Lesson 1: Introduction to Portfolio Construction

Introduction

- Introduction to portfolio management
- Expected returns and risk for a portfolio
- Portfolio construction with two-security case
- Portfolio construction with *N*-security case
- Risk diversification with portfolios

Portfolio Construction with Two Securities: Expected Returns

Portfolio Construction with Two Securities

What is a portfolio and why to invest in it?

- What happens to the (1) expected return and (2) risk when you combine two securities (or multiple securities)?
- What is diversification?
- Investing in mutual funds and index investing
- What is the difference in risk of investing in Nifty-50 vs. HDFC?

Expected Returns for Two-Security Case

Consider a portfolio constructed from two-security case with actual return distributions as R_1 and R_2

- The proportionate amounts invested in these assets are w_1 and w_2, where w_1+w_2 = 1
- Please also remember that expected returns E(R_1) = $\overline{R_1}$ and E(R_2) = $\overline{R_2}$
- Now, let us try to understand the return for the portfolio
- The actual return from the portfolio R_p
- $R_p = w_1 * R_1 + w_2 * R_2$ (1)

Expected Returns for Two-Security Case

What about expected returns?

- $\mathrm{E}(R_p) = E(w_1 * R_1 + w_2 * R_2)$ (2)
- $\mathrm{E}(R_p) = E(w_1 * R_1) + E(w2 * R_2)$ (3)
- $\mathrm{E}(R_p) = w_1 * E(R_1) + w2 * E(R_2)$ (4)

  where w_1 and w_2 are constants. Therefore, $\mathrm{E}(R_1 w_1) = w_1 \mathrm{E}(R_1)$.
- However, R_1 and R_2 are probabilistic variables with finite distributions.

Expected Returns for Two-Security Case

What about expected returns?

- For these variables, the expectation operator returns the probability weightage average. That is, $E(R_1) = \overline{R_1}$; therefore,

$$\overline{R_p} = w_1 * \overline{R_1} + w2 * \overline{R_2} \quad (5)$$

- Expected returns from the portfolio are simply the weighted average of expected returns of individual securities in the portfolio.

Expected Returns for Two-Security Case

What about expected returns?

- This can be generalized into three securities and multi-security as well

$$\overline{R_p} = w_1 * \overline{R_1} + w2 * \overline{R_2} + w3 * \overline{R_3}, \quad \text{where } w_1 + w_2 + w_3 = 1$$

- For "N" securities
- $\overline{R_p} = \sum_{i=1}^{N} w_i * \bar{R}_i$, where $\sum_{i=1}^{N} w_i = 1$ (6)

Expected Returns from Portfolio: A Simple Example

Expected Returns: Case 1 (Different Probabilities)

Pt	Ra	Rb	Wa*Ra	Wb*Rb	R_p=Wa*Ra+Wb*Rb	Pt*R_p
0.20	9.00%	6.00%	3.60%	3.60%	7.20%	1.44%
0.15	8.00%	5.00%	3.20%	3.00%	6.20%	0.93%
0.10	7.00%	8.00%	2.80%	4.80%	7.60%	0.76%
0.15	11.00%	9.00%	4.40%	5.40%	9.80%	1.47%
0.25	12.00%	10.00%	4.80%	6.00%	10.80%	2.70%
0.15	6.00%	11.00%	2.40%	6.60%	9.00%	1.35%
	Wa	**Wb**			**Total**	**8.65%**
	0.40	0.60	E(R_p)=P1*R_{p1}+P2*R_{p2} … … .+P6*R_{p6}			

Expected Returns: Case 2 (Equal Probabilities)

Ra	Rb	Wa*Ra	Wb*Rb	R_p=Wa*Ra+Wb*Rb
9.00%	6.00%	3.60%	3.60%	7.20%
8.00%	5.00%	3.20%	3.00%	6.20%
7.00%	8.00%	2.80%	4.80%	7.60%
11.00%	9.00%	4.40%	5.40%	9.80%
12.00%	10.00%	4.80%	6.00%	10.80%
6.00%	11.00%	2.40%	6.60%	9.00%
Wa	Wb		Average	8.43%
0.40	0.60	E(R_p)=(1/ N)* (R_{p1}+R_{p2} …. … .+R_{p6})		

