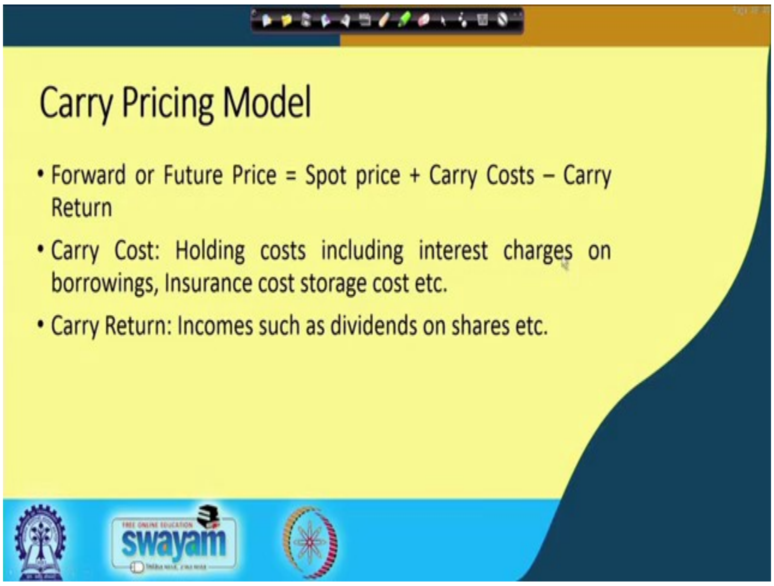
Financial Institutions and Markets
Prof. Jitendra Mahakud
Department of Humanities and Social Sciences
Indian Institute of Technology, Kharagpur

Lecture – 53
Derivatives Market – III

So, we have discussed certain concept related to the options and the futures. We have seen that the option premium has 2 components; one is your time value and other one is the intrinsic value. And the intrinsic value is nothing but the difference between the market value of the underlying asset on that day and as well as the strike price. Today we will be discussing certain models which are used pricing this derivatives instruments. We can start with the pricing of the futures or the forwards, then we can move into the pricing of the options in the upcoming sessions.

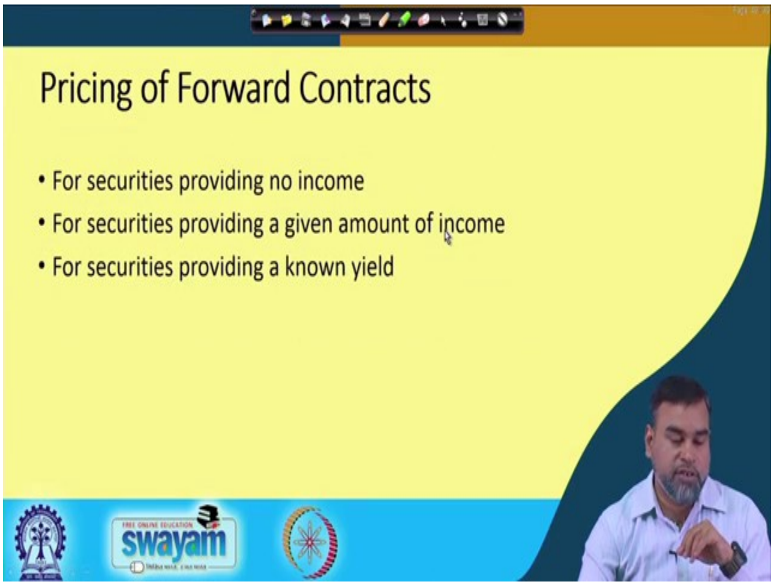
(Refer Slide Time: 01:12)



That whenever you talk about the pricing one model always used for pricing of the future that is basically we call it the carry pricing model. What do you mean by the carry pricing model? In a very simplistic way, the carry pricing model is nothing but the spot price + the carry cost - the carry return. How much cost we are incurring for this option or the future contract, what is the spot price today and how much return we are going to get it from this that is basically the price of the forward or the future. That is the basic fundamental or theoretical model what we can establish or theoretical logic what we can establish for pricing of the future.

So, here the carry cost means what, it may be holding cost, it may be interest charges and borrowings insurance cost. If it is commodity derivatives, then it can be considered as a storage cost, and all these things. Then we have the carry return. The carry return is nothing but the income what basically we are expecting from this. It may be dividends, it may be any kind of cash flow what we can get it from that particular asset or particular instrument or underlying assets, so that is the way basically the carry pricing model works. So, this is the theoretical understanding, but how exactly this carry pricing model works in the practical sense that we have to see in the actual sense how it is worked.

