IIT Madras

ONLINE DEGREE

Mathematics for Data Science 1
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Lecture - 18
Representation of a Line-1

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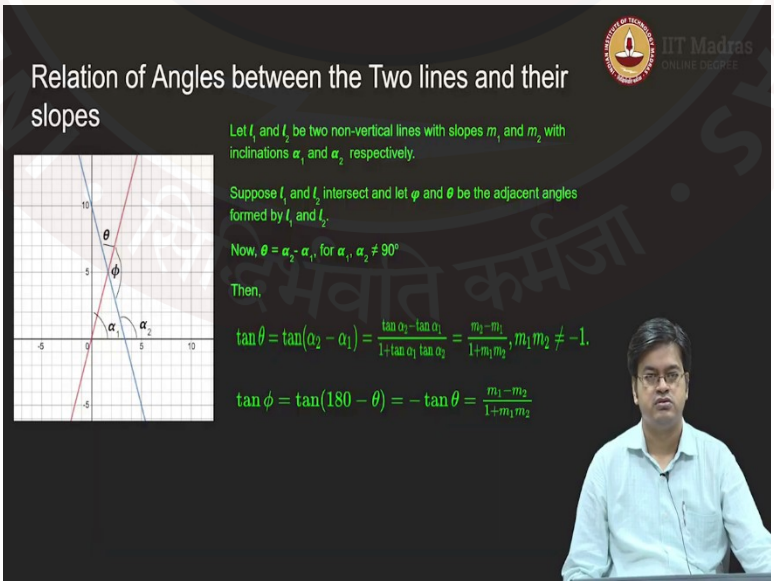

So, what we have seen so far is, what is a relation of the slope with respect to line and we have exploited certain how we can use the slope to determine whether the lines are parallel perpendicular. And, if I know the slope of the line then how will I find a slope of two non - vertical lines, then how will I find the relation between angles and other properties.

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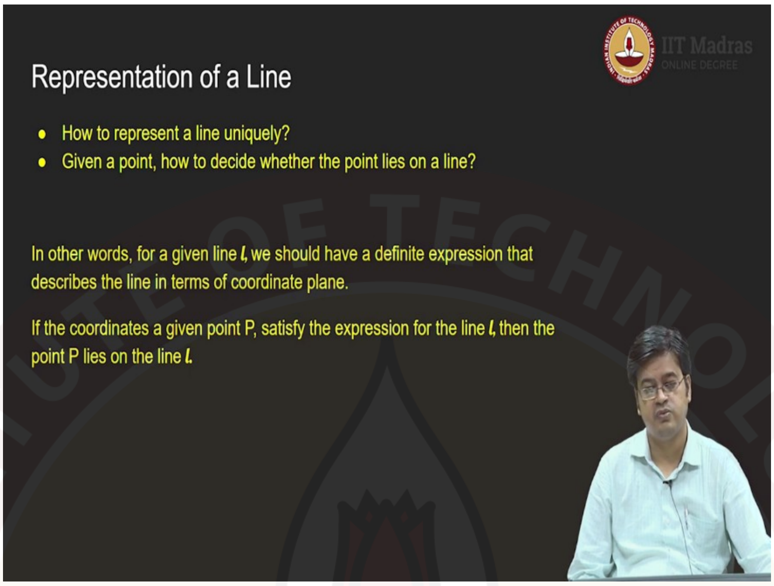

Now, we will come to the Representation of a Line, as we have already seen slope cannot represent a line uniquely. So, what is it that, that is required for representing a line uniquely? So, this raises two questions, how to represent a line uniquely? And the second question is, given any point of how will you decide whether that point lies on the line or not?

So, in order to answer these two questions, let us take the first question first and rephrase it. So, if I want to represent a line uniquely, then what I need to figure out is, I need to figure out a condition or a definite expression which will describe the line in terms of its coordinate plane. So, for a given line l I should be able to find a definite condition or expression which describes the line in terms of coordinate plane. That is in terms of the coordinates or to be more precise what should be the condition on the coordinates in order to describe the line l.

