Financial Institutions and Markets
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Lecture - 54
Derivatives Market – IV

So, we are discussing about the Derivatives Market. In the previous class, we discussed about the different type of instruments which are available in the derivatives market and as well as what is the use of the derivatives. And we started the discussion on the pricing of the derivatives. And here also we discussed about the concept of moneyness and as well as the intrinsic value and as well as the time value of the option premium. Today, we will be discussing about the different models, which are used for the option pricing or the calculation of the option premium.

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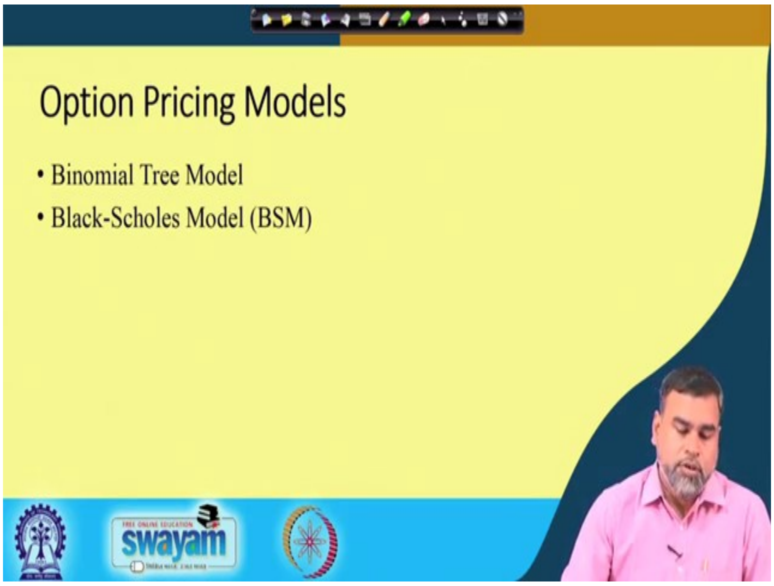

So, if you see that in popular sense, there are two popular models which are used for pricing the options; one is your Binomial Tree Model, then we have the famous the Black-Scholes model. So, these are the two popular models which are used for pricing the offsets. And one by one we will see that how these models basically work whenever we try to calculate the pricing of the options.

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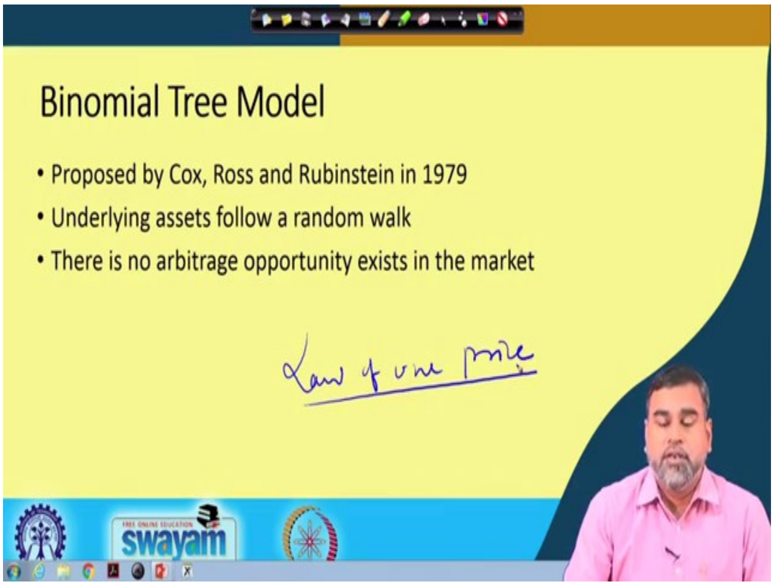

So, let us see that what this binomial tree model is all about. Binomial tree model was proposed by Cox, Ross and Rubinstein in 1979. And here what are the assumptions what this binomial tree model always takes that the asset prices follows a random walk, and there is no arbitrage opportunity exist in the market. What do you mean by the random walk? That means, here we are assuming that any kind of information which is coming to the market which drives the price of this particular underlying asset that basically is random, so that is a prediction of the prices using the past data is not possible.

That means, if you are able to use the past information or past data to predict this future, then we can say that that particular data series is not following the random walk. But here we are assuming that the assets whether the asset can be stock, it can be bond it can be any other asset that basically follow a random walk, and there is no arbitrage opportunity exist in the market. What do mean by the arbitrage opportunity? Here we are talking about arbitrage opportunity means the law of one price should hold good.

So, law of one price how to define the law of one price that price of a particular asset should be same in two different markets at a particular point of time. So, if there is a price differences, then the investor can always create the arbitrage opportunity or they can generate some riskless profits, that means, in one market they can buy this particular asset with a lower price, and at the same time they can take a reverse position in other market, by that without any risk they can create certain return in the market segments.

So, that is why here we are assuming that the law of one price also holds good, that means, there is no arbitrage opportunity exist in the market and as well as the underlying assets basically is following the random walk. So, these are the major assumptions what the binomial tree model takes. And now we will see that how this particular model works.

