Lesson 2: Portfolio Theory and Asset Pricing Models

INDIAN INSTITUTE OF TECHNOLOGY KANPUR

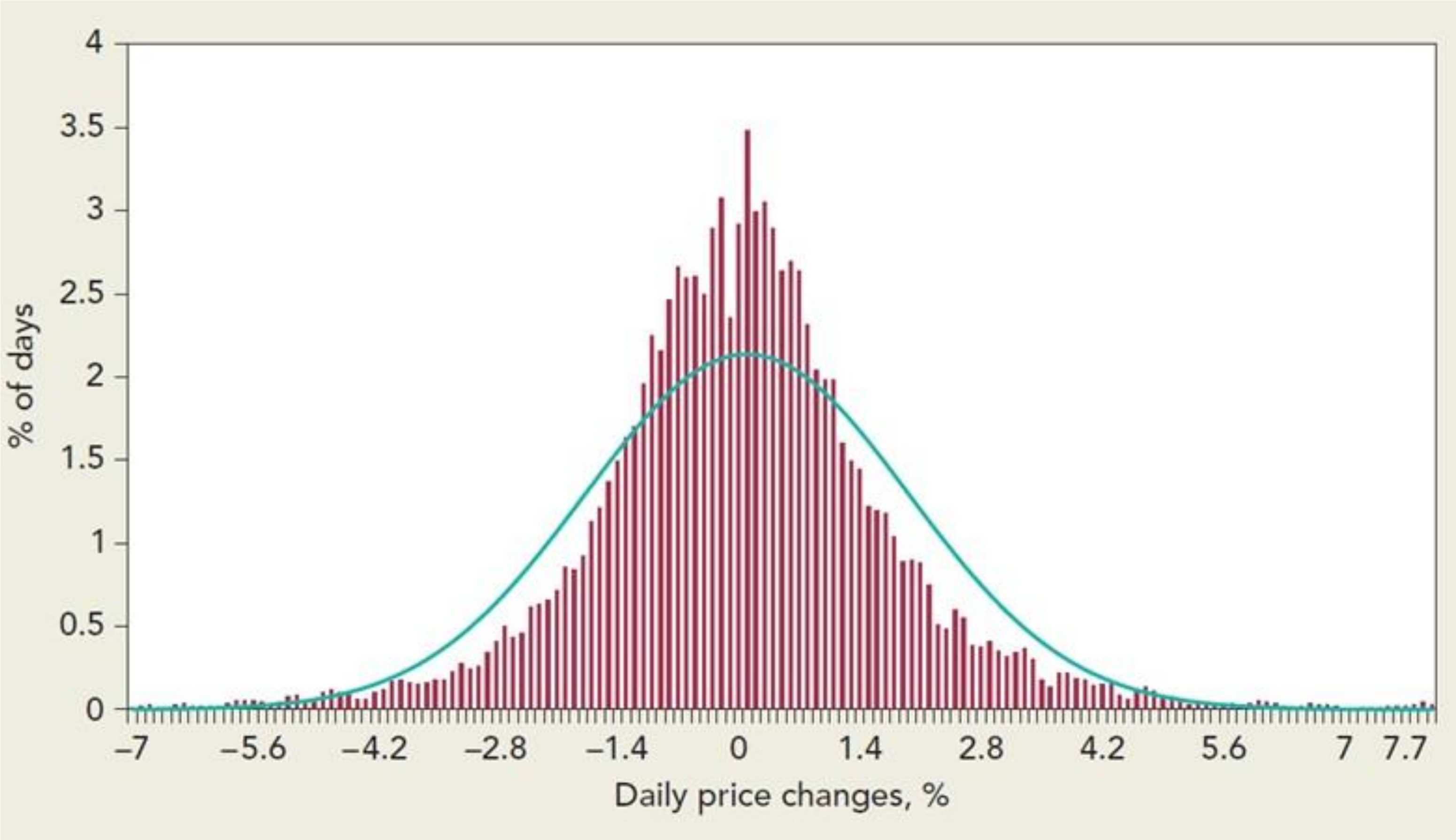
Introduction

In this lesson we will cover the following topics:

- Investment performance and return distribution
- Combining stocks with portfolios
- Introduction to CAPM
- Validity of CAPM
- Alternative theories of asset pricing
- Summary and concluding remarks

Investment Performance and Return Distribution

Investment Performance and Return Distribution



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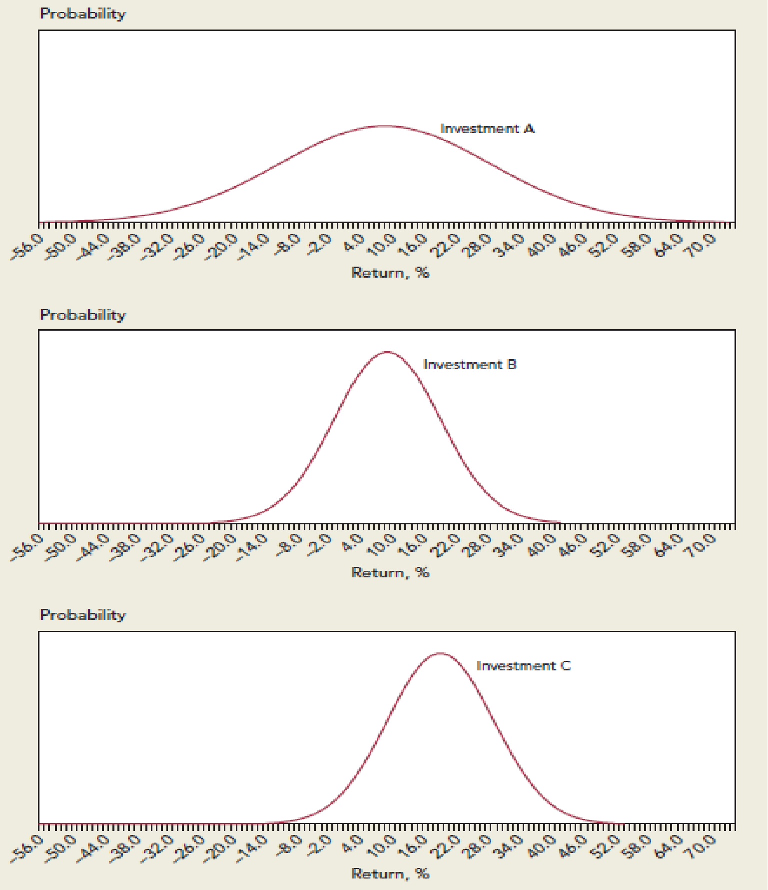
Investment Performance and Return Distribution

