Financial Institutions and Markets
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Lecture - 44
Bond Analysis - IV

So, in the previous class, we discussed about the different types of yield measures of the bond. And here the yield can be a current yield, it can be nominal yield, it can be yield to maturity, it can be yield to call, yield to put all these things. So, what we basically here, we have observed that the yield is nothing but whatever return you are going to or you are extracting from that particular bond. But, investment you are making on the bond, whatever return you are expecting that is basically represented as the yield.

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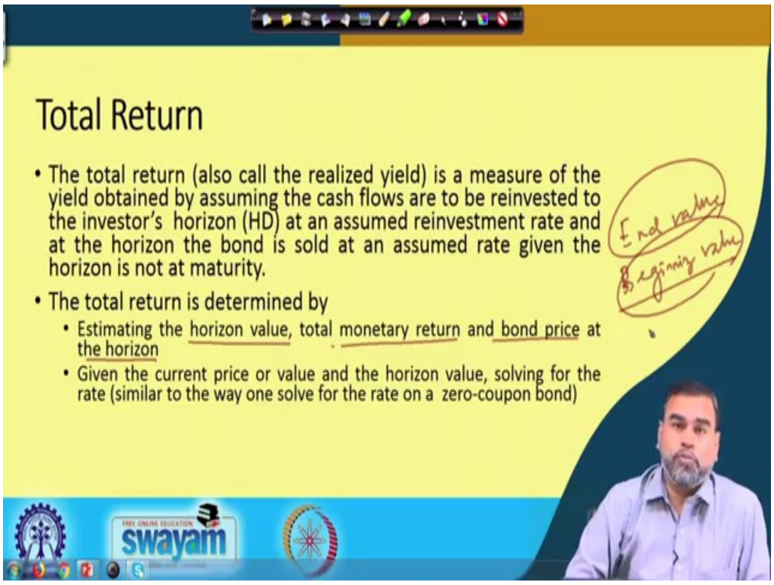

Then let us see one particular hypothetical case that whenever you are holding a bond and the maturity period of that particular bond is let 5 years, but the investor or the bondholder need some cash in between or he or she wants to redeem that particular bonding between. So, in that particular point of time, how we can calculate the total return or the yield of that bond, so that is basically defined as the realized yield.

So, whenever we are calculating the realized yield, what it measure, it measures basically how the particular coupons are reinvested in the market and as well as how much coupon you have received, and that how much price you have sold the bond in between that means after your horizon period, your holding period that is why, the bond yield in terms of realized yield is basically a measure obtained by assuming the cash flow are to be reinvested to the investor's horizon at an assumed reinvestment rate and at the horizon the bond is sold at an assumed rate given the horizon is not at maturity that means, the horizon period is not at maturity, horizon period is below the maturity period.

So, in that case, how the return of that particular bond can be calculated that means, exactly how much return you are going to realize, if you are going to sell that bond or you are going to redeem that bond before the maturity. So, here if you see, there are certain factors a certain determinants, which determine this return from that particular bond in terms of the total return or the realized return.

What are those? One is your horizon value, and what value basically you are selling that one, then the bond price at the horizon, then total monetary value or monetary return what you are getting. Then if you are getting that if you are going to sell that particular bond at that particular point of time, then what is the total value of that particular bond, and at what price you have purchased the bond. So, you have end value then you have a beginning value or the selling value or the purchase value. So, once you have the end value and this purchase value, then what you can do, you can find out the return from that by simply whatever way the return from the zero-coupon bond, we were calculating.

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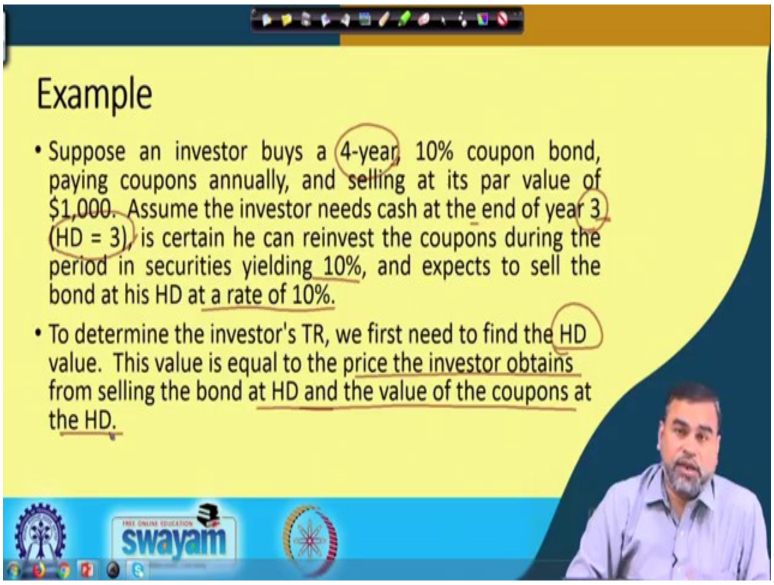

So, let us see that how that particular mechanism works. Suppose one investor buys a 4-year, 10 percent coupon bond, paying the coupon annually, and selling at its par value of 1,000. Assume the investor needs cash at end of 3 years. Here you remember this particular bonds maturity period is 4-years, but investor needs cash at the end of year 3 because of that what is happening, that means the horizon period is 3 years.

So, it is certain you are assuming that he can reinvest the coupon whatever he has received up to that particular 3-years period, because you are holding the bond of 2-3 years, and expects to sell the bond at the horizon at the rate of 10 percent. The yield basically was 10 percent, and that yield remain constant of that particular period of time. So, then in that particular contest how basically we can calculate the total return or the realized return?

