INDIAN INSTITUTE OF TECHNOLOGY MADRAS

सिद्धिर्भवति कर्मजा

IIT Madras

ONLINE DEGREE

Mathematics for Data Science 2
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Week 03 - Tutorial 03

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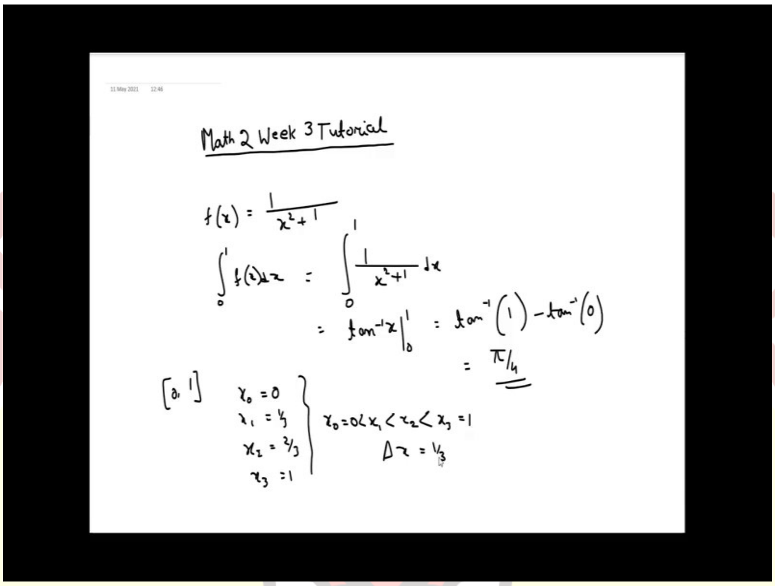

Hello everyone, welcome to Maths 2, week 3 Tutorial video. So, this is the third video. So, in this video, we will take an example $f(x) = \frac{1}{x^2+1}$ and we will try to find this integral $\int_0^1 f(x)dx$ in the interval 0 and 1. So, in the last video, we have seen the left Riemann sum and right Riemann sum and how they approximate this integration.

Actually we know the formula of this integration $\int_0^1 \frac{1}{x^2+1} dx$. So, this will give us $\tan^{-1}(x)|_0^1$, so we will get $\tan^{-1}(1) - \tan^{-1}(0)$. So, this is, this will give us $\frac{\pi}{4}$. Now, if I divide this interval, [0, 1] into 3 sub interval that is, my $x_0 = 0, x_1 = \frac{1}{3}, x_2 = \frac{2}{3}, x_3 = 1$. So, if I take this as my sub interval where this condition is satisfied, so $x_0 = 0 < x_1 < x_2 < x_3 = 1$. So, $\Delta x = \frac{1}{3}$ for each sub interval.

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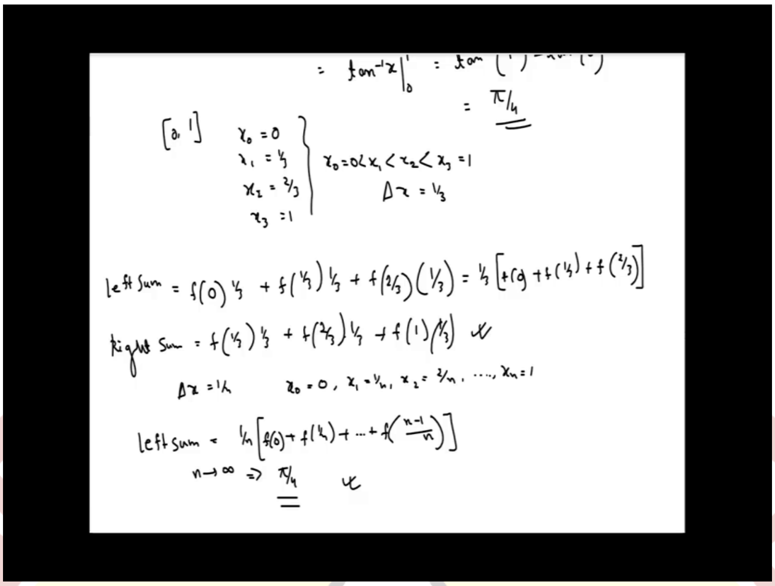

And if I write the left sum, then it will give me $f(0)\frac{1}{3} + f(\frac{1}{3})\frac{1}{3} + f(\frac{2}{3})\frac{1}{3}$. So, this will give us the left Riemann sum, so I am writing it as left sum and if you take the right sum, it will give $f(\frac{1}{3})\frac{1}{3} + f(\frac{2}{3}) + f(1)\frac{1}{3}$. So, in the first case, you can take 1 by 3 common and it will give us $\frac{1}{3}[f(0) + f(\frac{1}{3}) + f(\frac{2}{3})]$ and you know the function which is $\frac{1}{x^2+1}$ So, we can calculate this value. Similarly, for the right sum also.

