Financial Institutions and Markets
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Lecture - 43
Bond Analysis – III

So, in the previous class we discussed about the different concepts related to the bond and as well as the issues related to bond valuation. And today, we will be discussing about another important issue which is always we come across whenever we invest in the bond market that is basically your bond yield.

So, whenever we invest in the bond market, basically the basic objective is to maximize the yield from the market. So, whenever we talk about the yield there are the different types of yield we come across. And today, we will be discussing about what are those different types of yields and how these yields are basically calculated.

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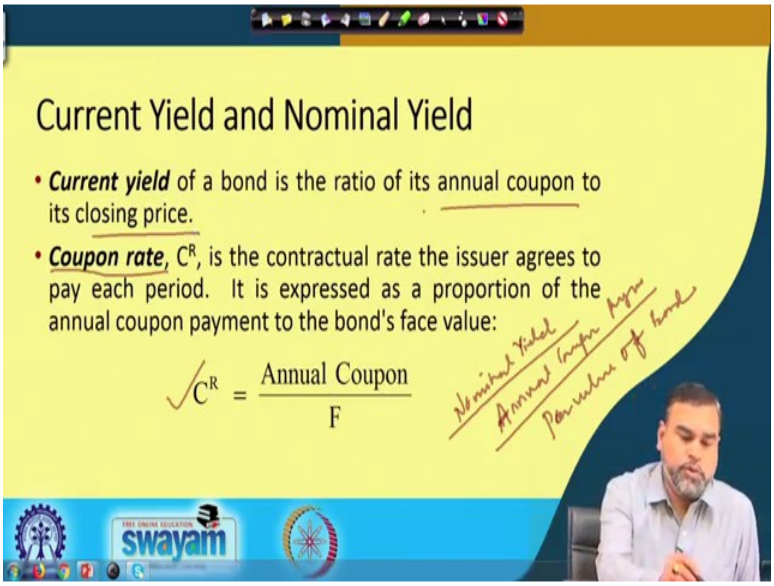

So, if you see that first of all, already all of you have heard about the concept of the coupon. The coupon is also one type of yield which is basically called a nominal yield. So, the coupon rate is basically called as the nominal yield. And the nominal yield, if you talk about the nominal yield, nominal yield is nothing but what is the annual coupon payment? The annual coupon payment divided by the face value of the bond or the par value of the bond.

It may be 1000, it maybe 100; depending upon the value of the bond, we can calculate the coupon rate and the coupon rate is nothing but the nominal yield. But whenever we talk about the current yield, current yield is basically what? Again the same thing, it is annual coupon to the price, the market price. If you remember, we are calculating this market price using the discounted cash flow formula.

So, whatever market price we calculate, the current yield is nothing but the coupon divided by the market price of that particular bond. So, the basic difference then between current yield and nominal yield is, the nominal yield is coupon divided by the face value of the bond, but whenever we talk about the current yield into the coupon divided by the market price of the bond. So, that is the basic difference between the current yield and the nominal yield.

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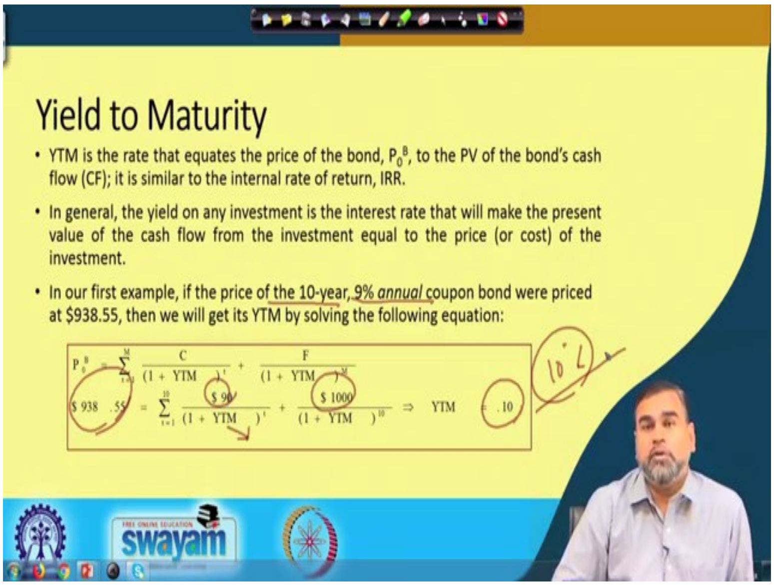

Then we have another concept that is called the yield to maturity, which is highly used whenever we are using this concept of devaluation, the yield to maturity is quite important.

Then what is the yield to maturity? Yield to maturity is nothing, but the particular yield which equates the price of the bond to the present value of the bonds cash flow. It is more or less similar to the internal rate of return what we are using in the investment analysis or we can say that project evaluation part. So, that is why, in general the yield of any investment is nothing but the interest rate that will make the present value of the particular cash flow from that investment equal to the price of the investment.

So, if you go back to our previous example, if you remember our previous example, we have calculated using an annual coupon payment where your coupon rate was 9 percent and the maturity period was 10 years. And let, that time the yield was given that is the required rate of return that was 10 percent, but now here let the price is given and we have calculated the price that time 938.55.

And this 938.55 is basically if you go and discount this 90 rupees in each period and as well as the par value of the bond is 1000, whatever if you solve this equation then whatever YTM you will find or whatever required rate of return you will find, that is basically we call it the required rate of return or the yield to maturity. That means the particular interest rate which equates the price of the bond with the present value of the cash flows.

So, that is basically we call it the yield to maturity which is always we use it whenever we go for the valuation of the bond.

