Statistics for Data Science -1

Lecture 8.1: Random Variable: Introduction

Usha Mohan

Indian Institute of Technology Madras

Learning objectives

Learning objectives

1. Define what is a random variable.

Learning objectives

1. Define what is a random variable.
2. Types of random variables: discrete and continuous.

Learning objectives

1. Define what is a random variable.
2. Types of random variables: discrete and continuous.
3. Probability mass function, graph, and examples.

Learning objectives

1. Define what is a random variable.
2. Types of random variables: discrete and continuous.
3. Probability mass function, graph, and examples.
4. Cumulative distribution function, graphs, and examples.

Learning objectives

1. Define what is a random variable.
2. Types of random variables: discrete and continuous.
3. Probability mass function, graph, and examples.
4. Cumulative distribution function, graphs, and examples.
5. Expectation and variance of a random variable.

Random variable

Example: Rolling a dice twice

Example: Tossing a coin three times

Random Variable

Random Variable

- When a probability experiment is performed, often we are not interested in all the details of the experimental result, but rather are interested in the value of some numerical quantity determined by the result.

Random Variable

- When a probability experiment is performed, often we are not interested in all the details of the experimental result, but rather are interested in the value of some numerical quantity determined by the result.
- For example, in rolling a dice twice, often we care about only their sum of outcomes and are not concerned about the values on the individual dice.

Random Variable

- When a probability experiment is performed, often we are not interested in all the details of the experimental result, but rather are interested in the value of some numerical quantity determined by the result.
- For example, in rolling a dice twice, often we care about only their sum of outcomes and are not concerned about the values on the individual dice.
  - That is, we may be interested in knowing that the sum is 7 and may not be concerned over whether the actual outcome was $(1, 6), (2, 5), (3, 4), (4, 3), (5, 2)$, or $(6, 1)$.

Random Variable

- When a probability experiment is performed, often we are not interested in all the details of the experimental result, but rather are interested in the value of some numerical quantity determined by the result.
- For example, in rolling a dice twice, often we care about only their sum of outcomes and are not concerned about the values on the individual dice.
  - That is, we may be interested in knowing that the sum is 7 and may not be concerned over whether the actual outcome was $(1, 6), (2, 5), (3, 4), (4, 3), (5, 2)$, or $(6, 1)$.
- These quantities of interest, or, more formally, these real-valued functions defined on the sample space, are known as random variables.

Random Variable

- When a probability experiment is performed, often we are not interested in all the details of the experimental result, but rather are interested in the value of some numerical quantity determined by the result.
- For example, in rolling a dice twice, often we care about only their sum of outcomes and are not concerned about the values on the individual dice.
  - That is, we may be interested in knowing that the sum is 7 and may not be concerned over whether the actual outcome was $(1, 6), (2, 5), (3, 4), (4, 3), (5, 2)$, or $(6, 1)$.
- These quantities of interest, or, more formally, these real-valued functions defined on the sample space, are known as random variables.
- Because the value of a random variable is determined by the outcome of the experiment, we may assign probabilities to the possible values of the random variable.

Rolling a dice: Sample space

Rolling a dice: Sample space

- Experiment: Roll a dice twice

Rolling a dice: Sample space

- Experiment: Roll a dice twice
- The sample space for this experiment is

$$S = \left\{ \begin{array}{l} (1,1),(1,2),(1,3),(1,4),(1,5),(1,6), \\ (2,1),(2,2),(2,3),(2,4),(2,5),(2,6), \\ (3,1),(3,2),(3,3),(3,4),(3,5),(3,6), \\ (4,1),(4,2),(4,3),(4,4),(4,5),(4,6), \\ (5,1),(5,2),(5,3),(5,4),(5,5),(5,6), \\ (6,1),(6,2),(6,3),(6,4),(6,5),(6,6), \end{array} \right\}$$

Rolling a dice: Sample space

- Experiment: Roll a dice twice
- The sample space for this experiment is

$$S = \left\{ \begin{array}{l} (1,1),(1,2),(1,3),(1,4),(1,5),(1,6), \\ (2,1),(2,2),(2,3),(2,4),(2,5),(2,6), \\ (3,1),(3,2),(3,3),(3,4),(3,5),(3,6), \\ (4,1),(4,2),(4,3),(4,4),(4,5),(4,6), \\ (5,1),(5,2),(5,3),(5,4),(5,5),(5,6), \\ (6,1),(6,2),(6,3),(6,4),(6,5),(6,6), \end{array} \right\}$$

- Consider the probabilities associated with the two questions

Rolling a dice: Sample space

- Experiment: Roll a dice twice
- The sample space for this experiment is

$$S = \left\{ \begin{array}{l} (1,1),(1,2),(1,3),(1,4),(1,5),(1,6), \\ (2,1),(2,2),(2,3),(2,4),(2,5),(2,6), \\ (3,1),(3,2),(3,3),(3,4),(3,5),(3,6), \\ (4,1),(4,2),(4,3),(4,4),(4,5),(4,6), \\ (5,1),(5,2),(5,3),(5,4),(5,5),(5,6), \\ (6,1),(6,2),(6,3),(6,4),(6,5),(6,6), \end{array} \right\}$$

- Consider the probabilities associated with the two questions
  1. Of the outcomes, how many outcomes will result in a sum of outcomes as 7?

Rolling a dice: Sample space

- Experiment: Roll a dice twice
- The sample space for this experiment is

$$S = \left\{ \begin{array}{l} (1,1),(1,2),(1,3),(1,4),(1,5),(1,6), \\ (2,1),(2,2),(2,3),(2,4),(2,5),(2,6), \\ (3,1),(3,2),(3,3),(3,4),(3,5),(3,6), \\ (4,1),(4,2),(4,3),(4,4),(4,5),(4,6), \\ (5,1),(5,2),(5,3),(5,4),(5,5),(5,6), \\ (6,1),(6,2),(6,3),(6,4),(6,5),(6,6), \end{array} \right\}$$

- Consider the probabilities associated with the two questions
  1. Of the outcomes, how many outcomes will result in a sum of outcomes as 7?
  2. Of the outcomes, how many outcomes will have the smaller of the outcomes as 3?