Now, if we divide this [0, 1] in n interval, so our Δx will be 1/n and our sub interval will be $x_0 = 0, x_1 = \frac{1}{n}, x_2 = \frac{2}{n} \ldots$. and so on and which will give us eventually $x_n = 1$. So, if we take this and if you take the left sum again, so for n sub interval, our left sum will give 1 over $n[f(0) + f(\frac{1}{n}) + \ldots f(\frac{n-1}{n})]$. So, if you calculate this thing and take n tending to infinity, you will get this sum as $\frac{\pi}{4}$.

So, I want you to check this thing and similarly for the right sum you can do the same thing and you will get the limit as n tends to infinity, and the sum will tend to $\frac{\pi}{4}$. So, now let us try to visualize this thing in GeoGebra.

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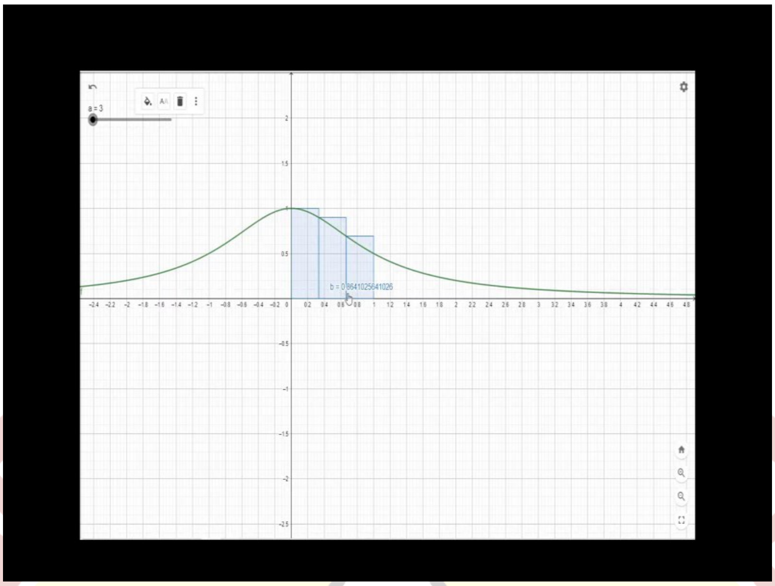

