Statistics for Data Science -1

Lecture 9.6: Standard deviation of a random variable

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Learning objectives

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2. Types of random variables: discrete and continuous.
3. Probability mass function, graph, and examples.
4. Cumulative distribution function, graphs, and examples.
5. Expectation and variance of a random variable.

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Remark

The standard deviation, like the expected value, is measured in the same units as is the random variable.

Properties of standard deviation

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1. If $Var(X) = 4$, what is $SD(3X)$? Answer: 6.
2. If $Var(2X + 3) = 16$, what is $SD(X)$?

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1. If $Var(X) = 4$, what is $SD(3X)$? Answer: 6.
2. If $Var(2X + 3) = 16$, what is $SD(X)$? Answer: 2.

Application: family bonus

Sanjay and Anitha are a married couple who work for the same company. Anitha's Diwali bonus is a random variable whose expected value is ₹15,000 and standard deviation is ₹3,000. Sanjay's bonus is is a random variable whose expected value is ₹20,000 and standard deviation is ₹4,000. Assume the earnings of Sanjay and Anitha are independent of each other. What is the expected value and standard deviation of the total family bonus.

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- Let X denote Anita's bonus. Given $E(X) = 15,000, SD(X) = 3,000.$
- Let Sanjay's bonus be Y. Given $E(Y) = 20,000, SD(Y) = 4,000$

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- Let Sanjay's bonus be Y. Given $E(Y) = 20,000, SD(Y) = 4,000$
- $E(X + Y) = E(X) + E(Y) =$ ₹35,000
- $SD(X + Y) = \sqrt{Var(X) + Var(Y)} =$ ₹5,000

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Section summary

- Notion of standard deviation of a random variable.
- Properties of standard deviation.
- Applications.