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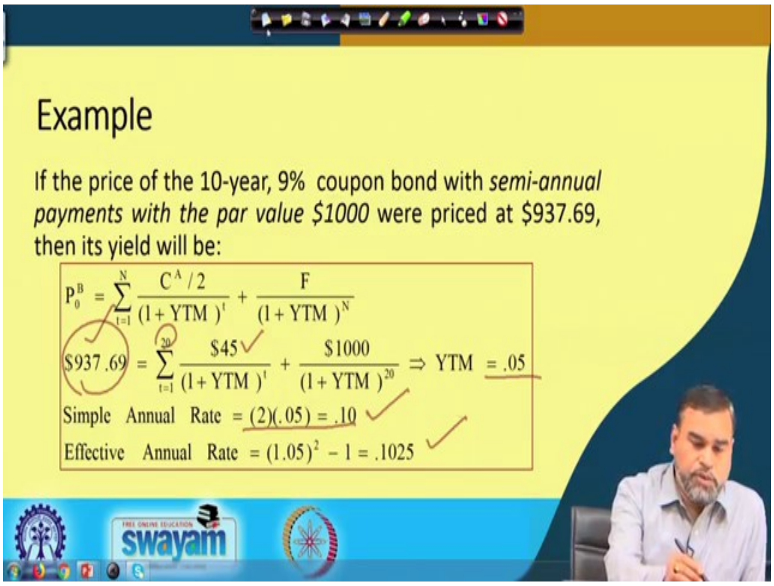

Then we will see that how this particular thing works basically in the market. So, for example, you are going the same example if you go back. The coupon payment was semiannually again, the 9 percent coupon, but the frequency of the payment of the coupon is semiannual. So, in that context what we find that, then semiannual coupon if you go back, our value was 937, the market value and this 45 rupees coupon because the 90 rupees per annum. Then every 6 months you are going to get 45, your period has been doubled, a total period of coupon payment.

Then finally what will find that, 5 percent which basically makes this value equal to present value of this particular cash flow. Then your simple annual interest rate or yield if you are going to calculate this is 2 ×0.05 that is 10 percent. And if you are going where effective annual rate what we have discussed in the previous class that is nothing, but the 1.05; that means, $1 + r / 2^{2} - 1$; that is 10.25 percent and that 10.25 percent we are getting because of the discounting of that particular cash flow with respect to that particular period and that payment is made basically semiannually.

So, this is what basically this yield to maturity and this is the way the yield to maturity is calculated.

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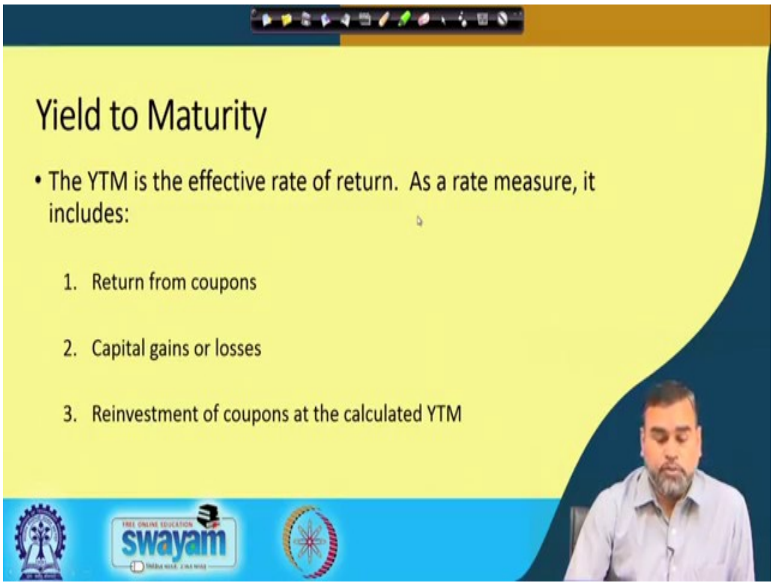

Now, we can observe that what those factors which are basically responsible for the calculation of the yield to maturity. So, if you see those factors which are responsible for calculation of yield to maturity, these are already we have observed. We what we need? We want return from the coupons. We want the return from the coupon; then we have the capital gains or losses, and the reinvestment of the coupons at the calculated YTM. So, these are different components or we can say the different factors which are responsible for the effective calculation of YTM in terms of the effective rate of return. This is what basically always you can remember you can keep in the mind.

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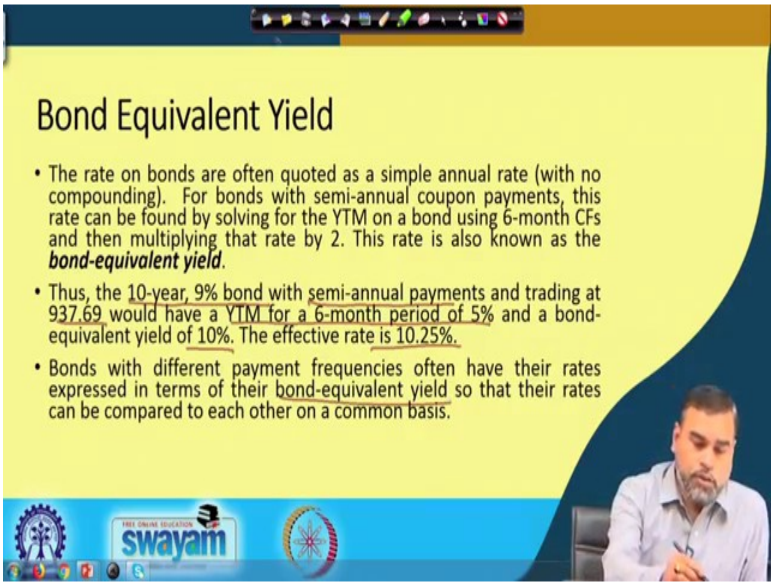

Then let us see that, that is another concept called the bond equivalent yield. What do mean by this bond equivalent yield? Bond equivalent yield is basically what? Most of the cases or most of the time what we have seen the yield of the bonds are quoted as a simple annual rate. And we do not consider the compounding part, the compounding frequency is not considered there.

