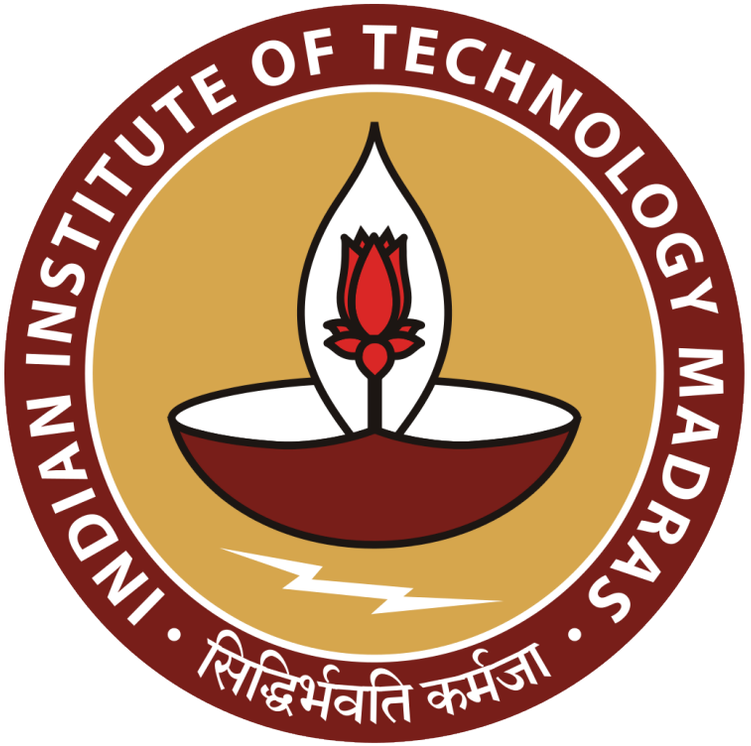

IIT Madras

ONLINE DEGREE

Is every collection a set?

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Mathematics for Data Science 1
Week 1

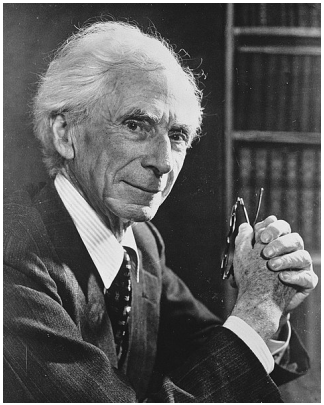
Set theory as a foundattion for mathematics

- A set is a collection of items
- Use set theory to build up all of mathematics
  - Georg Cantor, Richard Dedekind 1870s
- Natural numbers can be "defined" as follows
  - 0 corresponds to the empty set $\emptyset$
  - 1 is the set $\{0, \{0\}\} = \{\emptyset, \{\emptyset\}\}$
  - 2 is the set $\{1, \{1\}\}$
  - ...
  - $j + 1$ is the set $\{j, \{j\}\}$
- Define arithmetic operations in terms of set building

Georg Cantor

Russell's Paradox

- Set theory assumes the emptyset $\emptyset$ and basic set building operations
  - Union $\cup$, Intersection $\cap$, Cartesian product $\times$, ...
  - Set comprehension — subset that satisfies a condition
- Is every collection a set? Is there a set of all sets?
- Consider S, all sets that do not contain themselves
  - S is a set, by set comprehension
  - Does S belong to S?
  - Yes? But elements of S do not contain themselves
  - No? Any set that does not contain itself should be in S
- Russell's Paradox — also discovered by Ernst Zermelo
- Cannot have "set of all sets"



Bertrand Russell
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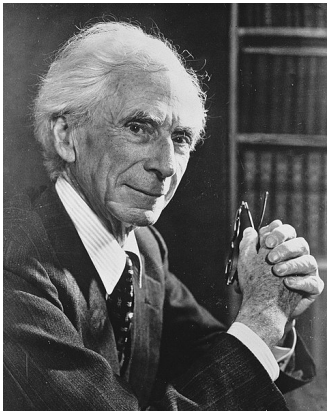
Sets and collections

- Russell's Paradox tells us that not every collection can be called a set
- Collection that is not a set is sometimes called a class
- The paradox had a major impact on set theory as a logical foundation of mathematics
- For us, just be sure that we always build new sets from existing sets
- Don't manufacture sets "out of thin air" — "set" of all sets



Bertrand Russell
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