INDIAN INSTITUTE OF TECHNOLOGY MADRAS

सिद्धिर्भवति कर्मजा

IIT Madras

ONLINE DEGREE

Statistics for Data Science - 1
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Lecture No. 10.1
Binomial Distribution- Bernoulli Random Variable

So, we will start understanding about a very important distribution today, and that is the binomial distribution. The binomial distribution is a discrete distribution, and it arises naturally in a lot of applications.

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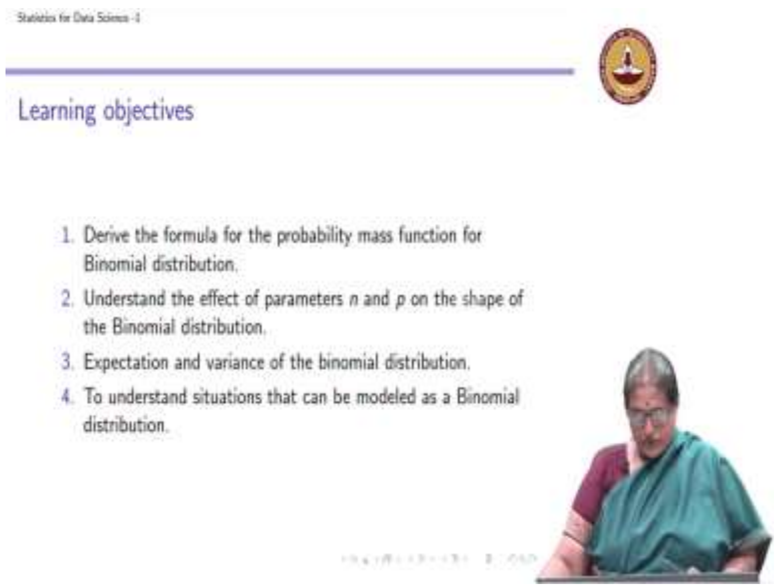

So, the learning objectives are first we go and we are we expect this week, we will first derive the probability mass function for a binomial distribution. How does the binomial distribution naturally arise? And then afterwards we will look at understanding the effect of the parameters of the binomial distribution on the shape of the distribution. We look at the expectation and variance and how do we answer certain applications based on expectation and variance and finally, we will understand situations that can be modelled as a binomial distribution.

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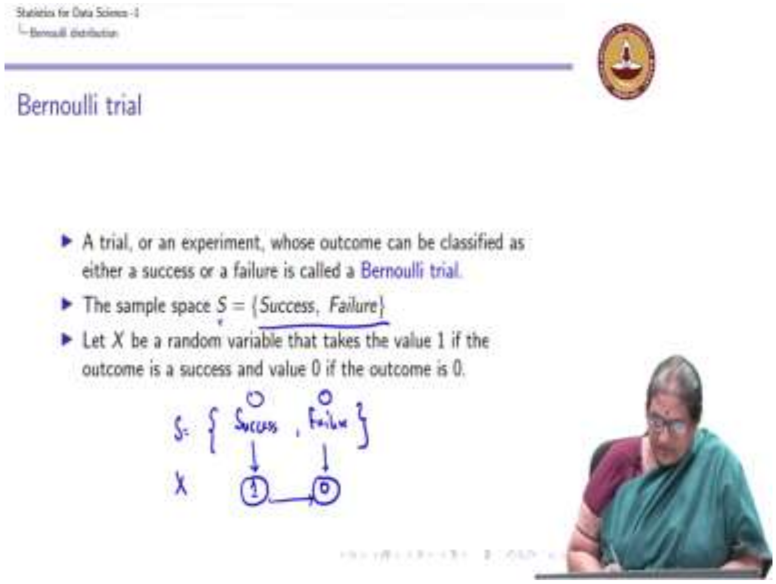

