Assignment - 7

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October 21, 2022

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1. Introduction:

In this particular assignment we have been given a task in which we have to take in a square matrix A and a vector b, and compute:

$$\text{x} = A^{-1}\text{b} \tag{1}$$

In this particular assignment we have to analyse the results obtained by the various methods that we use for the computation of x according to the given tasks.

2. QR Decomposition Method:

The QR decomposition (also called the QR factorization) of a matrix is a decomposition of the matrix into an orthogonal matrix and a triangular matrix. The QR decomposition of a real square matrix A is a decomposition of A as:

$$\text{A} = \text{QR} \tag{2}$$

where Q is an orthogonal matrix (i.e. $Q^TQ = I$) and R is an upper triangular matrix. If A is non singular, then this factorization is unique.

There are several methods for actually computing the QR decomposition. One of such method is the Gram-Schmidt process.

In this project report I have provided the program that takes in a square matrix A and a vector b, and computes $x = A^{-1}\ b$ using QR decomposition. The program file is attached with the folder of submission.

3. Plot of L∞ error in x v/s Matrix Size:

In the following section I have represented the plot of the L∞ error in x v/s Matrix Size ϵ [1, 1000] while approximating the value of $x = A^{-1}\, b$ by using the QR Decomposition Method and the Backslash "\" Method. The backslash "\" operator is used to solving a linear equation of the form $Ax = b$, where 'A' is a matrice, 'b' is a vector and 'x' is a vector. The solution of this equation is given by $x = A\backslash b$, but it works only if the number of rows in 'A' and 'b' is equal. The plot of the L∞ error in x v/s Matrix size ϵ [1, 1000] is shown below:

