INDIAN INSTITUTE OF TECHNOLOGY MADRAS

सिद्धिर्भवति कर्मजा

IIT Madras

ONLINE DEGREE

Mathematics for Data Science 1
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Week 9 Tutorial

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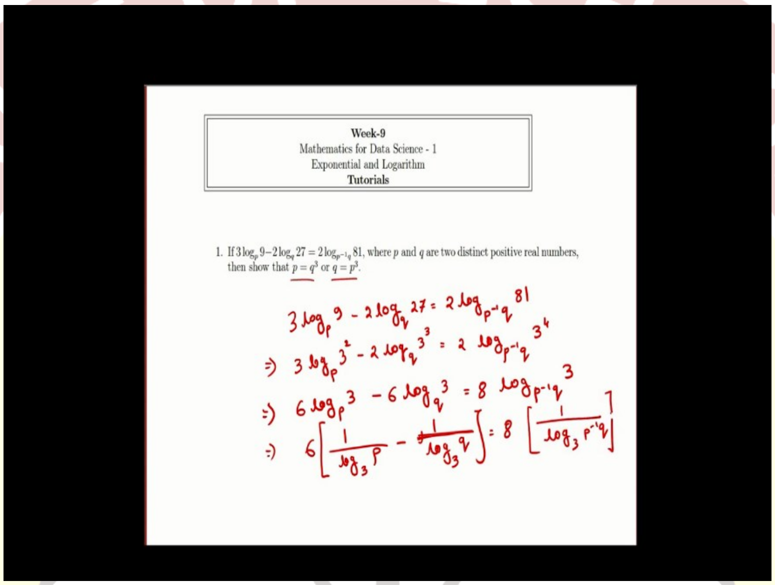

Hello, welcome to the tutorial session of this week 9. So, let us see the first question. So, in the first question we are seeing an equation which is given as $3\log_p 9$, this is base p, $2\log_q 27$ and 2 log here base is $p^{-1}q$ this is 81. So, here it is also given that p and $q$are two distinct positive real $n$umber and we have to show either $p = q^3$or $q = p^3$. So, let us try to solve this equation.

So, here we can see that we can write log 9 as 3^2, log 27 as $3^3$and we can write this as 3^4. So, let us take this power before this log, so it will become $6\log_p 3 - 6\log q$ is the base this is 3 and we can 8 here and it is $p^{-1}q$ and here it is 3. So, we transform everything in terms of log 3. So, our $m$otivation is to bring everything as the same base, so we can write it as so we can take 6 common and we can write $\log_p 3$ here we can write it as $\frac{1}{\log_3 p}$1 by log here base 3 and here it is $q$and we can write this as also $\frac{1}{\log_3 q}$ and here it is $p^{-1}q$.

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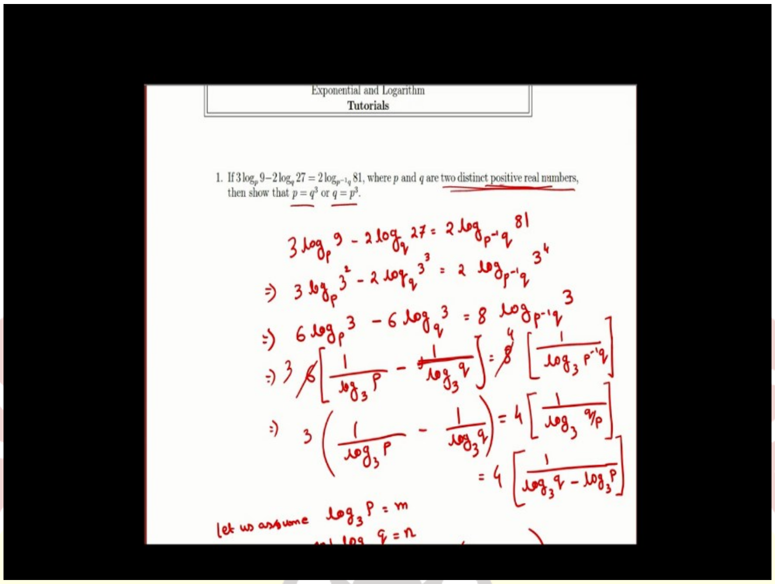

So, let us see what we can do here. So, one more step, so here 2 cancels the it is 3 and here it is 4 and one more step we can do before going ahead, so this is my log this portion remains same and this portion I can write it as so this is $p^{-1}q$, $p^{-1}q$ means $\frac{q}{p}$, so here it is given distinct positive real numbers, so it is non-zero, so this is the p inverse is valid basically and here also see that p and $q$cannot be negative also otherwise, this log will be undefined negative or 0, so otherwise, this log will be undefined. So, we are taking two distinct positive real numbers, so as p and q. So, here it is $\frac{1}{\log_3 q - \log_3 p}$.

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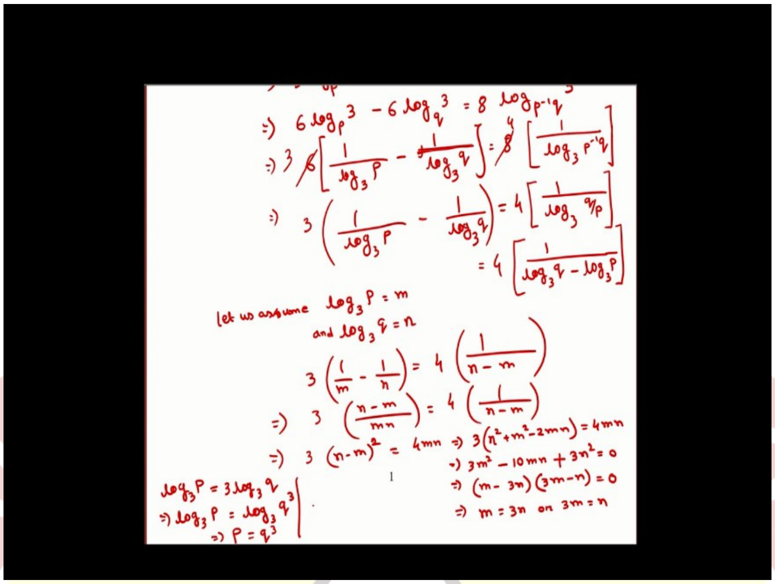

So, we have written every term as in terms of $\log_3 q$ or $\log_3 p$, so let us assume, $n$ow $\log_3 p$ as m and $\log_3 q$ as n. So, what we get? If we substitute these things we will get this is $\frac{1}{m}$, $\frac{1}{n}$, 4, 1 by n - m. So, let us see what this gives, so this will give n - m by mn, 4, 1 by n - m. So, if we take n - m in this side we will get $(n-m)^2 = 4mn$, so we will if we simplify this thing will get 3 let us break it down, $n^2 + m^2 - 2mn$, here it is $4mn$, so if we take everything in the same side we will get $3m^2 - 3 \times 2, 6$ and here it is - $4mn$, so $10mn + 3n^2$, so from here we can write this as $(m - 3n) \times (3m - n) = 0$.

So, if we factorize this thing you will get this term, because 3×3 is 9 and we can write this - 10 as $-9 - 1$, so we can simplify this thing. So, basically we get $m = 3n$ or $3m = n$. So, whatever m and n are so let us substitute here, so $\log_3 p = 3n$, so $3\log_3 q$ so it is giving us both side we have log 3 base and it is q^3, so log is an one on function, so we get $p = q^3$. And again if we substitute in the other term which is $3\,m = n$, we will get $q = p^3$. So, this proves that for the given equation what we have started with we get either $p = q^3$ or $q = p^3$.