- Compare investments A and B. These investments offer an expected returns of 10%. But A has much wider spread of possible outcomes (SD of A is 15% and that of B is 7.5%).
- Compare investments B and C. Both of them have the same standard deviation. However, the expected returns from B (10%) and C (20%) are different.



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Combining Stocks with Portfolios: Part 1

Combining Stocks with Portfolios

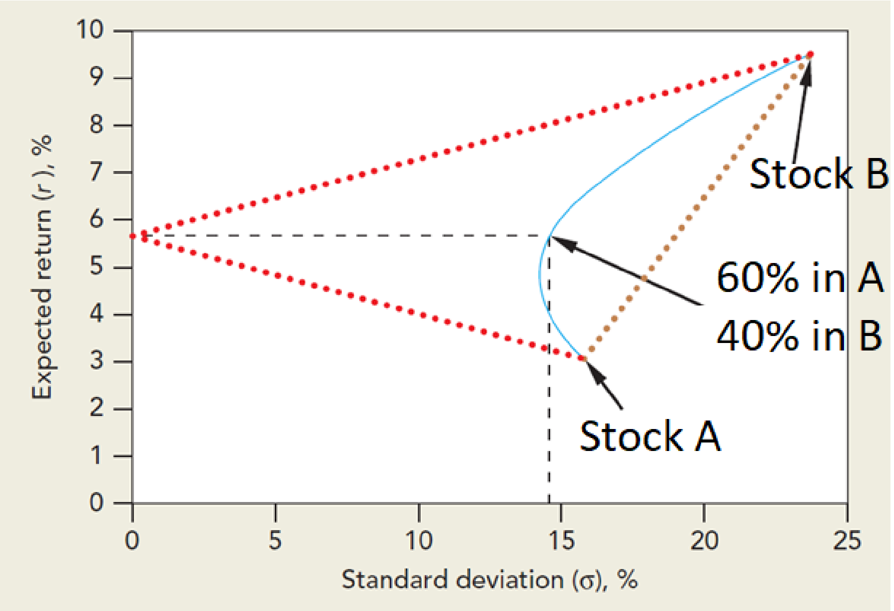
- Consider a scenario where you are examining stocks A and B as potential investments.
- Stock A offers 3.1% expected returns and Stock B offers 9.5% expected returns.
- Stock A has a standard deviation of 15.8% and stock B has a standard deviation of 23.7%.
- You can invest in a combination of these stocks.
- If you invest 60% in stock A and 40% in stock B then the expected return from this portfolio, is 0.60*3.1% + 0.40*9.5% = 5.66%.
- The same can not be said about the risk of the portfolio.

Combining Stocks with Portfolios

- The risk of a portfolio, that is standard deviation (SD), is less than the simple weighted average of individual stock SDs.
- $\text{Variance} = x_1^2\sigma_1^2 + x_2^2\sigma_2^2 + 2 * x_1 x_2 \sigma_1 \sigma_2 = 0.60^2 * 15.8^2 + 0.4^2 * 23.7^2 + 2 * (0.60 * 0.40 * 0.18 * 15.8 * 23.7) = 212.1;$ $\text{Standard Deviation} = \text{Sqrt}(212.1) = 14.6\%$
- The lower amount of SD reflects the diversification aspect, assuming a correlation of 0.18.

Combining Stocks with Portfolios

- The blue curve line shows all the possible expected risk and return combinations of these two stocks that one can achieve.
- A risk averse investor would hold A:B (50:50 or 60:40)
- A less risk-averse investor would invest most of their wealth in B.



- The brown line connecting A and B represents all portfolio combinations with correlation $(\rho) = 1.0$
- With $\rho = -1.0$ (red line), the stocks would move in exact opposite manner.