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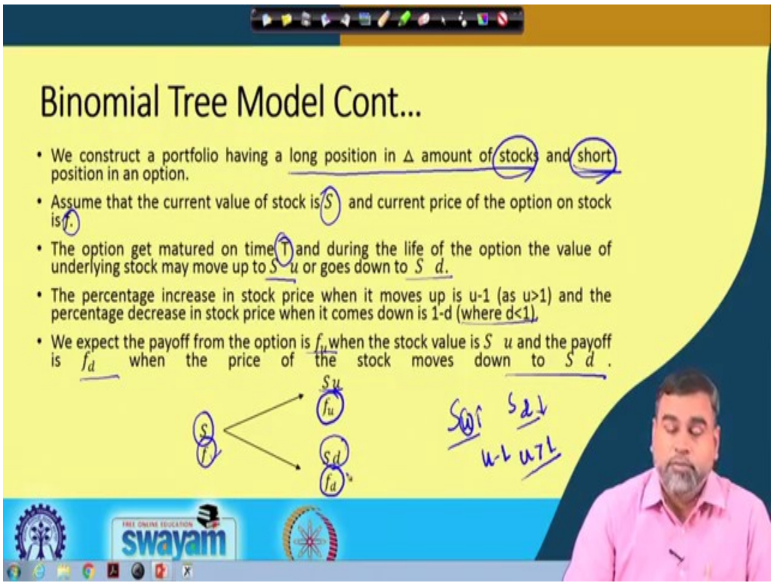

Here if you see that whenever we go for the binomial tree model, we try to construct a portfolio. And because our basic objective is to hedge the risk and we try to take the position or try to compose our portfolio in such a way, by that the total risk in the market can be hedged out. So, here what basically we are assuming, we are basically having a portfolio where we have taken a long position in the spot market and we have taken a short position in the option. That means, we are buying the delta amount of stocks here we are underlying asset we have taken the stocks, then we are selling the options which are based upon this particular stocks. That is why the short position, we are taking for the options and the long positions we are taking on the stocks.

And you assume the current market price or the market value of the stock is S, and the current price of the option on that particular stock is let f. So, our objective is to find out the f. And also we are assuming the option is going to be matured at the time T. And during the life of the option, the value of the underlying stock may go up to S u, or it can goes down to S d. So, either it can go up to S u or it can go up to S d; u means it is increasing; d means it is declining.

So, then the percentage increase in the stock price, when it moves off is u - 1, because it is increasing that means, it is more than 100 percent. So, here you are u > 1. And the percentage decrease in the stock price when it is comes down, it is 1 - d, because d <1. And here we are expecting a payoff from the option whenever the price is going off that is f u, and whenever the price is going down that is S u.

So, here if the stock price become S u, we are finding this option price is f u; and the payoff is f d when the price move down to S d. So, these are the notations what we are going to use. So, if I will explain it this way, the price was s it can go up to S u or it can go down to S d. So, here is the f that we are trying to find out. So, whenever it is going towards S u it the payoff will be f u; and whenever it is going down to S d the payoff will be f d. So, now using these notations, we have to see how that option prices can be or option price can be calculated from this.

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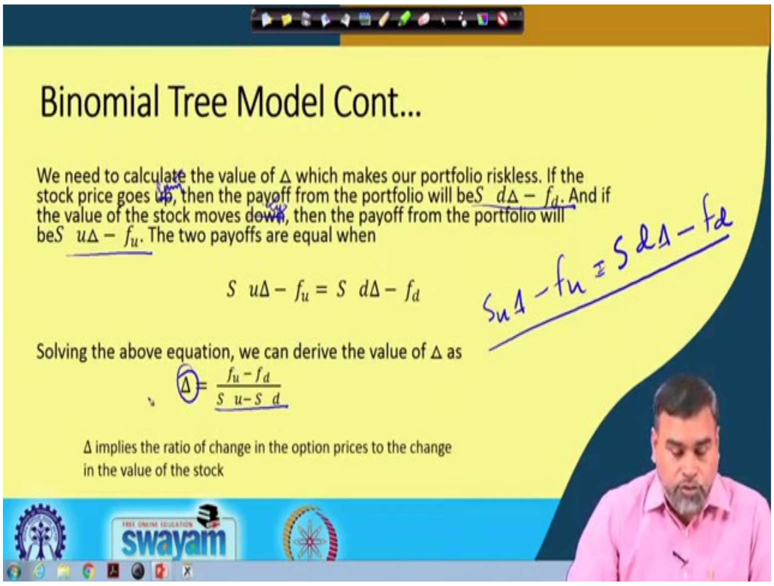

So, now, if you see using this notation, what basically our objective? We need to calculate the value of the delta. We need to calculate the value of the delta which makes our portfolio risk less that means you have to generate certain kind of return out of this which is nothing but the risk-free rate of return on the particular from this particular portfolio. If the stock price goes off, then the payoff from the portfolio will be S d Δ - fd;

and if the price of the stocks move down, then the payoff from the portfolio will be u Δ - f of u.

So, then the two payoff if you want to make it equal, then what basically you can find out S u Δ- f of u is equal to S d Δ - f of d. So, now, if you solve this equation your Δ will be f of u - f of d, that means, your S u Δ, basically it is S u Δ your S u Δ - f of u = S d Δ - f of d. So, in both the conditions basically that should be equal. If that is equal, then what basically you can find out here your Δ = (f of u - f of d) / (S u - S d); that means, we want to make this particular portfolio which is riskless. And we want to generate certain kind of return out of this which is basically your risk-free rate of return.

Here basically you see if the stock prices goes up and up or down depending upon that, the delta value will be changed. Here if you observe, if the stock prices goes up, the payoff from the portfolio will be s here basically it is you can make it goes up. If the stock prices goes up, then payoff will be goes up. Then payoff will be S u Δ - f of u; whenever it goes down the payoff will be S d Δ - f of d. So, it is goes down and it is moves up. So, now, we have S u Δ - f of u = S d Δ - f of d that equality has to be maintained. And here we are our objective is to find out the delta value.

