IIT ROORKEE

NPTEL ONLINE CERTIFICATION COURSE

QUANTITATIVE INVESTMENT MANAGEMENT

LECTURE 10

Bond Pricing Contd.

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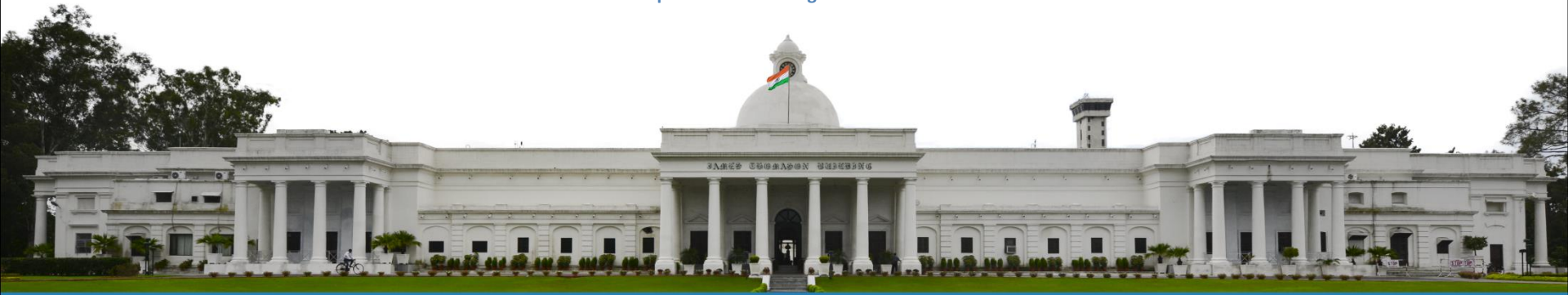
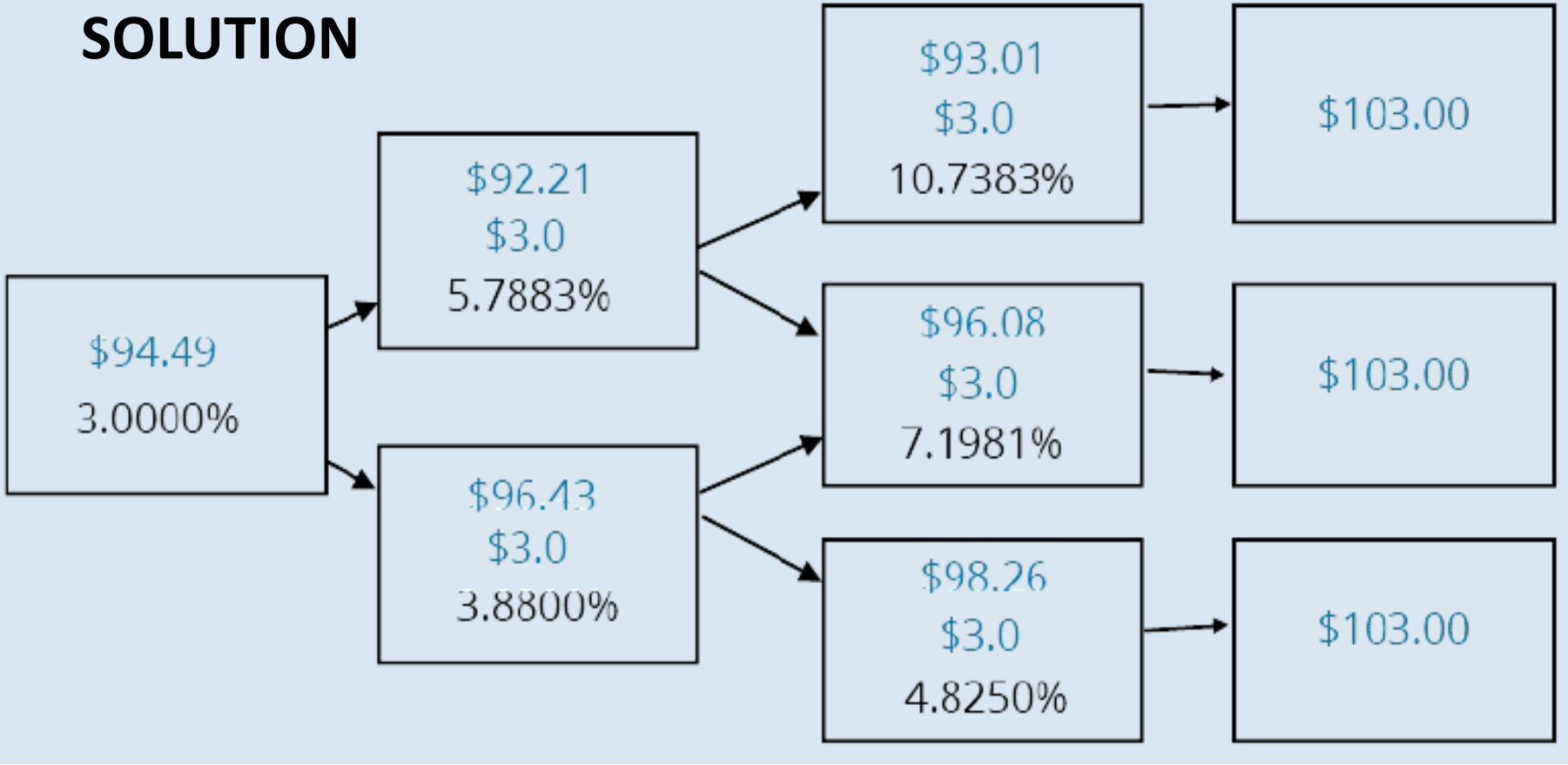