Portfolio Construction with Two Securities: Risk

Risk: Standard Deviation for Two Securities

The variance of a two-security portfolio

- Variance (σ_i^2)= $\sum_{t=1}^{T} P_t(R_{i,t} - \overline{R_i}\ )^2$
- Again, for past observations that are equally likely
- That is, $P_1 = P_2 = P_3 = P_4 \dots \dots . = P_T$. Since $\sum_{i=1}^{T} P_i$ =1, we have $P_1 = P_2 = P_3 = P_4 \dots \dots . = P_T = \frac{1}{T}$
- Variance (σ_i^2)= $\frac{1}{T}\sum_{t=1}^{T}(R_{i,t} - \overline{R_i}\ )^2$

Risk: Standard Deviation for Two Securities

The variance of a two-security portfolio

- Think of $(A+B)^2 = A^2 + B^2 + 2AB$
- $\boldsymbol{\sigma_p^2 = w_1^2 * \sigma_1^2 + w_2^2 * \sigma_2^2 + 2 * (w_1 * \sigma_1)(w_2 * \sigma_2)\rho_{12}}$ (7)
- where σ_p is the portfolio standard deviation (SD). σ_1 and σ_2 are SD of the individual securities. w_1 and w_2 are the investment proportions in each of the securities. ρ_{12} is the correlation between the two securities, and varies from -1.0 to 1.0
- What if $\boldsymbol{\rho_{12}}$=1?

Risk: Standard Deviation for Two Securities

The variance of a two-security portfolio

- $\sigma_p^2 = w_1^2 * \sigma_1^2 + w_2^2 * \sigma_2^2 + 2 * w_1 * w_2 * \boldsymbol{\rho_{12}} * \boldsymbol{\sigma_1} * \boldsymbol{\sigma_2}$ (7)
- $\boldsymbol{\rho_{12}} * \boldsymbol{\sigma_1} * \boldsymbol{\sigma_2}$ is called the covariance between securities 1 and 2, also $\boldsymbol{\rho_{12}} = \boldsymbol{\rho_{21}}$
- This variance (or SD) is less or more than the value given by Eq. (8)?
- For $\boldsymbol{\rho_{12}}$=1, $\sigma_p^2 = (w_1 * \sigma_1 + w_2 * \sigma_2)^2$
- $\boldsymbol{\sigma_p} = \boldsymbol{w_1} * \boldsymbol{\sigma_1} + \boldsymbol{w_2} * \boldsymbol{\sigma_2}$ (8)
- For all the values of ρ_{12} (except ρ_{12} =1), the value of Eq. (7) will be less than that of Eq. (8); What are the implications?

Risk: Standard Deviation for Two Securities

The variance of a two-security portfolio

- $\sigma_p^2 = w_1^2 * \sigma_1^2 + w_2^2 * \sigma_2^2 + 2 * w_1 * w_2 * \boldsymbol{\rho_{12}} * \boldsymbol{\sigma_1} * \boldsymbol{\sigma_2}$ (7)
- For $\boldsymbol{\rho_{12}} = -1$, $\sigma_p^2 = (w_1 * \sigma_1 - w_2 * \sigma_2)^2$
- $\boldsymbol{\sigma_p = w_1 * \sigma_1 - w_2 * \sigma_2}$ (9)
- For all the values of ρ_{12} (except $\rho_{12} = -1$), the value of Eq. (7) will be more than Eq. (9); What are the implications?

