IIT ROORKEE

NPTEL ONLINE CERTIFICATION COURSE

QUANTITATIVE INVESTMENT MANAGEMENT

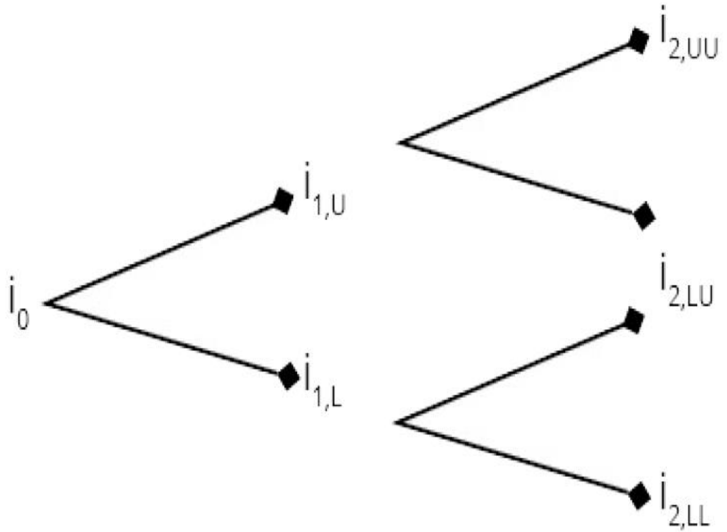
LECTURE 8

Binomial Interest Rate Tree

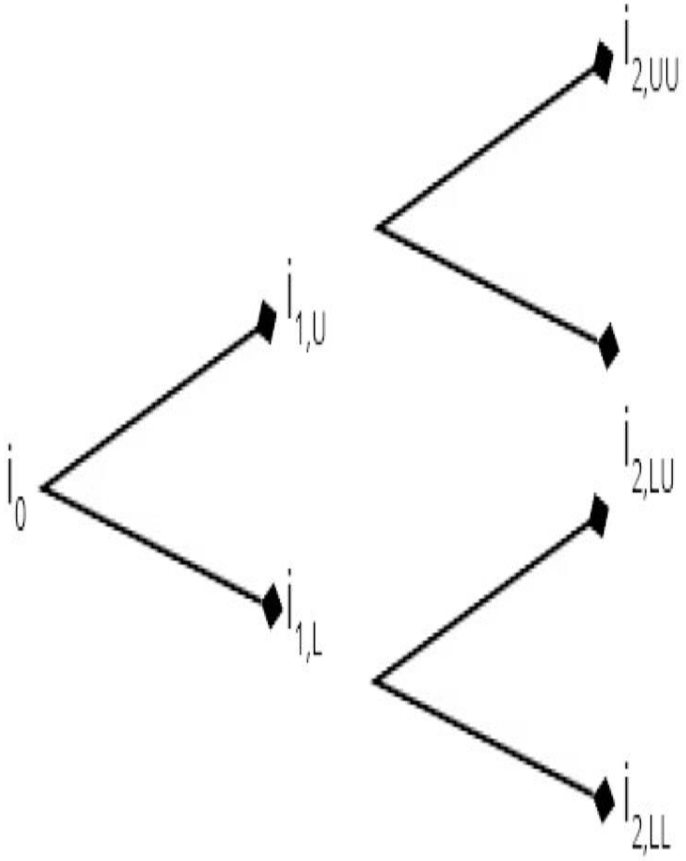
J P Singh

Department of Management Studies

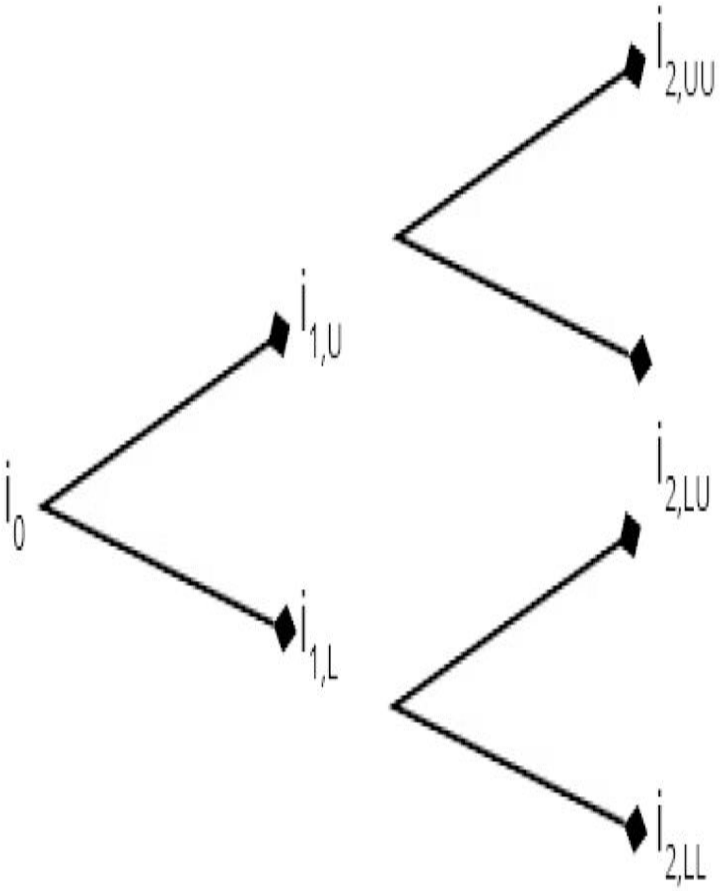
BINOMIAL INTEREST RATE TREE



- The binomial model envisages the following pattern for future interest rates.
- Interest rates have an equal probability of taking one of two possible values in the next period (hence the term binomial).