So, whenever the bond the coupons are paid semiannually, then how we can do it? The rates can be found by solving for the YTM on a bond using 6 months cash flows and then multiplying that rate by 2. And in that case, we call it, it is the bond equivalent yield. For example, if you say this 10 years bond with 9 percent bond 9 percent coupon, semiannual payment and it is trading at 937.69, then what we have, we find that we have a YTM for a 6 months period that is 5 percent and bond equivalent yield is 10 percent. But the effective rate is 10.25 percent that we have seen in the previous example.

So, the bonds with different frequency or payment frequency often have their rates expressed in terms of the bond equivalent yield. So, then the rates can be compared with each other on a common basis; that means, you have to convert everything either in the simple annual rate or you can calculate the effective rate. So, then what will happen? That this if you are comparing a particular bond, which pays this coupon in semiannually and another bond which is paid to the coupon annually, then for comparison purpose basically the bond equivalent yield concept is used. That is only a concept, but the thing is you have to only convert everything to annual basis to find out that.

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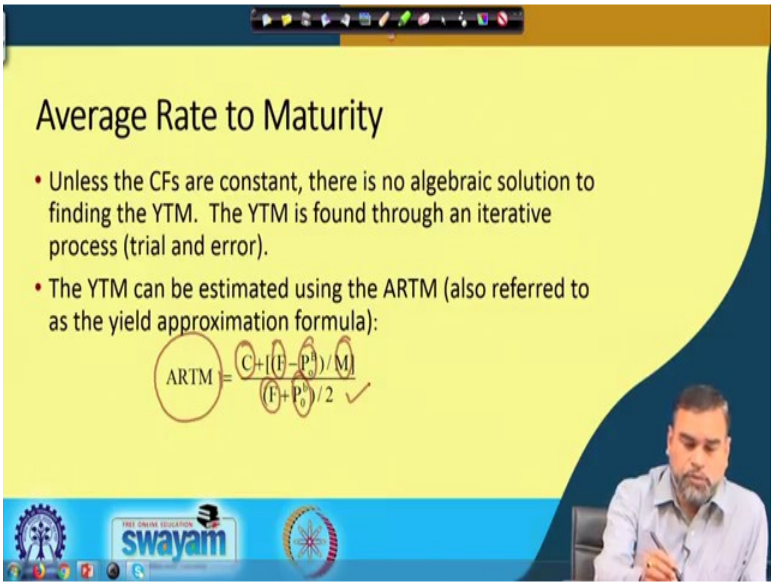

What do you see that, whenever we are calculating the yield to maturity, it is very difficult to find out a particular YTM which can make this value of the bond equal to the present value of the cash flows.

So in that case, we cannot find algebraic solution for that; a typical solution for that, it is not possible. In that context what basically we do? We can basically follow an iterative process, basically trial and error process to find out that YTM which makes that particular value equal to the present value of the cash flows. But it is relatively difficult to go for an iterative process every time to find out that value.

So, one of the ways to find out the YTM which may not be exactly giving you the actual exact answer, but you can find out a closed answer with respect to that yield to maturity. That is basically we call it the average rate to maturity. So, we are using a concept, the average rate to maturity to find out the closed value of YTM which can make this present value equal to the value of the bond. So, that is basically calculated in this formula.

So, here your C is the coupon, coupon payment F is the face value of the bond, this is the market value of the bond, this M is basically the maturity period, then again this is the face value of the bond, and this is the value of the market value of the bond divided by 2. So, if you take then you can find out an average rate to maturity which will be very close to yield to maturity. It may not be exactly equal to the yield to maturity, but it will be very close to yield to maturity. Then you can get this approximate answer from that; that which can be the yield to maturity for that particular bond which can make this value equal to the present value.

So, in that context if you see this example what basically we have discussed.

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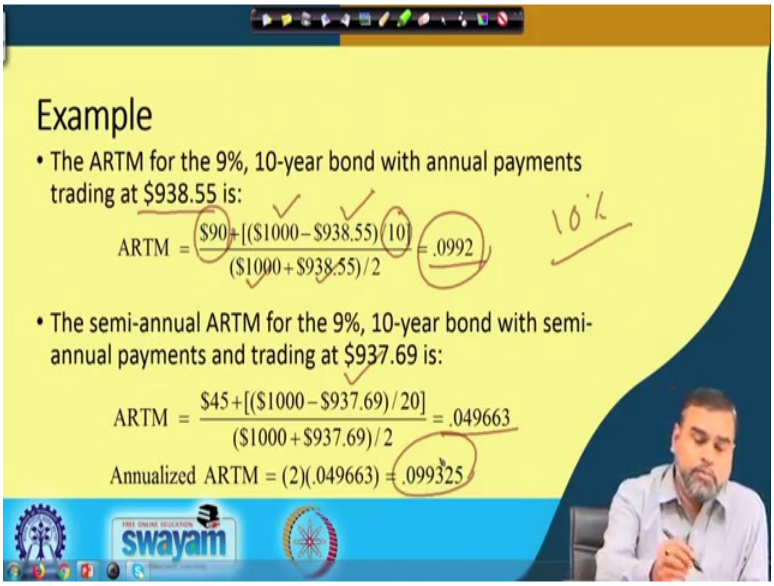

The same thing what we have seen that in the previous example in the annual payment the value of the bond was 938.55, that already you know. So, if you are going to calculate and you know that, the particular rate which makes this particular value equal to the present value that is 10 percent.