Then Δ = (f of u - f of d) / (S u - S d). And what is then delta the delta is nothing but the ratio of change in the option prices to the change in the value of the stock. So, let us take a numerical example to understand that how that particular delta value can be calculated.

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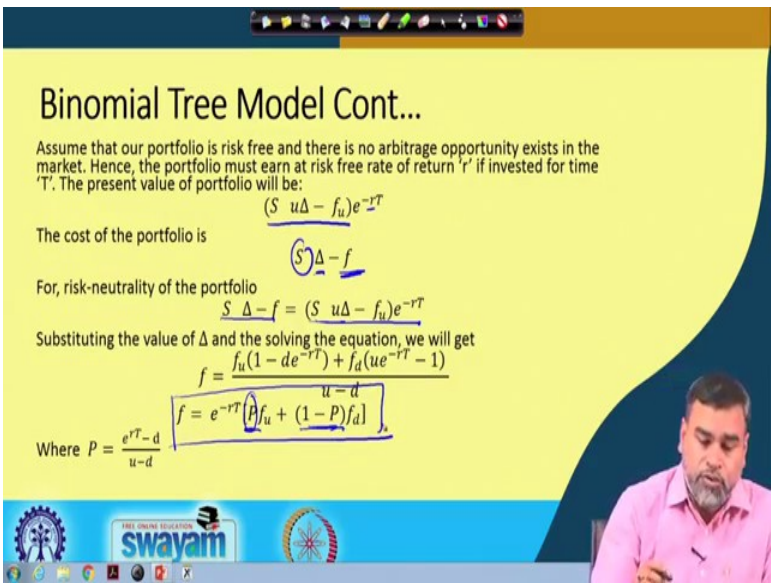

So, before going for this numerical example, for example, the portfolio is risk-free, already we have taken, there is no arbitrage opportunity that assumption we have taken. Then the portfolio must earn, there is risk-free rate of return r if invested for time T. So, then the present value if you want to calculate, then what basically you can calculate in different conditions that is your because we this is the payoff what basically we are getting that is S u Δ - f of u. Then if you want find out the value after the time T, and your r is equal to your rate of return, then it is basically that already you know that (S u Δ- f of u)× e^{-rT}.

And what is the cost of the portfolio? The cost of the portfolio is basically how much money we have spent on that particular stock, so that is why it is delta amount of the stock multiplied by the price of the stock minus the premium what were we have paid. So, now, if the particular post portfolio is risk neutral, then what basically you can see that S Δ - f = S u Δ - f u e^{-rT} if it is a risk neutral portfolio.

Then what you can do, already you know what is the delta, Δ = (f of u - f of d)/(S u -S d). You can put that particular value here, then you can find out f =f u 1 - de $^{-rT}$ + f d × u $^{-rT}$ - 1 / u - d. Then, obviously, your f = e $^{-rT}$ P of f u +1 - P f of d. This P and 1 -P basically shows the probability of increase of the price and probability of decreasing the price. The total probability is 1, P is basically shows you the increase, and 1 - P shows you the decrease.

Then here the P, how the P is calculated, the P is nothing but P = e ^{rT} -d / u - d. So, now, this is what basically we try to find out that f = e $^{-rT}$ P f u + 1 - P f d. So, now we will see that how basically it works.

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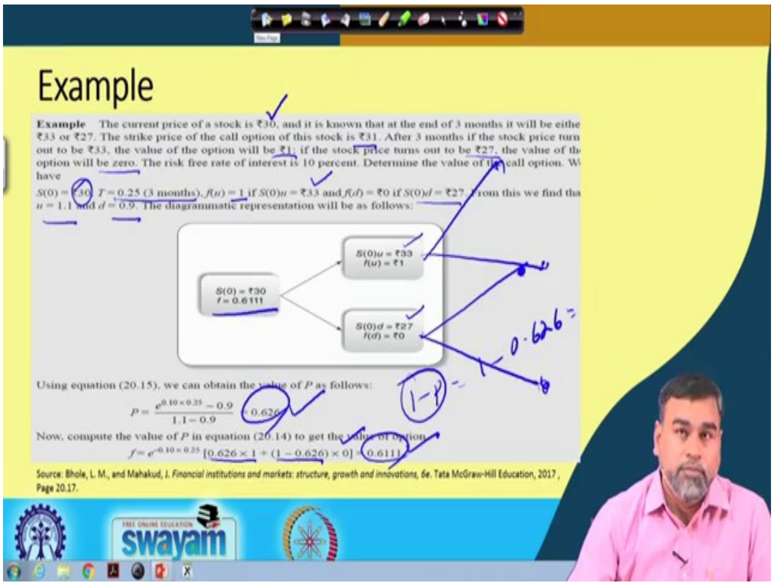

Let this is the example what you can take. Let that is the current price of the stock is 30 rupees. And it is known that at the end of 3 months, it will be either 33 or it will be 27 the strike price of the call option is 31. After 3 months if the stock price turns out to be 33, the value of the option will be 1 rupees; if the stock price turns out to be 27, then the value of the option will be 0, that means, the option will not be exercised, because it will be less than the call option.

The risk-free rate of return is 10 percent, then here find out the value of the call option that is the question. So, your S stock price in the beginning is 30, T is equal to 3 months that means 0.25 years; f of u is equal to 1 if the price goes up to 33, and f of d is equal to 0 obviously if the price goes down to 27. So, then your u will become 1.1, and your d become 0.9.