Rolling a dice: Sample space

- Experiment: Roll a dice twice
- The sample space for this experiment is

$$S = \left\{ \begin{array}{l} (1,1),(1,2),(1,3),(1,4),(1,5),(1,6), \\ (2,1),(2,2),(2,3),(2,4),(2,5),(2,6), \\ (3,1),(3,2),(3,3),(3,4),(3,5),(3,6), \\ (4,1),(4,2),(4,3),(4,4),(4,5),(4,6), \\ (5,1),(5,2),(5,3),(5,4),(5,5),(5,6), \\ (6,1),(6,2),(6,3),(6,4),(6,5),(6,6), \end{array} \right\}$$

- Consider the probabilities associated with the two questions
  1. Of the outcomes, how many outcomes will result in a sum of outcomes as 7?
  2. Of the outcomes, how many outcomes will have the smaller of the outcomes as 3?
- Notice, the experiment and sample space used to answer both the questions are the same.

- Let X denote the sum of outcomes of the two rolls.

- Let X denote the sum of outcomes of the two rolls.
- Let Y denote the lesser of the two outcomes. If the outcomes are the same, the value of the outcome is taken as value of Y.

- Let X denote the sum of outcomes of the two rolls.
- Let Y denote the lesser of the two outcomes. If the outcomes are the same, the value of the outcome is taken as value of Y.

Outcome	X	Y	Outcome	X	Y	Outcome	X	Y
(1,1)	2	1	(3,1)	4	1	(5,1)	6	1
(1,2)	3	1	(3,2)	5	2	(5,2)	7	2
(1,3)	4	1	(3,3)	6	3	(5,3)	8	3
(1,4)	5	1	(3,4)	7	3	(5,4)	9	4
(1,5)	6	1	(3,5)	8	3	(5,5)	10	5
(1,6)	7	1	(3,6)	9	3	(5,6)	11	5
(2,1)	3	1	(4,1)	5	1	(6,1)	7	1
(2,2)	4	2	(4,2)	6	2	(6,2)	8	2
(2,3)	5	2	(4,3)	7	3	(6,3)	9	3
(2,4)	6	2	(4,4)	8	4	(6,4)	10	4
(2,5)	7	2	(4,5)	9	4	(6,5)	11	5
(2,6)	8	2	(4,6)	10	4	(6,6)	12	6

Sum of rolls of the dice

Sum of rolls of the dice

- Let X denote the sum of outcomes of the two rolls.

Sum of rolls of the dice

- Let X denote the sum of outcomes of the two rolls.
- X takes the values 2,3,4,5,6,7,8,9,10,11, and 12.

Sum of rolls of the dice

- Let X denote the sum of outcomes of the two rolls.
- X takes the values 2,3,4,5,6,7,8,9,10,11, and 12.

Sum of rolls of the dice

- Let X denote the sum of outcomes of the two rolls.
- X takes the values 2,3,4,5,6,7,8,9,10,11, and 12.

Value of X	Relevant event

Sum of rolls of the dice

- Let X denote the sum of outcomes of the two rolls.
- X takes the values 2,3,4,5,6,7,8,9,10,11, and 12.

Value of X	Relevant event
2	

Sum of rolls of the dice

- Let X denote the sum of outcomes of the two rolls.
- X takes the values 2,3,4,5,6,7,8,9,10,11, and 12.

Value of X	Relevant event
2	$\{(1,1)\}$

Sum of rolls of the dice

- Let X denote the sum of outcomes of the two rolls.
- X takes the values 2,3,4,5,6,7,8,9,10,11, and 12.

Value of X	Relevant event
2	$\{(1,1)\}$
3	

Sum of rolls of the dice

- Let X denote the sum of outcomes of the two rolls.
- X takes the values 2,3,4,5,6,7,8,9,10,11, and 12.

Value of X	Relevant event
2	$\{(1,1)\}$
3	$\{(1,2),(2,1)\}$

Sum of rolls of the dice

- Let X denote the sum of outcomes of the two rolls.
- X takes the values 2,3,4,5,6,7,8,9,10,11, and 12.

Value of X	Relevant event
2	$\{(1,1)\}$
3	$\{(1,2),(2,1)\}$
4	

Sum of rolls of the dice

- Let X denote the sum of outcomes of the two rolls.
- X takes the values 2,3,4,5,6,7,8,9,10,11, and 12.

Value of X	Relevant event
2	$\{(1,1)\}$
3	$\{(1,2),(2,1)\}$
4	$\{(1,3),(2,2),(3,1)\}$

Sum of rolls of the dice

- Let X denote the sum of outcomes of the two rolls.
- X takes the values 2,3,4,5,6,7,8,9,10,11, and 12.

Value of X	Relevant event
2	$\{(1,1)\}$
3	$\{(1,2),(2,1)\}$
4	$\{(1,3),(2,2),(3,1)\}$
5	

Sum of rolls of the dice

- Let X denote the sum of outcomes of the two rolls.
- X takes the values 2,3,4,5,6,7,8,9,10,11, and 12.

Value of X	Relevant event
2	$\{(1,1)\}$
3	$\{(1,2),(2,1)\}$
4	$\{(1,3),(2,2),(3,1)\}$
5	$\{(1,4),(2,3),(3,2),(4,1)\}$
$\vdots$	$\vdots$

Sum of rolls of the dice

- Let X denote the sum of outcomes of the two rolls.
- X takes the values 2,3,4,5,6,7,8,9,10,11, and 12.

