IIT Madras

ONLINE DEGREE

Mathematics for Data Science 1
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Week - 04
Tutorial – 06

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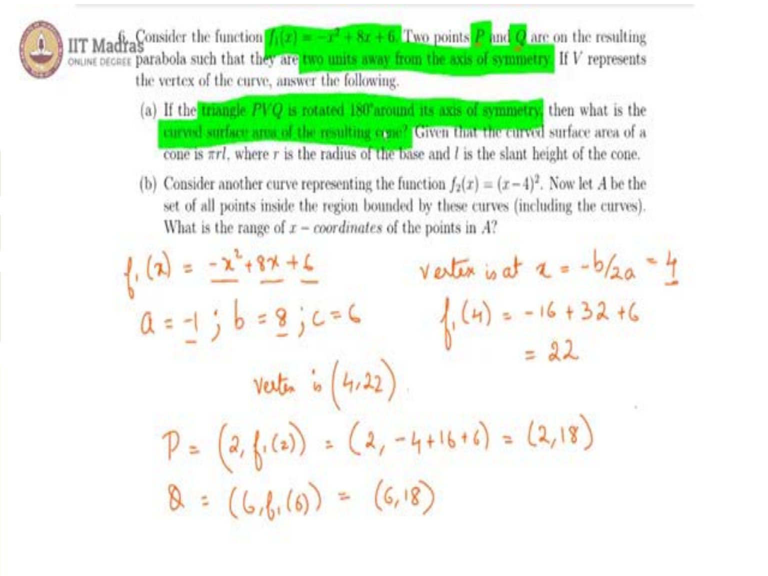

In our sixth question, we are given this particular quadratic function $f(x) = -x^2 + 8x + 6$. And we are told that two points P and Q, which are on this parabola such that they are two units away from the axis of symmetry. So, let us try to find out what the axis of symmetry is for this parabola. Our equation is $f_1(x) = -x^2 + 8x + 6$. And that would mean, in a standard form $a = -1$, $b = 8$, and $c = 6$.

And that would give us the vertex is at $x = \frac{-b}{2a}$, which in our case will then become $a = -1$, $b = 8$, so we will get 4. And the functions value at 4 is $f_1(4) = -(4)^2 + 8 \times 4 + 6 = 22$. So, the vertex is (4, 22). Further, we are told that P and Q are two units away from the axis of symmetry. So, the axis of symmetry is along $x = 4$, which means P and Q will be at $x = 2$ and $x = 6$, $4 - 2$ and $4 + 2$.

So, these points are going to be P$(2,\ f_1(2)) = (2, -4 + 16 + 6) = (2, 18)$. And the point Q is going to be $P(6,\ f_1(6))$ and from symmetry we know that this is also going to be 18, so $P(6,$ 18). And it is now told to us that the triangle PVQ is rotated 180 degrees about its axis of symmetry and we are being asked the curved surface area of the resulting cone. So, let us look at what this looks like.

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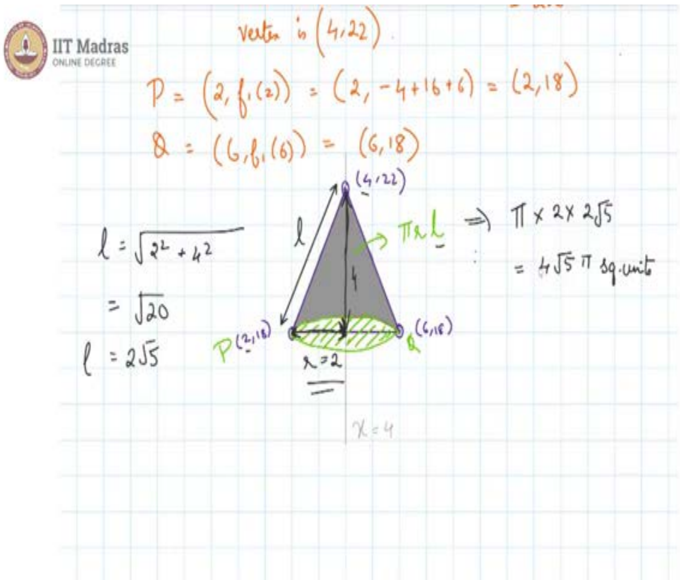

So, now let us suppose that this point here, let us call this our (4, 22), in that case 18 is 4 units below, so this will be the horizontal line passing through 18 and 2 will be here. So, (2,18) is here and this gives us (6,18) is here. This is (2,18) and this is (6,18). And that gives us a parabola which looks something like this, obviously a smoother curve than I have drawn, but something like this. And the triangle we are interested in is an isosceles triangle, which looks roughly like this.

