INDIAN INSTITUTE OF TECHNOLOGY MADRAS

सिद्धिर्भवति कर्मजा

IIT Madras

ONLINE DEGREE

Mathematics for Data Science 2
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Week 03 - Tutorial 02

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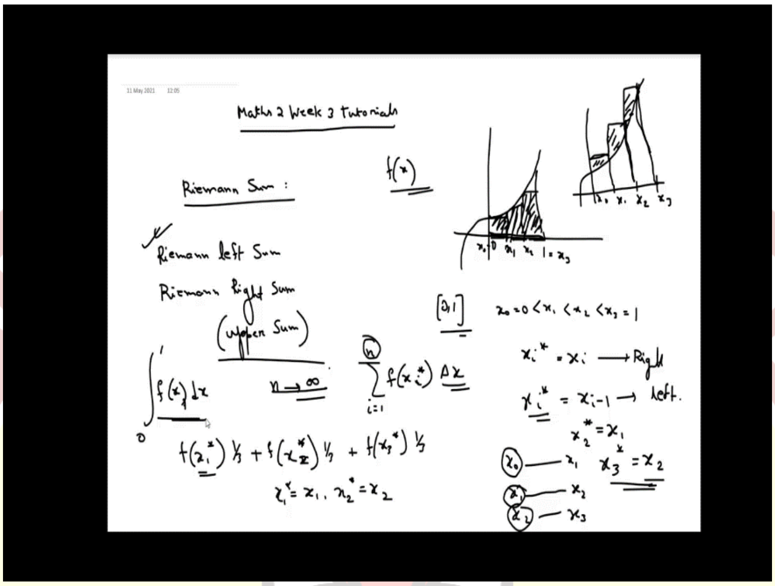

Hello everyone, welcome to the second tutorial video of week 3. So, in week 3 we have learnt a concept about Riemann sum. So, what we do in this case, we are basically, suppose we are given a function, so we have a curve corresponding to that function. Suppose I am drawing a rough picture, suppose this is my graph look like and I want to calculate the area under this curve bounded by this x axis, suppose from 0 to 1. In this interval I want to calculate this area under this curve.

So, what we do, we generally take this interval and we divide it in some parts, suppose here I am dividing it in 3 parts, so this is my point x_1, this is x_2 and 1 is my x_3 and x naught is 0. So, I am dividing my interval 0, 1. This is my interval 0, 1, I am dividing it in 3 parts, $x_0$which is 0, strictly less than x_1, strictly less than x_2, strictly less than x_3 which is 1. So, our interval is divided in 3 parts.

Now, in this 3 part, I can take two kind of events, one is Riemann left sum and one is Riemann right sum or sometime it is called upper sum. So, in this interval, we take this $\sum_{i=1}^{n} f(x_i^*)\Delta x$, what is x_i^*, I will say in a minute and Δx. So, and this i is going from 1 to n. Suppose I am dividing my interval in n parts, so here which is 3. So, $f(x_1^*)$ and Δx is nothing but the length of each interval, each sub interval.

So, if we divide in equal sub interval, then our length will be, so here total length is 1, so if I divide in 3, so it will be, Δx =1/ 3 for each case. So, this is $f(x_2^*)\frac{1}{3} + f(x_3^*)\frac{1}{3}$ This is our Riemann sum. Now, what is this x_1^*, x_2^* and x_3^*? So, we can take this x_i^*, if we take x_i^* as $x_i - 1$, that is the left point.

So, our x, this interval, this first sub interval is from to x_1 and if I take this x naught to be our x_i^*, then and the second sub interval is from x_1 to x_2 and our x_2^* is in that case x_1 and our x_3^* is x_2. If I take this convention, so I am taking the left point of each sub interval as you can see here.

So, this is my x_3^* which is x_2. So, if I take my, the left point of this sub interval, then I call it Riemann left sum, if I take the right point of this sub interval, that is my x_1^* is x1, my x2 star is x_2, that is my x_i^* is x_i, then we call it upper sum or right sum. So, this is right sum and this is left sum.

So, in this, so we can approximate, so if we take x_0, then this will be the area which is less than the actual area as you can see in the picture. It is less than the actual area. Now, if we take the right sum, then we will get… let me draw another picture here. So, this is my x_0, x_1, x_2 and here somewhere x_3. So, this is the curve. So, if I take the right sum, then I am getting this as my area as you can see here.