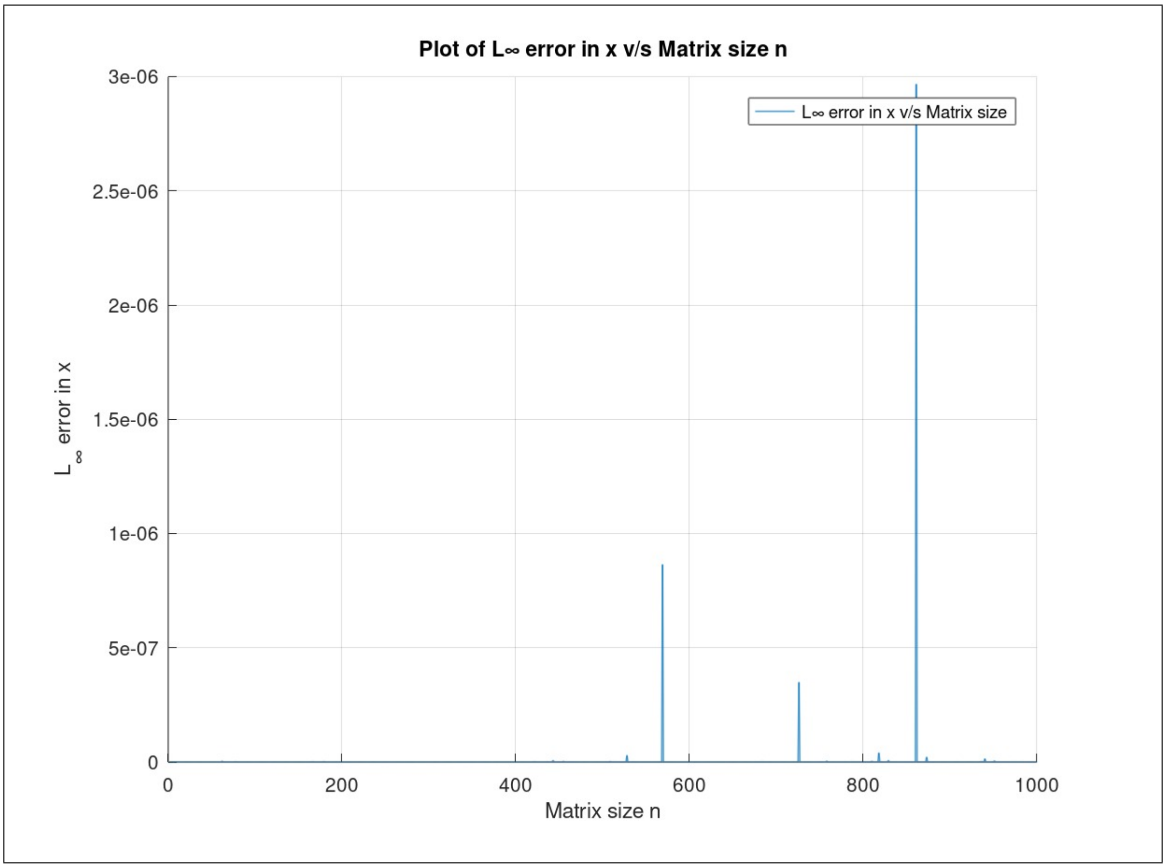

Figure 1: Plot of the L∞ error in x v/s Matrix Size ϵ [1, 1000]

4. Plot of Computational Time for x v/s Matrix Size:

In the following section I have represented the plot of the Computational Time for x v/s Matrix Size ϵ [1, 1000] while approximating the value of $x = A^{-1}\, b$ by using the four separate methods ie QR Decomposition Method, Octave's inbuilt QR Decomposition Method, Inverse "$inv()$" Method, Backslash "\" Method. The plot of the Computational Time for x v/s Matrix size ϵ [1, 1000] is shown below:

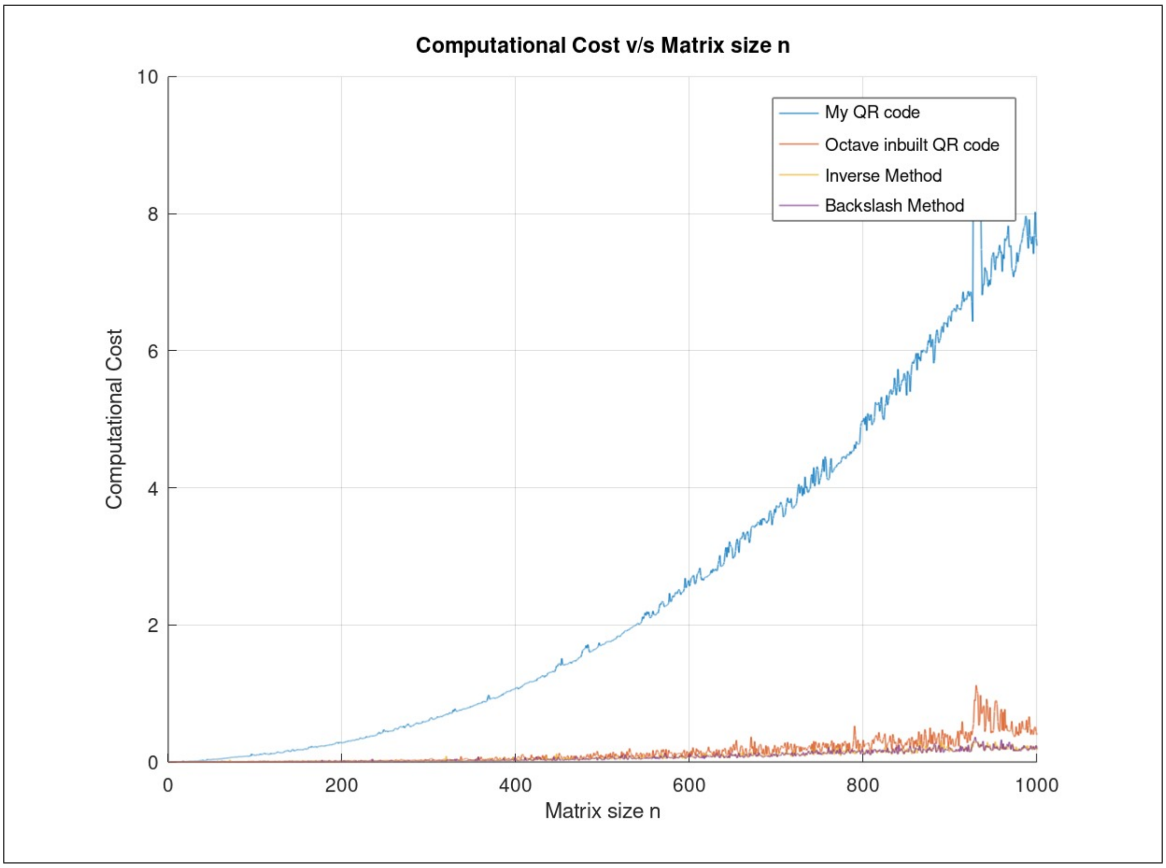

Figure 2: plot of the Computational Time for x v/s Matrix Size ϵ [1, 1000]

