**Financial Institutions and Markets**
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**Lecture - 42**
**Bond Analysis – II**

So, in the previous class we discussed certain features of the bond, like your coupon, you have the term to maturity, you have the your per value extra, then we started the discussion on the valuation. There what we have discussed that the coupon has to be discounted with respect to a particular record rate of return which is nothing but the discount rate.

Then finally, your par value or the maturity value will be discounted then we can find out the value of the bond and that particular point of time. That means, in general the value of the bond is nothing but the present value of the discounted cash flows.

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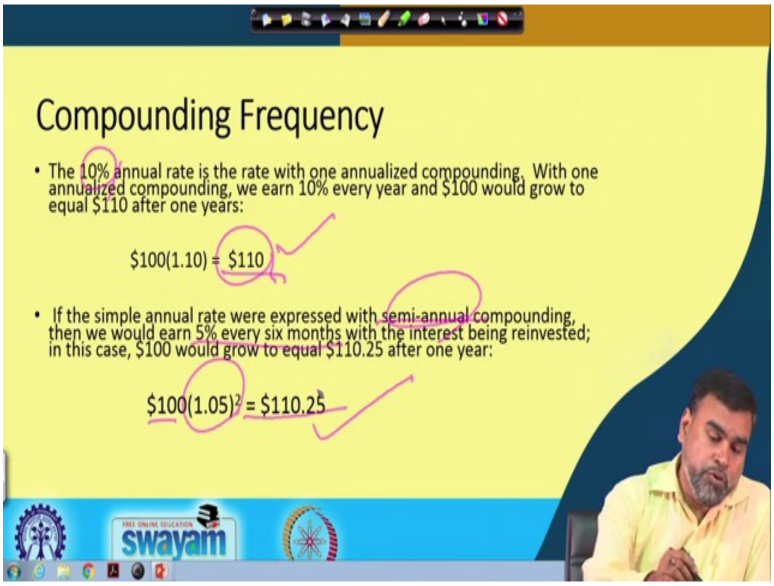


So, today if you see there are certain things which are related to this valuation. What are those? One by one if you see, first of all let us see how the concept of compounding frequency works in that particular case.

If you minutely observe let on the previous example also we are taking the annual interest rate was 10 percent. Then, if it is annually compounding then after 1 year the 100 rupees or 100 dollar will become 110, but late this particular coupon you are paid semi annually and this is compounded semi annual basis. Then there is a 5 percent we would earn 5 percent every 6 months, and that money can be reinvested in the market for another 6 months. You get back your 45 rupees, that 45 rupees can be reinvested in the market again for another 6 months because that 45 rupees you may not be keeping idle.

So, in that case what will happen? First year after end of the first year you are getting 110 rupees, second year your particular value will increase accordingly whenever your money is reinvested in the market. So, if the coupon is paid semi annually then the money whatever you got that can be reinvested in the market for the second half, what you have got in the first half; so in that contest how the value can be calculated? This is nothing but $100 \times 1.05^{2}$, $1 + R^{2}$ that is called 110.25 rupees.

So, the compound you see if it is 10 percent it is annualized compounding then you are getting 110, but if it is semi annually compounded then you are getting 110.25. That means, the money whatever you have received after 6 months that money also again can be or has been reinvested in the market because of that the value of this particular cash flow has increased of the present value of the cash flow can increase. So, the compounding frequency has a very strong role whenever we go for the valuation of any type of bond in the market.

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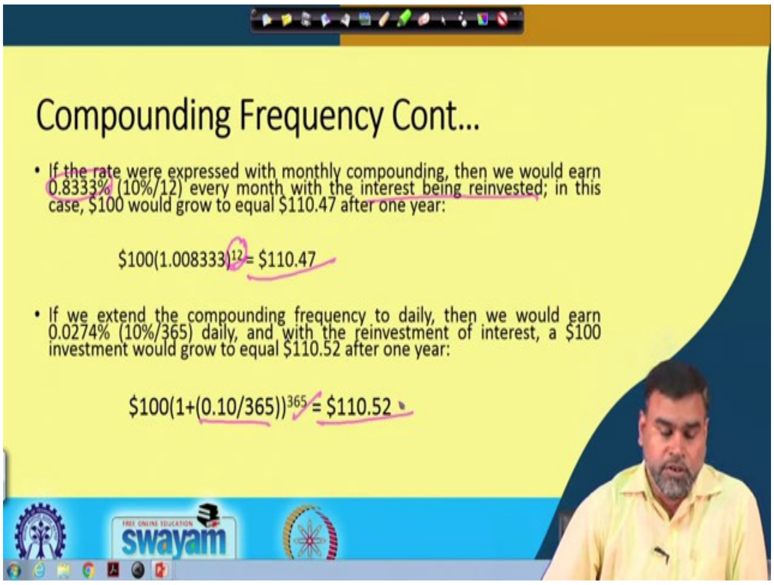

So, let it is monthly compounded. Then let 10 percent of the annual then each month how much you are getting 10 percent divided by 12 that is your 0.8333 percent. And every month the interest being reinvested in the market, then how the value is going to be? 100 dollar into 1.008333 percent means 1.008333 pars 3 3 3 to the power 12 because 12 months then you are getting 110.47.