If I can understand what is this condition then the second question is automatically answered because if the coordinates of P are given to me and they satisfy the condition or expression for the line l then they must lie on the line l otherwise they do not lie on the line l, then it is just a simple job of checking whether that condition is satisfied or not. So, with this in mind we will try to answer the first question that is how to represent a line uniquely?

Now, what kind of lines we have seen so far? We have seen lines which are similar to X - axis, lines which are similar to Y - axis; those are typically horizontal and vertical lines.

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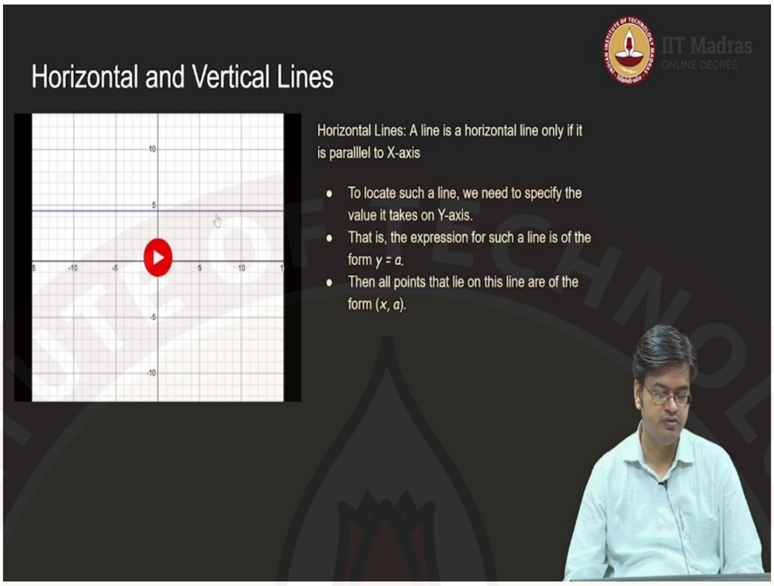

Let us first understand, what is a horizontal line. So, a line is said to be a horizontal line if it is of this form, now this line can be infinitely many. So, you can have infinitely many horizontal lines as can be seen from the video. Now, how to represent this line uniquely is my question. So, let us say I need to find this line or the condition for this line, how can I find the condition for this line?

So, let us first define this line as a horizontal line and let us say horizontal line is a horizontal line if and only if it is parallel to X - axis, this is our definition of a horizontal line. Now, if I want to specify this line uniquely what do I need to know? I need to know the distance of this line or the location of this line from X - axis, that is I need to know the y coordinate of this line you can see here. So, I want to locate this line or the value that it takes on Y - axis if I want to specify this line.

Let us say this value is given to be a then I know it is a horizontal line. So, all points will lie at a same distance from X - axis therefore, all points will satisfy the condition $y=a$. You take any point on this line it will satisfy the condition $y=a$.

So, in case of horizontal lines what I have done is I have identified the condition that is $y=a$. So, what will be the condition on points? The points will be of the form (x,a), x can be any value, but the y coordinate of that point will be fixed that is a. In a similar manner we can consider vertical lines.