Value of X	Relevant event
2	$\{(1,1)\}$
3	$\{(1,2),(2,1)\}$
4	$\{(1,3),(2,2),(3,1)\}$
5	$\{(1,4),(2,3),(3,2),(4,1)\}$
$\vdots$	$\vdots$
9	

Sum of rolls of the dice

- Let X denote the sum of outcomes of the two rolls.
- X takes the values 2,3,4,5,6,7,8,9,10,11, and 12.

Value of X	Relevant event
2	$\{(1,1)\}$
3	$\{(1,2),(2,1)\}$
4	$\{(1,3),(2,2),(3,1)\}$
5	$\{(1,4),(2,3),(3,2),(4,1)\}$
$\vdots$	$\vdots$
9	$\{(3,6),(4,5),(5,4),(6,3)\}$

Sum of rolls of the dice

- Let X denote the sum of outcomes of the two rolls.
- X takes the values 2,3,4,5,6,7,8,9,10,11, and 12.

Value of X	Relevant event
2	$\{(1,1)\}$
3	$\{(1,2),(2,1)\}$
4	$\{(1,3),(2,2),(3,1)\}$
5	$\{(1,4),(2,3),(3,2),(4,1)\}$
$\vdots$	$\vdots$
9	$\{(3,6),(4,5),(5,4),(6,3)\}$
10	

Sum of rolls of the dice

- Let X denote the sum of outcomes of the two rolls.
- X takes the values 2,3,4,5,6,7,8,9,10,11, and 12.

Value of X	Relevant event
2	$\{(1,1)\}$
3	$\{(1,2),(2,1)\}$
4	$\{(1,3),(2,2),(3,1)\}$
5	$\{(1,4),(2,3),(3,2),(4,1)\}$
$\vdots$	$\vdots$
9	$\{(3,6),(4,5),(5,4),(6,3)\}$
10	$\{(4,6),(5,5),(6,4)\}$

Sum of rolls of the dice

- Let X denote the sum of outcomes of the two rolls.
- X takes the values 2,3,4,5,6,7,8,9,10,11, and 12.

Value of X	Relevant event
2	$\{(1,1)\}$
3	$\{(1,2),(2,1)\}$
4	$\{(1,3),(2,2),(3,1)\}$
5	$\{(1,4),(2,3),(3,2),(4,1)\}$
$\vdots$	$\vdots$
9	$\{(3,6),(4,5),(5,4),(6,3)\}$
10	$\{(4,6),(5,5),(6,4)\}$
11	

Sum of rolls of the dice

- Let X denote the sum of outcomes of the two rolls.
- X takes the values 2,3,4,5,6,7,8,9,10,11, and 12.

Value of X	Relevant event
2	$\{(1,1)\}$
3	$\{(1,2),(2,1)\}$
4	$\{(1,3),(2,2),(3,1)\}$
5	$\{(1,4),(2,3),(3,2),(4,1)\}$
$\vdots$	$\vdots$
9	$\{(3,6),(4,5),(5,4),(6,3)\}$
10	$\{(4,6),(5,5),(6,4)\}$
11	$\{(5,6),(6,5)\}$

Sum of rolls of the dice

- Let X denote the sum of outcomes of the two rolls.
- X takes the values 2,3,4,5,6,7,8,9,10,11, and 12.

Value of X	Relevant event
2	$\{(1,1)\}$
3	$\{(1,2),(2,1)\}$
4	$\{(1,3),(2,2),(3,1)\}$
5	$\{(1,4),(2,3),(3,2),(4,1)\}$
$\vdots$	$\vdots$
9	$\{(3,6),(4,5),(5,4),(6,3)\}$
10	$\{(4,6),(5,5),(6,4)\}$
11	$\{(5,6),(6,5)\}$
12	

Sum of rolls of the dice

- Let X denote the sum of outcomes of the two rolls.
- X takes the values 2,3,4,5,6,7,8,9,10,11, and 12.

Value of X	Relevant event
2	$\{(1,1)\}$
3	$\{(1,2),(2,1)\}$
4	$\{(1,3),(2,2),(3,1)\}$
5	$\{(1,4),(2,3),(3,2),(4,1)\}$
$\vdots$	$\vdots$
9	$\{(3,6),(4,5),(5,4),(6,3)\}$
10	$\{(4,6),(5,5),(6,4)\}$
11	$\{(5,6),(6,5)\}$
12	$\{(6,6)\}$

- We say X is a random variable taking on one of the values 2,3,4,5,6,7,8,9,10,11, and 12 with respective probabilities

- We say X is a random variable taking on one of the values 2,3,4,5,6,7,8,9,10,11, and 12 with respective probabilities

Probability of X	Probability of relevant event	Probability

- We say X is a random variable taking on one of the values 2,3,4,5,6,7,8,9,10,11, and 12 with respective probabilities

Probability of X	Probability of relevant event	Probability
$P\{X = 2\}$	$P(\{(1,1)\})$	$\frac{1}{36}$

- We say X is a random variable taking on one of the values 2,3,4,5,6,7,8,9,10,11, and 12 with respective probabilities

Probability of X	Probability of relevant event	Probability
$P\{X = 2\}$	$P(\{(1,1)\})$	$\frac{1}{36}$
$P\{X = 3\}$	$P(\{(1,2),(2,1)\})$	$\frac{2}{36}$

- We say X is a random variable taking on one of the values 2,3,4,5,6,7,8,9,10,11, and 12 with respective probabilities

Probability of X	Probability of relevant event	Probability
$P\{X = 2\}$	$P(\{(1,1)\})$	$\frac{1}{36}$
$P\{X = 3\}$	$P(\{(1,2),(2,1)\})$	$\frac{2}{36}$
$P\{X = 4\}$	$P(\{(1,3),(2,2),(3,1)\})$	$\frac{3}{36}$

- We say X is a random variable taking on one of the values 2,3,4,5,6,7,8,9,10,11, and 12 with respective probabilities

