IIT ROORKEE

NPTEL ONLINE CERTIFICATION COURSE

QUANTITATIVE INVESTMENT MANAGEMENT

LECTURE 5

Arbitrage Free Pricing

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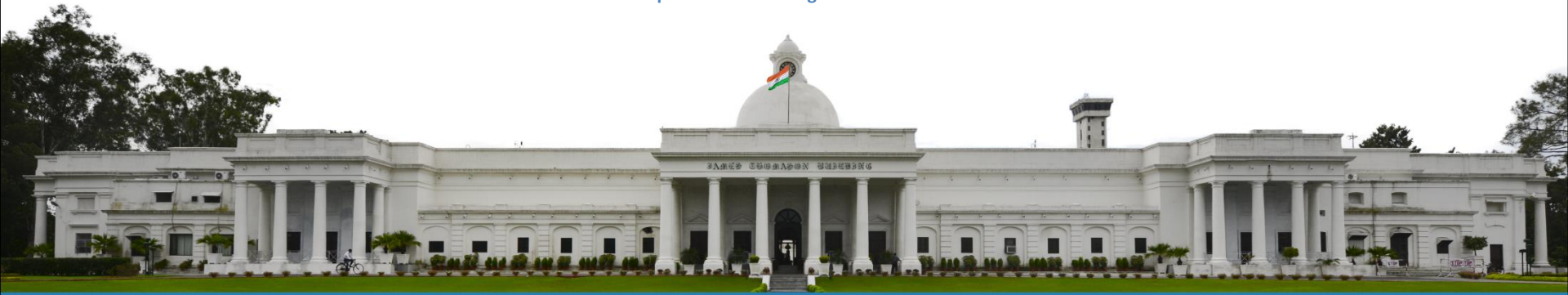


RISK & ARBITRAGE: RECAP

- **Risk:** If an asset has an uncertain future value, then the apprehension created due to the possibility of the asset failing to take a targeted value is called risk.
- **Arbitrage:** The process of generating profits due to difference in the prices of two assets with identical risk-return characteristics is called arbitrage.

TYPES OF ARBITRAGE OPPORTUNITIES

- There are two types of arbitrage opportunities:
- Value additivity (when the value of whole differs from the sum of the values of parts) and
- Dominance (when one asset trades at a lower price than another asset with identical characteristics).

VALUE ADDITIVITY

- If the principle of value additivity does not hold, arbitrage profits can be earned by stripping or reconstitution.

EXAMPLE OF VALUE ADDITIVITY

- A five-year, 5% Treasury bond should be worth the same as a portfolio of its coupon and principal strips.
- **Reconstitution:** If the portfolio of strips is trading for less than an intact bond, one can purchase the strips, combine them (reconstituting), and sell them as a bond.
- **Stripping:** Similarly, if the bond is worth less than its component parts, one could purchase the bond, break it into a portfolio of strips (stripping), and sell those components.

EXAMPLE

BOND	t=0	t=T
X	99	100
Y	990	1010

Securities X and Y are identical in every other respect.

Construct a portfolio of a long position in one Y bond and a short position in 10 X bonds.

Cost of setting up this portfolio at t=0 is NIL.

Payoff at maturity at t=T is 10.

Thus, we earn a riskless profit.

DOMINANCE

- Dominance occurs when the payoff from one security
- exceeds that of the other at maturity
- while they are priced equally.

EXAMPLE

	t=0	t=T
ASSET X	100	110
ASSET Y	100	120

Construct a portfolio of a long position in one Y bond and a short position in 1 X bond.

Cost of setting up this portfolio at t=0 is NIL.

Payoff at maturity at t=T is 10.

Thus, we earn a riskless profit.

ARBITRAGE FREE VALUATION OF SECURITIES

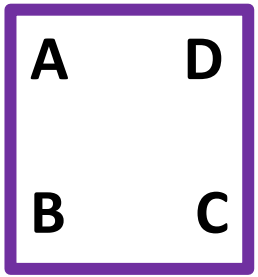
- Arbitrage-free valuation methods value securities such that no market participant can earn an arbitrage profit in a trade involving that security.
- As mentioned earlier, an arbitrage transaction involves no initial cash outlay but a positive riskless profit (cash flow) at some point in the future.