EXAMPLE

- X is interested in valuing a three-year, 3% annual-pay Treasury bond using the adjacent binomial tree. Value the bond.

0	1	2
3%	5.7883%	10.7383%
	5.7883%	7.1981%
	3.8800%	7.1981%
	3.8800%	4.8250%

SOLUTION


$94.49
3.0000%
$92.21
$3.0
5.7883%
$96.13
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3.8800%
$93.01
$3.0
10.7383%
$96.08
$3.0
7.1981%
$98.26
$3.0
4.8250%
$103.00
$103.00
$103.00

- $V_{2,UU} = \frac{103}{(1.107383)} = \$93.01$
- $V_{2,UL} = V_{2,LU} = \frac{103}{(1.071981)} = \$96.08$
- $V_{2,LL} = \frac{103}{(1.048250)} = \$98.26$
- $V_{1,U} = \frac{1}{2} \times \left[\frac{93.01+3}{1.057883} + \frac{96.08+3}{1.057883}\right] = \$92.21$
- $V_{1,L} = \frac{1}{2} \times \left[\frac{93.08+3}{1.038800} + \frac{98.26+3}{1.038800}\right] = \$96.43$
- $V_0 = \frac{1}{2} \times \left[\frac{92.21+3}{1.03} + \frac{96.43+3}{1.03}\right] = \$94.485$

NEED & RELEVANCE OF BINOMIAL MODEL

- For bonds with embedded options, the future cash flows are uncertain as they depend on whether the embedded option will be in the money (and hence exercised).
- Whether the option will be exercised depends on the interest rates & the consequential bond value at exercise dates.
- Thus, the underlying cash flows from the option are dependent on the future interest rates.

- Therefore, the value of the option & hence, that of the bond depends on uncertain future interest rates.
- Hence, to value bonds with embedded options, we have to allow for rates to fluctuate.
- One way to accomplish this is to use the binomial interest rate tree.
- We shall take up the valuation of bonds with embedded options in a later section.

PATHWISE VALUATION

PATHWISE VALUATION

- This is a method mathematically equivalent to the backward induction of the binomial tree.
- In this approach, the value of the bond is calculated corresponding to each path that interest rates could trace over the given number of periods and then an appropriate average is taken of all these path values.

