precise, let me draw a line here. This is a line; it does not look like a line, but assume that this is a line.

This is the point 1, then I am talking about $0 < a < 1$ that this zone. In this zone, the domain of a function is $\mathbb{R}$; range of a function is $(0, \infty)$. There are no $x -$ intercepts, no; y −intercept is 0, 1. Horizontal asymptote $y = 0$ is there. The function is decreasing. The end behavior as $x \to \infty$, $f(x) \to 0$; as $x \to -\infty$, $f(x) \to \infty$ correct.

Then, you look at the function which is $a > 1$, domain is real line, range is (0, ∞), nil; (0, 1), $y -$ intercept is (0, 1). Horizontal asymptote is $y = 0$. The only distinguishing feature is the function is increasing here and a function is decreasing here and because it is increasing and decreasing, the end behavior changes that is because it is decreasing, it will decrease to 0 because it is bounded below by 0 and because this is increasing, it will increase to infinity, but here it will go to 0 ok.

(Refer Slide Time: 19:21)



Then finally, you see the prototypes, just look at the graphs of these two functions ok. This ends our topic on exponential functions. Now, we will introduce something which is called natural exponential function in the next video.