(Refer Slide Time: 02:54)

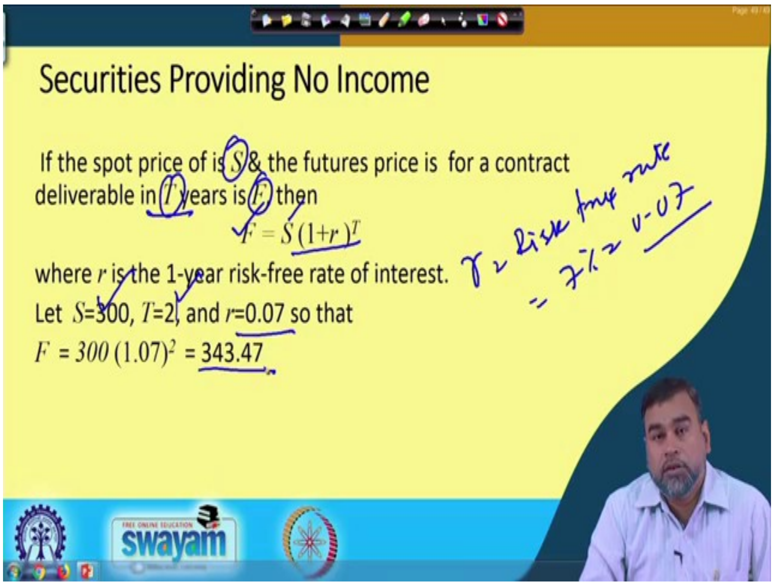


So, here we can observe that we have a different type of assets always we get, different type of instruments underlying instruments we have on that basis the future contracts are design. So, you have the different kind of assets are available in the market or we are designing this future contracts on the basis of the different type of assets, how the different assets are defined or different assets are classified.

The different assets are classified. There are some assets they provide no income; in between you do not generate any income out of this. And there are some assets they provide certain income which is given a fixed amount of dividend or fixed amount of return periodically you are getting from that asset.

And another kind of asset where the amount is not fixed or they know that how much yield they can get, the return percentage is given; percentage of certain kind of value will be given to you. So, this is the way the particular kinds of assets are classified. Then whenever you calculate the future price of those assets that future prices of those assets depends upon the nature of the particular cash flow what you are getting or the nature of assets whatever we have, so that is the way the pricing of the future contracts are made or forward contracts are made.

(Refer Slide Time: 04:34)

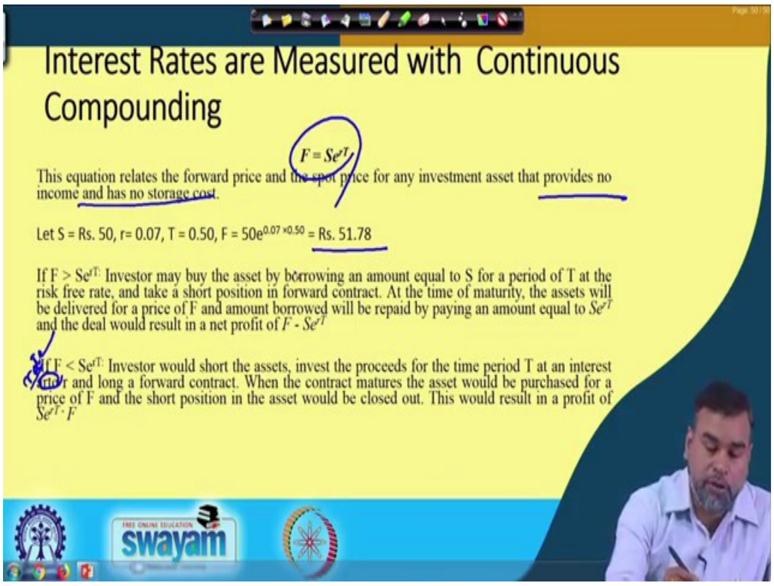


For examples, you see the securities which provide no income. If there is a security which provides no income, then the how the pricing of this particular security can be made. It is simple let here in the derivatives whenever we take always you remember, we consider the return is always a risk free rate of return that basically you can keep in mind.

So, let this spot price is S and the future price is F and the contract is deliverable in T, then $F = S(1 + r)^T$ that is why in the normal market we assume that the future price is more than the spot price. So, for example, you are here the T is equal to 2 and r is equal to the risk free rate of return which let in our case we have taken the 7 percent.

Let the r is equal to your risk-free rate, your risk-free rate is the r, then which is 7 percent that mean 0.07 then S = let 300, T = 2 years r = 7 percent then $F = 300\,(1.07)^2$ that will give you 343.46. The future price will be 343.47. You remember here this asset is not giving you any kind of income in between directly we are calculating with respect to the period of maturity that is your 2 years.

(Refer Slide Time: 06:38)



Here we are talking about another concept let particular interest rate is compounded. If the interest rate is compounded, then your F is equal to Se^{rT}. Here we have taken $S1 + r^T$. But here if the interest rates are measured with continuous compounding, then we have to consider your future price is equal to spot price e^{rT}. And here already I told you that provides more income, no storage cost that is the assumption whatever we have taken.

Let we have S is equal to 50, r is equal to 7 percent T is equal to let 6 months that means, 0.5 years, then F is equal to 50 $e^{0.07 \times 0.5}$ that you got it 51.78. But here two things you can observe, let you find that F > Se^{rT}, then what will happen? If F > Se^{rT}, then what the investor can do that means, the future price is more than the spot price.