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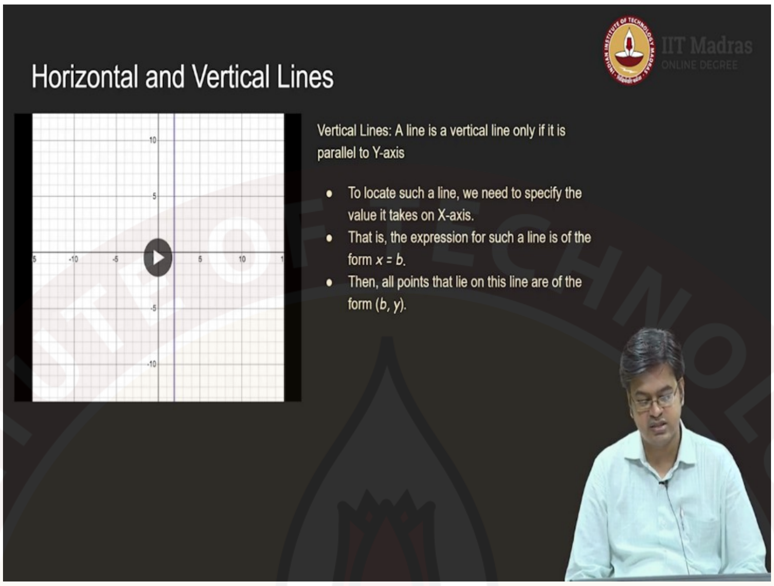

So, what is a vertical line? You can see this in this image and in this video, we can see all these kind lines are called vertical lines. So, how will I identify these vertical lines? First, I will define the vertical lines a line is a vertical line if and only if it is parallel to Y - axis. Now, to specify the location what do I need? To locate a line, I need to know the distance of this line from Y - axis ok. So, that essentially means what value it takes on x coordinate or X - axis.

So, how will you identify this? You just need to identify the one point in this particular line let us say this is the point and I need to see what is the distance of this point from X - axis, if that is b, then all points of the form (b, y) will be lying on this line; all points of the form (b, y) will be lying on this line. And therefore, the equation of the line the expression for the line will have a form $x=b$.

I mentioned the all points will be of the form (b, y). So, if I get two points where the y coordinate where the x coordinate is fixed and I know it is a line then I know it is parallel to Y - axis or it is a vertical line right. In a similar manner the other one is parallel to X - axis and it will be a horizontal line. Let us make it more crystal clear by solving one example.

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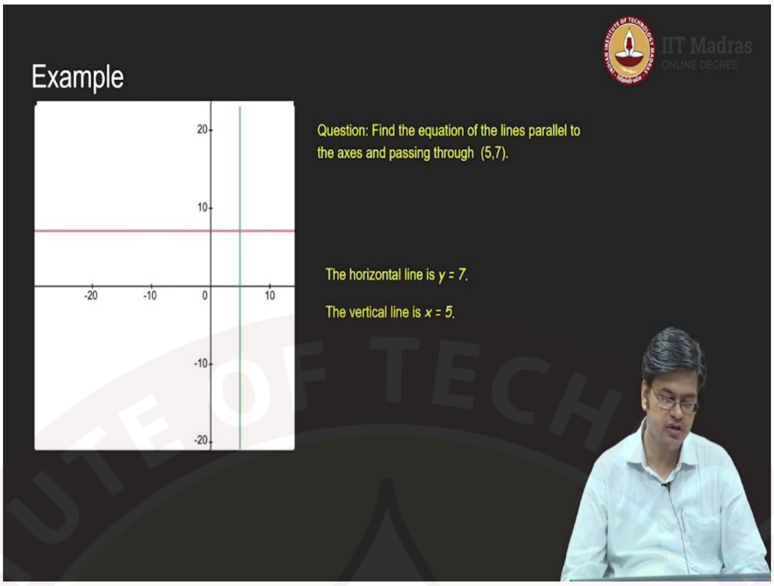

So, here is an example where a question is given to you want to find the equation or expression for the lines parallel to the axis and passing through point $(5,7)$. Now, the lines are passing through point $(5,7)$ and it is also given that they are parallel to axes. So, a line which is parallel to X - axis is known as horizontal line, a line which is parallel to Y - axis is known as vertical line. So, essentially this question asks you to find one horizontal line and one vertical line.

