Statistics for Data Science -1

Lecture 6.4: Probability- Properties of Probability

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7. Distinguish between independent and dependent events.
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Properties of Probability

Equally likely outcomes

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2. Relative frequency (Aposteriori or empirical): The probability of an event in an experiment is the proportion (or fraction) of times the event occurs in a very long (theoretically infinite) series of (independent) repetitions of experiment. In other words, if $n(E)$ is the number of times E occurs in n repetitions of the experiment, $P(E) = \lim_{n\to\infty} \frac{n(E)}{n}$

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3. Subjective: The probability of an event is a "best guess" by a person making the statement of the chances that the event will happen. The probability measures an individual's degree of belief in the event.

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3. For a sequence of mutually exclusive (disjoint) events, $E_1, E_2, \ldots,$

$$P\left(\bigcup_{i=1}^{\infty} E_i\right) = \sum_{i=1}^{\infty} P(E_i)$$

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In other words, if events E_1 and E_2 cannot simultaneously occur, then the probability that the outcome of the experiment is contained in either E_1 or E_2 is equal to the sum of the probability that it is in E_1 and the probability that it is in E_2.

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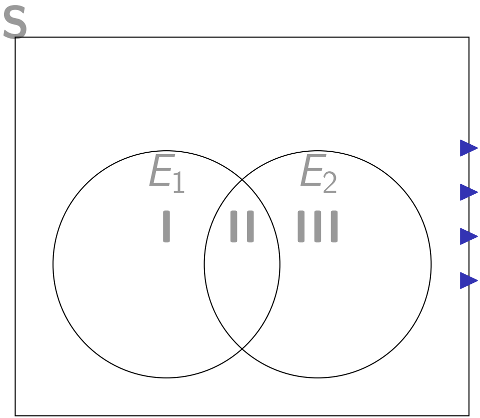
Addition rule of probability

The following formula relates the probability of the union of events E_1 and E_2, which are not necessarily disjoint, to $P(E_1), P(E_2)$, and the probability of the intersection of E_1 and E_2. It is often called the addition rule of probability.

For any events E_1 and E_2,

$$P(E_1 \cup E_2) = P(E_1) + P(E_2) - P(E_1 \cap E_2)$$

Proof of addition rule



- $E_1 \cup E_2 = I \cup II \cup III$
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- $E_2 = II \cup III$
- $E_1 \cap E_2 = II$

Section summary

- Probability axioms
- Properties of Probability
  - Addition rule