5. Plot of Computational Time for x v/s Matrix Size for my QR Method Code:

In the following section I have represented the plot of the Computational Time for x v/s Matrix Size ϵ [1, 1000] while approximating the value of $x = A^{-1}\, b$ for the Total Computational Time, Time taken for generating Q and R, Time taken for the Back-Substitution Method for solving x against Matrix Size ϵ [1, 1000]. The plot of the Computational Time for x v/s Matrix Size ϵ [1, 1000] while approximating the value of $x = A^{-1}\, b$ for the Total Computational Time, Time taken for generating Q and R, Time taken for the Back-Substitution Method for solving x against Matrix Size ϵ [1, 1000] is shown below:

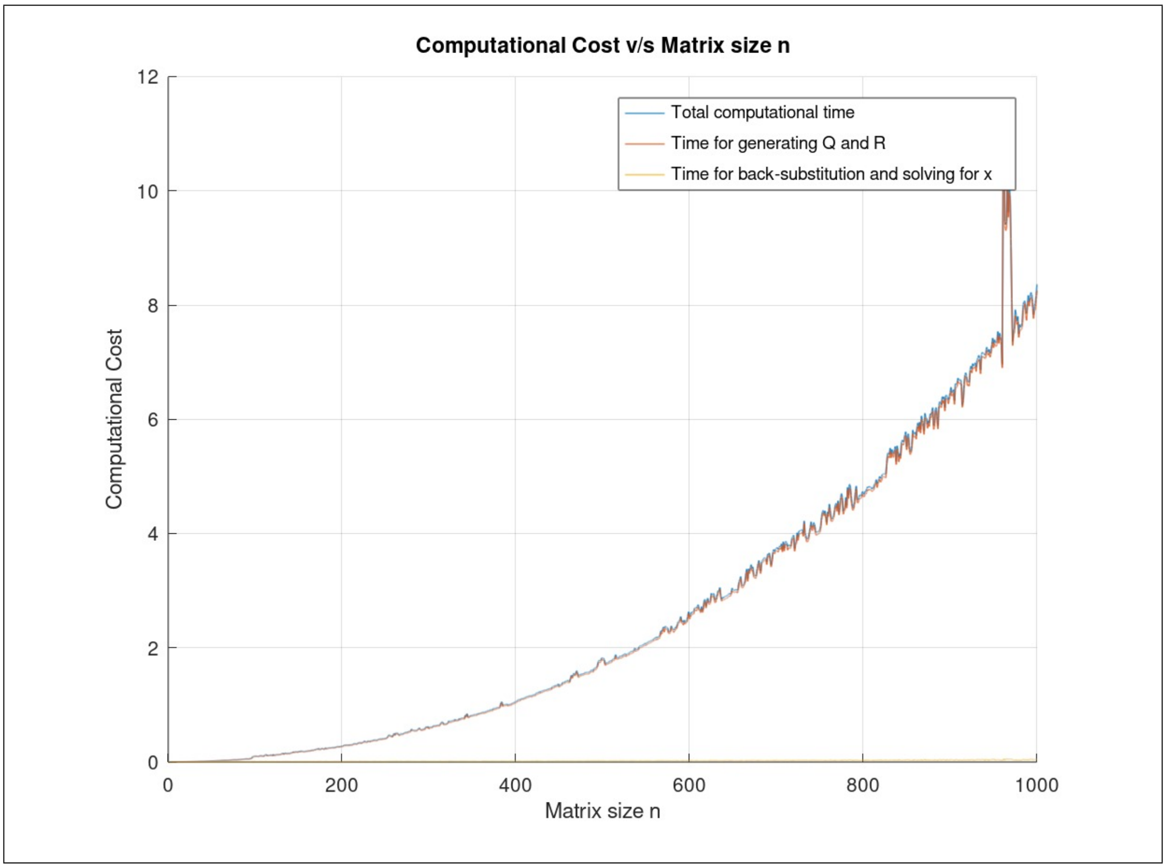

Figure 3: Plot of Computational Time for x v/s Matrix Size for my QR Method Code