But if you see, if you are going by ARTM formula, then your coupon is 90, your par value is 1000, value of the bond is 938, then maturity period is 10 years. So, if you see then it is the face value of the bond or par value and this is your market value. You will find a rate that is 9.92 percent. It is very close to the yield to majority, but not exactly the 10 percent what you are getting. So, if you want to get a very exact rate, maybe once you can get a point where you can have some kind of trial and error which relatively easier to find out an exact YTM.

So, the ARTM is a proxy which can be used for YTM, whenever it is difficult to find out the exact YTM or yield to maturity which can make this particular value equal to the present value of the cash flows. So, then exactly if you see now whenever it is a semiannual payment, you have seen that our market price was 937 or the present value of the bond was 937, then we got it, it is 4.9633 percent; then finally, we got it 9.93. Little bit deviation, it was 9.92, it is 9.9325 which is also again very close to that.

Already I told you that, it is not exactly equal to the YTM what is you are supposed to get. But you are getting something, which is very close to the YTM and after that if you want you can use some iterative process to find out the exact YTM, that actually you have to keep in the mind.

So, that is one of the ways through which the yield to maturity can be calculated.

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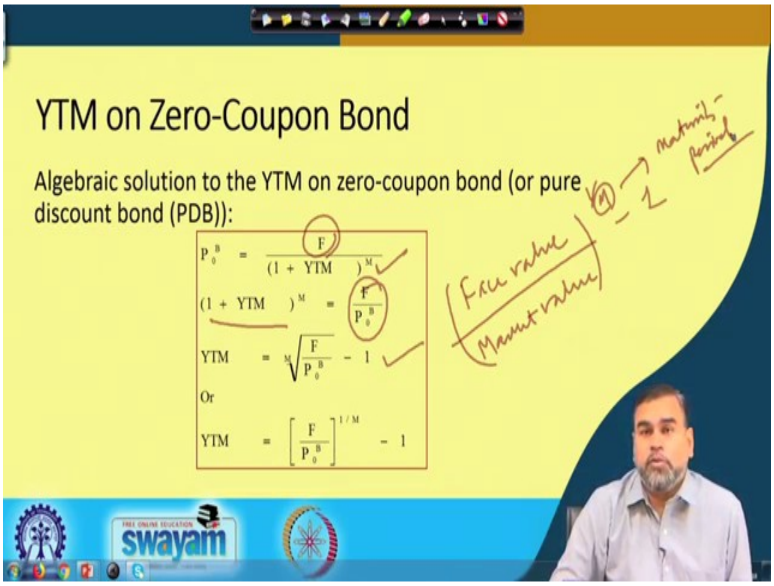

So, then we have another bond already we have discussed about this in the previous class that is basically the zero-coupon bond. So, zero-coupon bond means there is no such coupon involved in that and the bond is basically always issued at a discount and a redeemed at a par and also we call it the pure discount bond. So, for the pure discount bond, already you know the formula. How basically we calculate it? This price of a pure discount bond is your face value of the bond or the maturity value of the bond divided by 1 plus YTM to the power M. You are only discounting that thing with respect to the YTM, with respect to that period.

That means, that is only one component of the cash flow and the other component of the cash flow is not available in this case. Then if you solve it, then $1 + YTM^M = F / P_0$, then we have

$$YTM = \sqrt[M]{\frac{F}{P_0}} - 1 \vee YTM = \left[\frac{F}{P_0}\right]^{1/M} - 1$$

That means, your face value or the par value divided by the market value or the present value of the cash flows to the power 1 by M minus 1 which is nothing, but the M is nothing, but the maturity period. M is nothing, but the maturity period.

So, straight forward, we can calculate the yield to maturity of a zero-coupon bond. If you know that purchase price and as well as the face value. If that thing is known, then the yield to maturity of the zero-coupon bond can be easily calculated.

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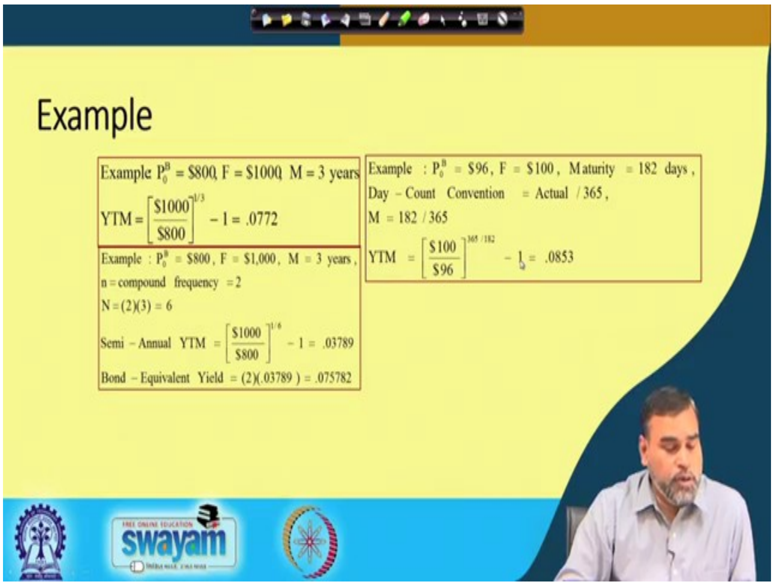

So, then we will see that some example of the zero-coupon bond, let the price of the zero-coupon bond was 800 dollar, then the face value was 1000, then maturity period is 3 years then your YTM is 7.72 percent. If the frequency is 2; that means, that is a semiannual it is, it is compounded basically N is equal to the compound frequency; that means, if you want to calculate it in that way, then your N is equal to 6, then your semiannual YTM is equal to this and bond equivalent yield means 2 multiplied by this, that is 7.57 percent; but let, the maturity period is only 182 days.