So, these pictures are really rough pictures. So, it will give you can idea that we are getting this much error. Now, as the number of sub interval increases, this error will become less and less and we will, we can approximate the area under this curve, that is the concept of Riemann sum and as this n, that is the number of interval, as this n tends to infinity, we can come closer and closer to the area under this curve and that is nothing but the integration of that function which is $\int_0^1 f(x)dx$.

So, as n tending to ∞, this Riemann sum will tends to this expression which is $\int_0^1 f(x)dx$. Now, let us try to visualize the whole thing in a GeoGebra using some graph of a function and try to see how this Riemann sum approximates the area under the curve or how it calculates the integration of that function.

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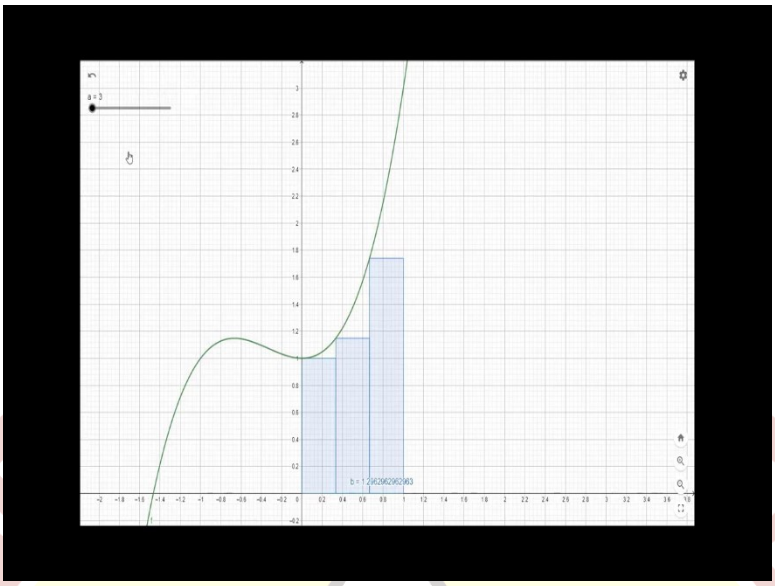

So, we are using the same function x cube plus x square plus 1 as we have used in tutorial 1. So, this is the graph of the function and we will try to calculate the area under this curve in the interval 0 and 1. Now as you can see, if we consider, so, this a will represent the number of intervals in which we are dividing this interval, this whole interval 0 and 1 and let us try to calculate the left sum.

So, suppose I am dividing a 3 interval, 3 sub intervals then you can see the area under the curve which we calculated, calculate by Riemann sum will be this. So, there are sum error as we can see. The white area under the curve is basically the error which we come across. So, if we take our number of sub intervals to be 3.

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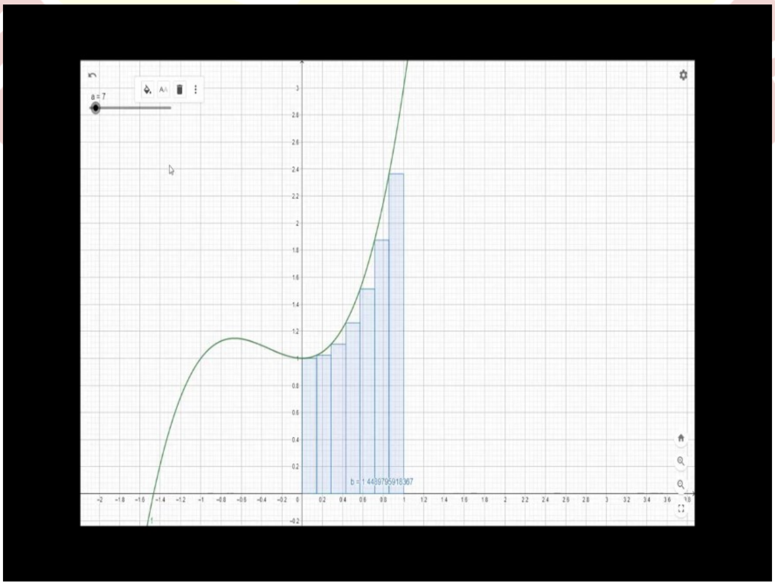

