Assignment - 7

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1. Introduction:

In this particular assignment we have been given a task in which we have to take in a square matrix A and a vector b, and compute:

$$\mathrm{x} = A^{-1}\mathrm{b} \tag{1}$$

In this particular assignment we have to analyse the results obtained by the various methods that we use for the computation of x according to the given tasks.

1.1 QR Decomposition Method:

The QR decomposition (also called the QR factorization) of a matrix is a decomposition of the matrix into an orthogonal matrix and a triangular matrix. The QR decomposition of a real square matrix A is a decomposition of A as:

$$\mathrm{A} = \mathrm{QR} \tag{2}$$

where Q is an orthogonal matrix (i.e. $Q^TQ = I$) and R is an upper triangular matrix. If A is non singular, then this factorization is unique.

There are several methods for actually computing the QR decomposition. One of such method is the Gram-Schmidt process.

In this project report I have provided the program that takes in a square matrix A and a vector b, and computes $x = A^{-1}\ b$ using QR decomposition. The program file is attached with the folder of submission.

2. Plot of L∞ error in x v/s Matrix Size:

In the following section I have represented the plot of the L∞ error in x v/s Matrix Size ϵ [1, 1000] while approximating the value of $x = A^{-1}\ b$ by using the QR Decomposition Method and the Backslash "\" Method. The plot of the L∞ error in x v/s Matrix size ϵ [1, 1000] is shown below:

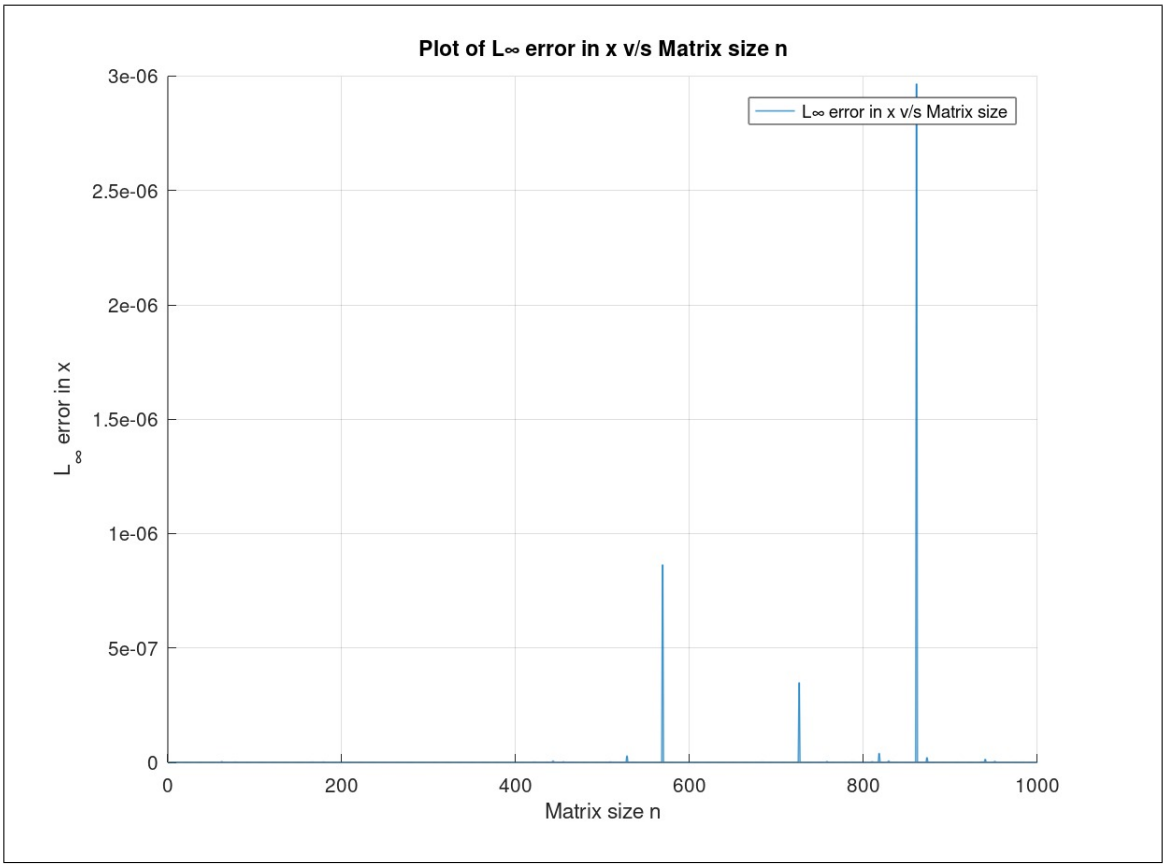

Figure 1: Plot of the L∞ error in x v/s Matrix Size ϵ [1, 1000]