This is the triangle that is being rotated 180 degrees about its axis of symmetry and its axis of symmetry is $x = 4$. I am erasing the parabola in order to focus on the triangle alone. If this triangle were to be rotated, this point which is our P, this is our Q, this point P basically goes around and reaches Q, whereas Q comes around and reaches P. And in this way, we have a cone in our hands and we want the curved surface area and that would be this region and the base circle is this flat surface below this is the base circle.

And we are interested in the curved region whose surface area is given to be $\pi r l$. So, what is r, r is the radius of the base circle. Which is basically then this quantity, this is r, which we can tell is $4 - 2$, so it is 2. And what is l over here, that is the slant height, which is basically this height, that height can be obtained as the hypotenuse of this base radius and height here, which is as we can see 4 units. So, $l = \sqrt{2^2 + 4^2} = \sqrt{20} = 2\sqrt{5}$.

So, we have $r = 2$ and $l = 2\sqrt{5}$, this gives us a curved surface area is $\pi \times 2 \times 2\sqrt{5} = 4\sqrt{5}\pi$ square units.

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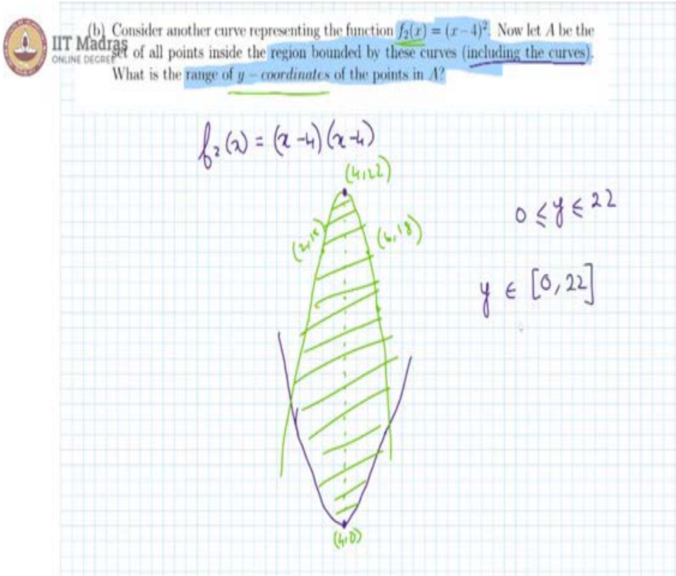

For the part B of our question we have another curve which is also quadratic and whose roots are basically 4 repeated. So, $f_2(x) = (x-4)(x-4)$. So, x being equal to 4 makes $f_2(4)$ =0. So, therefore, our root is 4 and it is repeated because coming twice here. So, let us now try to look at what they are asking. Now, let A be the set of all points inside the region bounded by these curves, including the curves. So, we are saying the region bounded by these curves and including the curves.

And they would like the range of y coordinates of points in it. We know already that (4, 22) is the vertex for our previous parabola. And it also passed through (2, 18) and (6, 18). And about this new parabola, the $f_2(x)$, we know that 4 is repeated root so there is only 1 root and therefore, at 4, that is 22, this would be 21, this is 20, this is 19, 18 17, 16, 15, 14, 13, 12, 1, 11, 10, 9, 8, 7, 6, 5, 4, 3, 2, 1, and 0. So, this is going to be the repeated root and the vertex of our other parabola.

So, if one parabola is like this, f_1 had negative x^2 coefficient so it is a downturned parabola, then the other parabola $f_2(x) = (x-4)^2$ is an upturned parabola which is going to be something like this. So, these curves are going to intersect in some way this way. And we are interested in the range of $y-coordinates$. So that would be, what are all the $y-coordinates$ possible in this region.

So, if this is the region we are looking at, then clearly this is the upper bound of our $y-coordinates$ and this is a lower bound. So, $y-coordinates$ in our region range between 0 and 22. And they said including the curve, so 0 is also included, 22 is also included, so we can write the same thing as $y \epsilon [0, 22]$.