So, now what basically you can find out you can use that equation; and that equation if you use then you can find out P = (e $^{0.1 \times 0.25}$ - 0.9) / (1.1 -0.9) that is 0.626. So, the P basically you got if the P you got, then you can find out your 1 -P then 1 - P = 1 - 0.626 that will become basically you will find out that 1 - P values. So, now, what you can do this in this equation, you have the f = e $^{-rT}$ × P × f of u + 1 - P × f of d. Then f of d =0, then obviously, your 0.626 × 1 + 1 - 0.626 × 0 that you got it 0.6111. So, the option premium of the option price of this particular example will be 0.611.

So, you can have also two stage model further again it can go up to something and go down something after a certain period. Then again whenever it has 27 it can again go up to something go down something, then like that you can find out each node the probability. Then if each node you can find out the probability, then the backward calculation you can make. And finally, the f can be calculated from that, so that is the way basically the price of the option can be calculated using the binomial tree model.

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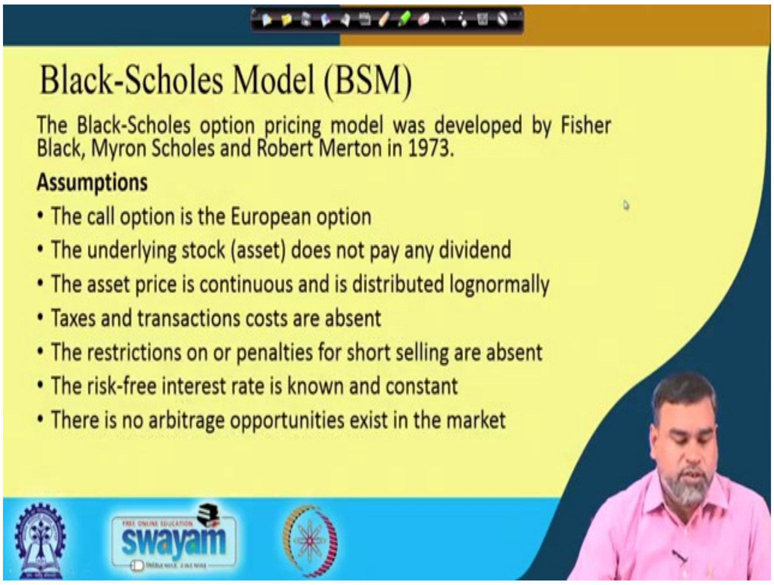

Then if you see in this context, we can have another model; we have the Black-Scholes model. And in the Black-Scholes model, it is basically developed by Fisher Black and Schole and Robert Merton in 1973. And here the assumption is basically they have taken that the option is the European option. And the underlying stock does not pay any dividend, then the asset prices is continuous and distributed lognormally. Taxes and transaction cost are absent. The restrictions on or penalties for short selling, there is no point of short selling here, no short selling is allowed. And risk-free rate of interest is constant or also known to us. And there is no arbitrage opportunity exists in the market. These are the assumption for the Black-Schole model has taken.

So, now what is our objective, our objective is to see that it needs lot of derivations in terms of log normal distributions, then your distribution in terms of the option prices, the process like Ito's lemma generalized linear process and all kinds of thing. So, this is basically beyond the scope of this. But here using these assumptions what this Black-

Schole basically has taken that general Brownian motion, this generalized linear process, then we have the Ito's lemma, all kind of concepts are used because those things are depends upon the properties of that particular underlying asset and as well as the options how they are going to be distributed over the time. So, we are not discussing those things.

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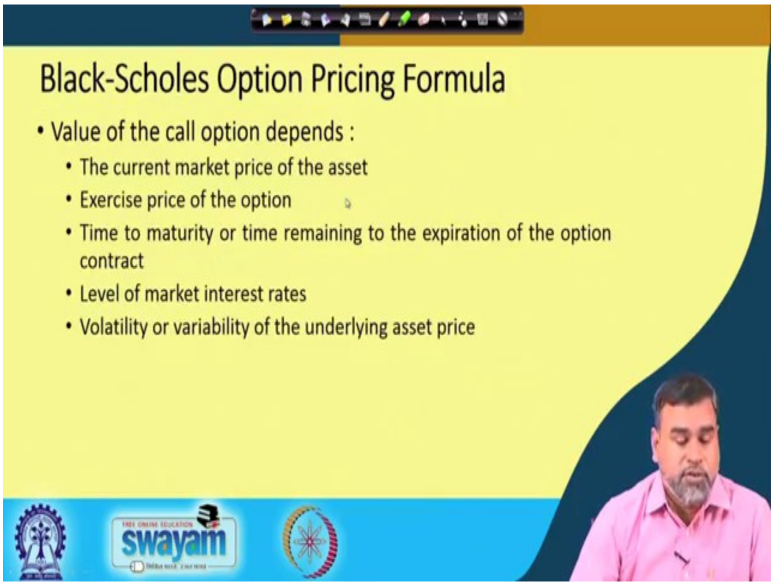

But using these assumptions basically Black-Schole was trying to find out what are those factors which affecting the call option. And here they said that the price basically determined by the market price of the asset, price of the option, exercise price of the option, time to maturity, market interest rate or the risk-free rate of return, and the volatility of the asset prices. These are the factors which are affecting the price of the call option.