So, we are using if you remember, we are using a day count convention. Your day count convention means, it is actual days divided by 365 or you can go by the 360, where the actual is not considered; we consider that every month is equal to 30 days.

So, in that context, if your maturity period is 182 days, the name is equal to 182 /365, then YTM is equal to $[100 / 96^{365}]/ 182 - 1$, that is 8.53 percent. So, it is a straightforward calculation, whenever we calculate the yield of a zero-coupon bond. But, in the case of the other bonds, you know there is a coupon involved in that. We have to always make this price of that particular bond. We are equating the price of the bond with respect to the present value of the cash flows.

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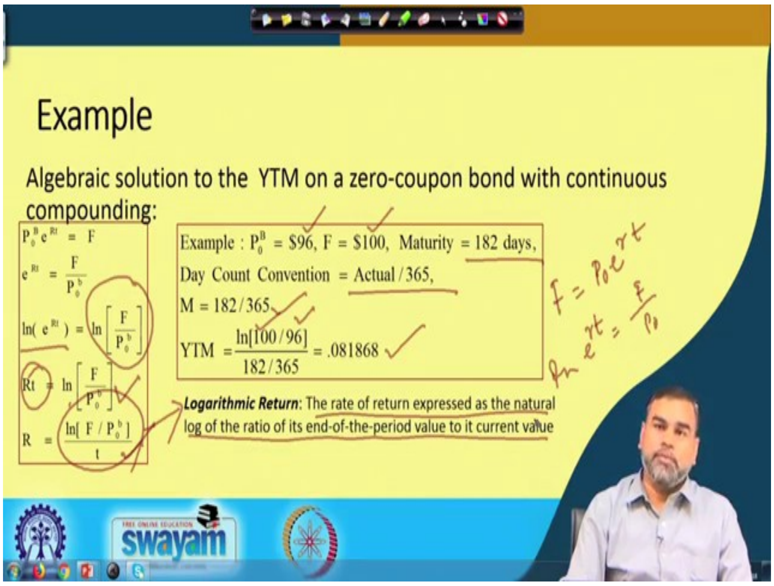

So, if you go for a continuous compounding, then it can be used in this way. You know already what do you mean by the continuous compounding? That means, your F is equal to in that case $P_0 e^{rt}$ and; that means, your e^{rt} is nothing but F / P_0 and; obviously, if you take log in both the sides whatever we have taken $\ln e^{rt} = \ln F / P_0$. Then $\ln e^{rt}$ is nothing, but the R t is equal to ln of F / P_0, then it is nothing but $R = [\ln F / P_0]/ t$. So, that is basically the way through which, if there is a continuous compounding, then the YTM of a zero-coupon bond can be calculated in this way.

The example is let, price is 96, F is 100, maturity 182 days, day count convention we have taken actual, that is actual by 365; that means, 182 / 365. Then you are, if you are assuming there is a continuous compounding involved in this, then your YTM will be l n 100, F = 100, your $P_0 = [96 / 182]/ 365$ that you are getting 8.18 percent.

So, the logarithmic return what we are considering here, this is basically the rate of return expressed as a natural log of the ratio of the end of the period value to the current value. So, that actually you have to keep in the mind. So, this is the way the YTM on a zero-coupon bond with continuous compounding can be calculated. That means, your R is equal to ln; the F / P_0 divided by the time period which is basically the t.

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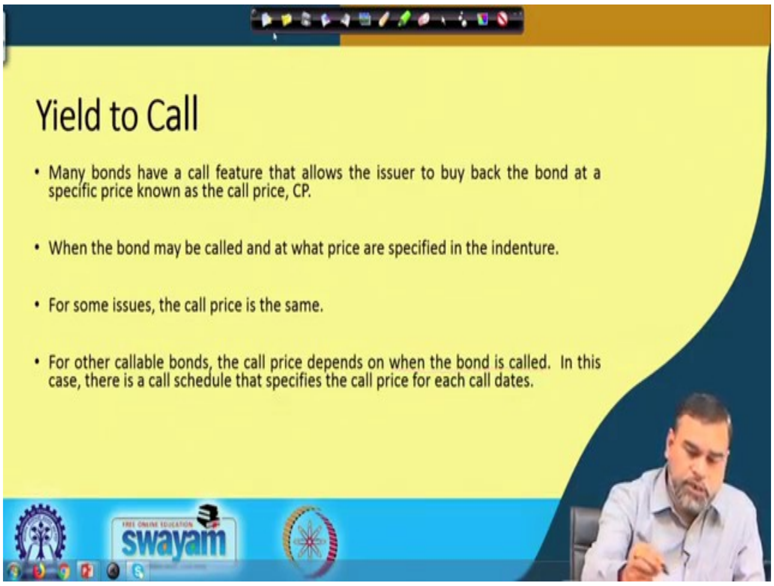

Then we can move into the other type of bonds which are available in the market. You know that there let there is a bond which has a call feature. What do you mean by the call feature? The call feature of a bond means, if there is a call feature in the bond, it allows the issuer to buy back that bond at a specific price known as the call price. For example, if one wants maturity period is 10 years and the par value of the bond is 1000 rupees.

So, if the bond has a call feature, then what will happen? That, let this investor always in that indenture provision it will be mentioned, that it has a call feature and after 3 years or 4 years, the bond can be called back here by the issuer. And whenever the bond will be called back, that particular point of time the issuer basically from the beginning has fixed a price. Let that price can be 1100, it can be 1050 or it can be 1200.

