IIT ROORKEE

NPTEL ONLINE CERTIFICATION COURSE

QUANTITATIVE INVESTMENT MANAGEMENT

LECTURE 4

Arbitrage

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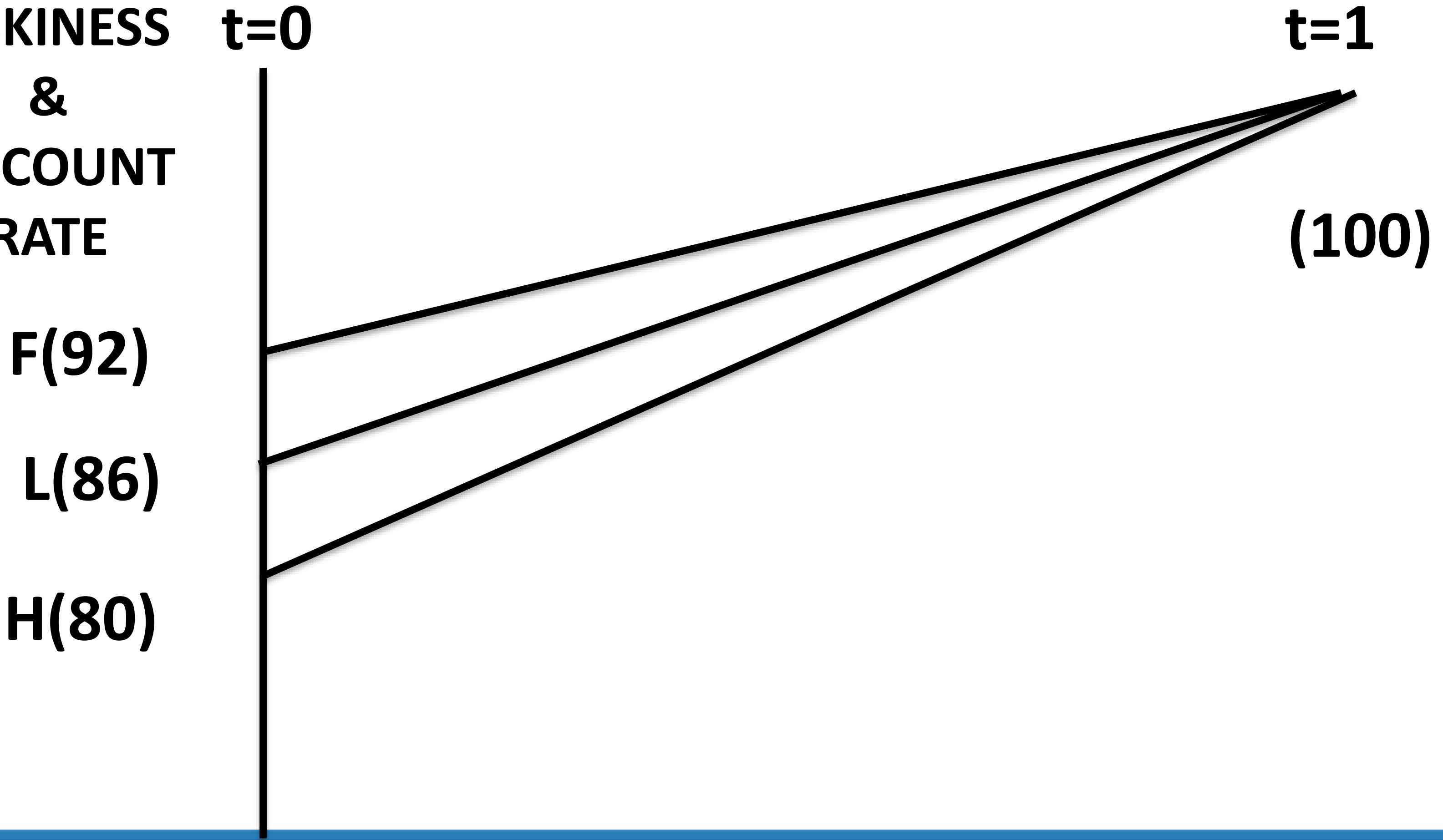
RISKINESS, PRICE & RETURN

- Part of the information the market has about an asset is its riskiness.
- Thus the riskiness is already included in the price, and
- Because rational investors view riskiness as an undesirable attribute of a security, riskiness will reduce the price.
- Thus, the price of a risky asset taking into account its riskiness must be lower than that of a risk-free asset.
- Indeed, greater the riskiness, lower the price.