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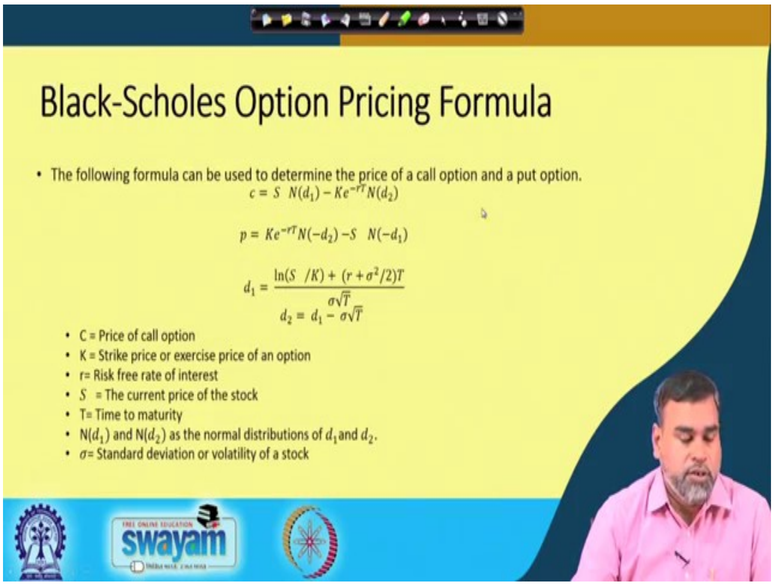

So, now what basically you can do if you see that this is the formula what basically Black-Scholes was derived that c is equal to this is your price of the underlying asset S N d 1 minus K e to the power minus r T N d 2 that is for the call option. For the put option is equal to it is reverse, it is K e, K e means it is the strike price, K e to the power minus r T N minus d 2 minus S N minus d 1.

Now, what you can do your d 1 is equal to if your d 1 also it has been derived d 1 is equal to log of S by K plus r for r plus r is equal to rate of interest, sigma square basically the standard deviation or the volatility of the stock or for any underlying asset divided by 2 into T divided by sigma root of T, T is equal to the time period. Then d 2 is equal to d 1 minus sigma root of T. So, here there are notation C is equal to price of call option; K is equal to strike price; r is equal to risk-free rate; S is equal to current price of the stock; time T is equal to time to maturity; N d 1 and N d 2 are the normal distribution of d 1 and d 2. And your sigma is equal to standard deviation or volatility of this stock. So, these are the notations and this is the formula.

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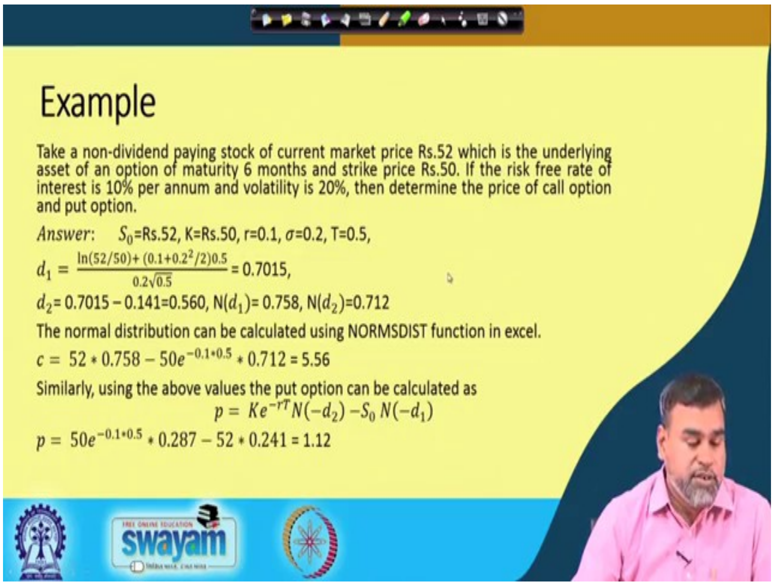

So, if you see this example, let there is a non-dividend paying stock current market price is 52 underlying asset of an option of maturity 6 months, strike price is 50. If the risk-free rate of interest is 10 percent per annum and volatility is 20 percent, then determine the price of the call option and the put option. So, now, your S is equal to or S 0 is equal to 52, K is equal to 50, r is equal to 10 percent, sigma is equal to 20 percent, T is equal to 0.5, because it is 6 months, then you can find out a d 2 d 1 values already you can put this formula ln 52 by 50 plus 0.1 into 0.2 square divided by 2 to the power 0.5 divided by 0.2 root of T that is 0.5, then you find out 0.7015.

Then your d 2 is equal to 0.7015 minus 0.141, it is basically your root sigma and root of T, then it is 0.560. Then you can go to the table normal distribution table of d 1 and d 2, you can find out N d 1 0.758; N d 2 is equal to 0.712. Then you can also use excel for calculating this norm NORMDIST function for this value of d 1 and d 2. Then c is equal to already formula the call option you know that there is S into what S into N d 1 N d 1 minus K e to the power minus r T into your N and d 2, then you can find out 5.56. The call option in this particular case is 5.56.

Similarly, you can putting these values, you can find out the put option. And here your put option price is 1.12. So, this is the way the Black-Schole model has been derived or has been used. So, although the derivation is very lengthy, but if anybody wants to go through that you can go through any books on derivatives like Hall and other books which talks about the derivation of this particular formula, which is used for the valuation of the options.