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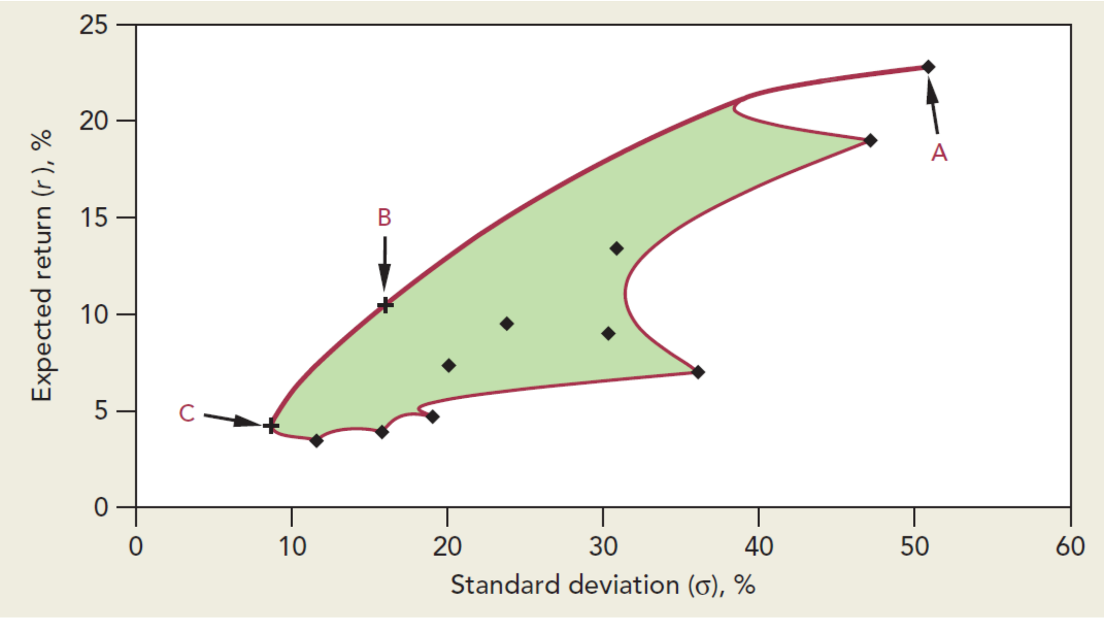
Combining Stocks with Portfolios

- In practice, you invest in many socks, by examining their historical risk-return related properties.
- For example, consider a portfolio of ten securities plotted here using risk-return data.
- The shaded green region shows the possible combinations of expected return and standard deviation by investing in a mixture of these stocks.



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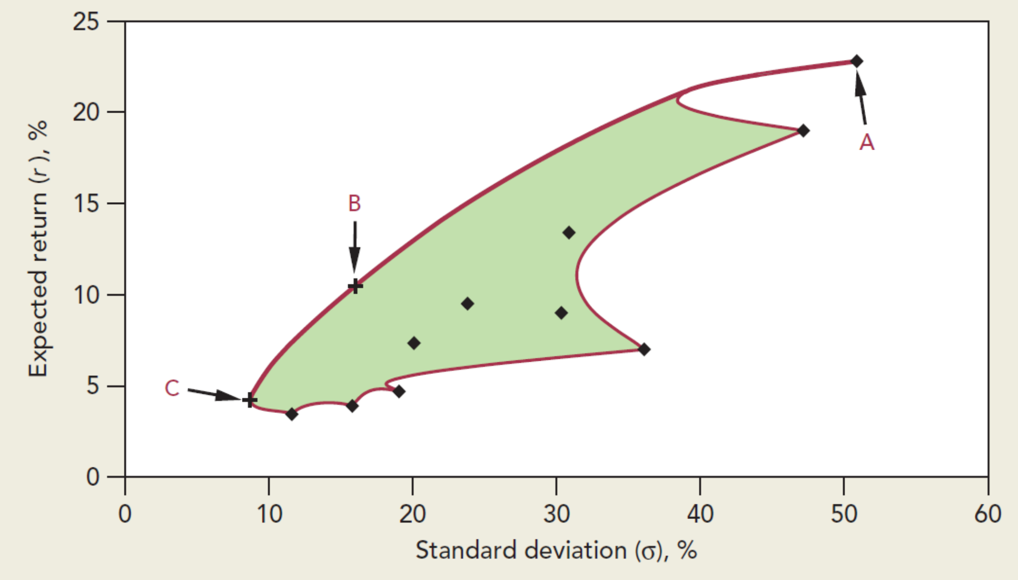
Combining Stocks with Portfolios

- Where would you want to be in that shaded region?
- You would want to go up, that is, increase the expected returns. You would also want to go left, that is, to reduce risk.
- As you move up and left, you end up at the solid dark brown line.



- The portfolio on this solid dark outer surface is often referred to as an efficient portfolio.

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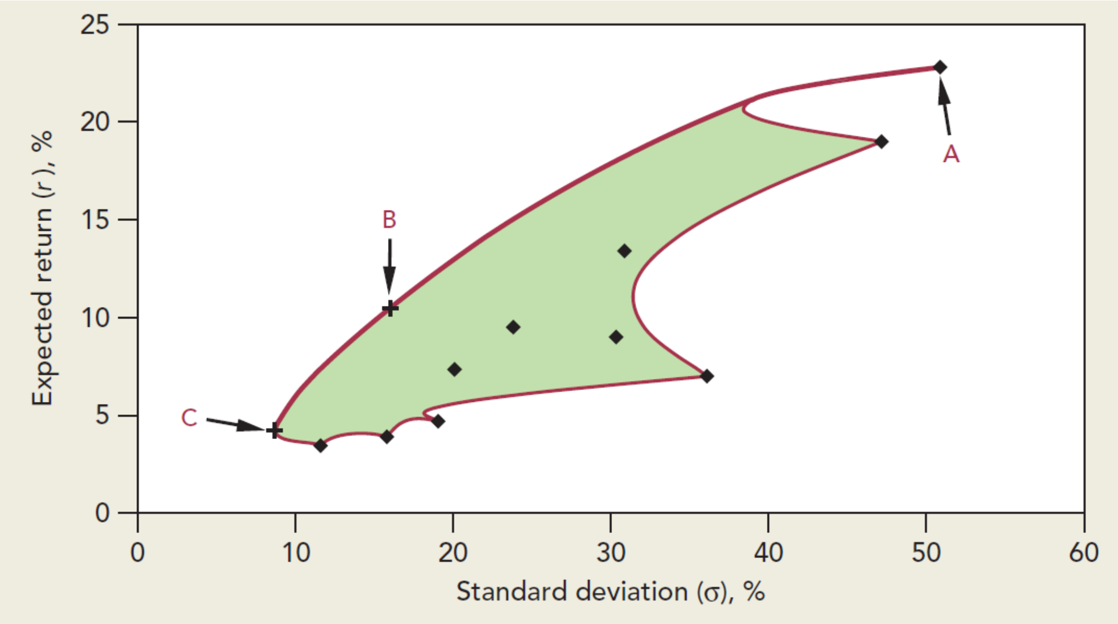
Combining Stocks with Portfolios