So, like that if it is daily compounding then further the value is increasing because 0.10 divided by 365 to the power 365 you are getting 110.52. Whenever it is annually compounded you are getting 10 percent, if it is half yearly compounded you are getting 10 percent by 2, 5 percent, but the value has become 110.25. Whenever it is monthly compounded you are getting 110.47, and whenever it is daily compounded you are getting 110.52. So that means, the compounding of that particular interest rate or compounding concept is very much playing the role for the valuation of the bond, that is what basically what we are trying to discuss here.

So, you have to keep in the mind that at what period whether it is compounded or it is simple. So, if it is compounded then obviously, we have to consider that whether this particular money or the cash flow what you are getting in the periodical basis that thing is reinvested in the market or not. If it is reinvested then on the basis of the frequency your value of that particular investment is going to be changed, that actually you have to keep in the mind.

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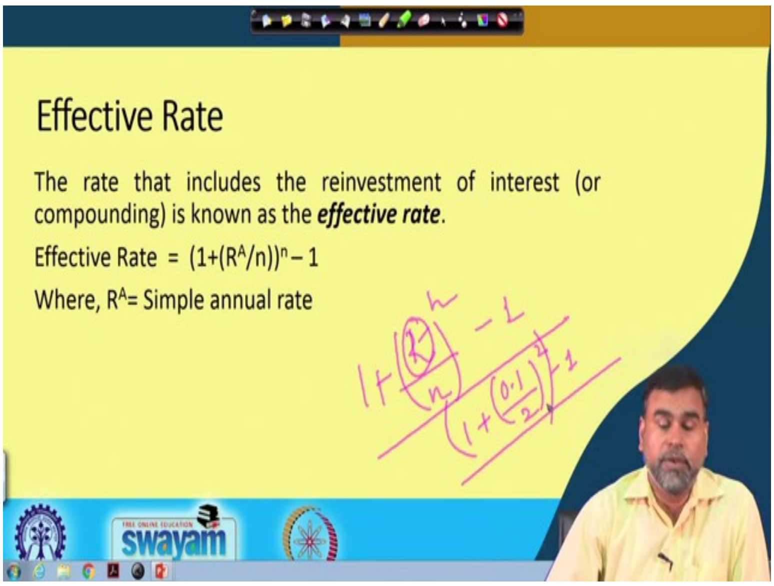

Then, in that basis what basically we can discuss one thing that is called effective rate. So, what is effective rate? The effective rate basically includes the reinvestment of interest or the compounding concept and it is calculated at basically your 1 plus your annual rate of interest divided by n to the power n minus 1. This R is basically those simple annual rate of interest. So, if n is equal to 2 then R is equal to 10, then it is 1 plus 0.1 divided by 2 to the power 2 minus 1. So, this is the way basically this what we can say the effective rate can be calculated or the effective interest rate can be calculated.

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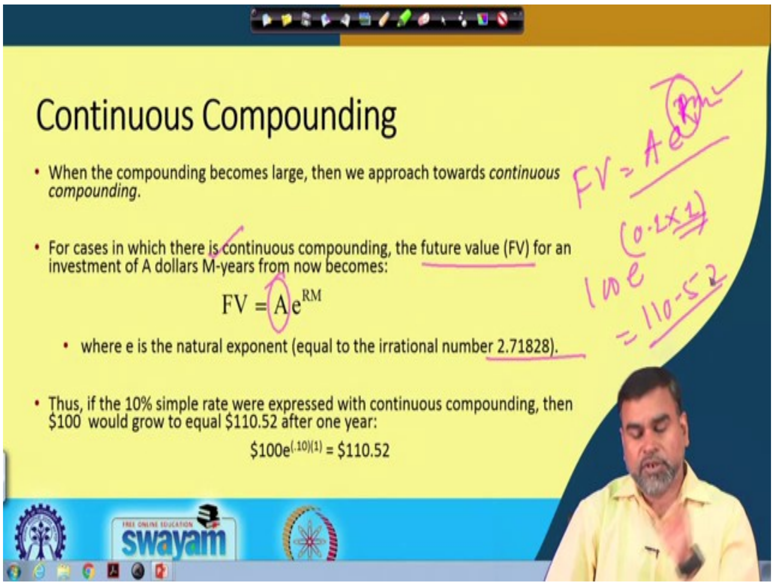


In this case if you see, another concept is it may be continuously compounded. It may be there is a possibility that particular coupon amount or the interest amount can be continuously compounded. So, when the compounding becomes very large then we approach towards the continuous compounding.

Generally, whenever it is daily compounded, we can think of the use of the continuous compounding. That mean the frequency is very small, the payment the coupon frequency is very small, in that particular context the continuous compounding can be used. So, how this continuous compounding basically works here? If you see the continuous compounding if you are going to use that F basically what, the future value of that particular asset the FV future value of that particular bond is let you have started investing dollar A for the M years and R is equal to your rate of interest, then your future value will be $FV \times Ae^{RM}$.

And already you know what do you mean by the e? e is basically a natural exponent which is basically a rational number with values 2.71828, the value of e is equal to 2.71828. So, that concept is used that means, anything whenever it is continuously compounded we always go for e to the power R M. That e to the power R M means this R is basically your rate of interest or the required rate of return and M is equal to basically your maturity period.

So, if you go by this example let it is continuously compounded, if the 10 percent simple interest rate were expressed with continuous compounding then after 1 year the 100 dollar will go to 110.52. So, here how it is calculated? This $100e^{0.1 \times 1}$, 1 year maturity that is why it is 110.52. That means, in the continuous compounding process the value is always more than whenever we are compounding it in the simple basis.