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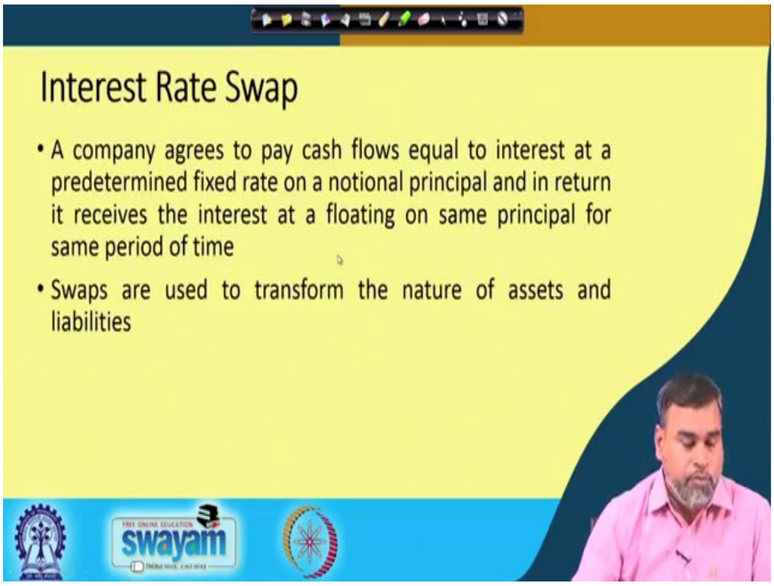

Then we have another instrument called the swap; already we discussed about the swap. Swap is nothing but it is a kind of instrument which is used to transact in terms of the cash flows in the periodical manner. So, here we have two types of measure swaps always we come across, one is your interest rate swap and another one is the currency swap. So, if a company agrees to pay cash flows equal to interest at a predetermined fixed rate on a notional principal and in return, it receives the interest at a floating rate on same principal for the same time period, then we can call that this is basically the interest rate swap. Then why this swaps are used, this swaps are basically used to transform the nature of the assets and its liabilities.

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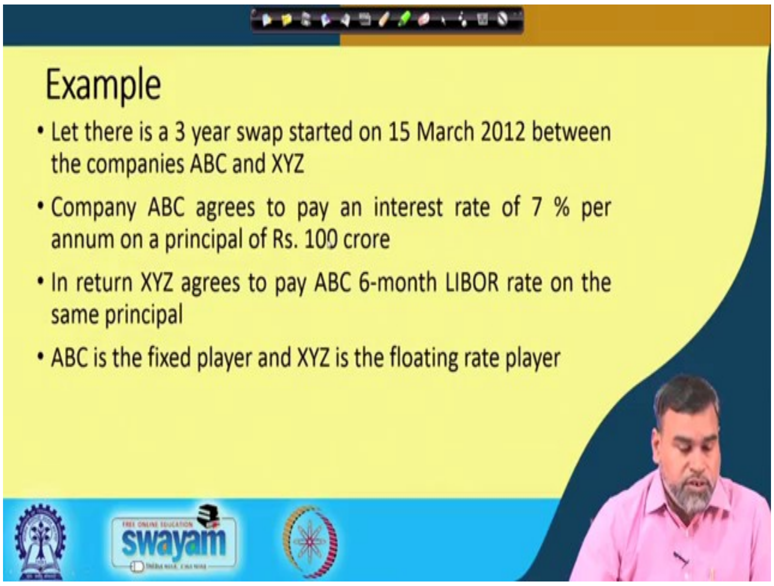

We will see this example, then it will be more clear for you that how the swap is used. Let there is a 3 year swap started on 15th March 2012 between the companies let ABC and XYZ. Company ABC agrees to pay an interest rate of 7 percent per annum on a principal of 100 crore. And in return XYZ agrees to pay ABC the 6-months LIBOR rate on the same principal and you see ABC is basically going for the fixed rate that is 7 percent, and XYZ agrees to pay 6 months LIBOR rate on the same principal. And here the principal is not x since that is why we call it is a notional principal amount. So, this is basically floating, and this is fixed, so that is why ABC is a fixed player and XYZ is a floating player in this case, because XYZ is playing its paying on the basis of the floating rate interest and ABC is paying on the basis of the fixed rate interest. So, now, you see that what is basically how the cash flow basically here looks like.

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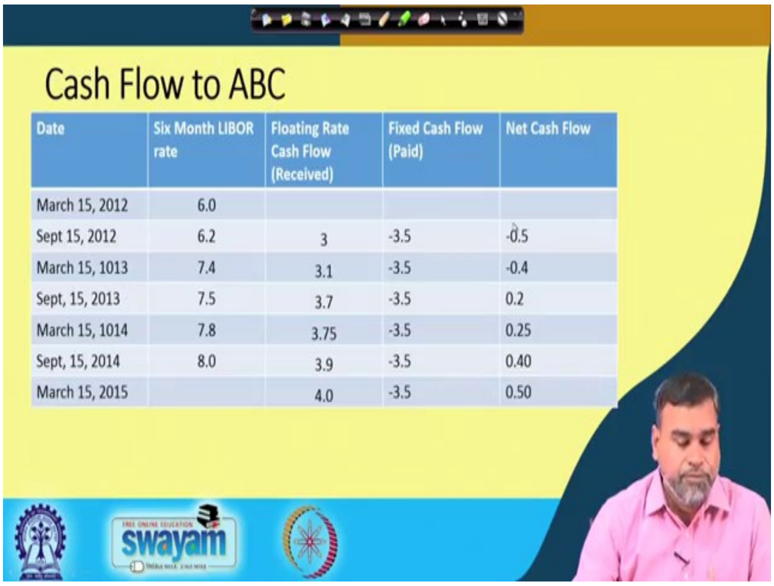

If you see the cash flow to the ABC; now because this is basically a 3 years contract. If the 4 years contract, then you have the 6 cash flows started in March 15 2012, then 6 months LIBOR rate let assume, these are the LIBOR rates which are given. So, therefore, the floating rate cash flow, what is the cash flow to the ABC first, again if you see cash flow to the ABC basically will be getting, because ABC is getting cash flow on the floating rate basis, and paying the cash flow on the fixed rate basis. So, in the March 15, it is basically they are getting the 6 percent of the total money that is 3, there is 3.1 on the basis of this interest rate. These are the interest rates.