So, let us go to the coordinate plane, this is the coordinate plane let us locate the point $(5,7)$, it will be somewhere here. Now let us first focus on identifying the horizontal line. What is a horizontal line? A line which is parallel to X - axis is a horizontal line. So, a line which is parallel to X - axis, then what do I need to know? Its distance from X - axis, the distance is 7 according to this particular expression because $(5,7)$ is a point on that line.

So, the distance is 7 so, the line must appear somewhere here, now further the next question is I want to find a vertical line that passes through point $(5,7)$. So, now I need to know the distance of a line from X - axis. So, I will locate point 5 over here and all points on the line on that particular line will be of the form $(5, y)$. So, $(5,7)$ will also fall on that line. So, this is the line; so, this is how we will find the lines.

Now what are the typical equations of the line? So, the horizontal line will be $y=7$ and the vertical line will be $x=5$. This is how we will study horizontal and vertical line. So, what is a vertical line? Vertical line has inclination as at 90 degrees, and therefore, the slope of this line is not defined remember this in mind.

Another point which is horizontal line it never intersects actually X - axis, but the inclination of this line with respect to X - axis is 0 degrees therefore, it will have a slope 0. So, we have eliminated the cases where the slope does not exist or slope is 0, now we need to identify similar kind of expressions for lines which are not vertical. So, let us go further and identify such expressions.

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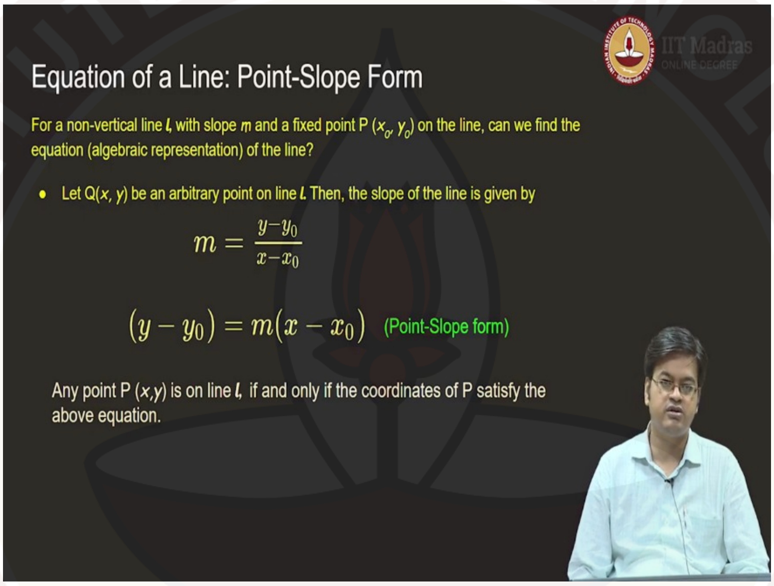

So, here as we already know that slope cannot uniquely determine line, then the question is which slope if I give you some more information can you determine the line? So, here in this case what we are identifying is we are giving a point and giving a slope and then we are asking a question can we solve this problem or can we find a unique expression for a given line? So, the question is for a non-vertical line l vertical line we do not have to consider because the slope is not defined. So, for a non-vertical line l with slope m and a fixed-point P (x_0, y_0) on the line can we find the equation or the algebraic representation of a geometric object that is line is the question.

So, here what are the things that we know? We know slope and we know a point on the line. So, in order to answer this question, we know that two points uniquely determine a line. So, let us take another point Q (x, y). I do not know the coordinates of these points, but I assume that this point lies on line l. Now, I know from the definition of slope that I have defined change in y by change in x the slope of a line is given by. So, what are the two points now? Q and P.

So, change in y will be $y-y_o$ and change in x will be $x-x_0$. So, I know $m=\frac{y-y_0}{x-x_0}$, this is what I know from my definition. It has nothing to do with $\tan\theta$ even if you have it you can find out what is $\tan\theta$, but since nothing is known in specific we cannot find the $\tan\theta$, but $\tan\theta$is anyway given to you in terms of slope.