- Over multiple periods, the set of possible interest rate paths is called a binomial interest rate tree.



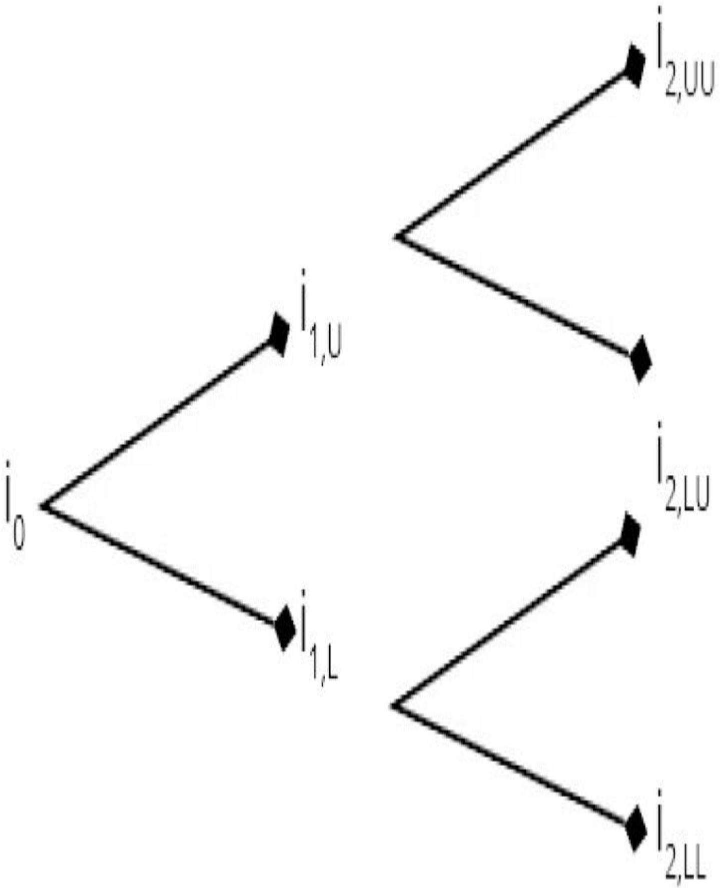
- A node is a point in time when interest rates can take one of two possible paths, an upper path, U, or a lower path, L.
- The tree is constructed by joining the various nodes across time to give interest rate paths.
- The interest rates at each node in this interest rate tree are one-period forward rates corresponding to the nodal period.