ARBITRAGE FREE PRICING OF FORWARDS

- We start with the simplest case:
- No income from the underlying during life of forward;
- No carrying cost of underlying during this period;
- No transaction costs & market frictions (bid-ask spread, lending-borrowing spread, commissions etc.).

	t=0	t=T
BORROW	$+S_0$	$-S_0exp(rT)$
BUY STOCK	$-S_0$	0
SHORT FORWARD	0	F_0
TOTAL	0	$F_0-S_0exp(rT)$
$F_0=S_0exp(rT)$		

FORWARD PRICING…



- AB: Buy one unit of underlying in spot market for S_0.
- BC: Carry the asset for delivery against forward obligation at t=T.
- CD: Deliver the asset S and receive F_0 units of M at t=T under the short forward contract.
- DA: Adjust the borrowings S_0 against the present value of the forward proceeds $F_0 exp(-rT)$.
- Net cash flow: $F_0 exp(-rT) - S_0 = 0$ or $F_0 = S_0 exp(rT)$ for no arbitrage.

PUT CALL PARITY FOR EUROPEAN OPTIONS

	t=0	t=T	
PORTFOLIO		$S_T<K$	$S_T>K$
SELL CALL	c	0	$-(S_T-K)$
BUY STOCK	$-S_0$	S_T	S_T
BUY PUT	-p	$(K-S_T)$	0
TOTAL	$c-S_0-p$	K	K
BORROWPV(K)=Ke^{-rT}	Ke^{-rT}	-K	-K
$c-S_0-p+Ke^{(-rT)}=0$			

BOND VALUATION

DEFINITION OF A BOND

- A bond is a legally binding agreement between a borrower (bond issuer) and a lender (bondholder):
- The agreement specifies
- the principal amount of the loan.
- the size and timing of the cash flows:
- in dollar terms (fixed-rate borrowing) OR
- as a formula (adjustable-rate borrowing)

SOME TERMINOLOGY

- Maturity/term to maturity
- Face value
- Coupon rate, frequency
- Premium/discount on redemption
- Premium/par/discount bonds

MATURITY/TERM TO MATURITY

- The maturity date of a bond is the date on which the principal is to be repaid.
- Once a bond has been issued, the time remaining until maturity is referred to as the term to maturity or tenor of a bond.

FACE/ PAR VALUE

- **The face value of a bond is the amount that is specified as such in the contract of issue.**
- **The amount of redemption proceeds as well as the coupon interest are determined by reference to face value. Though, they may not equal the face value.**
- **The face value is also referred to as the *par value*, of the bond.**

REDEMPTION AT PAR/PREMIUM DISCOUNT

- In case of bonds redeemable at par, the face value coincides with *maturity value*, *redemption value*, or *principal value* of the bond.
- If a bond is redeemable above par value, it is said to be redeemable at a premium.
- If a bond is redeemable below par value, it is said to be redeemable at a discount.

BOND QUOTATIONS

- Bonds can have a face value of any amount, and their prices are quoted as a percentage of par.
- A bond with a par value of $1,000 quoted at 98 is selling for $980.

BOND PRICES & PAR VALUES

- A bond that is selling for more than its par value is said to be trading at a premium to par.
- A bond that is selling at less than its par value is said to be trading at a discount to par.
- A bond that is selling for exactly its par value is said to be trading at par.

COUPON RATES & FREQUENCY

- **The coupon rate on a bond is the annual percentage of its par value that will be paid to bondholders.**
- **Some bonds make coupon interest payments annually, while others make semiannual, quarterly, or monthly payments.**

EXAMPLE

- A $1,000 par value semiannual-pay bond with a 5% coupon would pay 2.5% of $1,000, or $25, every six months.