- For a given level of risk, these portfolios offer the highest return. And for a given level of return, these portfolios offer the lowest amount of risk.
- Three such portfolios (A, B, and C) are shown in the figure here.



- You want to deploy the investor's funds to generate maximum expected returns for a given level of risk.
- This solution to this problem requires quadratic programming (QP).

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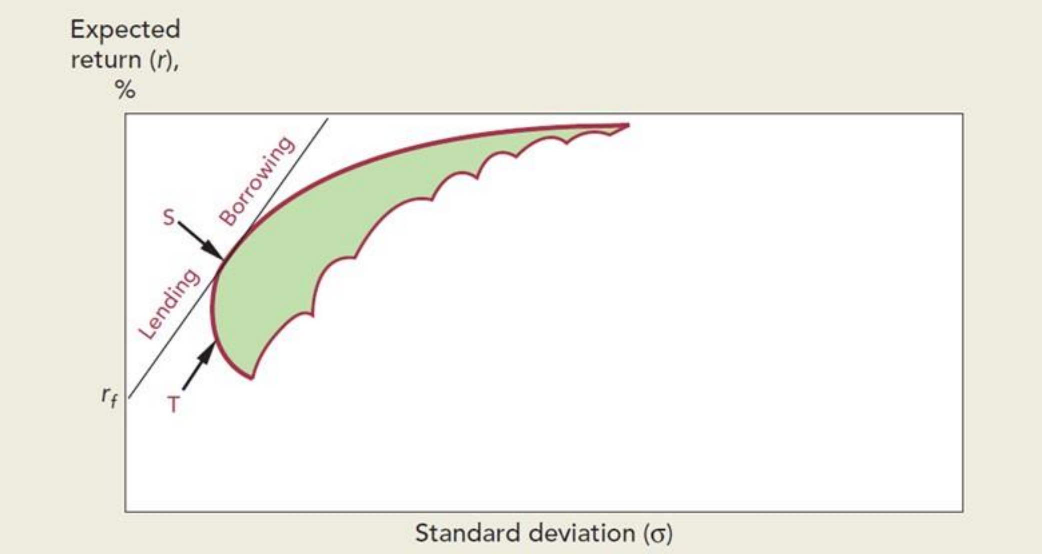
Combining Stocks with Portfolios: Part 2

Combining Stocks with Portfolios

- Now we introduce the possibility of lending and borrowing at a risk-free rate of interest (rf).
- Is this possibility a practical scenario?
- A combination of rf and any efficient portfolio (e.g., S) can offer various risk-return possibilities on the line rf-S.



- Investing in rf and S leads a portfolio on the line segment between rf and S.
- Borrowing at rf and investing the entire amount in S leads a position on rf-S towards the right of S.

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Combining Stocks with Portfolios

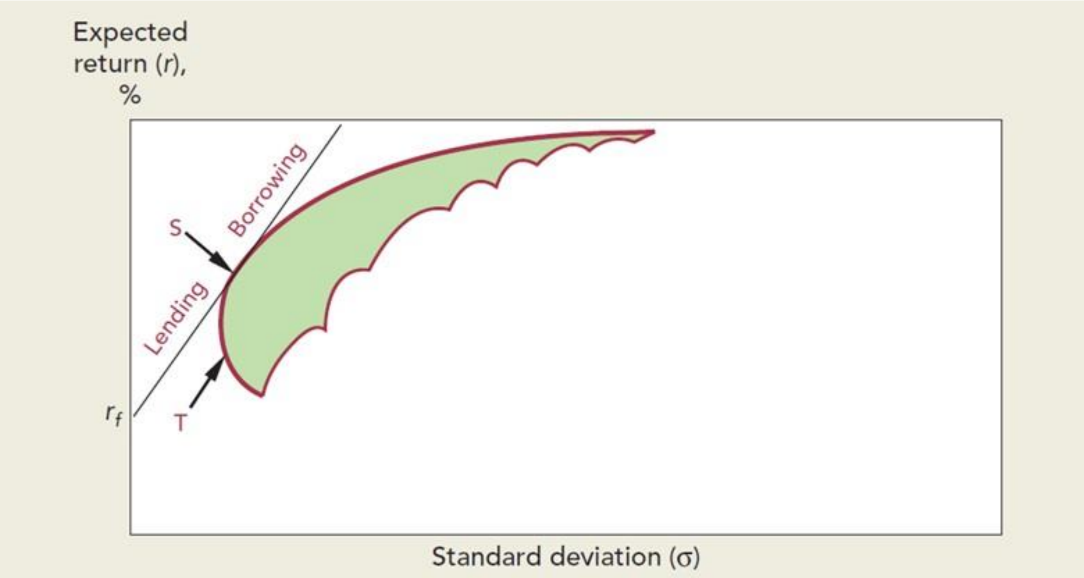
- Suppose that portfolio S has an expected return of 15% and a standard deviation of 16%.
- For risk-free instrument rf = 5% and risk = 0.
- If you decide to invest 50% in S and 50% in rf, the expected return and risk as computed here.
- $r = \frac{1}{2} * Expected\ Return\ on\ S + \frac{1}{2} * Interest\ Rate\ on\ Risk - free$ = 10%
- The formula for computation of risk: $SD = \sqrt{(x_1^2\sigma_1^2 + x_2^2\sigma_2^2 + 2x_1x_2\rho_{12}\sigma_1\sigma_2)}$ [Here, σ_2 = 0]
- $\sigma = \frac{1}{2} * SD\ of\ S$ = 0.5*15% = 8%

