- Zip all your files and label the zip file as **[Roll number in lower case]_hw6.zip**
- The scripts will be executed and compared against the submitted PDF file.
- Submit a single zip file containing `.tex, .py, .pdf` and image files only.
- Generic instructions from previous homeworks stand.
- **This assignment is to be done entirely in Python**
- Use `scipy.special.roots_legendre` to get the values of w_i and x_i

All tasks are performed on the following integral

$$I = \int_{-1}^{1} f(x)dx, \quad \text{where } f(x) = e^{-x}\sin^2(4x) \tag{1}$$

and define the error as

$$\text{Error} = \left| \int_{-1}^{1} f(x)dx - \sum_{i=0}^{N} f(x_i)w_i \right| \tag{2}$$

Answer the following (state your inferences for each),

1. Plot the first five Legendre polynomials on the same plot.
2. Plot log(error) as the number of terms in the series, N. Comment on the trend observed. Do you observe a monotonic decrease in error with N? Why or why not.
3. Plot a graph that shows the number of quadrature points required (N) for a given target error (e.g., 10^{-6}, 10^{-8} $\cdots$). On the same plot, also show the number of number of segments required from the "best" method (Trap, LE, RE or MP) of the previous assignment.
4. Redo the above plot with the computational time required for a given target error using the two methods (Quadrature vs "best" from previous assignment).

For those who are inclined (not graded):

Given the following properties of the Legendre polynomials

$$\int_{-1}^{1} P_n(x)P_m(x)dx = \frac{2}{2n+1}\delta_{\text{mn}}$$

where δ_{mn} is the Kronecker delta, plot the approximate function between -1 and 1 obtained by using the Legendre polynomials ($P_n(x)$) as basis functions for different values of N.

The approximate function is given by

$$f(x) \approx \sum_{i=0}^{N} P_i(x)w_i \tag{3}$$

To get started, multiply the above equation with $P_m(x)$ and integrate from -1 to 1 to arrive at the values of the weights.