So, now I have $m=\frac{y-y_0}{x-x_0}$. So, how will I find the condition on x and y? Just cross multiply this $x-x_0$, you will get an expression which is $y-y_0=m(x-x_0)$. This condition uniquely identifies my line, there cannot be any other line satisfying this condition.

So, therefore, any point that lies on this particular line that is P (x, y) that lies on this particular line, it must satisfy the condition that is given here. This form of expression is called point slope form. So, this is a point slope form of equation which essentially says that give me one point and slope of the line I will give you the equation of a line.

The beauty is the geometric object now can be represented in terms of the equation, initially when we started, we tried to represent a point which is a geometric object in terms of coordinate plane and the coordinates of the point. Now we are giving infinite set of points having certain condition that is a geometric object of line how you can represent it algebraically using the equation of a line. So, this is point slope form. Let us try to see how we can use the point slope form in our problem solving.

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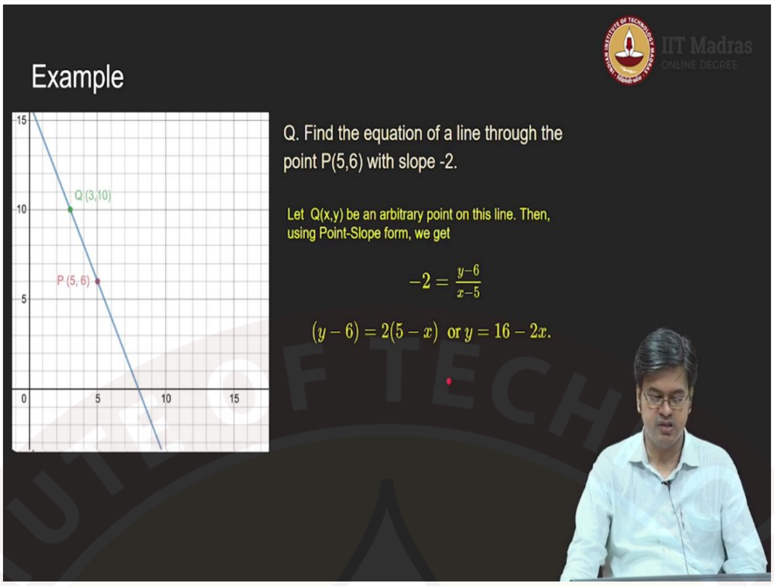

So, now, I have been asked to find the equation of a line which passes through point $(5,6)$ and has slope of -2. Here the interesting thing is slope is negative. So, let us identify the point $(5,6)$ on the coordinate plane and now I want to identify the line that passes through this with slope of -2. So, now I have a formula for point slope form, I can use that formula and I can straight away derive it for let us try Q (x,y) is an arbitrary point.

So, I need two points to identify a line. So, Q (x,y) with the arbitrary point on this line then using point slope formula we simply substitute $-2=\frac{y-6}{x-5}$, slightly rearrange the terms; what you will get is $y-6=2(5-x)$. If you simplify this you will get the expression $y=16-2x$.

Now, let us try to see, if I want to know this value of x what point what value of y will satisfy this equation. Let us put x is equal to 3 here if I put x is equal to 3 here then I get y is equal to 10 after simplifying this I will get $y=10$.So, that means, the point $(3,10)$ should lie on this particular line. So, let us see that $(3,10)$ is here and now you know from basic geometry that two points uniquely identify a line. So, you can just draw a line using your ruler passing through these two points this is the line that we are expecting.

So, the question did not ask you to draw a graph, but drawing graph always verifies whether you have found a correct answer or not. So, it is better to cross check using graphs. So, the answer to the question is the equation of the line passing through point $(5,6)$ and slope -2 is

$y = 16 - 2x$. Now, suppose somebody decides not to give me slope and somebody says that now you have been given only two points; can you find the equation of line?