So, continuous compounding a straight basis, basically whenever we are calculating the value that value will be different than whenever the cash flows are compounded with respect to that interest rate in a continuous basis. So, that is what basically we want to explain here that is continuous compounding is very much important whenever you go for the valuation or we calculate the present value of any asset including the point.

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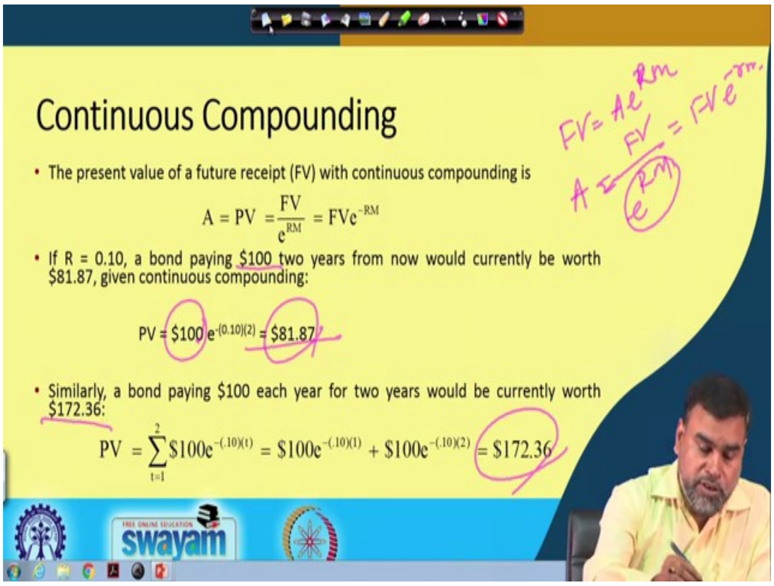


Then now so how you can calculate the present value of that particular asset? Which is nothing A you have to calculate, you know the future value then how you can present value? Simply you can go by this F, your FV was Ae $^{RM}$. So, now, here A is equal to what A is equal to F /e $^{RM}$, then it is nothing but your FV e $^{-RM}$

So, now if you take this example if R is equal to 10 percent a bond paying 100 dollar in 2 years from now, the value of the bond after 2 years will be 100 dollar, then it would currently how much is the value of that particular bond today, if you are giving a continuous compounding. Then what is the thing? You got this future value that is 100 dollar which is nothing but the power e $^{-0.1}$ × 2 divided it is basically you got it 81.87. That means, if you want to get 100 dollar after two years and there is a continuous compounding process involved in that then you can buy the bond at the price of 81.87 dollar.

That means, similarly if a bond paying 100 $ each year for 2 years then the value will be little bit different, then here you have to take t is the summation t is equal to 1 to 2, 100 $ e $^{-10}$ × t that is nothing but 100 $ e $^{-0.1 \times 1}$ + 100 $ e $^{-0.1 \times 2}$ then the value will be 172. So, you have invested 172 rupees today or each year yours basically for 2 years then you are worth of that particular investment will be 172 that means, to get 200 rupees you have to invest 172.36 rupees. So, that is the way basically the concept of continuous compounding always works.

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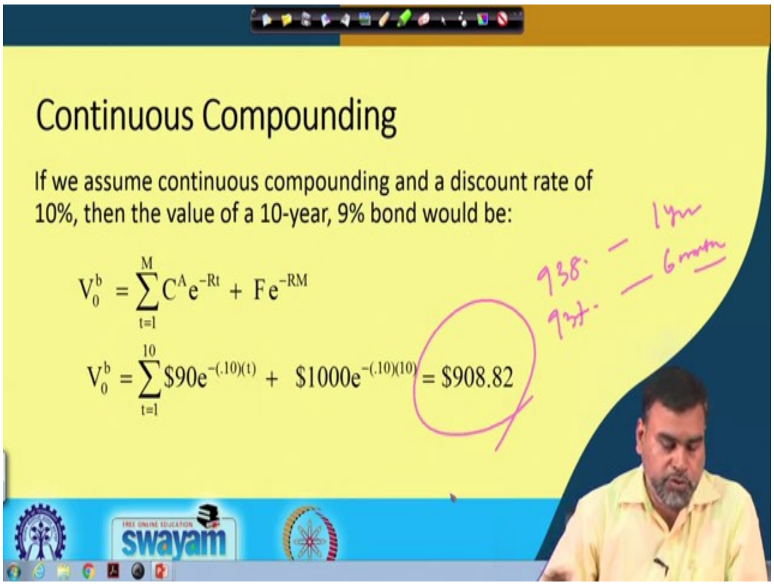


So, now if you assume that there is a continuous compounding and discount rate 10 percent then value of a 10 years bond, coupon of 9 percent what example we have taken from the beginning. So, if you go by calculating that thing in the continuous compound way then the value of the bond will be 908. If you remember this was something around 938 rupee point something in the case of 1 year coupon and 937 point something in terms of the 6 months coupon or 2 times coupon. But even if here what we are getting if it is continuously compounded then the value of the bond will be 908.82.

So, this is what that is why whether it is continuously compounded or not that we have to examine before if it is continuously compounded then the value of the bond calculation is different from than the straight forward discounting concept what you are using whenever you calculate the plain vanilla bonds. So, continuous compounding has a significant role for the valuation of the bond.