The investor at the time of maturity, the investor may buy the asset now by borrowing an amount equal to S for a period of T at the risk free rate of return and take a short position in the forward contract. Short position in the sense a sold the forward contract, he had sold the forward contract and bought the underlying asset today let for buying that asset does not have the money, then what we can do he has borrowed that money.

So, at the time of maturity, what will happen, the asset will be delivered for a price of F. He has sold it, he got the price. And the amount whatever he has borrowed that will be

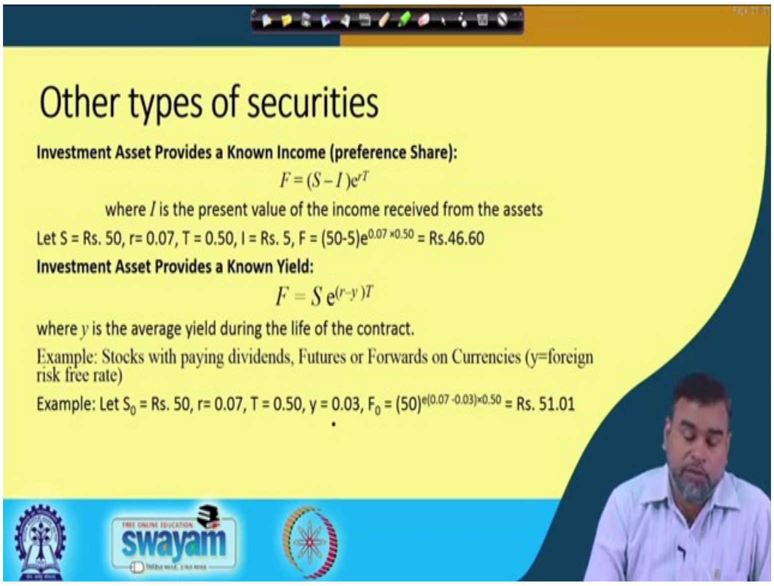
Se^{rT}, then obviously, the profit he can earn that is F - Se^{rT}. The borrowed amount is S, interest is compounded, then how much money he will repay he will repay Se^{rT}

But already he has taken a selling position short position in the future market that is why the contract will be matured, he will get back his price on that particular day and he will get F. And if F is greater than Se^{rT}, then even if he has brought that money at the rate of interest of which is 7 percent in this case risk free rate of return, then what he can do he can generate a profit of F - Se^{rT}.

It can be reverse also. If F < Se^{rT}, then what the investor can do, he would sort the asset. He may sell the asset in which the process for the time period T, with interest rate r and buy the long period contract. And when the contract matures, the asset would be purchased for a price of F and the short position in the asset would be closed out and finally, the property we can generate that is Se^{rT} -F.

So, if at any point of time, this particular condition does not hold, F = Se^{rT} does not hold, then there is a chance of profit generation at the time of maturity. That means, the investor can create certain profit if the identity between the condition like F = Se^{rT} does not hold that basically we have to keep in the mind.

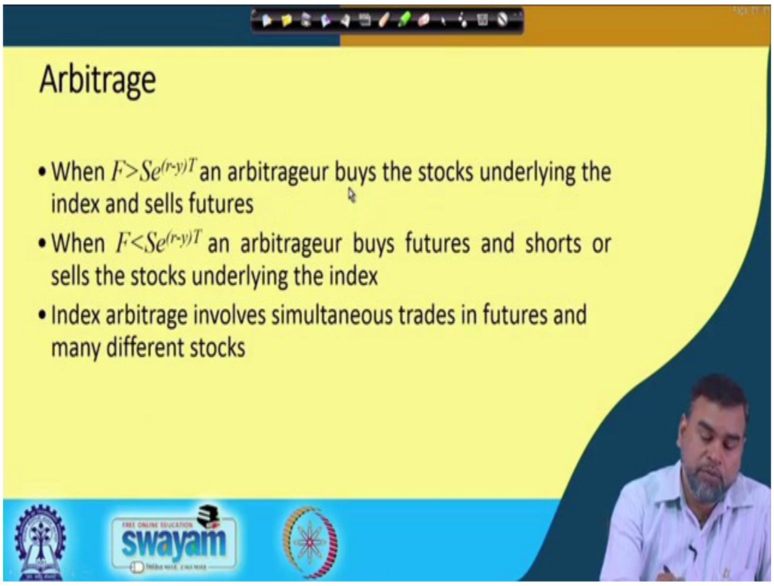
(Refer Slide Time: 11:14)



Then we can see that other type of securities. We discussed about a security, we does not provide any income. Let there is another security which is giving a known income let you assume preference shares. You know preference shares, which provides the certain amount of income periodically, certain amount of dividends periodically which is known to you.