3. Plot of Computational Time for x v/s Matrix Size:

In the following section I have represented the plot of the Computational Time for x v/s Matrix Size ϵ [1, 1000] while approximating the value of $x = A^{-1}\ b$ by using the four separate methods ie QR Decomposition Method, Octave's inbuilt QR Decomposition Method, Inverse "$inv()$" Method, Backslash "\" Method. The plot of the Computational Time for x v/s Matrix size ϵ [1, 1000] is shown below:

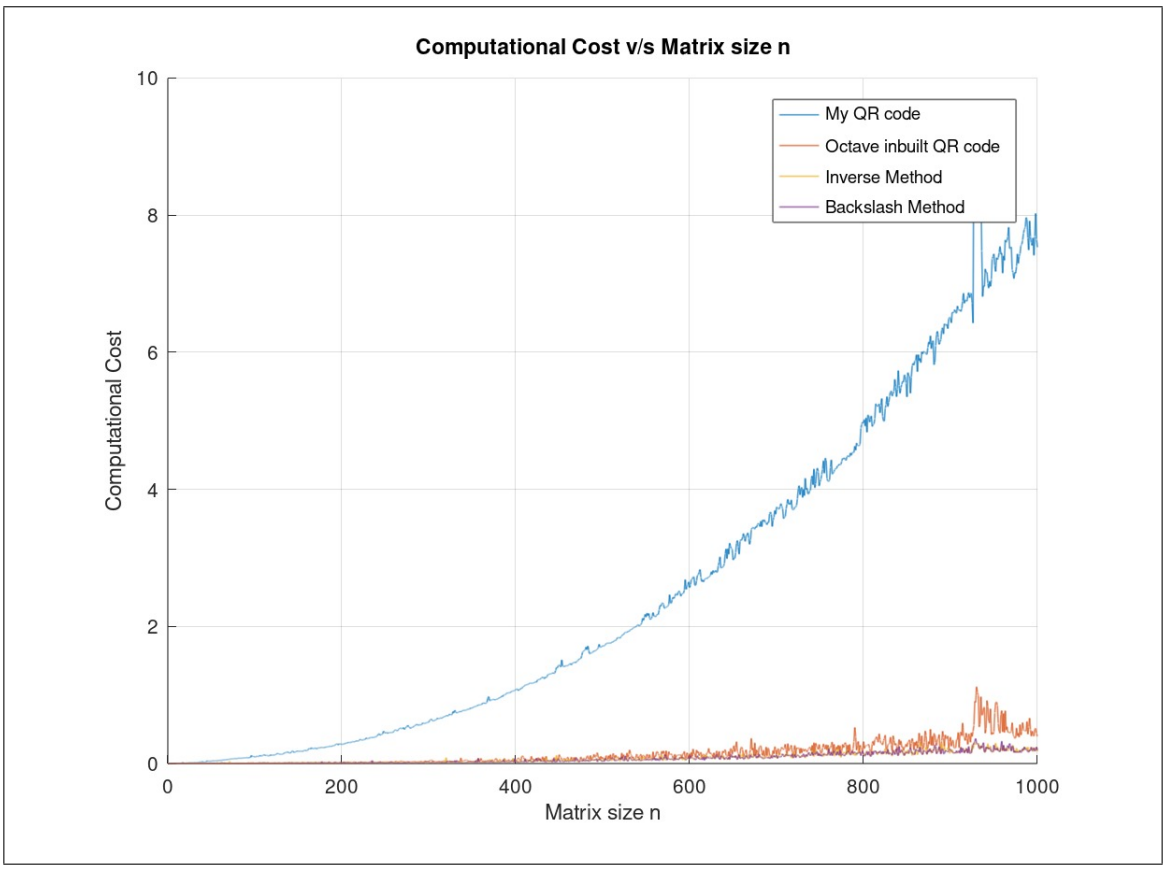

Figure 2: plot of the Computational Time for x v/s Matrix Size ϵ [1, 1000]