So, the bond is or what they will do, they will pay that particular price to the investor or the bond holder and can buy back that particular bond from the bond holder. And when the bond is maybe called at what price it is already specified in the indenture provision from the beginning whenever they bond has been issued. For some of the issues, the call price is the same. Because, there are different call dates in a call feature there are different type of call rates.

So, the call dates either the called, on the basis of the call date the call price will vary or the call dates on the basis of call date the call price may not be varying. The call price, may be same for all type of call dates or maybe there are led after 3 years, it can be called on this date. May be, if it is not called by that, then another call date will be there. Let there are series of the call dates. For each call date, the particular price of the bond is fixed by the issuer. So, either the price may be same for all the call dates or the price may be different for the different type of call dates.

So, the call price basically depends upon when the bond is called. So, in this case if there are different call dates of that particular bond and the prices are different in the different dates, then there is a call schedule that specifies the call price for each call date. The bond can be called on so and so date, the price will be this. If the bond will be called after 4 years, the price will be this. If the bond will be called after 7 years, the price will be this. So, all kind of schedule of that particular call price of the bond is mentioned from the beginning whenever the particular bond was issued.

So, on that basis the indenture provision are basically the legal provisions already explained, that what kind of call price schedule is available for that particular bond and the bond holder. The investor is fully aware about that what kind of price they are going to get if the bond will be bought back by the investor.

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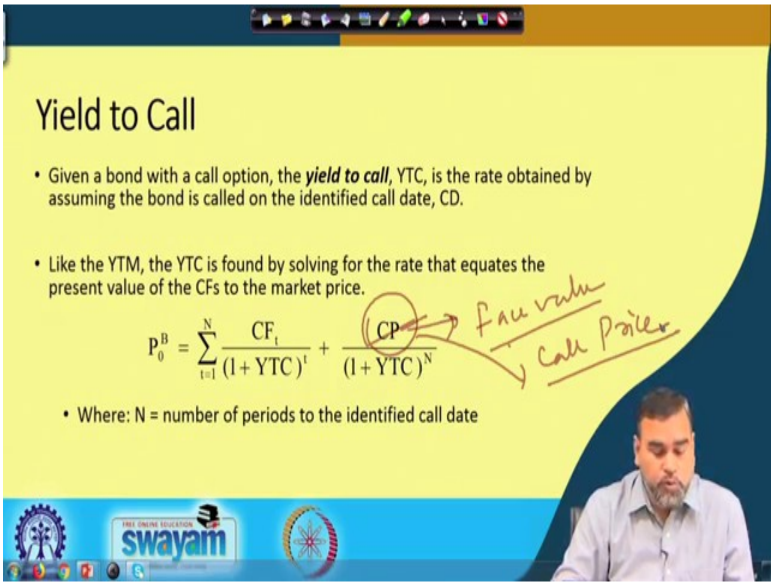

Then given a bond with a call option the yield to call basically we have to calculate like your yield to maturity and all these things. In case of call feature we calculate the yield to call. Then what is yield to call? The yield to call is basically obtained by assuming the bond is called on an identified call date.

So, yield to call can be calculated in the different call dates. So, we have to know that at which date we are trying to calculate the yield to call. So, like the yield to maturity, the yield to call basically can be found by solving the same thing which equates the present value of the cash flows to the market price. But here, one thing you remember, whenever we are talking about the yield to maturity, we are here, we are discounting the par value. This particular discount this part is basically we are using the par value of the bond or we can say that face value the bond.

The face value was considered in this case, but now it is the call price. It is the call price what basically we are considering. And n is equal to the number of periods to the identified call date. If you see this example, maybe it will be more clear for you that how the yield to call is calculated.

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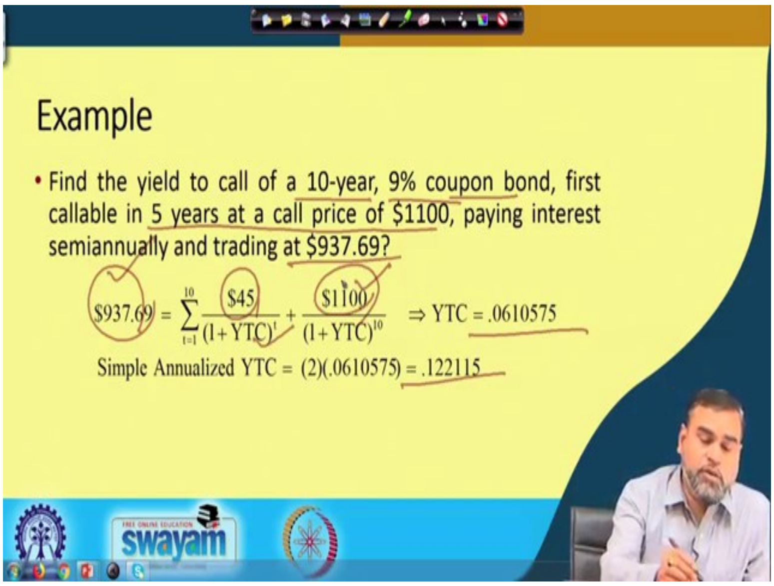

For example, there is a bond whose maturity period is 10 years, coupon is 9 percent and first callable in 5 years at a call price of 1100. And the interest rate spread semiannually and now it is trading at 937.69; that means the market price of that particular bond at that particular point of time is 937.69.

Then what you have to do? Then we have to basically equate this market price with respect to the present value of the cash flows. Then, what are those cash flows? The cash flows are basically the coupon that is your 45 dollar, because 9 percent of the coupon rate every year; that means for 1000 dividing 90 rupees and for every 6 months, we are getting 45 that already you know.