The answer is; obviously, yes because given two points I can always determine the slope right for example, in our earlier case when we defined slope I need to figure out what is change in y and what is change in x using these two points and that will give me slope to be equal to -2. And therefore, I can always use this formulation to find the equation of the line, but you can use this knowledge and derive another form that is equation of a line two - point form.

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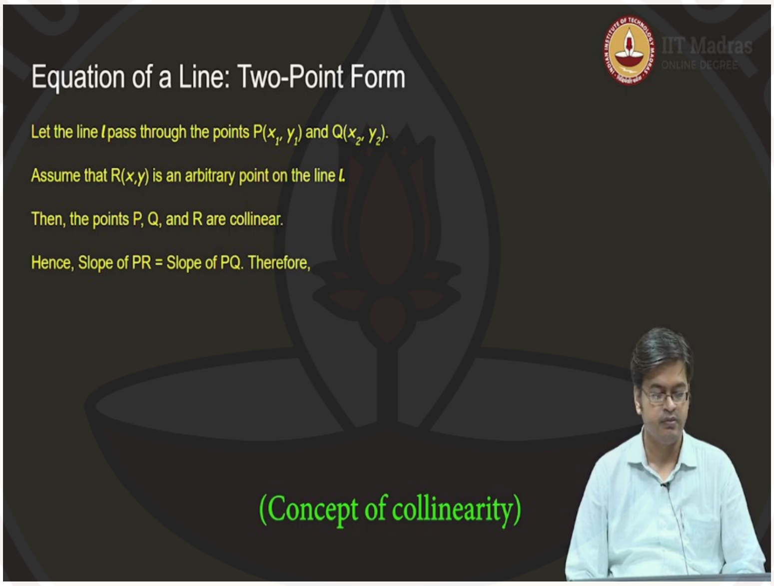

So, given two points the question is can you determine the line uniquely which should be possible and through our basic knowledge of geometry we already know that two points uniquely determine a line, now we will see that in our coordinate geometry. So, the assumption is let the line l pass through points P and Q with coordinates (x_1, y_1) and (x_2, y_2).

To start with this, I will take another point R which is arbitrary point because I want to find the condition in terms of coordinates. So, whenever I want to find the equation of line I will start with an arbitrary point. So, R (x, y) is an arbitrary point on the line l. Now, look at these three points P, Q, and R they all lie on one line therefore, the points P, Q, and R are collinear yes; so, points P, Q, and R are collinear.

Therefore, suppose I consider only these two points P and R, using these two points P and R, I can easily figure out the slope of a line. If I consider points P and Q, I also know the slope of a line; now because these points are collinear what can you say about slope of line PR and slope of line PQ, both must be same or equal? So, slope of PR is equal to slope of PQ because they are collinear.

So, if this is the case, then what is slope of PR? You can easily figure out P is this $(x_1, y_1)$and R is (x, y). So, the slope of PR first you consider change in y, $y-y_1$ upon change in x that is $x-x_1$that is slope of PR. What is slope of PQ? PQ is(x_1, y_1) and (x_2, y_2). So, y change in y is y_2-y_1 and change in x is x_2-x_1.

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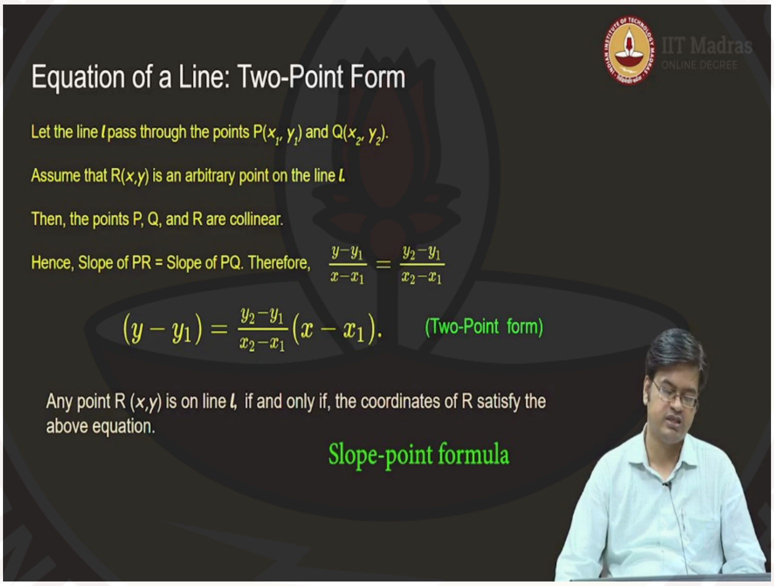