We see then what basically we have to first calculate, we have to first calculate the HD value. What is the value of the bond at the horizon period? And this value is nothing but how you can calculate this value, this value is nothing but the price the investor obtains from selling the bond at HD value. And the value of the coupons at the HD that means, if you want to calculate the value of that bond at that particular point of time, then what is the cash flow we are going to get? We are going to get the cash flow that is 100 rupees, because 10 percent is the coupon. And we have the value of the bond, what basically at price we are selling this one. These are the two cash flows, which are involved in this.

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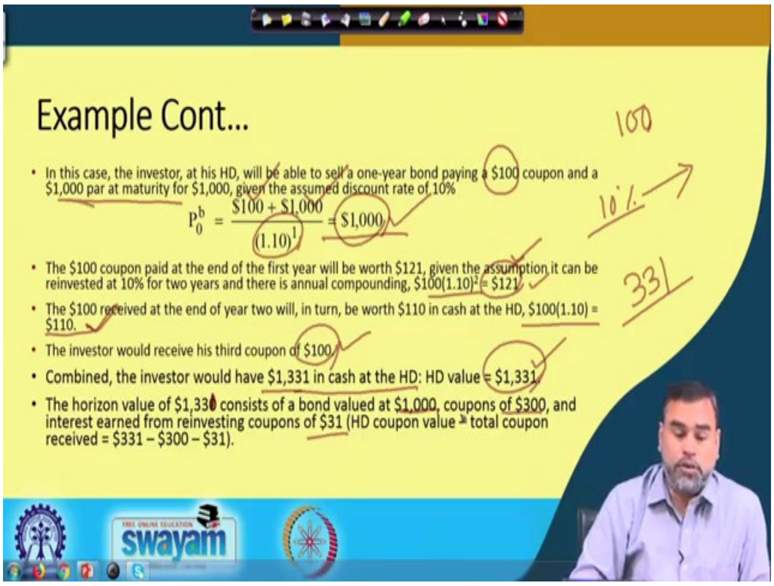

Let us see how that particular thing works that in this case if you observe, the investor can sell the bond only the bonds, maturity period is 4-years, already 3-years are gone only one year is left out. Then how much coupon you will be getting, you will be getting a coupon of 100 dollar though you will be getting a coupon of 100 dollar and a 1,000 par at the maturity.

So, if you are going to calculate the price of the bond in this case, because the yield and the coupon rate are same, then the value what you are getting that is the 1,000 dollar. 100 rupees that is the coupon is remaining or future you are going to get that is the par value of the bond, and one year is left out. So, if you discount it, then you are getting that particular value of the bond as 1000 rupees, why you are getting exactly 1000, because the reason the coupon rate and the yield rate are same. The 10 percent of the coupon and 10 percent is the yield.

And now what is happening how basically you can calculate, because you are holding the bond after 3-years. If you are holding the bond of 2-3 years, you got 1000 rupees, whatever you are sold that bond. Then how much total value you got, because 100 rupees you got after the first year, which is the coupon. If this 100 rupees, you got and this market interested or yield is 10 percent, then what you have done? You have invested that particular 100 rupees in the market right.

For another 2 years you might have invested then you got 100×1.1^{2} that is 121 rupees. Whatever 100 rupees, you have got in the first year. After first year, you have invested it for another 2 years, you got 121 dollar. Then in the end of the second year, you got another 100, so that 100, you can invested for another 1 year. Then how much you got 100×1.1 that basically will give you 110. So, whatever 100 rupees, you have got after the first year from there, you got 121. And whatever 100 you have got, after second year you got 110. Then what is the total you got, then in his third coupon up to third year you are investing that particular bond, you are holding that bond.

So, in the end of the third year, also you will be getting 100 rupees, but you did not have time to invest it. You have got 100, 100 dollar you got, but that money is not reinvested in the market. Then effectively how much you got, effectively you have got 121 plus 110 plus 100, then total you got 331. You got 331, and you have sold the bond the investors has sold that bond at the price of 1000 rupees or 1000 dollars.

Then total the investor would have 1, 331 in the cash at the HD. So, the HD value has become 1, 331. So, the coupon is reinvested, and whatever coupon in the end of the period he got. So, if you add of everything 1000 + 331, it will give you 1, 331. Now, so here what is basically happening that the horizon value of 1, 331, your basically 1, 331. 331 consists of the bond value of 1000 coupon is 300. And the reinvestment what you got that is 31, because the coupons are again reinvested in the market, you got that reinvestment amount 31. So, total value got 1, 331.

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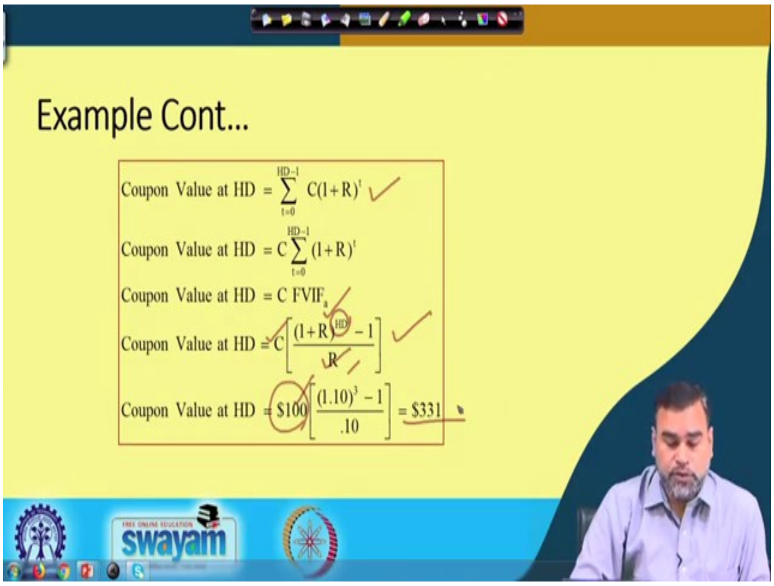

And now you can calculate the total return from this, then how you can calculate. So, now if you see, the total return is calculated in this way. So, you have this is the formula for calculation of this the coupon value at HD $C \times 1 + R^{t}$, t is equal to 0 to HD minus 1, because last year you cannot reinvest that money.