Then how the formula basically looks like here $F = (S - I)e^T$ remember we are all now using it in a continuous compounding case in this case. And what is I, I is nothing but the present value of the income received from the asset for that particular period. If your S is equal to 50, r is equal to 7 percent, T is equal to 0.5, let you I is 5 present value of this particular cash flow what you are getting from that particular security. In that particular period let that is 5, then your F is equal to 46.6, that means, that has to be deducted.

(Refer Slide Time: 12:17)



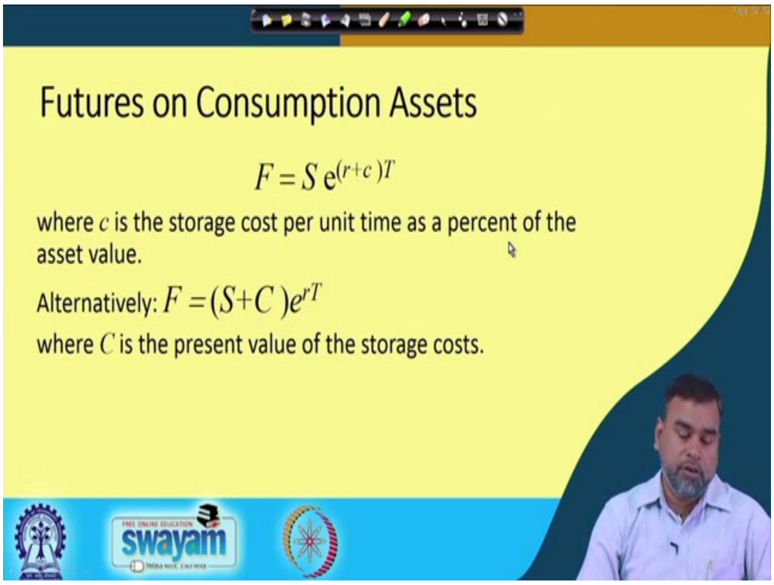
You remember that in the carry forward module whatever we have seen we have to the cost underlying asset price has to be deducted the cost what you are in covering that is the payment what you are making in terms of the dividends, so that is why $(S - Ie)^{rT}$, then you can get the future price of that particular contract. If it is a known yield, let the investment assets provides a known yield in terms of absolute value, they are giving certain interest certain kind of rate of return with a certain percentage. Then here basically let these stocks with paying dividends a pattern percentage, features or forwards on currencies like foreign risk free rate. If you are taking, then how basically you can calculate the price of that particular contract.

The same example if your y is equal to 0.03, then basically this formula will be this your $F = Se^{r-yT}$. The r - y is basically the but the yield or the if it is a forwards for the currencies and all, this is foreign risk free rate. So, here y = 0.3 if it is given, then you can find out $50e^{0.07} - 0.03 \times 0.505 = 51.01$. So, this is basically happening of a particular asset, where the asset provides a known yield.

Then we can see that what are those other things related to this. So, now, what we have seen if F should be equal to Se^{r-yT}, if there is a difference either $F < Se^{r-yT}$ or $F > Se^{r-yT}$ or F less than does not matter then there is a chance of arbitrators.

Then how the arbitrary then works, then when $F > Se^{r-yT}$, then what the arbitrators can do the arbitrators can buy the stocks and sell the future and when $F < Se^{r-yT}$ then the arbitrators can buy the future and sell the stocks underlying the index. This particular arbitrators involves the simultaneous trades in futures and many different stocks in the market.

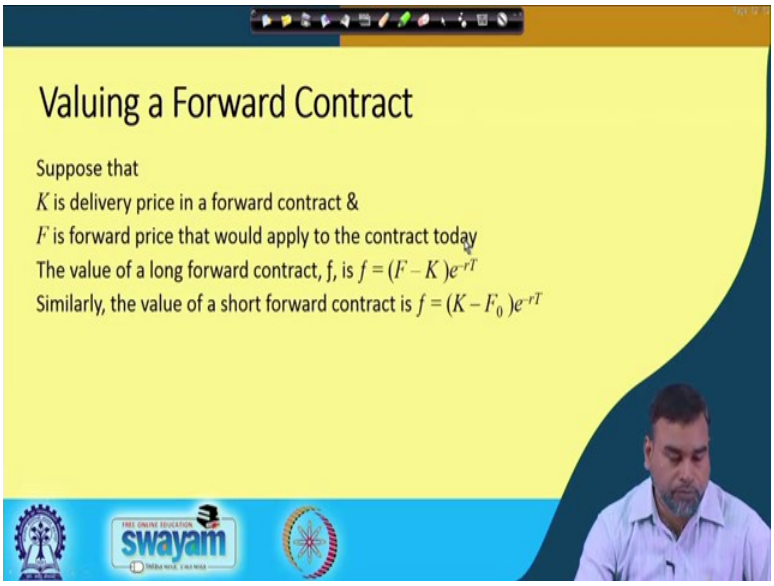
(Refer Slide Time: 15:30)



So, let us see that how basically it works. So, whenever we are talking about this arbitrators opportunity, already we have seen that if that identity does not hold good, then this kind of thing can be possible by taking the different positions in the market. Then if it is a consumption assets, let it is a commodity.