Combining Stocks with Portfolios

- Consider another scenario where you borrow at the risk-free rate an amount equal to 100% of your initial wealth.
- You invest your initial 100% wealth along with these borrowings in Portfolio S. That is double the amount of your initial wealth.
- Expected returns: $r = (2 * expected\ return\ on\ S) - (1 * Interest\ rate) = 25\%$
- Risk $\sigma = 2 * SD\ of\ S = 32\%$

Combining Stocks with Portfolios

- On the efficient region, you can always find a portfolio S that is the best efficient portfolio.
- How to find this portfolio?
- The steepest line (from rf) on the curve representing efficient portfolios: tangent line



- This tangent line has the highest ratio of risk-premium to standard deviation: Sharpe Ratio
- $Sharpe\ ratio = \frac{Risk-Premium}{Standard\ Deviatio} = \frac{r-r_f}{\sigma}$

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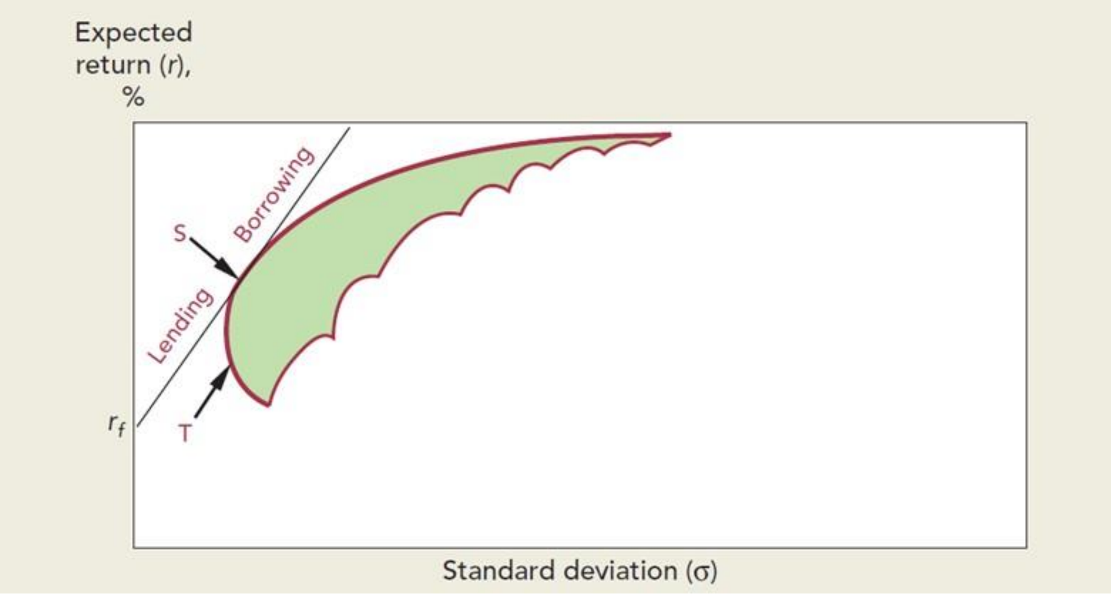
Combining Stocks with Portfolios

- In a competitive market, it is extremely difficult to find undervalued securities.
- Professional investors often investment in benchmark indices (e.g., S&P 500).
- This is often referred to as the passive strategy of investment.