Then finally, it is nothing but C into the future value of interest factor. Then total value the coupon value at HD, you can calculate any of the ways $C \times [\{(1 + R)^{HD} - 1\}/ R]$. Any of the formula you can use to find out the coupon value at HD. And here in this case, if you put your coupon was C, which is 100 dollar, your rate yield was 10 percent. Then 1.1, you are holding it for 3 years or to the power 3 minus 1 divided by 0.1, which is the R 10 percent. Then we are exactly getting 331 that is what we have extended, so 300 rupees we got it.

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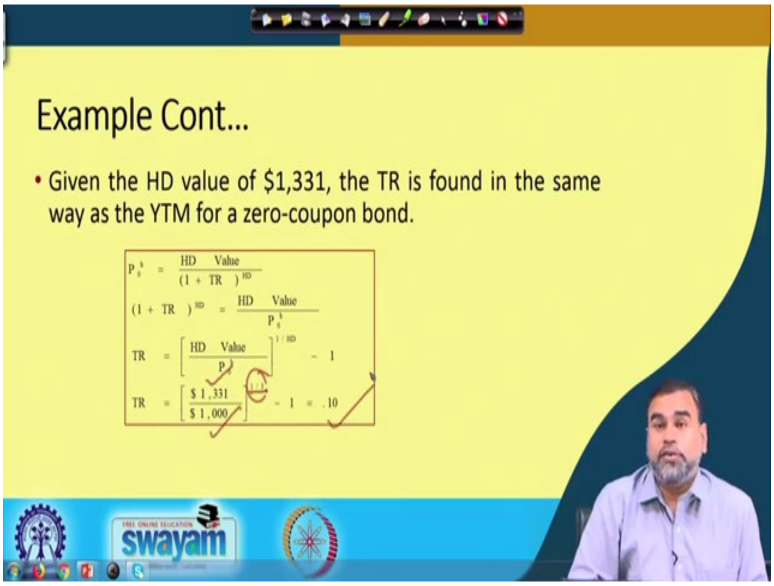

So, now the total value we got that is basically how much 1, 331. Then what is the return? If you see the return, now your HD value is 1, 331 if you see here, and what is the price, that is 1000 dollar. Then 1000 how much here you are for how many years you are holding it, 1.3, then one point to the power 1.3 minus 1 that is basically you are getting 10 percent here.

So, here we are getting 10 percent rate on, the reason is everything remains same. Your coupon and you will assign rate can also vary depending upon the change in the yield rate in the interest rate in the market. So, this concept is basically called the total return or the realized return that actually you can keep in the mind that means, the bond investors have not hold the bond up to the maturity, but then how much return you can expect from this.

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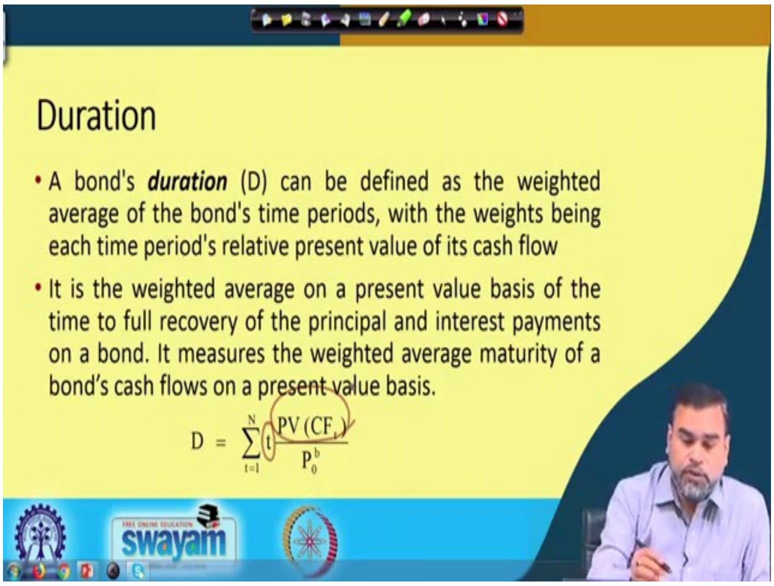

Then we can discuss some concepts related to the bond market or the bond instrument, which are used for the bond investment strategy, which is not a part of this particular syllabus. But, still you can have the idea that those kind of things are used extensively for minimization of the risk in the bond market, so one of the concept is duration. So, what do you mean by the duration, the duration is basically what it is a weighted average of the bonds time period.

In the weights are basically given on the basis of the present value of the gas flows or it is a weighted average on a present value basis of the time to full recovery of the principal and interest payments on a bond, and it measures the weighted average maturity of a bond cash flow on a present value. Whatever way you can define it, it is basically a measure of the time, but it is a weighted value. And the weights are given on the basis of the present value of the cash flows. So, if you see this, the weights are given, you are calculating the present value of the cash flows, and here t is equal to the time. So, we are multiplying t with respect to that weights and finally this duration is calculated.