Risk: Standard Deviation for Two Securities

$$\sigma_p^2 = w_1^2 * \sigma_1^2 + w_2^2 * \sigma_2^2 + 2 * w_1 * w_2 * \rho_{12} * \sigma_1 * \sigma_2$$

	1 (w_1, σ_1)	2 (w_2, σ_2)
1 (w_1, σ_1)	$w_1^2 * \sigma_1^2$	$\rho_{12} * w_1 * \sigma_1 * w_2 * \sigma_2$
2 (w_2, σ_2)	$\rho_{12} * w_1 * \sigma_1 * w_2 * \sigma_2$	$w_2^2 * \sigma_2^2$

Portfolio Construction with Multiple Securities: Risk

Risk: Standard Deviation for Multiple Securities

$$\sigma_p^2 = w_1^2 * \sigma_1^2 + w_2^2 * \sigma_2^2 + 2 * w_1 * w_2 * \rho_{12} * \sigma_1 * \sigma_2$$

	1 (w_1, σ_1)	2 (w_2, σ_2)	3 (w_3, σ_3)
1 (w_1, σ_1)	$w_1^2 * \sigma_1^2$	$\rho_{12} *$ $w_1 * \sigma_1 * w_2 * \sigma_2$	$\rho_{13} *$ $w_1 * \sigma_1 * w_3 * \sigma_3$
2 (w_2, σ_2)	$\rho_{12} *$ $w_1 * \sigma_1 * w_2 * \sigma_2$	$w_2^2 * \sigma_2^2$	$\rho_{23} *$ $w_2 * \sigma_2 * w_3 * \sigma_3$
3 (w_3, σ_3)	$\rho_{13} *$ $w_1 * \sigma_1 * w_3 * \sigma_3$	$\rho_{23} *$ $w_2 * \sigma_2 * w_3 * \sigma_3$	$w_3^2 * \sigma_3^2$

Risk: Standard Deviation for *N*-Security

	1 $(\mathbf{w_1}, \boldsymbol{\sigma_1})$	2 $(\mathbf{w_2}, \boldsymbol{\sigma_2})$	…..	……	N $(\mathbf{w_N}, \boldsymbol{\sigma_N})$
1 $(\mathbf{w_1}, \boldsymbol{\sigma_1})$					
2 $(\mathbf{w_2}, \boldsymbol{\sigma_2})$					
…..					
…..					
N $(\mathbf{w_N}, \boldsymbol{\sigma_N})$					

Risk: Standard Deviation for *N*-Security

The variance of *N*-security portfolio

- There will be "*N*" such boxes with entries of $w_i^2\sigma_i^2$
- Variance terms = $\sum_{i=1}^{N} w_i^2\sigma_i^2$
- Also, let us assume that all these stocks we have amounts invested in equal proportion (1/*N*).
- $\sum_{i=1}^{N} w_i^2\sigma_i^2 = \sum_{i=1}^{N} \frac{1}{N^2}\sigma_i^2 = \frac{1}{N}\sum_{i=1}^{N} \frac{\mathbf{1}}{\mathbf{N}}\, \boldsymbol{\sigma_i^2}$ because $w_i = \frac{1}{N}$
- Define $\sigma_{\text{avg}}^2 = \sum_{i=1}^{N} \frac{1}{N}\, \sigma_i^2$, Variance terms= $(\frac{1}{N}) * \sigma_{\text{avg}}^2$

Risk: Standard Deviation for *N*-Security

The variance of *N*-security portfolio

- There will also be "$N^2 - N$" boxes with covariance terms and cross products of weights invested in both the securities with the following entries: $w_i w_j \sigma_i \sigma_j \rho_{ij}$
- Covariance terms = $\sum_{i=1}^{N} \sum_{\substack{j=1 \\ i \neq j}}^{N} w_i w_j \sigma_i \sigma_j \rho_{ij}$, also $w_i = w_j = \frac{1}{N}$
- Covariance terms = $\sum_{i=1}^{N} \sum_{\substack{j=1 \\ i \neq j}}^{N} (\frac{1}{N^2}) \sigma_i \sigma_j \rho_{ij} = \frac{1}{N^2} \sum_{i=1}^{N} \sum_{\substack{j=1 \\ i \neq j}}^{N} \sigma_i \sigma_j \rho_{ij}$
- $\sigma_{\text{avg-cov}} = \frac{1}{N(N-1)} \sum_{i=1}^{N} \sum_{\substack{j=1 \\ i \neq j}}^{N} \sigma_i \sigma_j \rho_{ij}$