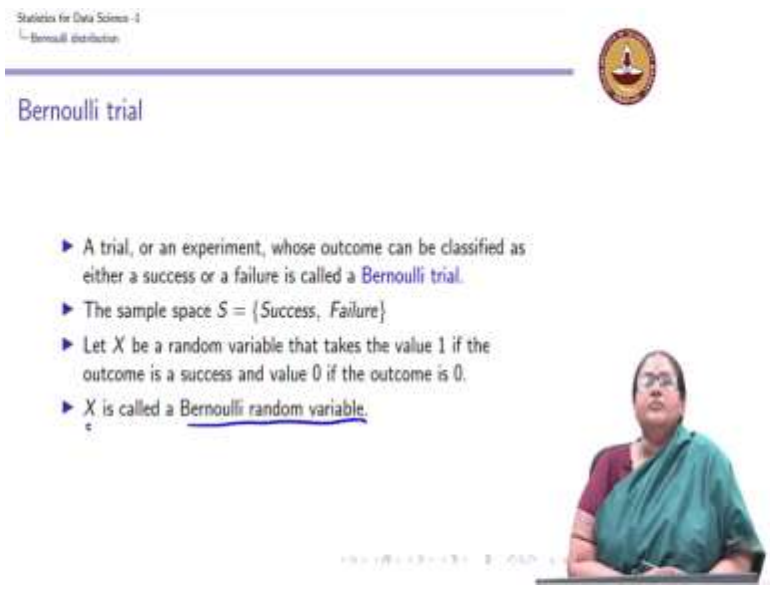

So, first we start with a brief review of what is a Bernoulli random variable. If you recall, we introduced a Bernoulli random variable when we discussed about discrete random variables. Now, we are going to talk about it in slightly greater detail. Now, a Bernoulli trial or a Bernoulli experiment is a trial or an experiment whose outcome can be classified as either a success or a failure. Now, what do we mean by a success or a failure. So, for example, I have a lot of instances where I can label my outcomes or I can map my outcome to be a success or a failure. So, if I have a success or a failure, the sample size is going to be success or a failure.

So, recall again I have my sample space which takes the value success. So, recall, I have a sample space which takes the value success and failure I have a random variable a mapping success to take the value 1 and failure to take the value 0. So, in other words, a trial or an experiment which gives me a success or a failure and can be mapped to 1 or 0 for convenience sake we keep 1 or 0 that is a mapping an outcome to the random variable. So, this X is what we refer to as our Bernoulli random variable.

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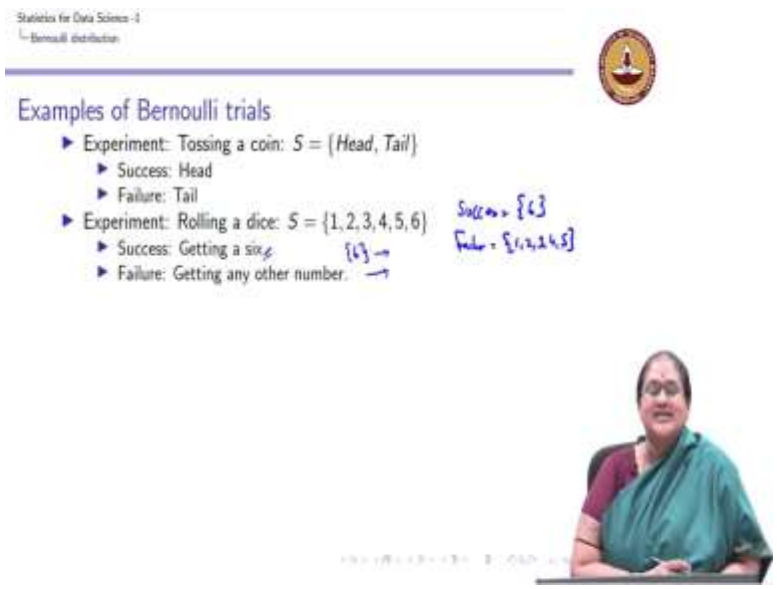