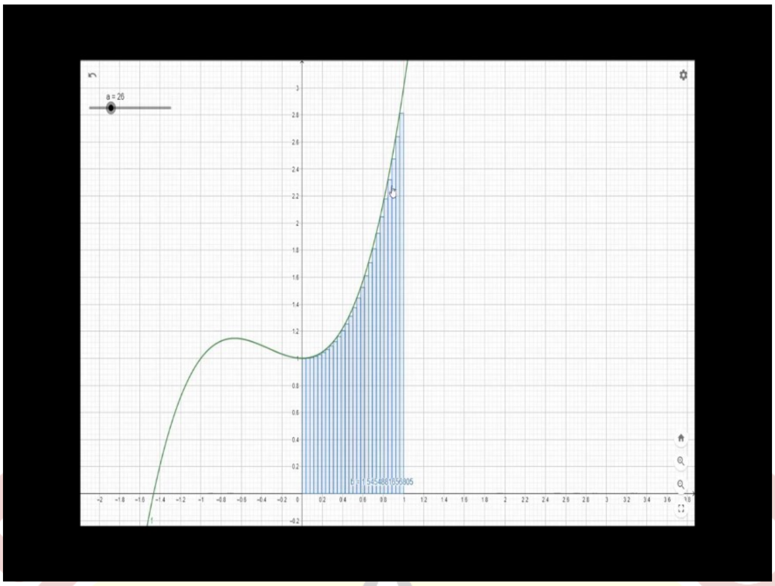

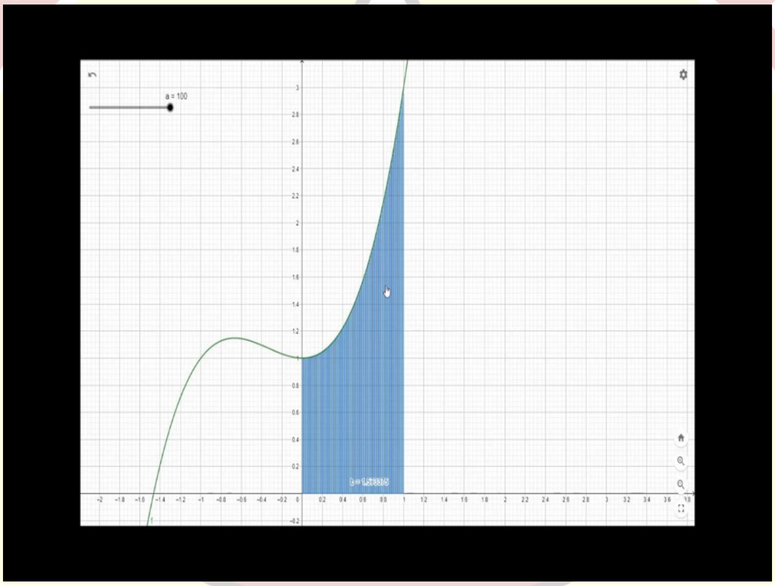

Now, if we increase the sub interval, you can see the error will be less. So, if $a = 7$, we can get this kind of picture, so you can see the error is decreasing and if we increase the number of sub interval again, suppose I am taking a to be 26 now, so you can see the error is very much less and we can get, we are getting closer to approximate the area under this curve. Using this left sum.

So, now if I increase this further using this animation so, as you can see the number of sub interval increases and it go to a, the number of sub interval to be 100 and you can see we can nearly approximate the area, there is still some gaps. So, it clearly it is much closer to approximate the area under the curve. So, I can show the animation again, so the number of sub interval decreases now and see the error increases and that is how we can get closer approximation of area under this curve using left Riemann sum.

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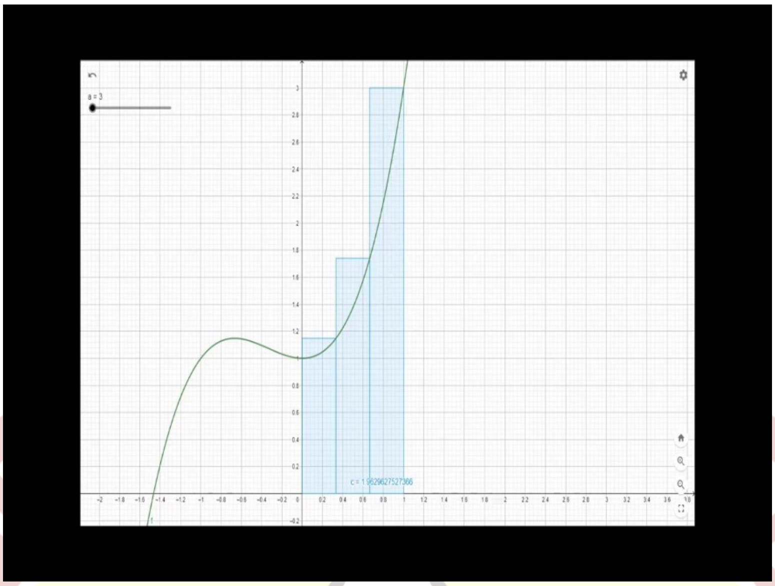