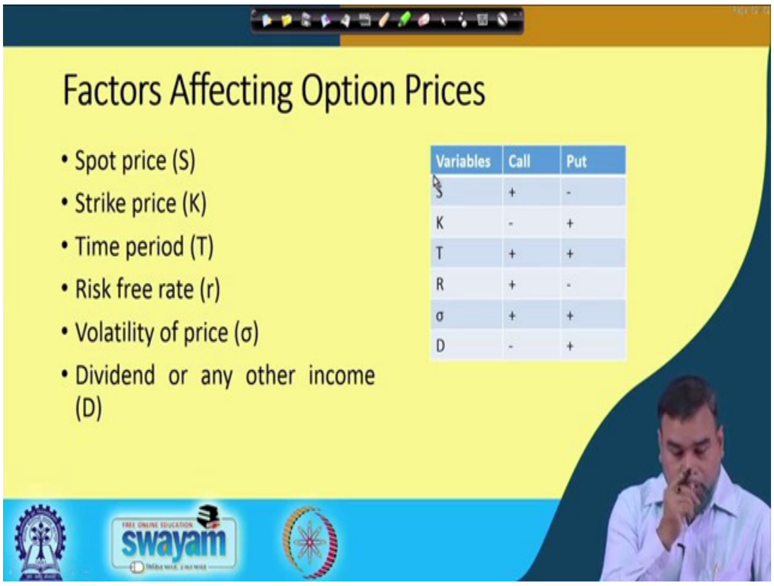
Then what basically you can do, you can go for let c is the storage per unit, then your F = $Se^{(r+c)T}$ c is the storage cost for the unit or you can find out F = S + T S + Xe^{rTC} is the present value of the storage cost of the period of time. So, in case of y, we are taking the C which is per unit storage cost or here C is nothing but the cost what we are incurring over a period of time and we are calculating present value of that.

(Refer Slide Time: 16:33)



Then if you going for a value in the period future forward contract, then if the K is the delivery price and F is the forward price, then value of the long forward contract is let $(F - K)e^{-rT}$ and the value of the short forward contract $F = (K - F_0)e^{-rT}$. And here K is equal to the delivery price, and F is equal to the forward price that is the way the valuation of the forward contract is done. It is $(F - K) e^{-rT}$ and r, if it is a short forward contract, then it is $(K - F) e^{-rT}$.

(Refer Slide Time: 17:20)



Then if you talk about the option prices, there are many factors which affect the option prices here the future prices are mostly affected by the cash flow and as well as the delivery price of that particular contract and as well as the time period and the risk free rate. And here option prices also is driven by the spot price of the contract, the strike price of the contract, the time period, risk free rate of return, then you have the volatility of the price that is sigma and another one dividend or any other income what you can get it from the underlying asset in that particular period of time.

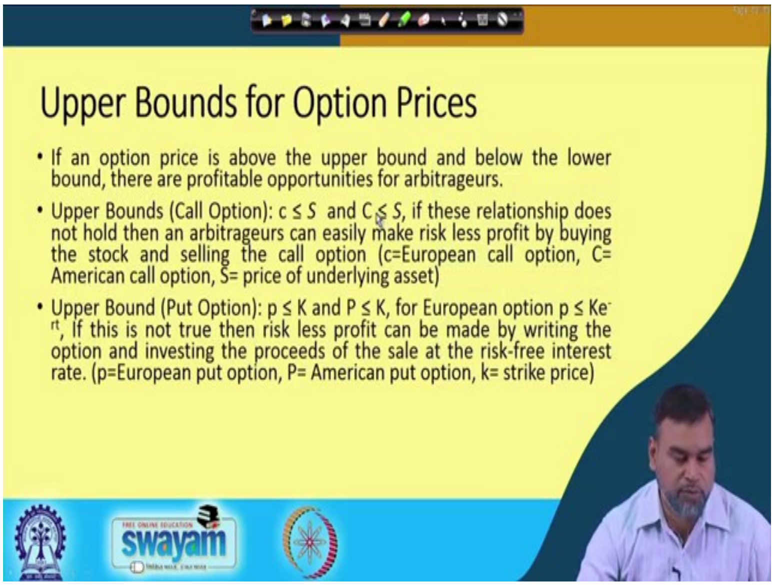
What kind of relationship we can expect between this fundamentals with option prices, a spot with the spot price increases, that means, the market price increases, then the price of the call option will increase that already we have seen that optional will be exercised. And the pay offs will be positive, then the price also of the option will increase. But if it is a put option, the price will be negative or price will be declining.

So, there is a positive relationship or direct relationship between the call option price with this spot price of the particular asset or the market price of the particular asset. And if it is a put option, there is an inverse relationship between these two. But if it is a strike price, then obviously if the strike price is more, then it has a positive impact on the put option and it will have a inverse impact on the call option pricing, because for the put option and the payoff is always decided on the basis of K - S and in case of call, it is S - K. So, keeping that thing in the mind, we have a negative relationship with call and positive relationship with put which is just reverse relationship what to expect for the spot price or underlying assets price.

