IIT Madras

ONLINE DEGREE

Mathematics for Data Science-1
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Lecture - 13
Distance formula

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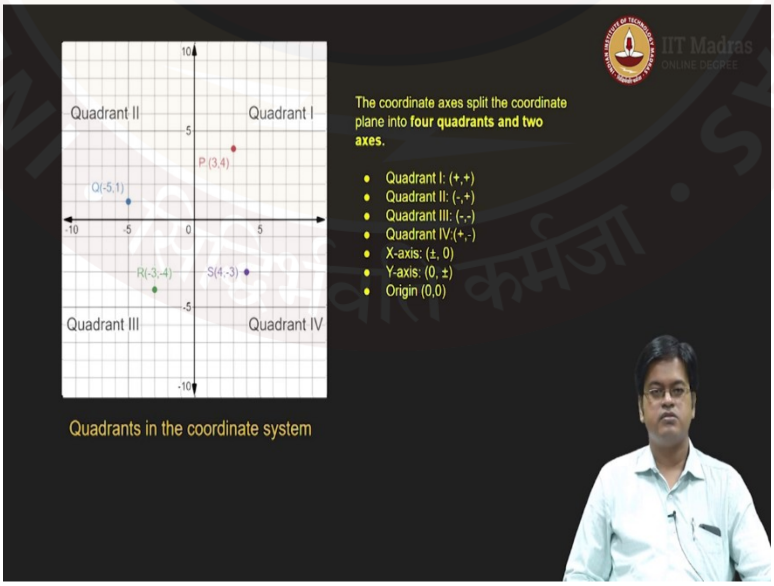

So, after coordinate system, let us try to identify one classical problem.

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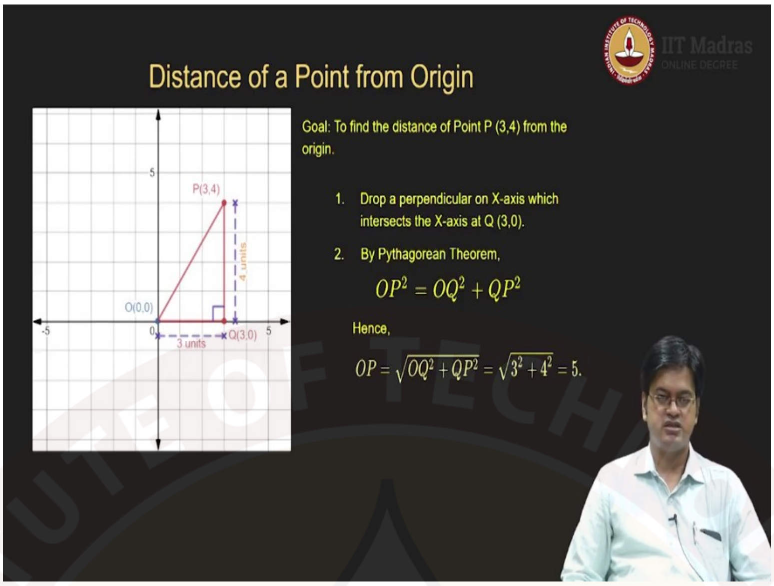

That is if I have a point and somebody ask me a point is located here; let us say point is (3,4). And somebody ask me what is the distance of this point from the origin? So, in this particular slide our goal is to find the distance of a point P which is (3,4) from the origin.

That essentially reduces to finding the length of this line segment which is joining points O and P. So, is there any classical tool that is of my help? Suppose, now if this point is either lying on X axis or Y axis, let us say if this point is say (3,0) ok. If this is the point that is of interest to me; do I know how to find the distance of this point? The answer is yes I know, I just need to calculate the units that are in horizontal direction.

Suppose the point is on Y axis, then do I know how to calculate the distance of this particular point from Y axis? The answer is again yes I know, I just need to calculate the number of units that I need to travel to reach this point. So, if the point lies on X axis and Y axis, I know how to calculate the distance of a point. Now, if the point is lying anywhere in the coordinate plane, how to find a distance is a question.

For that, let us try to understand the situation, that if I know if somehow I can understand this with respect to this coordinate axis. This particular position with respect to these coordinate axis then I will be able to give the answer to find the distance between the two points. So, let us try to do one thing that is let us try to get the image of this point (3, 4) on X axis.

So, how will I get the image of this point (3,4) onto the X axis? The easiest way is you drop a perpendicular on X axis, that intersects the X axis at point (3,0). Once this is done then you can actually drop a perpendicular and see that it forms a right angled triangle with X axis in place and a vertical line in place; you have a right angled triangle. Do you know any theorem in our conventional geometry that relates this particular structure?

You know Pythagoras Theorem or Pythagorean Theorem, that relates this particular structure. In a right angled triangle the hypotenuse length of the hypotenuse is given by square root of its adjacent sides; square root of squares of the lengths of the adjacent sides. So, we will try to use this for finding the distance of a point from the origin. So, by the Pythagorean Theorem, I know $OP^2$is actually equal to OQ^2+QP^2.

Now, the exercise that we did orally just before starting this problem will help us to understand what is OQ^2. So, what is OQ? OQ is a part of X axis, OQ is a line segment which is a part of X axis. What is the length of OQ? We have already discussed that, that length is 3 units. Similarly, if you look at QP; what is QP? QP is parallel to Y axis. So, it is as good as projection of Y axis projection onto Y axis.

