



IIT Madras


ONLINE DEGREE

Mathematics for Data Science 1

Week-03

Tutorial-01

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Week - 3
Tutorial
Straight line - 2
Mathematics for Data Science - 1

Syllabus Covered:

- General equation of line
- Equation of parallel and perpendicular lines in general form
- Equation of a perpendicular line passing through a point
- Distance of a line from a given point
- Straight line fit

Hello, mathematics students. This is a tutorial for week 3, where we will be doing more straight line concepts problems. Primarily, this is the syllabus that has been covered here. Let us begin with our first question.

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$W_A = 3$; $W_B = 4$

A company provides two kinds of equipment A and B which have work lives of 3 years and 4 years respectively. The values of equipment A and B decrease yearly according to equations $6x + 12.5v_A - 62.5 = 0$ and $6x + 12v_B + 72 = 0$ respectively, where v_A and v_B are the values (in thousands) of A and B respectively, and x is the number of years from the date of purchase.

(a) What are the costs of the equipments? $C_A = ₹5000$, $C_B = ₹6000$

(b) What are the yearly depreciations of the two equipments?

(c) If the company will buy back an equipment after its work life, and Vijay has a requirement of such equipment for 12 years, which kind of equipment will cost him lesser?

$x = 0$
 $12.5v_{A0} - 62.5 = 0 \Rightarrow v_{A0} = \frac{62.5}{12.5} = 5$
 $12v_{B0} + 72 = 0 \Rightarrow v_{B0} = \frac{-72}{12} = -6$

$x \rightarrow \text{no. of years}$
 $y \rightarrow \text{value}$

Slope = $\frac{\Delta y}{\Delta x}$

There is a company with two kinds of equipment, A and B. And they have work lives of 3 years and 4 years respectively. So, work life of A is, let us call it W_A is 3, W_B is 4 years. Further, the values of equipment A and B decrease yearly according to these equations. These are our

equations, where v_A is supposed to be the value of A and v_B is supposed to be the value of B in thousands, respectively, and x is the number of years for which that value is applicable.

So, what are the costs of the equipments? So, the cost of the equipments would be v_A and v_B values when x is equal to 0, that is, when you just bought it, what is the value of the equipment. So, we just take x is equal to 0 and from this we get $0.5v_A - 62.5 = 0$, this would give us v_A is equal to, to indicate that this is the initial time I am going to make it A_0, v_{A0} so yes, this is v_{A0} and that is $62.5 / 12.5$, which is equal to 5.

Therefore, the cost of A, I will call it C_A is rupees 5000. Now, let us work with B. Same thing again, we take x is equal to 0. So, we have $12v_B - 72 = 0$, this will imply v_B again we are calling v_{B0} to indicate the initial cost that would be $72 / 12$ which is equal to 6. So, C_B , the cost of B is rupees 6000. Going further, we are asked what are the yearly depreciations of the two equipments.

So, yearly depreciation basically means how much value is decreasing each year. So, let us look at that. Here, in this case, x is number of years, whereas y is the value. So, what is being asked in a yearly depreciation is the change in y for a unit change in x , which is basically just a slope. Because slope is changing y , Δy by changing x . So, when Δx is equal to 1, Δy is equal to the slope.

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and 4 years respectively. The values of equipment A and B decrease yearly according to equations $5x + 12.5v_A - 62.5 = 0$ and $6x + 12v_B - 72 = 0$ respectively, where v_A and v_B are the values (in thousands) of A and B respectively, and x is the number of years from the date of purchase.

(a) What are the costs of the equipments? $C_A = ₹5000, C_B = ₹6000$

(b) What are the yearly depreciations of the two equipments?

(c) If the company will buy back an equipment after its work life, and Vijay has a requirement of such equipment for 12 years, which kind of equipment will cost him lesser?

$x = 0$
 $12.5v_{A0} - 62.5 = 0 \Rightarrow v_{A0} = \frac{62.5}{12.5} = 5$
 $12v_{B0} - 72 = 0 \Rightarrow v_{B0} = \frac{72}{12} = 6$

$x \rightarrow$ no. of years
 $y \rightarrow$ value
 $y = mx + c$

Slope = $\frac{\Delta y}{\Delta x}$

$5x + 12.5v_A - 62.5 = 0$
 $\Rightarrow 12.5v_A = -5x + 62.5$
 $\Rightarrow v_A = -\frac{5}{12.5}x + 5$

$-0.4 \times 1000 = -400$

5000
 -4600

 400

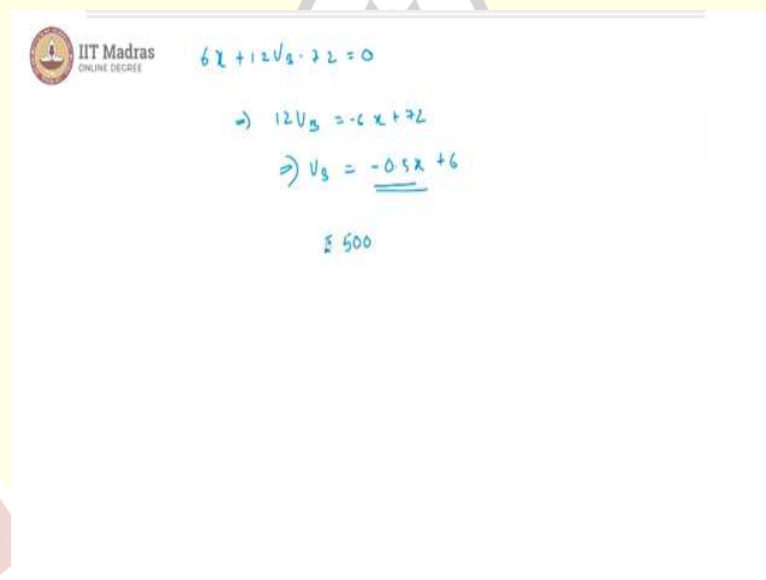
So, we can find this by just finding the slope for each of those two linear equations. And for the slope, we convert our equations to the $y = mx + c$ form, then the m is going to be the slope.