Then we have time to maturity, the time to maturity or time period increases, then obviously, the price of both type of so and so will increase so that is why we can have a positive relationship for the both. Then we have risk free rate, this is basically also is used as a discount rate, so that will have a positive impact on call and negative impact on the put. And if it is fluctuations or standard deviation, then it will be more the standard deviation or more the fluctuations of the pricing of this spot pricing, it will have positive impact on the option pricing, because more the volatility the risk will be more. So, depending upon that the premium also will be more.

So, dividend is basically we are deducting it from this that is why it is a call option and I will have a negative relationship. If it is a put option, it will have a positive relationship. So, these are the expected relationship what we can get it from the different kind of factors or how the expected relationship can be established between the different fundamentals or the factors with respect to the pricing of the options. Then you will see that how basically this particular concept works in the market in the future.

(Refer Slide Time: 21:03)

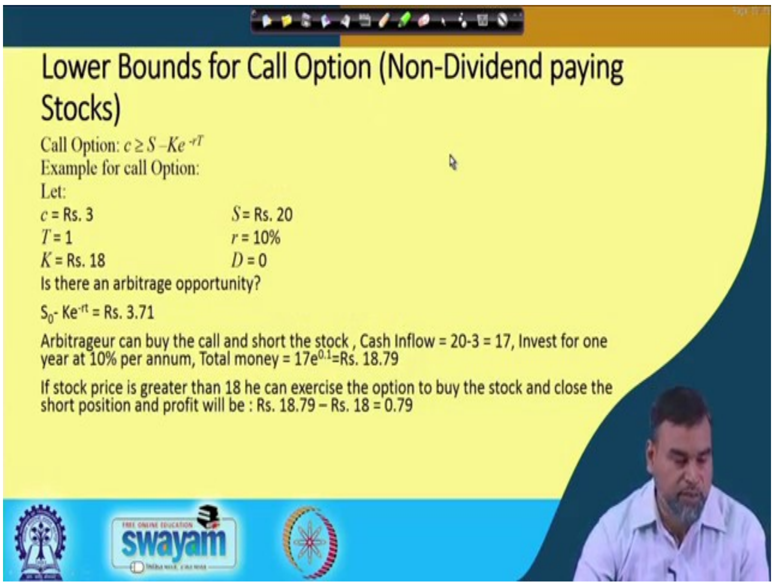


So, here you see for the option prices, we have some upper bounds, we have some lower bounds. How much the option prices can be maximum and how much the option prices can be the minimum? So, if an option; if an option price is above the upper bound or below the lower bound and there are profitable opportunities for the arbitrageurs that always we see. And we will see that how this particular arbitrageur opportunity can be possible.

Let there is an upper bound for the call option, how much maximum the call option price maybe, either it is a European option or it is American option that small c represents European option and capital C presents the American option. For both the cases, the maximum call option price will be the price of the underlying asset, the price of the underlying asset. So, either C will be less than the underlying assets price or it can be equal to the underlying assets price, it cannot go beyond that.

So, if this relationship does not hold good, then the arbitrageurs can easily make the risk less profit by buying this stock and selling the call option. For upper bound for the put option, it can go maximum to the strike price. The put option price either less than or equal to k for both American and European options, if this is not true then risk less profit can be made by writing the option or by selling the option and investing the proceeds whatever money you can get it of the sale of the risk free interest rate. Let us see how that particular concept works.

(Refer Slide Time: 23:17)

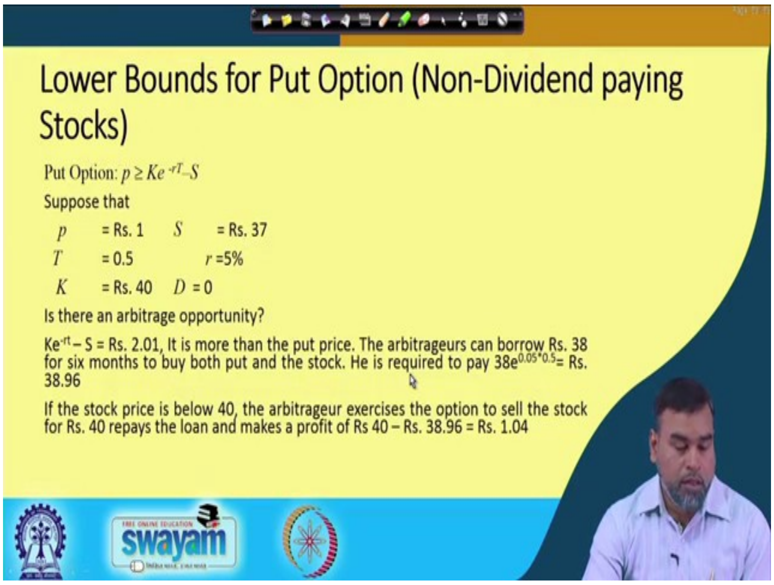


If you take this example, let there is a lower bound for call option non-dividend paying stocks the call option is basically what you have c greater than or equal to S -Ke ^{rT} that is the lower bound. Let example is c =3 rupees, T = 1 year, K = 18, S = 20, r = 10 percent, no dividend paying stocks we have taken. So, is there any arbitrageur opportunity?