4. Plot of Computational Time for x v/s Matrix Size for my QR Method Code:

In the following section I have represented the plot of the Computational Time for x v/s Matrix Size ϵ [1, 1000] while approximating the value of $x = A^{-1}\ b$ for the Total Computational Time, Time taken for generating Q and R, Time taken for the Back-Substitution Method for solving x against Matrix Size ϵ [1, 1000]. The plot of the Computational Time for x v/s Matrix Size ϵ [1, 1000] while approximating the value of $x = A^{-1}\ b$ for the Total Computational Time, Time taken for generating Q and R, Time taken for the Back-Substitution Method for solving x against Matrix Size ϵ [1, 1000] is shown below:

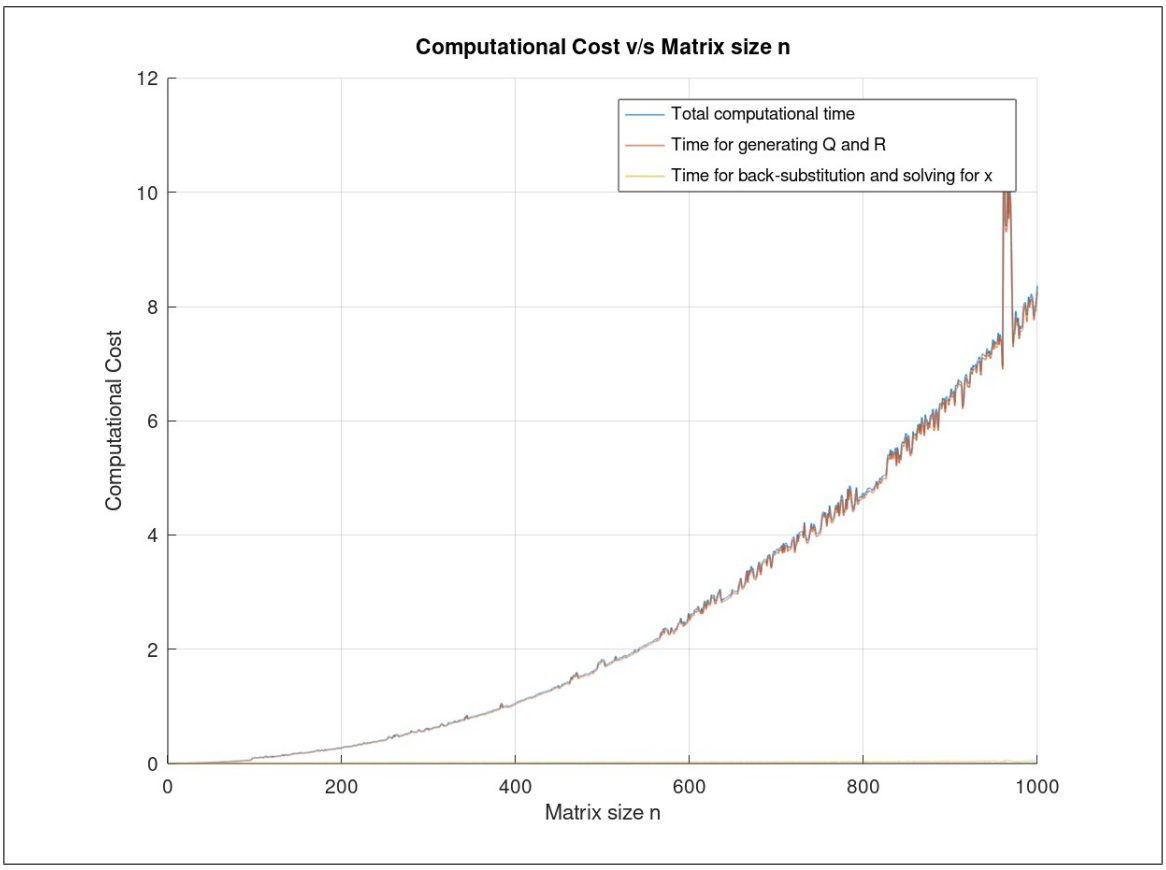

Figure 3: Plot of Computational Time for x v/s Matrix Size for my QR Method Code