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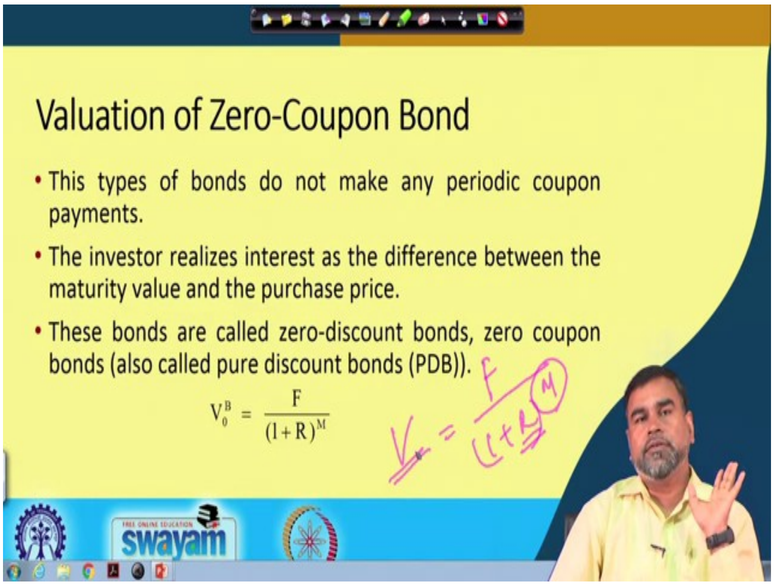


Then there is another type of bond we call it the zero-coupon bond. Then what is that zero-coupon bond? The zero-coupon bond means it does not pay any coupon in between. It has a purchase price and it has a maturity price. They do not make any periodic coupon, the investor basically what they get they get the interest as the difference between the maturity value and the purchase price on that basis the interest rate can be calculated from these or yield of that particular bond also can be calculated from this. The bonds also can be called the zero-discount bond, zero-coupon bond or also pure discount bond. There are different ways there are different names of the zero-coupon bond we can observe or we can say in the market.

So, then how the valuation of this particular bond or the value of that particular bond in the current market price of the bond can be calculated? So, this is basically calculated in this way the value is equal to your F means that is basically your maturity value divided by 1 +R $^{M}$, R is equal to the discount rate or the rate of interest involved in that and M is equal to your maturity period. So, there is no cash flow involved in that. So, because there is no cash flow involve in that this particular value of the bond can be calculated by F /1 +R $^{M}$ that actually you keep in mind.

So, then let us see that how basically it works in practical sense.

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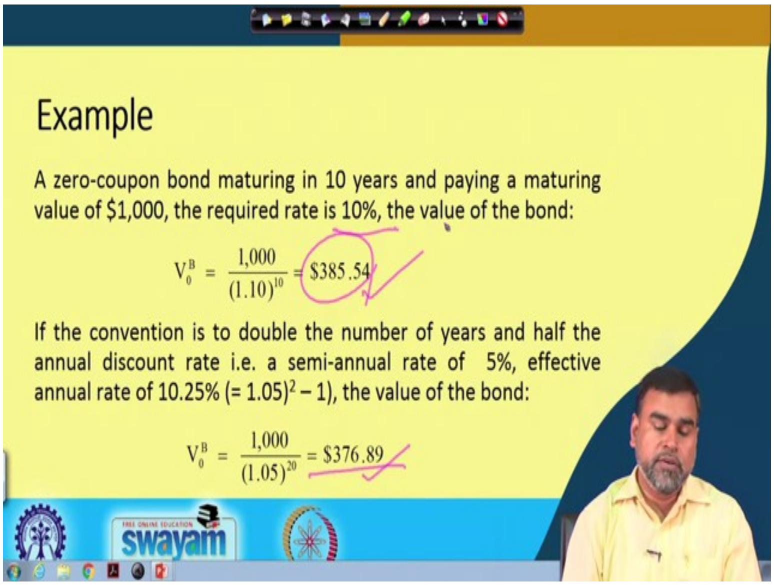


So, if you take this example a zero-coupon bond maturing in 10 years and paying a maturing value or maturity value of 1000 dollar and the required rate of return is 10 percent in the same example, then you can get 1000 / 1.8, 1.1 $^{10}$ because the required rate of return is 10 percent. So, now 10 year is the maturity period then the value of the bond is 385.54.

So, if the convention is to double the number of years and half the annual discount rate that means, the semi annual rate of 5 percent and already you know what is effective annual rate, the effective annual rate will be 1 + R /2 $^{n}$ - 1 that means, it is your in this case 10 % / 2 % that is being 0.5 percent then 1.5 %, 1.05 $^{2}$ -1 in the case of it is 2 times minus 1 then that will give you the value of the bond will be 376.

So, here also what we have seen that conventionally if you go by doubling the number and half the annual discount rate and a semi annual rate of 5 percent. We can say that the effective rate is 10 point 2, 5 percent then the value of the bond will be 376.89, and in case of annual payment or in case of annual discounting we get it 385.54; so this why the valuation of zero-coupon bonds takes place in the market.