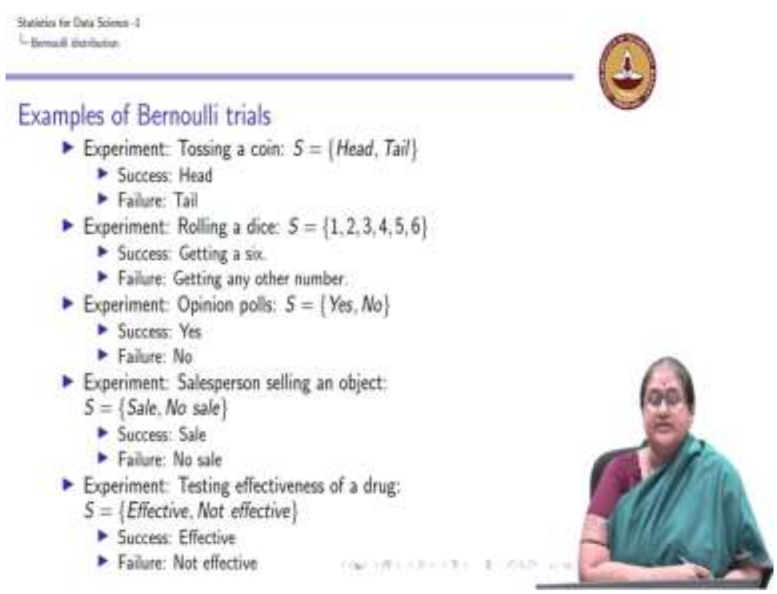

Now, let us look at examples of a Bernoulli trials. We have already looked at a lot of these but we will review them in the context of a Bernoulli experiment. So, let us start with tossing a coin I know the outcomes are head and tail. Now, the success can be a head, I can define failure to be a tail. So, again, this I can call the tossing of a coin once to be a Bernoulli trial.

Now, let us roll a dice I know the outcomes are 1, 2, 3, 4, 5, 6 again I do not have a two outcomes. But I can define a success to get a 6, in which case the outcomes are going to be just the singleton 6 and the failure to be any other number. So, you can see that I can define an event of getting a 6 to be a success and a failure to be getting any other number. So, I can map the success, success is going to be just the event 6 and the failure is going to be any other number 1, 2, 3, 4, 5.

So, I can define a Bernoulli trial where I define what is a success and failure. Again, opinion polls, typically you are asking questions where the answers could be yes or no I can define success to be yes and failure to be no this could be a again applied in a lot of situations for example political electoral polls, where I am asking whether we are voting for a particular candidate or not. In that case, yes could mean a success and no could mean a failure.

Again, when you are having a salesperson may selling a particular object a success could be a sale, a failure could be a no sale. So, again you can see outcome is selling or not selling. So, I can map it again. I can consider this as a Bernoulli trial. Again testing the effectiveness of a drug in a pharmaceutical trial, you want to know whether the drug is effective or non effective. Again, I can define success in this case to be effectiveness of the drug, and failure to be not effectiveness of drugs. So, you can see that a lot of trials which arise in our day to day lives are actually Bernoulli trials.

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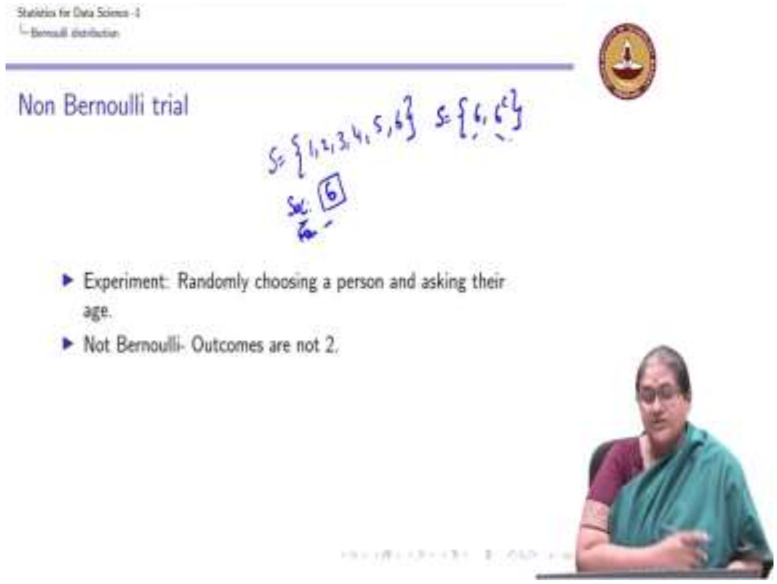