Probability of X	Probability of relevant event	Probability
$P\{X = 2\}$	$P(\{(1,1)\})$	$\frac{1}{36}$
$P\{X = 3\}$	$P(\{(1,2),(2,1)\})$	$\frac{2}{36}$
$P\{X = 4\}$	$P(\{(1,3),(2,2),(3,1)\})$	$\frac{3}{36}$
$P\{X = 5\}$	$P(\{(1,4),(2,3),(3,2),(4,1)\})$	$\frac{4}{36}$
$\vdots$	$\vdots$	$\vdots$

- We say X is a random variable taking on one of the values 2,3,4,5,6,7,8,9,10,11, and 12 with respective probabilities

Probability of X	Probability of relevant event	Probability
$P\{X=2\}$	$P(\{(1,1)\})$	$\frac{1}{36}$
$P\{X=3\}$	$P(\{(1,2),(2,1)\})$	$\frac{2}{36}$
$P\{X=4\}$	$P(\{(1,3),(2,2),(3,1)\})$	$\frac{3}{36}$
$P\{X=5\}$	$P(\{(1,4),(2,3),(3,2),(4,1)\})$	$\frac{4}{36}$
$\vdots$	$\vdots$	$\vdots$
$P\{X=9\}$	$P(\{(3,6),(4,5),(5,4),(6,3)\})$	$\frac{4}{36}$

- We say X is a random variable taking on one of the values 2,3,4,5,6,7,8,9,10,11, and 12 with respective probabilities

Probability of X	Probability of relevant event	Probability
$P\{X=2\}$	$P(\{(1,1)\})$	$\frac{1}{36}$
$P\{X=3\}$	$P(\{(1,2),(2,1)\})$	$\frac{2}{36}$
$P\{X=4\}$	$P(\{(1,3),(2,2),(3,1)\})$	$\frac{3}{36}$
$P\{X=5\}$	$P(\{(1,4),(2,3),(3,2),(4,1)\})$	$\frac{4}{36}$
$\vdots$	$\vdots$	$\vdots$
$P\{X=9\}$	$P(\{(3,6),(4,5),(5,4),(6,3)\})$	$\frac{4}{36}$
$P\{X=10\}$	$P(\{(4,6),(5,5),(6,4)\})$	$\frac{3}{36}$

- We say X is a random variable taking on one of the values 2,3,4,5,6,7,8,9,10,11, and 12 with respective probabilities

Probability of X	Probability of relevant event	Probability
$P\{X=2\}$	$P(\{(1,1)\})$	$\frac{1}{36}$
$P\{X=3\}$	$P(\{(1,2),(2,1)\})$	$\frac{2}{36}$
$P\{X=4\}$	$P(\{(1,3),(2,2),(3,1)\})$	$\frac{3}{36}$
$P\{X=5\}$	$P(\{(1,4),(2,3),(3,2),(4,1)\})$	$\frac{4}{36}$
$\vdots$	$\vdots$	$\vdots$
$P\{X=9\}$	$P(\{(3,6),(4,5),(5,4),(6,3)\})$	$\frac{4}{36}$
$P\{X=10\}$	$P(\{(4,6),(5,5),(6,4)\})$	$\frac{3}{36}$
$P\{X=11\}$	$P(\{(5,6),(6,5)\})$	$\frac{2}{36}$

- We say X is a random variable taking on one of the values 2,3,4,5,6,7,8,9,10,11, and 12 with respective probabilities

Probability of X	Probability of relevant event	Probability
$P\{X=2\}$	$P(\{(1,1)\})$	$\frac{1}{36}$
$P\{X=3\}$	$P(\{(1,2),(2,1)\})$	$\frac{2}{36}$
$P\{X=4\}$	$P(\{(1,3),(2,2),(3,1)\})$	$\frac{3}{36}$
$P\{X=5\}$	$P(\{(1,4),(2,3),(3,2),(4,1)\})$	$\frac{4}{36}$
$\vdots$	$\vdots$	$\vdots$
$P\{X=9\}$	$P(\{(3,6),(4,5),(5,4),(6,3)\})$	$\frac{4}{36}$
$P\{X=10\}$	$P(\{(4,6),(5,5),(6,4)\})$	$\frac{3}{36}$
$P\{X=11\}$	$P(\{(5,6),(6,5)\})$	$\frac{2}{36}$
$P\{X=12\}$	$P(\{(6,6)\})$	$\frac{1}{36}$

Lesser of the two values

Lesser of the two values

- Let Y denote the lesser of the two outcomes. If the outcomes are the same, the value of the out come is taken as value of Y.

Lesser of the two values

- Let Y denote the lesser of the two outcomes. If the outcomes are the same, the value of the out come is taken as value of Y.
- Y takes the values 1,2,3,4,5, and 6.

Lesser of the two values

- Let Y denote the lesser of the two outcomes. If the outcomes are the same, the value of the out come is taken as value of Y.
- Y takes the values 1,2,3,4,5, and 6.

Y	

Lesser of the two values

- Let Y denote the lesser of the two outcomes. If the outcomes are the same, the value of the out come is taken as value of Y.
- Y takes the values 1,2,3,4,5, and 6.

Y	Relevant event

Lesser of the two values

- Let Y denote the lesser of the two outcomes. If the outcomes are the same, the value of the out come is taken as value of Y.
- Y takes the values 1,2,3,4,5, and 6.

Y	Relevant event
1	

Lesser of the two values

- Let Y denote the lesser of the two outcomes. If the outcomes are the same, the value of the out come is taken as value of Y.
- Y takes the values 1,2,3,4,5, and 6.

Y	Relevant event
1	$\{(1,1),(1,2),(1,3),(1,4),(1,5),(1,6),(2,1),(3,1),(4,1),(5,1),(6,1)\}$

Lesser of the two values

- Let Y denote the lesser of the two outcomes. If the outcomes are the same, the value of the out come is taken as value of Y.
- Y takes the values 1,2,3,4,5, and 6.