But, instead of 1000, which is the face value of the bond? Here we are considering 1100. Why we were considering 1100? Because that 1100 will be the cash flow whenever the bond will be callable. Because it is a call feature, after 5 years the issuer will give you 1100 and say that the bond can be bought back from the investor from the bond holder.

So, then what you have to do? We have to basically find out that YTC that YTC which can equate this one with respect to the present value of this cash flows. So, in that context what will happen that, we find that the YTC become 6.10575 then, you have to go for the annualized YTC, then you multiply by 2; that is 12.21 percent.

So, here you see with the same example, we are getting a yield whenever you are discounting with respect to the face value, we are getting around 10.25 percent whenever we are using or 10 percent, whenever we are going for the yield to maturity. But whenever it is yield to call, the yield is 12.21 percent because that is the premium what the bond invest issuer wants to give for to the bond holder because there is a risk involved.

Because there is a possibility, whenever the interest rate is very low, the bond can be also called back. Although it is not profitable for investor that particular point of time, but the bond issuer can call back the bond because they can generate certain revenue in that particular point of time by discounting the bond we see that particular discount rate which is already available in the market in the lower rate.

So, there is a risk involved in the callable bonds; so because of that the interest rate is basically we have seen this way.

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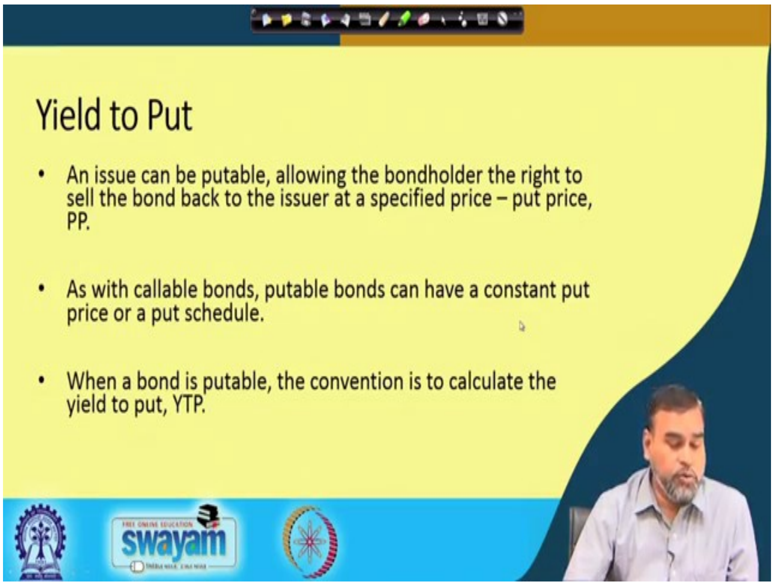

Then we have another one yield to put. Yield to put means, this reverse. It gives the bondholder the right to sell the bond back to the issuer at a specified price which is called the put price. Putable bonds also can have a schedule. At what time the price of the put is what? That particular like callable bonds for the putable bonds also the put prices are mentioned or the put dates are mentioned and accordingly the yield to put can be calculated.

So, whenever I talk about the yield to put, then with what we have to do?

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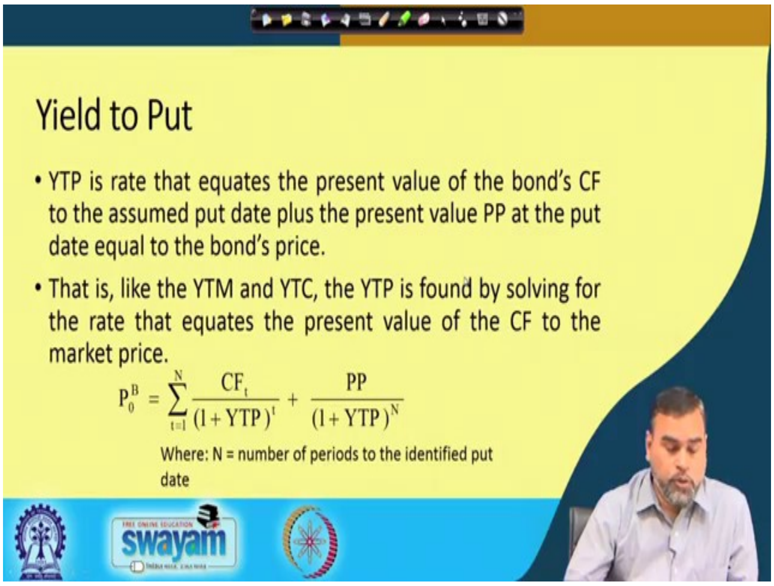

Again the same logic you can apply. But here if you see, we have to discount it with respect to the put price. We have to discount it with respect to put price. In the case of call option, we are using the call price. In case of the put option, we are using the put price. And here, if you see this example then it will be clearer for you.