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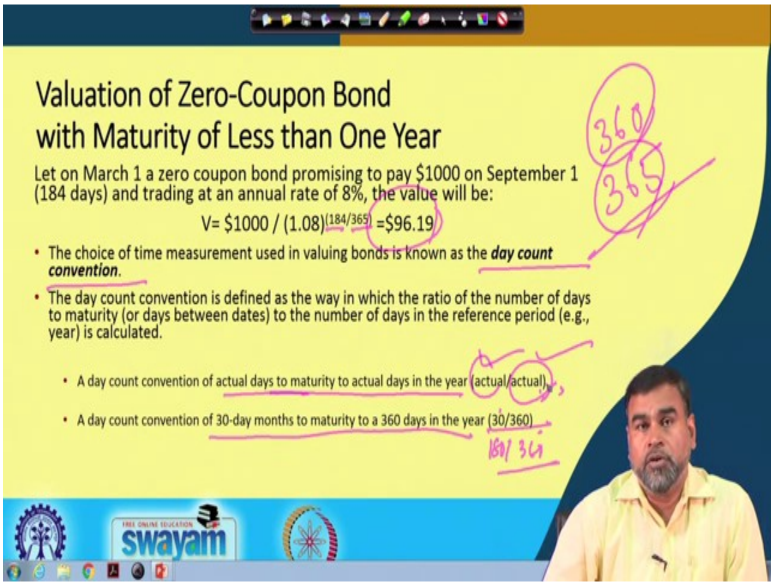


But here another thing if let the maturity period is less than one year of the zero-coupon bond, right. Let on March first zero-coupon bond promising to pay 1000 $ on September 1, that means, your March 1 to September 1, 184 days and trading at an annual rate of 8 percent. Then what is the value? That means, your 1000 / 1.08 $^{184/365}$, you remember we are here assuming 1 year is equal to 365 days, but somebody can also use 360 days, then your value becomes 96.19.

So, the choice of time measurement used in value of the bonds in known as the day count convention. What do you mean by the day count convention? The day count convention means either you can take 360 days or you can take 365 days. If you are assuming every month as 30 days, then conventionally you can talk about 360 days, 360 days. If you are assuming that actual the particular year is 365 days, we can go for the 365 days. So, concept is called the day count convention.

So, the day count convention is defined as the way in which the ratio of number of days to maturity to the number of days in the reference period is calculated. Here in this case majority period 184 days and we have taken 365 days in the whole year, then it is 184 by 365. So, that is why if it is actual days too much actual days to maturity to actual days in the year it is actual by actual. If it is 30 days months to maturity to a 360 days in the calendar it is the 30 / 60 it is 180 it can be 180 /360, it also can be 180 / 360, 6 months or it can be 184 /365.

So, depending upon the convention what you are considering your calculation would be different or will be varying. So, that is basically the way through which we can calculate the value of a bond which maturity period is less than 1 year. That actually you keep in mind the same way where we are calculating the value of the bond for the maturity more than 1 year the same way basically, we cannot calculate we have to consider the day count convention for this.

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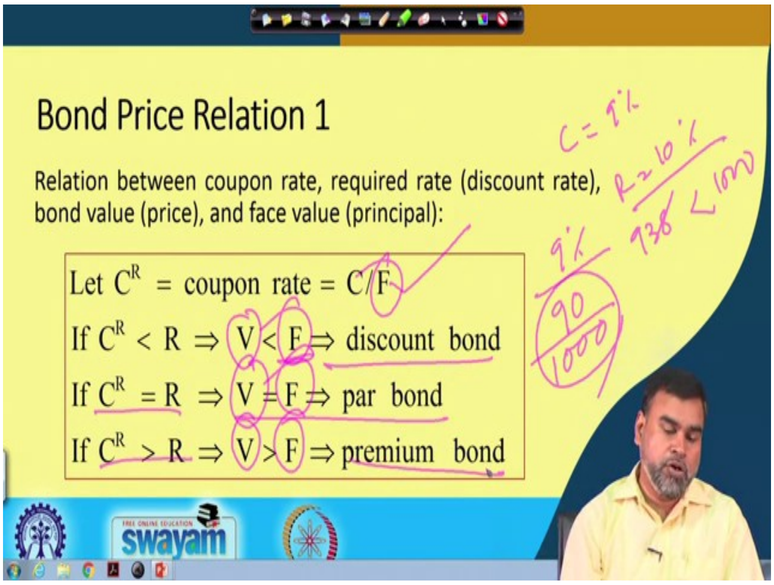


Then, we have let us see that what the relationship between coupon, your discount rate and the bond value, and also the face value is. Already we know that what do you mean by the coupon, coupon is nothing but the coupon divided by the face value. If you see this example what we have seen 9 percent. How it you got? 90 rupees is the coupon divided by your 1000, ok, that is basically you got it the coupon rate.

So, coupon rate is C / F. F means it is the face value or the par value. But one thing if your coupon rate is less than R, then the value of the bond will be less than the face value. In the previous example what we have seen? Your coupon was 9 percent C and R was 10 percent. So, because of that with 1 year coupon payment the value was 938, that means, it is less than 1000 rupees. So, that is called the discount bond. But if the coupon is equal to the R discount rate then the bond is at par it is called the par bond, the bond is valued at par that means, the value of the bond is equal to the par value of the bond or the phase value bond; in that contest we call it the par bond. But if your coupon is greater than R then the value of the bond will be more than the face value, then that time we call it is a premium bond, the bond is basically valued at premium.

So, depending upon the relationship between the coupon and the face value of the bond, we can say that whether the bond is a discount, bond value the discount, bond is redeemed at par or the bond will be redeemed with premium that actually bond the value at premium par or the discount. That thing we can judge on the basis of the relationship between the coupon and the discount rate that actually my intention, that is called the bond price relation 1.