Y	Relevant event
1	$\{(1,1),(1,2),(1,3),(1,4),(1,5),(1,6),(2,1),(3,1),(4,1),(5,1),(6,1)\}$
2	

Lesser of the two values

- Let Y denote the lesser of the two outcomes. If the outcomes are the same, the value of the out come is taken as value of Y.
- Y takes the values 1,2,3,4,5, and 6.

Y	Relevant event
1	$\{(1,1),(1,2),(1,3),(1,4),(1,5),(1,6),(2,1),(3,1),(4,1),(5,1),(6,1)\}$
2	$\{(2,2),(2,3),(2,4),(2,5),(2,6),(3,2),(4,2),(5,2),(6,2)\}$

Lesser of the two values

- Let Y denote the lesser of the two outcomes. If the outcomes are the same, the value of the out come is taken as value of Y.
- Y takes the values 1,2,3,4,5, and 6.

Y	Relevant event
1	$\{(1,1),(1,2),(1,3),(1,4),(1,5),(1,6),(2,1),(3,1),(4,1),(5,1),(6,1)\}$
2	$\{(2,2),(2,3),(2,4),(2,5),(2,6),(3,2),(4,2),(5,2),(6,2)\}$
3	

Lesser of the two values

- Let Y denote the lesser of the two outcomes. If the outcomes are the same, the value of the out come is taken as value of Y.
- Y takes the values 1,2,3,4,5, and 6.

Y	Relevant event
1	$\{(1,1),(1,2),(1,3),(1,4),(1,5),(1,6),(2,1),(3,1),(4,1),(5,1),(6,1)\}$
2	$\{(2,2),(2,3),(2,4),(2,5),(2,6),(3,2),(4,2),(5,2),(6,2)\}$
3	$\{(3,3),(3,4),(3,5),(3,6),(4,3),(5,3),(6,3)\}$

Lesser of the two values

- Let Y denote the lesser of the two outcomes. If the outcomes are the same, the value of the out come is taken as value of Y.
- Y takes the values 1,2,3,4,5, and 6.

Y	Relevant event
1	$\{(1,1),(1,2),(1,3),(1,4),(1,5),(1,6),(2,1),(3,1),(4,1),(5,1),(6,1)\}$
2	$\{(2,2),(2,3),(2,4),(2,5),(2,6),(3,2),(4,2),(5,2),(6,2)\}$
3	$\{(3,3),(3,4),(3,5),(3,6),(4,3),(5,3),(6,3)\}$
4	

Lesser of the two values

- Let Y denote the lesser of the two outcomes. If the outcomes are the same, the value of the out come is taken as value of Y.
- Y takes the values 1,2,3,4,5, and 6.

Y	Relevant event
1	$\{(1,1),(1,2),(1,3),(1,4),(1,5),(1,6),(2,1),(3,1),(4,1),(5,1),(6,1)\}$
2	$\{(2,2),(2,3),(2,4),(2,5),(2,6),(3,2),(4,2),(5,2),(6,2)\}$
3	$\{(3,3),(3,4),(3,5),(3,6),(4,3),(5,3),(6,3)\}$
4	$\{(4,4),(4,5),(4,6),(5,4),(6,4)\}$

Lesser of the two values

- Let Y denote the lesser of the two outcomes. If the outcomes are the same, the value of the out come is taken as value of Y.
- Y takes the values 1,2,3,4,5, and 6.

Y	Relevant event
1	$\{(1,1),(1,2),(1,3),(1,4),(1,5),(1,6),(2,1),(3,1),(4,1),(5,1),(6,1)\}$
2	$\{(2,2),(2,3),(2,4),(2,5),(2,6),(3,2),(4,2),(5,2),(6,2)\}$
3	$\{(3,3),(3,4),(3,5),(3,6),(4,3),(5,3),(6,3)\}$
4	$\{(4,4),(4,5),(4,6),(5,4),(6,4)\}$
5	

Lesser of the two values

- Let Y denote the lesser of the two outcomes. If the outcomes are the same, the value of the out come is taken as value of Y.
- Y takes the values 1,2,3,4,5, and 6.

Y	Relevant event
1	$\{(1,1),(1,2),(1,3),(1,4),(1,5),(1,6),(2,1),(3,1),(4,1),(5,1),(6,1)\}$
2	$\{(2,2),(2,3),(2,4),(2,5),(2,6),(3,2),(4,2),(5,2),(6,2)\}$
3	$\{(3,3),(3,4),(3,5),(3,6),(4,3),(5,3),(6,3)\}$
4	$\{(4,4),(4,5),(4,6),(5,4),(6,4)\}$
5	$\{(5,5),(5,6),(6,5)\}$

Lesser of the two values

- Let Y denote the lesser of the two outcomes. If the outcomes are the same, the value of the out come is taken as value of Y.
- Y takes the values 1,2,3,4,5, and 6.

Y	Relevant event
1	$\{(1,1),(1,2),(1,3),(1,4),(1,5),(1,6),(2,1),(3,1),(4,1),(5,1),(6,1)\}$
2	$\{(2,2),(2,3),(2,4),(2,5),(2,6),(3,2),(4,2),(5,2),(6,2)\}$
3	$\{(3,3),(3,4),(3,5),(3,6),(4,3),(5,3),(6,3)\}$
4	$\{(4,4),(4,5),(4,6),(5,4),(6,4)\}$
5	$\{(5,5),(5,6),(6,5)\}$
6	

Lesser of the two values

- Let Y denote the lesser of the two outcomes. If the outcomes are the same, the value of the out come is taken as value of Y.
- Y takes the values 1,2,3,4,5, and 6.