Let us, look at an example of a non Bernoulli trial, I just randomly choose a person and as their age, I am see that it is not just two outcomes outcomes are not two age can be anything. So, this is not a Bernoulli trial, you might argue that okay when a rolled a dice the outcomes are again not two, number of outcomes are 6. But here I define success to be obtaining a 6 and a failure to be obtaining anything else. So, the outcomes where mapped to just 2 where I termed success is to get 6.

So, in other words, I could have written the samples space as getting 6 and not getting 6, two outcomes. So, a Bernoulli trial depends on how you are defining the outcome. So, getting a 6 and not 6 are the outcomes, which will define a Bernoulli trial. So, in summary a Bernoulli trial is an experiment or a trial, which has two outcomes.

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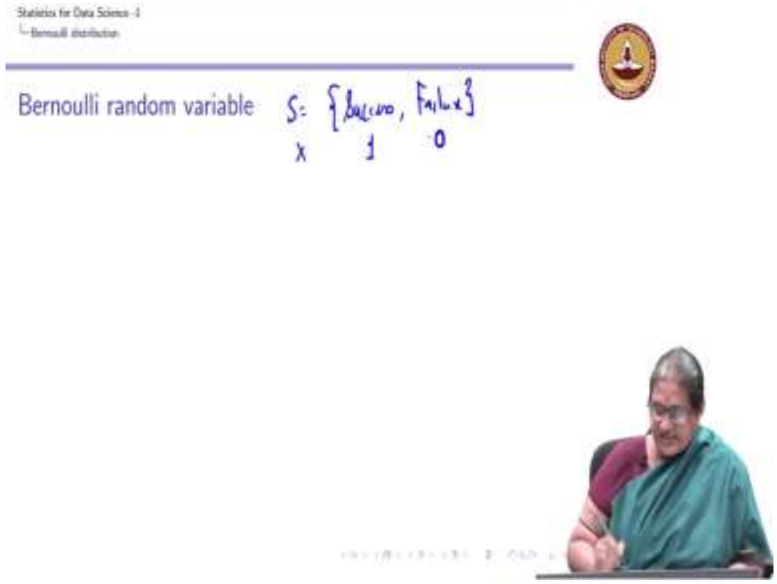

So, random variable. So, now when I have a sample space I have two outcomes, I am defining it as a success and the failure I am defining the random variable takes the value 1 if it is a success and 0 it is a failure.

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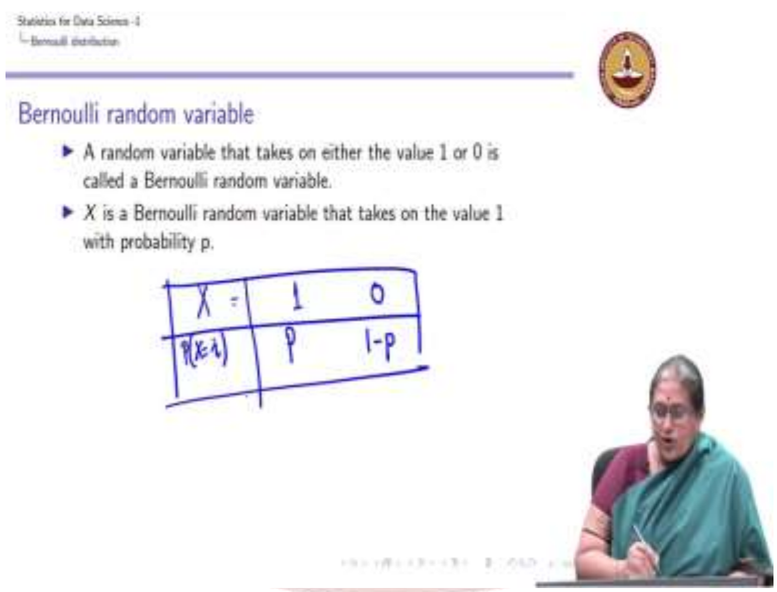