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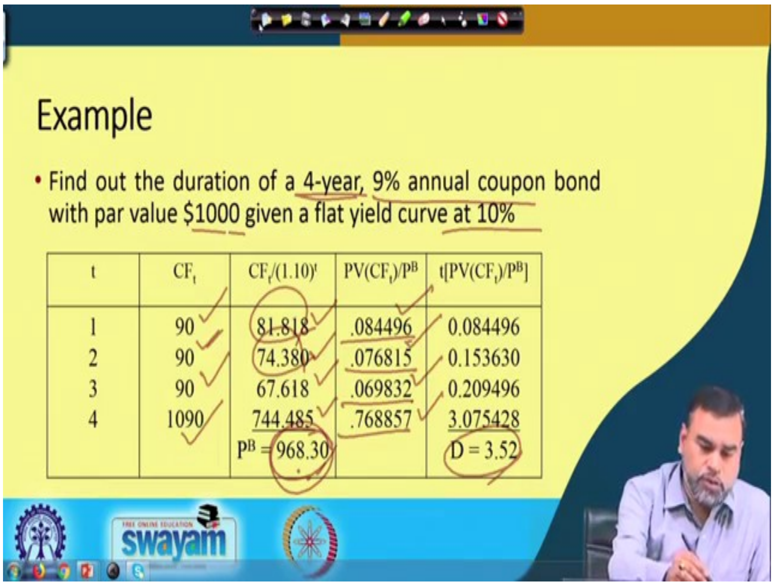

So, let us see one example that how the duration is calculated. Let there is a bond with maturity period is 4-years, coupon is 9 percent per value is 1000, curve is 10 percent means, the yield is 10 percent. So, if you see that, first year how much cash flow you are getting 90 rupees, second year 90 rupees, third year 90 rupees, and the fourth year the curve value is 1000 plus 90, 1090.

Then you find out this the present value of this 90 /1.1, you got is 81.18. 90 divided by 1.1 1 square, you got this 1.1 to the power 3, you got this. Then 1090 / 1.1^4, you got this. Then the total present value of the bond is one 968.3. Then you are finding out the weights, so this divide by total will give you, this divided by give you this the total will give you this, this divide by this will give you this, this divide by this will give you this.

Now, what you can do, this one you can multiply with the time period, so these are the weights now you got it. So, now 1 multiplied by this plus 2 multiplied by this plus 3 multiplied by this plus 4 multiplied by this that will give you 3.52. So for a 4-years maturity period term to maturity 1 the duration is 3.52. And use of the duration is basically to minimize the interested risk in the market you should hold a bond up to where the horizon period is matching with the deviation not with the termed maturity that is the investment strategy always we adopt. If a horizon period is 3.5 years, do not invest in a bond whose term to maturity 3.5 years, invest in your bond which duration is

3.5 years by that you are interested risk in the market can be minimized, so that is the use of the duration concept.

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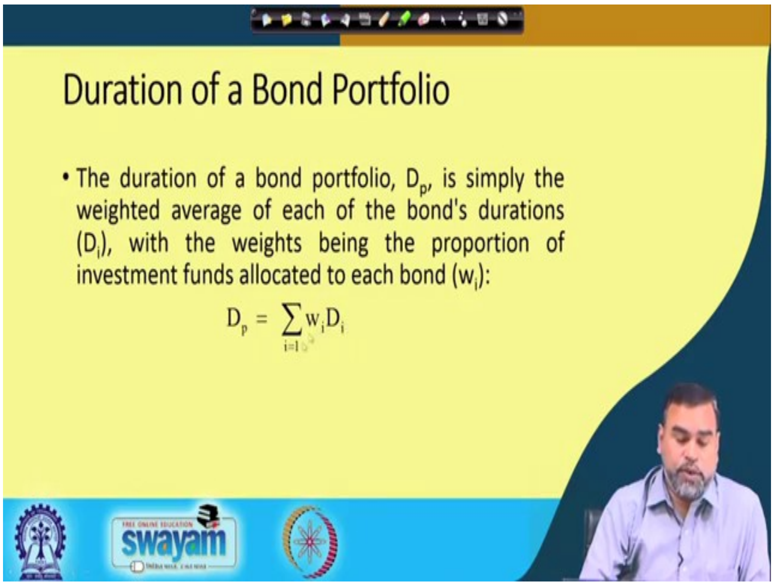

So, now if you are calculating the duration of a portfolio remember the duration of a portfolio is simply the weighted average of each of the bonds duration, with the weights giving the proportion investment funds allocated to each bond. If you have 100 rupees, you have spent is allocated 5.5 5 rupees for 1 bond, 20 rupees for another bond. Accordingly, we can find out the weights. And each bond you can find out the duration, if you multiply that, then that will give you duration of the portfolio.