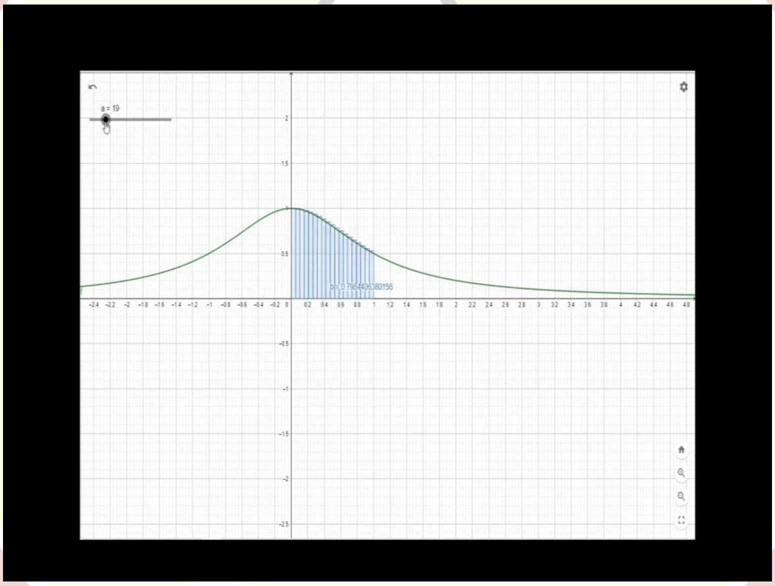

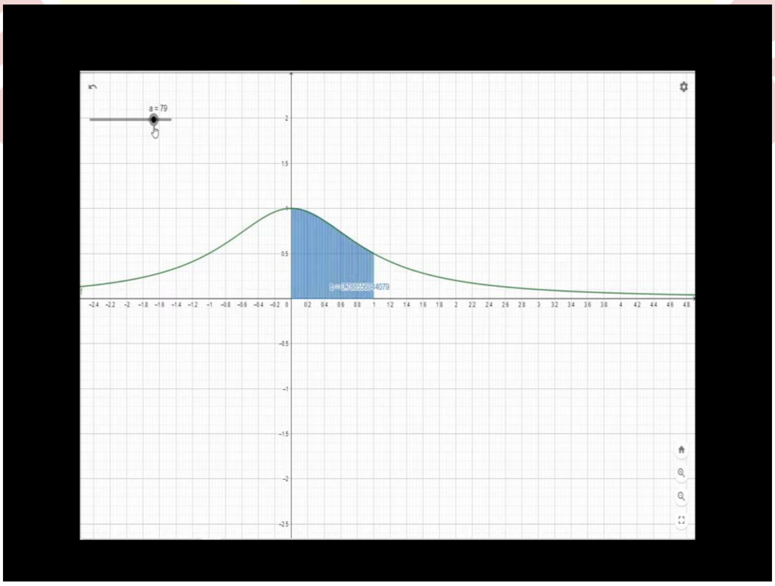

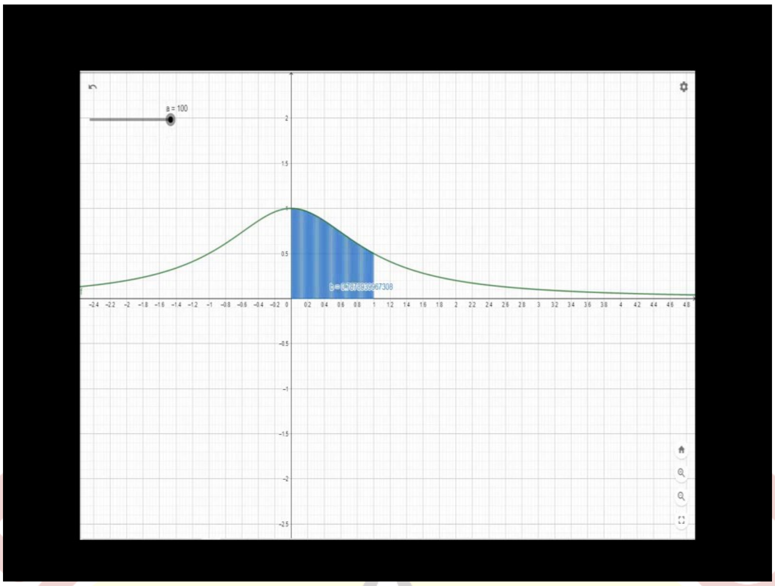

So, this is the graph of function $\frac{1}{x^2+1}$ and we are trying to calculate the area under this curve over this x axis in the interval 0 and 1. So, this a will represent the number of sub intervals in which we are dividing this interval 0 and 1. And if we see, and at first we are taking the left sum and if we see for a = 3, that is, there are 3 sub interval, so this will be the approximated area, this will give the Riemann sum. So, it is something 0.8641 and so on.

So, as we are increasing the number of sub intervals, we can see that it really approaches to the area under this curve. And as we can see when the number of sub intervals increasing sufficiently, it will get closer and closer to the area under the curve. So, let us see the animation again. So, at first, we are decreasing the number of sub intervals and we can get some error as we can see. Now if we increase this number of sub interval, we will get closer and closer to the required value. So, this is for the sub interval.