The lower bound is basically c is greater than or equal to S - Ke $^{-rT}$ that is the call option case. Then what things they can do the arbitrageur can buy the call S 0 - Ke ^{rT} if you can calculate that has come 3.71. The arbitrageur can buy the call and short the stock. And the inflow will be your 20 - 3 that is 17. Then invest for 1 year at 10 percent per annum, then how much you are getting 17e $^{0.1}$ that is 18.78, because your T = 1.

Then if the stock price is greater than 18, K is equal to strike price is 18, if the stock price is greater than 18, then he can exercise the option to buy the stock and close the short position and the profit will be 18.79 -18 that is 0.79. So, that 0.79 what we consider that is a risk less profit what we can earn from this.

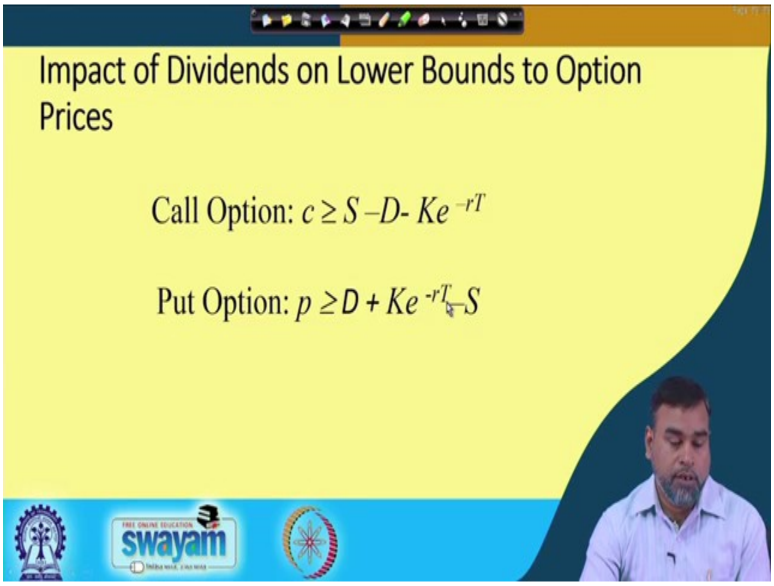
(Refer Slide Time: 25:08)



For example, for put options the p is greater than or equal to K e $^{-rT}$ - S. So, if that is the case, then if you take this example, you can see how this particular arbitrageur profit is possible p is equal to 1 we have taken suppose S is equal to 37, T = let us six months - 0.5, r = 5 percent, K = 40, and we have taken D = 0.

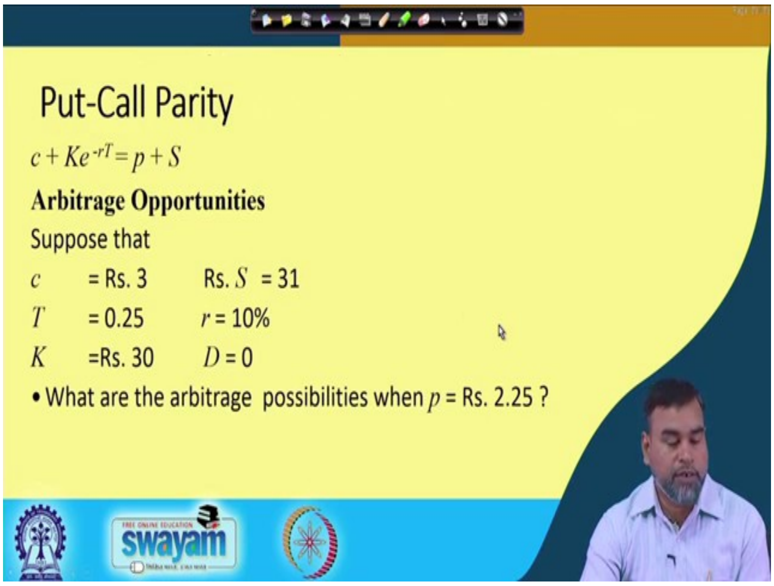
You can calculate K $^{-rT}$ - S that is 2.01, it is more than the put price p = 1. Then what the arbitrageur can do, can borrow 38 rupees for 6 months to buy both put and stock yes, 37 plus 1. Then how much he required to pay, he required to pay 38 e $^{0.05}$ into 0.0 into 5 is equal to 38.96. And stock price below 40, then the arbitrageur are exercise the option to sell the stock for 40 rupees and repay the loan and makes a profit of 40 - 38.96 that is 1.04 rupees, so that is the arbitrageurs profit what they can generate out of this.