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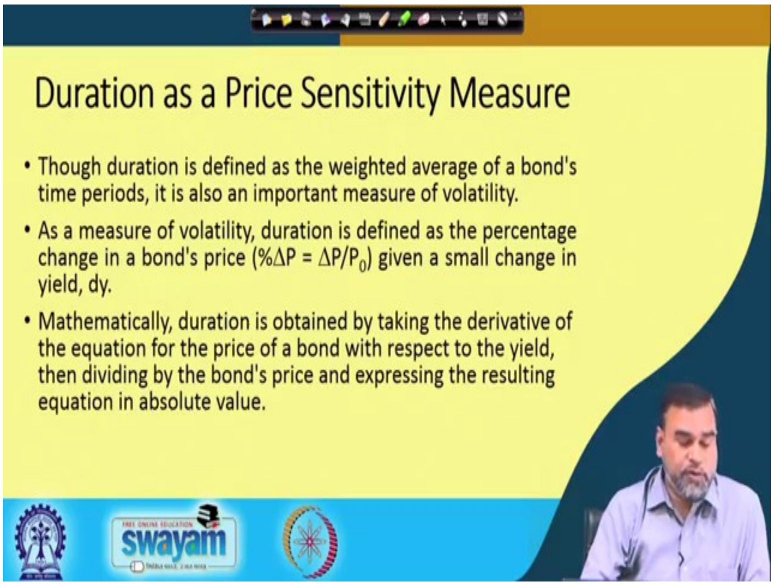

Duration also is used as a price sensitivity measure, so that is why, it is important measure of the bond price volatility. Duration also can be used as a measure of the price volatility of the bond, so that is why if you are defining in that way, duration is defined as the percentage change in the bond prize given a small change in the yield. Mathematically, if you want to find out the deviation, it is nothing but the first order derivative of the equation of the price of a bond with respect to the yield, then dividing by the bonds price and expressing the result in the equation in the absolute value.

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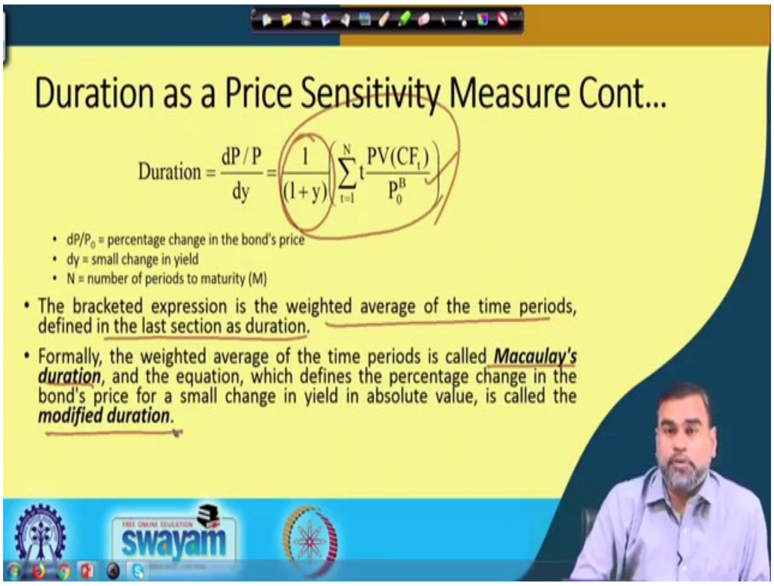

So, how it is basically represented, it is represented in this way. Duration is equal to already I told you (dP / dy) / P. So, then if you are going to find out from the present value formula, P is equal to you know what is that value of the P, the present value formula. Then if you find out the derivatives of that, then you can find out this one. And divided by P if you take, then this is basically the duration what you can find out.

But, one thing you remember, whenever in the beginning of the example, we are talking about the duration that is basically this bracketed expression, if you see this one what that basically talks about, if the weighted average of the time period define in the last section of the duration, what we have defined.

Now, we are getting another term that is 1/1 + y, whenever we have gone for the using the derivative concept, so that part is basically called the Macaulay's duration, and the overall talk whenever you are dividing 1 + y with to that it is called the modified duration. The Macaulay duration divided by 1 + y or into 1 / 1 + y that will give you the modified duration, which can be derived from the first order derivative of that particular price equation with respect to a change in the yield.

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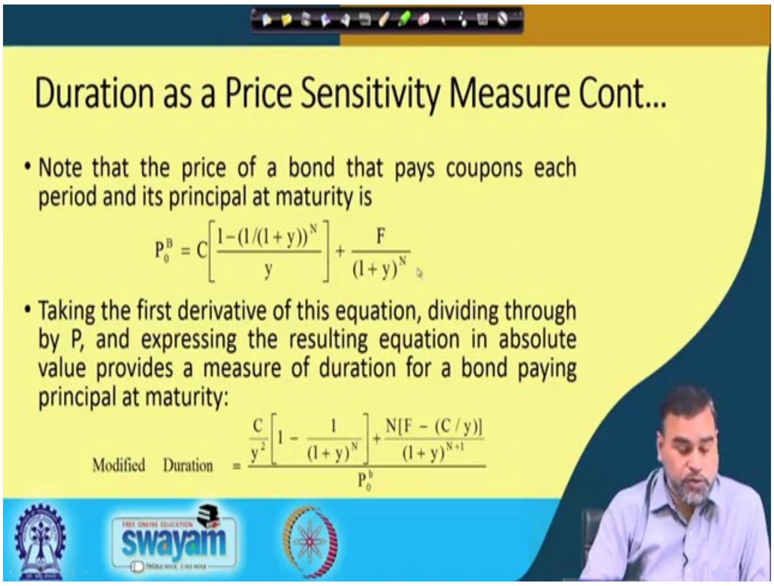

Then you see that if you go by the price of a bond, which pays the coupon its period and principal at maturity. This is the basically P, which is the formula that already we know. And if you are taking of the first order derivative of the equation, then your modified deviation will be this one.

$$\text{Modified Duration} = \frac{\dfrac{C}{y^2}\left[1 - \dfrac{1}{(1+y)^N}\right] + \dfrac{N[F - (C/y)]}{(1+y)^{N+1}}}{P_0^b}$$

This is the equation what you can find out, whenever we go for the first order derivative of this particular equation with respect to y.