Therefore, I will get the equation of this form $\frac{y-y_1}{x-x_1} = \frac{y_2-y_1}{x_2-x_1}$.

So, again if you look at it closely this particular thing is nothing, but the slope of a line m and we are doing things which are very similar to slope point form. But instead of counting it explicitly we are counting it as a ratio and then you rearrange the term and you will get this expression because you just take this denominator on the other side and you will get this expression.

Now, this line is again uniquely characterized and therefore, any point that lies on this line must satisfy this condition. So, if your point is R lies on this line then it must satisfy this condition and this form is called two - point form. So, remember these are the formulas that we are deriving; first was slope line formula, second is two - point form.

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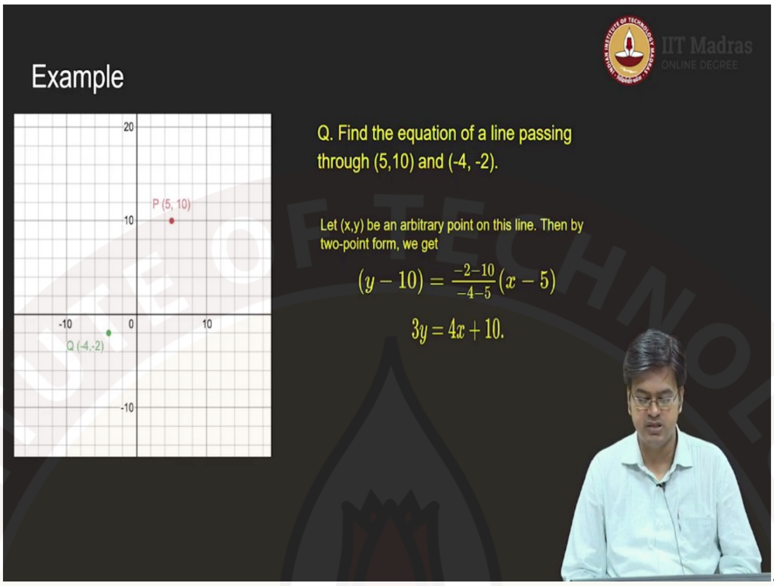

Let us understand this formula better by solving some examples. So, let us take one example where I want to find the equation of a line that is passing through two points $(5,10)\wedge(-4,-2)$. Let us identify these two points on a coordinate plane $(5,10),(-4,-2)$. I want to find the equation of this line.

So, I will use another point Q which is an arbitrary point and it has a coordinate(x,y), I will use the two - point form. So, using two- point form what should I get? So, I am taking, this point P. So, $(y-10)=\frac{-2-10}{-4-5}(x-5)$. So, always remember this order does not matter I can always start with this as well. So, change in y is $10-(-2)$ and change in x is $5-(-4)$ in both cases my answer to this particular fraction will be $\frac{12}{9}$ which is $\frac{4}{3}$.

So, it does not matter whether you take this as (x_1,y_1) or you take this as (x_1,y_1), you will always get the same answer. So, if you simplify this you will get the expression of a line because as I mentioned the slope was $\frac{4}{3}$. So, you just simplify this you will get the expression of a line$3y=4x+10$. So, this will be the line that is passing through these two points.