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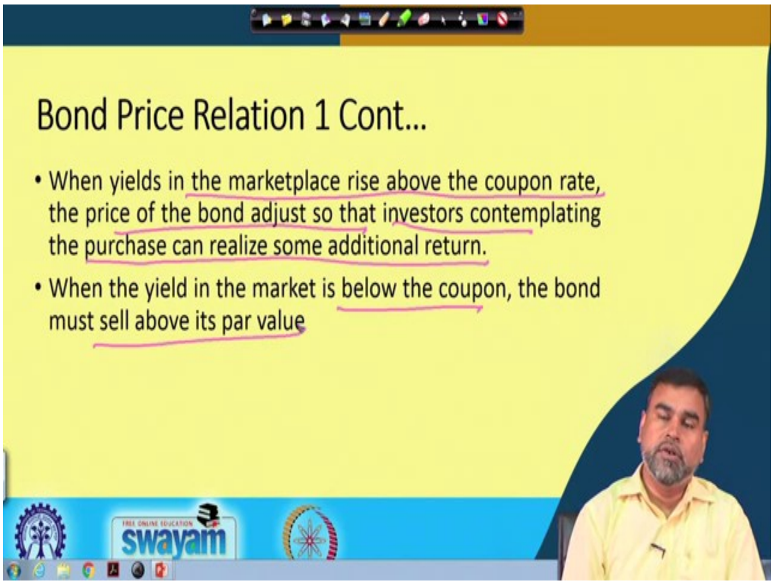


So, there are some other relations involved in this that let us see that how those things work. So, here what basically we got it? When the yield in the market price rise above the coupon rate the price of the bond adjust, so that the investors who purchase can realize some additional return, when the yield in the market below the coupon the bond must sell above its par value. That actually the implications what we got from the relationship between the coupon and the value of the bond with respect to the par value of the bond, that is the way basically it is interpreted or this is basically analyzed.

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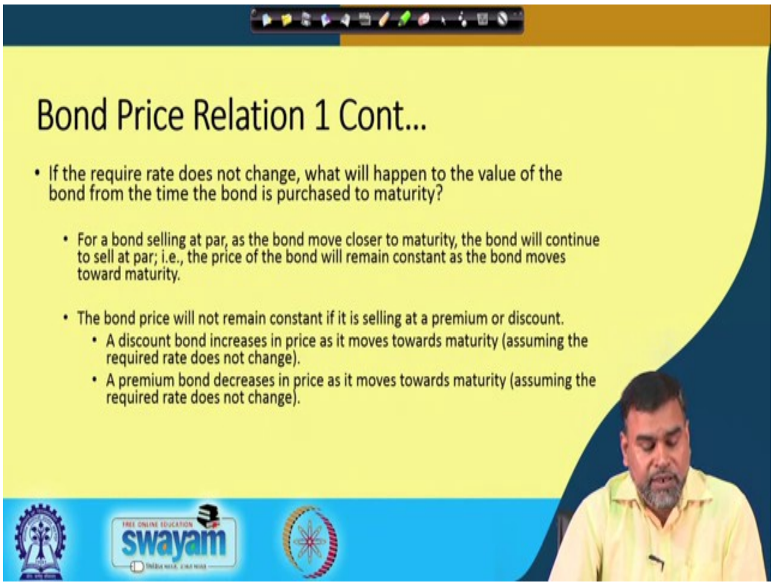


Then, let us see that how if the required rate of return does not change then what will happen to the value of the bond from the time the bond is purchased to maturity. Let the discount rate does not change. If the discount rate does not change then for a bond selling at par as the bond physically closer to the maturity the bond will continue to sell at par that means, the price of the bond will remain constant as the bond moves towards maturity.

But if the bond price is selling at a premium or the discount at that particular point of time then the bond price will not remain constant. A discount bond increases its price as it moves towards the maturity assuming the required rate does not, and a premium bond decreases its price as it moves towards maturity assuming the required rate does not change. So, in the beginning whether it is a discount bond or premium bond that will decide that whether the price is going to increase or going to decrease of that particular bond. So, a discount bond increases its price as it moves towards the maturity and a premium bond decreases its price as it moves towards the maturity. If you see you can observe this thing.

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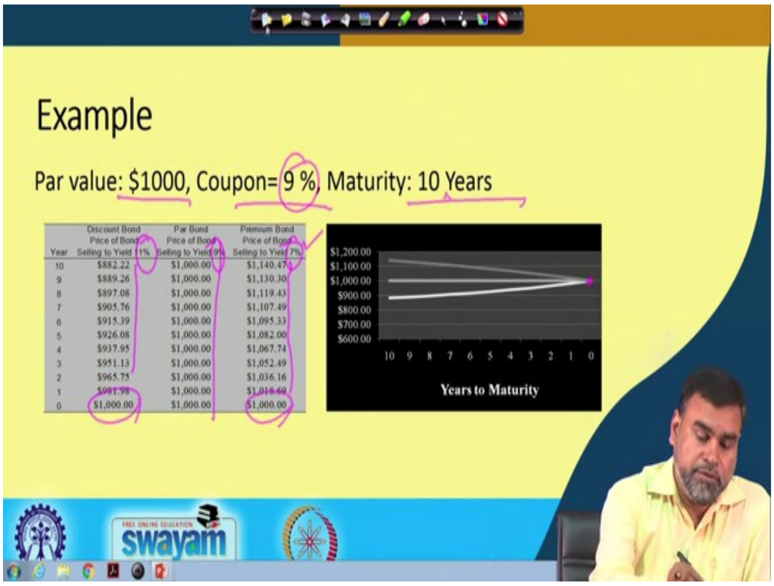