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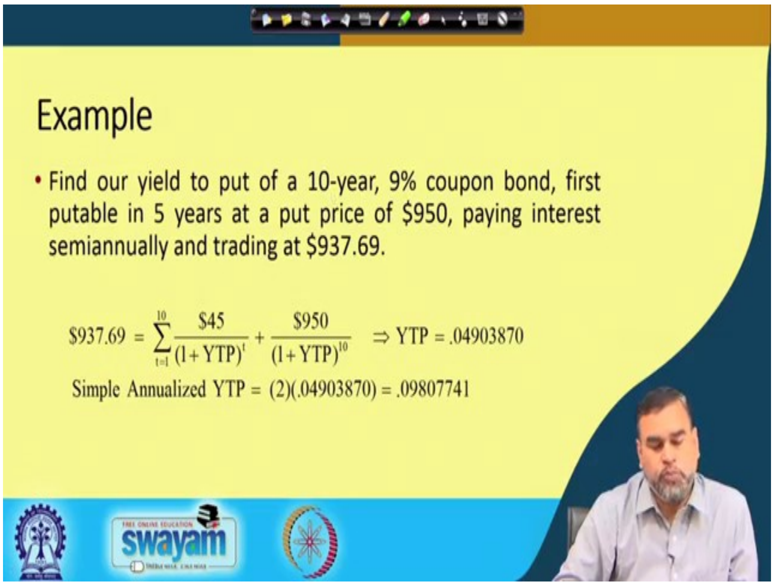

If you see this; there is yield to put you have to find out for a 10 years bond same 9 percent coupon paying the interest semiannually and after 5 years let the put prices 950 which has been mentioned from the beginning. Then, you have to equate it with respect to the market price or the price at which the bond is trading and that particular point of time, that is 937.69.

If you go and here, it is 950 other things remain same. Then what you get it, 5 years are remaining, then you are getting that yield to put is 4.90. Then in few semi analyzed means, you have to multiply the 2 to find out the yield to put for the annual yield to put; then it is 9.807741 percent. This is the way, the yield to put can be calculated on the differences you have to consider the face value instead of face value, we are using the put price.

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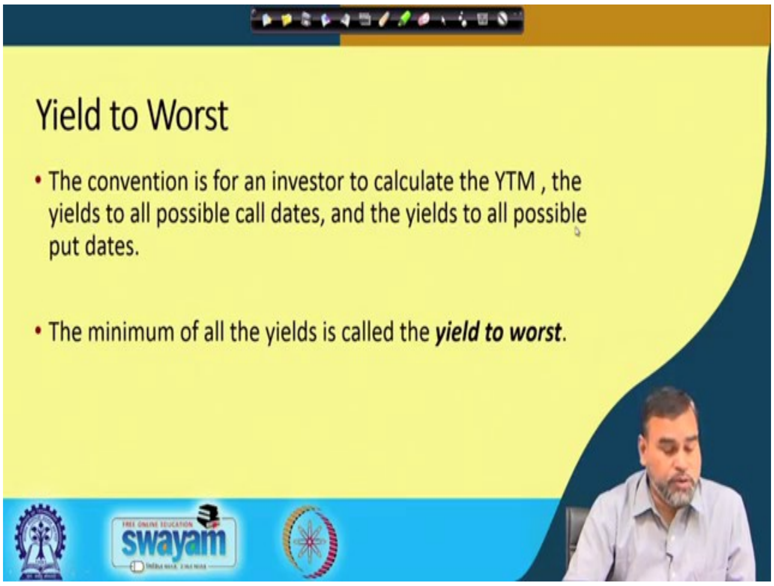

Then there is another one is yield to worst concept. It is basically, whenever there is a different call dates or different put dates. You have the yield you can calculate. Then the minimum of all the yields is called the yield to worst. At which date the yield is the minimum; that is called the yield to worst. This is available whenever there is a call schedule for that particular put option or the call option.

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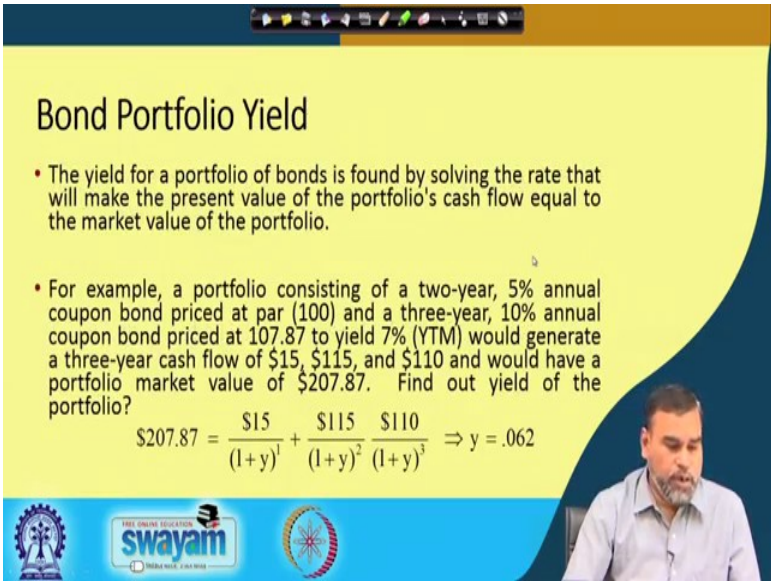

Then if you are going for a bond portfolio yield, then it is not basically the summation of the yields of that particular portfolio whatever bonds are there. This is again you have to solve the rate at which make the present value of the portfolios cash flow equal to the market value of the portfolio.

So, that is why; for example, if your portfolio consisting of a 2 years 5 percent annual coupon bond priced at par 100, in 3 years 10 percent annual coupon bond price priced at 107.87 then, it would generate a 3 year cash flow of 15, 115, 110 and would have a portfolio market value of this. Then what is the yield? Then the yield will be basically, this the cash flow what you are getting that is 15 dollar, 150 dollar, then 110 dollar, then your 207 is the total portfolio value. Then you have to discount it with respect to y $(1 + y)^1 + (1 + y)^2 + (1 + y)^3$, then finally, we got it y is equal to 6.2 percent.

So, it is not a simple summation of the weighted summation of the yield of the different bonds this calculation of the portfolio yield is different and it can be calculated in this way; so, this is about your portfolio yield.

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Please go through this particular references for this particular system. And next class we will be talking about certain issues related to the bond price volatility and the total rate on.

Thank you.