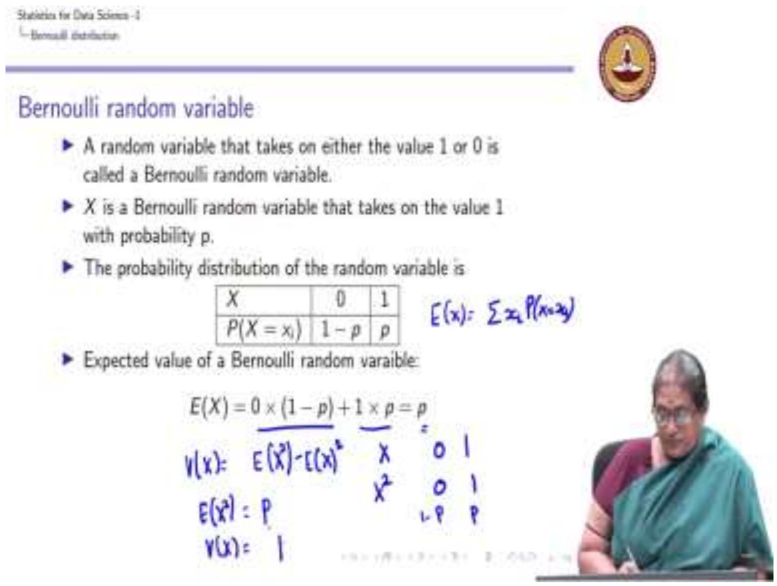

So, this is what I referred to as a Bernoulli random variable. Further I say X takes the value 1 and 0 it takes only two values so I can say a probability X takes the value 1 let it be p, then this probability has to be 1 - p because the sum of probabilities have to be equal to 1. So, this gives me what I refer to as a probability mass function or the probability distribution where probability X takes the value. So, x_i takes x_1 equal to 0, x_2 equal to 1 it takes the probability p and 1-p.

Further, we also know what would be the expected value recall expected value of a random variable $E[X] = \sum x_i P(X = x_i)$. So, I have an expectation of X, X takes the value 0 with probability (1 – p) + 1 * p which will give me the expected value to be p. Now, the $Var(X)$ again recall the computational formula is $E[X^2] - (E[X])^2$. Now, X takes the value 0, 1 X^2 also takes the value 0, 1 with the same probability 1-p and p. Hence, $E[X^2] = p$. If $(E[X])^2 = p^2$, I know $Var(X) = p - p^2 = p(1 - p)$.

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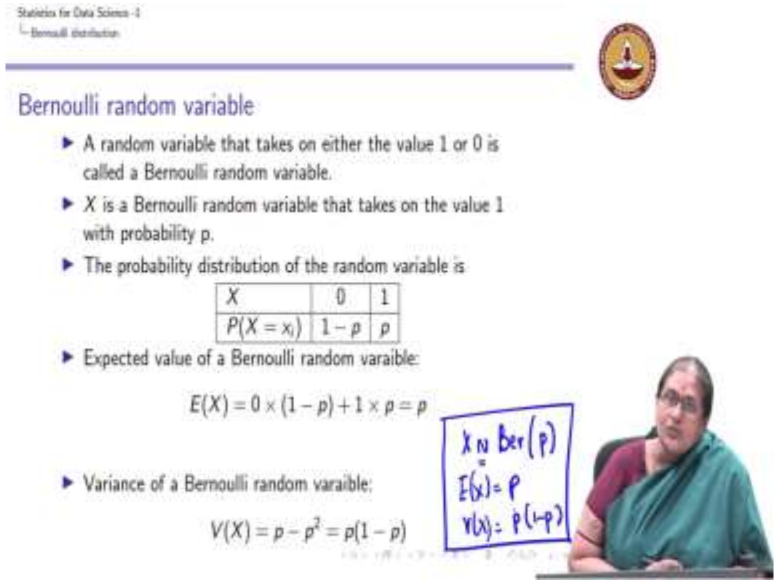