So, what is the length of this particular line segment which is QP? That is 4 units; so I know the length of OQ and I know the length of QP. Therefore, by Pythagorean Theorem, I know the length of OP. So, what will be the length of OP? It will be $\sqrt{\square}$. So, $3^2$is 9, $4^2$is 16 therefore, this will give me 25; 16+9 and positive square root of it will give me number 5.

Now, has it anything special to do with point (3,4) or can I generalize this? The answer is yes, it has nothing special to do with point (3,4). I could have started with point P which is (x , y) and then projected this onto X axis or I figured out the image onto X axis which will be (x , 0). And therefore, the length of OQ will be x and the length of QP will be from 0 to y units; that means, y units.

So, length of QP will be y units and therefore, the formula OP = $\sqrt{\square}$ would have been possible. So, let us try to take this particular example and try to generalize this problem to finding the distance between any two points.

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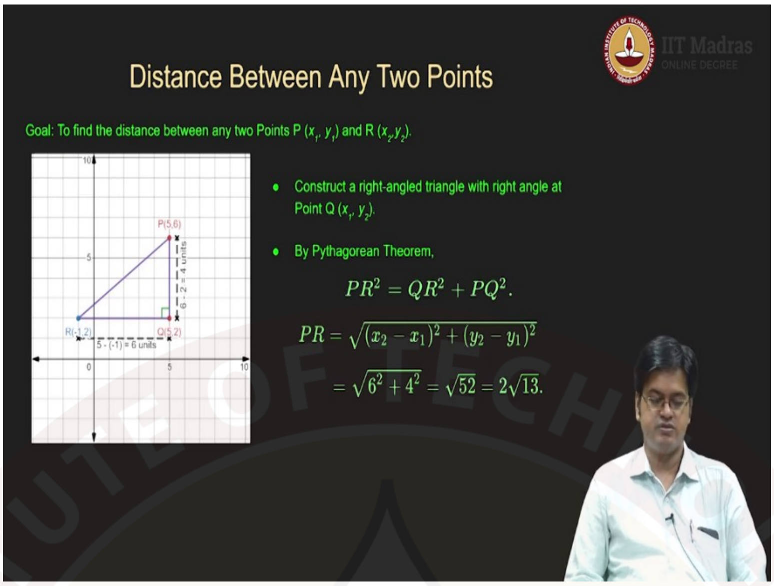

So, distance between any two points. So, again the setup is pretty common. Our goal is to find the distance between any two points P (x_1, y_1) and R (x_2, y_2). How will you find the distance between any two points? Let us see the points on the graph, then the things will be more specific. My (x_1, y_1) is (5, 6) and (x_2, y_2) is (-1 , 2).

Now, if I look at these two points, I want to find the distance between these two points. So, once easy way to find a distance between these two points is to construct a right angle triangle. But, now because the point is not located on X axis, this (-1 , 2) is not located on any of the axis; I cannot say drop a perpendicular to X axis.

So, the actual way that I should do here is I will drop a perpendicular to X axis which will intersect at (5 , 0). And, then to this line I will drop a perpendicular from the point R minus (1,2) and which will intersect this, this particular line which will be the perpendicular to X axis, at where the y coordinate will be 2 and x coordinate will be 5. So, this point will be (5,2) and then I will get a right angled triangle.

By skipping these steps, we can straight away say that you construct a right angled triangle with a right angle at point Q which is (x_1, y_2). Just relate this (x_1, y_2), if you use this terminology is the point (5,2). So, you can draw a right angle triangle using (5,2) ok.

So, this way we need not have to specify steps that you have to draw two perpendiculars and all; because the point may as well lie in the third quadrant. And, in that case dropping perpendicular to X axis may not help, you have to extend the perpendicular beyond X axis. So, it is always better to consider this kind of structure, that is construct a right angled triangle with right angle at point Q which is $(x¿¿1, y_2)¿$.

Then it does not matter where the point actually lies. Now, once the right angle triangle is in place, the same theory that we used Pythagorean Theorem will come into play. And, by Pythagorean Theorem if I want to find the length of PR, I know $PR^2 = QR^2 + PQ^2$. Can I calculate the length of QR and PQ? The answer is yes I can calculate, because, the line segment QR is actually parallel to X axis and the line segment PQ is parallel to Y axis.

Therefore, this is as good as computing the length on X axis and this is as good as computing the length on Y axis; hence what we will get is. So, how to compute the length? It is basically the change in x coordinates. So, how far the x coordinates have changed? So, while computing the length parallel to X axis always remember you should go from left to right, that is when you are subtracting you should take the highest value first that is 5 – (-1).

So, the length of this will be 6 units and while subtracting or while finding the length in a vertical direction go from bottom to up. That means, you subtract the value that is highest in Y direction to the value that is lowest in Y direction. So, here 6 - 2 will give me 4 and in the X direction 5 – (-1) will give me 6 units. So, this is how we will calculate the length of these two line segments.

And therefore, I can easily find the length of PR; while calculating the length because we are in this particular case, we are considering squares. It does not matter whether you consider x_1 first or $x_2$first because, anyway we are squaring even if you get the negative value, you will be squaring it.

So, in particular in this case where the coordinates are $(x¿¿1, y_1)¿$and $(x¿¿2, y_2)¿$, I will take $(x¿¿2 - x_1)^2¿$; does not matter which one is bigger. And $(y¿¿2 - y_1)^2¿$, does not matter which one is bigger.

And I will take a positive square root of it. Therefore, my length PR for this particular example will be 6^2+ 4^2; $6^2 = ¿$ 36, $4^2 = ¿$ 16 together they will give 52 is $2\sqrt{\square}$. So now, we have established a general formula which is called distance formula for finding the distance between any two points on a coordinate plane.