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NPTEL ONLINE CERTIFICATION COURSE

QUANTITATIVE INVESTMENT MANAGEMENT

LECTURE 6

Arbitrage Free Pricing of Bonds

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INTRINSIC VALUE OF AN INSTRUMENT

WHAT IS INTRINSIC VALUE?

- Intrinsic value of an asset is the ingrained worth of an asset
- as computed by a potential investor using an objective model
- e.g. DCF analysis, Black Scholes model of option pricing etc.

- **Intrinsic value is arrived at by means of**
- **an objective calculation or**
- **complex financial model**
- **rather than using the currently trading market price of that asset.**

INTRINSIC VALUE IS INVESTOR SPECIFIC

- Intrinsic value is not only security-specific but also investor-specific.
- Every investor makes his own estimates about the inputs required for the model that he chooses to apply and arrives at his assessment of intrinsic value.

- **If we use the DCF model for intrinsic value, we discount the cash flows at the rate that encapsulates the investor's own risk perception of the security.**

INTRINSIC VALUE & MARKET TRADES

- A COMPARISON OF INTRINSIC VALUE WITH MARKET PRICE ENABLES IDENTIFICATION OF MISPRICED SECURITIES AND HENCE, POTENTIAL INVESTMENT OPPORTUNITIES.

KEY TAKEAWAYS

- In financial analysis, intrinsic value is the calculation of an asset's worth based on a financial model.
- By comparing intrinsic value with CMP, mispriced securities that may constitute potential investment opportunities are identified.

APPROACHES TO BOND VALUATION
ARBITRAGE FREE PRICING

INTRINSIC VALUE IN THE AFP MODEL

- **Intrinsic value as per the Arbitrage Free Pricing model of a financial security is the present value of all future cash flows attributable to that security discounted at the rate that is representative of the risk profile of these cash flows**

$$V_0 = \sum_{t=1}^{T} \frac{C_t}{\left(1 + S_{0t}\right)^t}$$

RATIONALE OF THE AFP FORMULA

- Consider two investment portfolios:
- Portfolio A:
- A 2-year bond valued at P_A at t=0 that yields:
- Cash flow of C_1 at the end of first year (t=1)
- Cash flow C_2 at the end of the second year (t=2)

- Portfolio B:
- A deposit of an amount $P_1 = C_1/(1+S_{01})$ at t=0 for one year @ S_{01} yielding $C_1 = P_1(1+S_{01})$ at the end of one year (t=1) and
- A deposit of an amount $P_2=C_2/(1+S_{02})^2$ at (t=0) for two years @ S_{02} yielding $C_2 = P_2(1+S_{02})^2$ at the end of two years (t=2)
- We assume for the moment that receipt of C_1 at the end of the first year and C_2 at the end of the second year is default free from both portfolios A & B.

- Then, for both portfolios A & B:
- The cash flows at the end of first year (t=1) & second year (t=2) are both identical.
- The riskiness of the recovery of cash flows from both portfolios is the same (riskfree).
- There are no cash flows except at t=0, t=1 and t=2 years
- Thus, the portfolios A & B must cost the same at all times so that:
- $P_A = P_1 + P_2 = C_1/(1+S_{01}) + C_2/(1+S_{02})^2$

GENERALIZATION

MATURITY RISK

We can split our initial investment V_0 in the bond into T parts V_t, $t = 1, 2, \ldots T$ with part V_t being invested for t years and yielding the cashflow C_t at the end of year t so that

$$C_1 = V_1(1+S_{01}),\ C_2 = V_2(1+S_{02})^2, \ldots, C_T = V_T(1+S_{0T})^T$$

with $V_0 = \sum_{t=1}^{T} V_t = \sum_{t=1}^{T} \frac{C_t}{(1+S_{0t})^t}$

WHY INDICATE INTEREST RATES WITH INDICES? TERM STRUCTURE OF INTEREST RATES

$$C_1 = V_1(1+S_{01}),\; C_2 = V_2(1+S_{02})^2,\ldots, C_T = V_T(1+S_{0T})^T$$

$$\text{with } V_0 = \sum_{t=1}^{T} V_t = \sum_{t=1}^{T} \frac{C_t}{(1+S_{0t})^t}$$

INTEREST RATES ARE USUALLY A FUNCTION OF MATURITY. THIS PHENOMENON IS CALLED TERM STRUCTURE OF INTEREST RATES

STEPS IN VALUATION OF BONDS

- To value bonds we need to:
- Estimate future cash flows
- Size (how much) and
- Timing (when)
- Assess the risk of realizing these cash flows
- Select the appropriate discount rate based on risk assessment
- Discount future cash flows at an appropriate rate:

- **Bond cash flows are largely non-discretionary and determined by the contract of issue.**
- **For assessing the riskiness of the realizability of these cash flows and default probabilities recourse may be had to the instrument's creditr ratings.**

SEMIANNUAL COUPONS

- **Adjust the coupon payments by dividing the annual coupon payment by 2**
- **Adjust the discount rate by dividing the annual discount rate by 2**
- **The time period *t* in the present value formula is treated in terms of 6-month periods rather than years, hence double the number of periods.**

VALUE OF BOND WITH SEMI-ANNUAL COUPONS

$$V_0 = \frac{C_{1/2}}{\left(1+\frac{S_{0,1/2}}{2}\right)^1} + \frac{C_1}{\left(1+\frac{S_{0,1}}{2}\right)^2} + \frac{C_{3/2}}{\left(1+\frac{S_{0,3/2}}{2}\right)^3} + \ldots + \frac{C_T}{\left(1+\frac{S_{0,T}}{2}\right)^{2T}}$$

$$= \sum_{t=1}^{2T} \frac{C_{t/2}}{\left(1+\frac{S_{0,t/2}}{2}\right)^t}$$

The factor of ½ appears in the denominator because even the half-yearly rates are quoted on annualized (per annum) basis.

IMPORTANT

- **The factor of ½ appears in the denominator because even the half-yearly rates are quoted on annualized (per annum) basis.**
- **The compounding time point is assumed to coincide with the coupon payments.**

EXAMPLE

- Calculate the intrinsic value of a 10% semi-annual bond of the face value of INR 1,000 that has exactly 1.50 years to maturity. The bond has just made a coupon payment and the spectrum of risk adjusted interest rates is as follows:
- 6 months maturity: 8% p.a.
- 12 months maturity: 9% p.a.
- 18 months maturity 10% p.a.

Intrinsic Value of the Bond

$$V_0 = \frac{50}{\left(1+\frac{0.08}{2}\right)} + \frac{50}{\left(1+\frac{0.09}{2}\right)^2} + \frac{1050}{\left(1+\frac{0.10}{2}\right)^3}$$

$$= 48.0769 + 45.7865 + 907.0295 = 1,000.8929$$

BONDS VALUATION WITH FORWARD RATES

FORWARD RATES

- Forward rates are yields for future periods.
- A forward rate is a borrowing/lending rate for a loan to be made at some future date.

NOTATION FOR FORWARD RATES

- The notation used must identify both
- when in the future the money will be loaned/borrowed and
- the length of the ending/borrowing period.
- Thus, f_{12} is the rate for a 1-year loan one year from now; f_{23} is the rate for a 1-year loan to be made two years from now; f_{35} is the 2-year forward rate three years from now; and so on.

RELATIONSHIP BETWEEN SPOT & FORWARD RATES

- **To avoid arbitrage, *borrowing for T years at the T-year spot rate, or borrowing for one-year periods in T successive years, should have the same cost.***