Y	Relevant event
1	$\{(1,1),(1,2),(1,3),(1,4),(1,5),(1,6),(2,1),(3,1),(4,1),(5,1),(6,1)\}$
2	$\{(2,2),(2,3),(2,4),(2,5),(2,6),(3,2),(4,2),(5,2),(6,2)\}$
3	$\{(3,3),(3,4),(3,5),(3,6),(4,3),(5,3),(6,3)\}$
4	$\{(4,4),(4,5),(4,6),(5,4),(6,4)\}$
5	$\{(5,5),(5,6),(6,5)\}$
6	$\{(6,6)\}$

- We say Y is a random variable taking on one of the values 1,2,3,4,5, and 6 with respective probabilities

- We say Y is a random variable taking on one of the values 1,2,3,4,5, and 6 with respective probabilities

- We say Y is a random variable taking on one of the values 1,2,3,4,5, and 6 with respective probabilities

Y	Relevant event	Probability

- We say Y is a random variable taking on one of the values 1,2,3,4,5, and 6 with respective probabilities

Y	Relevant event	Probability
1	$\{(1,1),(1,2),(1,3),(1,4),(1,5),(1,6),(2,1),(3,1),(4,1),(5,1),(6,1)\}$	$\frac{11}{36}$

- We say Y is a random variable taking on one of the values 1,2,3,4,5, and 6 with respective probabilities

Y	Relevant event	Probability
1	$\{(1,1),(1,2),(1,3),(1,4),(1,5),(1,6),(2,1),(3,1),(4,1),(5,1),(6,1)\}$	$\frac{11}{36}$
2	$\{(2,2),(2,3),(2,4),(2,5),(2,6),(3,2),(4,2),(5,2),(6,2)\}$	$\frac{9}{36}$

- We say Y is a random variable taking on one of the values 1,2,3,4,5, and 6 with respective probabilities

Y	Relevant event	Probability
1	$\{(1,1),(1,2),(1,3),(1,4),(1,5),(1,6),(2,1),(3,1),(4,1),(5,1),(6,1)\}$	$\frac{11}{36}$
2	$\{(2,2),(2,3),(2,4),(2,5),(2,6),(3,2),(4,2),(5,2),(6,2)\}$	$\frac{9}{36}$
3	$\{(3,3),(3,4),(3,5),(3,6),(4,3),(5,3),(6,3)\}$	$\frac{7}{36}$

- We say Y is a random variable taking on one of the values 1,2,3,4,5, and 6 with respective probabilities

Y	Relevant event	Probability
1	$\{(1,1),(1,2),(1,3),(1,4),(1,5),(1,6),(2,1),(3,1),(4,1),(5,1),(6,1)\}$	$\frac{11}{36}$
2	$\{(2,2),(2,3),(2,4),(2,5),(2,6),(3,2),(4,2),(5,2),(6,2)\}$	$\frac{9}{36}$
3	$\{(3,3),(3,4),(3,5),(3,6),(4,3),(5,3),(6,3)\}$	$\frac{7}{36}$
4	$\{(4,4),(4,5),(4,6),(5,4),(6,4)\}$	$\frac{5}{36}$

- We say Y is a random variable taking on one of the values 1,2,3,4,5, and 6 with respective probabilities

Y	Relevant event	Probability
1	$\{(1,1),(1,2),(1,3),(1,4),(1,5),(1,6),(2,1),(3,1),(4,1),(5,1),(6,1)\}$	$\frac{11}{36}$
2	$\{(2,2),(2,3),(2,4),(2,5),(2,6),(3,2),(4,2),(5,2),(6,2)\}$	$\frac{9}{36}$
3	$\{(3,3),(3,4),(3,5),(3,6),(4,3),(5,3),(6,3)\}$	$\frac{7}{36}$
4	$\{(4,4),(4,5),(4,6),(5,4),(6,4)\}$	$\frac{5}{36}$
5	$\{(5,5),(5,6),(6,5)\}$	$\frac{3}{36}$

- We say Y is a random variable taking on one of the values 1,2,3,4,5, and 6 with respective probabilities

Y	Relevant event	Probability
1	$\{(1,1),(1,2),(1,3),(1,4),(1,5),(1,6),(2,1),(3,1),(4,1),(5,1),(6,1)\}$	$\frac{11}{36}$
2	$\{(2,2),(2,3),(2,4),(2,5),(2,6),(3,2),(4,2),(5,2),(6,2)\}$	$\frac{9}{36}$
3	$\{(3,3),(3,4),(3,5),(3,6),(4,3),(5,3),(6,3)\}$	$\frac{7}{36}$
4	$\{(4,4),(4,5),(4,6),(5,4),(6,4)\}$	$\frac{5}{36}$
5	$\{(5,5),(5,6),(6,5)\}$	$\frac{3}{36}$
6	$\{(6,6)\}$	$\frac{1}{36}$

Tossing a coin three times: Sample space

Tossing a coin three times: Sample space

- Experiment: Toss a coin three times.

Tossing a coin three times: Sample space

- Experiment: Toss a coin three times.
- The sample space for this experiment is

$$S = \{HHH, HHT, HTH, HTT, THH, THT, TTH, TTT\}$$

Tossing a coin three times: Sample space

- Experiment: Toss a coin three times.
- The sample space for this experiment is

$$S = \{HHH, HHT, HTH, HTT, THH, THT, TTH, TTT\}$$

- Consider the probabilities associated with the two questions:

Tossing a coin three times: Sample space

- Experiment: Toss a coin three times.
- The sample space for this experiment is

$$S = \{HHH, HHT, HTH, HTT, THH, THT, TTH, TTT\}$$

- Consider the probabilities associated with the two questions:
  1. Of the three tosses, how many tosses will be heads?

Tossing a coin three times: Sample space

- Experiment: Toss a coin three times.
- The sample space for this experiment is

$$S = \{HHH, HHT, HTH, HTT, THH, THT, TTH, TTT\}$$

- Consider the probabilities associated with the two questions:
  1. Of the three tosses, how many tosses will be heads?
  2. Of the three tosses, which toss results in a heads first?, i.e first, second or third toss is a head?