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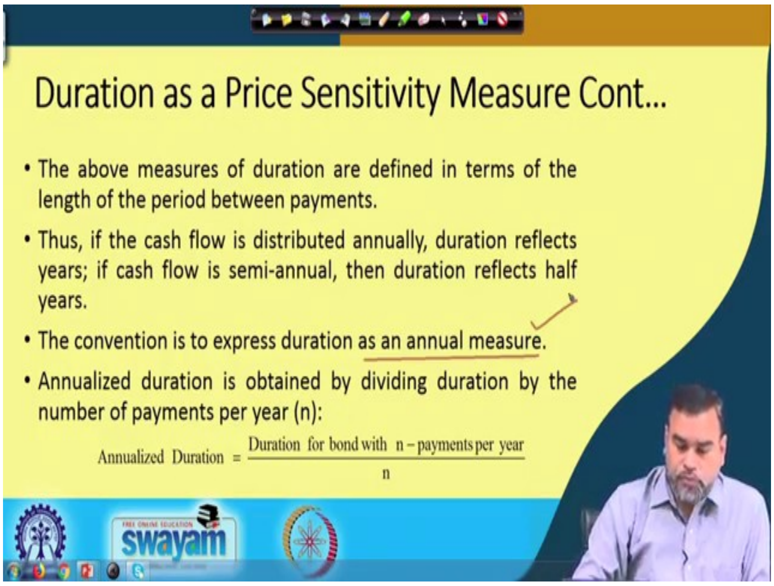

So, always you see the derivations are measured in terms of the years, and this is always reported in terms of the annualized value. So, if the deviation, if the cash flow is distributed annually, the duration reflex in years. If the cash flow is semi-annually, then duration reflects half of the year. But, even if you are calculating this thing half of the year so the years, finally the annualized duration has to be calculated. In the annualized duration is nothing but duration for their bond with n payments per year divided by n that is why, the duration are defined in terms of the length of the period between the payments.

And the convention is to express the duration as an annual measure that actually you keep in mind, this is always reported as a part of the annual measure. So, the annualized duration is obtained by dividing deviation by the number of payments per year. And if it is two times it is paid, then you find out that then finally divided by 2 that will give you the annualized deviation that actually we have to keep in the mind.

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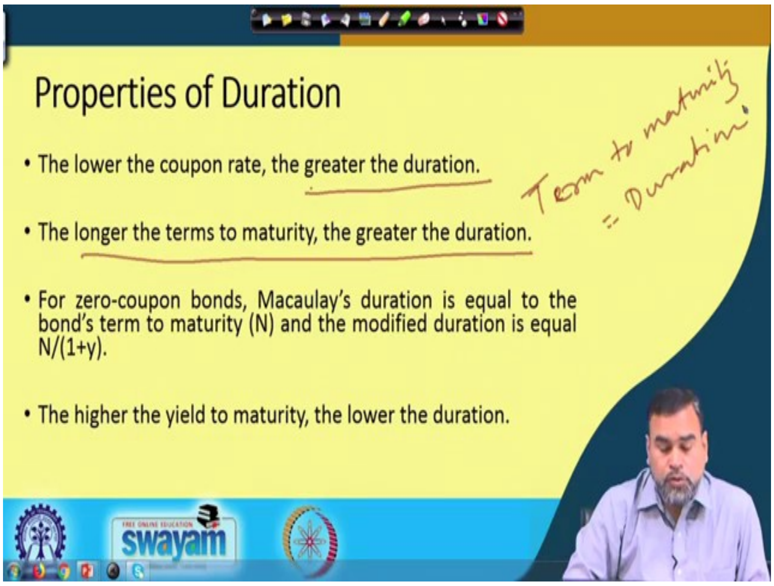

Then we have to see that, what are those properties of the duration because, you see the basic features of the bond is what coupon, maturity and yield. Then on that basis if you want to relate the duration with respect to all those born fundamentals like coupon, maturity, and the yield, then what kind of relationship you can establish of duration with respect to all the three variables.

What basically we have observed or it is observed, the lower the coupon rate the greater the duration. If you are taking two bonds, the term to maturity it is same, and other the par value is same, everything is same, but only coupon there is a difference in terms of the coupon. Then what you will observe, which our bond the coupon rate is lower. The duration of that particular bond will be higher.

The lower the coupon rate, greater the duration lower the coupon rate greater the duration. Then everything remain constant, everything remains same. If you see longer the term to maturity, greater the duration. And for a zero-coupon bond, where there is no such kind of coupon, which are periodically available. The Macaulay duration is equal to bonds term to maturity that means the term to maturity is equal to duration.

The term to maturity is equal to duration anywhere in between there is no cash flow involved in that so because of that obviously in the end we are getting the cash flow, because there is a zero-coupon bond in that context. What we can observe that it is equal to the duration is equal to term to maturity.

And the modified duration is equal to n / 1 + y, y is basically the yield. The modified duration is whatever is the duration you are getting divided by 1 plus y, duration is nothing but term to maturity, then let yield is let 5 percent of 3 percent. So, if you divide that with respect to that particular maturity period, then you can find out the modified duration.

Then another observation also we can find. The higher the yield to maturity, lower the duration. Everything remain constant, everything remains same. If you compare between the two different bonds, whichever bonds maturity period the higher yield to maturity is there or yield is more. And for that particular type of bond, the duration will be lower that also has been observed. So, these are the different properties of the duration.