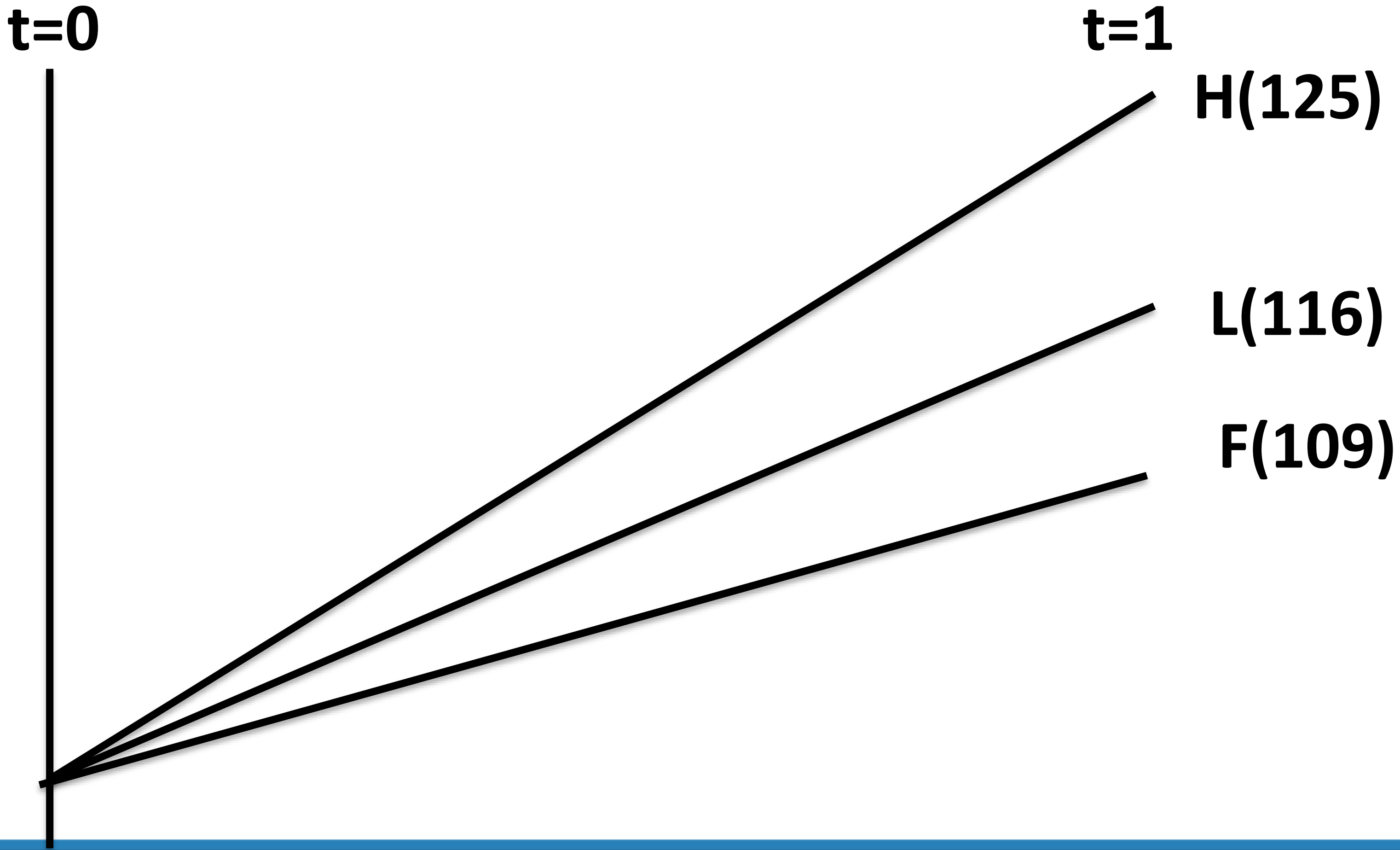
RISKINESS
&
DISCOUNT
RATE



DISCOUNT RATE & RISKINESS

- Thus, an asset's price i.e. its present worth reflects the value it is likely to have in the future reduced by a factor depending upon its riskiness called the discount rate.
- $For\ Asset\ F: r_F = \frac{100-92}{92} = 8.69\%$
- $For\ Asset\ L: r_L = \frac{100-86}{86} = 16.28\%$
- $For\ Asset\ H: r_H = \frac{100-80}{80} = 25.00\%$