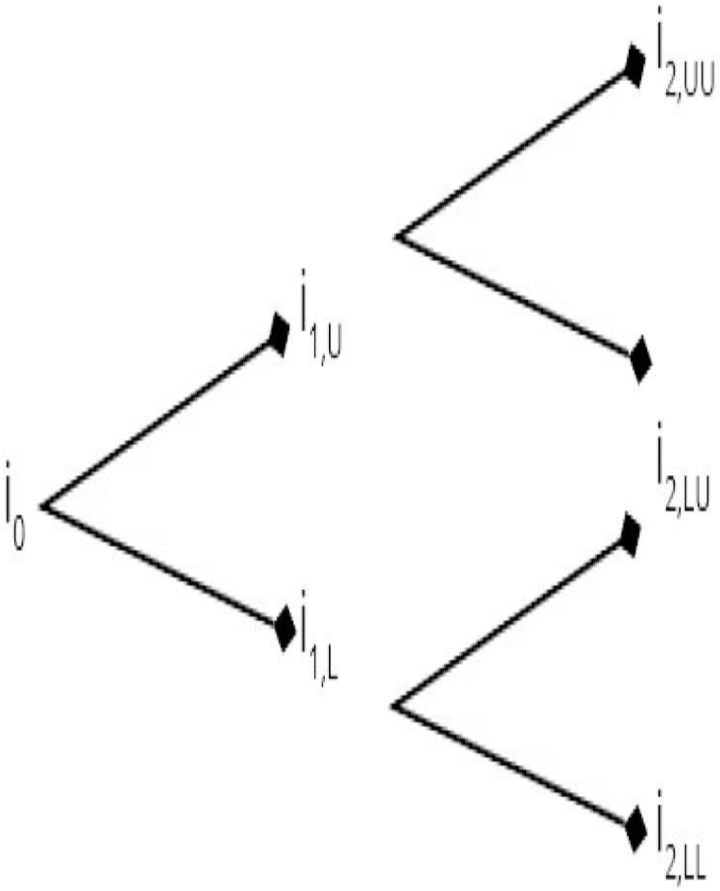
- For example, consider the node on the right side of the diagram where the interest rate $i_{2,LU}$ appears. This is the rate that will occur if the initial rate, i_0, follows the lower path from node 0 to node 1 to become $i_{1,L}$, then follows the upper of the two possible paths to node 2, where it takes on the value $i_{2,LU}$.

TREE CALIBRATION

- We usually calibrate the tree in such a way that:
- $i_{2,LU} = i_{2,LL}e^{2\sigma} \sim i_{2,LL}(1 + 2\sigma + \cdots)$
- where σ is the standard deviation of interest rates (i.e., the interest rate volatility used in the model).



- Thus, each forward rate is a multiple of the other forward rates in the same nodal period.
- Adjacent forward rates (at the same period) are approximately two standard deviations apart.



RECOMBINANT TREE

- **In this model, an upward move followed by a downward move, or a down-then-up move, produces the same result e.g. $i_{2,LU} = i_{2,UL}$.**
- **This is called a recombinant tree.**

RECOMBINANT TREE

- For the first period, there are two forward rates and hence: $i_{1,U} = i_{1,L}e^{2\sigma}$
- Beyond the first nodal period, adjacent forward rates are a multiple of $e^{2\sigma}$:
- $i_{2,UU} = i_{2,UL}e^{2\sigma} = i_{2,LU}e^{2\sigma} = i_{2,LL}e^{4\sigma}$ etc.
- Thus, the relationship among the set of rates associated with each individual nodal period is a function of the interest rate volatility assumed to generate the tree.

- Volatility estimates:
- can be based on historical data or
- can be implied volatility derived from interest rate derivatives.
- The binomial interest rate tree framework is a lognormal random walk model with two desirable properties:
- higher volatility at higher rates and
- non-negative interest rates.

VALUING AN OPTION-FREE BOND WITH THE BINOMIAL MODEL: BACKWARD INDUCTION

- The term "backward" is used because in order to determine the value of a bond today at node 0, we need to know the values that the bond can take at the Year 1 nodes.
- But to determine the values of the bond at the Year 1 nodes, we need to know the possible values of the bond at the Year 2 nodes and so on.
- Thus, for a bond that has N compounding periods, the current value of the bond is determined by computing the bond's possible values at period N and working backwards to Node 0.

- **Because the probabilities of an up move and a down move from any node of a binomial tree are both 50%, the value of a bond at a given node in a binomial tree is the average of the present values of the two possible values from the next period.**
- **The appropriate discount rate is the forward rate associated with the node.**