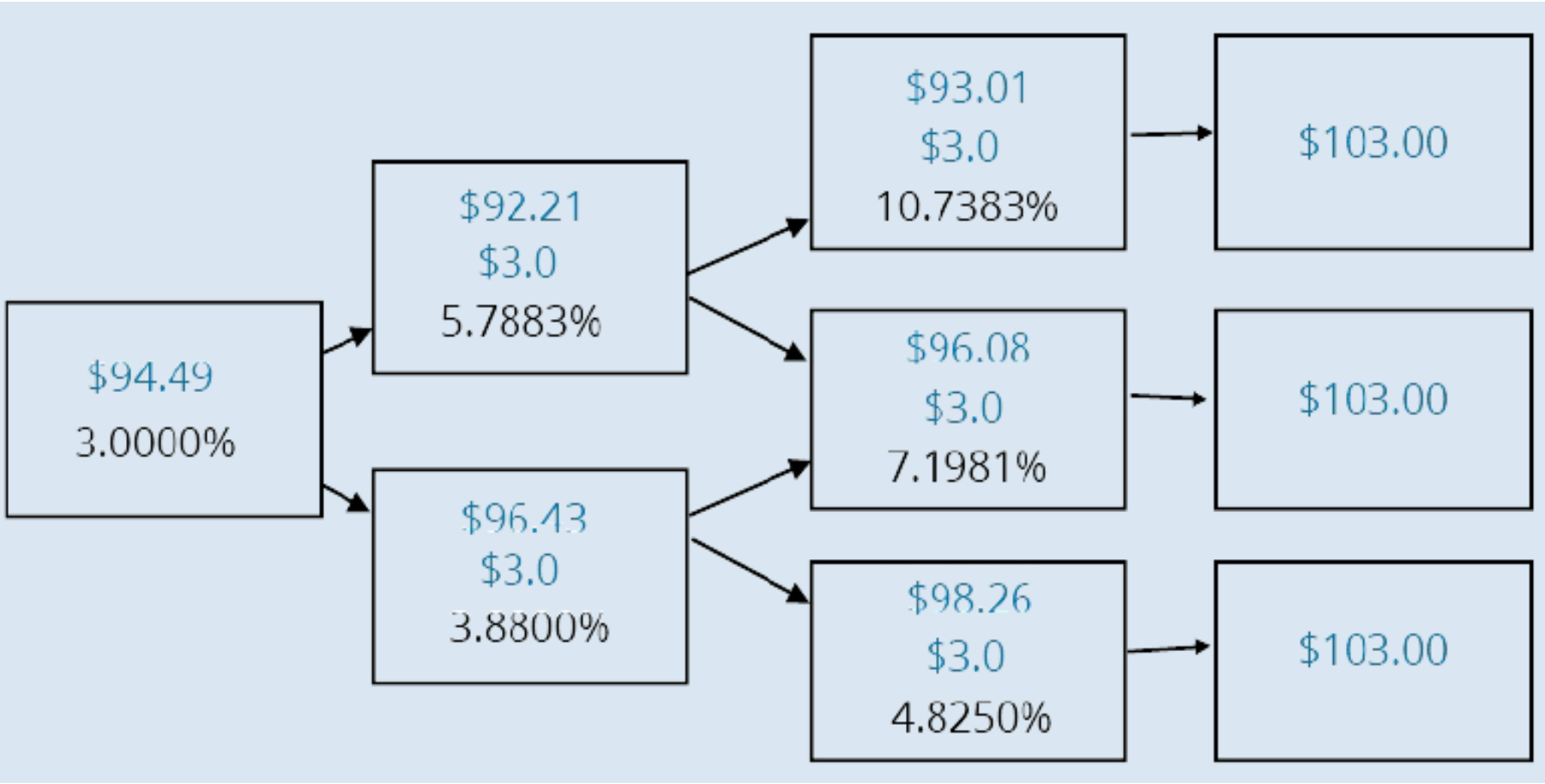
EXAMPLE

X wants to value a three-year, 3% annual-pay Treasury bond using path-wise valuation. The interest rate tree is shown in the adjacent box.