So, if X is a Bernoulli random variable, I can refer to it as X this is usually used to show the distribution, it is a Bernoulli random variable the parameter is p the $E[X] = p$ the $Var(X) = p(1-p)$. So, this way of stating a random variable is say that X is a Bernoulli random variable with parameter p, the expectation is p and the variances is p(1-p).

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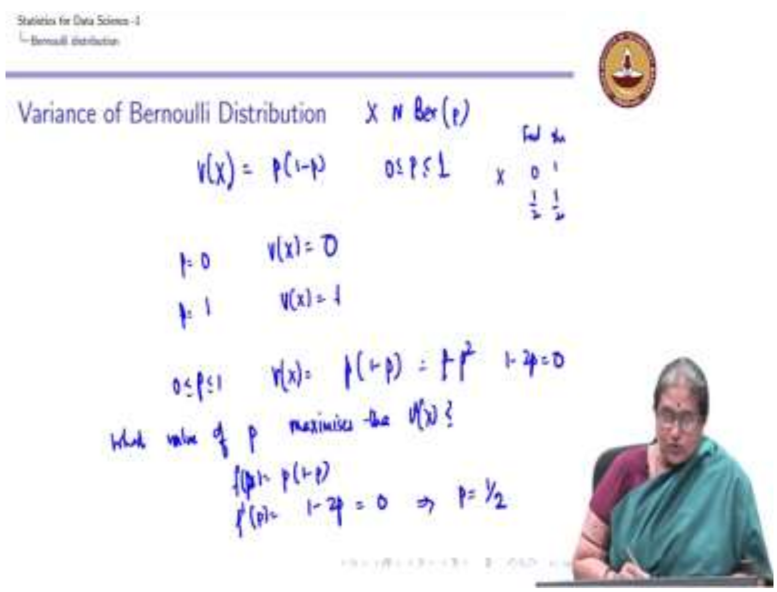

Why a Bernoulli random variables very important and important property as we have seen the variance of a Bernoulli random variable with parameter p we have just seen was p(1-p). I know p is a probability with $0 \leq p \leq 1$. We know at the extreme case when $p = 0$ the $Var(X) = 0$ when $p = 1$ again the $Var(X) = 0$.

So, between $0 < p < 1$, the $Var(X) = p(1-p) = p - p^2$, you see it is a quadratic. So, what value of p maximises the variance? So, I can just look at it I know that $1 - 2p = 0$ is what is if f$f(p) = p(1-p)$ the first derivative would be $1 - 2p$, I equate to at 0 which gives me $p = \frac{1}{2}$.

Now, what is $p = \frac{1}{2}$ imply? $p = \frac{1}{2}$ implies again recall there are two outcomes, this is a failure and this is a success with probability half each. So, this tells that the outcomes or the it is equally likely to fail as it is equally likely to succeed or failure and success has the same probability they are equally likely to happen. So, when there is an equal likelihood of these outcomes to happen, you see that is where the variances.