Risk: Standard Deviation for *N*-Security

The variance of *N*-security portfolio

- Covariance terms = $\sum_{i=1}^{N} \sum_{\substack{j=1 \\ i \neq j}}^{N} (\frac{1}{N^2}) \sigma_i \sigma_j \rho_{ij} = \frac{1}{N^2} \sum_{i=1}^{N} \sum_{\substack{j=1 \\ i \neq j}}^{N} \sigma_i \sigma_j \rho_{ij}$
- $\sigma_{\text{avg-cov}} = \frac{1}{N(N-1)} \sum_{i=1}^{N} \sum_{\substack{j=1 \\ i \neq j}}^{N} \sigma_i \sigma_j \rho_{ij}$
- $\sum_{i=1}^{N} \sum_{\substack{j=1 \\ i \neq j}}^{N} \sigma_i \sigma_j \rho_{ij} =$ Covariance terms*N^2 = $\sigma_{\text{avg-cov}}{}^{*} N(N-1)$
- Covariance terms=$(N^2 - N) * \left(\frac{1}{N}\right)^2 * \sigma_{\text{avg-cov}} = (\frac{N-1}{N}) * \sigma_{\text{avg-cov}}$

Risk: Standard Deviation for *N*-Security

The variance of *N*-security portfolio

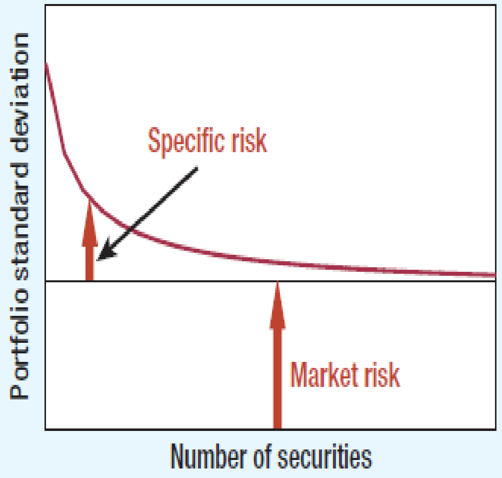
- Variance terms= $(\frac{1}{N}) * \sigma_{\text{avg}}^2$; Covariance terms=$(\frac{N-1}{N}) * \sigma_{\text{avg-cov}}$
- $\sigma_P^2 = (\frac{1}{N}) * \sigma_{\text{avg}}^2 + (\frac{N-1}{N}) * \sigma_{\text{avg-cov}}$
- Now, if *N* is very large ($N \to \infty$), then variance term will be close to zero
- Covariance term will be close to the average covariance
- The portfolio variance will be close to the average covariance
- $\sigma_P^2 = \sigma_{\text{avg-cov}}$
- What are the implications?

Risk Diversification with Portfolios

Risk Diversification with Portfolios

- For a well-diversified portfolio with a large number of securities, the variance terms will be close to zero
- Only the average covariances across the stocks will contribute to the portfolio risk

Diversification eliminates specific risk. But there is some risk that diversification *cannot* eliminate. This is called *market risk.*



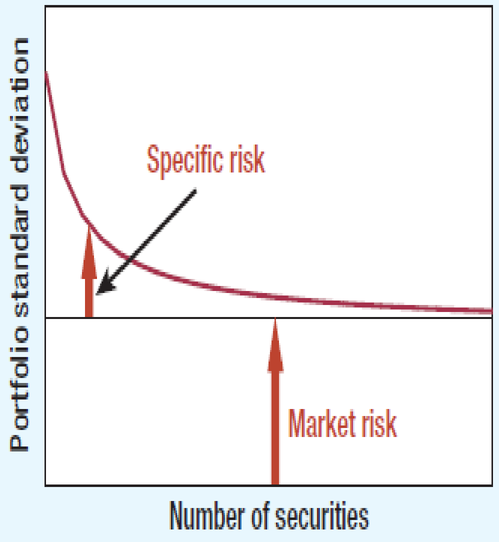
- These covariances arise due to the correlations between the security returns
- For a portfolio with low correlations across securities, the portfolio risk can be lower