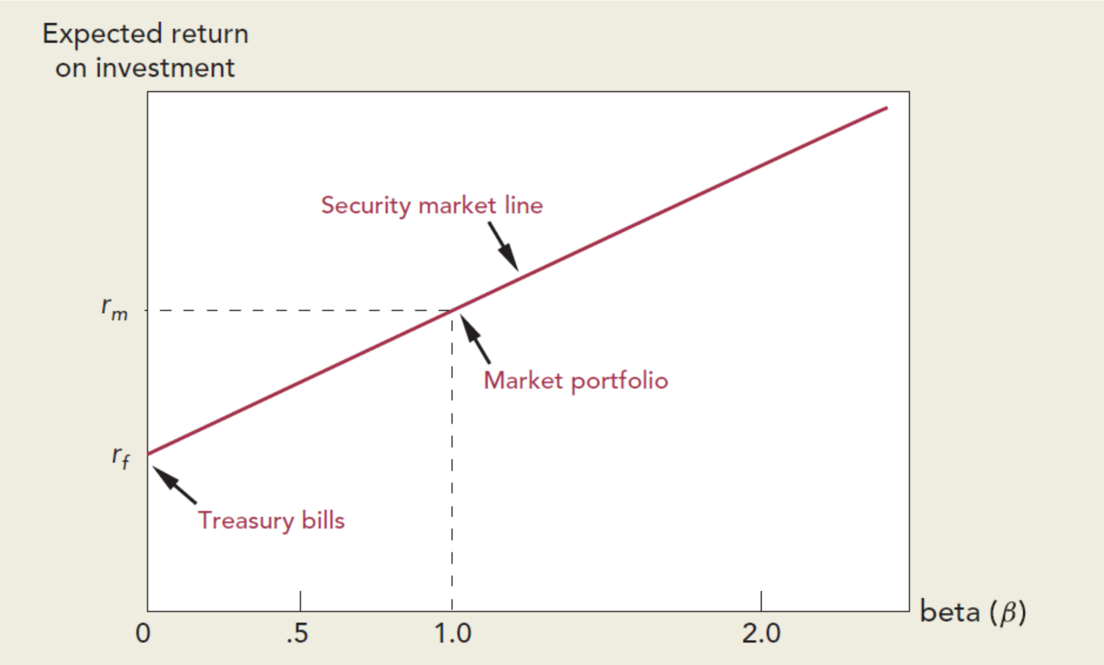
Introduction to CAPM

Introduction to CAPM

- We have previously examined the returns on different instruments.
- T-Bills have a beta = 0, and the market portfolio has a beta = 1.
- Difference between market risk (rm) and risk-free rate (rf) is often referred to as market risk premium.
- Using these benchmarks, we can determine the risk-premium for instruments for which beta is neither 0 nor 1.

Introduction to CAPM

- In 1960s, three economists, Sharpe, Lintner, and Treynor came-up with this model called Capital Asset Pricing Model (CAPM) that provides an extremely simple and easy to use solution for the asset pricing problem.
- In a competitive economy, the risk-premium is directly proportional to beta.



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Introduction to CAPM

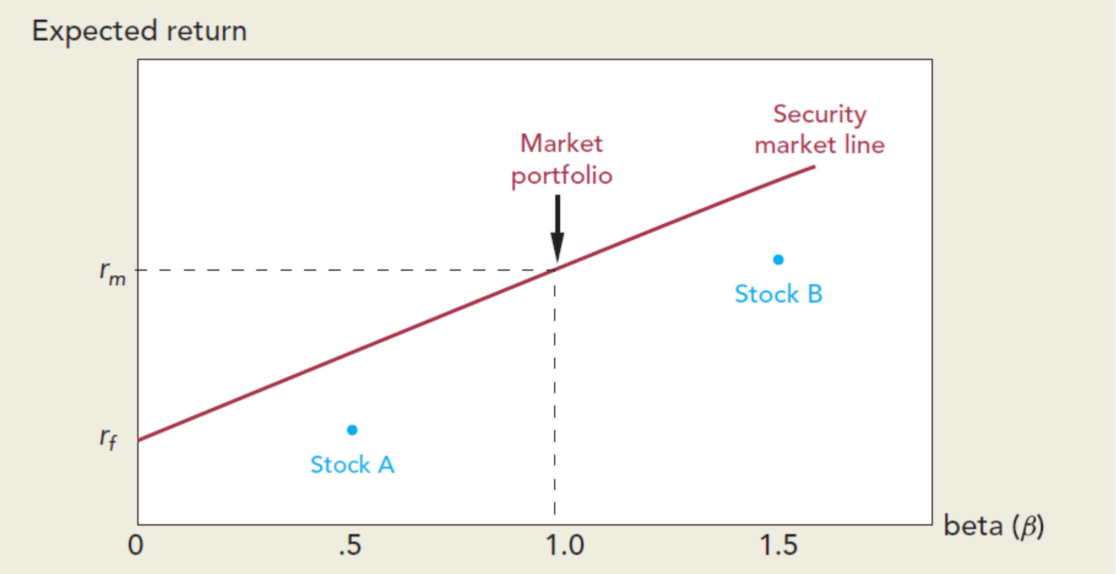
- The risk-premium on an investment with beta of 0.5 should be half of that available on the market.
- The expected risk-premium on an investment with beta of 2 is twice the risk-premium expected on the market.
- The resulting relationship is shown here: $r - r_f = \beta * (r_m - r_f)$
- Consider two stocks with beta of 0.30 (Stock A) and 2.16 (Stock B). You also observe that the market is offering a current risk-premium of 7% $(r_m - r_f)$ and the current treasury bill rate is 0.2%.
- $r_A = r_f + \beta * (r_m - r_f) = 0.20\% + 0.30 * 7\% = 2.30\%$
- $r_B = r_f + \beta * (r_m - r_f) = 0.20\% + 2.16 * 7\% = 15.32\%$

Introduction to CAPM

- CAPM can also be employed to estimate discount rates for risky projects and companies.
- To estimate discount rates, different risk factors, appropriate benchmark for risk-free rates needs to be estimated.
- The following principles are sacrosanct:
  - Investors like higher expected returns and low risk.
  - If the investors can lend and borrow at risk-free rate of interest, then one portfolio is better than all the other portfolios.
  - This best efficient portfolio depends on (a) expected returns, (b) standard deviation, and (c) correlations across securities.
  - In a well-diversified portfolio, only systematic risk matters.

Introduction to CAPM

- If stocks A and B (overvalued) do not fall on this line, then you will not buy them.
- Given the less demand and excess supply, the prices of A and B will fall until the expected returns lie on SML.
- The same logic applies to undervalued stocks as well.