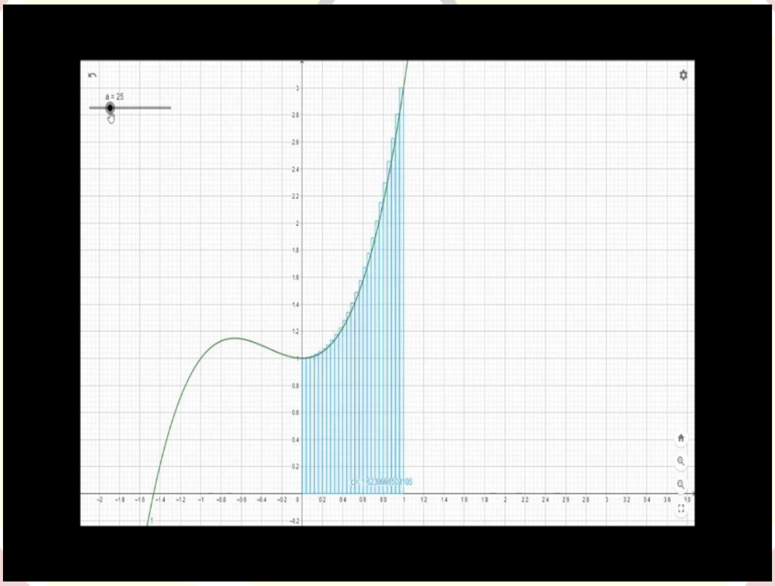

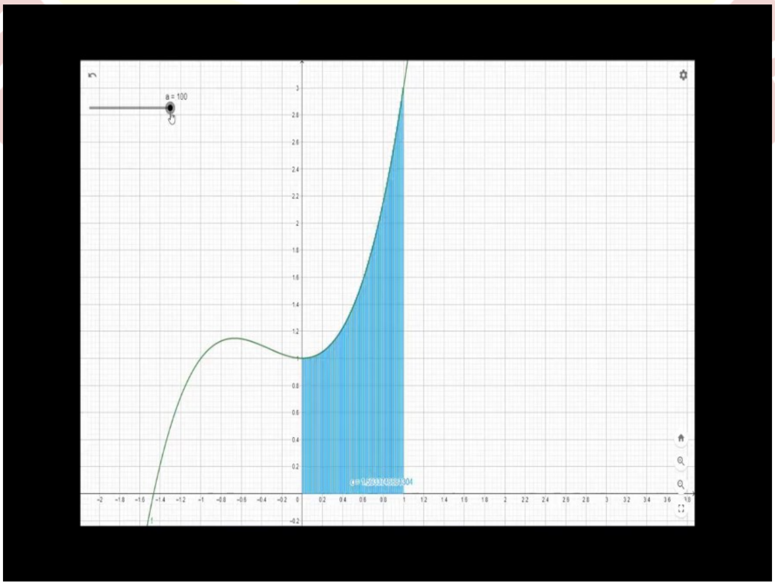

Now again we come back to 0 and now we are going to see the right Riemann sum. So, this is left. So, this is my, when $a = 3$, this is our right Riemann sum. So, you can see we can, we will approximate from the above so that is why it is called upper sum. So, we will approximate from the upper side of the curve. So, here we are calculating some more area when using 3 sub interval. So, this area is more than the area under the curve.

And as we increase our a, that is increase our number of sub interval, you can see we can come closer and closer to it from the upper side. So, you can see the animation how it comes closer and closer as the number of sub interval increases. And again, the number of sub interval decreases which you will, the error will increase now. So, it will give you a visual representation how the Riemann sum work and how the area under the curve is calculated using Riemann sum or integration. Thank you.