(Refer Slide Time: 26:34)



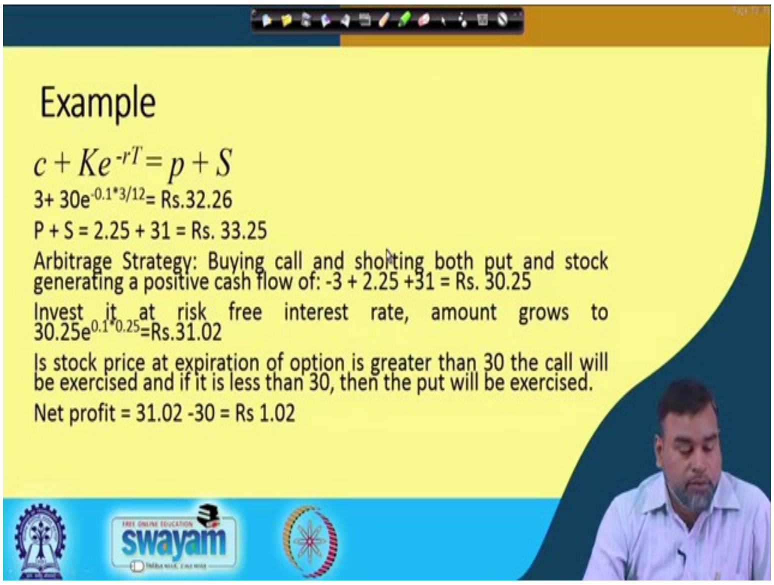
So, that is why you have our call option case, it should be if it is a dividend paid, then the conditions should be S - D - Ke ^{rT} for call option. For a put option it should be D + Ke^{-rT} - S that actually you can keep in the mind.

(Refer Slide Time: 26:55)



Then we have another condition, there is a put call parity, if you know put price you can find out the call price; if you know the call price, you can find out the put price. So, that the put called parity condition $c + Ke^{-rT} = p + S$; c = the call option, K = strike price, p = put price, S = spot price. And if that condition does not satisfy, then there is a chance of arbitrage. And how it is let c = 3, S = 31, T = 0.25, r = 10 percent, K = 30 and D = 0. What are the arbitrage responsibilities when p is equal to 2.25? Let this is the question.

(Refer Slide Time: 27:52)

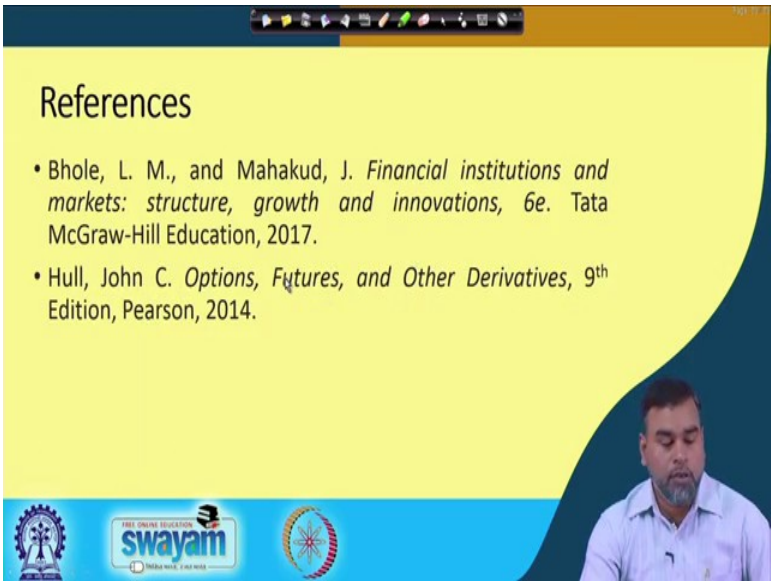


Then if you want to see then what you can find out because this condition has to be satisfied $c + Ke^{-rT} = p + S$. Then c = 3, K = 30, e^r = 10 percent, T = 0.25, 3 / 12, then you can get the 32.26. And p + S =, p = 2.25, S = 31, and it is 2.25 and you get 33.25.

So, what is the arbitrageur strategy you can create buy the call short both put and stock, and how much cash flow you can generate that is - 3. Buy call means you have paid 3 rupees which is the premium for the call option, then plus 2.25 plus 31, then your cash flow will be 302. Then invest that 302.5 at a risk rate of return, you get 31.02. And if the stock price at expiration of option is greater than 30, the call will be exercised; and if it is less than 30, then the put will be exercise.

Now, either of the cases the net profit will be 31.02 - 30 = 1.02, so then that means, what we have seen that if this condition does not satisfy in both the cases, the arbitrageur opportunity exist. So, that is why we have to always ensure not to create an advertiser opportunity in the put call parity conditions should holds.

(Refer Slide Time: 29:35)



These are the references what you have to follow for the session. And another things you keep in the mind in the future sessions, we will be discussing about the different models used for the option pricing like your Binomial Tree mode, and the Black Scholes model. Then we can move into the discussion on the swap. Then finally, we will discuss about the developments which have happened in the derivatives market with respect to India.

Thank you.