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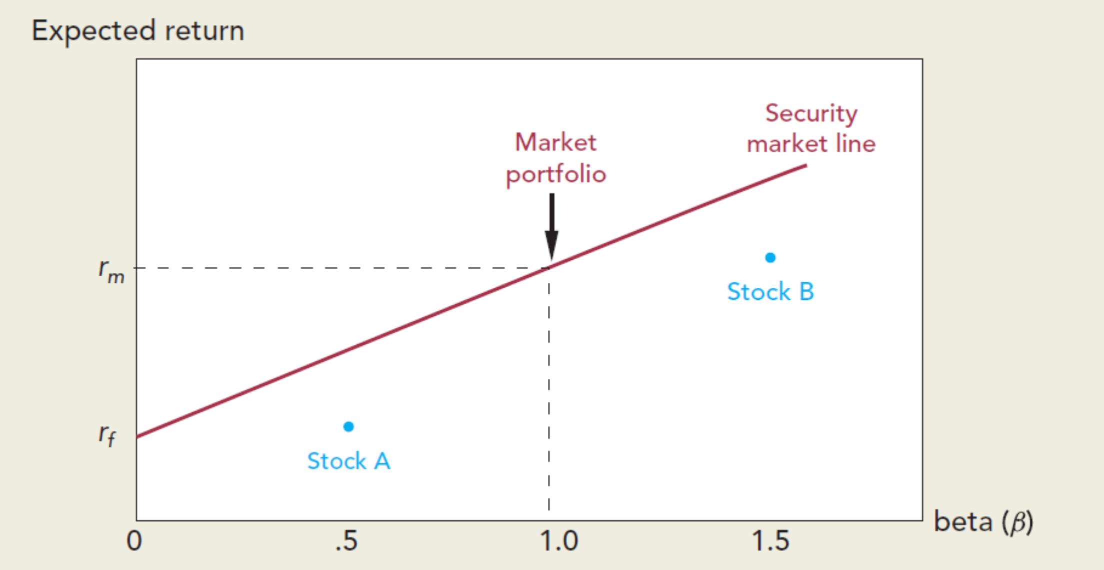
Introduction to CAPM

- Investors can hold a combination of market portfolio M and risk-free rate rf, to obtain an expected return $\bar{R} = r_f + \beta(r_m - r_f)$
- In well-functioning liquid and efficient markets, nobody will hold a stock that offers anything less.



- Equilibrium is obtained from the arbitrage mechanism, which drives prices towards efficient values, that is, towards this SML.

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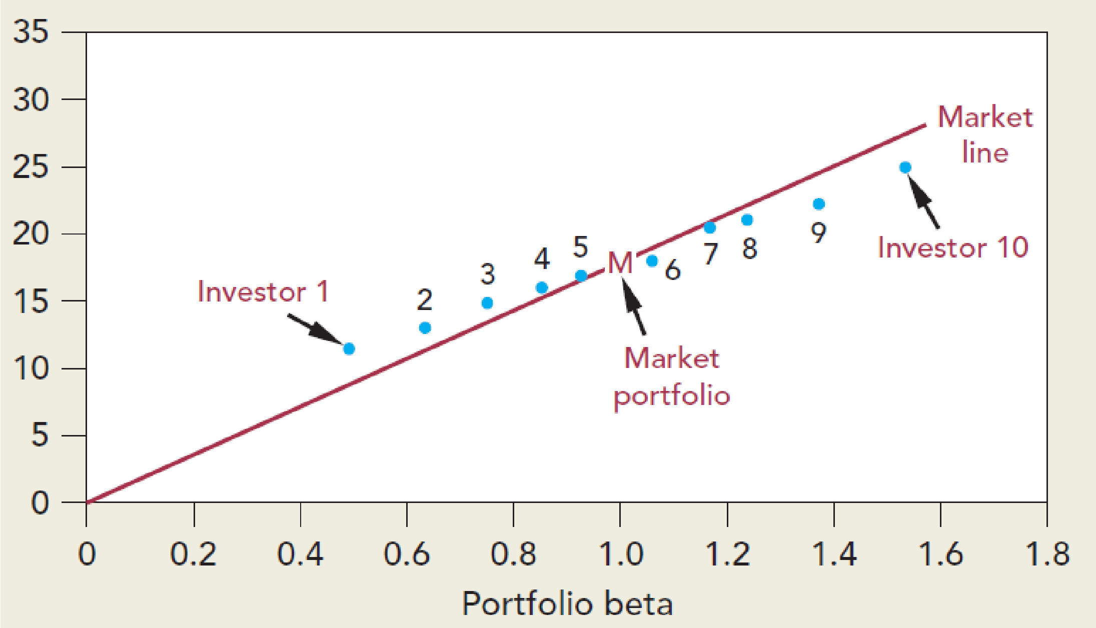
Validity of CAPM

Validity of CAPM

- Any economic model aims to provide a simple view of actual and real-world scenarios.
- There is a trade-off that the real thing may be far-away from the model if the model is too simple.
- Otherwise, the complexity has to be increased to make it closer to the real thing.
- Investors are rational, risk-averse individuals that require extra-return for taking on additional risk.
- Investors do not worry about those risks that can be diversified.
- The power of CAPM lies in its extreme simplicity, and it also has some pitfalls.

Validity of CAPM

- Ten investors portfolio returns are plotted.
- Investor 1 has a portfolio of mostly small stocks and Investor 10 has a portfolio of large-cap stocks.
- One can obtain by combining Investor 1 (long) and investor (10) to generate a zero-risk portfolio that offers excess abnormal returns.