So, these are the cash flow what basically they will be receiving and these are calculated on the basis of the principal amount that means, here on the 6 months 6 percent interest; that means 100 crore that means, 6 crore divided by 2 3 crore, then it is 6.2 3.1, crore; then 7.4 3.7 crore, 7.8 so like that there will be getting 7.5 3.75, 3.9, 4 like that they will get the cash flow what they will be receiving. And how much they are paying they are paying, it is fixed because that is 7 percent interest. So, they are every 6 months, they will be paying 3.5, 3.5, 3.5. So, the net cash flow if you see then end of the day the net cash flow is positive that is 0.50 for the company ABC.

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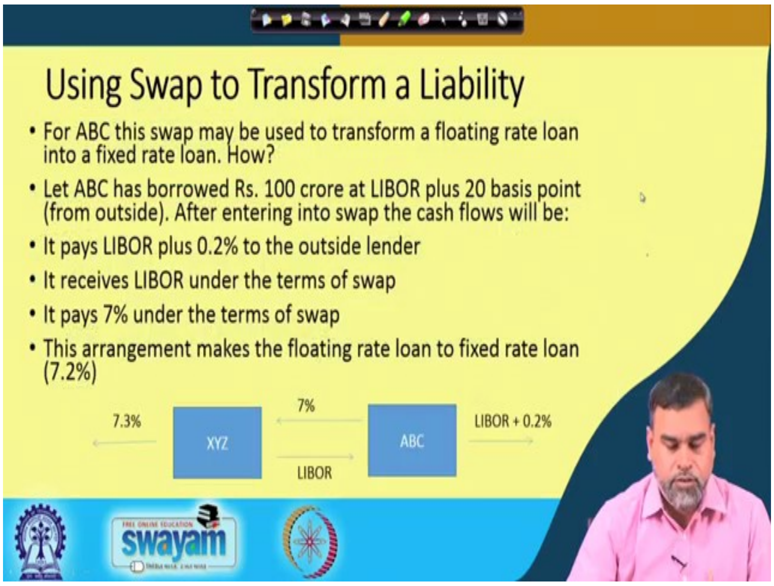

Now, if you see in this context how it is basically working or we can say that how it is helping these two companies who are within basically going for this kind of swap contract. So, now, you assume the ABC for ABC the swap may be used to transform a floating rate loan into a fixed rate loan. What does it mean? Let the company ABC has borrowed 100 crore at LIBOR plus 20 basis point from outside from any financial institutions they have a borrowed amount, and their obligation is 600 crore and that rate is LIBOR plus 20 basis point.

So, after entering this cash flow then how basically it is converted, let this is XYZ, and this is your ABC. So, now, what is happening ABC is already paying LIBOR plus 0.2 percent to some outsider he is paying. And now what he is doing it pays LIBOR plus 2 percent to the outsider, it receives LIBOR from the XYZ, because XYZ is paying on the basis of the LIBOR. Then he is paying 7 percent to XYZ. So, if that is the case then finally, the LIBOR will be cancelled out, then what is final is happening that previously it has a floating rate loan. Now, whenever they have entered into the swap the floating rate loan becomes a fixed rate loan of 7.2 percent for ABC.

So, let ABC wanted that they want to pay in the fixed rate basis, then that particular loan has become 7.2 percent after they have entered into the swap. So, the same thing can also happen XYZ. Let XYZ its paying in a fixed rate and they want to go for a floating rate, they want to convert their fixed rate loan into floating rate, then here if you observe also there is if you see they are paying 7.3 percent, getting 7 percent. And then what is happening they are paying LIBOR, then finally what is happening, it is LIBOR plus 0.3 percent basically what finally they will be paying so that means, the fixed rate loan has been converted into the floating rate loan. So, this is because of that this particular thing can be converted from the fixed to floating or floating to fixed once they have entered into the swap.

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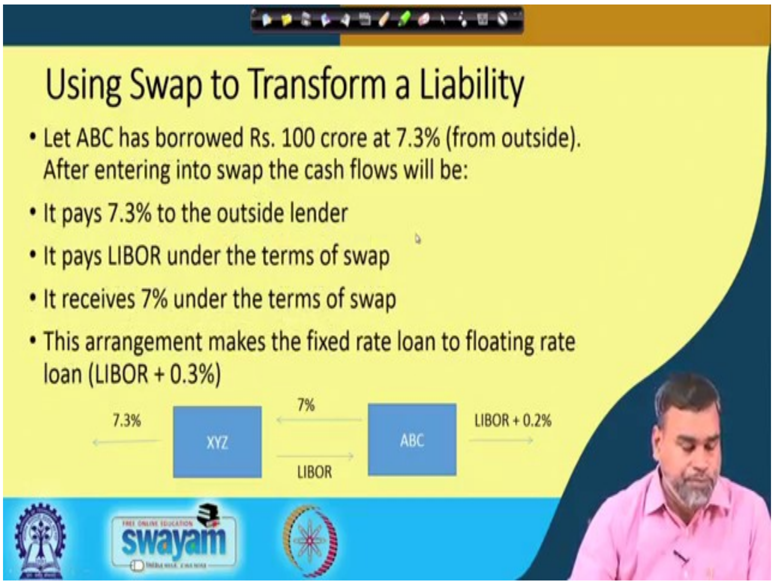

The same thing it pays 7.3 percent to the outsider pays LIBOR, it receive 7 percent under the terms of swap, this arrangement makes the fixed rate loan to the floating rate that is basically LIBOR lost 0.3 percent. 7 percent, 7 percent cancel, you will get 7 LIBOR plus 0.3 percent, so that is basically the conversion of the fixed rate to LIBOR plus 0.3 percent floating rate. This is the way the swaps are used in the market.