Tossing a coin three times: Sample space

- Experiment: Toss a coin three times.
- The sample space for this experiment is

$$S = \{HHH, HHT, HTH, HTT, THH, THT, TTH, TTT\}$$

- Consider the probabilities associated with the two questions:
  1. Of the three tosses, how many tosses will be heads?
  2. Of the three tosses, which toss results in a heads first?, i.e first, second or third toss is a head?
- Notice the experiment and sample space used to answer both the questions are the same.

- Let X denote the number of heads that appear. Let Y denote the toss in which a head appears first.

- Let X denote the number of heads that appear. Let Y denote the toss in which a head appears first.

Outcome	X	Y
HHH	3	1
HHT	2	1
HTH	2	1
HTT	1	1
THH	2	2
THT	1	2
TTH	1	3
TTT	0	NIL

Number of tosses that will be heads

Number of tosses that will be heads

- Let X denote the number of heads that appear.

Number of tosses that will be heads

- Let X denote the number of heads that appear.
- X takes the values 0,1,2,3

Number of tosses that will be heads

- Let X denote the number of heads that appear.
- X takes the values 0,1,2,3

Value of X	Relevant event

Number of tosses that will be heads

- Let X denote the number of heads that appear.
- X takes the values 0,1,2,3

Value of X	Relevant event
0	

Number of tosses that will be heads

- Let X denote the number of heads that appear.
- X takes the values 0,1,2,3

Value of X	Relevant event
0	$\{(TTT)\}$

Number of tosses that will be heads

- Let X denote the number of heads that appear.
- X takes the values 0,1,2,3

Value of X	Relevant event
0	$\{(TTT)\}$
1	

Number of tosses that will be heads

- Let X denote the number of heads that appear.
- X takes the values 0,1,2,3

Value of X	Relevant event
0	$\{(TTT)\}$
1	$\{(HTT), (THT), (TTH)\}$

Number of tosses that will be heads

- Let X denote the number of heads that appear.
- X takes the values 0,1,2,3

Value of X	Relevant event
0	$\{(TTT)\}$
1	$\{(HTT), (THT), (TTH)\}$
2	

Number of tosses that will be heads

- Let X denote the number of heads that appear.
- X takes the values 0,1,2,3

Value of X	Relevant event
0	$\{(TTT)\}$
1	$\{(HTT), (THT), (TTH)\}$
2	$\{(HHT), (HTH), (THH)\}$

Number of tosses that will be heads

- Let X denote the number of heads that appear.
- X takes the values 0,1,2,3

Value of X	Relevant event
0	$\{(TTT)\}$
1	$\{(HTT), (THT), (TTH)\}$
2	$\{(HHT), (HTH), (THH)\}$
3	

Number of tosses that will be heads

- Let X denote the number of heads that appear.
- X takes the values 0,1,2,3

Value of X	Relevant event
0	$\{(TTT)\}$
1	$\{(HTT), (THT), (TTH)\}$
2	$\{(HHT), (HTH), (THH)\}$
3	$\{(HHH)\}$

Number of tosses that will be heads

- Let X denote the number of heads that appear.
- X takes the values 0,1,2,3

Value of X	Relevant event
0	$\{(TTT)\}$
1	$\{(HTT), (THT), (TTH)\}$
2	$\{(HHT), (HTH), (THH)\}$
3	$\{(HHH)\}$

- We say X is a random variable taking on one of the values 0, 1, 2, and 3 with respective probabilities

Number of tosses that will be heads

- Let X denote the number of heads that appear.
- X takes the values 0,1,2,3

Value of X	Relevant event
0	$\{(TTT)\}$
1	$\{(HTT), (THT), (TTH)\}$
2	$\{(HHT), (HTH), (THH)\}$
3	$\{(HHH)\}$

- We say X is a random variable taking on one of the values 0, 1, 2, and 3 with respective probabilities
  - $P\{X = 0\} = P\{(TTT)\} = \frac{1}{8}$

Number of tosses that will be heads

- Let X denote the number of heads that appear.
- X takes the values 0,1,2,3

Value of X	Relevant event
0	$\{(TTT)\}$
1	$\{(HTT), (THT), (TTH)\}$
2	$\{(HHT), (HTH), (THH)\}$
3	$\{(HHH)\}$

- We say X is a random variable taking on one of the values 0, 1, 2, and 3 with respective probabilities
  - $P\{X = 0\} = P\{(TTT)\} = \frac{1}{8}$
  - $P\{X = 1\} = P\{(HTT), (THT), (TTH)\} = \frac{3}{8}$

Number of tosses that will be heads

- Let X denote the number of heads that appear.
- X takes the values 0,1,2,3

Value of X	Relevant event
0	$\{(TTT)\}$
1	$\{(HTT),(THT),(TTH)\}$
2	$\{(HHT),(HTH),(THH)\}$
3	$\{(HHH)\}$

- We say X is a random variable taking on one of the values 0, 1, 2, and 3 with respective probabilities
  - $P\{X=0\} = P\{(TTT)\} = \frac{1}{8}$
  - $P\{X=1\} = P\{(HTT),(THT),(TTH)\} = \frac{3}{8}$
  - $P\{X=2\} = P\{(HHT),(HTH),(THH)\} = \frac{3}{8}$

Number of tosses that will be heads

- Let X denote the number of heads that appear.
- X takes the values 0,1,2,3

Value of X	Relevant event
0	$\{(TTT)\}$
1	$\{(HTT),(THT),(TTH)\}$
2	$\{(HHT),(HTH),(THH)\}$
3	$\{(HHH)\}$

- We say X is a random variable taking on one of the values 0, 1, 2, and 3 with respective probabilities
  - $P\{X=0\} = P\{(TTT)\} = \frac{1}{8}$
  - $P\{X=1\} = P\{(HTT),(THT),(TTH)\} = \frac{3}{8}$
  - $P\{X=2\} = P\{(HHT),(HTH),(THH)\} = \frac{3}{8}$
  - $P\{X=3\} = P\{(HHH)\} = \frac{1}{8}$

Which toss results in a heads first

Which toss results in a heads first

- Let Y denote the toss in which a head appears first.