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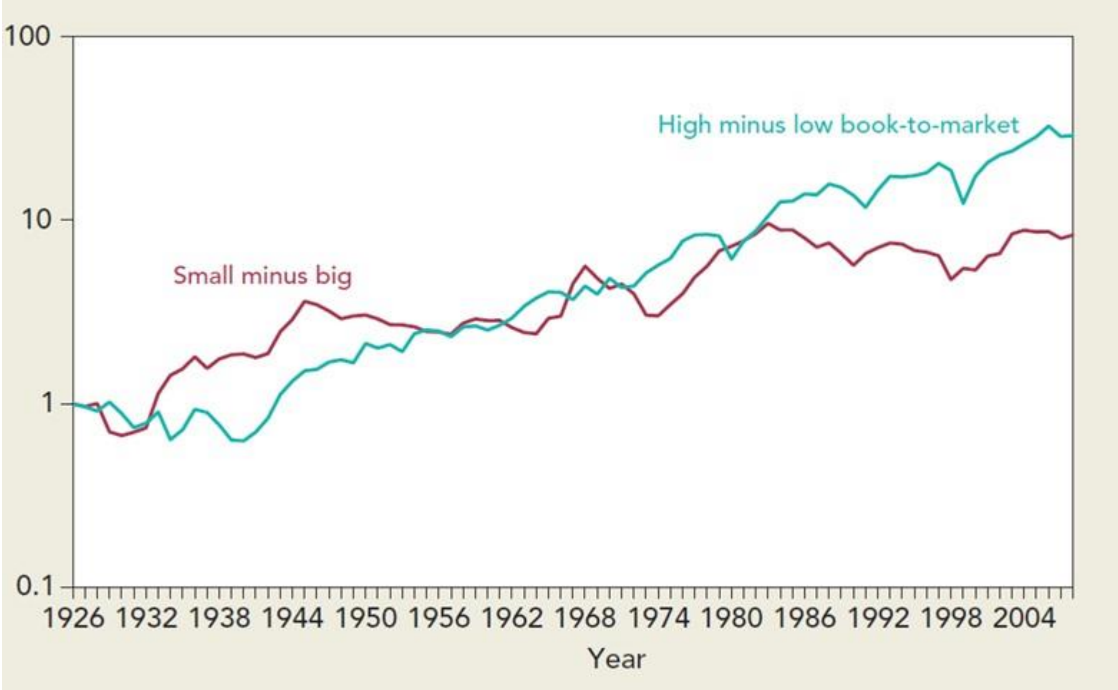
Validity of CAPM

- The red line shows the cumulative difference between small and large cam firms.
- The green line shows the cumulative difference between high book to value (Value stocks) minus low book to value stocks (Growth stocks).
- The figure does not fit well with CAPM postulations: that is beta is the only factor causing returns to differ across instruments.



Validity of CAPM

- Value stocks are underpriced cheap stocks. They may be underpriced at current P/E ratios for different reasons.
- Growth stocks are not cheap stocks at current P/E levels.
- The returns on value stocks minus growth stocks, on average, are often positive, and significant over long-term.
- This does not fit well with CAPM.

Alternative Theories of Asset Pricing

Alternative Theories of Asset Pricing

- CAPM considers investors are rational risk-averse investors that only consider expected return, risk, and correlation structure as relevant factors.
- However, investors often behave in irrational manner.
- Arbitrage Pricing Theory (APT) incorporates broad macroeconomic factors in asset pricing.
- It does not require efficient portfolios.
- $Return = a + b_1(r_{factor_1}) + b_2(r_{factor_2}) + b_3(r_{factor_3}) + \cdots + noise\ term$

Alternative Theories of Asset Pricing

- The APT theory does not provide any information on what these factors may be
- One set of risks, that are on account of these APT factors can not be eliminated with diversification
- PT theory suggests that expected risk premium on a stock should depend on the risk-premium associated with each of these factors and the stock's sensitivity $(b_1, b_2, b_3..)$
- $Expected\ risk - premium = r - r_f = b_1(r_{factor_1} - r_f) + b_2(r_{factor_2} - r_f) + \cdots b_n(r_{factor_n} - r_f)$

Alternative Theories of Asset Pricing

- As per APT, a well diversified portfolio that is not sensitive to any risk factor must be priced to offer a return that is same as risk-free rate.
- A portfolio’s expected return is directly proportional to its sensitivity to these risk factors.
- A stock’s contribution to a portfolio depends upon its sensitivity to the broad macroeconomic influences, often referred to as factors in APT parlance.
- CAPM and APT give similar results if the factors considered in APT have sensitivity to market portfolio.

Alternative Theories of Asset Pricing

- In CAPM, market portfolio plays a very important role as it is supposed to capture all the relevant influences.
- Identifying this portfolio is difficult, however, APT does not require identification of this market portfolio.
- APT can be tested only with a small number of risky assets.
- APT does not tell any information about these factors.
- Fama-French three-factor model is a very prominent example of APT.
- $r - r_f = b_{market}(r_{market}) + b_{size}(r_{size}) + b_{btm}(r_{btm})$

Summary and Concluding Remarks

Summary and Concluding Remarks

- Investors try to increase the expected returns and reduce the risk on their portfolios.
- A portfolio that gives the highest expected return for a given standard deviation, or the lowest standard deviation for a given expected return, is known as an efficient portfolio.
- The best efficient portfolio (tangent) has the highest risk-premium to standard deviation, i.e., Sharpe ratio.
- As per CAPM, the expected return and risk-premium are defined by the following model: $\bar{R}_i - R_F = \beta(\bar{R}_M - R_F)$
- A stock's marginal contribution to portfolio risk is measured by its sensitivity to changes in the value of the portfolio.

Summary and Concluding Remarks

- The capital asset pricing theory is the best-known model of risk and return
- However, other risk factors appear to explain the returns as well
- APT offers an alternative theory of risk and return, i.e., expected risk premium depends on the exposure of a portfolio to various macroeconomic systematic factors
- One example of APT is Fama-French three factor model which considers: (a) Market, (b) Size, (c) Book-to-market (BTM).

Thanks!