0	1	2
3%	5.7883%	10.7383%
	5.7883%	7.1981%
	3.8800%	7.1981%
	3.8800%	4.8250%

SOLUTION

- For a three-year bond, there are four potential interest rate paths.
- The value of the bond for each path is computed as the sum of the present values of each cash flow discounted at its respective path-specified rate.
- Pathwise valuation discounts cash flows one year at a time using one-year forward rates (similar to backward induction) rather than spot rates.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$3.0
7.1981%
$98.26
$3.0
4.8250%
$103.00
$103.00
$103.00

Path	Year 1	Year 2	Year 3	Value
$1(V_{UU})$	3%	5.7883%	10.7383%	91.03
$2(V_{UL})$	3%	5.7883%	7.1981%	93.85
$3(V_{LU})$	3%	3.8800%	7.1981%	95.52
$4(V_{LL})$	3%	3.8800%	4.8250%	97.55
			Average	94.49

- For example, the value of the bond in Path 1 is computed as:
- $V_{UU} = \frac{3}{(1.03)} + \frac{3}{(1.03)(1.057883)} + \frac{103}{(1.03)(1.057883)(1.107383)} = 91.03$

VALUATION OF BONDS WITH EMBEDDED OPTIONS

TYPES OF EMBEDDED OPTIONS

- Embedded options in a bond allow an issuer to:
- manage interest rate risk and/or
- issue the bonds at an attractive coupon rate.
- The embedded options can be a simple call or put option, or more complex options such as estate put or provisions for a sinking fund.

SIMPLE CALLABLE BONDS

- **Callable bonds** give the *issuer of the bond,* the option to call back the bond.
- Most callable bonds have a lockout period during which the bond cannot be called.

TYPES OF CALLABLE BONDS

- The call option can be
- a European-style option (whereby the option can only be exercised on a single day immediately after the lockout period),
- an American-style option (whereby the option can be exercised at any time *after* the lockout period), or
- a Bermudan-style option (whereby the option can be exercised at fixed dates after the lockout period).

PUTTABLE BONDS

- Puttable bonds allow the investor to put (sell) the bond back to the issuer prior to maturity.

EXTENDIBLE BONDS

- A related bond is an extendible bond, which allows the investor to extend the maturity of the bond.
- An extendible bond can be evaluated as a puttable bond with longer maturity (i.e., the maturity if the bond is extended).
- A two-year, 3% bond extendible for an additional year at the same coupon rate would be valued the same as an otherwise identical three-year puttable (European style) bond with a lockout period of two years.

COMPLEX OPTIONS: ESTATE PUT

- An estate put includes a provision that allows the heirs of an investor to put the bond back to the issuer upon the death of the investor.
- The value of this contingent put option is inversely related to the investor's life expectancy; the shorter the life expectancy, the higher the value.

SINKING FUND BONDS

- Sinking fund bonds (sinkers) require the issuer to set aside funds periodically to retire the bond (a sinking fund). This provision reduces the credit risk of the bond.
- Sinkers typically have several related issuer options (e.g., call provisions, acceleration provisions).

RELATIONSHIPS BETWEEN THE VALUES OF A CALLABLE OR PUTTABLE BOND, THE UNDERLYING OPTION-FREE (STRAIGHT) BOND, AND THE EMBEDDED OPTION.

CASE OF CALLABLE BOND

- In essence, the holder of a callable bond owns an option-free (straight) bond and is also short a call option written on the bond.
- The value of the callable bond $(V_{callable})$ is, therefore, simply the difference between the value of a straight bond $(V_{straight})$ and the value of the embedded call option (V_{call}) :
- $(V_{callable}) = (V_{straight}) - (V_{call})$

CASE OF PUTTABLE BOND

- Conversely, investors are willing to pay a premium for a puttable bond, since its holder effectively owns an option-free bond plus a put option.
- The value of a puttable bond can be expressed as:
- $V_{puttable} = V_{straight} + V_{put}$
- Rearranging, the value of the embedded put option can be stated as:
- $V_{put} = V_{puttable} - V_{straight}$