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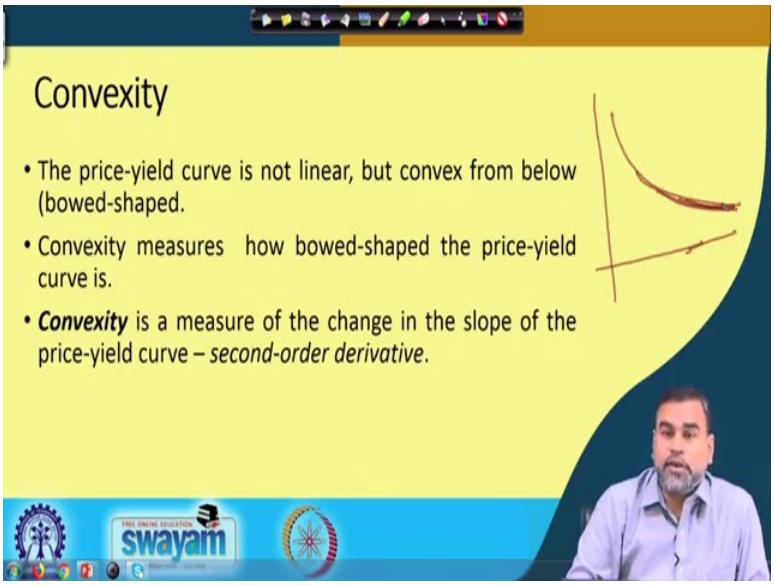

Then another concept, which is also important from the bond investment point of view that is called convexity. Why the convexity comes, because if you remember whenever you are having a relationship with a price and the yield, what we have seen price and yield curve is basically a convex curve or convex to the origin, so it bowed shaped. So, if it is there, so there is some kind of arc which is involved in that the curvature.

So, convexity is basically nothing but, it measures the curvature. The convexity is basically nothing measure the how the bowed-shaped the price-yield curve is or what is the curvature of that price yield curve that is basically measured by the convexity. So, what it then exactly in the mathematical since it measures, it basically nothing but the change in the slope of the price yield curve. The slope of the price yield curve basically is nothing but the duration.

And if you are going for change in the slope of the price yield curve, then we are going for the second order derivative of that particular equation, then we can find out the convexity. So, duration can be calculated by taking the first order derivative of the price equation with respect to change in the heat. And the convexity can be measured by taking the second order derivative of the price equation, whenever there is change in the yield that is the basic difference between convexity and the duration.

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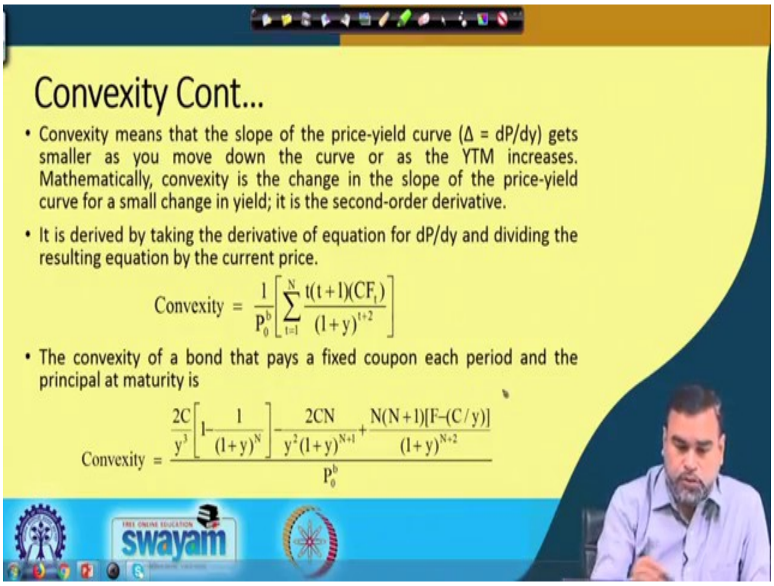

Then if you see that how the convexity can be measured, so already I told you the convexity means that the slope of the price yield curve like ΔdP / dy get smaller as you move down the curve or as the YTM increases. So, mathematically, convexity is the change in the slope of the price-yield curve for a small change in the yield, and it is the second order derivative.

So, if you trying to find out the second order derivative, then you can find this

$$\text{Convexity} = \frac{1}{P_0^b}\left[\sum_{t=1}^{N}\frac{t(t+1)(CF_t)}{(1+y)^{t+2}}\right]$$

so that is basically convexity. If you are talking about a bond of a fixed coupon each period, and the principal at maturity is also fixed, then what you can do, you can find out the convexity formula with this way.

Because, if you go for the second order derivative of that equation, then you can find out this value also

$$\text{Convexity} = \frac{\dfrac{2C}{y^3}\left[1-\dfrac{1}{(1+y)^N}\right]-\dfrac{2CN}{y^2(1+y)^{N+1}}+\dfrac{N(N+1)[F-(C/y)]}{(1+y)^{N+2}}}{P_0^b}$$

C means it is the coupon, N means the time. I mean what is the period divided by y square into 1 plus y to the power N plus 1 into N into N plus 1 into F minus C by y divided by 1 plus y to the power N plus 2 divided by the price, so that is the formula for the convexity of a coupon of a bond which pays the fixed coupon periodical basis, so that actually we can use it whenever we go for calculating the convexity of a particular bond.