So, let the yield or the required rate of return from the bond here it is a 11 percent, here it is 9 percent, here it is 7 percent, par value is 1000, coupon is 9 percent, maturity is 10 years. So, whenever now the coupon is 9 percent at require rate of return is 9 percent, so the bond is at par, then everywhere the price 1000 over the period the price is 1000. But whenever you have seen that the yield rate has gone out from or the discount rate has gone out from 10 percent to 11 percent what basically we have observed, the value have changed then, but at the end the value basically has merged with the particular initial value that is 1000 rupees. There is a convergence which can happen in this case.

The premium bond also the same thing whenever the coupon is basically 9 percent now the l is 7 percent you get the value was increased and then for the decline, decline then finally, there is a conversion that is 1000 rupees, 1000 dollar. So, years to maturity then you will find there is a convergence basically happening here at the end. This is what basically always how many times the cash flow you are getting and whenever the cash flow end you will find that the price of the bond will be equal to the maturity value whatever it is mentioned in the beginning. So, if the bond is issued a discount or bond is issued at premium that does not make any sense if you are holding the bond up to the maturity, that actually always happens in the market.

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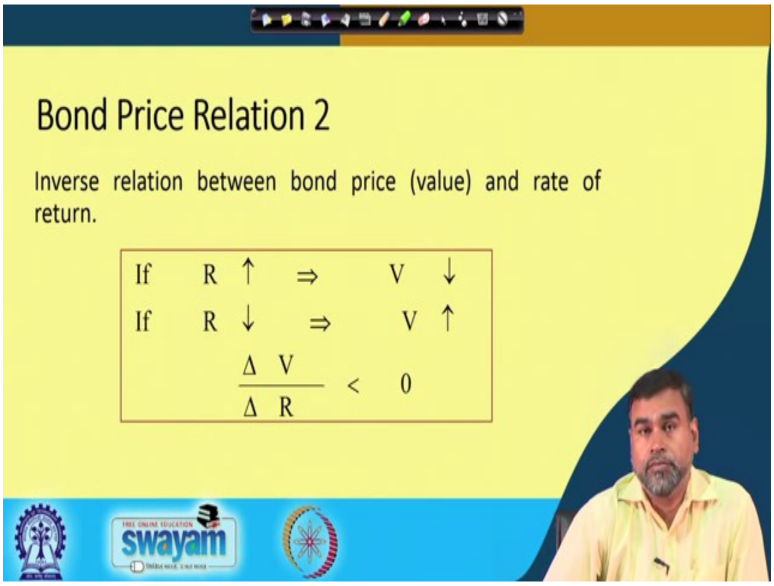


Then we have bond price relation 2, what basically it means it is very clear if the rate of interest or rate of return will increase the value will go down and rate of interest will decline the value will go up. So, if you represent it in a change way then delta V by delta R is always less than 0, delta V by delta R is less than 0.

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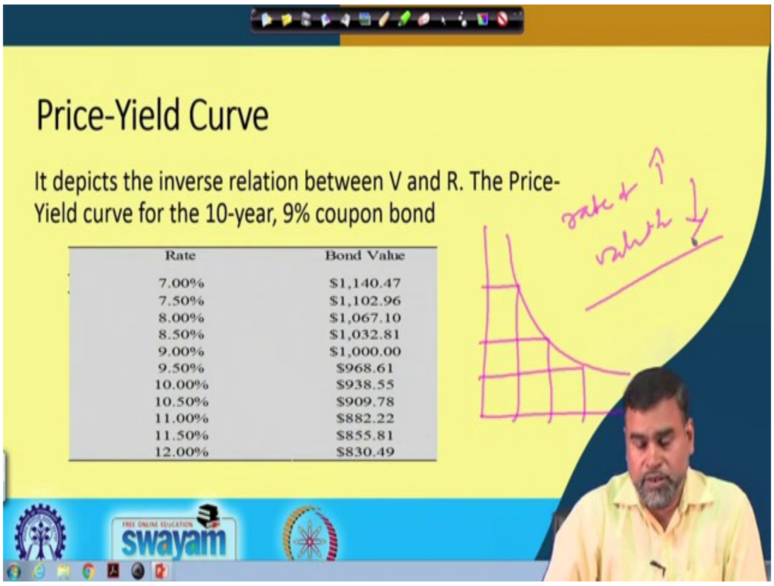


You can see this thing that there is inverse relation between the value and the required rate of return with a bond of 10 years maturity, 9 percent coupon par value is 1000, then at 7 percent bond value is this at 9.5, at 9 percent is exactly 1000 because coupon is equal to discount rate, then once the rate has increased the value of the bond has gone down that is basically we call it the price yield curve. So, that is always represented in this way. So, this is the relationship basically what you can show. So, if your term to maturity are the rate of interest increases then the value of the bond goes down, that is called the price yield curve.

