IIT Madras

ONLINE DEGREE

Sets: Examples

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Mathematics for Data Science 1
Week 1

Sets

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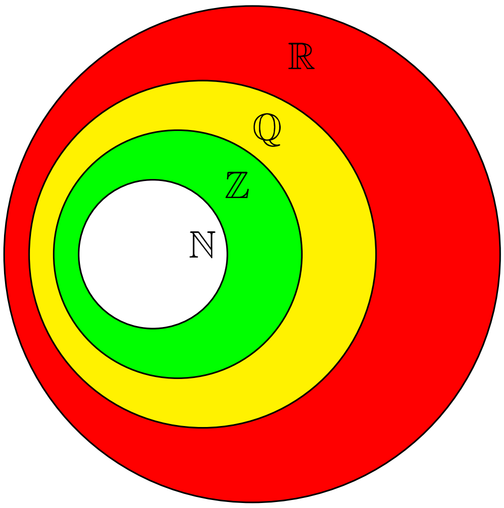
- Membership $x \in X$, Subset $X \subseteq Y$
  - $5 \in \mathbb{Z}$, $\sqrt{2} \notin \mathbb{Q}$
  - Primes $\subseteq \mathbb{N}$, $\mathbb{N} \subseteq \mathbb{Z}$, $\mathbb{Z} \subseteq \mathbb{Q}$, $\mathbb{Q} \subseteq \mathbb{R}$

Venn Diagram



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- Powerset — set of subsets of a set
  - $X = \{a, b\}$, powerset $\{\emptyset, \{a\}, \{b\}, \{a, b\}\}$
  - Set with n elements has 2^n subsets

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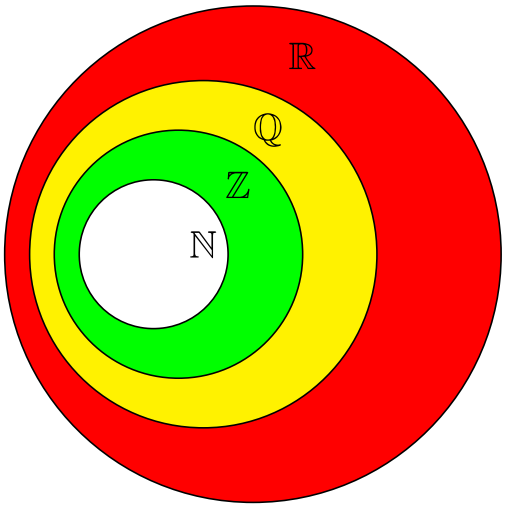

Set Comprehension

- Squares of the even integers
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$\cdots \quad (-2)^2 \quad 0^2 \quad 2^2 \quad 4^2$

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    $\{p/q \mid p/q \in \mathbb{Q}, gcd(p,q) = 1\}$
  - Reals in interval $[-1, 2)$
    $\{r \mid r \in \mathbb{R}, -1 \leq r < 2\}$

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- Now, a more readable version
  $X = \{n \mid n \in \mathbb{N}, n < 500\}$
  $Y = \{n^3 \mid n \in X\}$

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- Choose the generator as required