So, one equation is $5x + 12.5v_A - 62.5 = 0$. This would indicate that $12.5v_A = -5x + 62.5$. Going further then, it will have $v_A = -5x / 12.5 + 62.5 / 12.5$, we had already seen it to be equal to 5.

So, that is equal to $-0.4x + 5 = v_A$. So here, we are, our m in the equation is basically -0.4 . So, this is the reduction in one year, -0.4×1000 because we are taking everything in thousands, so, that is basically -400 . So, this is the depreciation, 400 is the depreciation every year for the company one, we can also verify this by looking at the values of v_A for year one.

So, when $x = 1$ we have $5 + 12.5v_A - 62.5 = 0$, this gives us $v_A = 57.5 / 12.5$ which is equal to 4.6. So, v_A was originally 5, that means it was originally 5000 rupees and after 1 year it became 4.6 which is 4600 rupees. So, the difference is 400 rupees. So, that is the yearly depreciation for the first equipment.

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$6x + 12v_B - 72 = 0$
 $\Rightarrow 12v_B = -6x + 72$
 $\Rightarrow v_B = \underline{-0.5x + 6}$
 $\text{₹ } 500$

Now, let us look at the second equipment now second equipment the equation was $6x + 12v_B - 72 = 0$. Again, if we put this to the $y = mx + c$ form, the slope intercept form we will be getting first we have to do $12v_B = -6x + 72$. This indicates $v_B = -0.5x + 6$, thus -0.5 is the slope here. Which means 500 rupees is the yearly depreciation.

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are the values (in thousands) of A and B respectively, and x is the number of years from the date of purchase.

(a) What are the costs of the equipments? $C_A = ₹5000$, $C_B = ₹6000$

(b) What are the yearly depreciations of the two equipments?

(c) If the company will buy back an equipment after its work life, and Vijay has a requirement of such equipment for 12 years, which kind of equipment will cost him lesser?

$$x=0 \quad 12.5V_A - 62.5 = 0 \Rightarrow V_A = \frac{62.5}{12.5} = 5$$

$$12V_B - 72 = 0 \Rightarrow V_B = \frac{72}{12} = 6$$

$x \rightarrow$ no. of years
 $y \rightarrow$ value

Slope = $\frac{dy}{dx}$

$$y = mx + c$$

$$-0.4 \times 1000 = -400$$

$$5x + 12.5V_A - 62.5 = 0$$

$$\Rightarrow 12.5V_A = -5x + 62.5$$

$$\Rightarrow V_A = -\frac{5}{12.5}x + 5$$

$$V_A = -0.4x + 5$$

In the last part, they said that the company will buy back the equipment after its work life. And Vijay has a requirement of such equipment for 12 years. Which kind of equipment will cost him lesser.

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Case A $2y + 3y$

$$\begin{array}{r} ₹5000 + ₹1200 \\ - ₹400 \quad y_1 \\ - ₹400 \quad y_2 \\ - ₹400 \quad y_3 \\ \hline ₹3800 \end{array}$$

12 years with A

$$5000 + 3(1200) = ₹8600 - 3800 = ₹4800$$

Case B $4y + 4y + 4y$

$$\begin{array}{r} ₹6000 + 2000 + 2000 \\ - ₹500 \\ - ₹500 \\ - ₹500 \\ \hline ₹4000 \end{array}$$

$₹10000 - 4000 = ₹6000$

So, in the case of the first equipment, let us call it case A, and here let us have case B to consider. And in case A the initial cost was 5000 rupees and each year there is a decrease of 400 rupees. So, in first year we lose this much, in the second year we lose another 400 rupees and at the end of the third years, there is a loss of another 400 rupees. And we are aware that 3 years is a worklife for A, whereas for B it is 4 years. This is to say that at the end of 3 years, the value of the machine is 3800 rupees.

So, if now, Vijay buys the equipment afresh, then and the company is buying back this 3800. All that Vijay needs to spend now is rupees 1200 and this way he gets an additional 3 years. So, with 5000 he got 3 years and now another 3 years this way. So, in order to get 12 years with equipment A, the total money that Vijay will require to spend is 5000, which is the initial first 3 years and from then on 3 years plus 3 years plus 3 years because it is totally 12 years.

So, 3 times 1200, that is rupees 3600 in case of A, whereas in B, B is more expensive. So, we have 6000 and every year there is a loss of 500 rupees in value and this is required to be done 4 times because the work life for B is 4 times. So, we are effectively subtracting 2000 rupees from the original value. So, we have 4000 at the end of it, which means for the first 4 years there is an expenditure of 6000 but then, for the remaining 8 years, there has been only 2000 each.

This is so because the product's value is already 4000 rupees and in order to get a new version of equipment B, Vijay only has to spend 2000 rupees. So, total expenditure in this case is going to be 10,000 rupees because $6000 + 2000 + 2000$. Here, we are not supposed to forget one thing though, that is the end of, after these 3 years, 3 years pass, at the end of 12 years, he can sell it off for 3800. So, we are supposed to further subtract 3800 here and likewise here, we can sell it off for 4000. So, here we get rupees 4800 whereas, here we get rupees 6000. So, the expenditure is clearly lesser for A. So, A would be the good choice for Vijay.