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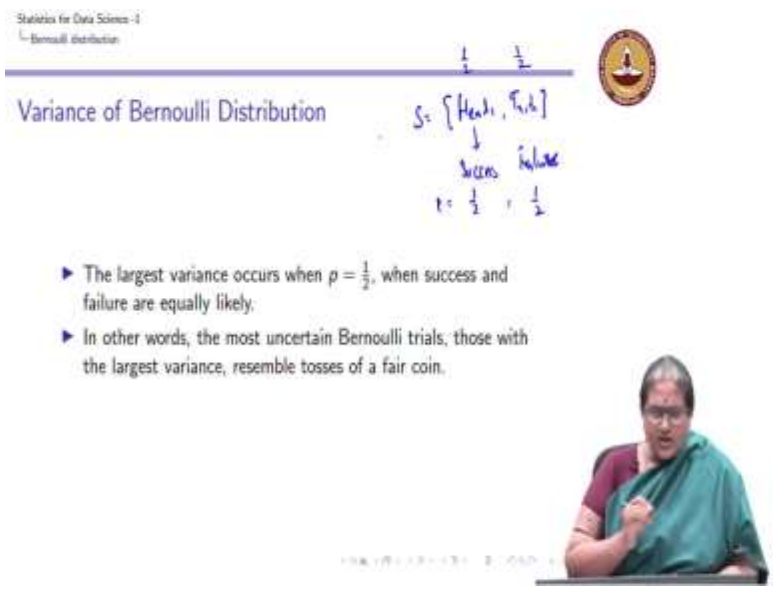

So, the largest variance of a Bernoulli distribution occurs when $p = \frac{1}{2}$, in other words when success and failure are equally likely to occur. So, in other words when so the largest variance say that the uncertainty is very high. So, in most uncertain Bernoulli trials resemble the tosses of a fair coin because again if you go to a toss of a fair coin I know my sample space is going to be heads and tails and we also referred this to be success and this to be failure and I know if I have a fair coin the chance of getting a head is the same as getting as a tail which is half and that is what this my p which is equally likely to happen. So, the most uncertain Bernoulli trials resemble outcome or tosses of fair coin.

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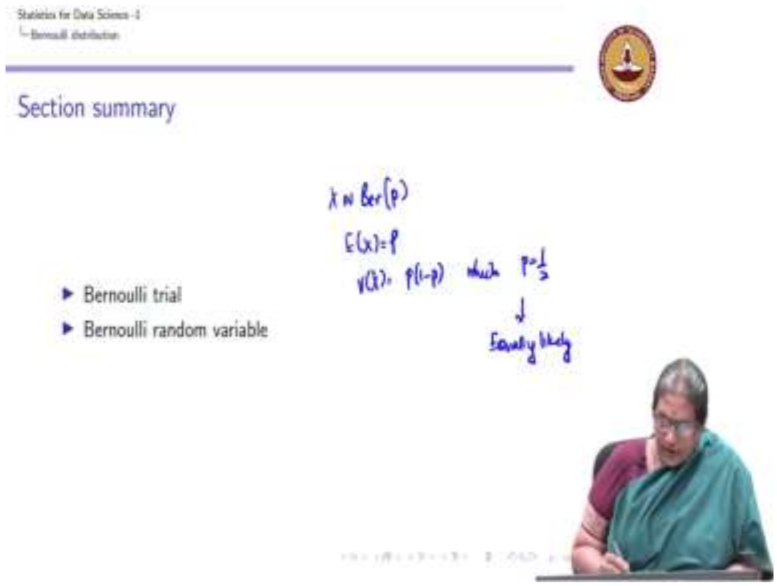

So, what we have learned in this section was, we reviewed or revisited the Bernoulli trial excess a Bernoulli random variable with parameter p, the $E[X] = p$, the $Var(X) = p(1 - p)$, which is maximised at $p = \frac{1}{2}$. This is in other words the outcomes are equally likely to happen give me the most uncertain Bernoulli trials and from this Bernoulli random variable we are going to now extend the notion of this Bernoulli random variable with two mutually exclusive outcomes to a binomial random variable.

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But before going to a binomial random variable, we need to understand what do we mean by independent and identically distributed Bernoulli trials.