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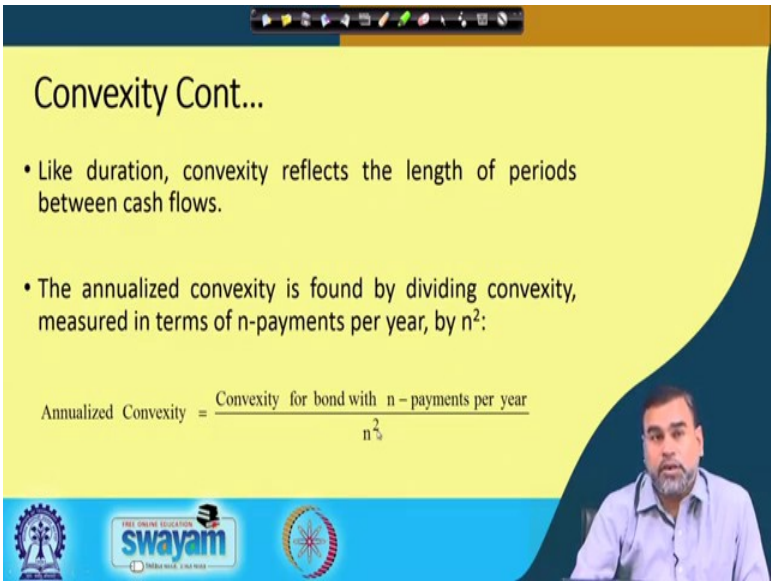

So, like duration, convexity reflects the length of period between two cash flows. And the annualized convexity is found by dividing the convexity, measured in terms of n-payments per year by n square. There were dividing with n, here we are dividing with n square, because we are talking about the second order part. So, the annualized convexity is convexity for bonds with n-payments per year 2 times, 3 times whatever frequency the coupon payments has divided by the n square that is the way the annualized convexity can be calculated.

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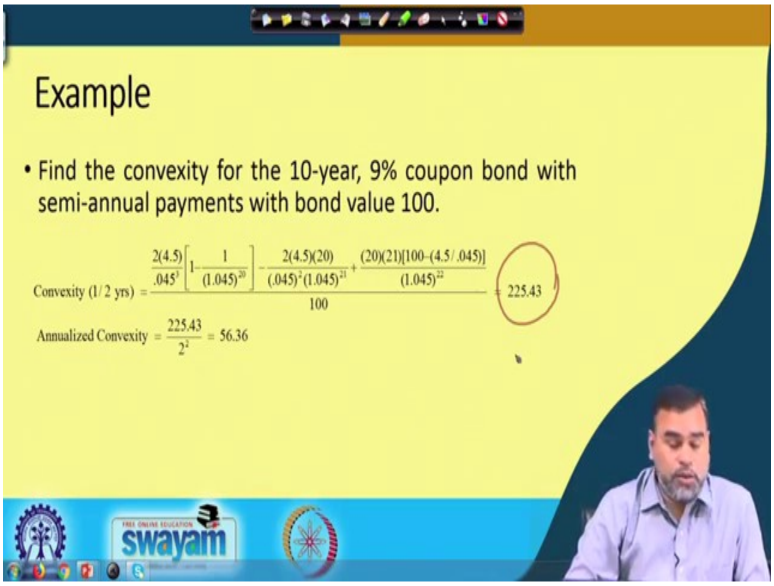

See this example of a 10-years bond of 9 percent coupon with semi-annual payment with a bond value 100, then obviously 9 rupees is the coupon. Semi-annual means 4.5, then 2 C you have taken 2 C divide by y q 2 C 2 into 4.5, Y is equal to four point 0.045, it is 4.5 rupees, it is 4.5 percent into 1 minus 1 by 1.045 to the power what to the power n, n means it the 10-years bond and semi-annual coupon. Then obviously, to the power 20 minus 2 CN period is 20. 4.5 into 20 divided by this is your interest rate zero point 0.45 square into 1.045 to the power 21, because it is basically N plus 1. Then plus your 20 into 21 into 100 minus 4.5, formula you can put it.

Then find out that particular value that is 225.43. And now the period is N is equal to 2 that how many time this coupon is paid that is basically two times, then you are annualized convexity is 225.43 divided by square of the two that is 4, you can get 56.36. For this particular one, the convexity is 56.36.

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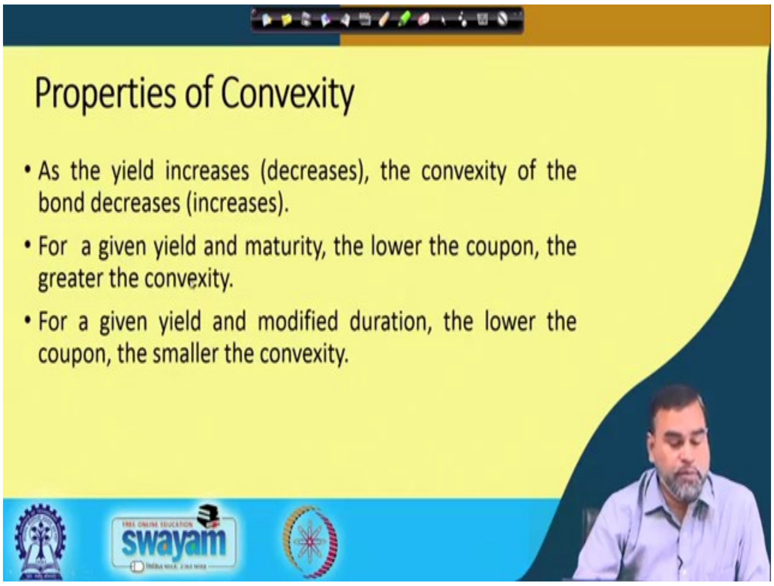

Then what is the properties of convexity. As the yield increases, the convexity of the bond decreases. For a given yield and maturity, the lower the coupon, the greater the convexity, it is something with duration also. For a given yield and modified deviation, the lower the coupon, the smaller is the convexity, so that is what basically we can always find out in terms of the properties of the convexity.

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Please go through this particular reference for this particular session.

Thank you.