Which toss results in a heads first

- Let Y denote the toss in which a head appears first.
- Y takes the values 1,2,3, and NIL

Which toss results in a heads first

- Let Y denote the toss in which a head appears first.
- Y takes the values 1,2,3, and NIL

Which toss results in a heads first

- Let Y denote the toss in which a head appears first.
- Y takes the values 1,2,3, and NIL

Value of Y	Relevant event

Which toss results in a heads first

- Let Y denote the toss in which a head appears first.
- Y takes the values 1,2,3, and NIL

Value of Y	Relevant event
1	

Which toss results in a heads first

- Let Y denote the toss in which a head appears first.
- Y takes the values 1,2,3, and NIL

Value of Y	Relevant event
1	$\{(HHH), (HHT), (HTH), (HTT)\}$

Which toss results in a heads first

- Let Y denote the toss in which a head appears first.
- Y takes the values 1,2,3, and NIL

Value of Y	Relevant event
1	$\{(HHH), (HHT), (HTH), (HTT)\}$
2	

Which toss results in a heads first

- Let Y denote the toss in which a head appears first.
- Y takes the values 1,2,3, and NIL

Value of Y	Relevant event
1	$\{(HHH), (HHT), (HTH), (HTT)\}$
2	$\{(THH), (THT)\}$

Which toss results in a heads first

- Let Y denote the toss in which a head appears first.
- Y takes the values 1,2,3, and NIL

Value of Y	Relevant event
1	$\{(HHH), (HHT), (HTH), (HTT)\}$
2	$\{(THH), (THT)\}$
3	

Which toss results in a heads first

- Let Y denote the toss in which a head appears first.
- Y takes the values 1,2,3, and NIL

Value of Y	Relevant event
1	$\{(HHH), (HHT), (HTH), (HTT)\}$
2	$\{(THH), (THT)\}$
3	$\{(TTH)\}$

Which toss results in a heads first

- Let Y denote the toss in which a head appears first.
- Y takes the values 1,2,3, and NIL

Value of Y	Relevant event
1	$\{(HHH), (HHT), (HTH), (HTT)\}$
2	$\{(THH), (THT)\}$
3	$\{(TTH)\}$
NIL	

Which toss results in a heads first

- Let Y denote the toss in which a head appears first.
- Y takes the values 1,2,3, and NIL

Value of Y	Relevant event
1	$\{(HHH), (HHT), (HTH), (HTT)\}$
2	$\{(THH), (THT)\}$
3	$\{(TTH)\}$
NIL	$\{(TTT)\}$

Which toss results in a heads first

- Let Y denote the toss in which a head appears first.
- Y takes the values 1,2,3, and NIL

Value of Y	Relevant event
1	$\{(HHH), (HHT), (HTH), (HTT)\}$
2	$\{(THH), (THT)\}$
3	$\{(TTH)\}$
NIL	$\{(TTT)\}$

- We say Y is a random variable taking on one of the values 1, 2, 3, and NIL with respective probabilities

Which toss results in a heads first

- Let Y denote the toss in which a head appears first.
- Y takes the values 1,2,3, and NIL

Value of Y	Relevant event
1	$\{(HHH), (HHT), (HTH), (HTT)\}$
2	$\{(THH), (THT)\}$
3	$\{(TTH)\}$
NIL	$\{(TTT)\}$

- We say Y is a random variable taking on one of the values 1, 2, 3, and NIL with respective probabilities
  - $P\{Y = 1\} = P\{(HHH), (HHT), (HTH), (HTT)\} = \frac{4}{8}$

Which toss results in a heads first

- Let Y denote the toss in which a head appears first.
- Y takes the values 1,2,3, and NIL

Value of Y	Relevant event
1	$\{(HHH), (HHT), (HTH), (HTT)\}$
2	$\{(THH), (THT)\}$
3	$\{(TTH)\}$
NIL	$\{(TTT)\}$

- We say Y is a random variable taking on one of the values 1, 2, 3, and NIL with respective probabilities
  - $P\{Y = 1\} = P\{(HHH), (HHT), (HTH), (HTT)\} = \frac{4}{8}$
  - $P\{Y = 2\} = P\{(THH), (THT)\} = \frac{2}{8}$

Which toss results in a heads first

- Let Y denote the toss in which a head appears first.
- Y takes the values 1,2,3, and NIL

Value of Y	Relevant event
1	$\{(HHH), (HHT), (HTH), (HTT)\}$
2	$\{(THH), (THT)\}$
3	$\{(TTH)\}$
NIL	$\{(TTT)\}$

- We say Y is a random variable taking on one of the values 1, 2, 3, and NIL with respective probabilities
  - $P\{Y = 1\} = P\{(HHH), (HHT), (HTH), (HTT)\} = \frac{4}{8}$
  - $P\{Y = 2\} = P\{(THH), (THT)\} = \frac{2}{8}$
  - $P\{Y = 3\} = P\{(TTH)\} = \frac{1}{8}$

Which toss results in a heads first

- Let Y denote the toss in which a head appears first.
- Y takes the values 1,2,3, and NIL

Value of Y	Relevant event
1	$\{(HHH), (HHT), (HTH), (HTT)\}$
2	$\{(THH), (THT)\}$
3	$\{(TTH)\}$
NIL	$\{(TTT)\}$

- We say Y is a random variable taking on one of the values 1, 2, 3, and NIL with respective probabilities
  - $P\{Y = 1\} = P\{(HHH), (HHT), (HTH), (HTT)\} = \frac{4}{8}$
  - $P\{Y = 2\} = P\{(THH), (THT)\} = \frac{2}{8}$
  - $P\{Y = 3\} = P\{(TTH)\} = \frac{1}{8}$
  - $P\{Y = NIL\} = P\{(TTT)\} = \frac{1}{8}$

Section summary

1. Definition of a random variable and examples