Brealey, Myers, and Allen, *Principles of Corporate Finance*, 10th, 11th, or 12th edition (Chapter 7)

Risk Diversification with Portfolios

- The component associated with variances is called diversifiable risk or specific risk
- Later, we will see that market does not reward this risk
- The risk that is associated with covariances is often called market risk or non-diversifiable risk

Diversification eliminates specific risk. But there is some risk that diversification *cannot* eliminate. This is called *market risk*.



- Market only rewards for bearing this non-diversifiable risk (market risk)

Brealey, Myers and Allen; Principles of Corporate Finance. 10th, 11th, or 12th editions. Chapter 7

Example: Computation of Expected Portfolio Returns

- For example, if we invest 60% of the money in security 1 and 40% of the money in security 2, and the expected returns from security 1 and security 2 are, respectively, 8% and 18.8%. Then, the expected returns from the portfolio are computed as follows:

  $\overline{R_p} = w_1 * \overline{R_1} + w2 * \overline{R_2}$

- $R_p = 0.60 * 8.0\% + 0.40 * 18.8\% = 12.30\%$

Example: Computation of Expected Portfolio SD

- Consider the same previous example (w_1 = 60%, w_2 = 40%). Now, some additional information is given to compute the portfolio variance: σ_1 = 13.2% and σ_2 = 31.0%. Consider five cases of correlation coefficients: ρ_{12} = −1.0, −0.5, 0, 0.5, and 1. Now, let us compute the SD of the portfolio for all the five scenarios
- $\sigma_p^2 = w_1^2 * \sigma_1^2 + w_2^2 * \sigma_2^2 + 2 * w_1 * w_2 * \rho_{12} * \sigma_1 * \sigma_2$

Example: Computation of Expected Portfolio SD

Case	Variance (σ_P^2)	Standard Deviation (σ_P)
ρ_{12}=1	$0.6^2 * 0.132^2 + 0.4^2 * 0.31^2 + 2 * 0.6 * 0.4 * 1 * 0.132 * 0.31 = 0.0413$	20.32%, which is same as = 0.6*13.2%+0.4*31.0%
ρ_{12}=0.5	$0.6^2 * 0.132^2 + 0.4^2 * 0.31^2 + 2 * 0.6 * 0.4 * 0.50 * 0.132 * 0.31 = 0.0315$	17.74%
ρ_{12}=0.0	$0.6^2 * 0.132^2 + 0.4^2 * 0.31^2 + 2 * 0.6 * 0.4 * 0.00 * 0.132 * 0.31 = 0.0217$	14.71%
ρ_{12}=-0.5	$0.6^2 * 0.132^2 + 0.4^2 * 0.31^2 + 2 * 0.6 * 0.4 * -0.5 * 0.132 * 0.31 = 0.0118$	10.88%
ρ_{12}=-1.0	$0.6^2 * 0.132^2 + 0.4^2 * 0.31^2 + 2 * 0.6 * 0.4 * -0.5 * 0.132 * 0.31 = 0.0020$	4.48%

Summary and Concluding Remarks

Summary and Concluding Remarks

- Adding more securities that are less correlated (have lower covariance) in the portfolio leads to diversification
- Diversification here means the reduction of stock-specific risk
- The part of the risk that is non-diversifiable is on account of the covariances across securities
- Often this risk is called market risk or systematic risk

Summary and Concluding Remarks

- Markets do not reward for bearing stock-specific diversifiable risks
- Since these risks can be easily mitigated, when we say that we expect certain return for bearing risk, that risk is systematic/non-diversifiable/market risk

Thanks!