RISKINESS & EXPECTED RETURN

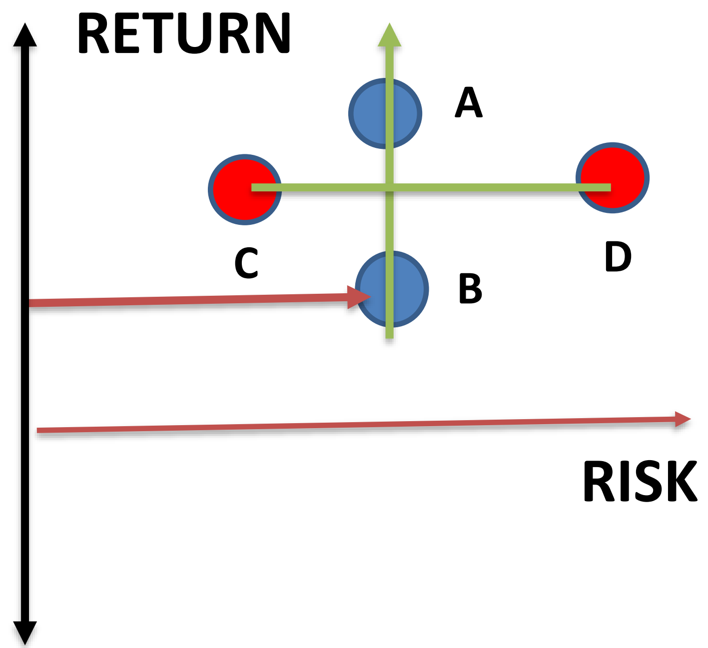


- **This means that for equally priced assets now, in a year from now we can expect the risky asset to be worth more than the less risky one.**
- **So increased riskiness means greater returns, but only on average - it also means a greater chance of losing money.**

ARBITRAGE

ARBITRAGE

- If $R_A > R_B$
- Invest in A, disinvest B
- Demand for A goes up,
- Demand for B declines.
- Price of A shoots up,
- Price of B slumps
- Returns on A decrease ($r = dP/P$)
- Returns on B increase.



LAW OF ONE PRICE

- **In the absence of interfering & distorting factors (e.g., liquidity, financing, taxes, credit risk), identical sets of cash flows should sell for the same price.**
- **“Identical” implies “perfect substitutes”.**

WHAT IS “IDENTICAL”

- “Identical” implies identical in all respects including:
- identical outcome in every possible state of nature and hence, an identical probability distribution of outcomes.
- “Identical” also implies identical interactions with the entire universe of securities.

ARBITRAGE BASED RELATIONSHIPS

EXAMPLE 1	t=0	t=T	
STATES OF NATURE		ALPHA	BETA
ASSET X	100	0	110
ASSET Y	100	0	120
EXAMPLE 2			
ASSET P	100	0	20
ASSET Q	100	10	0

EXAMPLE 3	t=0	t=T	
STATES OF NATURE		ALPHA	BETA
ASSET A	P_A	0	100
ASSET B	P_B	90	90
ASSET A	P_A	0	100
PORTFOLIO C =ASSET B + BOND B	P_B+ PV(10)	(90+10)=100	(90+10)=100

$$P_B+PV(10)>P_A$$

EXAMPLE 4	t=0	t=T	
STATES OF NATURE		ALPHA	BETA
ASSET A	P_A	90	100
ASSET B	P_B	100	90
ASSET A+BOND	$P_A+PV(10)>P_B$	100	110
ASSET B+BOND	$P_B+PV(10)>P_A$	110	100
NO UNIQUE PRICE $P_B \in (P_A-PV(10),P_A+PV(10))$ e.g. $P_A=95,P_B=88$			

ARBITRAGE: FUNDAMENTAL ISSUES

- A set of transactions can be classified as arbitrage if and only if either (i) the risk remains unchanged or (ii) the return remains unchanged.
- If both the risk and return change, the issue of risk-return trade off may crop up.
- There is no limit to the number of transactions that can be entered into for arbitrage. However, with the increase in the number of transactions, arbitrage profit may be eaten by market frictions.

ARBITRAGE PORTFOLIO

- A portfolio is said to be an arbitrage portfolio, if
- today it is of non-positive value, and
- in the future it has zero probability of being of negative value,
- and a non-zero probability of being of positive value.

- **Simply stated, if a portfolio that has**

1. **zero probability of returning a negative cash flow and**
2. **a non-zero probability of returning a positive cash flow at any future date**
3. **Then, it must entail a cash outflow (cost) to construct today.**
4. **If it does not do so, then it is an arbitrage portfolio.**

THEOREM

- ***If P and Q are riskless zero-coupon bonds with the same face value & maturity time, T, then they are of equal value at all previous times.***

- Without loss of generality, let P be dearer at arbitrary time $t<T$.
- We construct an arbitrage portfolio consisting of P short and Q long i.e. short sell P and buy Q.
- Since, $P_P>P_Q$, we have positive cash flow at time t.
- The cash flows on the long and short positions cancel at maturity.
- Both cash flows at maturity are riskless i.e. certain.
- Hence, the net cash flow at maturity is zero with certainty.
- Hence, the portfolio yields an arbitrage riskless profit