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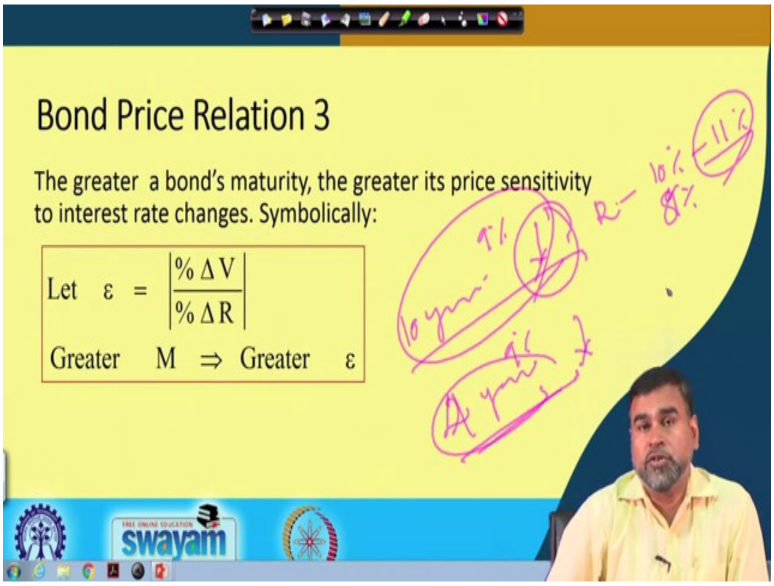


Then we can see there is another relation we can establish. Greater the bonds maturity the greater its price sensitivity to interest rate changes. That means, percentage of change in the value with respect to the percentage change in interest rate is always higher if the maturity of the bond is very long term in comparison to a certain bond. Greater the term to maturity greater the variations, greater the price sensitivity.

If you see this exam you can find out that for example, if you are going for a bond which maturity period is 10 years and another bond which maturity period is 4 years, then what you can find let coupon is 9 percent. So, here also the coupon in a coupon remain same. Then what is happening let that time you are required rate of return R was 10 percent then the R has let come down to 9 percent does for example, 8 percent or R has become 11 percent whatever it may be the R, R has become 11 percent. If R will become 11 percent then obviously, the value of the particular bond will go down and for 4 years bond also it will go down other things remain same.

But if you calculate whatever price change we have observed in case of 10 years maturity bond and whatever price since you have observed for the 4 years maturity bond, you will find for the 10 years maturity bond the percentage change in the price is relatively more with respect to that 1 percent change of the interest rate than the percentage price for the value of the bond for the 4 years maturity whenever the bond interest rate or the required rate of return was 10 percent. So, from the base value the fluctuations if you observe it is more fluctuating or we can observe more fluctuations in terms of the long-term matured bonds than the short-term maturity bonds. That is basically another observations or another relationship what we can observe.

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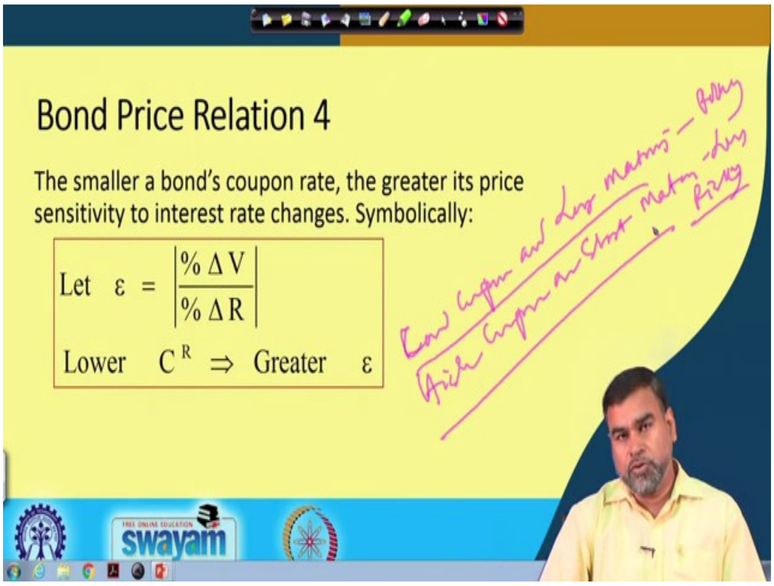


Then another thing also with respect to that if you see smaller the bonds coupon rate greater its price sensitivity, whenever there is a change in the interest rate. Previously what you have seen, greater the maturity, longer the maturity the change in the price is more, with respect to a shorter maturity bond. The same thing also can be applicable other things remain same maturity period is same, everything is same. But if you see that there is a change in interest rate, but there is a coupon a let were 1. 1 the coupon rate is 10 percent, another bond the coupon is coupon rate is 8 percent you will find which bonds coupon rate is less. The price changes of that type of bond is always more. Percentage change in the price of that bond with respect to change in interest rate of that bond is always more than the particular bond whose coupon rate is relatively higher.

So that means, whenever you are going for investing in the market keep in the mind the bonds having low coupon and high or the long maturity are riskier bonds than the bonds having high coupon and low maturity. So, the bonds having again I am repeating, the bonds having low coupon and long maturity longer maturity are more risky than the high coupon and short maturity bonds, it is less risky. So, from the investment perspective you can say that always those kind of bonds are riskier than this kind of bonds. So, those things always reflected from the bond price relations or the relationship between coupon, maturity period, interest rate and the par value and as well as the value of the bond.

So, this is what basically about the other concepts which are related to the bonds. Then the other classes will be discussing about the different type of yields, and returns, and as well as the development in the bond market mostly with reference to India. Please go through this particular references, for this particular session.

Thank you.