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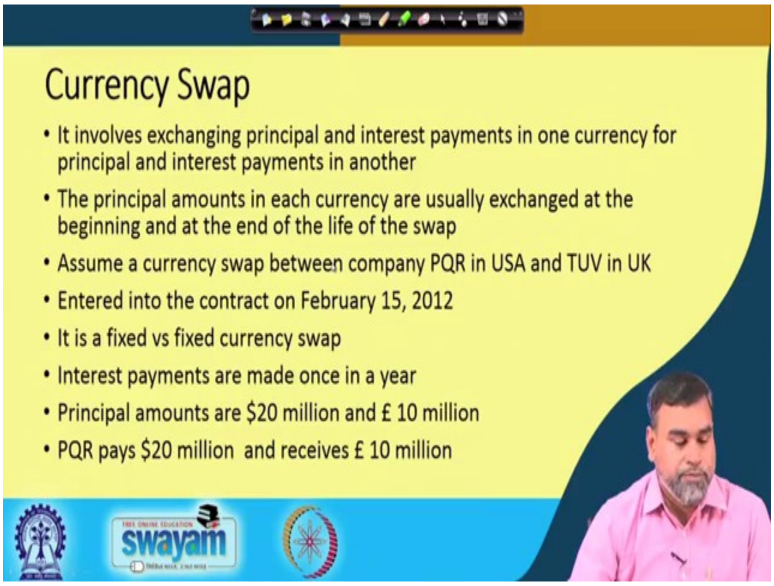

Then you have the currency swap it involved the exchanging principal and interest payments in one currency for principal and interest payment in another. The principal amount in each currency as usually extents to at the beginning at the end of the life of the swap. So, let if you take this example, there is a swap between company PQR in USA and TUV in UK enter into the swap contract in February 15, 2012. So, it is basically fixed versus fixed currency swap we have taken this example. Interest payments are made in a year. Principal amount is 20 million dollar and 10 million dollar. PQR pays 20 million and receives 10 million dollar, and receives 10 million pound.

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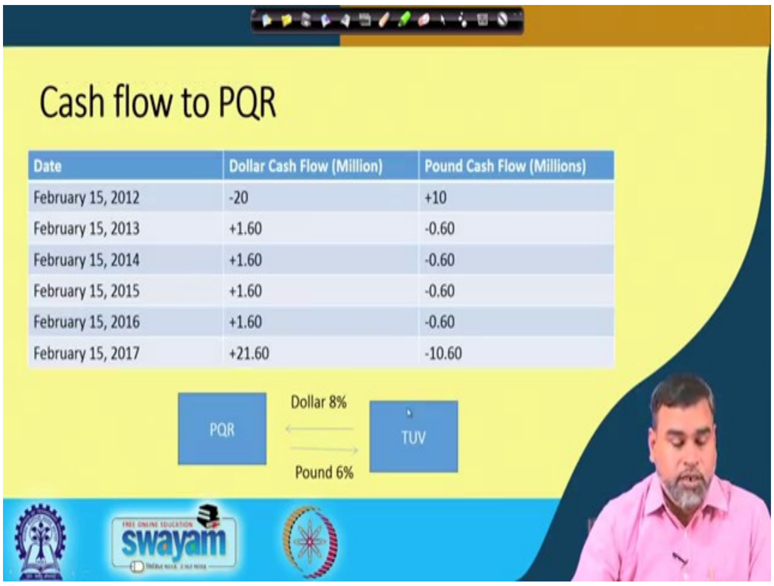

Then how the cash flow to the PQR looks like it will be in the beginning they have paid my 20 dollar, that means minus 20 and the positive basically they have paid this much, they received this much. So, then on the basis of the interest rate the 6 percent and 8 percent interest what we have considered on the basis of that this is the cash flow for the dollar cash flow and this is the pound cash flow which can happen to the company PQR. So, here if you see in the beginning, it is the principal amount is transacted, in the end also it is transacted in that particular cash flow statement.

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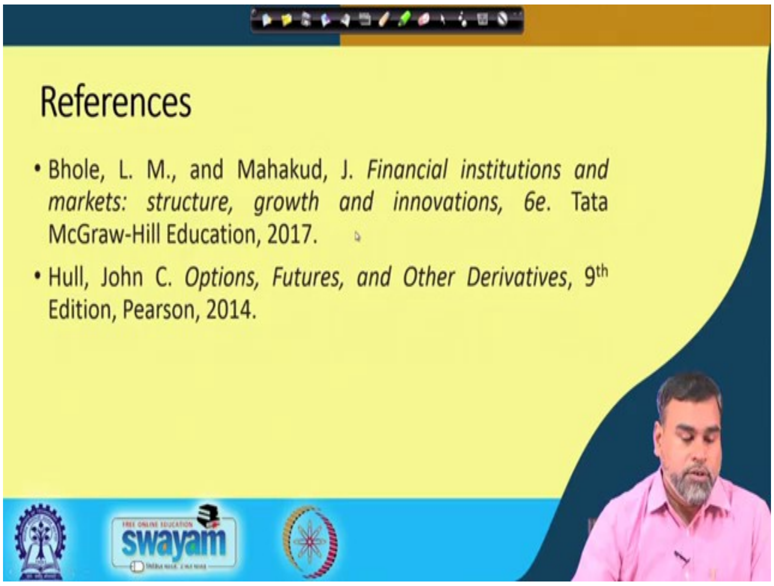

So, this is what basically the basic idea about the concept of the pricing of the options, and the use of the swap in the market. And these are the references what basically you can use for this particular session.

Thank you.