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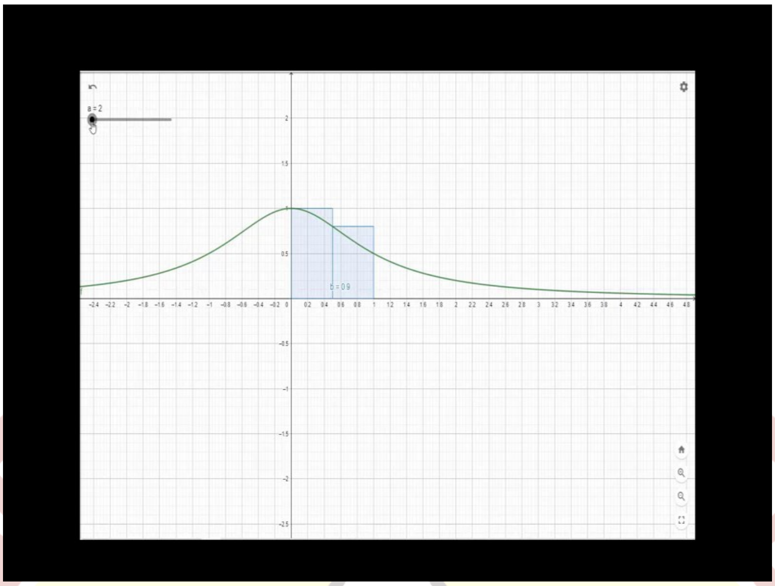

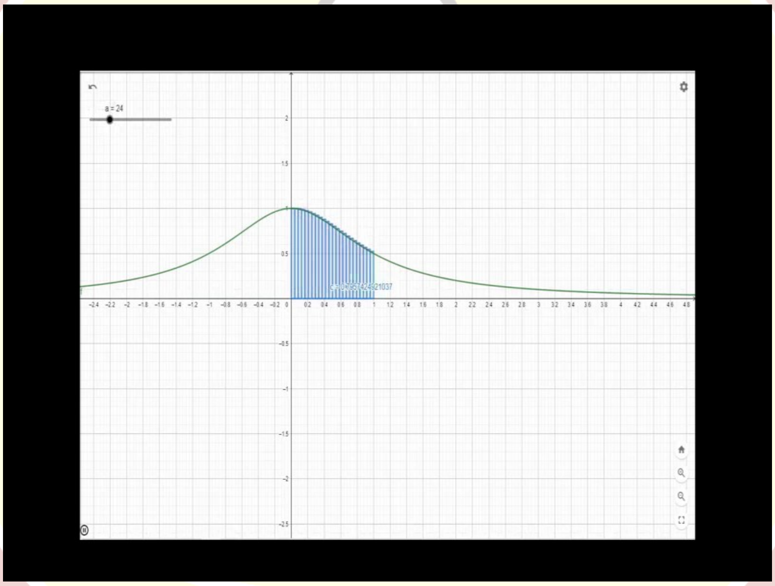

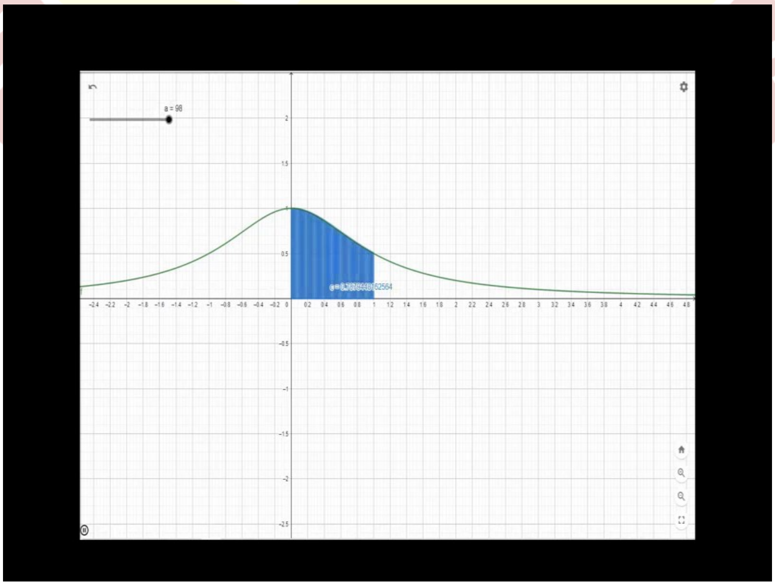

Now, if we take the upper sum and again, we can see that, so this is the upper sum. Again, if we increase the number of sub intervals, we will get the approximate value of the area under the curve. So, here the number of sub interval decreases. So, error increases and now again, the number of sub interval increases, so error decreases and we will get an, we will approaches to the actual value of the integration f(x)dx in the interval 0 and 1. Thank you.